# Investor Protection, Diversification, Investment, and Tobin's $q^*$

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## Abstract

The separation of ownership and control allows controlling shareholders to pursue private benefits under imperfect investor protection. We study the effects of managerial agency and under-diversification frictions on investment and firm value. While the under-diversification concern leads to under-investment, private benefits encourage over-investment. Both over-investment and under-investment can occur depending on firm size. Due to frictions the investment-capital ratio is stochastic and non-monotonic in firm size. The controlling shareholder's option to cash out/exit provides an important channel to manage significant exposure to the idiosyncratic business risk. Frictions drive a wedge between the controlling shareholder's private average $q$ and private marginal $q$, as well as a wedge between the public average $q$ and marginal $q$ for diversified outside investors. We derive simple formulas for the cost of capital for outside equity and show that conditional CAPM holds. Under imperfect investor protection, firm value is lower than the first-best benchmark. However, surprisingly, the cost of capital for outside equity under imperfect investor protection can be either higher or lower than the first-best benchmark, depending on firm size. Our model also allows us to calculate the controlling shareholder's idiosyncratic risk premium. Finally, we show that weaker investor protection leads to more concentrated ownership and smaller firm size. The quantitative effects of these frictions are significant.

# 1 Introduction

Investor protection is critical for the financing of investments, economic growth, and welfare. By "investor protection," we broadly refer to features of institutional, legal, political, regulatory, and market environments as well as corporate governance mechanisms at the firm level, which facilitate financial contracting and contractual enforcement, and protect investors against expropriation by corporate insiders.

Around the world, corporations, even large publicly traded companies, have controlling shareholders such as founders, founding family members, and States, in contrast to the common belief that corporations are widely held (Berle and Means (1932)). La Porta, López-de-Silanes, and Shleifer (1999) document controlling shareholders' concentrated ownership in large firms around the world.[1] With weak investor protection, controlling shareholders become entrenched and pursue private benefits at the expense of investors. By holding a concentrated ownership, the controlling shareholder alleviates agency conflicts with outside investors, but incurs an under-diversification cost due to imperfect risk sharing under incomplete markets.

The expropriation of minority shareholders by the controlling shareholder is at the core of agency conflicts in most countries and thus the extent of investor protection is an important determinant of corporate finance and governance around the world. Private benefits take a variety of forms including outright stealing from the firm, selling the firm's output or assets to a related party at below market prices, and hiring unqualified relatives/friends, to just name a few. Agency problems also often manifest themselves through non-value-maximizing investment choices.[2] It is difficult to verify and contract on corporate investment decisions, which often involve managerial discretion and judgment. Penalizing the controlling shareholder based on self-serving value-destroying investment is difficult, if not infeasible, especially when investor protection is weak.

We incorporate imperfect investor protection and incomplete markets, two key frictions for many firms around the world, into an integrated dynamic framework. The controlling shareholder makes *interdependent* household decisions (consumption/portfolio choice) and business decisions (private benefits, corporate investment/exit). Using this framework, we address the following questions:

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[1] Claessens, Djankov, and Lang (2000) and Faccio and Lang (2002) document concentrated ownership for large public firms in East Asian countries and Western European countries, respectively.

[2] For example, see La Porta et al. (2000a) for such a statement in an influential survey on investor protection and corporate governance.

- What determines corporate investment in firms run by controlling shareholders?

- How do private benefits of control influence corporate investment and valuation?

- What determines the firm's private valuation for the controlling shareholder and public valuation (Tobin's average $q$) for outsiders?

- What drives the marginal valuation wedge between private marginal $q$ for insiders and public marginal $q$ for outsiders?

- What determines the idiosyncratic risk premium for the controlling shareholder?

- What are the effects of frictions on the cost of (outside equity) capital?

- How do frictions influence firm size, inside ownership, growth, and welfare?

Investor protection and incomplete markets frictions have opposing effects on firm investment. Incomplete markets make the controlling shareholder under-diversified and the controlling shareholder's precautionary saving motive influences corporate investment and value.[3] In order to lower the idiosyncratic risk exposure to the firm's business, the controlling shareholder under-invests in the firm.

In contrast, lacking investor protection encourages over-investment. The weaker investor protection, the more private benefits that the controlling shareholder pursues in each period. A forward-looking controlling shareholder thus has a preference to build a bigger firm (invest more) under weaker investor protection, *ceteris paribus*. Over-investment/empire building in our paper is an outcome of the controlling shareholder's attempt to acquire future private benefits, and is not due to the assumption that the controlling shareholder values investment more than outside investors do.

Both over-investment and under-investment may occur as these two frictions work in opposing directions. Intuitively, for a firm with sufficiently large size, the controlling shareholder's concern about under-diversification outweighs incentives to pursue private benefits, leading to under-investment. In contrast, for a sufficiently small firm, the opposite holds and hence the controlling shareholder over-invests.

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[3]Panousi and Papanikolaou (2011) find that the firm's investment falls as its idiosyncratic risk rises, and more so when the manager owns a larger fraction of the firm and hence is more exposed to the firm's non-diversifiable idiosyncratic risk.

Since we focus on corporate investment, we naturally build on the neoclassical (Tobin's) $q$ theory of investment, which studies the effects of productivity shocks and capital illiquidity on intertemporal corporate investment and the value of capital under the Modigliani-Miller (MM) assumption.[4] Specifically, our first-best benchmark extends the seminal Hayashi (1982), a widely-used neoclassical $q$-theoretic model to a stochastic setting with risk premia. We introduce independently and identically distributed (iid) productivity shocks and shocks to capital into Hayashi (1982) to construct a stochastic first-best benchmark. In this benchmark, the optimal investment-capital ratio is constant, Tobin's average $q$ equals marginal $q$, and the capital asset pricing model (CAPM) holds. Because the implications of our first-best benchmark are so strikingly simple, any interesting new dynamics and properties that our model generates are thus attributed to the *interaction* of the frictions, investor protection and incomplete markets.

The controlling shareholder can time the business exit. By relinquishing control, the controlling shareholder forgoes future private benefits and also gives up an efficient production technology, but fully diversifies the idiosyncratic business risk. The controlling shareholder uses this exit option to manage the business risk exposure.

We develop an incomplete-markets-based $q$ theory of investment for firms run by non-diversified controlling shareholders under imperfect investor protection. We show that the *interaction* of managerial agency, diversification motive, and the exit option causes the investment-capital ratio to be stochastic and non-monotonic even when the firm faces iid investment opportunities with a constant return to scale and incurs homogeneous adjustment costs as in Hayashi (1982).

Provided that firm size is not very large, investment-capital ratio decreases in firm size because the controlling shareholder's under-diversification concern lowers investment, causing the firm to behave as if it faces a decreasing returns to scale in an otherwise neoclassical environment. Importantly, the exit option is quite valuable for the controlling shareholder to manage the business idiosyncratic risk causing the investment-capital ratio to increase in firm size as the exit option is sufficiently in the money.

Unlike in neoclassical $q$ models, neither public marginal $q$ nor average $q$ for outside

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[4]Brainard and Tobin (1968) and Tobin (1969) define the ratio between the firms market value to the replacement cost of its capital stock, as $q$ and propose to use this ratio to measure the firms incentive to invest in capital. This ratio has become known as Tobin's average $q$. Hayashi (1982) provides conditions under which average $q$ is equal to marginal $q$. Abel and Eberly (1994) develop a unified $q$ theory of investment in neoclassic settings. Lucas and Prescott (1971) and Abel (1983) are important early contributors. See Caballero (1999) for a survey on investment.

investors predicts corporate investment. We show that the controlling shareholder's private marginal $q$ determines investment. Additionally, the wedge between the controlling shareholder's private marginal $q$ and average $q$, as well as the wedge between average $q$ and marginal $q$ for outside investors, are stochastic and non-monotonic.

With imperfect investor protection, public firm value is unambiguously lower than the first-best value. However, the effect of investor protection on the cost of capital for outside equity is much more subtle. We derive a simple formula to calculate conditional beta and the cost of capital using public average $q$ and marginal $q$. We show that the cost of capital under imperfect investor protection can be either higher or lower than the cost of capital in our first-best framework. Small firms are riskier under imperfect investor protection than under the first-best benchmark. Large firms are less risky under imperfect investor protection than under the first-best benchmark. We also calculate the controlling shareholder's idiosyncratic risk premium. For our calibration, at the moment of exit, the annual idiosyncratic risk premium is about 1.5%.

Additionally, we endogenize ownership and the initial firm size. We show that firm size is larger and ownership is less concentrated under weaker investor protection. We also find that the startup cost for entrepreneurship has a significant effect on firm size, ownership concentration, and social welfare. The startup cost can be either direct or indirect costs that may deter entrepreneurship.

After outside equity is raised, conflicts of interest arise due to time inconsistent behavior of the controlling shareholder. We construct a full-commitment benchmark, where the controlling shareholder fully commits to maximize the sum of inside private equity and outside public equity valuation. Using this benchmark, we find that the agency cost between inside and outside equity is quantitatively significant.

A large empirical literature documents that under weaker investor protection, (1) private benefits of control are higher (Zingales (1994), Dyck and Zingales (2004), and Nenova (2003)); (2) dividend payout is smaller (La Porta et al. (2000a)); (3) firm value is lower (La Porta et al. (2002) and Claessens et al. (2002)); (4) corporate ownership is more concentrated (La Porta et al. (1999) and Claessens et al. (2000)); (5) financial markets are smaller and less developed (La Porta et al. (1997) and Demirgüc-Kunt and Maksimovic (1998));[5] and (6) firm size is smaller (La Porta et al. (1999)). Our model's predictions are consistent with these empirical findings. Additionally, our model also

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[5]See La Porta et al. (2000b) for a survey. Gompers et al. (2003) and Black et al. (2006) study the impact of firm-level corporate governance on firm value.

generates predictions on time-varying investment dynamics that is purely due to the frictions, rather than changing investment opportunities. We show that frictions matter for corporate investment and valuation, both conceptually and quantitatively.

**Related literature.** Shleifer and Wolfenzon (2002) develop a static equilibrium model of an entrepreneur's going public decision under imperfect investor protection. La Porta et al. (2002) provide a static model to explain their empirical findings of lower firm values in countries with weaker investor protection.[6] In both papers, the controlling shareholder's key decision is the amount of cash diversion. Because both papers use a static framework with a risk neutral controlling shareholder, they are thus silent on corporate investment decisions, the cost of capital, and the relation between investment and Tobin's $q$. Himmelberg, Hubbard, and Love (2002) develop a two-period model with imperfect investor protection. In their model, the risk-averse entrepreneur chooses ownership concentration by trading off the benefit of diversification with the cost of raising capital, but they do not model the controlling shareholder's investment, exit, consumption-saving and portfolio choice. Additionally, unlike Himmelberg et al. (2002), we also study how investor protection affects firm value, investment, cost of capital for outside equity, as well as the idiosyncratic risk premium for the controlling shareholder's inside equity in a unified dynamic incomplete-markets $q$ theory framework.

Albuquerque and Wang (2008) develop an equilibrium model of investment and asset pricing under imperfect investor protection.[7] They show that the firm over-invests, the cost of capital is higher, and Tobin's $q$ is lower when investor protection is weaker. The investment-capital ratio, risk premium, and Tobin's $q$ for both insiders and outside investors are all constant. Unlike their model, we focus on a single firm's investment, cost of capital, and firm valuation for both insiders and outsiders. While their work focuses on the steady-state properties in an equilibrium setting, our model generates dynamic investment and exit decisions for a single firm run by a controlling shareholder in an incomplete-markets framework. Li (2010) studies the effects of corporate governance on cross-sectional stock returns in a managerial agency model where the manager derives non-pecuniary private benefits from empire building.

Our model also relates to the growing dynamic corporate finance literature including

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[6]Stulz (2005) constructs a twin agency model where rulers of sovereign states and corporate insiders pursue their own interests to explain the limit of financial globalization.

[7]Dow, Gorton, and Krishnamurthy (2005) study the effects of agency conflicts on asset prices and investment by integrating managerial empire building into a neoclassical asset pricing model.

both investment-based models[8] and continuous-time capital structure models.[9] Unlike our work with risk aversion and idiosyncratic risk premium, much research in this literature assumes risk neutrality or a systematic risk premium for the firm. Moreover, most work in this literature does not explicitly model managerial agency. Exceptions include Zwiebel (1996), Morellec (2004), and Lambrecht and Myers (2008). These papers develop continuous-time capital structure models where financing policy is chosen by management, building on Stulz (1990), which is a model of financing policies with managerial empire building as in Jensen (1986).[10] Unlike existing work, we model the *interactive* effects of managerial agency and risk aversion in a generalized incomplete-markets-based $q$ theory of investment framework. Lambrecht and Myers (2011) assume risk-averse managers and generate a Lintner-type payout, but do not study investment dynamics, firm valuation, and the cost of capital for outside equity.

## 2 Model

We incorporate imperfect investor protection and non-diverisifiable idiosyncratic risk for the controlling shareholder into a neoclassical $q$-theoretic model of investment.

**Physical production and investment technology.** Let $K$ and $I$ denote the firm's capital stock and investment, respectively. Capital stock accumulates and stochastically depreciates.[11] We write the dynamics of capital stock as follows,

$$dK_t = (I_t - \delta_K K_t)dt + \sigma_K K_t dZ_t^K \,, \tag{1}$$

where $Z^K$ is a standard Brownian motion, $\delta_K \geq 0$ is the expected rate of depreciation, and $\sigma_K$ is volatility for capital shocks/depreciation. The firm's operating revenue over $(t, t + dt)$ is proportional to its time-$t$ capital stock $K_t$, and is given by $K_t dA_t$, where $dA_t$ is the firm's productivity shock over the same time period. The productivity shock $dA_t$ is assumed to be independently and identically distributed (iid), and is given by

$$dA_t = \mu_A dt + \sigma_A dZ_t^A, \tag{2}$$

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[8]See Gomes (2001), Hennessy and Whited (2007), and Riddick and Whited (2009), among others.

