What Drives Anomaly Returns?

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Abstract

We decompose the returns of five well-known anomalies into cash flow and discount rate news. Common patterns emerge across the five factor portfolios and their mean-variance efficient (MVE) combination. Whereas discount rate news predominates in market returns, systematic cash flow news drives the returns of anomaly portfolios and their MVE combination with the market portfolio. Anomaly cash flow and discount rate shocks are largely uncorrelated with market cash flow and discount rate shocks and business cycle fluctuations. These rich empirical patterns restrict the joint dynamics of firm cash flows and the pricing kernel, thereby informing models of stocks' expected returns.

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1 Introduction

Researchers in the past 30 years have uncovered robust patterns in stock returns that contradict classic asset pricing theories. A classic example is that value stocks outperform growth stocks, even though these stocks are similarly exposed to fluctuations in the overall stock market. To exploit such anomalies, investors can form long-short portfolios (e.g., long value and short growth) with high average returns and near-zero market risk. These long-short anomaly portfolios are an important part of the mean-variance efficient (MVE) portfolio and thus the stochastic discount factor (SDF). In the five-factor Fama and French (2015) model, non-market factors account for 85% of the variance in the model's implied SDF.[1]

Researchers sharply disagree about the source of these non-market factors. Several different models, both risk-based and behavioral, can explain why long-short portfolios based on valuation ratios and other characteristics earn high average returns.[2] In this paper, we introduce an efficient empirical technique for decomposing anomaly portfolio returns, as well as their mean-variance efficient (MVE) combination, into cash flow (CF) and discount rate (DR) shocks (news) as in Campbell (1991). These decompositions provide a wide array of new facts that guide specifications of asset pricing theories.

To see how this CF-DR decomposition relates to theories, consider at one extreme the model of noise trader risk proposed by De Long et al. (1990). In this model, firm cash flows are constant, implying that all return variation arises from changes in discount rates. At the other extreme, consider the simplest form of the Capital Asset Pricing Model (CAPM) in which firm betas, the market risk premium, and the risk-free interest rate are constant. In this setting, expected returns (discount rates) are constant, implying that all return variation arises from changes in expected cash flows. More generally, models that explain how firm characteristics like book-to-market (BM) or investment are related to expected firm returns

[1] Using data from 1963 to 2017, a regression of the mean-variance efficient combination of the five Fama-French factors on the market factor yields an R^2 of 15%.

[2] See, e.g., Barberis, Shleifer, and Vishny (1998), Berk, Green, and Naik (1999), Hong and Stein (1999), Daniel, Hirshleifer, and Subrahmanyam (2001), Zhang (2005), Lettau and Wachter (2007), and Kogan and Papanikolaou (2013).

have implications for the joint distribution of firm cash flows and the pricing of these cash flows. Applying our empirical methodology to simulated data from any such theory provides a test of whether the model matches the empirical properties of CF and DR shocks to anomaly portfolios and their MVE combination.

Our empirical work focuses on the annual returns of five well-known anomalies—value, size, profitability, investment, and momentum—from 1929 to 2017. We uncover three sets of novel findings for theories to explain. First, for all five anomalies, CF news explains most (64% to 80%) of the variation in anomaly returns. This finding builds on studies by Cohen, Polk, and Vuolteenaho (2003) and Campbell, Polk, and Vuolteenaho (2010; hereafter CPV) that find cash flows explain most of the variance in the returns of the value anomaly. It also builds on the study by Fama and French (1995) that shows that portfolios formed on size and value experience systematic shocks to earnings. We find that such systematic earnings shocks occur not only in size and value factor portfolios but also in profitability, investment, and momentum portfolios. Moreover, unlike Fama and French (1995), we are able to explicitly link systematic shocks to firms' earnings to the returns of the anomaly portfolios. To evaluate implications for the SDF, we combine all five anomalies into an MVE anomaly portfolio and still find that CF shocks explain most (73%) of this MVE portfolio's return variance. This finding contrasts with the stylized fact that DR shocks explain most of market return variance (see, e.g., Campbell (1991) and Cochrane (2011))—a fact that we replicate. The CF shock to the anomaly MVE portfolio represents a large and common source of variation in firms' CF shocks that spans anomaly boundaries, as opposed to the Vuolteenaho (2002; hereafter V02) conclusion that "cash-flow information is largely firm specific" (page 259).

Our second main result is that the CF and DR components in anomaly returns exhibit only weak correlations with the corresponding components in market returns. Conceptually, there are four correlations of interest between anomaly and market CF and DR components, all of which affect an anomaly's market beta. The correlations between market cash flows and the five anomaly cash flows range from −0.22 to 0.13. We can reject the hypothesis that CF shocks to the MVE portfolio consisting of all five anomalies are positively correlated (above 0.11) with market CF shocks, indicating that the anomaly MVE portfolio and the market portfolio are exposed to distinct fundamental risks. In addition, we estimate that the correlation between anomaly MVE DR news and market DR news is just 0.06 (SE = 0.12).

Our third finding is that, for most anomalies, CF and DR shocks are negatively correlated. That is, firms with negative news about future cash flows tend to experience persistent increases in discount rates. This association contributes significantly to return variance in anomaly portfolios. A notable caveat is that this result applies to anomaly portfolios based on stocks with market capitalizations that are not in the bottom quintile of New York Stock Exchange (NYSE) stocks, which roughly corresponds to excluding stocks popularly known as microcaps. In an alternative specification that includes microcaps, these stocks exert a large influence on some findings because they are numerous and have volatile characteristics and returns. Although our first two findings are essentially unchanged in this alternative specification, the correlation between anomaly CF and DR news becomes positive.[3]

Our main findings cast doubt on three types of theories of anomalies. First, theories in which DR news is the primary source of anomaly returns, such as De Long et al. (1990), are inconsistent with the evidence that CF news predominates over returns. The main reason that anomaly portfolios' returns are volatile is that CF shocks are highly correlated across firms with similar characteristics. For example, the long-short investment portfolio is volatile mainly because the cash flows of a typical high-investment firm are more strongly correlated with the cash flows of other high-investment firms than with those of low-investment firms. The small variance of anomaly DR news does not imply small variance in the conditional expected returns to anomaly portfolios. Indeed, we find substantial variation in anomalies'

[3]At the firm level, our results with and without microcaps are consistent with the finding in V02 that the correlation between cash flows and discount rates is highest for the smallest firms and the finding in Mendenhall (2004) that post-earnings-announcement drift is concentrated in the smallest firms.

one-year expected returns, consistent with, e.g., Haddad, Kozak, and Santosh (2018). However, because this expected return variation is not highly persistent, it has a small impact on stock prices and thus realized anomaly returns.

Second, theories that emphasize commonality in discount rates, such as theories of time-varying risk aversion (e.g., Santos and Veronesi (2010)) and theories of common investor sentiment (Baker and Wurgler (2006)), are difficult to reconcile with the low correlations between anomaly and market discount rate shocks. Third, theories in which anomaly CF news is strongly correlated with market CF news—i.e., cash flow beta stories such as Zhang (2005)—are inconsistent with empirical correlations that are close to zero. To investigate other sources of commonality predicted by these theories, we relate components of anomaly returns to measures of macroeconomic activity, such as consumption growth, and proxies for time-varying risk aversion, such as the default spread, and investor sentiment. We find little evidence of a relation between anomaly return components (CF or DR) and measures of macroeconomic activity and only weak relationships between anomaly DR shocks and proxies for risk aversion and sentiment.

In contrast, some theories of firm-specific biases in information processing and theories of firm-specific changes in risk are consistent with our three main findings. Such theories include behavioral models in which investors overreact to news about firms' long-run cash flows (e.g., Daniel, Hirshleifer, and Subrahmanyam (2001)) and risk-based models in which firm risk increases after negative news about long-run cash flows (e.g., Kogan and Papanikolaou (2013)). In these theories, DR shocks amplify the effect of CF shocks on returns, consistent with the negative empirical DR-CF correlation in our main specification. These theories are also consistent with low correlations between anomaly return components and market return components. As noted above, for microcaps we instead find that the CF and DR correlation is positive. This suggests that for the smallest firms, underreaction to CF news or positive correlation between firm risk and CF news is the dominant force, in contrast to the theories mentioned above. Thus, theories that aim to understand the cross-section of firms' cash flows and returns should clearly state how they apply to firms of different sizes.

Our approach builds on the log-linear approximation of stock returns in Campbell and Shiller (1988). Campbell (1991) uses this approximation to decompose overall stock market returns into CF and DR components, while V02 decomposes individual firms' returns. We directly estimate firms' DR shocks using an unbalanced panel vector autoregression (VAR) in which we impose the present-value relation to derive CF shocks. Unlike most prior work, we analyze the implications of our firm-level estimates for priced (anomaly) factor portfolios to investigate the fundamental drivers of these factors' returns. The panel VAR, as opposed to a time-series VAR for each anomaly portfolio, fully exploits information about the cross-sectional relation between shocks to characteristics and returns. Our panel approach allows us to consider more return predictors, substantially increases the precision of the return decomposition, and mitigates small-sample issues.[4] Motivated by Chen and Zhao's (2009) finding that VAR results can be sensitive to variable selection, we show that our return decompositions are robust across many different specifications.

The V02 study finds that CF news is the main driver of firm-level returns, which we confirm in our sample. V02 further argues that DR news is the main driver of market-level returns, which we also confirm. Cohen, Polk, and Vuolteenaho (2003) and CPV use various approaches to argue that CF news is the main driver of returns on the long-short value-minus-growth portfolio, consistent with our findings for value. Our study is unique in that we analyze multiple anomalies along with the market and most importantly the MVE portfolio, enabling us to uncover robust patterns across anomalies and the MVE portfolio.

Lyle and Wang (2015) estimate the discount rate and cash flow components of firms' BM ratios by forecasting one-year returns using return on equity and BM ratios. They focus

[4]More subtly, inferring CF and DR shocks directly from a VAR estimated using returns and cash flows of rebalanced anomaly portfolios (trading strategies) obfuscates the underlying sources of anomaly returns. Firms' weights in anomaly portfolios can change dramatically with the realizations of stock returns and firm characteristics. In Internet Appendix A, we provide extreme examples in which, for example, firms' expected cash flows are constant but direct VAR estimation suggests that all return variation in the rebalanced anomaly portfolio comes from CF shocks.

on stock return predictability at the firm level and do not analyze the sources of anomaly returns. Our work is also related to studies that use the log-linear approximation of Campbell and Shiller (1988) for price-dividend ratios, typically applied to the market portfolio (see, e.g., Campbell (1991), Larrain and Yogo (2008), and van Binsbergen and Koijen (2010)).

The paper proceeds as follows. Section 2 discusses theories' implications for anomaly cash flows and discount rates and their relationship to our empirical model. Section 3 introduces the data. Section 4 reports and discusses the baseline VAR estimation. Section 5 presents and analyzes decompositions of firm- and portfolio-level returns into CF and DR news. Section 6 shows robustness tests and discusses how and why our results differ from those in earlier studies. Section 7 interprets the results and highlights insights into asset pricing models. Section 8 concludes.

2 Theory

Empirical research identifies several asset pricing anomalies in which firm characteristics, such as profitability and investment, predict firms' stock returns even after controlling for market beta. Modern empirical asset pricing models therefore postulate multiple factors (e.g., Fama and French (1993, 2015) and Carhart (1997)), including non-market factors defined as long-short portfolios sorted on firm characteristics.

In this paper, we decompose the returns to long-short anomaly portfolios and their MVE combination into updates in expectations of future cash flows, CF news, and updates in expectations of future returns, DR news. The MVE combination of pricing factors is of interest as shocks to this portfolio's return are proportional to shocks to the SDF M_t (e.g., Roll (1976) and Cochrane (2005)):

$$M_t - E_{t-1}[M_t] = b(R_{MVE,t} - E_{t-1}[R_{MVE,t}]), \quad (1)$$

where $R_{MVE,t} = \sum_{h=1}^{H} \omega_h R_{F_h,t}$ is the return to the MVE portfolio at time t, expressed as a linear function of H factor returns ($R_{F_h,t}$), and where $b < 0$. In this interpretation, shocks to the MVE portfolio reflect the risks most highly correlated with marginal utility of the marginal investor, which is linear in M_t.[5] Understanding the properties of CF and DR shocks to the MVE portfolio and its components is therefore informative for all asset pricing models.

2.1 The Return Decomposition

Recall from Campbell (1991) that we can decompose shocks to log stock returns into shocks to expectations of cash flows and returns:[6]

$$r_{i,t+1} - E_t\left[r_{i,t+1}\right] \approx CF_{i,t+1} - DR_{i,t+1}, \tag{2}$$

where

$$CF_{i,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa^{j-1} \Delta d_{i,t+j}, \tag{3}$$

$$DR_{i,t+1} = (E_{t+1} - E_t) \sum_{j=2}^{\infty} \kappa^{j-1} r_{i,t+j}, \tag{4}$$

and where $\Delta d_{i,t+j}$ ($r_{i,t+j}$) is the log of dividend growth (log of gross return) of firm i from time $t + j - 1$ to $t + j$, and κ is a log-linearization constant slightly less than 1.[7] In words, return innovations are due to updates in expectations of future cash flows or future expected returns.

[5] The logic here assumes M_t is spanned by the set of traded assets. More generally, there exists a unique minimum variance SDF that is the projection of investors' SDF onto the space of traded asset payoffs.

[6] The operator $(E_{t+1} - E_t)\, x$ represents $E_{t+1}[x] - E_t[x]$: the update in the expected value of x from time t to $t + 1$. The equation relies on a log-linear approximation of the price-dividend ratio around its sample average.

[7] One can derive a similar decomposition based on earnings instead of dividend growth under the assumptions that clean-surplus accounting holds and there is no net issuance (see Ohlson (1995) and V02). In this case, the relevant cash flow is the log of gross return on equity. The DR shock takes the same form as in Equation (4).

We define anomaly returns as the value-weighted returns of stocks ranked in the highest quintile of a given firm characteristic minus the value-weighted returns of stocks ranked in the lowest quintile. We define anomaly CF news as the CF news for the top quintile portfolio minus the CF news for the bottom quintile portfolio. We similarly define anomaly DR news. In the empirical section, we describe our method in detail. Next we discuss the implications of this decomposition of anomaly and MVE portfolio returns for specific models of the cross-section of stock returns.

2.2 Relating the Decomposition to Anomalies

Theories of anomalies propose that investor beliefs and firm cash flows vary with firm characteristics. The well-known value premium provides a useful illustration. De Long et al. (1990) and Barberis, Shleifer and Vishny (1998) are examples of behavioral models that could explain this anomaly, while Berk, Green, and Naik (1999), Zhang (2005), and Lettau and Wachter (2007) are examples of risk-based explanations.[8]

First, consider a multiple-firm generalization of the De Long et al. (1990) model of noise trader risk. In this model, firm cash flows are constant but stock prices fluctuate because of random demand from noise traders, driving changes in firm BM ratios. Since expectations in Equation (3) are rational, there are no CF shocks in this model. By Equation (2), all shocks to returns are due to DR shocks. The constant cash flow assumption is clearly stylized. However, if one in the spirit of this model assumes that value and growth firms have similar cash flow exposures, the variance of net CF shocks to the long-short portfolio would be small relative to the variance of DR shocks. Thus, an empirical finding that DR shocks explain only a small fraction of return variance to the long-short value portfolio would be inconsistent with this model.

[8]In these risk-based models, the stochastic discount factor is exogenous, potentially consistent with both behavioral and rational investors. The models focus on the cross-section of firms' cash flow dynamics, which together with the exogenous pricing kernel account for the value premium.

Barberis, Shleifer, and Vishny (1998; hereafter BSV) develop a model in which investors overextrapolate from long sequences of past firm earnings when forecasting future firm earnings. A firm that repeatedly experiences low earnings will be underpriced (a value firm) because investors are too pessimistic about its future earnings. The firm will have high expected returns as its average future earnings are better than investors expect. Growth firms will have low expected returns for analogous reasons. In this model, CF and DR shocks are closely linked. Negative CF shocks cause investors to expect low future cash flows. But these irrationally low expectations manifest as positive DR shocks in Equations (3) and (4), which are based on rational expectations. Thus, this theory predicts a strong negative correlation between CF and DR shocks at the firm and anomaly levels.

Berk, Green, and Naik (1999) and Zhang (2005) provide risk-based explanations of the value premium based on firms' dynamic investment decisions. In Zhang's model, persistent idiosyncratic productivity (earnings) shocks by chance make firms into either value or growth firms. Value firms, which have low productivity, have more capital than optimal because of adjustment costs. These firms' values are very sensitive to negative aggregate productivity shocks as they have little ability to smooth such shocks through disinvesting. Growth firms, on the other hand, have high productivity and suboptimally low capital stocks and therefore are not as exposed to negative aggregate shocks. Value (growth) firms' high (low) betas with respect to aggregate shocks justify their high (low) expected returns. Similar to BSV, this model predicts a negative relation between firm CF and DR shocks. Different from BSV, the model predicts that the value anomaly portfolio has CF shocks that are positively related to market CF shocks because value stocks are more sensitive to aggregate technology shocks than growth stocks.

Lettau and Wachter (2007) propose a risk-based explanation of the value premium based on the duration of firms' cash flows. In their model, growth firms are, relative to value firms, more exposed to shocks to market discount rates and long-run cash flows, which are not priced, and less exposed to shocks to short-run market cash flows, which are priced.

This model implies that long-run DR and CF shocks to the value portfolio are negatively correlated with long-run DR and CF shocks to the market, respectively.

2.3 Relating the Decomposition to the Stochastic Discount Factor

Prior studies (e.g., Campbell (1991) and Cochrane (2011)) decompose market returns into CF and DR news. They argue that the substantial variance of market DR news has deep implications for the joint dynamics of investor preferences and aggregate cash flows in asset pricing models. For instance, the Campbell and Cochrane (1999) model relies on strong time-variation in investor risk aversion—i.e., the price of risk—which is consistent with the high variance of market DR news.

The modern consensus is that the MVE portfolio and thus the SDF includes factors other than the market. By the logic above, decomposing MVE portfolio returns into CF and DR news also can inform specifications of asset pricing models. For example, large time-variation in investor risk aversion, as in the Campbell and Cochrane (1999) model, suggests a strong common component in DR shocks across the factor portfolios in the SDF.

All models that feature a cross-section of stocks have implications for the return decomposition of anomaly portfolios and the MVE portfolio. As an example, Kogan and Papanikolaou (2013) propose a model in which aggregate investment-specific shocks, uncorrelated with market productivity shocks that affect all capital, have a negative price of risk. Value and growth firms have similar exposure to market productivity shocks, but growth firms have higher exposure to the investment-specific shock. These two CF shocks are the primary drivers of returns to the MVE portfolio in their economy. Since BM ratios increase with discount rates, their model also implies a negative correlation between CF and DR shocks.

2.4 The Empirical Model

Most theories of anomalies, including those above, apply to individual firms. To test these theories, one must analyze firm-level CF and DR news and then aggregate these shocks into anomaly portfolio news. As we explain in Internet Appendix A, the CF and DR news of rebalanced portfolios, such as the Fama-French value and growth portfolios, depend on the rebalancing process and therefore garble the underlying firms' CF and DR shocks.[9]

We assume that firm-level expected log returns are linear in observable variables (X):

$$E_t[r_{i,t+1}] = \delta_0 + \delta_1' X_{it}^{ma} + \delta_2' X_t^{agg}. \tag{5}$$

Here X_{it}^{ma} is a vector of market-adjusted characteristics, such as the BM ratio of firm i at time t, where we demean each characteristic by its value-weighted average at time t; X_t^{agg} is a vector of aggregate characteristics, such as the value-weighted average BM ratio at time t. The coefficient δ_1 captures cross-sectional variation in expected return related to characteristics, much like characteristic-based portfolio sorts and Fama-MacBeth regressions; δ_2 captures time-series variation in expected return that is common to all stocks. Following CPV and V02, we allow the cross-sectional and time-series relationships between a characteristic and expected return to be different.

