

Idiosyncratic and Aggregate Risk in the Presence of Government's Moral Hazard

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Abstract

Fiscal policy and taxation in particular play an important role in the insurance of local agents against income fluctuations. Government's power to impose taxes is a key tool for optimal redistribution among residents. And, since public debt represents future local tax income, fiscal policy also plays a role in the international risk sharing. I find that government's moral hazard introduces a trade-off between pooling idiosyncratic risk and diversifying aggregate country uncertainty. As a result, local agents face excessive consumption risk. This paper also explores how institutions can be designed as to overcome this moral hazard problem.

1 Introduction

Fiscal policy and taxation in particular play an important role in the insurance of local agents against income fluctuations. Government's power to impose taxes is a key tool for optimal redistribution among residents. Indeed, if agents cannot pledge their future income in the financial market, a domestic insurance market cannot privately arise. Local agents rely on government's tax power to diversify their idiosyncratic domestic income risk. Moreover, Government's fiscal policy also plays a role in the international risk sharing. Public debt represents future local tax income. Then, trading public debt in the international financial market allows country risk sharing.

Optimal risk sharing involves foreign investors holding domestic public debt, which introduces government's moral hazard. Because the government prevails local interests over foreign ones, the identity of the bond holders affects the ex-post optimal fiscal policy. This paper looks at how the government's lack of commitment technology affects the capacity of resident agents to optimally diversify risk. I find that government's moral hazard introduces a trade-off between pooling idiosyncratic risk and diversifying aggregate country uncertainty. As a result, local agents face excessive consumption risk.

The model in this paper represents risk averse consumers, who want to diversify domestic idiosyncratic risk and share the aggregate country risk in the international financial market. They are unable to pledge their future random return, which introduces the government's role. The government, who has the tax power, commits on behalf of local agents to deliver the promised goods to the share holders. In other words, domestic and international risk sharing are mediated by government's intervention.

Opening the capital account allows local agents to diversify aggregate country risk but also introduces government's moral hazard. Since taxes are levied exclusively on residents while bond returns are accrued in part by foreign investors, the government has incentives to lower taxes and reduce the return on public debt. Participants in the financial market adjust their demand for domestic bonds according to their credible return. Their equilibrium price also reflects government's future optimal policy. As a result, government's moral hazard does not affect international risk diversification. Domestic and foreign agents minimize aggregate risk at actuarially fair prices. However, government's moral hazard does affect internal tax policy, which is insufficient from an ex-ante point of view. Domestic fiscal policy pools risk suboptimally.

I explore how institutions can be designed as to overcome this moral hazard problem. It is optimal for the government to impose private non-transferable savings account composed of domestic bonds. By forcing residents to hold government bonds, the government restricts itself from expo-

priating bond holders in the future and can credibly commit to follow a Pareto superior policy. This commitment device is costly: it results in a suboptimal international risk sharing. The first best allocation will not be attained and the optimal restriction results from the trade-off between idiosyncratic and aggregate risk diversification.

I extend the baseline model to an infinitely repeated economy with overlapping generations. I analyze the conditions under which reputation can work as a commitment device for the government. In those cases that it does, the government implements the ex-ante optimal policy under the threat that any deviation will be punished by reversion to the Markov Perfect Equilibrium. The ability to commit depends on the instantaneous gains from deviating from the promised policy, versus the cost in terms of a suboptimal one thereafter. As expected, reputation works as a commitment technology if the government's intertemporal discount is sufficiently high. More interesting, reputation is less likely to support the ex-ante policy in economies with high idiosyncratic risk. When the idiosyncratic risk is higher, the government implements a policy closer to the ex-ante optimal after abandoning the rule. Then, the reversion to the Markov Perfect Equilibrium does not represent a sufficient threat. Along the same lines, imposing minimum requirements of domestic bonds on the pension funds improves fiscal policy, but it may deprive the government from a costless commitment technology.

The rest of the paper is organized as follows. Section 2 describes the static version of the model with a single generation of agents interacting with the government for two periods. I characterize here the commitment problem of the government and provide rationality for the use of minimum requirements on bond holdings as a commitment device. In section 3, I extend the basic static framework into a dynamic economy. I characterize the stationary Markov Perfect Equilibrium and analyze the conditions under which reputation can work as a commitment device for the implementation of the ex-ante optimal fiscal policy. Finally, section 4 concludes.

2 Static Framework

The model presented here describes a world with two countries: home and abroad. Each country is populated by a unit measure of consumers, alive for two periods. Agents have CARA preferences over consumption at time 1 and 2:

$$\begin{aligned} U &= u(c_1) + \beta E u(c_2) \\ u(c) &= -\frac{1}{\gamma} e^{-\gamma c} \end{aligned} \tag{2.1}$$

There is a single good, used as numeraire, which is received by agents as

endowment in the first and second periods. At date 1, domestic endowment is a deterministic amount w_1 (w_1^* for foreigners), while endowment in the second period is uncertain: each resident agent $i \in [0, 1]$ receives $w_{is} = w_s + \varepsilon_i$ units of goods in the second period (foreign agents get w_s^*). The aggregate risk is given by the realization of (w_s, w_s^*) , while ε_i corresponds to pure idiosyncratic domestic risk:

$$\begin{aligned} (w_s, w_s^*) &: N(\bar{w}, \bar{w}^*; \sigma^2, \sigma^{*2}, \sigma_{fd}) \\ \varepsilon_i &: iid.N(0, \sigma_i^2) \end{aligned} \tag{2.2}$$

At date 1, before the realization of the aggregate and idiosyncratic shocks, agents make their consumption/saving decision and choose their portfolio allocation. Each agent i can save in three ways: shares of future domestic endowment of each domestic agent j ($B_{id,j}$), a share of future foreign endowment (B_{if}), and a riskless storage technology that transform one unit of date-1 good into $1 + r$ units of date-2 good (B_{i0}). That is, the strategy of each local agent $i \in [0, 1]$ is a vector $\mathcal{B}_i = \{B_{i0}, \langle B_{id,j} \rangle_{j=0}^1, B_{if}\} \in \mathbb{R}_+^3$, while for each foreign agent is $\mathcal{B}^* = \{B_0^*, \langle B_{d,j}^* \rangle_{j=0}^1, B_f^*\} \in \mathbb{R}_+^3$.

Finally, to assure a constant consumption schedule, I assume that $\beta = \frac{1}{1+r}$.

2.1 Optimal allocation

As a benchmark, I characterize here the financial market equilibrium that corresponds to the first best allocation. In this frictionless economy foreign and domestic agents are able to sell (and buy) their future endowments in the financial market.

In period 1, each local agent $i \in [0, 1]$ receives her date-1 endowment w_1 and sells her future endowment w_{is} at a market price p_{id} . She consumes an amount c_{i1} and buys shares of foreign and domestic (from each local agent $j \in [0, 1]$) endowments. The remaining resources are invested in the riskless technology. At date 2, and in each state of nature s , the agent consumes c_{is} according to the return on her assets: shares on foreign and domestic (from each agent $j \in [0, 1]$) endowments have returns w_s^* and w_{js} respectively; and the return on the storage technology is $1 + r$.

Therefore, financial market equilibrium consistent with the first best allocation is defined as follows:

Definition 1 *The first best financial market equilibrium is a combination of strategies and market prices $\{\langle \mathcal{B}_i \rangle_{i=0}^1, \mathcal{B}^*, \langle p_{id} \rangle_{i=0}^1, p_f\}$ such that:*

(i) $\mathcal{B}_i = \{B_{i0}, \langle B_{id,j} \rangle_{j=0}^1, B_{if}\} \in \mathbb{R}_+^3$ maximizes (2.1) subject to:

$$c_{i1} + B_{i0} + \int_0^1 B_{id,j} p_{jd} dj + B_{if} p_f = w_1 + p_{id}$$

$$c_{is} = B_{i0} (1 + r) + \int_0^1 B_{id,j} w_{js} dj + B_{if} w_s^*$$

(ii) $\mathcal{B}^* = \{B_0^*, \langle B_{d,j}^* \rangle_{j=0}^1, B_f^*\} \in \mathbb{R}_+^3$ maximizes (2.1) subject to:

$$c_1^* + B_0^* + \int_0^1 B_{d,j}^* p_{jd} dj + B_f^* p_f = w_1^* + p_f$$

$$c_s^* = B_0^* (1 + r) + \int_0^1 B_{d,j}^* w_{js} dj + B_f^* w_s^*$$

(iii) And $\{ \langle p_{id} \rangle_{i=0}^1, p_f \}$ are such that the market clearing conditions are satisfied:

$$j \in [0, 1] : \int_0^1 B_{id,j} di + B_{d,j}^* = 1$$

$$B_f + B_f^* = 1$$

Since idiosyncratic risk is perfectly diversifiable, the price of individual domestic endowments w_{is} is identical for all $i \in [0, 1] : p_{id} = p_d$. Therefore, all domestic agents are ex-ante identical. Moreover, the optimal allocation on risky assets is independent of the level of wealth. It follows that all agents (foreign and domestic) have the same optimal portfolio, characterized by the following first order conditions:

$$foc(B_d) : -p_d + \beta [\bar{w} - \gamma (B_d \sigma^2 + B_f \sigma_{fd})] = 0 \quad (2.3)$$

$$foc(B_f) : -p_f + \beta [\bar{w}^* - \gamma (B_f \sigma^{*2} + B_d \sigma_{fd})] = 0 \quad (2.4)$$

where $B_d = \int_0^1 B_{d,j} dj$ is the perfectly diversified domestic asset. Finally, because the intertemporal discount is the reciprocal of the return on the risk-free asset, the optimal demand for risk-free assets equalizes marginal utilities of consumption over time:

$$foc(B_0) : -U'(c_1) + EU'(c_{is}) = 0 \quad (2.5)$$

where for $c_1 = c_{i,1}$ for all i .