[9]Leland (1994) and Goldstein, Ju, and Leland (2001) are important theoretical contributions on dynamic capital structure and valuation.

[10]Morellec, Nikolov, and Schurhoff (2011) and Nikolov and Whited (2011) estimate dynamic capital structure models with managerial agency.

[11]For stochastic capital depreciation in macro and finance, see Cox, Ingersoll, Jr., and Ross (1985), Obstfeld (1994), Albuquerque and Wang (2008), and Barro (2009) for example.

where $Z^A$ is a standard Brownian motion, $\mu_A > 0$ and $\sigma_A > 0$ are the mean and volatility parameters of the productivity shock. [12]

Changing capital stock often incurs adjustment costs. For example, installing new equipment or upgrading capital may disrupt production lines, and require additional time and resources. Costly capital adjustments are empirically plausible and widely assumed in the investment literature.[13] Let $\Phi(I, K)$ denote the capital adjustment cost function. As is standard in the $q$-theory literature, we assume that $\Phi(I, K)$ is convex in investment $I$. Moreover, following Hayashi (1982) and Lucas and Prescott (1971), we assume that $\Phi(I, K)$ is homogeneous of degree one in investment $I$ and capital $K$. That is, we may write $\Phi(I, K) = \phi(i)K$, where $i$ denotes the investment-capital ratio and $\phi(i)$ is increasing and convex. While our model applies to a well-behaved function $\phi(i)$, for simplicity, we specify $\phi(i)$ as follows,

$$\phi(i) = \frac{\theta_i}{2} i^2, \tag{3}$$

where the constant $\theta_i > 0$ is the adjustment cost parameter. A higher value $\theta$ implies a more costly adjustment. With homogeneity, average $q$ is equal to marginal $q$ without frictions. The non-diversifiable risk drives a wedge between average $q$ and marginal $q$ for the controlling shareholder. For simplicity, we assume that the firm's capital shock $Z^K$ and its productivity shock $Z^A$ are uncorrelated.

**Investor protection.** Because investor protection is imperfect and the controlling shareholder has control rights, firm profits are not shared on a *pro rata* basis between the controlling shareholder and outside shareholders. The controlling shareholder can pursue private benefits at a cost, which is socially inefficient. It may take a variety of forms such as excessive salary, transfer pricing, employing unqualified relatives and friends, to name just a few.[14] We assume that the controlling shareholders' discretion does not depend on their cash flow rights, provided that their equity ownership in the firm exceeds $\underline{\alpha}$, a lower bound.[15]

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[12] This continuous-time specification is a stochastic formulation of the "$AK$" technology in Hayashi (1982). For applications of this technology specification in dynamic corporate finance, see Bolton, Chen, and Wang (2011) and DeMarzo, Fishman, He, and Wang (2011).

[13] See Hayashi (1982), Abel (1983), and Abel and Eberly (1994) on the role of adjustment costs on investment and the value of capital. See Caballero (1999) for a survey.

[14] See Barclay and Holderness (1989), Dyck and Zingales (2004), and Albuquerque and Schroth (2010) on the empirical evidence in support of private benefits of control.

[15] We can extend our model to assume that the the controlling shareholder's power (and hence ability to pursue private benefits) also depends on ownership. The controlling shareholders can achieve full

By diverting the amount $sK$ from the firm, the controlling shareholder incurs a cost $\Psi(s, K)$, which is assumed to be increasing and convex in $s$ as in La Porta et al. (2002), Johnson et al. (2000), and Stulz (2005). Additionally, we assume that $\Psi(s, K)$ is homogeneous in diversion amount $sK$ and $K$, $\Psi(s, K) = \psi(s)K$, analogous to the one for the capital adjustment cost function $\Phi(I, K)$. For simplicity, we specify that the cost of diversion $\psi(s)$ is quadratic in $s$,

$$\psi(s) = \frac{\theta_s}{2} s^2 , \tag{4}$$

where $\theta_s$ is the investor protection parameter as in La Porta et al. (2002).

**The controlling shareholder's exit option.** While enjoying private benefits of control, the controlling shareholder also faces significant non-diversifiable risk from the exposure to the firm. It may be optimal for the controlling shareholder to exit from the firm under certain circumstances. Upon exiting, the controlling shareholder gives up the control and collects the sales proceeds of the *pro rata* share in the firm. Under the new management, firm value (net of all transaction costs due to the exit) is a fraction of the firm size, $lK$, where $l$ is a constant. This exit may take the form of public offering or a private sale arrangement. The exit decision resembles an American-style call option on the underlying non-tradable firm. Since markets are incomplete for the controlling shareholder, we cannot use the standard financial option pricing model but, instead, need to value this exit option by solving the controlling shareholder's interdependent dynamic optimization problem.

**Payouts to outside shareholders and the controlling shareholder.** Let $\tau$ denote the stochastic and endogenously chosen time that the controlling shareholder exits from the firm. Before exiting $(t < \tau)$, the payout to outside shareholders is given by

$$dY_t = K_t dA_t - I_t dt - \Phi(I_t, K_t)dt - s_t K_t dt , \quad t < \tau , \tag{5}$$

where the last term is the firm's output diverted by the controlling shareholder. The cash flow accruing to the controlling shareholder, $dM_t$ over the period $(t, t+dt)$, is given by the sum of *pro rata* cash flow and diverted output less the cost of diversion,

$$dM_t = \alpha dY_t + s_t K_t dt - \Psi(s_t, K_t)dt , \quad t < \tau , \tag{6}$$

control of the firm with far less than majority cash flow rights via dual class shares, pyramidal structure, a controlled board, and/or other strategies.

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where $dY_t$ given by (5) is the payout to outside shareholders.

**Financial investment opportunities.**   As outside shareholders, the controlling share-holder has standard liquid financial investment opportunities, a risk-free asset which pays a constant rate of interest $r$ and a risky market portfolio. As in Merton (1971), the incremental return $dR_t$ of the market portfolio over period $(t, t + dt)$ is iid and given by

$$dR_t = \mu_R dt + \sigma_R dB_t \,, \tag{7}$$

where $\mu_R$ and $\sigma_R > 0$ are mean and volatility parameters of the market portfolio return process, and $B$ is a standard Brownian motion. Let $\eta$ denote the Sharpe ratio of the market portfolio, which is given by

$$\eta = \frac{\mu_R - r}{\sigma_R} \,. \tag{8}$$

Let $\rho_A$ be the correlation coefficient between the firm's productivity shock $dZ_t^A$ and the return of the market portfolio $dR_t$. Similarly, let $\rho_K$ denote the correlation between the firm's capital shock $dZ_t^K$ and $dR_t$. The firm faces both productivity and capital shocks. With either $|\rho_A| \neq 1$ or $|\rho_K| \neq 1$, the controlling shareholder cannot fully diversify the idiosyncratic business risk. Non-diversifiable idiosyncratic risk and investor protection jointly play critical roles in the controlling shareholder's decision making and implied private as well as public valuation.

Let $X$ and $\Pi$ denote the controlling shareholder's financial wealth and the investment amount in the market portfolio, respectively. Thus, $(X - \Pi)$ is the amount invested in the risk-free asset. Before exiting, the controlling shareholder's wealth $X$ evolves as,

$$dX_t \;=\; r\,(X_t - \Pi_t)\,dt + \mu_R \Pi_t dt + \sigma_R \Pi_t dB_t - C_t dt + dM_t \,, \qquad t < \tau \,, \tag{9}$$

where the first and second terms are the investment returns in the risk-free asset and the risky market portfolio respectively, the third term gives the consumption outflow, and the last term $dM_t$ given by (6) is the income (the sum of *pro rata* share of equity payout and private benefits of control net of diversion costs) for the controlling shareholder. At the exit time $\tau$, the controlling shareholder's wealth changes from $X_{\tau-}$ to

$$X_\tau = X_{\tau-} + \alpha l K_\tau \,. \tag{10}$$

After exiting, the controlling shareholder's wealth evolves as follows,

$$dX_t \;=\; r\,(X_t - \Pi_t)\,dt + \mu_R \Pi_t dt + \sigma_R \Pi_t dB_t - C_t dt \,, \qquad t > \tau \,. \tag{11}$$

**The controlling shareholder's optimization problem.** The controlling share-holder chooses consumption $C_t$, allocation to the market portfolio $\Pi_t$, corporate investment $I_t$, diversion $s_t$, and stochastic exit time $\tau$ to maximize utility given by

$$\max_{C,\Pi,I,s,\tau} \quad \mathbb{E}\left[\int_0^\infty e^{-\zeta t} U(C_t)\,dt\right],\tag{12}$$

where $\zeta > 0$ is the subjective discount rate and $U(C)$ is an increasing and concave function. Our setup applies to any well-behaved utility function $U(C)$. For tractability, we adopt the constant absolute risk averse (CARA) utility for the remainder of the paper, $U(C) = -e^{-\gamma C}/\gamma$, where $\gamma > 0$ is the CARA coefficient. Our model by construction misses the wealth effect implications due to the CARA utility assumption. However, the main insight of our paper is robust, since the key mechanism of our model operates through the *interactive* and opposing effects between the controlling shareholder's precautionary savings demand and incentives to pursue private benefits.

**The outside shareholders' perspective.** Outside shareholders hold a diversified investment portfolio and thus demand risk premia for systematic risks, not for *idiosyncratic* risks. There are two sources of systematic risks, the productivity shock induced cash flow risk and the capital shock induced capital gains risk. We will show that the conditional capital asset pricing model (CAPM) holds in our model.

# 3    The first-best benchmark

With perfect investor protection and complete markets, the MM theorem holds and the standard $q$ theory of investment characterizes the firm's investment and Tobin's $q$.

**Proposition 1** *The first-best firm value is given by $V^{FB}(K) = q^{FB}K$, where*

$$q^{FB} = 1 + i^{FB}\theta_i,\tag{13}$$

*and the first-best investment-capital ratio $i^{FB}$ is given by*

$$i^{FB} = (r+\delta) - \sqrt{(r+\delta)^2 - \frac{2}{\theta_i}(\nu_A - (r+\delta))}.\tag{14}$$

*Here, the risk-adjusted mean for the productivity shock $\nu_A$ is given by*

$$\nu_A = \mu_A - \rho_A\eta\sigma_A,\tag{15}$$

*and the risk-adjusted capital depreciation rate, $\delta$, is given by*

$$\delta = \delta_K + \rho_K \eta \sigma_K \,. \tag{16}$$

*The firm's expected return $\mu^{FB}$ is given by CAPM,*

$$\mu^{FB} = r + \beta^{FB} \left( \mu_R - r \right) \,, \tag{17}$$

*where*

$$\beta^{FB} = \beta_K^{FB} + \beta_A^{FB} \,, \tag{18}$$

*and*

$$\beta_K^{FB} = \frac{\rho_K \sigma_K}{\sigma_R} \,, \tag{19}$$

$$\beta_A^{FB} = \frac{\rho_A \sigma_A}{\sigma_R} \frac{1}{q^{FB}} \,. \tag{20}$$

The homogeneity property implies that marginal $q$ equals average $q$ as in Hayashi (1982). The risk-adjusted expected productivity $\nu_A$ is lower than the expected productivity $\mu_A$ by the risk premium $\rho_A \eta \sigma_A$. Similarly, capital is subject to shocks and thus the risk-adjusted capital depreciation rate $\delta$ is larger than the expected depreciation rate $\delta_K$ by the size of the risk premium $\rho_K \eta \sigma_K$. Investment is positive if and only if $\nu_A > r + \delta$. Intuitively, when the firm's risk-adjusted expected productivity $\nu_A$ is greater than $r + \delta$, the firm optimally invests $i^{FB} > 0$ and capital earns rents in equilibrium, $q^{FB} > 1$. The idiosyncratic risk has no effects on investment, value, and the cost of capital. CAPM holds and the firm's beta has two components, $\beta_K^{FB}$ and $\beta_A^{FB}$ given in (19) and (20) for productivity and capital shocks, respectively. Finally, the higher the adjustment cost parameter $\theta_i$, the lower the investment-capital ratio $i^{FB}$ and the lower Tobin's $q^{FB}$.

# 4   Solution: The general case

While lacking investor protection induces the controlling shareholder to pursue private benefits and hence encourages empire building, the diversification motive gives incentives to under-invest. Depending on the strength of the two motives, both over- and under-investment may occur under imperfect investor protection and incomplete markets.

## 4.1 The controlling shareholder's optimality

After exiting, the controlling shareholder solves the standard consumption and portfolio choice problem as in Merton (1971). We summarize the main results below.

**Proposition 2** *The controlling shareholder's post-exit value function $F_0(X)$ is given by*

$$F_0(X) = -\frac{1}{\gamma r} \exp\left[-\gamma r \left(X + \frac{\eta^2}{2\gamma r^2} + \frac{\zeta - r}{\gamma r^2}\right)\right]. \tag{21}$$

*The optimal consumption is given by*

$$C_0(X) = r\left(X + \frac{\eta^2}{2\gamma r^2} + \frac{\zeta - r}{\gamma r^2}\right). \tag{22}$$

*The optimal allocation to the risky market portfolio is given by a constant mean-variance demand $\Pi_0 = \eta/(\gamma r \sigma_R)$, where $\eta$ is the Sharpe ratio for the market portfolio.*

Next, we first characterize the controlling shareholder's decision making before exit. We show that the controlling shareholder's value function $F(X, K)$ is given by

$$F(X, K) = -\frac{1}{\gamma r} \exp\left[-\gamma r \left(X + \frac{\eta^2}{2\gamma r^2} + \frac{\zeta - r}{\gamma r^2} + \alpha P(K)\right)\right], \quad t < \tau, \tag{23}$$

where $P(K)$ is the controlling shareholder's private (certainty equivalent) valuation of the firm. The following theorem characterizes the solution of $P(K)$ and decision making.