To implement the return decompositions, we estimate two separate VAR(1) systems. First, we estimate an aggregate VAR to model dynamics in expected market returns and aggregate characteristics:

$$Z_{t+1} = \mu^{agg} + A^{agg} Z_t + \varepsilon_{t+1}^{agg}, \tag{6}$$

where $Z_t = [r_t^{agg};\ X_t^{agg}]$ is a $K^{agg} \times 1$ vector, ε_{t+1}^{agg} is a vector of conditionally mean-zero shocks, and r_t^{agg} denotes the value-weighted average log return at time t. We compute aggregate DR

[9] In Internet Appendix A, we provide an example of a value-based trading strategy. The underlying firms only experience DR shocks, but the traded portfolio is driven solely by CF shocks as a result of rebalancing.

shocks using the standard VAR formula from Campbell (1991):

$$\begin{aligned} DR_{t+1}^{agg} &= E_{t+1}\sum_{j=2}^{\infty}\kappa^{j-1}r_{t+j}^{agg} - E_t\sum_{j=2}^{\infty}\kappa^{j-1}r_{t+j}^{agg} \\ &= e_1'\kappa A^{agg}\left(I_{K^{agg}} - \kappa A^{agg}\right)^{-1}\varepsilon_{t+1}^{agg}. \end{aligned} \tag{7}$$

Here e_1 is a $K^{agg} \times 1$ column vector with 1 as its first element and zeros elsewhere, $I_{K^{agg}}$ is the $K^{agg} \times K^{agg}$ identity matrix, and $\kappa = 0.95$ as in CPV.

For the cross-section, we estimate a market-adjusted panel VAR:

$$Z_{i,t+1} = \mu^{ma} + A^{ma}Z_{i,t} + \varepsilon_{i,t+1}, \tag{8}$$

where $Z_{it} = [r_{it}^{ma};\ X_{it}^{ma}]$ is a $K^{ma} \times 1$ vector, $\varepsilon_{i,t+1}$ is a vector of conditionally mean-zero shocks, and $r_{it}^{ma} \equiv r_{it} - r_t^{agg}$. Similar to Equation (7), firms' market-adjusted DR shocks are:

$$DR_{i,t+1}^{ma} = \iota_1'\kappa A^{ma}\left(I_{K^{ma}} - \kappa A^{ma}\right)^{-1}\varepsilon_{i,t+1}, \tag{9}$$

where ι_1 is a $K^{ma} \times 1$ column vector with 1 as its first element and zeros elsewhere, and $I_{K^{ma}}$ is the $K^{ma} \times K^{ma}$ identity matrix.

We extract CF shocks from the VARs by combining the present-value Equation (2) for returns and the VAR Equations (7) and (9) for DR shocks:

$$\begin{aligned} CF_{t+1}^{agg} &= r_{t+1}^{agg} - E_t\left[r_{t+1}^{agg}\right] + DR_{t+1}^{agg} \\ &= e_1'\left(I_{K^{agg}} + \kappa A^{agg}\left(I_{K^{agg}} - \kappa A^{agg}\right)^{-1}\right)\varepsilon_{t+1}^{agg}, \end{aligned} \tag{10}$$

$$\begin{aligned} CF_{i,t+1}^{ma} &= r_{i,t+1}^{ma} - E_t\left[r_{i,t+1}^{ma}\right] + DR_{i,t+1}^{ma} \\ &= \iota_1'\left(I_{K^{ma}} + \kappa A^{ma}\left(I_{K^{ma}} - \kappa A^{ma}\right)^{-1}\right)\varepsilon_{i,t+1}. \end{aligned} \tag{11}$$

Thus, we impose the present-value relation when estimating the joint dynamics of firm cash flows and discount rates.

We simply combine the aggregate and market-adjusted return components to obtain firms' total DR and CF shocks as:

$$DR_{it} = DR_t^{agg} + DR_{it}^{ma}, \tag{12}$$

$$CF_{it} = CF_t^{agg} + CF_{it}^{ma}. \tag{13}$$

This two-step approach allows the predictive coefficients in the VAR to be different in the cross section (A^{ma}) and the time series (A^{agg}), which V02 and CPV show is important to match the data.

We analyze CF and DR shocks to five long-short anomaly portfolios. Each of these portfolios takes long (short) positions in the top (bottom) quintile of stocks sorted by one of the five anomaly characteristics. We construct the CF and DR shocks to the long and short portfolios by value-weighting the CF and DR shocks to the firms in these portfolios. We then compute the long-short portfolios' CF and DR shocks as the difference between the long and short legs. Value-weighting the components of log returns is not the same as value-weighting the components of simple returns. We use this procedure to follow V02 and CPV and simplify the interpretation of our results. In Internet Appendix D, we estimate the CF and DR components of simple portfolio returns using a second-order Taylor approximation of a simple return in terms of the corresponding log return's CF and DR components. Table D.1 shows that our main return decomposition results still hold with this approximation of anomalies' simple returns. In the paper, we use the parsimonious value-weighted aggregation of log returns that is consistent with previous literature.

3 Data

We estimate the CF and DR components of returns using data on publicly traded United States (US) stocks from Compustat and the Center for Research on Securities Prices (CRSP) from 1926 through 2017. Our analysis requires panel data on firms' returns, book values, market values, earnings, and other accounting information, as well as time-series data on factor returns, risk-free rates, and price indexes. Because some variables require three years of historical data, our VAR estimation focuses on the period from 1929 through 2017.

We obtain all accounting data from Compustat, except that we add book equity data from Davis, Fama, and French (2000). We obtain data on stock prices, returns, and shares outstanding from CRSP. We obtain one-month and one-year risk-free rate data from one-month and one-year yields of US Treasury Bills, respectively, which are available on Kenneth French's website and the Fama Files in CRSP. We obtain inflation data from the Consumer Price Index (CPI) series in CRSP.

We compute log stock returns in real terms ($lnRealRet$) by subtracting the log of inflation, the log change in the CPI, from the log nominal stock return. We compute annual returns from the end of June to the next end of June to ensure that investors have access to December accounting data prior to the ensuing June-to-June period over which we measure returns.

When computing a firm's BM ratio, we adopt the convention of dividing its book equity by its market equity at the end of the June immediately after the calendar year of book equity. This timing of market equity coincides with the beginning of the stock return period. We compute book equity using Compustat data when available, supplementing it with hand-collected data from Davis, Fama, and French (2000). We adopt the Fama and French (1992) procedure for computing book equity. Market equity (ME) is equal to shares outstanding times stock price per share. We sum market equity across all classes of common stock for each firm. We define $lnBM$ as the log of ratio of book equity to market equity.

We compute several firm characteristics that predict short-term stock returns in historical samples. The size variable in our VAR is the five-year change in log market equity ($d5.lnME$) following Gerakos and Linnainmaa (2018), rather than the level of log market equity, to ensure stationarity. Following Fama and French (2015), profitability is annual revenues minus costs of goods sold, interest expense, and selling, general, and administrative expenses, all divided by book equity from the same fiscal year.[10] We use the log of one plus profitability in the VAR ($lnProf$). Following Cooper, Gulen, and Schill (2008) and Fama and French (2015), investment is proxied by growth in total assets. In the VAR, we use five-year log asset growth as the investment characteristic ($lnInv$), to capture the long-horizon predictability of the investment characteristic as documented by Cooper, Gulen, and Schill (2008). Annual data presents a challenge for measuring the momentum anomaly. In Jegadeesh and Titman (1993), the maximum momentum profits accrue when the formation and holding periods sum to 18 months. Therefore, we construct a six-month momentum variable based on each firm's December to June return. The subsequent holding period implicit in the VAR is the 12 months from June to the following June. We transform each measure by adding one and taking its log, resulting in the following variables: $d5.lnME$, $lnProf$, $lnInv$, and $lnMom6$. When forming value-weighted portfolios, we compute value weights as a firm's ME divided by the total ME for all firms in the portfolio. In a robustness specification, we include the log of one plus return on equity (ROE), $lnROE = \ln(1 + ROE)$, where ROE is earnings divided by lagged book equity.

To compute variables as early as 1929 as in CPV, we use proxies for ROE, profitability, and investment when these items are unavailable, which is common before 1962. To impute missing ROE data, we assume earnings are equal to clean surplus earnings, which we define as the change in book equity plus net payouts to stockholders. Net payout is dividends and share repurchases minus issuance from Compustat. If any of these are missing, we set them to zero. For missing profitability data, we assume profitability equals $0.2 + 0.5ROE$, where the intercept and slope coefficients are based on the approximate linear relationship from the period in which both items are available.[11] For missing investment data, we assume investment equals the growth in book equity, which equals growth in assets if leverage is constant.

[10] Novy-Marx (2013) defines profitability with a denominator of total assets, not book equity.

[11] Before 1962 when profitability data become available, we cannot distinguish between ROE and profitability. Since we do not use ROE in our main analysis, this issue primarily affects our robustness analysis based on accounting ROE ($lnROE$) in Section 6.

If a firm goes bankrupt and its stock price is zero, its gross return is zero, which means its log return is undefined. Therefore, following V02, we analyze pseudo-firms in each year that are portfolios with a 10% weight in the real risk-free rate and a 90% position in the firms' stocks. We adjust the pseudo-firms market-to-book, ROE, and other firm characteristics accordingly. The results are not sensitive to small variation in the portfolio weight that defines pseudo-firms.

We impose sample restrictions to ensure the availability of high-quality accounting and stock price information. We exclude firms with negative book values and include only firms with nonmissing market equity data at the end of the most recent calendar year. In our main results, we also exclude firms in the bottom quintile of the prior year's distribution of NYSE market capitalization, which correspond to microcaps as defined by the SEC.[12] We exclude microcaps in order to estimate VAR coefficients using the most economically important stocks and those with high-quality data. Microcaps account for 1% to 2% of US stocks' total market capitalization. To compare our results with studies that include microcaps, such as V02 and CPV, we report robustness tests with no restriction on size.

We impose all sample and data availability restrictions ex ante and compute subsequent returns, earnings, BM ratios, and other characteristics as permitted by data availability in the following year. We use CRSP delisting returns and replace missing delisting returns with the average delisting return for the delisting event. To impute any missing data on dependent variables in the VAR, as in V02 and CPV, we assume BM, size, and profitability are constant, return is zero, dividends are zero, ROE satisfies clean surplus accounting, and investment is equal to growth in book equity. Missing data are rare because our ex ante sample does not contain microcaps. For any stock that becomes a microcap, we compute its actual realized returns, earnings, valuation ratios, and other characteristics in its final year of sample eligibility using the same procedures as for other stocks.

[12]The SEC (2013) defines a microcap stock as one "with a market capitalization of less than $250 or $300 million." In 2013, the cutoff for the bottom NYSE size quintile was a market capitalization of $266M. Since this cutoff of $266M is consistent with the size given in the SEC definition, we refer to stocks in the bottom NYSE quintile as microcaps.

Table 1 presents summary statistics for all firm-level variables. Panel A displays the number of observations, means, and standard deviations for each variable. In the average year from 1929 to 2017, the average firm has a log BM ratio of -0.24, which implies a market-to-book ratio of $e^{0.24} = 1.27$. Valuation ratios range widely across firms, as shown by the $lnBM$ standard deviation of 0.63. The cross-sectional variation in real stock returns is also substantial at 0.29 per year. Panel B shows cross-sectional correlations for the firm-level variables. There are only three correlations above 0.4 in absolute value and two are somewhat mechanical: the correlations between BM and five-year change in size (-0.44) and between six-month momentum and one-year returns (0.69). In addition, firm profitability is positively correlated with stock returns (0.56).

4 Baseline VAR Estimation

In our main estimation, we specify a VAR with panel regressions for firm-specific variables and time-series regressions for aggregate variables. To avoid seasonality and maximize data availability, we measure all variables at an annual frequency. As predictors of returns and cash flows, we include characteristics that are proxies for firms' risk exposures, stock mispricing, and measures of lagged cash flows and returns. Following the literature, we specify a VAR(1), which is a reasonable model of annual dynamics.

4.1 Specification

Following CPV, we adjust all firm-specific variables by subtracting the corresponding market-level variables. The panel regressions include six market-adjusted variables: annual log returns ($lnRet$) and the five anomaly characteristics, which are $lnBM$, $lnProf$, $lnInv$, $d5.lnME$, and $lnMom6$. The aggregate (market) variables are the value-weighted averages of the unadjusted versions of these six variables. In the panel regressions, we weight each year equally following CPV by applying weights to each firm-year observation equal to the inverse of the number of firms in the year.

In our main specification, the CF shock is the residual from the present-value relation–e.g., Equation (11) for the market-adjusted CF shock. In the robustness section (Section 6), we estimate an alternative VAR to predict $lnROE$ from firms' accounting statements and thereby obtain an alternative CF shock based on innovations in the discounted infinite sum of accounting ROE.

The panel regressions allow us to estimate the long-run dynamics of (market-adjusted) log returns and log earnings based on the short-run (one-year) properties of a broad cross section of firms. We do not need to impose restrictions on which firms survive for multiple years, thereby mitigating statistical noise and survivorship bias. Similarly, the aggregate VAR provides estimates of the long-run dynamics of market-wide variables based on their short-run properties. In Section 6, we show that the VAR's key autoregressive assumption provides a reasonable approximation of the long-run dynamics of returns and earnings.

Our VAR specification differs from specifications in prior studies, which could drive differences in our CF-DR decomposition as discussed in Chen and Zhao (2009). To illustrate how our CF-DR decomposition depends on specification choices, we replicate several prior specifications and compute CF-DR compositions based on these alternative specifications. We report these results in Section 6 after our main specification.

4.2 Panel Regressions

Table 2 reports weighted ordinary least squares (OLS) estimates of the predictive coefficients (A^{ma}) in the market-adjusted panel VAR, where the weighting ensures equal weight for each year. In parentheses, we report standard errors of coefficients that account for correlations between regression errors within years as described in Internet Appendix C.

The findings in the log return regressions are consistent with those of the large literature on short-horizon forecasts of returns. Log BM, profitability, and six-month momentum are positive predictors of firms' one-year log returns, whereas log investment and size are negative predictors of log returns. These coefficients are all statistically significant at the 1% level, except the BM coefficient which has a p-value slightly over 0.05. The fact that the BM ratio is insignificant when we include investment and profitability is consistent with findings in Fama and French (2015). The modest R^2 of 2.1% is typical for forecasts of firm-level returns.

Many of the other coefficients in the VAR are significant. The coefficients on the diagonal of A^{ma} show the persistence of each predictor. The BM ratio has the highest persistence coefficient at 0.905, while profitability, investment, size, and momentum have persistence coefficients of 0.584, 0.720, 0.743, and 0.058, respectively. Since persistent predictors tend to dominate in the DR and CF formulas in Equations (9) and (11), we infer that BM, investment, and size are likely the most important characteristics for explaining realized returns. In addition, cross predictions such as the fact that momentum predicts BM, which is highly persistent, can have a material impact on return decompositions, in this case particularly for returns of portfolios sorted on momentum.

Table 3 reports the predictive coefficients from the aggregate VAR (A^{agg}). The first column shows the forecasting regression for aggregate log one-year returns. The R^2 is high at 17.3%, which implies the estimated annual equity risk premium varies a lot over time. In our 89 year sample, expected log real returns on the market have a standard deviation of 0.085, range from -0.253 to 0.314, and are negative in 23 of 89 years. The p-value of the F-test for the joint significance of the regression coefficients is 0.003 (not reported in table). Due to relatively high correlations between the explanatory variables, however, only the five-year aggregate change in market size is individually significant with a negative coefficient, as expected. The signs of the other predictors are also as expected, with the BM ratio and profitability predicting with a positive sign, and investment with a negative sign.

The remaining columns in Table 3 show the forecasting regressions for the aggregate predictors. The most persistent predictor is aggregate log BM, which has a persistence coefficient of 0.961. Further investigation reveals that the autocorrelation coefficient of aggregate log BM is only 0.81, indicating that the presence of correlated regressors substantially increases the persistence coefficient. Because of its high persistence, log BM is a primary driver of long-run aggregate return predictability.

5 Decomposing Returns

5.1 Firm Return Decomposition

We now examine the implications of this VAR system for the DR and CF components of returns. The DR and CF components of firms' market-adjusted log returns come directly from substituting the VAR estimates into Equations (9) and (11). Similarly, the DR and CF components of log market returns come from Equations (7) and (10). We obtain the components of total firm returns as the sums of the respective components of market-adjusted returns and market returns as in Equations (12) and (13).

Table 4 reports the decomposition of log return variance into DR and CF components. Standard errors appear in parentheses below the point estimates of each variance component. The standard errors account for estimation uncertainty from both sampling variation and that from estimating the VAR coefficients, as well as heteroskedasticity and contemporaneous cross-correlation of residuals. Internet Appendix C provides further details.

The first row in Table 4 shows that DR news explains just 8% of variance in firms' market-adjusted returns, whereas CF news explains 72% of variance. The importance of CF news at the firm level confirms a key finding in V02. Interestingly, the third column shows that negative covariance between DR and CF news tends to amplify return variance, contributing a highly significant positive amount (20%) of variance. The last column shows that the correlation between DR and CF news is significantly negative (−0.42). This negative correlation means that low expected firm cash flows are associated with high firm discount rates.

The last row in Table 4 shows the decomposition of log market returns. Consistent with prior studies, such as Campbell (1991) and Cochrane (2011), DR news is the main driver of market returns, accounting for 74% of variance in our main specification. In contrast, CF news accounts for just 15% of market return variance. The covariance between market DR and CF components is slightly negative and accounts for the remaining 10% of return variance. The market DR-CF correlation of −0.15 is not significantly different from zero, which is broadly consistent with the literature.

The middle row in Table 4 reports the decomposition of total log firm returns. Because the total return components come from combining the market-adjusted and market return components, the middle row looks similar to an average of the top and bottom rows. Because market-adjusted CF news is more volatile than market DR news, CF news accounts for the majority, 55%, of total firm return variance. Variation in DR news accounts for 25%, and negative DR-CF covariance accounts for the remaining 20% of total variance in firm returns. Overall, the firm- and market-level results are consistent with earlier literature. The only exception is the negative correlation between firm-level CF and DR shocks, which is different from the positive correlation in V02. We show in Section 6 that this difference is due to our exclusion of microcaps.

5.2 Anomaly Return Decompositions

We now analyze the returns of long-short anomaly portfolios to bring new facts to the debate on the source of anomalies. We use the VAR to compute the DR and CF components of anomaly portfolio returns. We form anomaly portfolios using cross-sectional sorts on value, size, profitability, investment, and momentum. These sorts are based on characteristics used in the VAR, except for firm size and investment. Whereas the VAR uses firms' five-year change in log size and five-year investment, we sort by the level of firm size and one-year investment when forming portfolios to be consistent with empirical studies of anomalies.

As described earlier, we compute value-weighted averages of firm-level DR and CF estimates to obtain portfolio-level DR and CF estimates. When aggregating firm-level shocks to the portfolio level, only correlated shocks to firms remain. Thus, if cash flow shocks are largely uncorrelated but discount rate shocks are highly correlated, the portfolio return variance decomposition can be very different from the firm return variance decomposition.

Panel A in Table 5 reports the decompositions of return variance for the five anomaly portfolios. We compute standard errors for these decompositions using the same procedure as described earlier and in Internet Appendix C. The striking result in Table 5 is that CF news accounts for the vast majority of return variance for all five anomalies. As shown in column two of Table 5, the contribution of CF news to variance ranges from 64% of variance for the size anomaly to 80% of variance for the profitability anomaly. The high volatility of anomaly CF news shows that CF shocks to firms with similar anomaly characteristics exhibit a high degree of commonality. In contrast, DR news accounts for less than 15% of variance for all five anomalies.

The correlation in anomaly DR and CF news is significantly negative for three of the five anomalies—value, size, and momentum—with values ranging from −0.68 to −0.55. For the profitability and investment anomalies, this covariance is statistically indistinguishable from zero. However, as we will show in Table 10, if we exclude the first 10 years of our sample—1929 to 1938 or the Great Depression—the correlations between DR and CF news for profitability and investment are negative and significant, and the other three anomalies still have significantly negative correlations. Thus, the negative correlation between DR and CF news at the firm level generally drives a negative correlation at the anomaly level.