Combining the first order conditions with the market clearing conditions, the decentralized equilibrium that corresponds to the first best allocation is given by the following portfolio composition and prices:

$$B_d = B_d^* = \frac{1}{2} \quad (2.6)$$

$$B_f = B_f^* = \frac{1}{2} \quad (2.7)$$

$$p_f = \beta \left[\bar{w}^* - \frac{\gamma}{2} (\sigma^{*2} + \sigma_{fd}) \right] \quad (2.8)$$

$$p_d = p_{id} = \beta \left[\bar{w} - \frac{\gamma}{2} (\sigma^2 + \sigma_{fd}) \right] \quad (2.9)$$

Welfare depends on preferences and the country-endowment distribution:

$$U = (1 + \beta) u(c_1)$$

$$\text{where : } c_1 = \frac{1}{1 + \beta} \left[w_1 + \beta \bar{w} - \beta \frac{\gamma}{2} \text{Var}(c_{is}) - \beta \frac{\gamma}{4} (\sigma^2 - \sigma^{*2}) \right] \quad (2.10)$$

$$U^* = (1 + \beta) u(c_1^*)$$

$$\text{where : } c_1^* = \frac{1}{1 + \beta} \left[w_1^* + \beta \bar{w}^* - \beta \frac{\gamma}{2} \text{Var}(c_s^*) - \beta \frac{\gamma}{4} (\sigma^{*2} - \sigma^2) \right] \quad (2.11)$$

$$\text{Var}(c_s^*) = \text{Var}(c_{is}) = \frac{1}{4} (\sigma^2 + \sigma^{*2} + 2\sigma_{fd}) \quad (2.12)$$

As expected, consumption of both foreign and local agents increase in the agents' discounted expected total endowment. The covariance matrix affects consumption in two ways. First, precautionary savings increase with non-diversifiable risk –i.e. $\text{Var}(c_s)$ –. Since both local and foreign investors hold the same amount of risky assets, the variance of domestic and foreign consumption is identical. And second, through its effect on prices, residents' consumption decreases on σ^2 , since residents are net suppliers of domestic assets, and increases in σ^{*2} , because that reduces the price of the asset for which residents are net demanders. The symmetric opposite characterizes foreign agents' consumption.

2.2 Imperfect Financial Market

Because atomistic agents have no power to levy taxes, consumers cannot commit at time 1 to share their future endowment with other agents. However, the government has that tax power; that is, it can expropriate local agents of their endowment and commit on their behalf to deliver the date-2 domestic endowment to the share holders. In effect, a government intervention can replicate the first best allocation.

The government issues a state-contingent bond that pays R_s at time 2. It taxes residents at time 1 and 2, with a tax rate τ_1 and τ_2 chosen before the

shocks are realized. The storage technology is also available for the government: it can transform a unit of date-1 good into $(1 + r)$ units of date-2 goods. So the government chooses the policy $\mathcal{P} = \{\tau_1, \tau_2, R_s\} \in \mathbb{R}^3$ to maximize 2.1 subject to date 1 and date 2 budget constraints:

$$t = 1 : 0 \leq \tau_1 w_1 + p_d \quad (2.13)$$

$$t = 2 : R_s = \tau_2 w_s + (\tau_1 w_1 + p_d)(1 + r) \quad (2.14)$$

The law of large number holds, so the return on government bonds is only contingent on the aggregate risk.

For simplicity, I introduce this lack of commitment to domestic agents only. The return on foreign bonds is still given by w_s^* in period 2 and state s .

In this section I first characterize the financial market equilibrium for a given policy vector $\{\tau_1, \tau_2, R_s\}$. Then, I analyze the optimal government intervention. A credible government that maximizes residents' welfare can achieve the first best allocation. In this case, the government chooses the policy internalizing its effect on the equilibrium level of consumption. However, without a commitment device, the government has incentives to deviate from the ex-ante optimal rule. If date-2 tax rate τ_2 is chosen after date-1 financial decisions are set, the government has incentives to impose a sub-optimal (from an ex-ante point of view) tax policy. Because domestic bonds are held both by local and foreign investors, the government has incentives to reduce ex-post the return on bonds and lower the domestic tax burden accordingly. This policy is inefficient and results in an imperfect diversification of domestic idiosyncratic risk.

2.3 Financial Market Equilibrium

As has been described above, the strategy of each local agent consists of three actions $\mathcal{B} = \{B_d, B_f, B_0\} \in \mathbb{R}_+^3$ -correspondingly, the strategy of a foreign investor is $\mathcal{B}^* = \{B_d^*, B_f^*, B_0^*\} \in \mathbb{R}_+^3$. The financial market equilibrium for a given policy $\mathcal{P} = \{\tau_1, \tau_2, R_s\}$ is defined as follows:

Definition 2 *For a given policy $\mathcal{P} = \{\tau_1, \tau_2, R_s\}$ that satisfies (2.13) and (2.14), a financial market equilibrium is a combination of strategies and market prices $\{\mathcal{B}, \mathcal{B}^*, p_d, p_f\}$ such that:*

(i) $\mathcal{B} = \{B_0, B_d, B_f\} \in \mathbb{R}_+^3$ maximizes (2.1) subject to:

$$c_1 + B_0 + B_d p_d + B_f p_f = w_1 (1 - \tau_1) \quad (2.15)$$

$$c_{is} = (1 - \tau_2) w_{is} + B_0 (1 + r) + B_d R_s + B_f w_s^* \quad (2.16)$$

(ii) $\mathcal{B}^* = \{B_d^*, B_f^*, B_0^*\} \in \mathbb{R}_+^3$ maximizes (2.1) subject to:

$$c_1^* + B_0^* + B_d^* + B_f^* p_f = w_1^* + p_f \quad (2.17)$$

$$c_s^* = B_0^* (1 + r) + B_d^* R_s + B_f^* w_s^* \quad (2.18)$$

(iii) The equilibrium prices $\{p_d, p_f\}$ are such that the market clearing conditions are satisfied:

$$B_d + B_d^* = 1 \quad (2.19)$$

$$B_f + B_f^* = 1 \quad (2.20)$$

The first order conditions that characterize the optimal demands for domestic and foreign assets are, for local investors:

$$foc(B_f) : -p_f + \beta [\bar{w}^* - \gamma \{B_f \sigma^{*2} + [1 - \tau_2 (1 - B_d)] \sigma_{fd}\}] = 0 \quad (2.21)$$

$$foc(B_d) : -p_d + \beta [\bar{R} - \gamma \tau_2 \{[1 - \tau_2 (1 - B_d)] \sigma^2 + B_f \sigma_{fd}\}] = 0 \quad (2.22)$$

and for foreign investors:

$$foc(B_f^*) : -p_f + \beta [\bar{w}^* - \gamma \{B_f^* \sigma^{*2} + B_d^* \tau_2 \sigma_{fd}\}] = 0 \quad (2.23)$$

$$foc(B_d^*) : -p_d + \beta [\bar{R} - \gamma \tau_2 \{B_d^* \tau_2 \sigma^2 + B_f^* \sigma_{fd}\}] = 0 \quad (2.24)$$

where \bar{R} corresponds to the expected return on domestic bonds $\bar{R} = \int_{-\infty}^{\infty} R_s f(w_s) dw_s$.

The optimal exposure to risk is independent of the wealth level. Thus, domestic and foreign agents will have the same exposure to domestic and foreign uncertainty in equilibrium, as in the first best allocation. From (2.21) and (2.23), domestic and foreign consumers face foreign risk in the amount B_f and B_f^* respectively. Then, the credit market equilibrium satisfies the market clearing condition (2.20) and $B_f = B_f^*$, which implies that the demand for foreign shares is equal to the first best equilibrium:

$$B_f = B_f^* = \frac{1}{2} \quad (2.25)$$

In the case of domestic risk, the total holdings of future domestic endowment is not equal to the demanded domestic contingent bond. From (2.22), residents not only hold domestic endowment in the amount $B_d \tau_2$, they also have an amount $(1 - \tau_2)$ of their own risky endowment. Therefore, residents' total exposure to domestic risk is $[1 - \tau_2 (1 - B_d)]$, while foreigners' is only given by their share of domestic bonds: $B_d^* \tau_2$. As a result,

the market for domestic shares is in equilibrium when (2.19) is satisfied and $B_d^* \tau_2 = [1 - \tau_2 (1 - B_d)]$, which implies:

$$B_d = \frac{2\tau_2 - 1}{2\tau_2} \quad B_d^* = \frac{1}{2\tau_2} \quad (2.26)$$

Residents' demand for domestic bonds increases in the tax rate τ_2 . A larger tax rate decreases residents' exposure to their own endowment risk, which has a sovereign risk component -i.e. $Var(w_{is}) = \sigma_i^2 + \sigma^2$. Then, residents are willing to hold more domestic bonds when the exposure to their own endowment risk is lower. In the case of foreign investors, their demand of domestic bonds lowers in τ_2 , since higher tax rate increases the variance of domestic bonds.