**Theorem 1** *The controlling shareholder's private valuation of capital per unit of ownership, $P(K)$, solves the following ordinary differential equation (ODE),*

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\begin{aligned}
rP(K) &= (\nu_A + b(\alpha)) K - \delta K P'(K) + \frac{(P'(K) - 1)^2}{2\theta_i} K + \frac{\sigma_K^2 K^2 P''(K)}{2} \\
&\quad - \frac{\alpha \gamma r K^2}{2}\left[(1 - \rho_A^2)\sigma_A^2 - 2\rho_A \rho_K \sigma_A \sigma_K P'(K) + (1 - \rho_K^2)\sigma_K^2 P'(K)^2\right],
\end{aligned} \tag{24}
$$

*where the net private benefit of control per unit of ownership, $b(\alpha)$, is given by*

$$b(\alpha) = \frac{(1 - \alpha)^2}{2\alpha\theta_s}. \tag{25}$$

*We solve the ODE (24) subject to the following boundary conditions,*

$$P(0) = 0, \tag{26}$$
$$P(\overline{K}) = l\overline{K}, \tag{27}$$
$$P'(\overline{K}) = l. \tag{28}$$

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The optimal investment-capital ratio $i = I/K$ is given by

$$i(K) = \frac{P'(K) - 1}{\theta_i}.$$

(29)

The optimal diversion $s$ is given by (A.3), and the optimal consumption rule is given by

$$C(X, K) = r\left(X + \frac{\eta^2}{2\gamma r^2} + \frac{\zeta - r}{\gamma r^2} + \alpha P(K)\right).$$

(30)

The optimal investment amount in the risky market portfolio is given by

$$\Pi(K) = \frac{\eta}{\gamma r \sigma_R} - \frac{\rho_A \sigma_A}{\sigma_R} \alpha K - \frac{\rho_K \sigma_K}{\sigma_R} \alpha K P'(K).$$

(31)

The controlling shareholder collects private benefits at the rate of $(1 - \alpha)sK$ and incurs cost at the rate of $\psi(s)K$. Thus, per unit of capital and ownership, the net private benefit per period is $[(1 - \alpha)s - \psi(s)]/\alpha = b(\alpha)$. The consumption rule (30) is similar to other CARA-utility-based permanent-income/precautionary-saving models. The optimal investment-capital ratio is now determined by the controlling shareholder's (private) marginal $q$, $P'(K)$. The optimal portfolio rule (31) has the standard mean-variance demand as well as the dynamic hedging demands against both productivity and capital shocks. The ODE (24) characterizes $P(K)$, the controlling shareholder's certainty equivalent valuation of the firm in the interior region of $K$. The left boundary $K = 0$ is absorbing and hence $P(0) = 0$. Conditions (27)-(28) are the value-matching and smooth-pasting conditions for $P(K)$ at the optimal exit boundary $\overline{K}$.

## 4.2 A special case: Imperfect investor protection and CM

Now consider the case with imperfect investor protection and complete markets. With CM, the controlling shareholder demands no idiosyncratic risk premium. The following proposition summarizes the main results for the CM case.

**Proposition 3** *The controlling shareholder values the firm $P^{CM}(K) = p^{CM}(\alpha)K$, where $p^{CM}(\alpha)$ is the controlling shareholder's average $q$ as well as marginal $q$, given by*

$$p^{CM}(\alpha) = 1 + \theta_i i^{CM}(\alpha),$$

(32)

*and the investment-capital ratio $i^{CM}(\alpha)$ is given by*

$$i^{CM}(\alpha) = (r + \delta) - \sqrt{(r + \delta)^2 - \frac{2}{\theta_i}\left(\nu_A + b(\alpha) - (r + \delta)\right)}.$$

(33)

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*For outside investors, firm value is $V^{CM}(K) = q^{CM}(\alpha)K$, where Tobin's q is given by*

$$q^{CM}(\alpha) = \frac{\nu_A - s(\alpha) - i^{CM}(\alpha) - \phi(i^{CM}(\alpha))}{r + \delta - i^{CM}(\alpha)} . \qquad (34)$$

With CM, the controlling shareholder fully diversifies idiosyncratic business risk. Because the net private benefits increase with firm size, the controlling shareholder over-invests by choosing $i^{CM}(\alpha) > i^{FB}(\alpha)$, and the controlling shareholder's q is larger than the first-best q, $p^{CM}(\alpha) > q^{FB}(\alpha)$. Outside investors anticipate agency and thus price the firm at a discount, $q^{CM}(\alpha) < q^{FB}(\alpha)$. The controlling shareholder gains at the expense of outside investors, but private benefits and investment distortions are socially costly, with a net welfare loss of $q^{FB} - \left[\alpha p^{CM}(\alpha) + (1-\alpha)q^{CM}(\alpha)\right] > 0$. Strengthening investor protection or increasing inside equity ownership decreases private benefits, mitigates over-investment incentives, and reduces the welfare cost.

# 5 The controlling shareholder's private valuation and corporate investment

We now explore the implications of imperfect investor protection and non-diversifiable idiosyncratic risk on investment and the controlling shareholder's private valuation.

**Parameter choice.** When applicable, parameter values are annualized. The risk-free rate is $r = 5\%$. For the market portfolio return, the risk premium and the volatility are $\mu_R - r = 6\%$ and $\sigma_R = 20\%$, with an implied Sharpe ratio of $\eta = 30\%$. Using the sample of large firms in Compustat from 1981 to 2003, Eberly, Rebelo, and Vincent (2009) provide empirical evidence in support of Hayashi (1982). Using their work as a guideline, we set the expected productivity $\mu_A = 22.8\%$, the volatility of productivity shocks $\sigma_A = 25\%$, and the expected capital depreciation rate $\delta_K = 8\%$. We choose the adjustment cost parameter $\theta_i = 3$, which is in the range of estimates used in the literature.[16] We set the volatility of the capital shock $\sigma_K = 20\%$, and the correlation coefficients $\rho_A = \rho_K = 0.5$. The implied risk-adjusted productivity $\nu_A = 19\%$ and the risk-adjusted depreciation rate $\delta = 11\%$. In the first-best benchmark, $q^{FB} = 1.256$, $i^{FB} = 0.085$, $\beta_A^{FB} = 0.5 = \beta_K^{FB} = 0.5$, which implies the firm's beta, $\beta^{FB} = 1$.

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[16]See Whited (1992), Hall (2004), Riddick and Whited (2009), and Eberly, Rebelo, and Vincent (2009).

The controlling shareholder also has an exit option. The firm's productive technology makes this exit option worthless in the first-best environment. However, business exit becomes a valuable option for the purpose of diversification when incomplete-markets frictions are present. Exit may take the form of a public offering, a private sale arrangement, or a buyout. We set $l = 1.15$ so that investors recover 1.15 per unit of capital (net of transaction costs) upon exiting. A value of $l = 1.15$ captures the value of cashing out. While the value for each unit of capital is larger than unity, the firm's setup and other adjustment costs rule out arbitrage.

The controlling shareholder's ownership is set at $\alpha = 0.25$ (Dahlquist et al. (2003)). Using the calibration in Albuquerque and Wang (2008) as a reference, we set the investor protection parameter $\theta_s = 350$, which implies that the diversion amount in each period is $s(0.25) = (1 - 0.25)/350 = 0.21\%$ of the firm's capital stock, or equivalently 0.94% of the firm's average output.

We next turn to risk aversion. Researchers often have views about sensible values of *relative* risk aversion, but not *absolute* risk aversion. While researchers may disagree on the exact value of relative risk aversion, they agree that a sensible range of relative risk aversion is likely between one to five. For CARA utility, we may approximate the coefficient of relative risk aversion by $\gamma X_0$, where $X_0$ is the entrepreneur's initial wealth. We choose the CARA coefficient $\gamma = 2$, which implies that the coefficient of *relative* risk aversion is around 3.2 in our calibration. See Section 9 for details for the calibration of $\gamma$. Table 3 summarizes all variables and baseline parameter values used in the paper.

## 5.1 The controlling shareholder's private valuation

Figure 1 plots the controlling shareholder's (private) marginal $q$, $P'(K)$, which is decreasing in $K$ for $K \leq 22.9$ and increasing for $K > 22.9$ (see the solid line). The controlling shareholder's under-diversification cost increases with $K$. The controlling shareholder optimally exercises an "American-style" exit option when firm size $K$ reaches the endogenous exit boundary $\overline{K} = 30.8$. Intuitively, the controlling shareholder's optimal exit implies that the private marginal $q$ equals $l = 1.15$ at the moment of exiting.

For $K \leq 22.9$, the under-diversification increases with firm size $K$, making $P'(K)$ decrease in $K$. For $22.9 < K \leq 30.8$, the exit option is sufficiently deep in the money, causing $P'(K)$ to increase in $K$. Importantly, incomplete markets, the exit option, and investor protection jointly generate rich dynamics, despite the homogeneity property
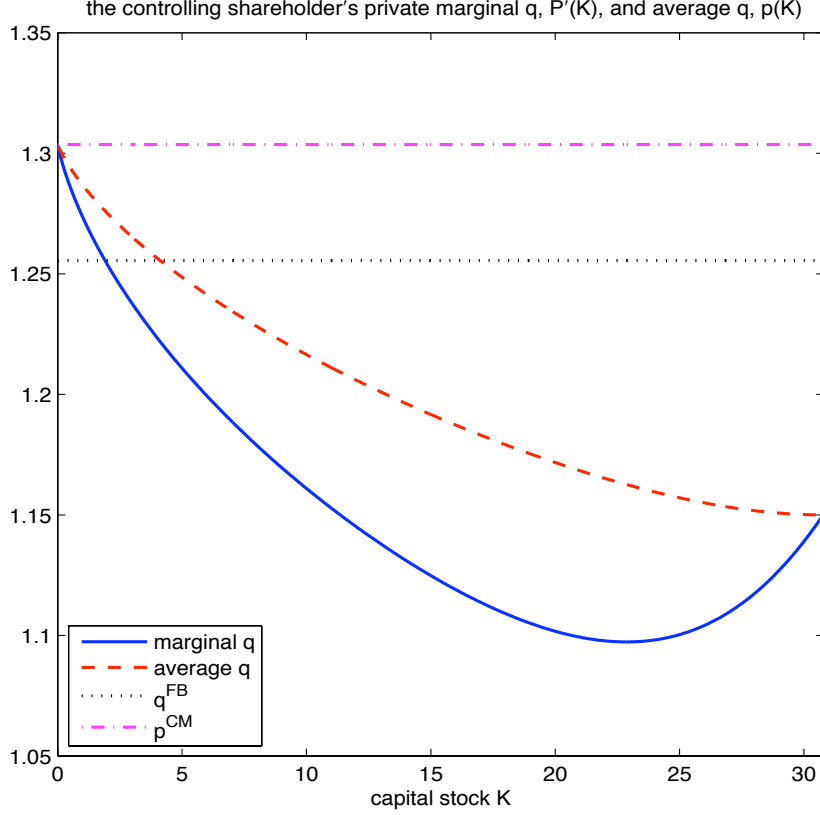
Figure 1: The controlling shareholder's private marginal $q$, $P'(K)$, and average $q$, $p(K)$.

and the constant-return-to-scale production technology.

We note that the firm behaves as if it faces a decreasing-return-to-scale production for $K \leq 22.9$ in our example. Despite assuming the constant-return-to-scale technology, our agency-based model generates firm investment that resembles a neoclassical production based model with a *decreasing*-return-to-scale technology. In contrast, for a firm with sufficiently large size, $22.9 < K \leq 30.8$ in our example, the investment-capital ratio now *increases* with firm size because the controlling shareholder's exit option causes the private firm valuation $P(K)$ to be convex in $K$.