Figure 1 concisely summarizes the anomaly return decompositions for all five anomalies. The blue bars show that CF news contributes most of anomaly return variance. The red bars indicate the contributions from DR news are small. Lastly, the light green bars show that negative covariance between DR and CF news is an important contributor to return variance for the value, size, and momentum anomalies. Standard error bounds appear at the top of each bar.

Panel B of Table 5 reports the CF and DR decompositions of the MVE and market portfolios. The MVE portfolio applies weights to the factor portfolios that maximize the

MVE portfolio's in-sample Sharpe ratio. We compute two versions of the MVE portfolio: "MVE ex market" optimally weights each of the five long-short anomaly portfolios; and "MVE cum market" optimally weights the market and the five anomaly portfolios. The weights of the MVE ex market (or anomaly MVE) portfolio on the five long-short anomalies are: 0.06 for value, 0.66 for profitability, −1.55 for investment, −0.80 for size, and 1.44 for momentum. The weights of the MVE cum market are 0.80 for market, −0.21 for value, 0.73 for profitability, −1.87 for investment, −0.35 for size, and 1.35 for momentum.

The key result in Panel B of Table 5 is that CF news is the main driver of returns for both MVE portfolios, particularly the anomalies-only MVE portfolio. Thus, there is a common component in CF news across all five anomalies that is not diversified away in the anomaly MVE portfolio. The DR components of anomaly returns exhibit weaker commonality as demonstrated by comparing $var(DR)$ for the five individual anomalies, which ranges from 7% to 14%, to $var(DR)$ for the anomaly MVE portfolio, which is just 7%.

The observed similarity of the anomaly and firm return decompositions is not mechanical. Aggregating firms into long-short portfolios diversifies away firm-specific cash flow and discount rate shocks, leaving only common cash flow and discount rate variation in anomaly portfolios. The relative importance of CF and DR news for each anomaly depends on the correlation of CF and DR shocks across the assets in anomaly portfolios, which in turn depends on the commonality in shocks to assets' characteristics. For example, common shocks to small firms' BM ratios relative to big firms' BM ratios could drive variation in CF and DR news for the long-short size portfolio. But neither firm-specific nor aggregate shocks to BM ratios have any impact on CF and DR news of anomaly portfolios.

Figure 2 summarizes the return decompositions of the MVE portfolios and the market. The MVE portfolio that combines the market and the anomalies inherits some properties from both sets of portfolios and has an interesting new property: zero DR-CF news correlation. But the main message from Figure 2 is that CF news is the primary source of fluctuations in both MVE portfolios and thus the SDF. In contrast, if one considers the market portfolio as the MVE portfolio, one would conclude that DR news is the primary driver of MVE returns. We next analyze the correlations across portfolios to improve our understanding of variation in MVE portfolio returns.

5.3 Correlations Across Portfolios

In Table 6, we report correlations between components of market returns and components of anomaly and MVE returns. For ease of interpretation, we multiply the long-short returns of the investment and size portfolios by -1 before computing correlations, so that these portfolios have positive premiums. The first column of Table 6 displays correlations between anomaly CF news and market CF news. Strikingly, only one of the five anomaly CF shocks (investment) exhibits a significant correlation with market CF news. All five correlations between anomaly and market cash flows are economically small, ranging between -0.22 for investment and 0.13 for value.

The fourth column in Table 6 reveals that market DR news is significantly positively correlated with the DR news for the value and (negative) size anomalies. But market DR shocks exhibit no statistically significant correlations with the other anomalies' DR shocks. In addition, we later show in Table 10 that the DR shocks to value and the market are not strongly correlated if we exclude the Great Depression period. The middle two columns in Table 6 show that there is little cross-correlation between market DR news and anomaly CF news or between market CF news and anomaly DR news. All such correlations are smaller in magnitude than 0.3.

The second to last row in Table 6 report the correlations between DR and CF components of market returns and those of the MVE ex market portfolio. The main finding is that none of the four correlations is economically large. Only the negative correlation (-0.24) between market CF news and anomaly MVE DR news is marginally significant with a t-statistic of 2.0. Notably, CF shocks to the MVE portfolio consisting of all five anomalies are slightly negatively correlated with market CF shocks: -0.17 correlation (standard error (SE) =

0.14). Thus, we can reject CF news correlations above 0.11. Similarly, the correlation between anomaly MVE and market DR shocks is very close to zero at 0.06 (SE = 0.12). This evidence suggests that distinct forces drive market and anomaly return components.

The bottom row of Table 6 shows the correlation between components of market returns on those of the MVE portfolio that includes the market factor. The CF shocks to this total MVE portfolio are uncorrelated with market CF shocks. Thus, non-market cash flow factors dominate CF news in this total MVE portfolio—a remarkable finding that we discuss in Section 7. On the other hand, market DR shocks account for nearly all of the DR shocks to the total MVE portfolio, as shown by the huge DR correlation of 0.90%, which is driven by the large DR component in market returns.

Figure 3 visually represents the weak correlation between the DR and CF components of market and anomaly returns. All six red bars showing correlations between anomaly and market CF news are quite small. None of these bars lies more than one standard error above zero, as shown by the standard error bounds. The six blue bars represent correlations between anomaly and market DR news. Although the correlations between market DR news and DR news for the BM and size anomalies are significantly positive, none of the other four correlations is more than one standard error above zero. Most importantly, the correlation between market DR news and DR news for the anomaly MVE portfolio is close to zero.

While the observed correlations between the market and anomaly return components are small on average, they could be significant during extreme crises. We examine this possibility by plotting CF and DR news for the market and the anomaly MVE (i.e., MVE ex market) portfolios in Figure 4. The top graph shows that anomaly and market CF shocks exhibit no discernible relationship. During periods when market CF news is low, anomaly MVE CF news tends to be slightly higher than average. But this tendency is weak and its direction is counterintuitive for standard risk-based models. In the financial crisis, when the market experienced negative CF news, the anomaly MVE portfolio experienced positive CF news, meaning that short legs of anomalies had higher CF news than the long legs. The bottom plot in Figure 4 shows DR news for the market and the anomaly MVE portfolio. Again, there is no clear correlation even if one focuses on periods in which market DR news is high. The most notable features in Figure 4 are the high volatility of anomaly CF news relative to anomaly DR news (blue lines), and the high volatility of market DR news relative to market CF news (red lines).

5.4 Correlations with Aggregate Shocks

In Table 7, we report correlations of DR and CF shocks to the market and anomaly portfolios with notable aggregate shocks. We estimate each aggregate shock as the residual from a first-order autoregressive (AR(1)) model of the relevant time series. One group of aggregate shocks reflects macroeconomic cash flow shocks: real per-capita consumption and GDP growth, three-year forward-looking consumption growth, and the labor share. These are constructed from annual June to June log growth rates based on quarterly data from 1947 to 2017. The other group represents shocks to aggregate risk aversion or discount rates: one-year change in the default spread (Baa - Aaa corporate bonds; 1929 to 2017); one-year change in the term spread (difference between the five- and one-year T-bond yields; 1954 to 2017); and the one-year change in investor sentiment (from Jeffrey Wurgler's website, 1965 to 2010). For these series, annual shocks are based on a June year-end to correspond to the timing of the VAR.

Consistent with intuition, market CF shocks are positively correlated with macroeconomic cash flows, namely consumption and GDP growth. Positive shocks to the labor share and thus negative shocks to the capital share are slightly negatively correlated with market CF shocks. Market CF shocks are negatively correlated with shocks to the default spread, which is a plausible measure of risk aversion or discount rates. Market DR shocks are significantly negatively correlated with GDP and consumption growth, indicating market discount rates increase in recessions. Market DR shocks are also strongly positively correlated with shocks to the default spread, consistent with the latter being a measure of bad times when risk and/or risk aversion is high. Overall, the correlations between market return components and the macroeconomic shocks are intuitive and consistent with earlier literature.

We observe several interesting patterns in the correlations between macroeconomic shocks and anomaly return components. The most striking finding is the lack of any significantly positive correlation between CF shocks to the anomaly MVE (ex market) portfolio and any of the four conventional measures of macroeconomic activity: GDP growth, one- or three-year consumption growth, and the labor share. There are also no significant correlations between anomaly MVE CF news and the three proxies for risk aversion: the default spread, the term spread, and investor sentiment. Furthermore, there are no significant correlations between DR shocks to the anomaly MVE portfolio and the four macroeconomic measures of cash flows. However, there is evidence that sentiment and the default spread, two measures of aggregate discount rates, are correlated with DR shocks to the anomaly MVE portfolio with the expected signs.

Some of the correlations between the components of individual anomaly returns and the four macroeconomic cash flow measures are statistically significant. But even these correlations have modest magnitudes, ranging from -0.35 to 0.33 and implying that macroeconomic cash flows explain no more than 12% of the variance in any component of any anomaly's return. We find similarly low and insignificant correlations between anomaly return components and the term spread, a variable that is associated with business cycle fluctuations. In addition, none of the anomaly CF and DR correlations with sentiment are large. The largest CF and DR correlations are between the BM anomaly and the default spread. But, since the CF and DR correlations have roughly equal and opposite signs (-0.45 and 0.53), we infer that the total return of the value anomaly is negatively correlated (roughly -0.5) with the default spread, and there is no special relationship with either the CF or DR component.

In Table F.1 in the Internet Appendix, we report correlations between anomaly and market return components for two subperiods: 1929 to 1962, representing early years without direct data on profitability and investment; and 1963 to 2017, representing the modern sample used in most empirical studies. Most correlations are qualitatively similar and statistically indistinguishable across the two subperiods. One exception is the book-to-market anomaly, which exhibits no significant correlations with market return components in the early years but some strong correlations in the modern years. This finding is related to the well-known positive market beta of value stocks in the early years—e.g., Campbell and Vuolteenaho (2004). The other exception is the negative correlation between DR news for the MVE cum market portfolio and market CF news that exists only in the modern sample. Since this finding is driven by the negative correlation between market DR and market CF news that exists only in the modern sample (0.05 early vs. −0.59 modern), it is not directly relevant for anomalies.

We discuss in Section 7 how the evidence presented in this section relates to theoretical models of anomaly returns. In the next section, we analyze the robustness of these results by exploring several alternative specifications.

6 Robustness

6.1 Testing VAR Assumptions

To estimate anomaly CF and DR shocks, we directly estimate short-run (one-year) firm-level dynamics of assets' expected returns and extrapolate these dynamics to infer long-run (infinite-horizon) expected returns of anomaly portfolios. We now evaluate whether our short-run firm-level regressions make accurate predictions of short-run anomaly-level returns and most importantly long-run anomaly returns, which form the basis of CF and DR shocks.

Table 8 compares realized anomaly returns to expected anomaly returns from the VAR. Panel A shows regressions of realized one-year log anomaly returns on expected one-year log anomaly returns, where the expected return comes from firm-level VAR predictions weighted by anomaly portfolio weights. Panel B shows the results from a regression of each anomaly's realized (unrebalanced) 10-year return on the anomaly's predicted long-run return from the

VAR. Consistent with theory, we apply κ^i weights to each year $i = 1, ..., 10$ when computing the 10-year realized return. We approximate the 10-year expected return using the infinite horizon expected return with the same κ^i weights, which is the basis for DR news. We adjust the regression standard errors for autocorrelation in the residuals using the Newey-West method with 10 lags.

Panel A in Table 8 shows that our baseline VAR model accurately fits the unconditional average of one-year log returns on all five anomaly portfolios. The differences in realized and expected returns are statistically insignificant even at the 10% level. In addition, the model successfully matches the premium on the MVE combination of anomalies (MVE ex market portfolio): average return = 12.8% versus expected return = 11.8% (t-stat of 0.41 on the difference).

From Panel A in Table 8, the model predicts 2% volatility in most anomalies' expected returns, implying that many anomalies exhibit negative expected returns in some years. The size anomaly is an example of an anomaly that has a near-zero unconditional expected return but exhibits conditional expected returns ranging from −5% to +5%, which is broadly consistent with recent work by Haddad, Kozak, and Santosh (2018). In the anomaly MVE portfolio, which combines all anomalies and is scaled to match market volatility, the volatility of expected returns is 5.5%. Even though this volatility is substantial, the anomaly MVE portfolio rarely has negative expected returns because its unconditional expected return is highly positive. This result contrasts with the market portfolio, which has negative expected returns in many years according to our VAR and others in the literature.

In Panel B of Table 8 we show that, for each of the five anomalies and two MVE portfolios, we cannot reject the hypothesis that the coefficient on the VAR prediction of long-run anomaly returns is 1.0. Notably, for all but one anomaly (BM), there is sufficient statistical power to reject the hypothesis that the coefficient is 0.0. There is no apparent bias in these coefficients judging by the anomaly MVE portfolio coefficient of 0.95, which is economically very close to 1.0. Thus, we conclude that the VAR does a decent job of capturing actual long-run expected returns and thus cash flows, since we impose the present value constraint.

6.2 Reconciling Prior Empirical Findings

Our baseline VAR's predictions are consistent with prior literature in that DR news explains most variation in market return and CF news explains most variation in firm-level returns. However, V02 and CPV find that the correlation between firm-level CF and DR news is positive, while we find that this correlation is negative.

To investigate how changes in sample selection, sample years, and VAR specification affect CF and DR news, we replicate and update the main findings in V02 and CPV. We directly build on the methodology in these studies and reconcile their findings with ours by making the following incremental changes:

1) Microcaps: Unlike V02 and CPV10, we exclude firms in the bottom NYSE quintile, which correspond to microcaps as defined by the SEC.

2) Young firms: V02 excludes firms without multiple years of accounting data history—i.e., young firms. For V02, this filter is necessary to avoid Compustat survivorship bias in early data since V02 does not use Davis, Fama, and French (2000) book equity data, as we and CPV do, or exclude small firms, as we do.

3) Market and anomaly portfolio weights: We consistently use value weights when aggregating across firms. V02 always uses equal weights for aggregates. CPV uses value weights for market-level variables; and CPV subtracts equal-weighted averages from firm variables when computing market-adjusted variables.

4) Return predictors: Our VAR specification uses firm-level and aggregate characteristics motivated by the Fama and French (2015) and Carhart (1997) models. V02 and CPV focus on a smaller set of characteristics: earnings, book-to-market, and past returns. For V02 firm-level and aggregate predictors, we use $lnROE$, $lnBM$, and $lnRealRet$.

For CPV firm-level predictors, we use the 5-year average of clean-surplus earnings, $lnBM$, and $lnRealRet$; and, for CPV aggregate predictors, we use term spread, small stock value spread, and cyclically-adjusted price-to-earnings (CAPE) ratio as measured in CPV.

5) Sample years: We examine all years from 1929 to 2017. CPV examines the years from 1929 to 2000, whereas V02 examines 1954 to 1996.

In Table 9, we report extensive VAR specifications to reconcile our findings with those of V02 and CPV. This table also reports three variants of our main specification in which we exclude the Great Depression years (1929-1938), include microcaps, and define CF shocks based on accounting ROE instead of the present-value relation. Table 9 shows the variance of the CF and DR components of firms' market-adjusted log returns. The three panels focus on three methodologies: V02, CPV, and ours (LT).

The most important result in Table 9 is that the correlation between DR and CF news is much higher in samples that include microcaps, regardless of which panel one examines. Excluding microcaps reduces the DR-CF correlation from 0.52 to −0.11, from 0.39 to −0.41, or from 0.36 to −0.42 in the three panels. Although microcap observations account for 46% of the number of firm-years, they represent less than 2% of total market value in most years. The fact that the DR-CF correlation decreases with firm size is consistent with V02.

We also analyze how including microcaps affects results other than the firm-level CF-DR correlation. In Table 10, we show that including microcaps substantially increases the anomaly-level CF-DR correlations, which become positive in most cases like the firm-level DR-CF correlation. There are two reasons for this increase. First, the inclusion of microcaps changes some VAR coefficients, particularly the coefficient on profitability in predicting firms' market-adjusted returns. Table F.2 compares the return predictability coefficients with and without microcaps. Although the raw return predictability coefficient on profitability is only 41% larger in the microcap specification, the standardized coefficient on profitability is 114% larger because microcaps have much more volatile profits than other firms. Since high past profits predict high returns and high future profits, the large profitability coefficient increases the DR-CF correlation for microcaps. Our finding of greater predictability from past profits in small firms is related to the finding that post-earnings announcement drift is larger in small firms as shown in Mendenhall (2004).

The second reason is that the inclusion of microcaps has a big impact on the composition and characteristics of anomaly portfolios, which govern the CF and DR shocks to these portfolios. Microcaps are numerous and have volatile characteristics. Since microcaps constitute 46% of all observations, the top and bottom quintiles of firms ranked by each anomaly characteristic (e.g., past profits) could consist entirely of microcaps. In addition, because microcaps have volatile characteristics, the changes in the characteristics of stocks in anomaly portfolios and thus the CF and DR shocks to these portfolios depend heavily on microcaps when these stocks are in the sample.

Interestingly, despite these considerations, Tables F.3 and F.4 in the Internet Appendix show that including microcaps does not have a large impact on how anomaly return components are correlated with market return components and measures of macroeconomic activity. Our key findings that anomaly CF shocks are uncorrelated with market CF shocks and macroeconomic activity remain robust.

Returning to Table 9, two other methodological changes have a material impact on the DR-CF correlation. First, using a broad set of predictors of returns and cash flow in the VAR, as we do and CPV does, decreases the DR-CF correlation from -0.22 to -0.42 in Panel A. The key is the inclusion of a persistent predictor of returns and cash flows beyond just the present value measure, $lnBM$. CPV also includes five-year clean-surplus earnings and we include five-year investment and change in size, whereas V02 has no additional persistent predictors.

The second methodology change that matters is redefining the CF shock to be based on accounting ROE rather than the present-value identity in Equation (11). Unless a firm happens to have equal market and book values, accounting ROE does not properly measure the relevant cash flow for stockholders, which is potential dividends. A firm with a negative $lnBM$ value with high ROE will earn far less than a firm with a positive $lnBM$ value that has the same ROE value, meaning that the latter firm's true cash flow to stockholders is higher. In contrast, the CF shock from our VAR, which is based on the present-value identity, appropriately accounts for stockholder payouts. We discuss this point and provide a numerical illustration in Internet Appendix E.

Despite this important difference, our main results hold even when using accounting ROE as the basis for CF shocks. Table 10 compares decompositions of anomaly return variance in our "Baseline" results with CF to those in our alternative results with "CF from accounting ROE," denoted by aCF.[13] The most important point is that CF news accounts for the biggest component of return variance in all anomalies, regardless of whether one uses CF or aCF. The correlation between CF and DR news does, however, depend on whether one considers CF shocks based on accounting ROE or the residual method. Specifically, the DR-CF correlation increases if one defines CF shocks based on accounting ROE, and it becomes greater than zero (0.15) for the anomaly MVE portfolio. This difference arises because accounting ROE ignores variation in net payouts to stockholders, such as repurchases and issuance, which can be correlated with discount rates as well as accounting ROE.

Table F.5 in the Internet Appendix shows that the correlations between anomaly and market CF news remain small even if one uses CF shocks based on accounting ROE. A minor exception is the positive and statistically significant 0.33 correlation between market and value anomaly CF shocks in the "Accounting ROE" column in Table F.5. This finding is broadly consistent with the finding in Cohen, Polk, and Vuolteenaho (2009) that market cash flow betas increase with firms' BM ratios—a finding also based on accounting ROE. Although the correlations between two anomalies' (BM and size) CF shocks with market CF shocks increase from roughly 0.1 to 0.3 when using accounting ROE, the remaining anomaly

[13] In the alternative VAR with accounting ROE, we predict aggregate and firm-level $lnROE$ but restrict the coefficients on lagged $lnROE$ variables to be zero in the A matrix since lagged $lnROE$ does not add predictive power beyond the other variables in the VAR.

and market CF correlations are close to zero. In addition, the correlation between CF shocks to the anomaly MVE portfolio and the market changes from -0.17 to -0.10 when using the accounting-based CF shock, demonstrating that there's little impact on this key result.