The equilibrium prices that sustain the market allocations are:

$$p_d = \beta \left[\bar{R} - \frac{\gamma}{2} \tau_2 (\sigma^2 + \sigma_{fd}) \right] \quad (2.27)$$

$$p_f = \beta \left[\bar{w}^* - \frac{\gamma}{2} (\sigma^2 + \sigma_{fd}) \right] \quad (2.28)$$

where \bar{R} denotes the expected return on domestic asset.

Finally, since the return on the risk-free asset is the reciprocal of the intertemporal discount rate, condition (2.5) is again satisfied for all agents.

2.4 Ex-ante optimal government intervention

The first best allocation can be attained if the government can credibly commit to follow the ex-ante optimal policy. That is, the government chooses a policy $\mathcal{P} = \{R_s, \tau_1, \tau_2\} \in \mathbb{R}^3$ that maximizes :

$$\begin{aligned} & \max_{\{R_s, \tau_1, \tau_2\}} u(c_1) + \beta E u(c_{is}) \\ & s.t. \\ & CE(c_{is}) = c_1 \end{aligned} \quad (2.29)$$

$$c_1 = \frac{1}{1 + \beta} \left[w_1 + \beta \bar{w} - \beta \frac{\gamma}{2} Var(c_{is}) - \beta \frac{\gamma}{4} (\sigma^2 - \sigma^{*2}) \right] \quad (2.30)$$

$$where : Var(c_{is}) = \frac{1}{4} (\sigma^2 + \sigma^{*2} + 2\sigma_{fd}) + (1 - \tau_2)^2 \sigma_i^2$$

Budget Constraints (2.13) and (2.14)

where, $CE(c_{is})$ denotes for Certainty Equivalent of c_{is} . Combining (2.5), (2.25)-(2.28), and (2.15)-(2.16), the equations (2.30) and (2.29) correspond to the equilibrium consumption schedule for residents given a policy vector $\{R_s, \tau_1, \tau_2\}$.

The ex-ante optimal tax schedule is therefore:

$$\begin{aligned}\tau_2 &= 1 \\ \tau_1 w_1 &= -\beta \left[\bar{w} - \frac{\gamma}{2} (\sigma^2 + \sigma_{fd}) \right]\end{aligned}$$

The optimal tax rate at date 2 prevents residents from facing any idiosyncratic risk. And government's transfers to the residents at date 1 correspond to the discounted certainty equivalent of date 2-tax revenues. The first best allocation is attained.

The expected return on domestic bonds \bar{R} (and therefore the optimal government's investment in the storage technology) is undetermined. It does not affect the covariance matrix, which only depends on the policy choice τ_2 , and it is fully accounted for in the equilibrium price (2.27). Summarizing, since agents and the government have the same storage technology, changes in \bar{R} do not alter the consumption schedule in (2.30) and (2.29).

2.5 Time inconsistent Government

The ex-ante government policy leads the economy to the first best allocation. However, this policy may not be optimal ex-post because, once investment decisions have been made, a lower date-2 tax rate τ_2 can increase local agents' expected consumption by expropriating foreign investors. Here, the government chooses date-2 tax rate τ_2 –and therefore the return on domestic shares– after the portfolio choice is set and before the realization of the shocks. The policy equilibrium is defined as follows:

Definition 3 *Policy Equilibrium is a combination of strategies and market prices $\{\mathcal{P}, \mathcal{B}, \mathcal{B}^*, p_d, p_f\}$ such that:*

(i) $\{\mathcal{B}, \mathcal{B}^*, p_d, p_f\}$ is a Financial Market Equilibrium given $\mathcal{P} = \{\tau_1, \tau_2, R_s\}$

(ii) τ_1 maximizes government's objective at time 1: $u(c_1) + \beta Eu(c_{is})$

given the budget constraint (2.13)

(iii) $\{\tau_2, R_s\}$ maximizes government's objective at time 2: $Eu(c_{is})$, given

$\{\mathcal{B}, \mathcal{B}^*, p_d, p_f\}$ and subject to the budget constraint (2.14).

The policy equilibrium is characterized by backwards induction. At date 2, for a given financial market equilibrium $\{\mathcal{B}, \mathcal{B}^*, p_d, p_f\}$ and date-1 policy

τ_1 , the government chooses $\{R_s, \tau_2\} \in \mathbb{R}^2$ to maximize

$$\begin{aligned} & \max_{\{\tau_2, R_s\}} EU(c_{is}) \\ & s.t. \\ & R_s = \tau_2 w_s + (\tau_1 w_1 + p_d)(1+r) \\ & c_{is} = w_{is}(1-\tau_2) + B_0(1+r) + B_d R_s + B_f w_s^* \end{aligned} \quad (2.31)$$

At the time of choosing $\{R_s, \tau_2\}$ investors already made their financial decisions and agreed on a price for domestic bonds. Then, the policy does not alter the holdings of domestic bonds or its price. In other words, the government chooses its policy without internalizing its effect on already taken financial decisions. Therefore, it maximizes the ex-post consumption level in (2.31), instead of the equilibrium consumption level in (2.30). However, the government's optimal policy feeds back into investors' expectation. As a result, from an ex-ante perspective, the government's optimal tax rate is suboptimal and results in a suboptimal diversification of domestic risk.

The first order condition that characterizes the optimal date-2 tax rate and the return on the domestic asset is:

$$-(1-B_d)\bar{w} + \gamma \left\{ \begin{array}{l} (1-\tau_2)\sigma_i^2 \\ + [1-\tau_2(1-B_d)](1-B_d)\sigma^2 \\ + (1-B_d)B_f\sigma_{fd} \end{array} \right\} = 0 \quad (2.32)$$

The return on assets R_s is financed through taxes. While taxes are levied entirely on residents, the return on assets goes to share holders, which only a proportion B_d are residents. Therefore, expected consumption of residents decreases on τ_2 in an amount $(1-B_d)\bar{w}$, which corresponds to the first term in equation (2.32). The government, who does not consider foreigners' utility in its welfare objective, has incentives to reduce taxes and return on domestic asset. However, the optimal tax rate and return on domestic asset will not be zero. Because domestic government bonds are used to diversify aggregate and idiosyncratic risk, a time-inconsistent government will still find it optimal to tax residents and pay a positive return on domestic shares. An increase in τ_2 reduces the variance of consumption, as can be observed in the second term in (2.32).

The optimal date-2 tax rate as a function of residents' portfolio is presented in equation (2.33). It is a positive function of residents' holdings of domestic assets and the variance of both aggregate and idiosyncratic risk:

$$\tau_2 = 1 - (1-B_d) \frac{\bar{w} - \gamma [B_d\sigma^2 + B_f\sigma_{fd}]}{\gamma [(1-B_d)^2\sigma^2 + \sigma_i^2]} \quad (2.33)$$

Domestic bonds are not only used for international risk diversification, they also play a key role in the domestic financial system: since agents cannot commit their future endowment, they use government bonds to diversify the idiosyncratic risk. As a result, the larger the diversifiable risk, the lower the time inconsistency problem of the government. Indeed, in the limit of infinite idiosyncratic risk, the optimal tax rate coincides with the ex-ante optimum: $\lim_{\sigma_i^2 \rightarrow \infty} \tau_2 = 1$.

The ex-post optimal policy $\{\tau_2, R_s\}$ feeds back into the agents expectation and the resulting market equilibrium at date 1. The policy equilibrium is the set of fix points for which the market's foreseen policy coincides with the ex-post government optimum. Combining (2.26) and (2.25) with (2.33), the equilibrium date-2 tax rate is implicitly defined by:

$$\tau_2(1 - \tau_2) = \frac{\bar{w} - \frac{\gamma}{2}(\sigma^2 + \sigma_{fd})}{2\gamma\sigma_i^2} \quad (2.34)$$

In order to assure an interior solution to the problem, I make the following parametric assumption:

$$\frac{\gamma}{2}(\sigma_i^2 + \sigma^2 + \sigma_{fd}) \geq \bar{w} \quad (2.35)$$

This assumption assures that there are values of τ_2 such that the benefits from retaining all the domestic endowment are lower than its cost in terms of variance of consumption. In particular, when $\tau_2 = \frac{1}{2}$, the government increases welfare by rising taxes and reducing consumption risk. It follows that the stable equilibrium corresponds to the positive root of (2.34) and the optimal date-2 tax rate is positive but lower than the ex-ante best policy: $\tau_2 \in [\frac{1}{2}, 1]$.

As in the ex-ante optimal policy, the expected return on the domestic bonds is undetermined as long as the budget constraint (2.13) is satisfied: $\bar{R} \geq \tau_2 \bar{w}$. Finally, combining (2.27) and (2.14), the date-1 transfer to the residents is also lower than the ex-ante optimal policy:

$$\tau_1 w_1 = -\beta \tau_2 \left[\bar{w} - \frac{\gamma}{2}(\sigma^2 + \sigma_{fd}) \right] \quad (2.36)$$

The equilibrium price for domestic bonds at date 1 takes into account the future optimal government response. In other words, the price incorporates the future incentive of the government to decrease taxes and reduce payments to share holders. As a result, date-1 revenues from selling the domestic bond are also reduced and so are the transfers to local agents. For that reason, the time inconsistency of the government does not affect expected consumption of residents or foreign investors. Moreover, the international risk diversification is not affected by the suboptimal tax rate. Both foreign and resident

investors adjust their demand for domestic bonds according to the after-tax covariance matrix. Total exposure to aggregate risk coincides with the first best allocation in (2.10) and (2.11).