We define the controlling shareholder's (private) average $q$ as follows,

$$p(K) = \frac{P(K)}{K} \,. \tag{35}$$

Figure 1 also plots the private average $q$, $p(K)$, and compares it with private marginal $q$, $P'(K)$. In the interior region, $0 < K < 30.8$, the average $q$, $p(K)$, decreases with $K$. As firm size $K$ increases, the idiosyncratic business risk exposure increases and thus firm
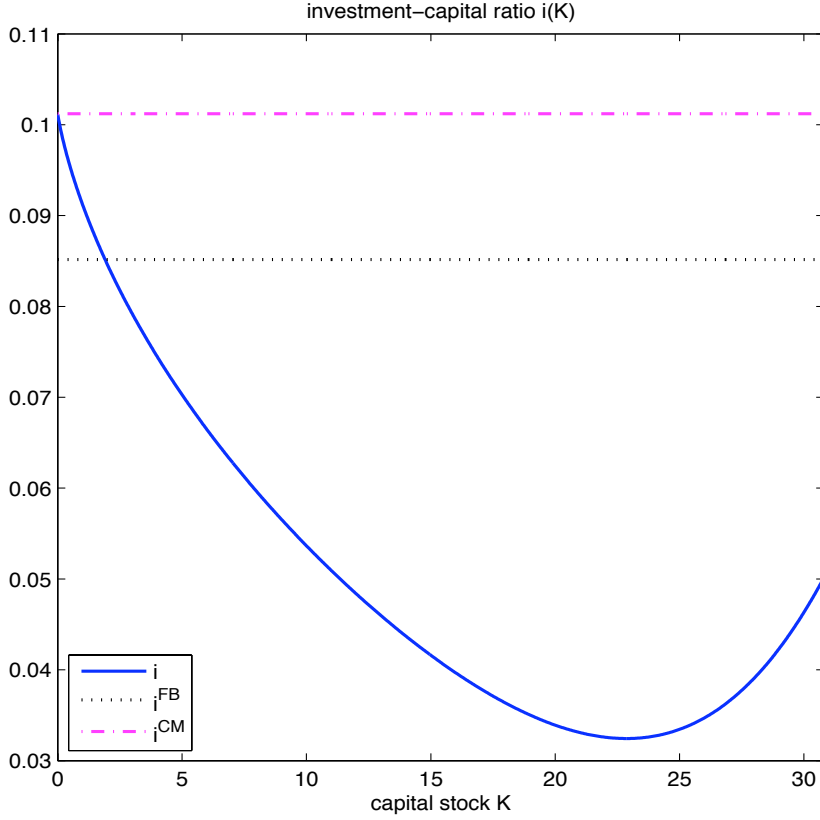
Figure 2: The optimal investment-capital ratio $i(K)$.

value per unit of capital for the controlling shareholder falls. Because the controlling shareholder's average $q$, $p(K)$, decreases with firm size $K$, the private marginal $q$ then must be lower than private average $q$, $P'(K) < p(K)$. This relation is analogous to the one between the marginal cost and average cost in micro theory.[17]

At the boundaries, $K = 0$ and $K = \overline{K}$, the private average $q$ and marginal $q$ are equal. As $K \to 0$, the idiosyncratic risk becomes negligible for the controlling shareholder. We thus have $p(0) = P'(0) = p^{CM} = 1.304$, the CM solution of Section 4.2. At the optimal exit boundary $\overline{K}$, the controlling shareholder is fully diversified and $p(\overline{K}) = P'(\overline{K}) = l = 1.15$. Because average $q$ is always larger than the marginal $q$ and the two $q$'s equal at both boundaries, $K = 0$ and $K = \overline{K}$, we have have a non-monotonic marginal $q$ as we see in Figure 1.

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[17]Using $p'(K) = (P'(K) - p(K))/K$, we have $p'(K) < 0$ if and only if $P'(K) < p(K)$.

## 5.2 Investment-capital ratio $i(K)$

Figure 2 plots the investment-capital ratio $i(K)$, which has the same shape as the controlling shareholder's (private) marginal $q$, $P'(K)$. In the standard investment FOC in $q$ models with convex adjustment costs, the firm equates its marginal $q$ with its marginal cost of investing. In our model, the controlling shareholder equates the private marginal $q$, $P'(K)$, with the marginal cost of investing, $1 + \theta_i\, i(K)$. Therefore, even with a simple time-invariant investment opportunity (with iid shocks), our model features time-varying investment policy. Both over- and under-investment (relative to the first-best level) occur in our model. The private benefits of control give incentives to over-invest while diversification motive encourages under-investment. When the private benefits motive is stronger than the diversification one (relatively low values of $K$), the firm over-invests. Otherwise, it under-invests.

The investment-capital ratio $i(K)$ is non-monotonic in firm size $K$ because the controlling shareholder's private marginal $q$ is also non-monotonic. In the region $K \leq 22.9$, as $K$ increases, the controlling shareholder becomes increasingly concerned with under-diversification and therefore decreases firm investment from the $i^{CM} = 0.101$ at $K = 0$.

For $22.9 < K \leq 30.8$, the firm gets close to the exit boundary $\overline{K}$, and the controlling shareholder's option value of exit becomes significant. The positive volatility effect on the exit option value gives the controlling shareholder incentives to increase investment. For our example, $i(K)$ increases in $K$ from $i(22.9) = 0.032$ to $i(30.8) = 0.050$.

# 6 Public firm value, average $q$, and marginal $q$

Taking the controlling shareholder's decisions as given, we now value the firm for outside investors. Outside investors rationally anticipate managerial agency and price the firm at its fair value. Unlike the controlling shareholder, well diversified investors only demand a systematic risk premium. Proposition 4 reports firm value $V(K)$.

**Proposition 4** *Firm value $V(K)$ for outside investors solves the ODE,*

$$rV(K) = (\nu_A - s(\alpha) - i(K) - \phi(i(K)))\, K + (i(K) - \delta)\, KV'(K) + \frac{\sigma_K^2 K^2 V''(K)}{2}\,, \quad (36)$$

*subject to the following boundary conditions,*

$$V(0) = 0\,, \quad (37)$$
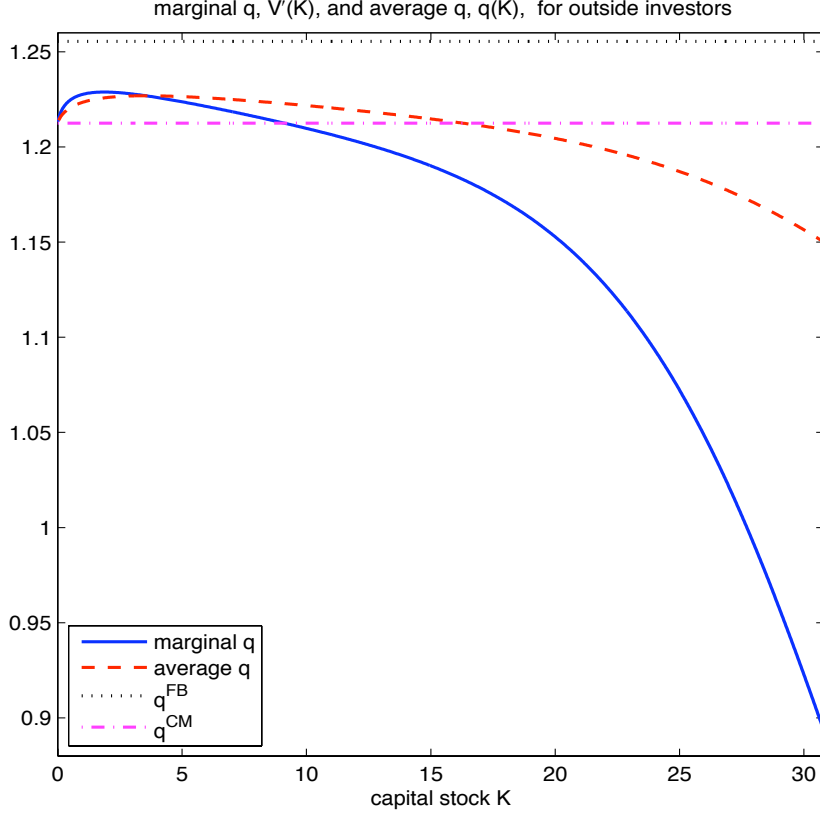$$V(\overline{K}) = l\overline{K}\,. \quad (38)$$

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Figure 3: Public marginal $q$, $q_m(K) = V'(K)$ and public average $q$, $q_a(K) = V(K)/K$

For outside investors, the firm's public average $q$ and public marginal $q$ are

$$q_a(K) \equiv \frac{V(K)}{K}, \quad \text{and} \quad q_m(K) \equiv V'(K). \tag{39}$$

Figure 3 compares public average $q$, $q_a(K)$, with public marginal $q$, $q_m(K)$. Conflicts of interest cause public average $q_a(K)$ to be lower than $q^{FB} = 1.256$ in the first-best benchmark. Outside investors' $q_a(K)$ and $q_m(K)$ are nonlinear and non-monotonic in firm size $K$ because the expected growth rate of capital chosen by the controlling shareholder is highly nonlinear and non-monotonic.

In the limit as $K \to 0$, the controlling shareholder faces no idiosyncratic business risk and thus both average $q$ and marginal $q$ equal $q^{CM}$, Tobin's $q$ under complete markets but imperfect investor protection, $q_a(0) = q_m(0) = q^{CM} = 1.213$. With complete markets, agency frictions are solely due to imperfect investor protection. Naturally, outside investors' average $q$ equals $q^{CM} = 1.213$, which is 7% lower than the controlling shareholder's average $q$ given by $p^{CM} = 1.304$. The wealth transfer from investors to

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the controlling shareholder is anticipated and priced. Therefore, $p^{CM} = 1.304 > q^{FB} = 1.256$, while $q^{CM} = 1.213 < q^{FB}$. At the optimal exit threshold $\overline{K} = 30.8$, by definition, both Tobin's average $q$ for outside investors, $q_a(\overline{K})$, and the controlling shareholder's average $q$, $p(\overline{K})$, equal the exit value $l$, in that $q_a(\overline{K}) = p(\overline{K}) = l = 1.15$.

Average $q$, $q_a(K)$, first increases in $K$ and then decreases in $K$. As firm size $K$ increases from the origin, the controlling shareholder's idiosyncratic business risk exposure increases and thus the investment-capital ratio decreases. Luckily, this decrease of $i(K)$ in $K$ near the origin is value enhancing for investors because investment is excessive near the origin, $i(0) = i^{CM} = 0.101 > i^{FB} = 0.085$. In our example, when $K \leq 3.6$, $q_a(K)$ increases with firm size $K$, and marginal $q$ must be higher than average $K$, $q_m(K) \geq q_a(K)$. This relation between average $q$ and marginal $q$ again is analogous to the average cost and marginal cost in micro theory.[18]

When $K \geq 3.6$, average $q$, $q_a(K)$, decreases with $K$. When that happens, marginal $q$ has to fall below average $q$, $q_m(K) < q_a(K)$, so that $q_a(K)$ decreases with firm size $K$. Note that at the exit threshold $\overline{K} = 30.8$, the marginal $q$ equals $q_m(30.8) = 0.89$, which is 22.6% lower than the average $q$, $q_a(30.8) = 1.15$.

# 7 Beta and the cost of capital for outside equity

We now explore the risk and return implications for outside investors. Using Ito's formula, we show that the incremental return $dR_t^V$ for outside equity is given by the sum of dividend yield $dY_t/V_t$ and capital gains $dV_t/V_t$,

$$dR_t^V \equiv \frac{dY_t + dV_t}{V_t} = \mu_V(K_t)dt + \frac{\sigma_A}{q_a(K_t)}dZ_t^A + \frac{q_m(K_t)}{q_a(K_t)}\sigma_K dZ_t^K, \qquad (40)$$

where the firm's expected return, $\mu_V(K)$, is also referred to as the cost of capital for outside equity.

**Proposition 5** *The conditional CAPM holds for outside equity and $\mu_V(K)$ satisfies*

$$\mu_V(K) = r + \beta(K)(\mu_R - r). \qquad (41)$$

*The conditional beta for outside equity, $\beta(K)$, equals the sum of productivity shock beta, $\beta_A(K)$, and capital shock beta, $\beta_K(K)$, in that*

$$\beta(K) = \beta_A(K) + \beta_K(K), \qquad (42)$$

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[18]Using $q_a'(K) = (q_m(K) - q_a(K))/K$, we have $q_a'(K) > 0$ if and only if $q_m(K) > q_a(K)$.
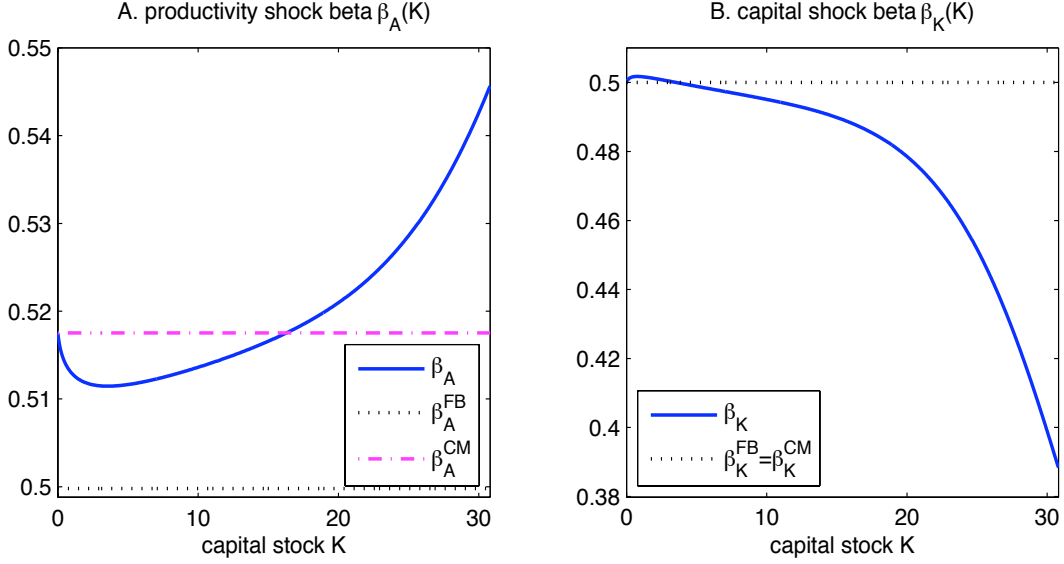
Figure 4: Productivity shock beta $\beta_A(K)$ and capital shock beta $\beta_K(K)$

*where productivity shock and capital shock betas are respectively given by*

$$\beta_A(K) = \beta_A^{FB} \frac{q^{FB}}{q_a(K)}, \tag{43}$$

$$\beta_K(K) = \beta_K^{FB} \frac{q_m(K)}{q_a(K)}. \tag{44}$$

While our model has both productivity and capital shocks, our conditional CAPM only has one factor, capital stock. This is due to the assumption that productivity shock is iid. Productivity shock carries a firm-size dependent risk premium, because average $q$, $q_a(K)$, depends on firm size $K$.