The last important result in Tables 9 and 10 is that CF news always accounts for the majority of firm- and anomaly-level return variance regardless of which methodology or sample one uses. In addition, the contribution of DR news to return variance is always small, accounting for less than 10% in most specifications. Table 10 shows that these results hold for the "No Depression" and "Including microcaps" samples. Tables F.4 and F.6 in the Internet Appendix show that the finding of weak correlations between anomaly CF and DR news and measures of macroeconomic activity is robust in these two samples.

6.3 Overfitting and Misspecifying Expected Returns

Here we consider two possible sources of misspecification in the VAR: spurious return predictability and omitted predictors of returns. This section summarizes a detailed analysis of these issues that appears in Internet Appendix G. Incorrectly specifying the predictors of returns, including estimating predictability coefficients with noise, induces an error in estimated DR news. Since we impose the present value relation ($r - E(r) = CF - DR$), any error in estimated DR news affects estimated CF news.

Spurious return predictability resulting from decades of research on anomalies is our foremost concern. By chance, some firm characteristics will be associated with future stock returns in historical samples, so estimates of return predictability are likely overstated, as argued in Harvey, Liu, and Zhu (2016) and Linnainmaa and Roberts (2018). Internet Appendix G shows that the use of data-mined characteristics in our VAR framework biases estimates of DR news variance upward. More subtly, data mining also increases the estimated covariance between CF and DR shocks because CF shocks must offset the impact of overstated DR shocks in total returns, which we observe directly. Our findings indicate that CF shocks are the dominant component of anomaly returns and that the correlation between DR and CF shocks is negative. Without data mining of VAR characteristics, these two conclusions would likely be even more pronounced. However, if instead our VAR equation for returns improperly omits key predictors of returns, the opposite biases could occur, as shown in Internet Appendix G. This analysis underscores the importance of correctly specifying return predictability.

7 Interpreting the Results

The stylized facts from the main tables are:

1) Most variation in firm and anomaly returns comes from variation in CF news, which has significant commonality across anomalies.

2) Anomaly DR and CF shocks are not significantly correlated with market DR and CF news or standard measures of macroeconomic activity.

3) Firm- and anomaly-level DR and CF news are negatively correlated.

Fact 3 applies only if we exclude microcaps and only if CF shocks satisfy the present-value relation. Otherwise the facts above are remarkably stable across methodologies and samples.

These findings can help guide asset pricing theories. For instance, the importance of CF shocks in Fact 1 indicates that the 10% annual volatility in a typical long-short anomaly portfolio returns comes mainly from shocks to a cash flow factor. Furthermore, anomaly characteristics are proxies for firms' different exposures to this cash flow factor. Fact 2 demonstrates that this cash flow factor is uncorrelated with market cash flows and standard macroeconomic aggregates like GDP and consumption growth.

These two facts present a high hurdle for two types of theories. First, the data do not support theories that rely on errors in firm valuations that are unrelated to actual cash flows, such as the De Long et al. (1990) model in which random noise trading drives price movement. To explain anomaly return decompositions, firms' exposures to shocks to investor risk aversion or sentiment, if they exist, cannot explain too much variance in returns at the firm or anomaly level. At the market level, on the other hand, risk aversion or sentiment shocks could be quite important given that 71% of market variance comes from DR shocks. Second, the weak link between anomaly and market CF shocks casts doubt on theories of anomalies that rely on differences in how sensitive firms' cash flows are to aggregate cash flows (market or macroeconomic), such as the investment-based model of Zhang (2005).

In addition, the generally weak correlation between anomaly DR shocks and market DR shocks in Fact 2 is inconsistent with theories that emphasize the role of common DR shocks across priced factor portfolios. Such a common DR shock could arise from time-variation in risk aversion as in Campbell and Cochrane (1999) or time-variation in investor sentiment as in Baker and Wurgler (2006). The near-zero correlation between anomaly MVE and market DR shocks is inconsistent with the idea that arbitrageurs who exploit anomalies are exposed to the same shocks to risk aversion as investors who hold the market. Instead, the evidence suggests distinct forces drive market and anomaly return components. Indeed, Figure 4 shows that there are no clear relationships between market and anomaly return components even during crisis periods.

That said, the DR correlations in Table 7 provide modest support for some theories of discount rate determination. First, DR shocks to the value-minus-growth portfolio are positively correlated with macroeconomic shocks, implying that the discount rate of the value factor portfolio increases during good macroeconomic times. For example, in the Internet boom of the 1990s when consumption and income growth were high, the valuation spread increased as growth firms' valuations diverged markedly from value firms' valuations, which in turn increased expected returns to the value factor portfolio. Figure 4 also shows that market discount rates declined during the Internet boom. This evidence is consistent with the discount rate aspect of duration-based explanations of the value premium in which growth firms are more sensitive to market DR shocks than value firms, as in Lettau and Wachter (2007). However, Table 7 also shows that DR shocks to the anomaly MVE portfolio (MVE ex market) are only slightly and not significantly negatively correlated with macro shocks. Thus, the duration-based theory is inconsistent with broader evidence on DR variation.

Anomaly MVE and market DR shocks are both positively correlated with shocks to the default spread, a possible proxy for aggregate risk aversion. However, since the overall correlation between anomaly MVE and market DR shocks is close to zero, as shown in Table 6, these DR shocks must exhibit other large and distinct sources of variation. At the individual anomaly level, the correlations with market DR shocks are generally small and the pattern is inconsistent. Thus, while there is some evidence of a common component related to the default spread, consistent with, e.g., Campbell and Cochrane (1999), this component is small: most fluctuations in anomaly CF and DR news shocks are unrelated to fluctuations in market CF and DR news.

Finally, the negative correlation in CF and DR shocks in Fact 3 could arise for behavioral or risk-based reasons. Investor overreaction to positive firm-level CF shocks could lower firms' effective discount rates. If anomaly characteristics are associated with firms' exposures to these CF shocks, CF and DR shocks would be negatively correlated at the anomaly level. For example, a shock to the productivity of new vs. old technology could increase growth firms' cash flows and decrease value firms' cash flows. Investor overreaction to this technology shock would reduce growth firms' discount rates and increase value firms' discount rates.[14] Alternatively, a risk-based theory in which this technology shock decreases growth firms' risks and increases value firms' risks could be consistent with the evidence. Such cash flow shocks cannot be market- or industry-level shocks, as they exhibit low correlations with market CF and DR shocks and industry exposures do not appear to be priced. The Kogan and Papanikolaou (2013) model is consistent with these facts.

[14] For the sample with microcaps, there is a positive DR-CF correlation, which is consistent with underreaction to CF news perhaps because investors devote little attention to these firms.

8 Conclusion

Despite decades of research on predicting short-term stock returns, there is no widely accepted explanation for observed cross-sectional patterns in stock returns. We provide new evidence on the sources of anomaly portfolio returns by aggregating firm-level CF and DR news from a panel VAR system, producing new insights into the components of anomaly returns. Any model that features stocks with heterogeneous cash flow dynamics has implications for the variance decomposition of the MVE portfolio return and thus the SDF that prices all assets. Forcing models to match these empirical moments restricts the shocks that drive investors' marginal utility and behavioral biases. The empirical patterns that we document also hold broadly across individual long-short anomaly portfolios, thus providing guidance for theories of individual anomalies.

Our empirical framework provides three new facts. First, CF shocks to the stocks underlying the MVE portfolio of anomalies account for 73% of this portfolio's return variance; DR shocks account for only 7% of this portfolio's variance. Even when including the market portfolio in the MVE portfolio, we still find that 69% of the return variation comes from CF news. These results contrast with the finding that 74% of the return variance in the market portfolio alone is due to DR news. Second, CF and DR shocks to anomalies exhibit little relation with market CF and DR shocks. In fact, DR shocks to the market are uncorrelated with DR shocks to the MVE combination of anomaly portfolios, casting doubt on theories that rely on common variation in the price of risk (or sentiment) as an important determinant of these portfolios' returns. Anomaly CF shocks are also largely uncorrelated with business cycle variables such as GDP and consumption growth. Third, there is a negative correlation between CF and DR shocks to the anomaly MVE portfolio. Based on this evidence, the most promising theories of anomalies and the MVE portfolio are those that feature cash flow factors with little relation to market returns or the business cycle, where firms' exposure to these factors are related to anomaly characteristics (e.g., investment or profitability) and where the CF shocks drive changes in firm risk or errors in investors' expectations.

References

Ang, Andrew, Robert Hodrick, Yuhang Xing, and Xioayan Zhang, 2006, The cross-section of volatility and expected returns, *Journal of Finance* 61, 259–299.

Barberis, Nicholas, Andrei Shleifer, and Robert Vishny, 1998, A model of investor sentiment, *Journal of Financial Economics* 49, 307–343.

Babenko, Ilona, Oliver Boguth, Yuri Tserlukevich, 2016, Idiosyncratic cash flows and systematic risk, *Journal of Finance* 81, 425–455.

Baker, Malcolm, and Jeffrey Wurgler, 2006, Investor sentiment and the cross-section of stock returns, *Journal of Finance* 61, 1645–1680.

Berk, Jonathan, Richard Green, and Vasant Naik, 1999, Optimal investment, growth options, and security returns, *Journal of Finance* 54, 1553–1607.

van Binsbergen, Jules, and Ralph Koijen, 2010, Predictive regressions: A present value approach, *Journal of Finance* 65, 1439–1471.

Campbell, John, 1991, A variance decomposition for stock returns, *Economic Journal* 101, 157–179.

Campbell, John, Christopher Polk, and Tuomo Vuolteenaho, 2010, Growth or glamour? Fundamentals and systematic risk in stock returns, *Review of Financial Studies* 23, 305–344.

Campbell, John, and Robert Shiller, 1988, Stock prices, earnings, and expected dividends, *Journal of Finance* 43, 661–676.

Campbell, John, and Luis Viceira, 2002, *Strategic Asset Allocation* (Clarendon Lectures in Economics, Cambridge, MA).

Chen, Long and Xinlei Zhao, 2009, Return Decomposition. *Review of Financial Studies* 22 (12), 5213–5249.

Cohen, Randolph, Christopher Polk, and Tuomo Vuolteenaho, 2003, The value spread, *Journal of Finance* 58, 609–641.

Cohen, Randolph, Christopher Polk, and Tuomo Vuolteenaho, 2009, The price is (almost) right, *Journal of Finance* 64, 2739–2782.

Cooper, Michael, Huseyin Gulen, and Michael Schill, 2008, Asset growth and the cross-section of stock returns, *Journal of Finance* 63, 1609–1651.

Davis, James, Eugene Fama, and Kenneth French, 2000, Characteristics, covariance, and average returns: 1929-1997, *Journal of Finance* 55, 389–406.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, 2001, Overconfidence, arbitrage, and equilibrium asset pricing, *Journal of Finance* 56, 921–965.

Daniel, Kent, and Sheridan Titman, 2006, Market reactions to tangible and intangible information, *Journal of Finance* 61, 1605–1643.

De Long, Bradford, Andrei Shleifer, Lawrence Summers, and Robert Waldmann, 1990, Noise trader risk in financial markets, *Journal of Political Economy* 98, 703–738.

Fama, Eugene, and Kenneth French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427–465.

Fama, Eugene, and Kenneth French, 1995, Size and book-to-market factors in earnings and returns, *Journal of Finance* 50, 131–155.

Fama, Eugene, and Kenneth French, 2015, A five-factor asset pricing model, *Journal of Financial Economics* 116, 1–22.

Fama, Eugene, and James MacBeth, 1973, Risk, return, and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607–636.

Gerakos, Joseph, and Juhani Linnainmaa, 2018, Decomposing value, *Review of Financial Studies* 31, 1825–1854.

Haddad, Valentin, Serhiy Kozak, and Shrihari Santosh, 2018, Predicting relative returns, UCLA working paper.

Hansen, Lars, John Heaton, Junghoon Lee, and Nikolai Roussanov, 2007, Chapter 61: Intertemporal substitution and risk aversion, in James J. Heckman and Edward E. Learner, eds.: *Handbook of Econometrics*, Volume 6A (Elsevier, Amsterdam).

Hong, Harrison, and Jeremy Stein, 1999, A unified theory of underreaction, momentum trading, and overreaction in asset markets, *Journal of Finance* 54, 2143–2184.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance* 48, 65–91.

Kelly, Bryan, and Seth Pruitt, 2013, Market expectations in the cross-section of present values, *Journal of Finance* 68, 1721–1756.

Kogan, Leonid, and Dimitri Papanikolaou, 2013, Firm characteristics and stock returns: The role of investment-specific shocks, *Review of Financial Studies* 26, 2718–2759.

Larrain, Borja, and Motohiro Yogo, 2008, Does firm value move too much to be justified by subsequent changes in cash flow? *Journal of Financial Economics* 87, 200–226.

Lettau, Martin, and Stijn van Nieuwerburgh, 2008, Reconciling the return predictability evidence, *Review of Financial Studies* 21, 1607–1652.

Lettau, Martin, and Jessica Wachter, 2007, Why is long-horizon equity less risky? A duration-based explanation for the value premium, *Journal of Finance* 62, 55–92.

Lyle, Matthew R., and Charles C.Y. Wang, 2015, The cross section of expected holding period returns and their dynamics: A present value approach, *Journal of Financial Economics* 116, 505–525.

Mendenhall, Richard R., 2004, Arbitrage risk and post-earnings-announcement drift, *Journal of Business* 77, 875–894.

Nickell, Stephen, 1981, Biases in dynamic models with fixed effects, *Econometrica* 49, 1417–1426.

Novy-Marx, Robert, 2013, The other side of value: The gross profitability premium, *Journal of Financial Economics* 108, 1–28.

Ohlson, James, 1995, Earnings, book values, and dividends in equity valuation, *Contemporary Accounting Research* 11, 661–687.

Petersen, Mitchell A., 2009, Estimating standard errors in finance panel data sets: Comparing approaches, *Review of Financial Studies* 22, 435–480.

Pontiff, Jeffrey, and Artemiza Woodgate, 2008, Share issuance and cross-sectional returns, *Journal of Finance* 63, 921–945.

Shiller, Robert, 1981, Do stock prices move too much to be justified by subsequent changes in dividends? *American Economic Review* 71, 421–436.

Santos, Tano, and Pietro Veronesi, 2010, Habit formation, the cross section of stock returns and the cash flow risk puzzle, *Journal of Financial Economics* 98, 385–413.

Stambaugh, Robert, 1999, Predictive regressions, *Journal of Financial Economics* 54, 375–421.

Vuolteenaho, Tuomo, 2002, What drives firm-level stock returns? *Journal of Finance* 57, 233–264.

Zhang, Lu, 2005, The value premium, *Journal of Finance* 60, 67–103.

Table 1 - Summary Statistics

Panel A shows summary statistics for firms' returns and characteristics. As stated in the text, we define $lnRealRet$ as one-year real return, $lnROE$ as one-year return on equity, $lnBM$ as book equity divided by market equity, $lnProf$ as revenues minus costs and expenses divided by book equity, $lnInv$ as five-year average of growth in assets, $d5.lnME$ as five-year change in market equity, and $lnMom6$ as six-month return. We measure all variables in logs, adding one before taking the log except for book-to-market and change in size. See text for details. The second column reports the average number of firm observations per year. The third and fourth columns give the average mean and standard deviation of the variables per year. Panel B provides the average yearly correlation matrix for these firm characteristics. The sample spans the years 1929 through 2017.

Panel A:	Avg. number of firms per year	Avg. Mean per year	Avg. St.Dev. per year
$lnRealRet$	1,399	0.030	0.293
$lnROE$	1,399	0.065	0.176
$lnBM$	1,399	−0.240	0.626
$lnProf$	1,399	0.197	0.143
$lnInv$	1,399	0.094	0.109
$d5.lnME$	1,399	0.389	0.693
$lnMom6$	1,399	0.035	0.204

Panel B:	1	2	3	4	5	6
$lnRealRet$ (1)	1.00					
$lnROE$ (2)	0.22	1.00				
$lnBM$ (3)	−0.34	−0.11	1.00			
$lnProf$ (4)	0.14	0.56	−0.12	1.00		
$lnInv$ (5)	−0.05	0.11	−0.11	0.06	1.00	
$d5.lnME$ (6)	0.38	0.24	−0.44	0.17	0.27	1.00
$lnMom6$ (7)	0.69	0.11	−0.23	0.07	−0.05	0.24

Table 2 - Market-adjusted Panel VAR

The table gives the results from estimating the panel VAR using market-adjusted firm returns and characteristics. The variables are: log one-year real returns ($lnRealRet$), log book-to-market ratio ($lnBM$), log profitability ($lnProf$), log five-year asset growth ($lnInv$), five-year change in log market equity ($d5.lnME$), and log six-month return ($lnMom6$). We also include log return on equity ($lnROE$) as a dependent variable in the VAR, but restrict all coefficients on this variable's lag to equal zero, as explained in the main text. The sample spans the years 1929 through 2017. Standard errors clustered by year and firm appear in parenthesis. N denotes the number of observations. The marks * and ** indicate significance at the 5 and 1 percent levels, respectively.

	Dependent Variables						
Regressors	$lnRealRet_t$	$lnBM_t$	$lnProf_t$	$lnInv_t$	$d5.lnME_t$	$lnMom6_t$	$lnROE_t$
$lnRealRet_{t-1}$	0.016	0.068*	0.034**	0.007**	0.244**	−0.012	0.095**
	(0.033)	(0.029)	(0.005)	(0.002)	(0.036)	(0.021)	(0.011)
$lnBM_{t-1}$	0.033	0.905**	−0.017**	−0.008**	−0.008	0.025*	−0.043**
	(0.017)	(0.004)	(0.003)	(0.001)	(0.020)	(0.012)	(0.019)
$lnProf_{t-1}$	0.157**	−0.029	0.584**	0.013**	0.190**	0.085**	0.269**
	(0.030)	(0.025)	(0.020)	(0.003)	(0.040)	(0.018)	(0.023)
$lnInv_{t-1}$	−0.145**	0.105**	−0.091**	0.720**	−0.048	−0.061**	−0.137**
	(0.023)	(0.022)	(0.008)	(0.007)	(0.028)	(0.019)	(0.013)
$d5.lnME_{t-1}$	−0.016**	0.032**	0.000	0.019**	0.743**	−0.017**	0.013**
	(0.006)	(0.006)	(0.001)	(0.001)	(0.013)	(0.004)	(0.002)
$lnMom6_{t-1}$	0.095**	−0.093**	0.009	−0.008**	0.071*	0.058**	−0.023
	(0.033)	(0.030)	(0.006)	(0.002)	(0.035)	(0.018)	(0.012)
R^2	0.021	0.747	0.373	0.797	0.632	0.017	0.126
N	124,535	124,535	124,535	124,535	124,535	124,535	124,535

Table 3 - Aggregate VAR

This table gives the results from estimating the aggregate VAR. The variable are all value-weighted averages of the firm-level variables used in the panel VAR in Table 2. The variables are: log real one-year returns ($lnRealRet$), log book-to-market ($lnBM$), log profitability ($lnProf$), log five-year asset growth ($lnInv$), five-year change in log market equity ($d5.lnME$), and log six-month momentum ($lnMom6$). We also include log return on equity ($lnROE$) in the VAR, but restrict all coefficients on this variable's lag to equal zero, as explained in the main text. The sample spans the years 1929 through 2017. Heteroskedasticity adjusted (White) standard errors appear in parenthesis. N denotes the number of observations. The marks * and ** indicate significance at the 5 and 1 percent levels, respectively.