However, the government distortion does have welfare implications. Residents' utility is still given by $U = (1 + \beta) u(c_1)$, but the variance of consumption for residents is now higher than in the first best allocation (2.12):

$$Var(c_{is}) = \frac{1}{4} (\sigma^2 + \sigma^{*2} + 2\sigma_{fd}) + (1 - \tau_2)^2 \sigma_i^2$$

Because domestic bonds are also used to diversify the domestic idiosyncratic risk, residents face an excessive volatility of consumption relative to the ex-ante optimal one. The incentive of the government to expropriate foreign share holders has a negative impact on the domestic financial system, which is unable to diversify the idiosyncratic endowment risk. The suboptimal tax rate has no effect on foreign investors' welfare, which is still characterized by equation (2.11) and (2.12).

These findings are summarized by the following proposition

Proposition 1 *For a given date-1 domestic and foreign endowment $\{w_1, w_1^*\}$ and date-2 risky endowment $\{w_{is}, w_s^*\}$ with distributions defined in (2.2) and (2.35), the policy equilibrium is characterized by:*

(i) *Sub-optimal tax on risky local endowment: $\tau_2 \in (\frac{1}{2}, 1)$ with $\frac{\partial \tau_2}{\partial \sigma^2} > 0$, $\frac{\partial \tau_2}{\partial \sigma_i^2} > 0$, and $\lim_{\sigma_i^2 \rightarrow \infty} \tau_2 = 1$*

(ii) *Excessive volatility of local consumption:*

$$c_1 = CE(c_{is}) = \frac{1}{1 + \beta} \left\{ w_1 + \beta \bar{w} - \beta \frac{\gamma}{2} Var(c_{is}) - \beta (\sigma^2 - \sigma^{*2}) \right\}$$

$$where : Var(c_{is}) = (1 - \tau_2)^2 \sigma_i^2 + \frac{1}{4} (\sigma^2 + \sigma^{*2} + 2\sigma_{fd})$$

(iii) *Foreign agent's consumption at its first best level:*

$$c_1^* = CE(c_s^*) = \frac{1}{1 + \beta} \left\{ w_1^* + \beta \bar{w}^* - \beta \frac{\gamma}{2} Var(c_s^*) - \beta (\sigma^{*2} - \sigma^2) \right\}$$

$$where : Var(c_s^*) = \frac{1}{4} (\sigma^2 + \sigma^{*2} + 2\sigma_{fd})$$

2.6 Commitment Device

The ex-ante optimal policy enables the financial market to replicate the first best allocation. However, the government cannot credibly commit to follow

it. This distortion arises because the government prevails domestic interests over foreign ones and, if anticipated, ends up reducing residents' welfare. The first best allocation will not be attained in this economy. Nevertheless, the government can credibly commit to follow a superior policy by forcing residents to hold domestic bonds above their individual optimal level. A Pareto better allocation is attained if the government gives part of the date-1 transfer in the form of non-transferable individual domestic-bond accounts.

From equation (2.33), the optimal date-2 tax rate increases with the holdings of domestic bonds by the representative local agent. However, the portfolio of a single investor does not affect government's incentives. That is, the individual optimum B_d in equation (2.26) does not internalize its effect on government's incentives.¹ Then, giving part of the initial transfer to local investors in the form of domestic bonds is a Pareto improvement. Or, in other words, the government uses date-1 transfer to residents as a commitment device for a better policy in period 2. Needless to say, that policy will have an impact only if those accounts are nontransferable and the amount transferred is above the privately chosen B_d in (2.26). Then, government's policy is a vector $\mathcal{P} = \{\bar{B}_d, \tau_1, \tau_2, R_s\} \in \mathbb{R}^4$, where $\{\bar{B}_d, \tau_1\}$ are chosen at date 1, while $\{\tau_2, R_s\}$ are decided at date 2.

At date 1, the government chooses $\{\bar{B}_d, \tau_1\}$ that maximizes ex-ante residents' welfare, taking into account its own ex-post incentives to impose a suboptimal tax rate τ_2

$$\begin{aligned} & \max_{\{\bar{B}_d, \tau_1\}} \{u(c_1) + \beta E(c_{is})\} \\ & s.t. \\ & \{\mathcal{B}, \mathcal{B}^*, p_d, p_f\} \text{ is a Financial Market Equilibrium for } \mathcal{P} = \{\bar{B}_d, \tau_1, \tau_2, R_s\} \\ & \tau_2 = 1 - (1 - \bar{B}_d) \frac{\bar{w} - \gamma [\bar{B}_d \sigma^2 + B_f \sigma_{fd}]}{\gamma [(1 - \bar{B}_d)^2 \sigma^2 + \sigma_i^2]} \end{aligned} \quad (2.37)$$

Budget Constraints (2.13) and (2.14)

For a given $\mathcal{P} = \{\bar{B}_d, \tau_1, \tau_2, R_s\}$ the financial market equilibrium is characterized by:

$$p_d = \beta \left[\bar{R} - \frac{\gamma}{2} \tau_2 (\sigma_{fd} + \sigma^2) \right] + \beta (\bar{B}_d - B_d) \gamma \tau_2^2 \sigma^2 (1 - \rho^2) \quad (2.38)$$

$$p_f = \beta \left[\bar{w}^* - \frac{\gamma}{2} (\sigma^{*2} + \sigma_{fd}) \right] \quad (2.39)$$

$$B_f = \frac{1}{2} - \frac{\sigma_{fd}}{\sigma^{*2}} \tau_2 (\bar{B}_d - B_d) \quad (2.40)$$

¹This mechanism is in the lines of Tirole (2003): local investors exert externalities on each other through their impact on the government's incentives.

where B_d is the privately optimum holding of domestic asset in (2.26), while \bar{B}_d is the amount transferred by the government. Foreign holdings of assets are $B_d^* = 1 - \bar{B}_d$ and $B_f^* = 1 - B_f$.

The transfer of domestic bonds to the local agents affects the price of the bonds in two opposite ways. First, as expected, the policy increases their demand and, as a result, the price rises with \bar{B}_d —computed in the second term in (2.38). And second, because higher local holdings of domestic bonds \bar{B}_d result in a larger date-2 tax rate τ_2 , the variance of the return on domestic bonds increases. This affects negatively the price of domestic bonds. Notice from (2.39) that the price of foreign bonds is not affected by the local policy.

The existence of individual domestic-bond accounts also affects the demand for foreign bonds, since the diversification strategy is altered (see (2.40)). The sign of this effect depends on the sign of the covariance between foreign and domestic endowment risks.

The optimal date-1 transfer of domestic bonds is \bar{B}_d that satisfies the following first order equation:

$$\left[\frac{\partial c_1}{\partial \bar{B}_d} + \frac{\partial c_1}{\partial \tau_2} \frac{\partial \tau_2}{\partial \bar{B}_d} \right] + \frac{\partial c_1}{\partial B_f} \left[\frac{\partial B_f}{\partial \bar{B}_d} + \frac{\partial B_f}{\partial \tau_2} \frac{\partial \tau_2}{\partial \bar{B}_d} \right] + \frac{\partial c_1}{\partial p_d} \left[\frac{\partial p_d}{\partial \bar{B}_d} + \frac{\partial p_d}{\partial \tau_2} \frac{\partial \tau_2}{\partial \bar{B}_d} \right] = 0$$

Notice from (2.22), that $\frac{\partial c_1}{\partial \bar{B}_d}$ corresponds to the first order condition for the individual local agent, which is zero for $\bar{B}_d = B_d$ and negative for $\bar{B}_d > B_d$. Also, since agents can freely choose the amount of foreign bonds, it follows that $\frac{\partial c_1}{\partial B_f} = \text{foc}(B_f)$ is equal to zero. Finally, the ex-post optimal tax level τ_2 satisfies the first order condition (2.32) and therefore: $\frac{\partial c_1}{\partial \tau_2} = -\beta \frac{\gamma}{2} \frac{\partial \text{Var}(c_{is})}{\partial \tau_2} = \beta (1 - \bar{B}_d) \bar{w}$.