Figure 4 plots $\beta_A(K)$ and $\beta_K(K)$, in Panels A and B, respectively. The productivity shock beta, $\beta_A(K)$, is stochastic and depends inversely on Tobin's $q_a(K)$. Under imperfect investor protection, average $q$ is lower than $q^{FB}$. Thus, the formula (43) for $\beta_A(K)$ implies $\beta_A(K) > \beta_A^{FB}$. The lower Tobin's $q_a(K)$, the higher $\beta_A(K)$. Intuitively, the weaker investor protection, the lower firm value, which in turn makes productivity shock riskier, and hence a higher $\beta_A(K)$. As we have discussed, $q_a(K)$ is non-monotonic in firm size $K$, thus productivity shock $\beta_A(K)$ is also non-monotonic in $K$. In the limit as $K \to 0$, the controlling shareholder faces no idiosyncratic business risk exposure, hence the productivity shock beta approaches $\beta^{CM} = \left(q^{FB}/q^{CM}\right)\beta^{FB} = 0.518$.
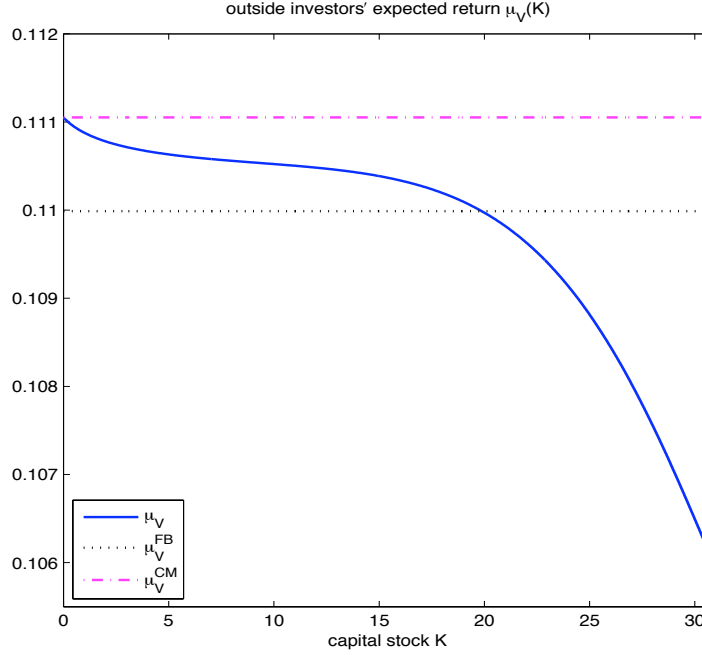
Figure 5: The cost of capital $\mu_V(K)$ for outside equity

The capital shock beta, $\beta_K(K)$ given in (44), depends on the ratio between the firm's marginal $q$, $q_m(K)$, and average $q$, $q_a(K)$. Equivalently, $\beta_K(K)$ depends on $d\ln V(K)/d\ln K = q_m(K)/q_a(K)$, the elasticity of firm value $V(K)$ with respect to capital $K$. We note that firm beta is often tied to the elasticity of firm value with respect to the underlying state variable, capital $K$ in our case. Intuitively, capital growth is stochastic and co-varies with the aggregate risk, which induces a risk premium.

When $K \leq 3.6$, $q_m(K) > q_a(K)$, hence $\beta_K(K) > \beta_K^{FB}$. In contrast, when $K > 3.6$, $q_m(K) < q_a(K)$ and $\beta_K(K) < \beta_K^{FB}$. We also note that neither productivity shock beta, $\beta_A(K)$, nor capital shock beta, $\beta_K(K)$, is monotonic in firm size $K$.

We now turn to the cost of capital for outside equity, which is also the firm's expected rate of return $\mu_V(K)$. Figure 5 plots $\mu_V(K)$. With imperfect investor protection and under incomplete markets, $\mu_V(K)$ is time-varying and stochastic. Recall that under the first-best benchmark, the firm's beta is constant and the expected return thus also remains constant. In our example, $\beta^{FB} = 1$ and $\mu_V^{FB} = 11\%$ (see the dotted line). Frictions cause the expected return $\mu_V(K)$ to be either higher or lower than the expected return $\mu_V^{FB} = 11\%$ under the first-best benchmark. For our example, the cost of capital for outside equity is higher than the first-best benchmark value $\mu_V^{FB} = 11\%$ for $K \leq 19.8$,

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and $\mu_V(K) < 11\%$ when $K \geq 19.8$.

While frictions lower firm value, the effect of frictions on the cost of capital is far from obvious. We show that productivity shock makes firm under imperfect investor protection riskier than under the first-best framework, $\beta_A(K) > \beta_A^{FB}$. Intuitively, the effect of a given productivity shock on return is greater if firm value is lower, *ceteris paribus*. Second, capital shock beta influences the cost of capital via the ratio between marginal $q$ and average $q$, or equivalently the elasticity of firm value with respect to capital. When average $q$ decreases, marginal $q$ must be lower than average $q$, which implies that the elasticity of firm value with respect to capital is less than one and the firm is less risky (against capital shocks) under imperfect investor protection than under the first-best benchmark. This scenario occurs when $K \geq 19.8$ in our example.

We next turn to the special case with complete markets but imperfect investor protection. As we show, for this special case, we may calculate beta for both types of shareholders explicitly.

**Corollary 1** *With CM, CAPM holds for both types of shareholders but with different betas. For capital shocks, the beta for both types equals the first-best value, $\beta_K^{CM} = \beta_K^{FB}$, where $\beta_K^{FB}$ is given in (19). For productivity shocks, the beta for outside shareholders, $\beta_{A,out}^{CM}$, and the beta for the controlling shareholder, $\beta_{A,in}^{CM}$, respectively equal*

$$\beta_{A,out}^{CM} = \frac{\rho_A \sigma_A}{\sigma_R} \frac{1}{q^{CM}}, \tag{45}$$

$$\beta_{A,in}^{CM} = \frac{\rho_A \sigma_A}{\sigma_R} \frac{1}{p^{CM}}. \tag{46}$$

Private benefits only accrue to the controlling shareholder even under complete markets. The productivity shock beta depends inversely on Tobin's $q$. Recall that with CM, the controlling shareholder's $q$ is larger than outside investors' $q$, $p^{CM}(\alpha) > q^{CM}(\alpha)$. Therefore, productivity shock beta for inside equity, $\beta_{A,in}^{CM}$, is lower than $\beta_{A,out}^{CM}$, the productivity shock beta for outside equity, in that $\beta_{A,in}^{CM} < \beta_{A,out}^{CM}$. Intuitively, for a given size of productivity (cash flow) shock, the firm with a higher $q$ is less risky, *ceteris paribus*. In our CM setting, an iid productivity shock is less risky for the controlling shareholder than for outside investors because the former values the firm more than the latter due to imperfect investor protection.

With CM, there is no wedge between marginal $q$ and average $q$ for either inside or outside equity. Therefore, capital shock beta is the same for both types of shareholders.

Therefore, with no idiosyncratic risk, the expected return for the controlling shareholder is lower than the expected return for outside shareholders.

# 8 Idiosyncratic risk premium for the controlling shareholder

With incomplete markets, the controlling shareholder requires both systematic and idiosyncratic risk premia for the significant exposure to illiquid business. We develop an analytically tractable and operational method to calculate the controlling shareholder's idiosyncratic risk premium. The controlling shareholder holds the illiquid business until choosing to exit. Therefore, a natural measure for the controlling shareholder's cost of capital is the internal rate of return (IRR) over the stochastic holding period. Let $\xi$ denote the IRR for the controlling shareholder, which solves

$$P(K_0) = \frac{1}{\alpha}\mathbb{E}\left[\int_0^\tau e^{-\xi t}dM_t + \alpha e^{-\xi\tau}lK_\tau\right], \tag{47}$$

where $\tau$ is the stochastic liquidation time. The right side of (47) is the present value (PV) of cash flows accruing to the controlling shareholder plus the PV from liquidation, discounted at the IRR $\xi$, per unit of ownership. The left side is the controlling shareholder's "private" certainty equivalent value $P(K_0)$. Since the IRR is a function of $K_0$ and we thus denote the IRR $\xi$ as $\xi(K_0)$. We obtain the value of IRR $\xi(K_0)$ by solving an ODE given in the appendix.

The IRR contains both the systematic and the idiosyncratic risk premia. If markets are complete, the cost of capital for inside equity, $\mu_{in}^{CM}$, is given by CAPM,

$$\mu_{in}^{CM} = r + \left(\beta_K^{CM} + \beta_{A,in}^{CM}\right)(\mu_R - r) = r + \left(\frac{\rho_K\sigma_K}{\sigma_R} + \frac{\rho_A\sigma_A}{\sigma_R}\frac{1}{p^{CM}(\alpha)}\right)(\mu_R - r). \tag{48}$$

The idiosyncratic risk premium $\omega(K_0)$, defined as the difference between $\xi(K_0)$ and $\mu_{in}^{CM}$, is given by

$$\omega(K_0) = \xi(K_0) - \mu_{in}^{CM}. \tag{49}$$

There is much debate in the empirical literature about the significance of this *private* equity risk premium. For example, Moskowitz and Vissing-Jorgensen (2002) document the risk-adjusted returns to investing in U.S. nonpublicly traded equity are not higher than the returns to private equity, while Mueller (2011) finds the opposite. Our model
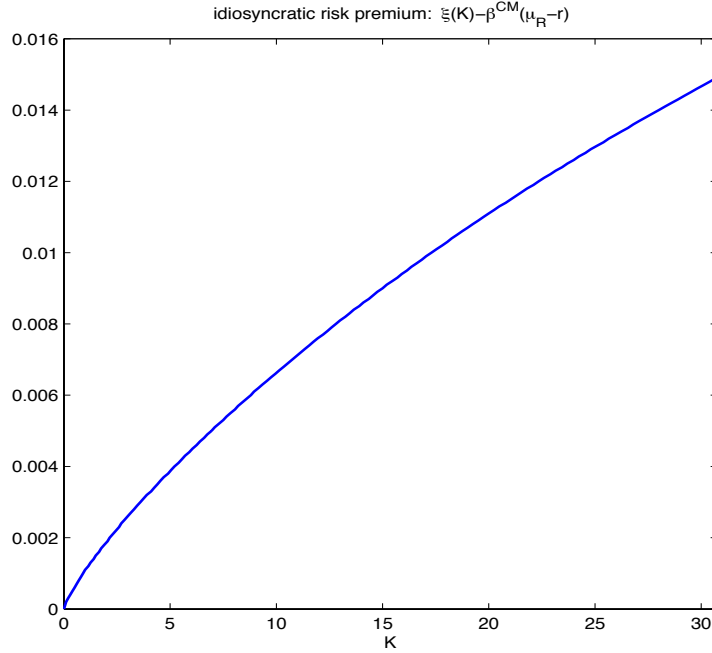
Figure 6: Idiosyncratic risk premium for controlling shareholder.

provides an analytical formula to calculate this private equity idiosyncratic risk premium for controlling shareholders. Figure 6 plots the idiosyncratic risk premium for the controlling shareholder. As $K \to 0$, the idiosyncratic risk premium disappears as the firm vanishes, $\omega(0) = 0$. As firm size increases, the idiosyncratic risk premium also increases because firm-specific risk increases. At the moment of exit, $\overline{K} = 30.8$, the idiosyncratic risk premium approaches 1.5%.

# 9  Endogenous ownership and firm size

We now endogenize the initial firm size $K_0$ and the controlling shareholder's ownership.

## 9.1  Model setup

We assume that the external source of financing is equity as in Shleifer and Wolfenzon (2002). Additionally, we assume that the entrepreneur needs to have a sufficiently high equity stake in the firm to retain full control after equity issuance. Let $\underline{\alpha}$ denote the minimal level of ownership for the entrepreneur to have full control rights. We focus on the economically interesting case where it is optimal for the entrepreneur to hold a

controlling stake in the firm, $\alpha \geq \underline{\alpha}$.[19]

Let $K_e$ and $K_m$ denote the controlling shareholder's and outside shareholders' contribution to the initial firm size $K_0$, respectively. The initial firm size $K_0$ is

$$K_0 = K_e + K_m. \tag{50}$$

Under perfectly competitive capital markets, outside shareholders break even in risk-adjusted present value, which implies

$$K_m = (1 - \alpha) V(K_0; \alpha). \tag{51}$$

Here, we explicitly note the dependence of firm value $V(K)$ on ownership $\alpha$.

Setting up the firm is costly for the entrepreneur. We take a broad interpretation for the setup cost. It can simply be the real setup cost including legal, accounting, and compliance costs. It can also represent indirect costs such as the forgone wages earned elsewhere. Additionally, the setup cost can also relate to the difficulty of raising funds or other barriers to be an entrepreneur.

Let $\Lambda(K_0)$ denote this setup cost as a function of initial firm size $K_0$. Intuitively the larger the firm size $K_0$, the higher the setup cost $\Lambda(K_0)$ and also the higher the marginal setup cost $\Lambda'(K_0)$. While our model applies to any increasing and convex cost function $\Lambda(K)$, for simplicity, we assume

$$\Lambda(K) = \lambda_1 K + \frac{\lambda_2 K^2}{2}, \tag{52}$$

where $\lambda_1 > 0$ and $\lambda_2 > 0$.

Let $X_0$ denote the entrepreneur's total liquid wealth just prior to setting up the firm. At time 0, the entrepreneur invests amount $K_e$ in the firm, raises external equity $K_m$ for the firm, chooses ownership $\alpha \geq \underline{\alpha}$, pays the setup cost $\Lambda(K_0)$, and allocates the remaining amount, $X_0 - K_e - \Lambda(K_0)$, between the risk-free asset and the risky market portfolio to maximize lifetime utility (12) subject to outside shareholders' break-even condition (51) and the accounting identity (50) for initial firm size $K_0$.

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[19]It is conceivable that the entrepreneur's control rights within the firm depend on ownership $\alpha$. In that case, the entrepreneur has an additional tradeoff margin between diversification and the degree of control. We leave this extension for future research.