	Dependent Variables						
Regressors	$lnRealRet_t$	$lnBM_t$	$lnProf_t$	$lnInv_t$	$d5.lnME_t$	$lnMom6_t$	$lnROE_t$
$lnRealRet_{t-1}$	−0.131	0.126	0.041*	0.006	0.061	−0.085	0.013
	(0.120)	(0.132)	(0.018)	(0.012)	(0.269)	(0.092)	(0.025)
$lnBM_{t-1}$	0.073	0.961**	−0.005	−0.002	−0.150	0.051	−0.073**
	(0.120)	(0.128)	(0.008)	(0.006)	(0.132)	(0.068)	(0.013)
$lnProf_{t-1}$	1.195	−1.265	0.883**	0.101*	2.961	0.789	0.698**
	(1.196)	(1.347)	(0.075)	(0.048)	(1.522)	(0.703)	(0.129)
$lnInv_{t-1}$	−0.351	0.774	0.059	0.826**	−1.621	−0.310	0.073
	(1.017)	(1.144)	(0.102)	(0.048)	(2.237)	(0.589)	(0.159)
$d5.lnME_{t-1}$	−0.128*	0.145*	−0.016*	0.007*	0.512**	−0.028	−0.018
	(0.059)	(0.062)	(0.007)	(0.003)	(0.156)	(0.035)	(0.011)
$lnMom6_{t-1}$	0.011	0.048	−0.044	−0.011	−0.043	0.025	−0.009
	(0.220)	(0.236)	(0.026)	(0.013)	(0.436)	(0.159)	(0.040)
R^2	0.173	0.686	0.793	0.916	0.470	0.115	0.599
N	89	89	89	89	89	89	89

Table 4 - Firm-level and Market Return Variance Decompositions

The table displays variance decomposition of firm- and market-level real returns. We decompose each log return into CF and DR news based on the panel VAR in Table 2 and the aggregate VAR in Table 3. "Firm market-adjusted return" refers to the decomposition of market-adjusted log firm returns from the panel VAR. "Market return" refers to the decomposition of log market returns from the aggregate VAR. "Firm return" refers to the decomposition of total firm returns, obtained by combining components of firm market-adjusted returns and market returns. The sample spans the years 1929 through 2017. Standard errors appear in parentheses.

	$var\,(DR)$	$var\,(CF)$	$-2cov\,(DR, CF)$	$corr\,(DR, CF)$
Firm market-adjusted return	8%	72%	20%	-0.42
	(4%)	(10%)	(4.9%)	(0.06)
Firm return	25%	55%	20%	-0.27
	(10%)	(7.6%)	(7.2%)	(0.11)
Market return	74%	15%	10%	-0.15
	(34%)	(7.6%)	(25%)	(0.38)

Table 5 - Anomaly Variance Decompositions

Panel A shows decompositions of the variance of log anomaly returns into cash flow (CF) and discount rate (DR) components, as described in the main text. The long-short anomaly return is the difference between the log return of the top quintile portfolio and the log return of the bottom quintile portfolio, where the quintiles are based on sorts of stocks by each anomaly characteristic. We apply value weights to stocks' log returns within each quintile portfolio. The anomaly characteristics are firm book-to-market ratio, profitability, size (market equity), momentum (six-month return), and investment (one-year asset growth) as defined in the text. Panel B shows variance decompositions of log returns to two in-sample mean-variance efficient (MVE) portfolios: MVE ex market combines only the five long-short anomaly portfolios; and MVE cum market comprises the five anomalies and the market portfolio. The sample spans the years 1929 through 2017. Standard errors appear in parentheses.

	Fraction of Portfolio Return Variance			
	$var\,(DR)$	$var\,(CF)$	$-2cov\,(DR, CF)$	$corr\,(DR, CF)$
Panel A: Individual Anomalies				
Book-to-market	7%	68%	25%	−0.56
	(5%)	(19%)	(10%)	(0.10)
Profitability	14%	80%	6%	−0.10
	(8%)	(27%)	(16%)	(0.14)
Size	7%	64%	29%	−0.68
	(5%)	(17%)	(10%)	(0.09)
Momentum	7%	70%	23%	−0.55
	(4%)	(21%)	(11%)	(0.11)
Investment	14%	78%	7%	−0.10
	(9%)	(19%)	(13%)	(0.14)
Panel B: MVE portfolios				
MVE ex market	7%	73%	20%	−0.43
	(4%)	(16%)	(10%)	(0.12)
MVE cum market	36%	69%	−5%	0.05
	(14%)	(18%)	(21%)	(0.19)

Table 6 - Correlations between Anomaly and Market Return Components

The table shows correlations of market cash flow and discount rate shocks with anomaly cash flow and discount rate shocks and with cash flow and discount rate shocks to mean-variance efficient (MVE) combinations of these portfolios. "MVE ex market" is the in-sample MVE combination of the five long-short anomaly portfolios. "MVE cum market" is the in-sample MVE combination of the five anomalies and the market portfolio. The sample spans the years 1929 through 2017. Standard errors appear in parentheses. The marks * and ** indicate significance at the 5 and 1 percent levels, respectively.

	Market CF		Market DR	
	Anomaly CF	Anomaly DR	Anomaly CF	Anomaly DR
Book-to-market	0.13	−0.23	−0.26	0.42**
	(0.15)	(0.17)	(0.13)	(0.12)
Profitability	−0.11	−0.03	0.02	0.04
	(0.11)	(0.13)	(0.11)	(0.12)
(−) Investment	−0.22*	−0.01	0.05	0.09
	(0.11)	(0.13)	(0.12)	(0.13)
(−) Size	0.09	−0.24	−0.29*	0.31**
	(0.17)	(0.15)	(0.13)	(0.11)
Momentum	−0.05	−0.12	0.28*	−0.21
	(0.18)	(0.16)	(0.14)	(0.12)
MVE ex market	−0.17	−0.24*	0.16	0.06
	(0.14)	(0.12)	(0.11)	(0.12)
MVE cum market	0.05	−0.20	0.23	0.90**
	(0.17)	(0.34)	(0.15)	(0.07)

Table 7 - Correlations of CF and DR News with Aggregate Metrics

The table shows correlations of the market and anomaly portfolios' cash flow and discount rate shocks with shocks to several aggregate metrics: one-year log real per capita consumption growth, one-year real per capita GDP growth, the one-year difference in the log labor share, three-year consumption growth (current and future two years), one-year log difference in Baker and Wurgler's (2006) sentiment index, the one-year change in the difference between Baa-rated and Aaa-rated corporate bond yields (default spread), and the one-year change in the difference between five-year and one-year zero-coupon Treasury bond yields (term spread). The shocks to the aggregate metrics are the residuals in a first-order autoregressive univariate model of the shock. The sample spans the years 1929 through 2017 but differs across variables because of data availability as described in the text. Standard errors that appear in parenthesis account for time-series variation in the shocks but do not account for estimation error in the VAR coefficients. The marks * and ** indicate significance at the 5 and 1 percent levels, respectively.

	1yr GDP Growth	1yr Cons. Growth	Labor Share	3yr Cons. Growth	Investor Sentiment	Default Spread	Term Spread
CF Correlations							
Market	0.37**	0.43**	−0.10	0.35**	0.14	−0.26**	−0.15
Book-to-market	0.21	0.12	−0.01	0.01	0.33*	−0.45**	−0.05
(−) Investment	−0.20	−0.28*	0.03	−0.20	0.33*	−0.09	0.13
Profitability	−0.26	−0.31*	−0.07	−0.18	0.06	0.06	0.19
(−) Size	−0.02	0.08	−0.21	0.11	−0.02	−0.33**	0.00
Momentum	−0.10	−0.09	0.14	0.08	−0.15	0.26*	0.00
MVE ex market	−0.27*	−0.30*	0.04	−0.15	0.08	0.04	0.14
MVE cum market	−0.26*	−0.29*	0.05	−0.09	0.11	0.11	0.14
DR Correlations							
Market	−0.29*	−0.37**	0.14	−0.18	−0.10	0.66**	−0.15
Book-to-market	0.27*	0.15	0.33**	0.33**	0.17	0.53**	−0.26*
(−) Investment	0.15	0.11	0.07	0.29*	−0.06	0.26**	−0.19
Profitability	−0.18	0.00	−0.09	−0.04	−0.18	0.03	−0.05
(−) Size	−0.09	−0.13	0.30**	0.07	−0.16	0.32**	−0.08
Momentum	−0.19	−0.13	−0.32**	−0.35**	−0.16	−0.12	0.24
MVE ex market	−0.19	−0.11	−0.15	−0.15	−0.31*	0.29**	0.06
MVE cum market	−0.38**	−0.39**	0.00	−0.26	−0.26	0.69**	−0.09

Table 8 - Realized versus VAR-implied Expected Anomaly Returns

The first row in Panel A displays the mean realized log returns of the long-short anomaly portfolios, the market portfolio, and the mean-variance efficient (MVE) combinations of these portfolios. The second row shows the expected log return of each of the portfolios from the baseline VAR in Tables 2 and 3. The third row shows a t-test of whether the return difference is significantly different from zero. The fourth row in Panel A reports the standard deviation of the conditional expected (annual) log return for each of the portfolios from the baseline VAR. Panel B shows the slope coefficients from regressions of future 10-year log returns on each of the portfolios on the VAR-implied long-run return estimate. Coefficient standard errors account for autocorrelation in the residuals up to 10 lags using the Newey-West method. The sample spans the years 1929 through 2017. The marks * and ** indicate significance at the 5 and 1 percent levels, respectively.

	Mkt.	B/M	Prof.	Inv.	Size	Mom.	MVE ex mkt.	MVE cum mkt.
Panel A: Mean log returns								
Anomaly mean return	3.4%	2.1%	3.3%	−3.2%	1.3%	4.6%	12.8%	16.2%
VAR expected return	3.4%	4.7%	2.6%	−2.7%	−0.8%	3.5%	11.8%	13.7%
t-stat for difference	0.00	−1.40	0.61	−0.52	1.51	0.71	0.41	1.03
St. dev. of expected return	8.5%	1.9%	2.2%	2.1%	2.4%	2.5%	5.5%	7.6%
Panel B: Long-run return prediction								
Slope coefficient	0.85	0.82	0.91	1.11	1.15	0.64	0.95	0.83
(standard error)	(0.11)	(0.75)	(0.20)	(0.46)	(0.29)	(0.19)	(0.31)	(0.19)
t-stat vs. 1	−1.42	−0.24	−0.46	0.25	0.51	−1.92	−0.17	−0.89
t-stat vs. 0	7.90**	1.09	4.59**	2.45*	4.01**	3.39**	3.08**	4.37**

Table 9 - Decompositions of Firm Return Variance from Alternative Specifications

The table displays variance decompositions of firm-level market-adjusted returns into CF and DR components for different specifications. Firms' market-adjusted returns are log firm returns minus market returns, where market returns are equal-weighted or value-weighted averages of log firm returns. Panel A reconciles Vuolteenaho (2002; V02) with the baseline results in this study (LT). Panel B reconciles Campbell, Polk, and Vuolteenaho (2010; CPV10) with LT. Panel C shows alternative specifications of LT. The first column is an explanatory note for the specification. In Panels A and B, we apply the explanatory notes sequentially. The 'Years' column shows the sample years; the 'Micro' column shows whether the sample includes firms in the bottom NYSE size quintile (microcaps); the 'Young' column shows whether the sample includes firms with brief accounting histories that V02 excludes; the column "Market weights" indicates whether market returns are equal-weighted or value-weighted averages of log firm returns. In Panel C, the "Baseline" specification repeats the results from Table 4; "No Depression" is a sample that excludes the Great Depression (1929-1938); "Accounting ROE" refers to computing cash flow news using expected accounting ROE rather than using the residual obtained from the present value restriction.

		Firms		Market	Variance Decomposition			
	Years	Micro	Young	weights	var(DR)	var(CF)	−2cov(DR,CF)	corr(DR,CF)
Panel A: V02 Specifications								
V02: Replication	1954-1996	Yes	No	EW	15%	118%	−33%	0.39
V02: All years	1929-2017	Yes	No	EW	7%	124%	−31%	0.52
V02: No microcaps	1929-2017	No	No	EW	4%	92%	4%	−0.11
V02: Young firms	1929-2017	No	Yes	EW	4%	89%	7%	−0.19
V02: Value-weight	1929-2017	No	Yes	VW	6%	84%	10%	−0.22
V02: LT predictors	1929-2017	No	Yes	VW	8%	72%	20%	−0.42
Panel B: CPV10 Specifications								
CPV10: Replication	1929-2000	Yes	Yes	EW	5%	105%	−10%	0.21
CPV10: All years	1929-2017	Yes	Yes	EW	5%	114%	−19%	0.39
CPV10: No microcaps	1929-2017	No	Yes	EW	4%	81%	15%	−0.41
CPV10: Value-weight	1929-2017	No	Yes	VW	6%	74%	20%	−0.48
CPV10: LT predictors	1929-2017	No	Yes	VW	8%	72%	20%	−0.42
Panel C: LT Specifications								
LT: Baseline	1929-2000	No	Yes	VW	8%	72%	20%	−0.42
LT: No Depression	1939-2017	No	Yes	VW	11%	71%	19%	−0.34
LT: Incl. microcaps	1929-2017	Yes	Yes	VW	14%	115%	−29%	0.36
LT: Accounting ROE	1929-2017	No	Yes	VW	8%	58%	3%	−0.06

Table 10 - Anomaly Variance Decompositions: Alternative Specifications

The table displays the variance decomposition of anomaly returns into CF and DR components for alternative specifications relative to Table 5. The "Baseline" specification repeats the results in Table 5. "No Depression" refers to a sample that excludes the Great Depression (1929-1938). "CF from accounting ROE" computes cash flow news based on expected return on equity from accounting statements instead of using the residual method that imposes the present value constraint. "Including microcaps" refers to a sample that includes firms in the bottom NYSE size quintile. Unless otherwise specified, the sample spans the years 1929 through 2017.

		Fraction of Portfolio Return Variance			
Anomaly	Specification	$var\,(DR)$	$var\,(CF)$	$-2cov\,(DR,CF)$	$corr\,(DR,CF)$
Book-to-market	Baseline: 1929-2017	7%	68%	25%	−0.56
	No Depression: 1939-2017	9%	77%	14%	−0.28
	CF from accounting ROE	7%	60%	10%	−0.24
	Including microcaps	7%	97%	−4%	0.08
Profitability	Baseline: 1929-2017	14%	80%	6%	−0.10
	No Depression: 1939-2017	11%	63%	26%	−0.50
	CF from accounting ROE	14%	119%	−27%	0.33
	Including microcaps	29%	125%	−52%	0.45
Size	Baseline: 1929-2017	7%	64%	29%	−0.68
	No Depression: 1939-2017	9%	63%	28%	−0.61
	CF from accounting ROE	7%	32%	11%	−0.36
	Including microcaps	5%	89%	6%	−0.14
Momentum	Baseline: 1929-2017	7%	70%	23%	−0.55
	No Depression: 1939-2017	10%	70%	20%	−0.39
	CF from accounting ROE	7%	41%	4%	−0.13
	Including microcaps	7%	102%	−9%	0.17
Investment	Baseline: 1929-2017	14%	78%	7%	−0.10
	No Depression: 1939-2017	8%	68%	24%	−0.50
	CF from accounting ROE	14%	79%	−26%	0.38
	Including microcaps	60%	184%	−143%	0.68
MVE ex market	Baseline: 1929-2017	7%	73%	20%	−0.43
	No Depression: 1939-2017	8%	70%	22%	−0.45
	CF from accounting ROE	7%	61%	−6%	0.15
	Including microcaps	13%	105%	−19%	0.25
MVE cum market	Baseline: 1929-2017	36%	69%	−5%	0.05
	No Depression: 1939-2017	60%	74%	−34%	0.26
	CF from accounting ROE	36%	85%	−17%	0.16
	Including microcaps	51%	98%	−49%	0.35

Figure 1 - Anomaly Return Variance Decompositions

This figure visually represents the return variance decomposition of the five individual long-short anomaly portfolios shown in Table 5. 'DR' stands for discount rate news, and 'CF' stands for cash flow news. The sample is 1929 to 2017.

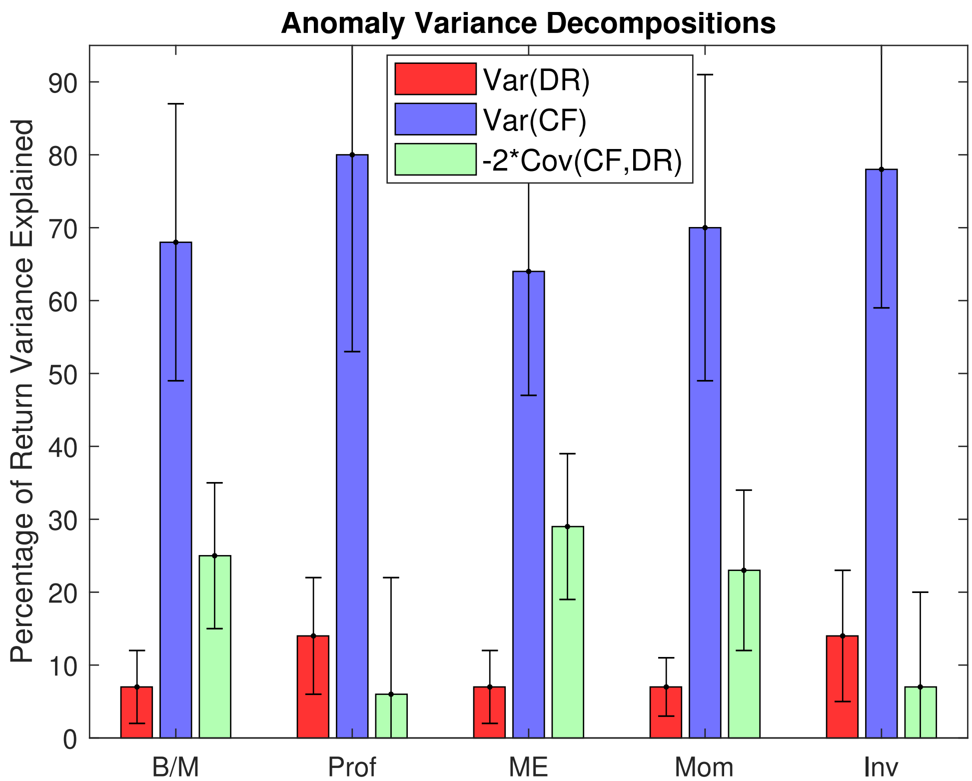


Figure 2 - Market and MVE Return Decompositions

This figure visually represents the return variance decomposition of the market portfolio and two version of the MVE portfolio, with and without the market as explained in the text, as shown in Tables 4 and 5. 'DR' stands for discount rate news, and 'CF' stands for cash flow news. The sample is 1929 to 2017.

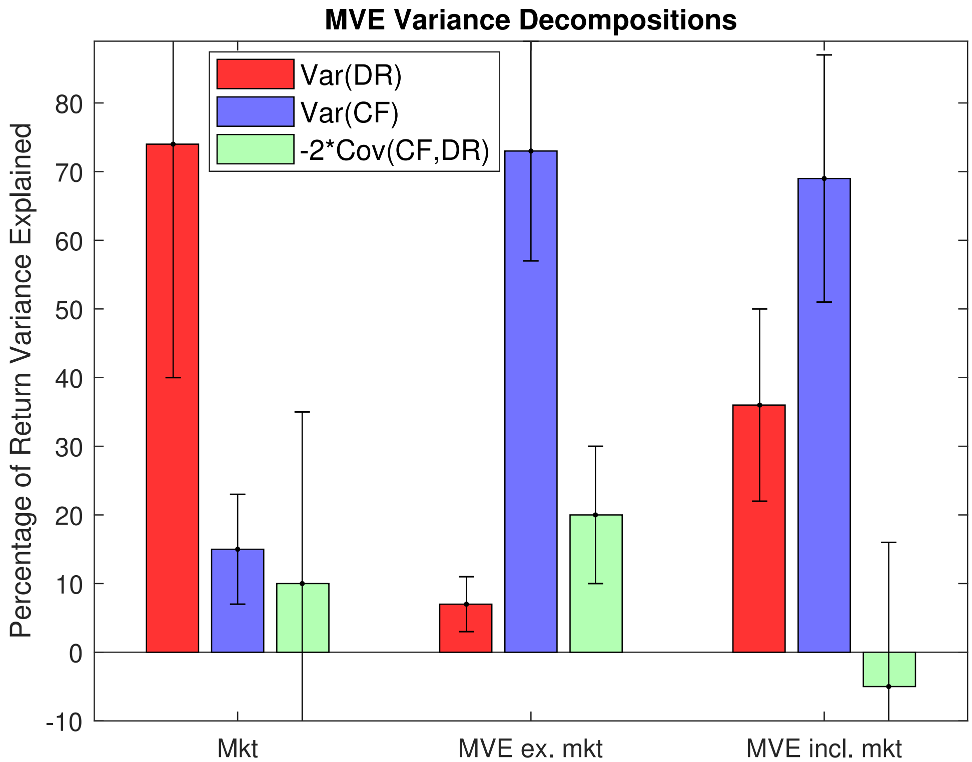


Figure 3 - Anomaly vs. Market CF and DR Correlations

This figure visually represents the correlations between anomaly and market cash flow (CF) and discount rate (DR) news shown in Table 6. The sample is from 1929 to 2017.