Replacing, the first order condition for \bar{B}_d is:

$$\begin{aligned} \text{foc}(\bar{B}_d) &= \text{foc}_{ind}(\bar{B}_d) + \beta (1 - \bar{B}_d) \gamma \tau_2^2 \sigma^2 (1 - \rho^2) \\ &+ \beta (1 - \bar{B}_d) \left[\bar{w} - \frac{\gamma}{2} (\sigma_{fd} + \sigma^2) + 2\gamma \tau_2 (\bar{B}_d - B_d) \sigma^2 (1 - \rho^2) \right] \frac{\partial \tau_2}{\partial \bar{B}_d} = 0 \end{aligned}$$

where $\text{foc}_{ind}(\bar{B}_d)$ corresponds to the individual first order condition in (2.22), evaluated at the government's optimum \bar{B}_d and $\frac{\partial \tau_2}{\partial \bar{B}_d} > 0$.²

It follows that the optimal \bar{B}_d is larger than the private optimum B_d but allows some international diversification: $\bar{B}_d \in (B_d, 1)$.³ Since foreign

²Replacing (2.40) in (2.37), follows that the ex-post optimal tax-rate is a function of \bar{B}_d only: $\tau_2 = 1 - (1 - \bar{B}_d) \frac{\bar{w} - \frac{\gamma}{2} (\sigma^2 + \sigma_{fd}) - \gamma (\bar{B}_d - \frac{1}{2}) \sigma^2 (1 - \rho^2)}{\gamma [(1 - \bar{B}_d)^2 \sigma^2 (1 - \rho^2) + \sigma_i^2]}$. The tax rate τ_2 is a positive function of \bar{B}_d : $\frac{\partial \tau_2}{\partial \bar{B}_d} = \frac{\tau_2 (1 - \bar{B}_d)^2 \sigma^2 (1 - \rho^2) + (1 - \tau_2) \sigma_i^2}{(1 - \bar{B}_d) [(1 - \bar{B}_d)^2 \sigma^2 (1 - \rho^2) + \sigma_i^2]} > 0$

³For $\bar{B}_d = B_d$: $\text{foc}(\bar{B}_d) = \beta (1 - B_d) \left[\gamma \tau_2^2 \sigma^2 (1 - \rho^2) + [\bar{w} - \frac{\gamma}{2} (\sigma_{fd} + \sigma^2)] \frac{\partial \tau_2}{\partial \bar{B}_d} \right] > 0$

investors still hold some domestic bonds, the date-2 tax rate is below its first best level. The first best tax level ($\tau_2 = 1$) is not optimal. From equation (2.37), that would require residents to hold the entire supply of domestic bond ($\bar{B}_d = 1$). In that case, the idiosyncratic risk would be perfectly diversified, but there would be suboptimal international risk sharing.

Summarizing, it is optimal for the government to impose private non-transferable savings account composed of domestic bonds. By forcing residents to hold public bonds, the government restricts itself from expropriating bond holders in the future and can credibly commit to follow a Pareto better policy. This commitment device is costly: it results in a suboptimal international risk sharing. As a result, agents will face lower idiosyncratic risk but the first best allocation will not be attained. The optimal restriction results from the trade-off between idiosyncratic and aggregate risk diversification.

3 Dynamic Model

In this section I extend the baseline model to an infinitely repeated economy. Based on the previous static framework, I develop an overlapping generation model with zero population and economic growth. The government is an infinitely lived agent, who internalizes the future benefits of implementing the ex-ante optimal policy.

I analyze the conditions under which "reputation" can work as a commitment device. In those cases where it does, the government implements the ex-ante optimal policy under the threat that any deviation will be punished by reversion to the Markov Perfect Equilibrium. Then, the economy attains its first best allocation. That is, residents diversify the domestic idiosyncratic risk and minimize the aggregate risk by holding the international portfolio of assets.

I characterize in this section the stationary Markov Perfect Equilibrium and analyze the cases in which the reversion to such equilibrium represents a sufficient threat.

3.1 Dynamic Environment

At each moment in time $t = 1, 2, 3, \dots$, two generations coexist in the local economy: a unit mass of young agents with endowment w_1 , who consume an amount $c_{1,t}$; and a unit mass of old agents with a random endowment $w_{is,t}$, who consume $c_{is,t}$. The state of the economy at each time t is given by the old investors' assets and the government's storage of goods $\{\mathcal{B}_{t-1}, \mathcal{B}_{t-1}^*, a_{t-1}\}$, where a_t corresponds to the government's investment in the storage technology, with a riskless return $(1 + r)$.

For $\bar{B}_d = 1 : \text{foc}_{ind}(\bar{B}_d) = \text{foc}(\bar{B}_d) < 0$. It follows that $\bar{B}_d \in (B_d, 1)$.

As in the static framework, young and old residents at time t consume according to (3.1) and (3.2) respectively:

$$c_{1,t} + B_{0,t} + B_{d,t}p_{d,t} + B_{f,t}p_{f,t} = (1 - \tau_{1,t}) w_1 \quad (3.1)$$

$$c_{is,t} = (1 - \tau_{2,t}) w_{is,t} + B_{0,t-1}(1 + r) + B_{d,t-1}R_{s,t} + B_{f,t-1}w_{s,t}^* \quad (3.2)$$

Similarly, young and old foreign investors at time t consume:

$$c_{1,t}^* + B_{0,t}^* + B_{d,t}^*p_{d,t} + B_{f,t}^*p_{f,t} = w_1^* + p_{f,t}^* \quad (3.3)$$

$$c_{s,t}^* = w_{s,t}^* + B_{0,t-1}^*(1 + r) + B_{d,t-1}^*R_{s,t} + B_{f,t-1}^*w_{s,t}^* \quad (3.4)$$

Notice that the generation born at time t is affected by the policy actions $\{\tau_{1,t}, \tau_{2,t+1}, R_{s,t+1}\}$.

Similar to the static case, the government has a period by period budget constraint. Differently, at each time t , government resources combine taxes levied to young and old agents, and revenues from selling the domestic bond.

$$\begin{aligned} R_{s,t} + a_t &\geq \tau_{2,t}w_{s,t} + \tau_{1,t}w_1 + p_{d,t} + a_{t-1}(1 + r) \\ a_t &\geq 0 \end{aligned} \quad (3.5)$$

3.2 Financial Market Equilibrium

As in the static framework, the strategy of each local agent born at time t consists of three actions $\mathcal{B}_t = \{B_{d,t}, B_{f,t}, B_{0,t}\} \in \mathbb{R}_+^3$ -correspondingly, the strategy of a foreign investors is $\mathcal{B}_t^* = \{B_{d,t}^*, B_{f,t}^*, B_{0,t}^*\} \in \mathbb{R}_+^3$. The financial market equilibrium is defined for a given policy path $\{\mathcal{P}_t\}_{t=t_0}^\infty$, where $\mathcal{P}_t = \{\tau_{1,t}, \tau_{2,t}, R_{s,t}, a_t\}$.

Definition 4 For a given policy path $\{\mathcal{P}_t\}_{t=1}^\infty$ such that for all t (3.5) is satisfied, a Financial Market Equilibrium is a combination of strategies and market prices $\{\mathcal{B}_t, \mathcal{B}_t^*, p_{d,t}, p_{f,t}\}$ such that:

(i) $\mathcal{B}_t = \{B_{d,t}, B_{f,t}, B_{0,t}\}$ maximizes $u(c_{1,t}) + \beta E u(c_{is,t+1})$ subject to (3.1) and (3.2)

(ii) $\mathcal{B}_t^* = \{B_{d,t}^*, B_{f,t}^*, B_{0,t}^*\}$ maximizes $u(c_{1,t}^*) + \beta u E(c_{s,t+1}^*)$ subject to (3.3) and (3.4)

(iii) Market clearing conditions are satisfied:

$$B_{d,t} + B_{d,t}^* = 1 \quad (3.6)$$

$$B_{f,t} + B_{f,t}^* = 1 \quad (3.7)$$

As in the static framework, agents live for only two periods, so the financial market equilibrium at each time t is analog to the static case:

$$\begin{aligned} B_{d,t} &= \frac{2\tau_{2,t+1} - 1}{2\tau_{2,t+1}} & B_{d,t}^* &= \frac{1}{2\tau_{2,t+1}} \\ B_{f,t} &= \frac{1}{2} & B_{f,t}^* &= \frac{1}{2} \end{aligned} \quad (3.8)$$

Similarly, the equilibrium prices are:

$$p_{f,t} = \beta \left[\bar{w}^* - \frac{\gamma}{2} (\sigma^{*2} + \sigma_{fd}) \right] \quad (3.9)$$

$$p_{d,t} = \beta \left[\bar{R}_{t+1} - \frac{\gamma}{2} \tau_{2,t+1} (\sigma^2 + \sigma_{fd}) \right] \quad (3.10)$$

The existence of a storage technology assures that for each generation, the consumption schedule satisfies (2.5). Then, replacing in the budget constraints (3.1) and (3.2), the consumption schedule for each generation born at time t is:

$$c_{1,t} = \frac{1}{1+\beta} \left\{ w_1 + \beta \bar{w} - \beta \frac{\gamma}{4} (\sigma^2 - \sigma^{*2}) - \beta \frac{\gamma}{2} \text{Var}(c_{is,t+1}) + T_t \right\} \quad (3.11)$$

$$\begin{aligned} \text{where : } & \begin{cases} \text{Var}(c_{is,t+1}) = (1 - \tau_{2,t+1})^2 \sigma_i^2 + \frac{1}{4} (\sigma^2 + \sigma^{*2} + 2\sigma_{fd}) \\ T_t = -\tau_{1,t} w_1 - \beta \tau_{2,t+1} \left[\bar{w} - \frac{\gamma}{2} (\sigma^2 + \sigma_{fd}) \right] \end{cases} \\ c_{1,t} &= CE(c_{is,t+1}) \end{aligned} \quad (3.12)$$

where $CE(c_{is,t+1})$ denotes for the Certainty Equivalent of $c_{is,t+1}$, and T_t is the net transfer received by the generation born at time t . In the static framework, with only one generation alive, the net transfer is necessarily zero. That is, the resources received when young are equal to the discounted certainty equivalent of future tax payments (see equation (2.36)). This is not necessarily the case in the dynamic framework.