## 9.2 Solution

In the appendix, we show that the entrepreneur's utility maximization can be simplified to the following optimization problem,

$$\max_{K_0, \, \alpha \geq \underline{\alpha}} W(K_0; \alpha) - (K_0 + \Lambda(K_0)) \,. \tag{53}$$

Here, $W(K)$ is the sum of the controlling shareholder's private valuation, $\alpha P(K)$, and outside investors' public valuation, $(1 - \alpha)V(K)$,

$$W(K; \alpha) = \alpha P(K; \alpha) + (1 - \alpha)V(K; \alpha) \,. \tag{54}$$

We refer to $W(K_0; \alpha) - (K_0 + \Lambda(K_0))$ as the net social surplus of setting up the firm. The entrepreneur's utility maximization is equivalent to the maximization of the net surplus given in (53), because outside investors break even *ex ante* and the controlling shareholder internalizes the net benefits of setting up the firm. The entrepreneur and investors nonetheless anticipate that agency conflicts will arise after the firm is set up.

**Proposition 6** *For a given $\alpha \geq \underline{\alpha}$, firm size $K_0^*$ as a function of $\alpha$, $K_0^*(\alpha)$, solves*

$$W_K(K_0^*; \alpha) = 1 + \Lambda'(K_0^*) \,. \tag{55}$$

*The controlling shareholder's optimal ownership $\alpha^*$ satisfies*

$$W_\alpha(K_0^*(\alpha^*); \alpha^*) \leq 0. \tag{56}$$

*Additionally, if $\alpha^* > \underline{\alpha}$, (56) holds with equality.*

The FOC (55) states that the controlling shareholder's marginal benefit of capital, $W_K(K_0^*; \alpha)$, equals the marginal setup cost, $1 + \Lambda'(K_0^*)$. Inequality (56) states that $W_\alpha(K; \alpha)$ cannot be positive at optimally chosen $K_0^*$. Otherwise, increasing ownership $\alpha$ raises $W(K; \alpha)$, contradicting the controlling shareholder's optimality. Moreover, if the optimal $\alpha$ is interior, $\alpha^* > \underline{\alpha}$, (56) holds with equality. Using the optimal ownership $\alpha^*$ and firm size $K_0^*$, we then obtain the optimal amount of external capital $K_m^*$ via the outside shareholders' break-even condition (51). Next, we choose parameter values.

## 9.3 Parameter choice and calibration

We set the minimal level of ownership for the entrepreneur to retain full control of the firm at $\underline{\alpha} = 20\%$. Various mechanisms such as dual class shares, cross holdings, and pyramidal structure allow the large shareholder with cash flow rights substantially less than a majority position to achieve full control.[20] For the setup cost, we choose $\lambda_1 = 8\%$ and $\lambda_2 = 6\%$. To choose the coefficient of absolute risk aversion $\gamma$, we use the invariance result in the following proposition.

**Proposition 7** *Define $\overline{\gamma} = \gamma X_0$ and $k = K/X_0$, where $X_0$ is the entrepreneur's initial wealth. We have $P(K) = g(k)X_0$, where $g(k)$ solves the following ODE,*

$$
\begin{aligned}
rg(k) &= \left(\nu_A + b(\alpha)\right)k - \delta k g'(k) + \frac{(g'(k) - 1)^2}{2\theta_i}k + \frac{\sigma_K^2 k^2 g''(k)}{2} \\
&\quad - \frac{\alpha \overline{\gamma} r k^2}{2}\left[(1 - \rho_A^2)\sigma_A^2 - 2\rho_A\rho_K\sigma_A\sigma_K g'(k) + (1 - \rho_K^2)\sigma_K^2 g'(k)^2\right],
\end{aligned} \tag{57}
$$

*subject to the following boundary conditions,*

$$
\begin{aligned}
g(0) &= 0, \tag{58} \\
g(\overline{k}) &= l\overline{k}, \tag{59} \\
g'(\overline{k}) &= l. \tag{60}
\end{aligned}
$$

*Here, $\overline{k}$ is the ratio between the optimal exit threshold $\overline{K}$ and initial wealth $X_0$.*

The entrepreneur's certainty equivalent value of business, as a fraction of the initial wealth $X_0$, is $\alpha P(K_0)/X_0 = \alpha g(k_0)$, where $k_0 = K_0/X_0$. Note that $g(k)$ depends only on the product of CARA coefficient $\gamma$ and the entrepreneur's initial wealth $X_0$, $\overline{\gamma} = \gamma X_0$, a proxy for the coefficient of *relative* risk aversion. This property is useful for our calibration and quantitative analysis because researchers often have views on values of *relative* risk aversion, rather than *absolute* risk aversion.

Proposition 7 also implies that the certainty equivalent valuation, $P(K) = g(k)X_0$, is proportional to initial wealth $X_0$, given $k = K/X_0$ and $\overline{\gamma} = \gamma X_0$. To illustrate, for a value of $\overline{\gamma}$, if an entrepreneur with $X_0 = \$200$M values a firm with capital $K_0 = \$100$M at

---

[20]La Porta et al. (1999) use ownership data on large corporations in 27 wealthy countries to show that the controlling shareholders often have power that significantly exceeds their cash flow rights, primarily through the use of pyramids and active managerial participation. Claessens et al. (2000) provide evidence for pyramidal class shares and cross holdings in nine Eastern Asian countries/regions: Hong Kong, Indonesia, Japan, South Korea, Malaysia, the Philippines, Singapore, Taiwan, and Thailand.
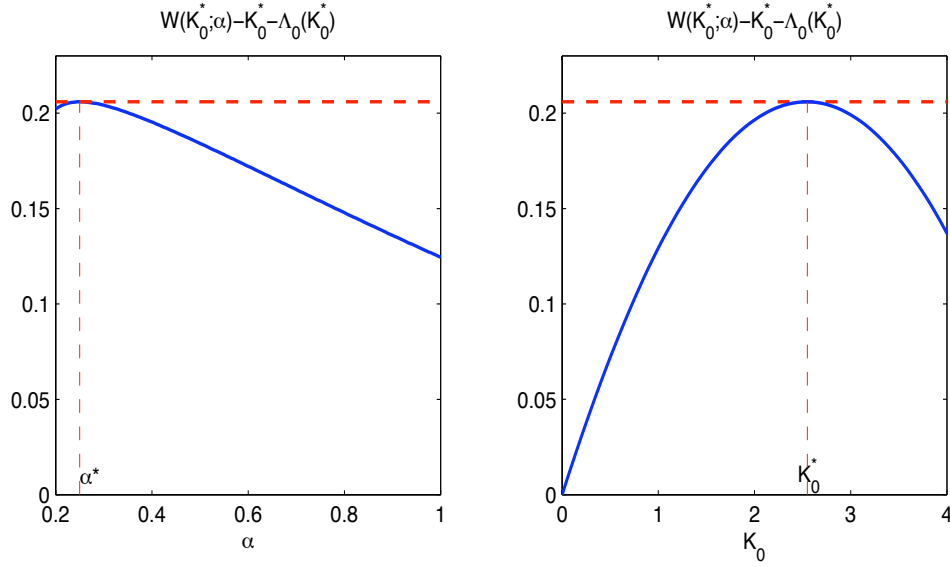
Figure 7: **The controlling shareholder's net surplus,** $W(K_0; \alpha) - K_0 - \Lambda(K_0)$, **ownership $\alpha$, and firm size $K_0$.** Panel A plots net surplus as a function of $\alpha$ by fixing $K_0 = K_0^* = 2.55$. Panel B plots net surplus as a function of $K$ by fixing $\alpha_0 = \alpha_0^* = 25\%$.

$P(K_0) = g(0.5)X_0 = \$120M$, our model implies that this entrepreneur with $X_0 = \$100M$ values the same firm with $K_0 = \$50M$ at $P(K_0) = g(0.5)X_0 = \$60M$. This invariance to the unit of account, in general, does not hold under incomplete markets, but it is a desirable property of our model from a valuation perspective.

We choose the CARA coefficient $\gamma = 2$ based on the following argument. For a given value of $\gamma$, say $\gamma = 2$, we use two empirically motivated identifying assumptions, (1) the entrepreneur's certainty equivalent wealth for the business is about 40% of total assets[21] and (2) the controlling shareholder's inside ownership is $\alpha = 25\%$.[22] The implied value for our proxy of relative risk aversion, $\overline{\gamma} = \gamma X_0 = 2 \times 1.8 = 3.6$, which is within the range of values for the coefficient of relative risk aversion commonly used in quantitative and calibration exercises.[23] All other parameter values are given in Table 3.

[21] Gentry and Hubbard (2004) report that active businesses account for about 41.5% of entrepreneurs' total assets using the survey of consumer finance (SCF). We use 40% for our calibration.

[22] This estimate is within the range reported by La Porta et al. (1999), Claessens et al. (2000), and Dahlquist et al. (2003).

[23] The first condition gives $40\% = \alpha P(K_0^*)/(\alpha P(K_0^*) + X_0 - K_e^* - \Lambda(K_0^*))$. Later in this section, we show that the optimal firm size $K_0^* = 2.55$, inside equity contribution $K_e^* = 0.20$, and the controlling shareholder's private average $q$ is $p(K_0^*) = 1.269$. Based on these numbers, the calibrated value for initial wealth is $X_0 = 1.8$, which implies $k_0 = K_0^*/X_0 = 1.41$ and $g(k_0) = 1.8$.

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## 9.4 Results

The controlling shareholder contributes $K_e^* = 0.20$ as inside equity, and raises a substantially larger amount, $K_m^* = 2.35$, from outside investors. The initial firm size is thus $K_0^* = K_e^* + K_m^* = 2.55$. The break-even condition for outside investors implies that 75% of firm equity belongs to outside investors, and the controlling shareholder retains the remaining 25% equity. The average $q$ for outside equity is $q_a(2.55) = 1.227$. The optimal exit boundary is $\overline{K} = 30.8$. The controlling shareholder internalizes the setup cost, $\Lambda(2.55) = 0.40$. The marginal setup cost is $\lambda_1 + \lambda_2 K_0^* = 0.233$. Evaluated at optimal insider ownership $\alpha^* = 25\%$ and firm size $K_0^* = 2.55$, the controlling shareholder's *net surplus* $W(K_0^*; \alpha^*) - K_0^* - \Lambda(K_0^*)$ equals 0.21.

We now discusses the tradeoff that the controlling shareholder faces when choosing ownership $\alpha$ and firm size $K_0$. In Panel A of Figure 7, we plot the controlling shareholder's net surplus as a function of $\alpha$ by holding firm size fixed at $K_0^* = 2.55$. The net surplus increases with $\alpha$ for $20\% \leq \alpha \leq 25\%$, and then decreases with $\alpha$ for $\alpha \geq 25\%$. Intuitively, for a given $K_0$, the higher the inside ownership, the more capital contribution by the controlling shareholder, hence the better incentive alignment but also the more idiosyncratic risk exposure. The controlling shareholder trades off these two effects, optimally selects $\alpha^* = 25\%$ and collects maximum net surplus of 0.21.

In Panel B of Figure 7, we plot the net surplus as a function of firm size $K_0$, by holding ownership fixed at $\alpha^* = 25\%$. The net surplus increases with $K_0$ for $K_0 \leq 2.55$ and then decreases for $K_0 \geq 2.55$. For a given $\alpha$, the larger firm size $K_0$, the more capital that the controlling shareholder contributes, the larger firm and the better incentive alignment but also the more idiosyncratic risk exposure. The controlling shareholder selects $K_0^* = 2.55$, and obtains a maximum net surplus of 0.21.

**Investor protection, ownership, and firm size.** La Porta et al. (1999) and Claessens et al. (2000) document that ownership is more concentrated under weaker investor protection. La Porta et al. (2000a) and Demirgüç-Kunt and Maksimovic (1998) find that financial markets are smaller and less developed in countries with weaker investor protection. Our model predictions are consistent with these empirical findings. Recall that a lower $\theta_s$ corresponds to weaker investor protection in our model.

Table 1 shows that the optimal insider's ownership $\alpha^*$ decreases with investor protection $\theta_s$. For example, $\alpha^* = 25\%$ when $\theta_s = 350$ and $\alpha^* = \underline{\alpha} = 20\%$ when $\theta_s = 700$.

Table 1: Investor protection, ownership, and firm size

| investor protection $\theta_s$ | 300 | 350 | 400 | 500 | 700 |
|---|---|---|---|---|---|
| ownership $\alpha^*$ | 28% | 25% | 23% | 21% | 20% |
| inside capital $K_e^*$ | 0.28 | 0.20 | 0.14 | 0.05 | 0.02 |
| external capital $K_m^*$ | 2.22 | 2.35 | 2.46 | 2.60 | 2.68 |
| firm size $K_0^*$ | 2.50 | 2.55 | 2.60 | 2.65 | 2.70 |
| net surplus | 0.20 | 0.21 | 0.21 | 0.22 | 0.22 |

The ownership constraint $\alpha \geq \underline{\alpha}$ for full control binds under sufficiently strong investor protection, $\theta_s = 700$. Firm size $K_0^*$ also increases with investor protection $\theta_s$. For example, $K_0^*$ increases by 6% from 2.55 to 2.70 as we increase $\theta_s$ from 350 to 700. While firm size increases with investor protection, the composition between inside capital and outside capital changes. Inside capital $K_e^*$ falls by 90% from 0.20 to 0.02 as we increase $\theta_s$ from 350 to 700, and outside capital, $K_m^*$, increases from 2.35 to 2.68. The net surplus increases approximately by 5% from 0.21 to 0.22 as $\theta_s$ increases from 350 to 700. Our exercises here demonstrate that the value of improving investor protection is quantitatively significant.

