



COLUMBIA
BUSINESS
SCHOOL

Spring 1999
Professor Costis Maglaras
B9801-33: Stochastic Processing Networks
409 Uris Hall

B9801-33 Stochastic Processing Networks

Course description

Stochastic processing network models may be used to represent service operations, manufacturing systems, or communication systems. These “stochastic processing networks” are highly complex and difficult to analyze exactly. A major insight that has emerged over the past 10-15 years, uses a hierarchy of approximate models as the basis for analysis and synthesis of such systems. In particular, the analytical theory associated with fluid approximations and Brownian approximations has produced important insights in understanding how the performance of a multiclass network depends on different design and control parameters.

Within this general framework, the two central themes of this course are: first, descriptive performance analysis for processing networks, where one strives to understand the factors that contribute to congestion and delay; and second, dynamic flow management, including problems of routing, sequencing and input control. The course starts with a description of classical queueing network models, but mainly focuses on their continuous idealizations based on fluid and Brownian model approximations.

Course objectives

The main objective for this course is to describe the hierarchical framework to stochastic network control problems based on fluid and Brownian approximations, to cover the basic theory behind it, exploit its power and generality, and finally cover some of its applications. In detail, we will study

- the analytical theory of fluid and Brownian models,
- stability analysis for multiclass networks based on fluid models,
- applications from diverse fields, and, finally,
- get overview of current research in these areas, both in theory and applications.

Textbook & Grading

There is no textbook. Most of the material covered in this course is not contained in a textbook or even a research monograph. Complete lecture notes will be handed out during the course, and will be available in postscript and adobe pdf via WWW from

<http://128.59.215.113/networks/B9801.htm>

Some texts will serve as auxiliary references:

- F. P. Kelly. *Reversibility and Stochastic Networks*. John Wiley & Sons, 1979.

- J. M. Harrison. *Brownian motion and stochastic flow systems*. John Wiley & Sons, 1985.

There will be 5 or 6 homework assignments (that will involve some Matlab programming – no prior knowledge of Matlab is required), and a term project. The grade: homework 50%, project 50%.

Course outline

The course is divided into five parts. A tentative outline is as follows:

- I. *Product-form networks* (2 lectures).
 1. Course introduction. Preliminaries on Markov Processes, reversibility, and the Output Theorem. Introduction to product-form networks.
 2. Jackson networks. Quasi-reversibility and Kelly networks. Closed queueing networks.
- II. *Fluid models and stability analysis* (2.5 lectures).
 3. Multiclass networks: system dynamics; examples of scheduling rules. Introduction to stability theory. Fluid models: formal derivation and basic properties.
 4. Dai's stability theorem. Stability analysis based on fluid model analysis. Lyapunov and trajectory decomposition techniques. Instability examples. Virtual stations.
- III. *Dynamic control based on fluid models* (2.5 lectures).
 5. Description of the basic methodology. Examples. Dynamic control of fluid models. Optimal control for fluid models. Algorithmic issues.
 6. Translating of the fluid control policy in the stochastic network. Discrete-review policies: stability and fluid-scale asymptotic optimality.
 7. Extensions to routing and admission control problems. Examples: flexible resources, communication networks, revenue management.
- IV. *Brownian models* (2 lectures)
 8. Introduction to Brownian models. One-dimensional reflected Brownian motion. Heavy-traffic limit for the $G/G/1$ queue.
 9. $G/G/1$ queue continued. Brownian model approximations for stochastic processing networks in heavy-traffic and the BIGSTEP approach to dynamic network control problems.
- V. *Applications and Special topics* (3 lectures). The last part of the course will focus on applications of the techniques developed so far, and in an overview of the current literature on these -or related- topics; this part will be tailored according to class interests.

(Each lecture will consist of roughly two 75 minute parts with a 10 minute break.)