3.3 Ex-ante Optimal Policy

If local agents could sell their future endowment, the decentralized equilibrium would be a repeated version of the static equilibrium presented in section 2.2. That is, prices would be constant over time and each generation born at time t would have the same consumption schedule and portfolio composition as in the decentralized equilibrium characterized by equations (2.6)-(2.12). As in the static framework, a credible government can replicate this first best allocation for the case where agents cannot pledge their future endowment. The ex-ante optimal policy is a path $\{\mathcal{P}_t\}_{t=1}^{\infty}$ with $\mathcal{P}_t = \{\tau_{1,t}, \tau_{2,t}, R_{s,t}, a_t\} \in \mathbb{R}^4$ that

maximizes

$$\begin{aligned} & \max_{\{\mathcal{P}_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1} [u(c_{1,t}) + \beta Eu(c_{is,t+1})] \\ & s.t. \\ & R_{s,t} + a_t = \tau_{1,t}w_1 + \tau_{2,t}w_{s,t} + p_{d,t} + a_{t-1}(1+r) \end{aligned} \quad (3.13)$$

(3.10), (3.11), and (3.12)

The ex-ante optimal policy leads the economy towards the first best risk diversification. That is, the idiosyncratic risk is diversified away and the aggregate risk is minimized:

$$\forall t : \tau_{2,t} = 1 \quad (3.14)$$

Since the intertemporal preference is the reciprocal of the return on the riskless technology, the utility is constant across generations:

$$u'(c_{1,t}) = Eu'(c_{is,t}) = u'(c_{1,t-1}) \quad (3.15)$$

Then, at any time t , the optimal transfer to the young agents is:

$$\tau_{1,t}w_1 = -a_0r - \beta \left[\bar{w} - \frac{\gamma}{2} (\sigma^2 + \sigma_{fd}) \right] \quad (3.16)$$

where a_0 corresponds to the government's initial holdings of assets in storage. Then, the net transfer to the young generation $T_t = -\tau_1w_1 - \beta \left[\bar{w} - \frac{\gamma}{2} (\sigma^2 + \sigma_{fd}) \right]$ is equal to the flow of returns on government's assets: a_0r .

Again, the level of expected return on the domestic asset \bar{R}_t is undetermined, together with the government's storage in the riskless technology, and does not affect agents' welfare:

$$\bar{R}_t = (a_{t-1} - a_0)(1+r) + \bar{w} \quad (3.17)$$

At each period t , the government gets $a_{t-1}(1+r)$ from its investment in the storage technology. An amount $(a_t - a_0)(1+r)$ is assigned to debt payments, $a_{0-1}r$ is transferred to the young, and the remaining a_{0-1} is reinvested.

3.4 Markov Perfect Equilibrium

A time inconsistent government cannot commit to follow the ex-ante optimal dynamic policy. Instead, it has incentives to impose a suboptimal tax on old residents and, by doing so, it prevents residents from fully diversifying the idiosyncratic risk. Moreover, in the dynamic framework, a new distortion arises: a time inconsistent government has incentives to redistribute resources across generations.

For each t , the timing of each stage-game is the following:

1. Local and foreign young agents choose strategies $\mathcal{B}_t = \{B_{d,t}, B_{f,t}, B_{0,t}\} \in \mathbb{R}_+^3$ and $\mathcal{B}_t^* = \{B_{d,t}^*, B_{f,t}^*, B_{0,t}^*\} \in \mathbb{R}_+^3$. The government implements a policy vector $\mathcal{P}_t = \{\tau_{1,t}, \tau_{2,t}, R_{s,t}, a_t\}$.
2. The aggregate and idiosyncratic shocks are realized: $\{w_{s,t}, w_{s,t}^*, \{\varepsilon_{i,t}\}_{i=0}^1\}$.
3. Consumption takes place: $\{c_{1,t}, \{c_{is,t}\}_{i=0}^1\}$

I characterize here the stationary Markov Perfect Equilibrium (MPE) for the dynamic game described above. In this type of equilibria, strategies can only be contingent on the payoff-relevant state of the world and the prior actions taken within the same period. The Markov Perfect Equilibrium for this economy is defined as follows:

Definition 5 *For a given state $\{\mathcal{B}_0, a_0\}$, a stationary Markov Perfect Equilibrium is a combination of strategies and market prices $\{\mathcal{P}_t, \mathcal{B}_t, \mathcal{B}_t^*, p_{d,t}, p_{f,t}\}_{t=1}^\infty$ such that the three strategies are best responses to the other three, and the asset markets clear. That is:*

(i) *for all $t = 1, 2, 3, \dots$, $\{\mathcal{B}_t, \mathcal{B}_t^*, p_{d,t}, p_{f,t}\}$ is a Financial Market Equilibrium given $\{\mathcal{P}_t\}_{t=1}^\infty$*

(ii) *for all $t = 1, 2, 3, \dots$, \mathcal{P}_t satisfies*

$$W(\mathcal{B}_{t-1}, a_{t-1}) = \max_{\mathcal{P}_t} \{u(c_{1,t}) + Eu(c_{is,t}) + \beta EW(\mathcal{B}_t, a_t)\}$$

s.t.

(3.5), (3.1), and (3.2)

$$\lim_{T \rightarrow \infty} \sum_{t=1}^T \beta^t \left\{ \tau_{1,t} w_1 + \beta \tau_{2,t} \left[\bar{w} - \frac{\gamma}{2} (\sigma^2 + \sigma_{fd}) \right] \right\} + a_0 \geq 0 \quad (3.18)$$

(iii) *the allocation is stationary: for all $t = 1, 2, 3, \dots$: $c_{1,t} = c_1$ and $E(c_{is,t}) = E(c_{is})$*

The government has incentives to transfer resources from the old to the new generations. For that reason, I restrict, in the limit, the budget constraint of the government. Condition (3.18) requires that transfers to the young generation are financed out of taxes on residents or initial government's savings. In other words, under condition (3.18) no bubble arises.

I guess the following equation for the government's future policy:

$$\bar{R}_{t+1} = (a_t - a_0)(1 + r) + \beta\tau_{2,t+1}\bar{w} + \frac{1 - B_d}{B_d}(\tau_{1,t}w_1 - \tau_1^*w_1)(1 + r) \quad (3.19)$$

Combining (3.10) and (3.19), the resulting financial market equilibrium price for the domestic bond is:

$$p_{d,t} = a_t - a_0 + \beta\tau_{2,t+1} \left[\bar{w} - \frac{\gamma}{2}(\sigma^2 + \sigma_{fd}) \right] + \frac{1 - B_d}{B_d}[\tau_{1,t}w_1 - \tau_1^*w_1] \quad (3.20)$$

The optimal tax rate on the old residents' endowment is suboptimal relative to the ex-ante optimum. Again, the tax rate corresponds to the optimal trade-off between variance and expected consumption. The reaction function describing the optimal tax level $\tau_{2,t}$ is identical to the static one:

$$\tau_{2,t} = 1 - (1 - B_{d,t-1}) \frac{\bar{w} - \gamma [B_{d,t-1}\sigma^2 + B_{f,t-1}\sigma_{fd}]}{\gamma [(1 - B_{d,t-1})^2\sigma^2 + \sigma_i^2]} \quad (3.21)$$

Combining (3.8) and (3.21), the MPE tax level and holdings of domestic assets by residents are constant over time and identical to the static equilibrium in (2.34) and (2.26). For all $t = 1, 2, 3, \dots$

$$\tau_{2,t} = \tau_2 : \tau_2(1 - \tau_2) = \frac{\bar{w} - \frac{\gamma}{2}(\sigma^2 + \sigma_{fd})}{2\gamma\sigma_i^2} \quad (3.22)$$

$$B_{d,t} = B_d = \frac{2\tau_2 - 1}{2\tau_2}$$

The optimal transfer to young residents satisfies the following condition:

$$u'(c_{1,t})w_1 = Eu'(c_{is,t})B_{d,t-1} \left[w_1 + \frac{\partial p_{d,t}}{\partial \tau_1} \right] \quad (3.23)$$

The first term in (3.23) corresponds to the marginal benefit of increasing the transfer to the young, while the second term is its marginal cost in terms of reducing the payments to the bond holders. Everything else constant, a unit of extra consumption to the young generation implies a reduction in today's payments to the elderly. However, because this payments take the form of returns on domestic bonds, this reduction only affects residents in a proportion $B_{d,t-1}$, while the remaining $(1 - B_{d,t-1})$ affects foreign investors not taken into account in the government's objective function. Then, if the price of future bonds did not react to current transfer to the young generation, the government would have incentives to redistribute resources from the elderly to the young above the ex-ante optimal.

In the stationary MPE, the price reaction in (3.20) exactly offsets government's incentives to redistribute resources across generations and the level of utility is constant. A constant utility over time together with a constant tax on old residents' endowment implies that for all t , the tax on the young generation is also constant: $\tau_{1,t}w_1 = \tau_1w_1$.

The price on domestic bonds increases with current tax burden on the young generations. The intuition is simple: any increase in the transfer to young residents is permanent over time and implies a reduction on future payments to domestic bond holders, which results in a lower price. Notice that the incentives of the government to redistribute resources to the young generation decreases in the share of domestic bonds held by residents. Indeed, in the limit of $B_d = 1$, the price of the domestic bond does not react to current taxes and the government has no incentives to further reduce payments.