**Setup cost, firm size, and welfare.** We broadly interpret the firm's setup cost $\Lambda(K)$ as a measure for the (formal and informal) institutions that facilitate entrepreneurship. This cost can be real setup costs, or indirect costs, such as potential briberies that may have to be paid, to start up a business. We treat the parameters $\lambda_1$ and $\lambda_2$ in the setup cost function $\Lambda(K)$ given in (52) as exogenous. It is conceivable that these parameters may (slowly) change over time and can be endogenous (at least in the long run). Intuitively, the more costly it is to start up a business, the smaller the firm size and the more concentrated the firm ownership.

Panel A in Table 2 holds $\lambda_2 = 6\%$ and varies $\lambda_1$ from 2% to 10%. As $\lambda_1$ increases from 2% to 10%, inside ownership $\alpha^*$ increases from 22% to 26%, inside capital $K_e^*$ increases from 0.16 to 0.21, outside capital decreases significantly by 40% from 3.39 to 2.04. As a result, firm size $K_0^*$ decreases by 37% from 3.55 to 2.25. These changes lead to a 60% decrease of the net surplus from 0.39 to 0.16.

Panel B in Table 2 holds $\lambda_1 = 8\%$ and varies $\lambda_2$ from 2% to 10%. As $\lambda_2$ increases

Table 2: The effects of setup cost $\Lambda(K) = \lambda_1 K + \lambda_2 K^2/2$.

|  | | Panel A. | $\lambda_2 = 6\%$ | | |
|---|---|---|---|---|---|
| $\lambda_1$ | 2% | 4% | 6% | 8% | 10% |
| $\alpha^*$ | 22% | 23% | 24% | 25% | 26% |
| $K_e^*$ | 0.16 | 0.18 | 0.20 | 0.20 | 0.21 |
| $K_m^*$ | 3.39 | 3.02 | 2.70 | 2.35 | 2.04 |
| $K_0^*$ | 3.55 | 3.20 | 2.90 | 2.55 | 2.25 |
| net surplus | 0.39 | 0.32 | 0.26 | 0.21 | 0.16 |
|  | | Panel B. | $\lambda_1 = 8\%$ | | |
| $\lambda_2$ | 2% | 4% | 6% | 8% | 10% |
| $\alpha^*$ | 20% | 22% | 25% | 28% | 31% |
| $K_e^*$ | 0.15 | 0.15 | 0.20 | 0.21 | 0.23 |
| $K_m^*$ | 6.95 | 3.65 | 2.35 | 1.74 | 1.32 |
| $K_0^*$ | 7.10 | 3.80 | 2.55 | 1.95 | 1.55 |
| net surplus | 0.57 | 0.30 | 0.21 | 0.16 | 0.13 |

from 2% to 10%, inside ownership $\alpha^*$ increases from 20% to 31%, inside capital $K_e^*$ increases from 0.15 to 0.23, outside capital decreases significantly by 81% from 6.95 to 1.32! As a result, firm size $K_0^*$ decreases by 78% from 7.10 to 1.55. These changes lead to a 77% decrease of the net surplus from 0.57 to 0.13!

Undoubtedly, increasing the setup cost has an overwhelmingly strong effect on welfare. Naturally, the policy implications are clear. Ideally, we should find ways to reduce the barrier for entrepreneurship and provide incentives to become an entrepreneur. However, the policy implementation may not be straightforward. For example, decision makers who are in power may not have interest in promoting entrepreneurship. They may be more interested in collecting rents by setting up various barriers to entry.

Finally, it is conceivable that investor protection and setup/entrepreneurship entry costs are correlated and jointly determined in the long run. Institutional variables tend to be slow moving, and can depend on the dynamically changing political economy. We leave these important issues for future studies.

# 10 Agency and time inconsistency

After outside capital is raised, there is an inevitable conflicts of interest between inside and outside equity. We show that the controlling shareholder's motives to pursue private benefits and diversify idiosyncratic business risk make the optimization problem time inconsistent and reflect dynamically evolving conflicts between inside and outside equity.

## 10.1 Full-commitment case: A comparison benchmark

First, we solve the entrepreneur's dynamic optimization under the assumption that the entrepreneur can commit to maximize total surplus $W(K; \alpha)$, given in (54), at all times. We refer to this case as the full-commitment case.

**Proposition 8** *With full commitment, the optimal diversion is $s = 0$, the optimal inside ownership is $\alpha^* = \underline{\alpha}$, and the optimal investment satisfies*

$$i(K) \;=\; \frac{W'(K) - 1}{\theta_i} = \frac{\alpha P'(K) + (1 - \alpha)V'(K) - 1}{\theta_i}\,, \tag{61}$$

*where $P(K)$ and $V(K)$ jointly solve*

$$rP(K) \;=\; (\nu_A - i(K) - \phi(i(K)))\,K + (i(K) - \delta)\,KP'(K) + \frac{\sigma_K^2 K^2 P''(K)}{2}$$
$$-\frac{\alpha\gamma r K^2}{2}\left[(1 - \rho_A^2)\sigma_A^2 - 2\rho_A\rho_K\sigma_A\sigma_K P'(K) + (1 - \rho_K^2)\sigma_K^2 P'(K)^2\right]\,, \tag{62}$$

$$rV(K) \;=\; (\nu_A - i(K) - \phi(i(K)))\,K + (i(K) - \delta)\,KV'(K) + \frac{\sigma_K^2 K^2 V''(K)}{2}\,, \tag{63}$$

*subject to the following boundary conditions,*

$$P(0) \;=\; V(0) \;=\; 0\,, \tag{64}$$
$$P(\overline{K}) \;=\; V(\overline{K}) \;=\; l\overline{K}\,, \tag{65}$$
$$W'(\overline{K}) \;=\; l\,. \tag{66}$$

Diversion transfers resources from outside investors to the controlling shareholder but at a cost, $\theta_s > 0$. With full commitment, the controlling shareholder optimally chooses not to divert, $s = 0$, which maximizes $W(K)$, the sum of the controlling shareholder's private valuation and outside investors' public valuation. The optimal ownership is the minimal level to retain full control, $\alpha^* = \underline{\alpha}$, because $V(K) < P(K)$. The investment policy is selected to equate the marginal cost of investing $1 + \theta_i\,i$ with $W'(K)$, as given

in (61). Unlike the ODE for public firm value $V(K)$, the ODE for the controlling shareholder's private valuation $P(K)$ has a nonlinear term (the last one), which reflects the idiosyncratic risk discount for the controlling shareholder.

The boundary condition (64) states that the firm is worthless to both inside and outside equity at $K = 0$. Equation (65) states that both private value $P(K)$ and public value $V(K)$ equal the exit value $lK$ at the exit boundary $\overline{K}$. Finally, (66) states that $W'(K)$ equals the exit value per unit of capital, $l$ at the optimal exit boundary $\overline{K}$.

The full-commitment solution is different from the one for the first-best benchmark. The controlling shareholder is required to hold a minimal $\underline{\alpha}$ in the firm to retain full control. If we do away with this constraint, then with full commitment, the controlling shareholder fully diversifies and achieves first-best allocation of resources.

## 10.2 Time inconsistency, investment, and liquidation

Figure 8 compares the optimal investment-capital ratio $i(K)$ under full commitment with $i(K)$ under no commitment. We fix $\alpha = \underline{\alpha} = 20\%$ for the comparison because the ownership constraint $\alpha \geq \underline{\alpha}$ binds under full commitment.

First, without commitment, the entrepreneur over-invests for $K \leq 3.0$ and under-invests for $K \geq 3.0$. For a small firm, the private benefits motive dominates under-diversification concern, causing over-investment. For a sufficiently large firm, the under-diversification concern becomes sufficiently strong and the controlling shareholder under-invests. With full commitment, the entrepreneur pursues no private benefits and only has an incentive to under-invest in order to reduce under-diversification.

Second, with commitment, the controlling shareholder only liquidates the firm as $K$ reaches $\underline{K} = 116.2$. In contrast, without commitment, liquidation occurs as firm size $K$ reaches $\underline{K} = 42.7$, which is much earlier than that under the full commitment case. The firm survives much longer under commitment as the agency conflict is much less severe.

The controlling shareholder's inability to commit implies a significant value loss measured in net surplus, $W(K^*; \alpha) - \Lambda(K^*)$. The net surplus decreases by 13% from 0.24 under full commitment to 0.21 under no commitment.

# 11 Conclusion

Many firms including large publicly traded ones around the world are run by controlling shareholders. By using pyramidal structure, cross holdings, dual class shares, and other
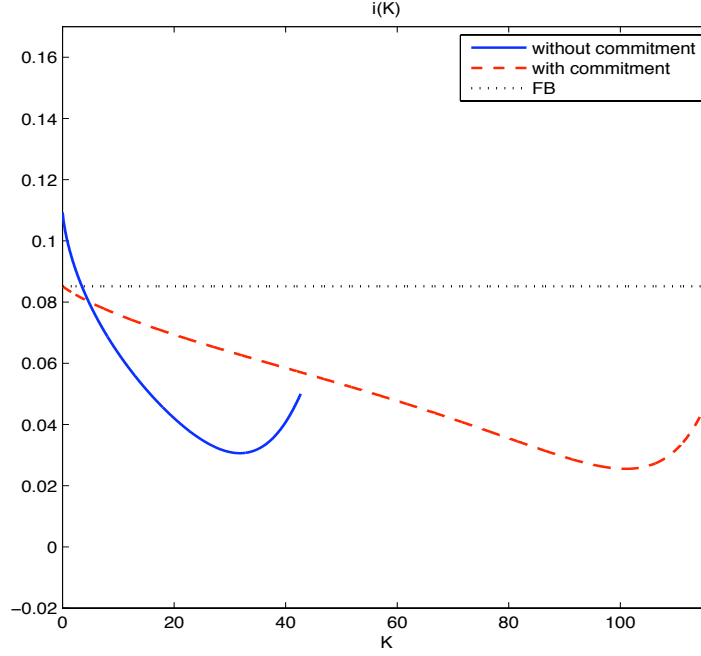
Figure 8: Investment under full commitment and no commitment when $\alpha = 20\%$

mechanisms, they obtain control rights which significantly exceed their cash-flow rights. With imperfect investor protection, controlling shareholders extract private benefits and choose non-value maximizing investment decisions. Business ownership concentration inevitably exposes substantial idiosyncratic risks to controlling shareholders, because they cannot fully diversify idiosyncratic business risks under incomplete markets.

We introduce investor protection and incomplete markets, two key frictions, into a tractable dynamic framework where the controlling shareholder makes *interdependent* consumption-saving, portfolio choice between a risky asset and a risk-free asset, private benefits, corporate investment, and business cash-out/exit timing decisions. Our model builds on the seminal Hayashi (1982) model which implies that investment-capital ratio, Tobin's average $q$, and marginal $q$ are all constant without frictions.

The weaker investor protection, the more private benefits the controlling shareholder pursues. Additionally, the larger the firm size, the more private benefits are pursued. In a dynamic framework with imperfect investor protection, such as Albuquerque and Wang (2008) and ours, the controlling shareholder naturally has incentives to over-invest. In contrast, incomplete markets discourage the controlling shareholder from investing in the firm because a larger firm implies a larger under-diversification cost for the control-

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ling shareholder, *ceteris paribus*. We show that the controlling shareholder over-invests when the firm is small and under-invests when the firm becomes large. Moreover, the investment-capital ratio is non-monotonic in firm size. Under-diversification causes the controlling shareholder to decrease the investment-capital ratio as firm size increases. However, as firm size continues to increase and the exit option becomes sufficiently in the money, the controlling shareholder chooses to increase investment due to the option value of exit.

In terms of valuation, we obtain analytically tractable formulas for the controlling shareholder's private marginal $q$ and private average $q$ as well as outside investors' marginal $q$ and average $q$. We show that the controlling shareholder's private marginal $q$ determines corporate investment. The frictions drive a wedge between the controlling shareholder's marginal $q$ and average $q$, as well as a wedge between the firm's public average $q$ and public marginal $q$.

Using the firm's average $q$ and marginal $q$, we have a simple formula to calculate the conditional beta and the cost of capital for outside equity. Conditional CAPM holds in our model. While conflicts of interest clearly lower firm value, the agency impact on the cost of capital is much less clear. We show that agency can make the cost of capital either higher or lower than the first-best benchmark, depending on firm size. Our model can be used to provide guidance for empirical work on corporate governance and cross-sectional returns.[24]

We further endogenize ownership and the initial firm size. We show that firm size is larger and ownership is less concentrated under weaker investor protection. We also study the effect of the firm's startup cost on entrepreneurship. We find that the startup cost has a significant effect on firm size, ownership concentration, and social welfare.

After outside equity is raised, conflicts of interest arise due to time inconsistent behavior of the controlling shareholder. We construct a benchmark where the controlling shareholder fully commits to maximize the sum of inside private equity and outside public equity valuation. Using this full-commitment case as the comparison benchmark, we quantify the agency cost between inside and outside equity, and find significant effects on corporate investment and liquidation decisions, as well as overall welfare.

Our model generates predictions that are broadly consistent with existing empirical findings, which include higher private benefits, smaller dividend payout, lower firm value, more concentrated corporate ownership, smaller and less developed financial markets,

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[24]See Gompers, Ishii, and Metrick (2003) for an influential empirical study using US data.

and smaller firm size under weaker investor protection, *ceteris paribus*. Moreover, our model generates rich predictions on time-varying investment dynamics purely driven by the two frictions, investor protection and incomplete markets, rather than changing investment opportunities. Quantitatively and conceptually, we show that these two frictions interactively influence corporate investment and valuation.