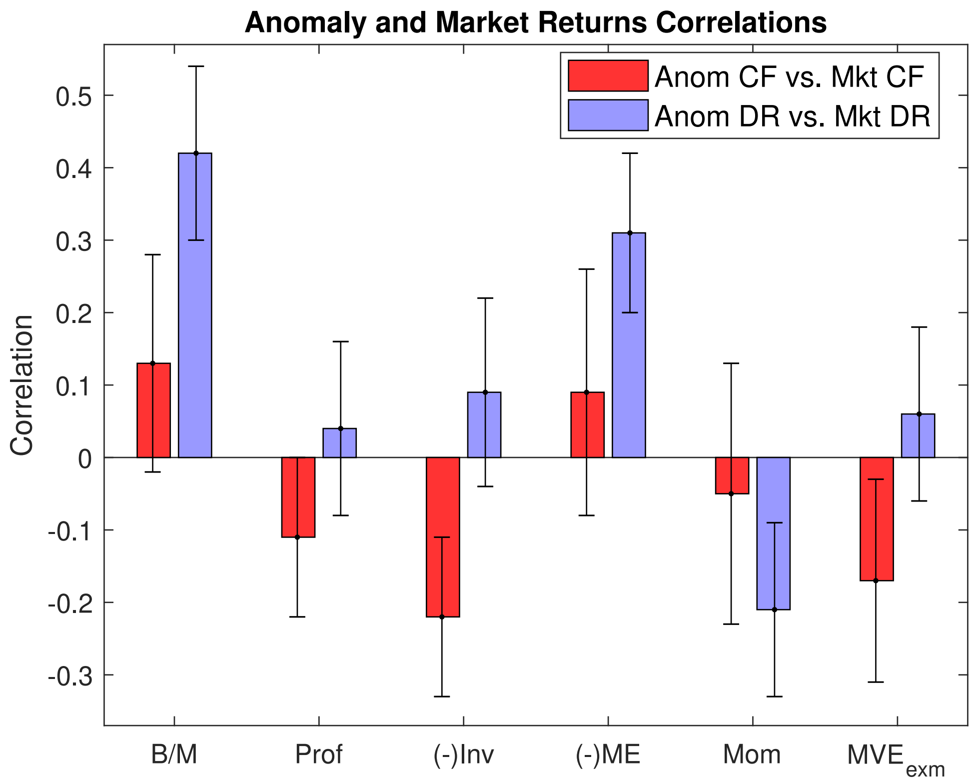
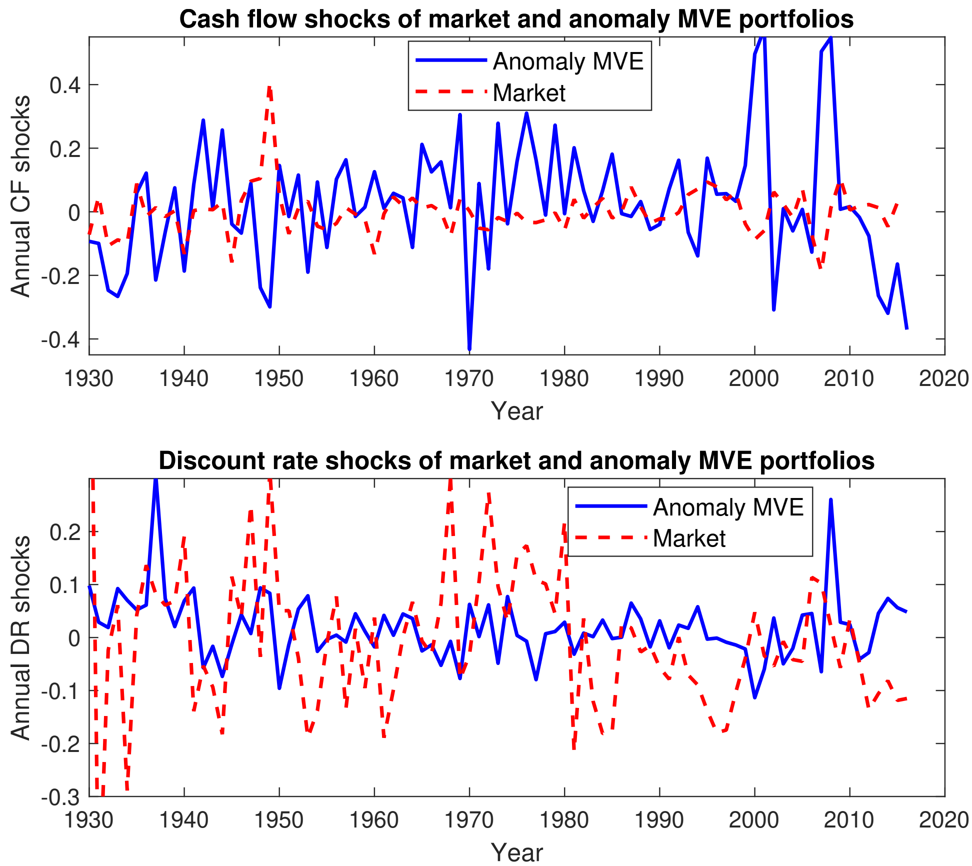


Figure 4 - Market CF and DR News vs. Anomaly MVE CF and DR News

The top plot shows the cash flow (CF) news from the market and the anomaly mean-variance efficient (MVE) portfolio, which combines only the long-short anomaly portfolios and uses in-sample MVE weights. The bottom plot shows analogous discount rate (DR) news. The sample is annual from 1929 through 2017.



Internet Appendix for "What Drives Anomaly Returns?"

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Table of Contents

A Cash Flows vs. Discount Rates of Trading Strategies

Here we show that the cash flows and discount rates of rebalanced portfolios, such as anomaly portfolios, can differ substantially from those of the underlying firms in the portfolios. We provide examples below in which firms have constant cash flows, but all variation in returns to the rebalanced portfolio comes from cash flow shocks.

We first consider a stylized behavioral model of stock returns and cash flows. Assume that all firms pay constant dividends:

$$D_{i,t} = \bar{d}. \tag{A.1}$$

Assume that investors in each period erroneously believe that any given firm's dividend is permanently either $d_L < \bar{d}$ or $d_H > \bar{d}$. We define the firms associated with low (high) dividend beliefs to be value (growth) firms. The pricing of these firms satisfies:

$$P^{\text{value}} = \frac{d_L}{R-1}, \tag{A.2}$$

$$P^{\text{growth}} = \frac{d_H}{R-1}, \tag{A.3}$$

where R is the gross risk-free rate. Each period, with probability q, investors switch their beliefs about each stock's dividends either from d_L to d_H or from d_H to d_L. Investors believe their beliefs will last forever, whereas in reality they will switch with probability q in each period.

Now consider a value fund that invests only in stocks that investors currently believe will pay dividends of d_L. Further assume that there are only two firms in the economy—a firm that currently is a growth firm and a firm that currently is a value firm. When beliefs switch, the growth firm becomes a value firm and vice versa. This switch therefore induces trading in the value fund as the fund has to sell firms that become growth firms and buy the new value firms.

Such trading has a significant impact on the fund's dividends. Suppose that the fund initially holds one share of the value stock, which implies that its initial wealth is $W_0 = P^{\text{value}}$. Assume investors do not switch beliefs in the next period. In this case, the fund's gross return is:

$$\begin{aligned} R_1^{\text{value}} &= \frac{P^{\text{value}} + \bar{d}}{P^{\text{value}}} \\ &= 1 + \frac{\bar{d}}{P^{\text{value}}}. \end{aligned} \tag{A.4}$$

Period 1 cum-dividend wealth is

$$W_1^{cum} = P^{\text{value}} + \bar{d}, \tag{A.5}$$

where ex-dividend wealth is P^{value} and dividend is $d_1 = \bar{d}$. Assume that beliefs switch in period 2. Then:

$$\begin{aligned} R_2^{\text{value}} &= \frac{(R-1)\left(\frac{d_H}{R-1} + \bar{d}\right)}{d_L} \\ &= \frac{d_H + (R-1)\,\bar{d}}{d_L} \\ &= \frac{d_H}{d_L} + \frac{\bar{d}}{P^{\text{value}}}. \end{aligned} \tag{A.6}$$

So fund wealth becomes:

$$W_2^{cum} = P^{\text{value}} \frac{d_H}{d_L} + \bar{d}.$$

Ex-dividend wealth is $W_2^{ex} = P^{\text{value}} \frac{d_H}{d_L}$, and the dividend is $d_2 = \bar{d}$ once again. The dividend price ratio of the strategy is now:

$$\frac{W_2^{ex}}{d_2} = \frac{P^{\text{value}}}{\bar{d}} \frac{d_H}{d_L} > \frac{P^{\text{value}}}{\bar{d}}. \tag{A.7}$$

The higher price-dividend ratio reflects high expected dividend growth next period.

Importantly, the fund now reinvests its capital gain into the current value stock and is able to purchase more than one share. Assuming beliefs do not switch in period 3, the fund's wealth increases to:

$$\begin{aligned} W_3^{cum} &= P^{\text{value}}\frac{d_H}{d_L}\left(1+\frac{\bar{d}}{P^{\text{value}}}\right) \\ &= P^{\text{value}}\frac{d_H}{d_L}+\frac{d_H}{d_L}\bar{d}. \end{aligned} \tag{A.8}$$

Now ex-dividend wealth is $W_3^{ex} = P^{\text{value}}\frac{d_H}{d_L}$ and $d_3 = \bar{d} \times d_H/d_L$, implying that dividend growth during this period is high as $d_3/d_2 = d_H/d_L > d_2/d_1 = 1$. The price-dividend ratio of the strategy is now:

$$\frac{W_3^{ex}}{d_3} = \frac{P^{\text{value}}\frac{d_H}{d_L}}{\bar{d} \times d_H/d_L} = \frac{P^{\text{value}}}{\bar{d}}, \tag{A.9}$$

meaning that the price-dividend ratio returns to its original value.

In summary, dividend growth of the dynamic value strategy varies over time, but expected returns to the strategy are constant and given by:

$$\begin{aligned} E(R^{\text{value}}) &= 1-q+q\frac{d_H}{d_L}+\frac{\bar{d}}{P^{\text{value}}} \\ &= 1-q+q\frac{d_H}{d_L}+\frac{\bar{d}}{d_L}(R-1). \end{aligned} \tag{A.10}$$

A symmetric argument applies to the analogous growth strategy, which also has time-varying dividend growth and constant expected returns. We conclude that return variation in the dynamic trading strategies arises solely because of cash flow shocks even though all firms in the economy incur only discount rate shocks. Firm-level return variation is driven by changes in firms' expected returns, not their dividends—which are constant.

There are no discount rate shocks to the returns of these dynamic strategies when viewed from the perspective of an investor who invests in the value or growth funds. However, unexpected returns to such funds are in fact, under the objective measure of the econometrician, due to discount rate shocks to the underlying firms. The firms' actual expected returns vary, whereas their dividend growth does not.

This feature of rebalanced portfolios is not limited to the case of time-varying mispricing. Consider a risk-based model in which value firms have riskier cash flows than growth firms. If time variation in a firm's cash flow risk causes it to switch between being a value firm and a growth firm, the risk-based model delivers the same insights as the behavioral model discussed above.

In this example, we assume firms' log dividend growth is:

$$\Delta d_{i,t+1} = -\frac{1}{2}\sigma^2 + \sigma\left(\rho_{s_{i,t}}\varepsilon_{m,t+1} + \sqrt{1-\rho^2_{s_{i,t}}}\varepsilon_{i,t+1}\right), \tag{A.11}$$

where $\varepsilon_{m,t+1}$ and $\{\varepsilon_{i,t+1}\}_i$ are uncorrelated standard normally distributed shocks representing aggregate and firm-specific dividend shocks, respectively. Firm exposure to aggregate dividend shocks is:

$$\rho_{s_{i,t}} = \begin{cases} \rho^H & \text{if } s_{i,t} = 1 \\ \rho^L & \text{if } s_{i,t} = 0 \end{cases}, \tag{A.12}$$

where $s_{i,t}$ follows a two-state Markov process where $\Pr\{s_{i,t+1} = 1 | s_{i,t} = 0\} = \Pr\{s_{i,t+1} = 0 | s_{i,t} = 1\} = \pi$. For ease of exposition, set $\rho^L = 0$ and $\rho^H = 1$. Initially, half of firms are in state 1, while the other half are in state 0. If a regime change occurs, all firms currently in state 1 switch to state 0, and vice versa.

The log stochastic discount factor is:

$$m_{t+1} = -\frac{1}{2}\gamma^2\sigma^2 - \gamma\sigma\varepsilon_{m,t+1}, \tag{A.13}$$

where we implicitly assume a zero risk-free rate and where $\gamma > 0$ represents risk aversion. These assumptions imply that the conditional mean and volatility of cash flow growth is constant. However, firm risk varies with $s_{i,t}$, which determines the covariance of cash flows with the pricing kernel, causing time-varying firm risk premiums.

Solving for the price-dividend ratio as a function of the state yields:

$$\begin{aligned} PD\left(s_{i,t}\right) &= E_t\left[e^{-\frac{1}{2}\gamma^2\sigma^2-\gamma\sigma\varepsilon_{m,t+1}-\frac{1}{2}\sigma^2+\sigma\left(\rho_{s_{i,t}}\varepsilon_{m,t+1}+\sqrt{1-\rho^2_{s_{i,t}}}\varepsilon_{i,t+1}\right)}\left(1+PD\left(s_{i,t+1}\right)\right)\right] \\ &= e^{-\gamma\sigma^2\rho_{s_{i,t}}}\left(1+\pi PD\left(s_{i,t+1}\neq s_{i,t}\right)+\left(1-\pi\right)PD\left(s_{i,t+1}=s_{i,t}\right)\right). \end{aligned} \tag{A.14}$$

Denote the price-dividend ratio in state j as PD_j. The price-dividend relation above is a system with two equations and two unknowns with the solution:

$$PD_1 = \frac{1}{e^{\gamma\sigma^2}-1} \tag{A.15}$$

$$PD_0 = 1/\pi + \frac{1}{e^{\gamma\sigma^2}-1} \tag{A.16}$$

These equations show that price-dividend ratios are higher in state 0 when dividend risk is low than in state 1 when dividend risk is high, implying that expected returns are higher in state 0 as expected dividend growth is constant across states. Firms' expected net returns are:

$$E_t\left[R_{i,t+1}|s_{i,t}=0\right]-1 = 0, \tag{A.17}$$

$$E_t\left[R_{i,t+1}|s_{i,t}=1\right]-1 = 2(e^{\gamma\sigma^2}-1). \tag{A.18}$$

Since $PD_0 > PD_1$, we see that $E_t\left[R_{i,t+1}|s_{i,t}=1\right] > E_t\left[R_{i,t+1}|s_{i,t}=0\right]$. Thus, firms' price-dividend ratios fluctuate because of shocks to discount rates, not cash flows. Although there are cash flow shocks in returns arising from the contemporaneous dividend shock $(\sigma\left(\rho_{s_{i,t}}\varepsilon_{m,t+1}+\sqrt{1-\rho^2_{s_{i,t}}}\varepsilon_{i,t+1}\right))$, dividends are unpredictable and therefore do not induce time-variation in the price-dividend ratio.

Now consider a value mutual fund that in each period buys firms that are currently in the low valuation state 1. With probability π, value firms held by the fund will switch to the high valuation state 0, meaning that they become growth firms. The fund sells all firms in each period and reinvests the proceeds in firms that are in the low valuation state 1.

The fund pays out all firm dividends as they occur. The expected return to this strategy is constant and equal to $E_t\left[R_{i,t+1}|s_{i,t}=1\right]-1=2(e^{\gamma\sigma^2}-1)$, even though all firms' expected returns vary over time.

We now analyze the growth of the value fund's dividends in each period. The first source of fund dividend growth is growth in the underlying firms' dividends, which satisfy:

$$\frac{D_{i,t+1}}{D_{i,t}}=e^{-\frac{1}{2}\sigma^2+\sigma\varepsilon_{m,t+1}}. \tag{A.19}$$

The second source of fund dividend growth is growth in the number of shares of value firms held by the fund. If value firms switch to growth firms, the fund will reap a capital gain and be able to buy more shares of the new value firms in the following period. Define the indicator variable $\mathbf{1}_{s_{i,t}\neq s_{i,t-1}}$ as equal to 1 if there was a regime shift from period $t-1$ to period t and 0 otherwise. Accounting for both sources of growth, fund dividend growth is:

$$\frac{D_{t+1}^{Fund}}{D_t^{Fund}}=\mathbf{1}_{s_{i,t}\neq s_{i,t-1}}\frac{PD_0}{PD_1}e^{-\frac{1}{2}\sigma^2+\sigma\varepsilon_{m,t+1}}+\left(1-\mathbf{1}_{s_{i,t}\neq s_{i,t-1}}\right)e^{-\frac{1}{2}\sigma^2+\sigma\varepsilon_{m,t+1}}, \tag{A.20}$$

where the term $\frac{PD_0}{PD_1}=1+\frac{e^{\gamma\sigma^2}-1}{\pi}$ represents the capital gain from the prior period. Dividends are predictably high after high capital gains and low after low capital gains. The predictability in dividend growth leads to a time-varying price-dividend ratio for the mutual fund, even though its expected return is constant. Thus, discount rate shocks to the underlying value firms are cash flow shocks for the mutual fund implementing a value trading strategy.

B Relation to Equilibrium Models

The VAR offers a parsimonious, reduced-form model of the cross-section of expected cash flows and discount rates at all horizons. Here we demonstrate that the VAR specification is related to standard asset pricing models. In well-known models such as Campbell and Cochrane's (1999) habit formation model and Bansal and Yaron's (2004) long-run risk model, the log stochastic discount factor is conditionally normally distributed and satisfies:

$$m_{t+1} = -r_{f,t} - \frac{1}{2} \left\| \lambda_t \right\|^2 + \lambda_t' \eta_{t+1}, \tag{B.1}$$

where λ_t is a $K \times 1$ vector of conditional risk prices, η_{t+1} is a $K \times 1$ vector of standard normal shocks, and $r_{f,t}$ is the risk-free rate. With conditionally normal log returns, applying the Law of One Price yields the following expression for the conditional expected log return of firm i:

$$\begin{aligned} E_t \left[r_{i,t+1} \right] &= r_{f,t} - \frac{1}{2} v_{i,t} + cov_t \left(m_{t+1}, r_{i,t+1} \right) \\ &= r_{f,t} - \frac{1}{2} v_{i,t} + \beta_{i,t}' \lambda_t, \end{aligned} \tag{B.2}$$

where $v_{i,t} \equiv var_t \left(r_{i,t+1} \right)$ is firm return variance, and $\beta_{i,t}^{(k)} = \frac{cov_t \left(\lambda_t^{(k)} \eta_{t+1}^{(k)}, r_{i,t+1} \right)}{var_t \left(\lambda_t^{(k)} \eta_{t+1}^{(k)} \right)}$ and $\beta_{i,t} = \left[\beta_{i,t}^{(1)} \;\; \beta_{i,t}^{(2)} \;\; ... \;\; \beta_{i,t}^{(K)} \right]'$ represent firm betas.