From (3.18), the resulting MPE is characterized by a constant level of utility over time, which requires constant transfers. The transfer to each young generation is given by the certainty equivalent of their future tax payments when old, and the flow of interest on the initial government's savings:

$$\tau_{1,t}w_1 = \tau_1^*w_1 = -a_0r - \beta\tau_2 \left[\bar{w} - \frac{\gamma}{2} (\sigma^2 + \sigma_{fd}) \right] \quad (3.24)$$

Finally, replacing the optimal tax schedule on (3.19), the government's choice of expected return on bonds is given by (3.19). Payments to bond holders are financed out of taxes on old generation's endowment and out of government's return on savings. At each period t , government's revenues from its investment in the storage technology are $a_{t-1}(1+r)$. An amount $(a_t - a_0)(1+r)$ is assigned to debt payments, $a_{0-1}r$ is transferred to the young, and the remaining a_{0-1} is reinvested. The MPE expected return on domestic bonds is again undetermined, together with the optimal level of government's storage, and does not affect agents' utility but only the price level.

$$\bar{R}_t = (a_{t-1} - a_0)(1+r) + \tau_2\bar{w} \quad (3.25)$$

Welfare value for the government is only affected by its initial wealth and the endowment distribution:

$$W(\mathcal{B}_{t_0-1}, a_{t_0-1}) = u(c_1) \frac{2}{(1-\beta)}$$

where :

$$c_1 = \frac{1}{1+\beta} \left[w_1 + \beta\bar{w} + a_0r - \beta\frac{\gamma}{4} (\sigma^2 - \sigma^{*2}) - \beta\frac{\gamma}{2} Var(c_{is}) \right]$$

$$Var(c_{is}) = (1-\tau_2)^2 \sigma_i^2 + \frac{1}{4} (\sigma^2 + \sigma^{*2} + 2\sigma_{fd})$$

These findings are summarized as follows:

Proposition 2 For a given domestic and foreign endowment $\{w_1, w_1^*, w_{is}, w_s^*\}$ with distributions defined in (2.2) and (2.35), the policy (stationary Markov Perfect) equilibrium is characterized by:

(i) Sub-optimal tax on risky local endowment: $\tau_2 \in (\frac{1}{2}, 1)$ with $\frac{\partial \tau_2}{\partial \sigma^2} > 0$, $\frac{\partial \tau_2}{\partial \sigma_i^2} > 0$, and $\lim_{\sigma_i^2 \rightarrow \infty} \tau_2 = 1$

(ii) Excessive volatility of local consumption:

$$c_1 = CE(c_{is}) = \frac{1}{1+\beta} \left\{ w_1 + \beta \bar{w} + a_0 r - \beta \frac{\gamma}{2} \text{Var}(c_{is}) - \beta (\sigma^2 - \sigma^{*2}) \right\}$$

$$\text{where : } \text{Var}(c_{is}) = (1 - \tau_2)^2 \sigma_i^2 + \frac{1}{4} (\sigma^2 + \sigma^{*2} + 2\sigma_{fd})$$

(iii) Foreign agent's consumption at its first best level:

$$c_1^* = CE(c_s^*) = \frac{1}{1+\beta} \left\{ w_1^* + \beta \bar{w}^* - \beta \frac{\gamma}{2} \text{Var}(c_s^*) - \beta (\sigma^{*2} - \sigma^2) \right\}$$

$$\text{where : } \text{Var}(c_s^*) = \frac{1}{4} (\sigma^2 + \sigma^{*2} + 2\sigma_{fd})$$

(iv) Price for domestic bonds reacts negatively to current transfers to the young generation: $\frac{\partial p_{d,t}}{\partial (-\tau_1 w_1)} = -\frac{1-B_d}{B_d} < 0$

3.5 Equilibrium with Reputation

I analyze in this section the conditions under which reputation can work as a commitment device. Here, I allow strategies to be contingent not only on actions taken within the same period, but also on the history of strategies. The optimal policy rule that the government can credibly commit to follow is given by the following program:

$$\max_{\{\mathcal{P}'_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t [u(c_{1,t}) + \beta E u(c_{is,t+1})]$$

s.t.

$$R_{s,t} + a_t = \tau_{1,t} w_1 + \tau_{2,t} w_{s,t} + p_{d,t} + a_{t-1} (1 + r)$$

(3.10), (3.8), (3.11), and (3.12)

$$\forall t : W(\mathcal{B}_{t-1}, a_{t-1}, \{\mathcal{P}'_{t'}\}_t^{\infty} | \{\mathcal{P}'_{t'}\}_0^{t-1}) \geq W(\mathcal{B}_{t-1}, a_{t-1}, \{\mathcal{P}'_{t'}\}_t^{\infty} | \{\mathcal{P}'_{t'}\}_0^{t-1})$$

The policy rule $\{\mathcal{P}'_t\}_t^{\infty}$ maximizes the ex-ante welfare subject to $\{\mathcal{B}_t, \mathcal{B}_t^*, p_{d,t}, p_{f,t}\}$ being a financial market equilibrium for every t , given the history of government's policy $\{\mathcal{P}'_t\}_0^{t-1}$. And, at each moment t , the incentive compatibility

constraint is satisfied. That is, the government has incentives to follow the promised rule $\{\mathcal{P}'_t\}_t^\infty$ instead of implementing the MPE policy $\{\mathcal{P}'_t\}_t^\infty$. The reversion to the Markov Perfect Equilibrium is used as a threat to sustain the ex-ante policy rule. The ability to commit depends on the instantaneous gains from deviating from the promised rule, versus the cost in terms of a suboptimal policy thereafter. Then, if the reversion to the MPE does not represent a sufficient threat, reputation cannot constitute a commitment technology.

The ex-ante optimal policy rule is $\{\mathcal{P}'_t\}_t^\infty$ such that $\mathcal{P}_t = \{\tau_{1,t}, \tau_{2,t}, R_{s,t}, a_t\}$ satisfies (3.14)-(3.17). So, if the government follows the ex-ante optimal rule, residents' portfolio is, in equilibrium, $\mathcal{B}^F = \{B_0^F, B_d^F, B_f^F\}$ such that:

$$B_d^F = B_f^F = \frac{1}{2}$$

$$B_0^F : c_{1,t} = E(c_{is,t+1})$$

The welfare value for the government if following the rule is:

$$W(\mathcal{B}_{t-1}, a_{t-1}, \{\mathcal{P}'_t\}_t^\infty | \{\mathcal{P}'_t\}_0^{t-1}) = u(c_1^F) \frac{2}{1-\beta}$$

where :

$$c_1^F = \frac{1}{1+\beta} \left[w_1 + \beta \bar{w} + a_0 r - \beta \frac{\gamma}{2} \text{Var}(c_{is}^F) - \beta \frac{\gamma}{4} (\sigma^2 - \sigma^{*2}) \right] \quad (3.26)$$

$$\text{Var}(c_{is}^F) = \frac{1}{4} (\sigma^2 + \sigma^{*2} + 2\sigma_{fd})$$

If the government abandons the rule, it succeeds in surprising the market for one period. After that, the economy goes back to its Markov Perfect Equilibrium described previously. In that case, welfare value for the government is:

$$W(\mathcal{B}_{t_D-1}, a_{t_D-1}, \{\mathcal{P}'_t\}_{t_D}^\infty | \{\mathcal{P}'_t\}_0^{t_D-1}) = \max_{\mathcal{P}'_t} \{u(c_{1,t}) + Eu(c_{is,t}) + \beta EW(\mathcal{B}_t, a_t)\}$$

s.t.

(3.20), (3.8), (3.11), and (3.12)

$\{\mathcal{B}_{t_D-1}, a_{t_D-1}\}$ correspond to \mathcal{B}^F, a^F

$$\lim_{T \rightarrow \infty} \sum_{t=t_D+1}^T \beta^{t-t_D} \left\{ \tau_{1,t} w_1 + \beta \tau_{2,t} \left[\bar{w} - \frac{\gamma}{2} (\sigma^2 + \sigma_{fd}) \right] \right\} + a_D \geq 0$$

where t_D is the time of abandoning the rule, and a_D corresponds to government's asset at that time.

When residents expect the government to follow the rule, their holdings of domestic bonds are larger than in the MPE ($B_d^F > B_d$). Therefore, at the time of deviating, the optimal tax rate is higher than the MPE tax rate in

(3.22). In other words, since residents' exposure to domestic risk is above the MPE levels, the government implements a tax rate τ_2^D higher than the MPE tax rate τ_2 :

$$t = t_D : \tau_{2,t} = \tau_2^D = 1 - \frac{1\bar{w} - \frac{\gamma}{2}(\sigma^2 + \sigma_{fd})}{2\gamma(\frac{1}{4}\sigma^2 + \sigma_i^2)} \quad (3.27)$$

Residents adjust their holdings of bonds as soon as the government deviates from the promised rule. From then on, both the tax rate and the portfolio choice are consistent with the MPE described above:

$$\begin{aligned} t > t_D : \tau_{2,t} &= \tau_2 < \tau_2^D \\ t \geq t_D : B_{d,t} &= B_d = \frac{2\tau_2 - 1}{2\tau_2} \end{aligned}$$

where τ_2 corresponds to the MPE tax rate in equation (3.22)).