We view our work as a first-step towards building a dynamic theory of controlling shareholders' decision making and firm valuation for both inside and outside investors. We intentionally choose our model specifications in a way to strike a balance between modeling the richness of the key frictions in the real world and keeping the model sufficiently tractable. Our model's predictions on corporate investment, the controlling shareholder's private average $q$ and private marginal $q$, as well as public average $q$ and marginal $q$ for outside investors are novel and potentially empirically testable.

In our model, inside equity ownership is chosen once and then permanently fixed when the firm is set up. We plan to incorporate dynamic ownership management into our incomplete-markets-based $q$-theory framework.[25] A critical issue in firms run by controlling shareholders is the succession of power.[26] We plan to incorporate this important decision into our dynamic framework in future work.

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[25]See DeMarzo and Urosevic (2006) for a model of ownership dynamics. Leland and Pyle (1977) is a classic (static) signaling model in Corporate Finance where the risk-averse entrepreneur with a higher quality of the project holds a more concentrated ownership.

[26]Burkart, Panunzi, and Shleifer (2003) develop a model of family firms.

Table 3: Summary of Key Variables and Parameters

This table summarizes the symbols for the key variables used in the model and the parameter values.

| Variable | Symbol | Parameter | Symbol | Value |
|---|---|---|---|---|
| Productivity shock | $dA$ | Depreciation rate | $\delta_K$ | 8% |
| Capital stock | $K$ | Expected output rate | $\mu_A$ | 22.76% |
| Business investment | $I$ | Volatility of output | $\sigma_A$ | 25% |
| Capital adjustment cost | $\Phi$ | Volatility of capital stock | $\sigma_K$ | 20% |
| Diversion cash-capital ratio | $s$ | Capital adjustment cost | $\theta_i$ | 3 |
| The controlling shareholder's diversion cost | $\Psi$ | Investor protection parameter | $\theta_s$ | 350 |
| The controlling shareholder's private value | $P$ | Risk-free rate | $r$ | 5% |
| The controlling shareholder's value function | $F$ | Market portfolio expected return | $\mu_R$ | 11% |
| Cash flow to controlling shareholder | $dM$ | Market portfolio volatility | $\sigma_R$ | 20% |
| Incremental return of market portfolio | $dR$ | Market portfolio Sharpe ratio | $\eta$ | 30% |
| The controlling shareholder's liquid wealth | $X$ | Correlation(productivity/market) | $\rho_A$ | 50% |
| Firm value | $V$ | Correlation(capital/market) | $\rho_K$ | 50% |
| Payout to outside shareholders | $dY$ | Risk-adjusted capital depreciation rate | $\delta$ | 11% |
| Utility function | $U$ | Risk-adjusted expected output rate | $\nu_A$ | 19% |
| Social welfare | $W$ | Ownership of equity | $\alpha$ | 25% |
| Consumption | $C$ | Cash-out parameter | $l$ | 1.15 |
| Portfolio allocation | $\Pi$ | Risk aversion | $\gamma$ | 2 |
| Public Tobin's average $q$ | $q_a(K)$ | Minimal ownership requirement for full control | $\underline{\alpha}$ | 20% |
| Public Tobin's marginal $q$ | $q_m(K)$ | Firm's setup cost parameters | $\lambda_1$ | 8% |
| Controlling shareholder's average $q$ | $p(K)$ | Firm's setup cost parameters | $\lambda_2$ | 6% |
| Firm's setup cost | $\Lambda(K)$ | | | |

# Appendices

## A    Technical details

**For Theorem 1.**    Let $F(X, K)$ denote the controlling shareholder's value function. The controlling shareholder chooses consumption $C$, investment $I$, diversion $s$, and allocation to the market portfolio $\Pi$ to solve the following Hamilton-Jacobi-Bellman (HJB) equation,

$$\zeta F = \max_{C,\Pi,I,s} U(C) + (I - \delta_K K)F_K + \frac{(\sigma_K K)^2}{2}F_{KK} + \rho_K \sigma_K \sigma_R K\Pi F_{KX}$$
$$+ [rX + \Pi(\mu_R - r) - C + \alpha(\mu_A K - I - \Phi(I, K)) + (1 - \alpha)sK - \Psi(s, K)]F_X$$
$$+ \frac{(\alpha\sigma_A K)^2 + 2\rho_A \sigma_A \sigma_R \alpha K\Pi + \sigma_R^2 \Pi^2}{2}F_{XX}. \tag{A.1}$$

The controlling shareholder's optimality implies that $\zeta F(X, K)$ is equal to the sum of utility flow $U(C)$ and the expected change of $F(X, K)$ given by the remaining terms.

The controlling shareholder's optimal diversion $s$, as a fraction of capital stock, has the following first-order condition (FOC),

$$\Psi_s(s, K) = (1 - \alpha)K, \tag{A.2}$$

which equates the marginal cost of diverting with the marginal benefit of doing so. For a quadratic diversion cost as given in (4), we have a simple diversion rule

$$s(\alpha) = \frac{1 - \alpha}{\theta_s}. \tag{A.3}$$

The FOC for corporate investment $I$ is given by

$$1 + \Phi_I(I, K) = \frac{F_K(X, K)}{\alpha F_X(X, K)}. \tag{A.4}$$

For a well diversified investor, the marginal cost of investing is $1 + \Phi_I(I, K)$. However, from the non-diversified controlling shareholder's perspective, the marginal cost of investing is $\alpha(1 + \Phi_I(I, K))F_X(X, K)$ after we adjust for ownership $\alpha$ and the controlling shareholder's under-diversification. The marginal benefit of investing is $F_K(X, K)$. Therefore, at optimality, we have the FOC (A.4). The FOC for consumption $C$ is

$$U'(C) = F_X(X, K), \tag{A.5}$$

which equates the marginal utility of consumption $U'(C)$ with the marginal value of wealth $F_X(X, K)$. The FOC for the market portfolio allocation $\Pi$ implies

$$\Pi = -\frac{\eta}{\sigma_R} \frac{F_X(X, K)}{F_{XX}(X, K)} - \frac{\rho_A \sigma_A}{\sigma_R} \alpha K - \frac{\rho_K \sigma_K}{\sigma_R} \frac{K F_{KX}(X, K)}{F_{XX}(X, K)}, \tag{A.6}$$

where the first term gives the mean-variance demand, and the last two terms give the dynamic hedging demand against productivity and capital shocks, respectively.

At the instant of exit, the controlling shareholder's value function is continuous,

$$F(X, K) = F_0(X + \alpha l K), \tag{A.7}$$

where the left side and the right side of (A.7) correspond to the controlling shareholder's pre-exit and post-exit value functions, respectively. The value-matching condition (A.7) defines the exit boundary for capital stock as a function of liquid wealth $X$, $\overline{K} = \overline{K}(X)$.

Because the exit boundary $\overline{K}(X)$ is optimally chosen by the controlling shareholder, we have the following smooth-pasting conditions along both $X$ and $K$ margins:

$$F_X(X, \overline{K}(X)) = F_0'(X + \alpha l \overline{K}(X)), \tag{A.8}$$

$$F_K(X, \overline{K}(X)) = \alpha l F_0'(X + \alpha l \overline{K}(X)). \tag{A.9}$$

The smooth-pasting conditions (A.8) and (A.9) state that both the controlling shareholder's marginal utility of wealth $F_X$ and the marginal value of capital $F_K$ are continuous at the instant of exit. In general, the controlling shareholder's wealth enters as a state variable in addition to capital $K$. However, with CARA utility, we may simplify the two-dimensional mixed control/stopping problem with free boundaries into a one-dimensional problem.

We conjecture that the controlling shareholder's value function is given by (23). Substituting the value function (23) into the FOCs (A.4), (A.5), and (A.6) for investment, consumption, and portfolio, respectively, we may express the firm's investment-capital ratio $i(K)$, consumption rule $C(X, K)$, and portfolio allocation rule $\Pi(K)$, in terms of the controlling shareholder's certainty equivalent valuation for the firm, $P(K)$. Substituting the consumption rule (30), portfolio allocation rule (31), and value function (23) in the HJB equation (A.1) and simplifying, we have

$$rP(K) = (\nu_A + b(\alpha) - i - \phi(i)) K + (i - \delta)K P'(K) + \frac{\sigma_K^2 K^2 P''(K)}{2}$$
$$- \frac{\alpha \gamma r K^2}{2} \left[ (1 - \rho_A^2)\sigma_A^2 - 2\rho_A \rho_K \sigma_A \sigma_K P'(K) + (1 - \rho_K^2)\sigma_K^2 P'(K)^2 \right], \tag{A.10}$$

where $\nu_A = \mu_A - \rho_A \eta \sigma_A$ and $b(\alpha) = (1-\alpha)^2/(2\theta_s \alpha)$.

Substituting the pre-exit and the post-exit value functions (23) and (21) into he controlling shareholder's value-matching condition (A.7) and the smooth-pasting conditions (A.8) and (A.9), we obtain (27) and (28) in terms of the certainty equivalent $P(K)$.

When the adjustment cost function $\phi(i)$ is quadratic as given in (3), we obtain ODE (24) and the optimal investment-capital ratio $i(K)$ can be further simplified as (29).

**For Proposition 1.** For exponential utility, we uncover the first-best benchmark results by taking the limit $\gamma \to 0$. We can also derive the first-best benchmark results by directly solving the complete-markets maximization problem.

**For Proposition 2.** See Merton (1971).

**For Proposition 3.** The proof is quite similar to that for Proposition 1.

**For Proposition 4.** Given the investment-capital ratio $i(K)$ given in (29), the exit boundary $\overline{K}$, and the diversion policy $s(\alpha)$ given in (A.3), we have

$$
\begin{aligned}
dY_t &= (\mu_A - s(\alpha) - i(K) - \phi(i(K))) K_t dt + \sigma_A K_t dZ_t^A \\
&= (\nu_A - s(\alpha) - i(K_t) - \phi(i(K_t))) K_t dt + \sigma_A K_t d\widetilde{Z}_t^A, \quad\quad \text{(A.11)}
\end{aligned}
$$

where $\nu_A = \mu_A - \rho_A \eta \sigma_A$, and $Z_t^A$ and $\widetilde{Z}_t^A$ are standard Brownian motions under physical and risk neutral measures, respectively. Note that

$$
d\widetilde{Z}_t^A = dZ_t^A + \rho_A \eta \sigma_A dt. \quad\quad \text{(A.12)}
$$

The firm's capital stock accumulates as follows,

$$
dK_t = (i(K_t) - \delta) K_t dt + \sigma_K K_t d\widetilde{Z}_t^K, \qu\quad \text{(A.13)}
$$

where $\delta = \delta_K + \rho_K \eta \sigma_K$, and $Z_t^K$ and $\widetilde{Z}_t^K$ are standard Brownian motions under physical and risk neutral measures, respectively. Here, we have

$$
d\widetilde{Z}_t^K = dZ_t^K - \rho_K \eta \sigma_K dt. \quad\quad \text{(A.14)}
$$

For outside investors, firm value $V(K)$ is then given by the present discounted value of all future cash flows under the risk neutral measure, in that

$$
V(K) = \widetilde{\mathbb{E}} \left( \int_0^\infty e^{-rt} d\widetilde{Y}_t \right). \quad\quad \text{(A.15)}
$$

Firm value then solves the ODE (36). At the boundary $\overline{K}$, firm value equals $l\overline{K}$, implying (38). Equation (37) states that firm is worthless at $K = 0$.

**For Propositions 5 and 6.** See the text.

**For Proposition 7.** Let $k = K/X_0$ and $g(k) = P(K)/X_0$, where $X_0$ is the entrepreneur's initial wealth. Substituting $k = K/X_0$ and $g(k) = P(K)/X_0$ into (24), we obtain ODE(57) for $g(k)$. Substituting $g(k) = P(K)/X_0$ into the boundary conditions (26)-(28) gives (58)-(60).

**For Proposition 8.** Because diversion is purely costly and lowers $W(K)$, the optimal diversion is $s = 0$. Using the pricing formulas (62) and (63) for inside equity and outside equity, we have the following equation for $W(K)$,

$$
\begin{aligned}
rW(K) &= (\nu_A - i(K) - \psi(s) - \phi(i(K))\,K + (i(K) - \delta)\,KW'(K) + \frac{\sigma_K^2 K^2 W''(K)}{2} \\
&\quad - \frac{\alpha^2 \gamma r K^2}{2}\left[(1 - \rho_A^2)\sigma_A^2 - 2\rho_A\rho_K\sigma_A\sigma_K P'(K) + (1 - \rho_K^2)\sigma_K^2 P'(K)^2\right] (A.16)
\end{aligned}
$$

Maximizing $W(K)$ gives the FOC (61) for $i(K)$. Simplifying the value matching and smooth pasting conditions for $W(K)$ at $\overline{K}$, we have (64)-(66).

**A valuation equation for the idiosyncratic risk premium.** For a given constant value of $\xi$, we denote $\widetilde{P}(K_t; \xi)$ the following present value,

$$
\widetilde{P}(K_t; \xi) = \frac{1}{\alpha}\mathbb{E}\left[\int_t^\tau e^{-\xi(v-t)}dM_v + \alpha e^{-\xi(\tau-t)}l\overline{K}_\tau\right]. \tag{A.17}
$$

Using the standard martingale representation, we have the following ODE for $\widetilde{P}(K; \xi)$,

$$
\xi\widetilde{P}(K) = (\mu_A + b(\alpha) - i(K) - \phi(i(K)))\,K + (i(K) - \delta_K)\,K\widetilde{P}'(K) + \frac{\sigma_K^2 K^2 \widetilde{P}''(K)}{2}, \tag{A.18}
$$

subject to the following boundary conditions,

$$
\begin{aligned}
\widetilde{P}(0) &= 0, &\text{(A.19)} \\
\widetilde{P}(\overline{K}) &= l\overline{K}. &\text{(A.20)}
\end{aligned}
$$

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