We make simplifying assumptions to relate this setup to the VAR specification. Define firm risk premiums as $z_{i,t}^{(k)} \equiv \beta_{i,t}^{(k)} \lambda_t^{(k)}$ and $z_{i,t} = \left[z_{i,t}^{(1)} \;\; z_{i,t}^{(2)} \;\; ... \;\; z_{i,t}^{(K)} \right]'$. Suppose that risk premiums, variances, and the risk-free rate evolve according to:

$$z_{i,t+1} = \bar{z} + A_z \left(z_{i,t} - \bar{z} \right) + \Sigma_{z,t} \varepsilon_{i,t+1}^{z}, \tag{B.3}$$

$$v_{i,t+1} = \bar{v} + A_v \left(v_{i,t} - \bar{v} \right) + \sigma_{v,t} \varepsilon_{i,t+1}^{v}, \tag{B.4}$$

$$r_{f,t+1} = \bar{r}_f + A_{r_f} \left(r_{f,t} - \bar{r}_f \right) + \sigma_{r,t} \varepsilon_{t+1}^{r_f}, \tag{B.5}$$

for all firms i. Assume firm log return on equity is also conditionally normal:

$$e_{i,t+1} = \mu + x_{i,t} + \sigma_{e,t}\varepsilon^{e}_{i,t+1}, \quad \text{(B.6)}$$

$$x_{i,t+1} = A_x x_{i,t} + \Sigma_{x,t}\varepsilon^{x}_{i,t+1}, \quad \text{(B.7)}$$

where $x_{i,t}$ is an $L \times 1$ vector of latent state variables determining expected return on equity. All shocks can be correlated.

Assuming the clean-surplus model described earlier, firm book-to-market ratios are given by:

$$bm_{i,t} = a_0 + a_1' r_{f,t} + a_2' z_{i,t} + a_3' x_{i,t} + a_4 v_{i,t}. \quad \text{(B.8)}$$

Define the $(2K + L + 1) \times 1$ vector $s_{i,t} = \left[r_{f,t}' \;\; z_{i,t}' \;\; v_{i,t} ... x_{i,t}' \right]'$ to consist of the stacked state variables. We assume there exist $(2K + L + 1)$ observed characteristics, $\xi_{i,t}$, that span $s_{i,t}$:

$$\xi_{i,t} = A_1 + A_2 s_{i,t}, \quad \text{(B.9)}$$

where A_2 is invertible. With the characteristic spanning assumption, firms' book-to-market become a function of the observed characteristics, resulting in a VAR representation of the present-value relation. In sum, the VAR specification concisely summarizes the dynamics of expected cash flows and discount rates, even when both consist of multiple components fluctuating at different frequencies. The VAR yields consistent estimates even though there is heteroskedasticity across firms and time.

When analyzing long-short portfolios, we obtain the anomaly cash flow (discount rate) shock as the difference in the cash flow (discount rate) shocks between the long and short portfolios. Taking the value anomaly as an example, suppose the long value portfolio and short growth portfolio have the same betas with respect to all risk factors except the value factor (say, $\lambda_t^{(2)}$). According to Equation $(B.2)$, discount rate shocks to this long-short portfolio can only arise from three sources: 1) shocks to the spread in the factor exposure between value and growth firms ($\beta^{(2)}_{\text{value},t} - \beta^{(2)}_{\text{growth},t}$); 2) shocks to the price of risk of the value factor ($\lambda_t^{(2)}$); or 3) shocks to the difference in return variance between the two portfolios. The third possibility arises because we analyze log returns. Similarly, cash flow shocks to this long-short portfolio only reflect these portfolios' differential exposure to cash flow factors.

C VAR and Return Decomposition Standard Errors

Each statistic, s, that we estimate, such as the correlation between cash flow and discount rate shocks, is a function of our VAR coefficient matrix (A), which we also estimate. We account for two sources of estimation uncertainty in $s(A)$. First, since we observe a finite sample of T years of aggregate shocks, there is sampling variation in s conditional on A, which we write as $Var(s|A)$. Second, to the extent that values of s depend on the values of A, uncertainty in A estimates induces uncertainty in $s(A)$, which we write as $Var(s(A))$. We compute $Var(s|A)$ using standard formulas for uncertainty in the appropriate statistic, depending on whether s is a mean, variance, covariance, or correlation, under the assumption that shocks to s are independent over time. We compute $Var(s(A))$ by applying the delta method to s and the covariance matrix of estimated VAR coefficients $Var(A)$. Lastly, we combine the two sources of estimation uncertainty to get the standard error of s from the relationship:

$$SE(s) = \sqrt{Var(s)} = \sqrt{Var(s|A) + Var(s(A))}, \tag{C.1}$$

where the second variance term satisfies the delta method equation:

$$Var(s(A)) = \left(\frac{\partial s}{\partial A}\right)' Var(A) \frac{\partial s}{\partial A}, \tag{C.2}$$

where $\frac{\partial s}{\partial A}$ is a gradient vector representing the sensitivity of s to each coefficient in the A matrix and $Var(A)$ is the covariance matrix for all VAR coefficients across all equations. We compute the sensitivity of s to each element in A using numerical derivatives. The next section explains how we compute the covariance matrix of VAR coefficients.

C.1 VAR Standard Errors

We estimate the covariance matrix of VAR coefficients, $Var(A)$, using standard regression methods. To obtain unbiased point estimates of each row of A coefficients, we separately estimate regressions for each firm-level and aggregate variable in the VAR. Denote the variables in an arbitrary row representing a firm-level regression by the subscript m, so we can express the regression as $y_m = X_m\beta_m + \varepsilon_m$. The point estimates of the A coefficients in this row satisfy the standard ordinary least squares (OLS) formulas:

$$b_{ma} = \left(X_m'X_m\right)^{-1} X_m'y_m \tag{C.3}$$

$$= \beta_m + \left(X_m'X_m\right)^{-1} X_m'\varepsilon_m. \tag{C.4}$$

The panel of firm-level observations is unbalanced with N_t firms at each time t. There are K regressors including an intercept. Let $x_{it}^{(m)}$ denote the $K \times 1$ vector with the regressors for firm i at time t for row m in the VAR. Let X_m be the set of all regressors for all firms and times in equation m:

$$\underset{(\tau\times K)}{X_m} = \left[\begin{array}{ccccccccc} x_{11}^{(m)} & \cdots & x_{N_1 1}^{(m)} & x_{12}^{(m)} & \cdots & x_{N_2 2}^{(m)} & \cdots & x_{1T}^{(m)} & \cdots & x_{N_T T}^{(m)} \end{array}\right]',$$

where $\tau = \Sigma_{t=1}^{T} N_t$.

The covariance matrix of the estimated row m in the A matrix, b_m, is then:

$$Var\left(b_m\right) = E\left[\left(X_m'X_m\right)^{-1} X_m'\varepsilon_m\varepsilon_m'X_m' \left(X_m'X_m\right)^{-1}\right]. \tag{C.5}$$

We compute the covariance between estimates of two rows, b_m and b_n, of the A matrix, where rows m and n both represent firm-level regressions, using:

$$Cov\left(b_m, b_n\right) = E\left[\left(X_m'X_m\right)^{-1} X_m'\varepsilon_m\varepsilon_n'X_n' \left(X_n'X_n\right)^{-1}\right]. \tag{C.6}$$

The A matrix also contains rows based on regressions with aggregate variables as the independent and the dependent variables. There can be different numbers of regressors in these aggregate regressions than in the firm-level regressions. Consider the coefficients, b_a, from an arbitrary row a in the A matrix representing an aggregate regression in the VAR, $y_a = X_a\beta_a + \varepsilon_a$, where X_a is a $T \times K_A$ matrix of aggregate regressors and ε_a is the $T \times 1$

vector of aggregate residuals. Then the $K \times K_A$ covariance matrix of firm-level coefficients, b_m, and aggregate coefficients, b_a, is:

$$Cov(b_m, b_a) = E\left[(X_m' X_m)^{-1} X_m' \varepsilon_m \varepsilon_a' X_a' (X_a' X_a)^{-1}\right]. \tag{C.7}$$

The covariance between two rows of aggregate coefficients, a and j, is:

$$Cov(b_j, b_a) = E\left[\left(X_j' X_j\right)^{-1} X_j' \varepsilon_j \varepsilon_a' X_a' (X_a' X_a)^{-1}\right]. \tag{C.8}$$

C.2 Equal-weighting the VAR Across Time

Since the panel is unbalanced, we assign weights to the firm-level observations so that the sum of weights across firms is equal at each time t. Let W be a $\tau \times \tau$ diagonal matrix with diagonal vector, w, defined as:

$$w = \begin{bmatrix} \frac{\mathbf{1}_{N_1}}{N_1} & \frac{\mathbf{1}_{N_2}}{N_2} & \cdots & \frac{\mathbf{1}_{N_T}}{N_T} \end{bmatrix},$$

where $\mathbf{1}_z$ is a $1 \times z$ vector of ones. We compute weighted versions of the firm-level estimates and standard errors by modifying the formulas with appropriate insertions of the W matrix. Specifically, we replace all $X_m' X_m$ terms with $X_m' W X_m$ and all $X_m' \varepsilon_m$ terms with $X_m' W \varepsilon_m$. The resulting weighted estimates of firm-level VAR coefficients are:

$$\begin{aligned} b_m &= (X_m' W X_m)^{-1} X_m' W y_m && \text{(C.9)} \\ &= \beta_m + (X_m' W X_m)^{-1} X_m' W \varepsilon_m. && \text{(C.10)} \end{aligned}$$

The weighted standard errors for firm-level coefficients are:

$$\begin{aligned} Var(b_m) &= E\left[(X_m' W X_m)^{-1} X_m' W \varepsilon_m \varepsilon_m' W X_m' (X_m' W X_m)^{-1}\right], && \text{(C.11)} \\ Cov(b_m, b_n) &= E\left[(X_m' W X_m)^{-1} X_m' W \varepsilon_m \varepsilon_n' W X_n' (X_n' W X_n)^{-1}\right]. && \text{(C.12)} \end{aligned}$$

The weighted covariance between firm-level estimates, b_m, and aggregate estimates, b_a, is:

$$Cov\left(b_m, b_a\right) = E\left[\left(X_m' W X_m\right)^{-1} X_m' W \varepsilon_m \varepsilon_a' X_a' \left(X_a' X_a\right)^{-1}\right]. \tag{C.13}$$

The firm-level weighting does not affect covariance terms involving just aggregate coefficients.

D Portfolio Aggregation

In the main paper, we aggregate from firm-level to various portfolios (the market and anomaly portfolios) by value-weighting log return components to be consistent with prior literature and to simplify the analysis. In reality, of course, a portfolio return is a weighted average of simple (not log) returns. In this section, we construct an aggregation method that works for simple returns and show that this alternative anomaly return decomposition produces the same conclusions as the simpler approach in the main paper.

In particular, we approximate each firm's simple return using a second-order Taylor expansion around its current expected log return and then aggregate shocks to firms' simple returns using portfolio weights in the usual way.

The first step in this process is to express simple (gross) returns in terms of the components of log returns using:

$$\begin{aligned} R_{i,t+1} &\equiv \exp\left(r_{i,t+1}\right) \\ &= \exp\left(E_t r_{i,t+1}\right)\exp\left(CF_{i,t+1}^{shock} - DR_{i,t+1}^{shock}\right), \end{aligned} \tag{D.1}$$

where $E_t r_{i,t+1}$ is the predicted log return and $CF_{i,t}^{shock}$ and $DR_{i,t}^{shock}$ are estimated shocks from firm-level VAR regressions in which we impose the present-value relation. A second-order expansion around zero for both shocks yields:

$$R_{i,t+1} \approx \exp\left(E_t r_{i,t+1}\right)\left\{1 + CF_{i,t+1}^{shock} + \frac{1}{2}\left(CF_{i,t+1}^{shock}\right)^2 - DR_{i,t+1}^{shock} + \frac{1}{2}\left(DR_{i,t+1}^{shock}\right)^2 + CF_{i,t+1}^{shock} DR_{i,t+1}^{shock}\right\}. \tag{D.2}$$

Later we show that this approximation works well in practice. Next we define the cash flow and discount rate shocks to simple firm returns as:

$$CF_{i,t+1}^{simple\_shock} \equiv \exp\left(E_t r_{i,t+1}\right)\left\{CF_{i,t+1}^{shock} + \frac{1}{2}\left(CF_{i,t+1}^{shock}\right)^2\right\}, \tag{D.3}$$

$$DR_{i,t+1}^{simple\_shock} \equiv \exp\left(E_t r_{i,t+1}\right)\left\{DR_{i,t+1}^{shock} - \frac{1}{2}\left(DR_{i,t+1}^{shock}\right)^2\right\}, \tag{D.4}$$

$$CFDR_{i,t+1}^{cross} \equiv \exp\left(E_t r_{i,t+1}\right) CF_{i,t+1}^{shock} DR_{i,t+1}^{shock}. \tag{D.5}$$

For a portfolio with weights $\omega_{i,t}^{p}$ on firms, we can approximate the simple portfolio return using:

$$R_{p,t+1} - \sum_{i=1}^{n} \omega_{i,t}^{p} \exp\left(E_t r_{i,t+1}\right) \approx CF_{p,t+1}^{simple\_shock} - DR_{p,t+1}^{simple\_shock} + CFDR_{p,t+1}^{cross}, \tag{D.6}$$

where

$$CF_{p,t+1}^{simple\_shock} = \sum_{i=1}^{n} \omega_{i,t}^{p} CF_{i,t+1}^{simple\_shock}, \tag{D.7}$$

$$DR_{p,t+1}^{simple\_shock} = \sum_{i=1}^{n} \omega_{i,t}^{p} DR_{i,t+1}^{simple\_shock}, \tag{D.8}$$

$$CFDR_{p,t+1}^{cross} = \sum_{i=1}^{n} \omega_{i,t}^{p} CFDR_{i,t+1}^{cross}. \tag{D.9}$$

Since they are sums over individual firms' simple cash flow and discount rate shocks, the portfolio cash flow and discount rate shocks account for the conditional covariance structure of the shocks. We decompose the variance of portfolio returns using

$$\begin{aligned} var\left(\tilde{R}_{p,t+1}\right) \approx\; & var\left(CF_{p,t+1}^{simple\_shock}\right) + var\left(DR_{p,t+1}^{simple\_shock}\right) \\ & -2cov\left(CF_{p,t+1}^{simple\_shock}, DR_{p,t+1}^{simple\_shock}\right) \\ & +var\left(CFDR_{p,t+1}^{cross}\right), \end{aligned} \tag{D.10}$$

where $\tilde{R}_{p,t+1} \equiv R_{p,t+1} - \sum_{i=1}^{n} \omega_{i,t}^{p} \exp\left(E_t r_{i,t+1}\right)$. We ignore covariance terms involving $CFDR_{p,t+1}^{cross}$ as these are very small in practice. When analyzing cash flow and discount rate shocks to long-short portfolios, we obtain the anomaly cash flow (discount rate) shock as the difference in the cash flow (discount rate) shocks to the long and short portfolios.

Table D.1 shows that the resulting return variance decompositions are close to those in Table 5 in the main paper. Furthermore, the second-order approximation above is accurate in the sense that the correlation between predicted anomaly returns from this second-order approximation ($r = E[r] + CF - DR$) is highly correlated with the actual anomaly portfolio return. In sum, we conclude that value-weighting log returns instead of simple returns does not materially affect our conclusions.

Table D.1 - Anomaly Variance Decompositions: Simple Returns

This table shows decompositions of the variance of simple anomaly returns into cash flow (CF) and discount rate (DR) components from the baseline panel VAR of Table 2. As described in Internet Appendix D, we apply a second-order Taylor approximation to estimate the CF and DR components of firms' simple returns from the CF and DR components of firms' log returns. We weight firms' simple CF and DR news to obtain simple anomaly CF and DR news. The anomaly return is the difference between the return of the top quintile portfolio and the return of the bottom quintile portfolio, where the quintile sort is based on the relevant characteristic. The main columns show the variance decomposition of anomalies' approximate simple returns. The last column shows correlations between our approximations of simple anomaly returns and actual simple anomaly returns. We also show the variance decomposition of simple returns to the in-sample MVE portfolio based on the quintile long-short anomaly portfolios. The sample spans the years 1929 through 2017. Standard errors appear in parentheses.

	Fraction of Portfolio Return Variance				
	$var\,(DR)$	$var\,(CF)$	$-2cov\,(DR, CF)$	$corr\,(DR, CF)$	$Corr(Pred, Act)$
Book-to-market	6%	46%	25%	−0.77	0.99
	(4%)	(21%)	(15%)	(0.08)	(0.02)
Profitability	11%	60%	8%	−0.08	0.99
	(4%)	(19%)	(13%)	(0.14)	(0.01)
Size	5%	48%	25%	−0.82	0.99
	(4%)	(23%)	(14%)	(0.08)	(0.02)
Momentum	4%	50%	19%	−0.69	1.00
	(2%)	(23%)	(11%)	(0.15)	(0.01)
Investment	8%	60%	8%	−0.19	0.99
	(4%)	(19%)	(13%)	(0.15)	(0.01)
MVE ex market	9%	72%	26%	−0.51	0.99
	(5%)	(16%)	(9%)	(0.13)	(0.02)

E Alternative Measure of Cash Flow News

Here we compare our main cash flow measure, CF, based on the present-value identity in Equations (10) and (11) to an alternative cash flow shock, aCF, equal to the discounted infinite sum of expected accounting ROE ($lnROE$). With this latter measure, one cannot precisely decompose log returns into cash flow and discount rate components because $aCF \neq CF$ implies that $aCF - DR \neq r$. However, by defining $iCF \equiv CF - aCF$, we can write a new return decomposition:

$$r = aCF + iCF - DR. \tag{E.1}$$

A simple numerical example illustrates why CF is the natural measure of payouts to stockholders, why aCF is inappropriate, and how to interpret iCF. Consider two firms, A and B, that have the same initial book values of equity of 100. Both firms also have the same expected accounting ROE ($lnROE$) and the same expected log returns (r) on their stocks in all future periods. Specifically, investors expect ROE to be 0% in year 1, year 3, and all periods thereafter, but in year 2 they expect to ROE to be 40%, implying $lnROE = \ln(1.4)$. For simplicity, assume investor expectations are rational, clean-surplus accounting holds, the risk-free rate is 0%, and future ROE and payout policies are known in advance. Since there is no uncertainty, expected returns on both firms equal the risk-free rate of 0%.

The only difference between the firms is that investors rationally expect firm A to issue 900 in new equity in year 1 and pay 1300 in dividends in year 2, whereas firm B will not issue any equity and will pay only 40 in dividends (D) in year 2. This difference will cause firm A to have much more capital in year 2 when the two firms have high expected ROE of 40%. Tables E.1 and E.2 summarize the two firms' financial conditions in years 0 to 3. Firms' market equity—i.e., their valuations—are simply the sum of their expected future dividends because expected returns are 0%.

The key point is that firm A's valuation of 500 is much higher than firm B's valuation of 140 because its expected future net payouts to stockholders are higher: $1300 - 900 = 400$

Table E.1 - Firm A Valuation

Year	0	1	2	3
Book Equity	100	1000	100	100
Earnings		0	400	0
Dividends		0	1300	0
Net Issuance		900	0	0
Net Payouts		−900	1300	0
ROE		0%	40%	0%
Return		0%	0%	0%
$lnROE$		0	0.336	0
r		0	0	0
Market Equity	500	1400	100	100
$lnBM$	-1.609	-0.336	0	0

Table E.2 - Firm B Valuation

Year	0	1	2	3
Book Equity	100	100	100	100
Earnings		0	40	0
Dividends		0	40	0
Net Issuance		0	0	0
Net Payouts		0	40	0
ROE		0%	40%	0%
Return		0%	0%	0%
$lnROE$		0	0.336	0
r		0	0	0
Market Equity	140	140	100	100
$lnBM$	-0.336	-0.336	0	0

for firm A versus just 40 for firm B. This difference in dividends arises despite no difference in the firms' accounting ROE measures in any future period.

Now we will decompose firm B's return under the assumption that it unexpectedly announces in year 0 that it will pursue firm A's issuance and payout policies. Firm B's stock price will immediately jump to 500 from 140, giving it a return of 257% or a log return of $r = \ln(5/1.4) = 1.273$. Firm B's expected returns will remain the same, so $DR = 0$. The firm's expected ROE will also remain the same, so $aCF = 0$. But the cash flow shock for firm B based on the present-value formula will be $CF = r + DR = 1.273$. This large positive CF news correctly reflects the fact that firm B's future net payouts to stockholders increase by 360 when it changes its issuance and payout policy. Intuitively, the reason is that the firms' management is increasing value by scaling up the operation of the business when it is most profitable.

Firm issuance and thus investment policies can create or destroy value, which has a real impact on firm cash flows and payouts to stockholders. The aCF shock based accounting ROE is insensitive to the scale of the firm's operations, whereas the CF shock based on the present-value formula properly takes this scale into consideration. We therefore interpret the difference between aCF and CF, which is iCF, as a measure of shocks to future net payouts (or equivalently net issuance) holding future accounting ROE constant.

F Additional Robustness Tests

In this section, we present additional robustness analyses. Table F.1 shows subperiod correlations between anomaly and market return components estimated from the full-sample baseline VAR. The remaining tables report findings from other VAR specifications. Table F.2 compares the VAR coefficients from the baseline and microcap samples. Table F.3 compares the correlations between anomaly and market return components in the baseline specification to those in three alternative specifications. Tables F.4 to F.6 report correlations of CF and DR news with aggregate metrics, where the CF and DR news come from three different VARs: Table F.4 includes microcaps; Table F.5 estimates the CF shock from accounting return on equity; and Table F.6 excludes the Great Depression.

Table F.1 - Correlations between Anomaly and Market Return Components: Early vs. Modern Years

The table shows correlations of market cash flow and discount rate shocks with anomaly cash flow and discount rate shocks and with cash flow and discount rate shocks to mean-variance efficient (MVE) combinations of these portfolios. "MVE ex market" is the in-sample MVE combination of the five long-short anomaly portfolios. "MVE cum market" is the in-sample MVE combination of the five anomalies and the market portfolio. The first entry in each cell is the correlation from 1929 to 1962 (early years), and the second entry is the correlation from 1963 to 2017 (modern years).

		Market CF		Market DR	
	Subperiod	Anomaly CF	Anomaly DR	Anomaly CF	Anomaly DR
Book-to-market	1929–1962	0.18	−0.43	−0.58	0.61
	1963–2017	0.06	0.19	0.20	0.00
Profitability	1929–1962	0.06	−0.09	0.06	0.14
	1963–2017	−0.33	0.05	0.04	−0.09
(−) Investment	1929–1962	−0.12	−0.07	−0.22	0.10
	1963–2017	−0.35	0.21	0.36	−0.05
(−) Size	1929–1962	0.20	−0.27	−0.60	0.43
	1963–2017	−0.05	−0.20	0.13	0.09
Momentum	1929–1962	0.00	−0.06	0.42	−0.26
	1963–2017	−0.13	−0.22	0.16	−0.13
MVE ex market	1929–1962	0.06	−0.30	−0.03	0.18
	1963–2017	−0.42	−0.16	0.41	−0.15
MVE cum market	1929–1962	0.41	−0.01	0.28	0.94
	1963–2017	−0.28	−0.59	0.27	0.82

Table F.2 - VAR Coefficients in the Baseline and Microcap Samples

The table reports regressions of firms' market-adjusted log returns on lagged market-adjusted firm characteristics with and without microcaps included in the sample. The dependent variable is market-adjusted log real one-year return ($lnRealRet$). The independent variables are one-year lagged $lnRealRet$, log book-to-market ($lnBM$), log profitability ($lnProf$), log five-year asset growth ($lnInv$), five-year change in log market equity ($d5.lnME$), and log six-month momentum ($lnMom6$). Standard errors of coefficients clustered by year appear in parenthesis. The 'Micro?' column indicates whether microcaps are included in the sample. The two 'Std. dev.' rows reports the average yearly standard deviation of each regressor with and without microcaps. The two 'Std. coef.' rows show the standardized coefficient of each regressor with and without microcaps. The 'Std. coef. % chg.' row shows the percentage change in the standardized regression coefficient from including microcaps. The sample spans the years 1929 through 2017. N denotes the number of observations. The marks * and ** indicate significance at the 5 and 1 percent levels, respectively.

	Regressors									
	$lnRealRet_{t-1}$	$lnBM_{t-1}$	$lnProf_{t-1}$	$lnInv_{t-1}$	$d5.lnME_{t-1}$	$lnMom6_{t-1}$	Micro?	Years	Obs.	R^2
$lnRealRet_t$ coef.	0.016	0.033	0.157**	−0.145**	−0.016**	0.095**	*No*	89	124,535	2.1%
Standard error	(0.033)	(0.017)	(0.030)	(0.023)	(0.006)	(0.033)				
t-statistic	0.494	1.933	5.278	−6.279	−2.833	−2.897				
$lnRealRet_t$ coef.	0.071**	0.043**	0.222**	−0.140**	−0.001	0.122**	*Yes*	89	228,571	4.2%
Standard error	(0.022)	(0.016)	(0.020)	(0.023)	(0.007)	(0.025)				
t-statistic	3.309	2.706	11.236	−6.149	−0.188	4.941				
Std. dev.	0.293	0.626	0.143	0.109	0.693	0.204	*No*	89	124,535	
Std. dev.	0.360	0.723	0.217	0.130	0.768	0.251	*Yes*	89	228,571	
Std. coef.	0.005	0.021	0.022	−0.016	−0.011	0.019	*No*	89	124,535	
Std. coef.	0.026	0.031	0.048	−0.018	−0.001	0.030	*Yes*	89	228,571	
Std. coef. % chg.	435%	52%	114%	14%	−91%	57%				

Table F.3 - Correlations between Anomaly and Market Return Components: Alternative samples

The table displays the correlations of anomaly CF news with market CF news and anomaly DR news with market DR news for different specifications relative to Table 6. The "Baseline" specification repeats the results in Table 6. "No Depression" refers to a sample that excludes the Great Depression (1929-1938). "Accounting ROE" computes cash flow news based on expected return on equity from accounting statements instead of using the residual method that imposes the present value constraint. "Including microcaps" refers to a sample that includes firms in the bottom NYSE size quintile. Unless otherwise specified, the sample spans the years 1929 through 2017.

		Specification			
Anomaly	Statistic	Baseline	No Depression	Accounting ROE	Microcaps
Book-to-market	$Corr(CF_{mkt}, CF_{b/m})$	0.13	−0.06	0.33	0.13
	$Corr(DR_{mkt}, DR_{b/m})$	0.42	0.18	0.42	0.27
Profitability	$Corr(CF_{mkt}, CF_{prof})$	−0.11	−0.05	−0.04	−0.12
	$Corr(DR_{mkt}, DR_{prof})$	0.04	−0.06	0.04	0.05
(−) Investment	$Corr(CF_{mkt}, CF_{inv})$	−0.22	−0.17	−0.04	−0.12
	$Corr(DR_{mkt}, DR_{inv})$	0.09	−0.25	0.09	0.16
(−) Size	$Corr(CF_{mkt}, CF_{size})$	0.09	0.03	0.28	0.06
	$Corr(DR_{mkt}, DR_{size})$	0.31	0.28	0.31	0.24
Momentum	$Corr(CF_{mkt}, CF_{mom})$	−0.05	−0.06	−0.21	−0.03
	$Corr(DR_{mkt}, DR_{mom})$	−0.21	0.04	−0.21	−0.09
MVE ex market	$Corr(CF_{mkt}, CF_{mve\ ex\ mkt})$	−0.17	−0.15	−0.10	−0.11
	$Corr(DR_{mkt}, DR_{mve\ ex\ mkt})$	0.06	0.01	0.06	0.23
MVE cum market	$Corr(CF_{mkt}, CF_{mve\ cum\ mkt})$	0.05	0.26	0.24	0.08
	$Corr(DR_{mkt}, DR_{mve\ cum\ mkt})$	0.90	0.94	0.90	0.82

Table F.4 - Correlations of CF and DR News with Aggregate Metrics: VAR Estimated Including Microcaps

This table shows the same correlations as in Table 7, except that the cash flow (CF) and discount rate (DR) shocks here come from a VAR based on a sample that includes stocks in the bottom NYSE size quintile (microcaps) whereas the Table 7 analysis excludes microcaps. This table shows correlations of the market and anomaly portfolios' CF and DR shocks with shocks to several aggregate metrics: one-year log real per capita consumption growth, one-year real per capita GDP growth, the one-year difference in the log labor share, three-year consumption growth (current and future two years), one-year log difference in Baker and Wurgler's (2006) investor sentiment index, the one-year change in the difference between Baa-rated and Aaa-rated corporate bond yields (default spread), and the one-year change in the difference between five-year and one-year zero-coupon Treasury bond yields (term spread). The shocks to the aggregate metrics are the residuals in a first-order autoregressive univariate model of the shock. The sample spans the years 1929 through 2017 but differs across variables because of data availability as described in the text. Standard errors that appear in parenthesis account for time-series variation in the shocks but do not account for estimation error in the VAR coefficients. The marks * and ** indicate significance at the 5 and 1 percent levels, respectively.

	1yr GDP Growth	1yr Cons. Growth	Labor Share	3yr Cons. Growth	Investor Sentiment	Default Spread	Term Spread
CF correlations							
Market	0.38^{**}	0.44^{**}	-0.10	0.35^{**}	0.15	-0.26^{**}	-0.15
Book-to-market	0.26^{*}	0.16	-0.02	0.01	0.36^{**}	-0.45^{**}	-0.05
(−) Investment	-0.17	-0.27^{*}	0.04	-0.16	0.36^{**}	-0.03	0.11
Profitability	-0.26	-0.33^{*}	-0.04	-0.14	0.09	-0.03	0.17
(−) Size	0.00	0.08	-0.24^{*}	-0.12	-0.12	-0.34^{**}	-0.03
Momentum	-0.12	-0.08	0.11	0.08	-0.21	0.22^{*}	0.03
MVE ex market	-0.22	-0.26^{*}	0.06	-0.07	0.10	-0.01	0.12
MVE cum market	-0.20	-0.25	0.09	0.00	0.14	0.06	0.12
DR Correlations							
Market	-0.30^{*}	-0.38^{**}	0.15	-0.19	-0.10	0.66^{**}	-0.14
Book-to-market	0.41^{**}	0.24	0.28^{*}	0.27^{*}	0.35^{**}	0.27^{**}	-0.24^{*}
(−) Investment	0.01	-0.18	0.08	0.20^{*}	0.20	0.23^{*}	-0.04
Profitability	-0.43^{*}	-0.26	-0.10	-0.19	-0.13	0.03	0.10
(−) Size	-0.19	-0.19	0.25^{*}	0.04	-0.29^{*}	0.20	-0.05
Momentum	-0.25	-0.18	-0.27^{*}	-0.27	-0.28^{*}	0.25^{**}	-0.36^{**}
MVE ex market	-0.32^{*}	-0.31^{*}	-0.15	-0.17	-0.20	0.34^{**}	0.20
MVE cum market	-0.41^{**}	-0.46^{**}	0.01	-0.25	-0.18	0.64^{**}	0.02

Table F.5 - Correlations of CF and DR News with Aggregate Metrics: Cash Flow News from Accounting ROE

This table shows the same correlations as in Table 7, except that the cash flow (CF) shocks here are based on expected accounting return on equity (ROE) instead of the present-value relation with log returns and discount rate (DR) shocks. This table shows correlations of the market and anomaly portfolios' CF and DR shocks with shocks to several aggregate metrics: one-year log real per capita consumption growth, one-year real per capita GDP growth, the one-year difference in the log labor share, three-year consumption growth (current and future two years), one-year log difference in Baker and Wurgler's (2006) investor sentiment index, the one-year change in the difference between Baa-rated and Aaa-rated corporate bond yields (default spread), and the one-year change in the difference between five-year and one-year zero-coupon Treasury bond yields (term spread). The shocks to the aggregate metrics are the residuals in a first-order autoregressive univariate model of the shock. The sample spans the years 1929 through 2017 but differs across variables because of data availability as described in the text. Standard errors that appear in parenthesis account for time-series variation in the shocks but do not account for estimation error in the VAR coefficients. The marks * and ** indicate significance at the 5 and 1 percent levels, respectively.

	1yr GDP Growth	1yr Cons. Growth	Labor Share	3yr Cons. Growth	Investor Sentiment	Default Spread	Term Spread
CF Correlations							
Market	0.13	0.17	0.00	0.35^{**}	0.02	-0.52^{**}	-0.28^{**}
Book-to-market	0.40^{**}	0.39^{**}	0.08	0.27^{*}	0.26	-0.46^{**}	-0.18
(−) Investment	-0.19	-0.32^{*}	0.02	-0.12	0.23	0.06	0.13
Profitability	-0.47^{**}	-0.48^{**}	-0.11	-0.32^{*}	0.03	-0.02	0.27^{**}
(−) Size	0.35	0.22	-0.07	0.12	-0.03	-0.35^{**}	-0.16
Momentum	-0.35^{**}	-0.32^{*}	0.09	-0.07	-0.11	0.34^{**}	0.07
MVE ex market	-0.44^{**}	-0.45^{**}	0.02	-0.22	-0.01	0.16	0.15
MVE cum market	-0.45^{**}	-0.47^{**}	0.01	-0.13	-0.02	0.04	0.12
DR Correlations							
Market	-0.29^{*}	-0.37^{**}	0.14	-0.18	-0.10	0.66^{**}	-0.15
Book-to-market	0.27^{*}	0.15	0.33^{**}	0.33^{**}	0.17	0.53^{**}	-0.26^{**}
(−) Investment	0.15	0.11	0.07	0.29^{*}	-0.06	0.26^{**}	-0.19
Profitability	-0.18	0.00	-0.09	-0.04	-0.18	0.03	-0.05
(−) Size	-0.09	-0.13	0.30^{**}	0.07	-0.16	0.32^{**}	-0.08
Momentum	-0.19	-0.13	-0.32^{**}	-0.35^{**}	-0.16	-0.12	0.24^{*}
MVE ex market	-0.19	-0.11	-0.15	-0.15	-0.31^{*}	0.29^{**}	0.06
MVE cum market	-0.38^{**}	-0.39^{**}	0.00	-0.26	-0.26	0.69^{**}	-0.09

Table F.6 - Correlations of CF and DR News with Aggregate Metrics: VAR Estimated 1939 - 2017 (No Depression)

This table shows the same correlations as in Table 7, except that the sample here excludes the Great Depression period (1929-1938). This table shows correlations of the market and anomaly portfolios' cash flow (CF) and discount rate (DR) shocks with shocks to several aggregate metrics: one-year log real per capita consumption growth, one-year real per capita GDP growth, the one-year difference in the log labor share, three-year consumption growth (current and future two years), one-year log difference in Baker and Wurgler's (2006) investor sentiment index, the one-year change in the difference between Baa-rated and Aaa-rated corporate bond yields (default spread), and the one-year change in the difference between five-year and one-year zero-coupon Treasury bond yields (term spread). The shocks to the aggregate metrics are the residuals in a first-order autoregressive univariate model of the shock. The sample spans the years 1939 through 2017 but differs across variables because of data availability as described in the text. Standard errors that appear in parenthesis account for time-series variation in the shocks but do not account for estimation error in the VAR coefficients. The marks * and ** indicate significance at the 5 and 1 percent levels, respectively.

	1yr GDP Growth	1yr Cons. Growth	Labor Share	3yr Cons. Growth	Investor Sentiment	Default Spread	Term Spread
CF Correlations							
Market	0.15	0.29*	−0.13	0.18	0.05	−0.12	−0.03
Book-to-market	0.22	0.13	0.01	0.02	0.35**	−0.23*	−0.06
(−) Investment	−0.18	−0.28*	0.04	−0.19	0.34**	0.12	0.12
Profitability	−0.27*	−0.32*	−0.08	−0.18	0.06	0.14	0.19
(−) Size	−0.04	0.06	−0.20	−0.11	−0.03	−0.04	−0.01
Momentum	−0.10	−0.09	0.06	0.07	−0.16	0.19	0.01
MVE ex market	−0.25	−0.30*	0.06	−0.12	0.11	0.22*	0.13
MVE cum market	−0.27*	−0.31*	0.03	−0.12	0.17	0.22*	0.16
DR Correlations							
Market	−0.27*	−0.37**	0.16	−0.21*	0.01	0.36**	−0.06
Book-to-market	0.27*	0.14	0.33*	0.32*	0.18	0.13	−0.26**
(−) Investment	0.16	0.12	0.08	0.30*	0.05	0.04	−0.19
Profitability	−0.18	0.01	−0.09	−0.04	−0.18	0.02	−0.05
(−) Size	−0.09	−0.14	0.31**	0.07	−0.17	0.12	−0.08
Momentum	−0.17	−0.11	−0.31	−0.35*	−0.17	0.12	0.25**
MVE ex market	−0.17	−0.07	−0.19	−0.16	−0.29*	0.17	0.07
MVE cum market	−0.32**	−0.38**	0.08	−0.25	−0.10	0.39**	−0.05

G The Effect of Misspecification on the Return Variance Decomposition

Here we consider two possible sources of misspecification in the VAR: omitted predictors of returns and spurious return predictability, which could come from data mining. Since we impose the present value relation in the VAR, any error in the estimated DR component of returns affects the estimated CF component.

Decompose estimated DR news (DR) into true DR (DR^{true}) news and an orthogonal error term (ε_{DR}):

$$DR_t = aDR_t^{true} + \varepsilon_{DR,t}, \tag{G.1}$$

where $a = 1$ and $Var(\varepsilon_{DR,t}) = 0$ if estimated DR shocks are correct. Omitting return predictors decreases the value of a with no effect on the error variance, while spurious predictability tends to increase or have no effect on a and increases the variance in the error term. We can evaluate the impact of misspecification on estimated CF news by using the present value restriction:

$$\begin{aligned} CF_t &= CF_t^{true} - DR_t^{true} + DR_t \\ &= CF_t^{true} - (1-a)\, DR_t^{true} + \varepsilon_{DR,t}. \end{aligned} \tag{G.2}$$

Now we can examine the bias in the covariance between estimated CF and DR, which equals:

$$\begin{aligned} Cov(CF_t, DR_t) &= aCov\left(CF_t^{true}, DR_t^{true}\right) - (1-a)\, aVar\left(DR_t^{true}\right) \\ &\quad + aCov\left(CF_t^{true}, \varepsilon_{DR,t}\right) + Var(\varepsilon_{DR,t}). \end{aligned} \tag{G.3}$$

As a useful benchmark, assume the true covariance between CF and DR is zero. The impact of misspecification on the estimated covariance is then:

$$ErrorCov = -(1-a)\, aVar\left(DR_t^{true}\right) + aCov\left(CF_t^{true}, \varepsilon_{DR,t}\right) + Var(\varepsilon_{DR,t}). \tag{G.4}$$

Data mining is likely to impart a positive bias on the estimated covariance. In particular, consider the effect of data-mining as $a \geq 1$ and $Var\left(\varepsilon_{DR,t}\right) > 0$. The first and last terms are then positive. In addition, spurious prediction of future cash flows suggests that the middle term—covariance of cash flows with spurious discount rate errors—is also positive, or at least not negative and large enough to undo the effect of the other two terms. So data mining induces a positive bias in the covariance of CF and DR news.

Now consider omitted return predictors, which manifest as $a < 1$ and $Var\left(\varepsilon_{DR,t}\right) = 0$. Since the first term in $ErrorCov$ will be negative, omitted return predictors can induce a negative bias in the CF and DR covariance.

In summary, VAR misspecification can lead to both a positive and negative bias in the estimated covariance of CF and DR shocks. However, if data mining is the main concern, the bias is likely positive, which strengthens the finding that most empirical estimates of this covariance are negative.