Deviating from the rule is not neutral over generations. The immediate gains from surprising the market will be spread over time but still, the initial old generation will be the most benefited from the departure. As before, the government has incentives to redistribute in favor of the young: increasing the transfer ($-\tau_1 w_1$) implies a one-to-one increase in young residents' consumption and a reduction of only B_d^F in old residents'. However, the negative reaction of the price level more than offsets this effect. From equation (3.20)), the price of domestic bonds at the time of deviation decreases in $\frac{1-B_d}{B_d}$ for every dollar transferred to the young. Because the price reflects the future performance of the domestic bond, its elasticity towards current transfer depends on B_d , the future holdings of the asset, which is lower than its current level, B_d^F . Therefore, the optimal transfer ($-\tau_{1,t} w_1$) to the young generation satisfies the following condition:

$$\begin{aligned} t = t_D : u'(c_{1,t}) &= Eu'(c_{is,t}) \frac{B_d^F}{B_d} \\ t > t_D : u'(c_{1,t}) &= Eu'(c_{is,t}) \end{aligned} \quad (3.28)$$

The resulting payment to bond holders is:

$$\begin{aligned} t = t_D : \bar{R}_t &= (a_F - a_D)(1+r) + \tau_2^D \bar{w} \\ t > t_D : \bar{R}_t &= (a_{t-1} - a_D)(1+r) + \tau_2 \bar{w} \end{aligned} \quad (3.29)$$

At the time of abandoning the rule, the government succeeds in surprising the financial market. It does so by deviating from the promised tax τ_2 and by altering the amount of government's savings designated to pay its debt. The government transfers $a_D r$ to the young (different from $a_0 r$ under the rule), and assigns $(a_F - a_D)(1+r)$ to debt payments instead of $(a_F - a_0)(1+r)$ committed under the rule.

The welfare value of the government if deviating is:

$$W\left(\mathcal{B}_{t-1}, a_{t-1}, \{\mathcal{P}'_{t'}\}_{t_D}^\infty \mid \{\mathcal{P}'_{t'}\}_0^{t_D-1}\right) = u(c_1) \left[\frac{2}{(1-\beta)} - \frac{(1-\tau_2)}{\tau_2} \right] \quad (3.30)$$

where :

$$c_1 = \frac{1}{1+\beta} \left[w_1 + \beta \bar{w} + a_D r - \beta \frac{\gamma}{4} (\sigma^2 - \sigma^{*2}) - \beta \frac{\gamma}{2} \text{Var}(c_{is}) \right] \quad (3.31)$$

$$\text{Var}(c_{is}) = (1-\tau_2)^2 \sigma_i^2 + \frac{1}{4} (\sigma^2 + \sigma^{*2} + 2\sigma_{fd})$$

where a_D is defined implicitly by (3.28), (3.31) and (3.29).

The instantaneous benefit from deviating can be observed in the second term in (3.30). Part of the gains from expropriating bond holders is shared with future generations, who are the beneficiaries of the flows of interests on a_D , as can be observed in (3.31). However, future generations will be jeopardized with suboptimal diversification of idiosyncratic risk.

The ability to commit to the ex-ante rule depends on the instantaneous gains from surprising the market, versus the cost in terms of a suboptimal policy thereafter. As expected, reputation works as a commitment technology if the intertemporal discount is sufficiently high (see figure 1.a.). In the limit of $\beta = 1$ (or equivalently $r = 0$), government's flow of return on a_D has no effect on consumption ($\lim_{\beta \rightarrow 1} a_D r = 0$). It follows that the $\lim_{\beta \rightarrow 1} c_1 < c_1^F$ and therefore:

$$\lim_{\beta \rightarrow 1} \frac{W\left(\mathcal{B}_{t-1}, a_{t-1}, \{\mathcal{P}'_{t'}\}_t^\infty \mid \{\mathcal{P}'_{t'}\}_0^{t-1}\right)}{W\left(\mathcal{B}_{t-1}, a_{t-1}, \{\mathcal{P}'_{t'}\}_t^\infty \mid \{\mathcal{P}'_{t'}\}_0^{t-1}\right)} = \frac{u(c_1)}{u(c_1^F)} > 1$$

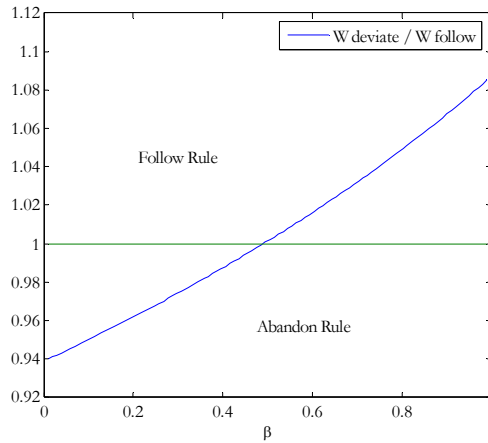


Figure 1.a

More interesting, reputation is less likely to support the ex-ante rule in economies with high idiosyncratic risk. When the idiosyncratic risk is higher, the government implements a policy closer to the ex-ante optimal after abandoning the rule. Then, the reversion to the Markov Perfect Equilibrium does not represent a sufficient threat. Indeed, the higher is σ_i^2 , the larger is the tax rate on the risky endowment (τ_2) and residents' holding of domestic bond (B_d). The improvement in the optimal policy more than compensates the increase in the idiosyncratic risk and residents end up facing a lower variance of consumption:

$$\frac{\partial Var(c_{is})}{\partial \sigma_i^2} = \frac{\partial \left[(1 - \tau_2)^2 \sigma_i^2 \right]}{\partial \sigma_i^2} = -\frac{(1 - \tau_2)^2}{(2\tau_2 - 1)}$$

Then, if the government is indifferent between following the rule or deviating, an increase in the idiosyncratic risk will make the rule less attractive.⁴ Figure 2.b. shows the case where such point of indifference exists, contrary to Figure 2.a. where pre-commitment is possible for all values of σ_i^2 .

In the limit of infinite idiosyncratic variance, the optimal policy without commitment technology coincides with the ex-ante optimal. As a result, the incentive compatibility constraint is satisfied with equality, as shown in Figure 2.a and 2.b..

$$\lim_{\sigma_i^2 \rightarrow \infty} \frac{W\left(\mathcal{B}_{t-1}, a_{t-1}, \{\mathcal{P}'_t\}_t^\infty \mid \{\mathcal{P}'_0\}_0^{t-1}\right)}{W\left(\mathcal{B}_{t-1}, a_{t-1}, \{\mathcal{P}'_t\}_t^\infty \mid \{\mathcal{P}'_0\}_0^{t-1}\right)} = 1$$

⁴If a point of indifference exists. At that point, the direct effect of an increase in the idiosyncratic risk (keeping a_D constant) on the welfare value after abandoning the rule is:

$$\frac{\partial W\left(\mathcal{B}_{t-1}, a_{t-1}, \{\mathcal{P}'_t\}_t^\infty \mid \{\mathcal{P}'_0\}_0^{t-1}\right)}{\partial \sigma_i^2} = \frac{\gamma}{2} u'(c_1) (1 - \tau_2)^2 \left[\frac{\beta}{1 - \beta} \frac{1}{2\tau_2 - 1} - \frac{B_d}{B_d^F} \frac{(1 - \tau_2^D)^2}{(1 - \tau_2)^2} \right]$$

which is positive for $\beta > 0.5$. Making a_D endogenous can only increase the welfare value even further.

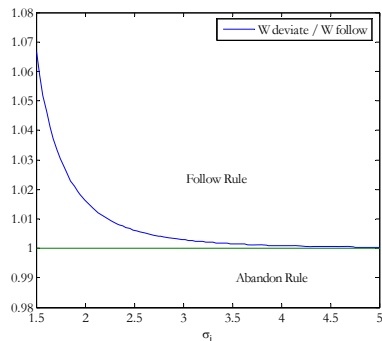


Figure 2.a.

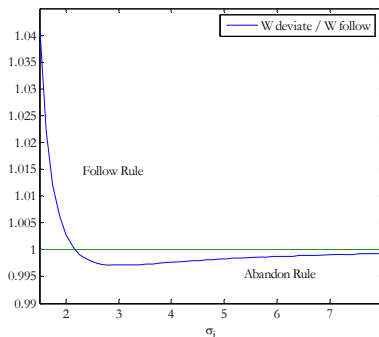


Figure 2.b

These findings are summarized in the following proposition:

Proposition 3 *The threat of a reversion to the stationary Markov Perfect Equilibrium can sustain the ex-ante optimal policy if*

$$u(c_1^F) \frac{2}{(1-\beta)} \geq u(c_1) \left[\frac{2}{(1-\beta)} - \frac{(1-\tau_2)}{\tau_2} \right] \quad (3.32)$$

where c_1^F is defined as in (3.26), c_1 as in (3.31), and a_D is implicitly given by (3.28).

At a point where (3.32) is satisfied with equality, a higher β increases the cost of the reversion to the stationary MPE, while a rise in σ_i^2 reduces it.

Summarizing, when the idiosyncratic risk is high, reputation is less likely to work as a commitment device for the implementation of the ex-ante optimal policy and the economy will not attain the first best allocation. Along the same lines, transfer to young residents in the form of domestic bonds improves fiscal policy. But, as a downside, they reduce the effectiveness of the reversion to the Markov Perfect Equilibrium as a threat and may deprive the government from a costless commitment technology.

4 Conclusions

Fiscal policy plays a double role in terms of risk diversification: government's bonds are used for international risk sharing, while taxes are key to pool domestic idiosyncratic risk. If the government can pre-commit to follow the ex-ante optimal policy, there is no trade-off between these two roles. The

ex-ante optimal fiscal policy succeeds in perfectly diversifying domestic idiosyncratic risk and local investors minimize their exposure to aggregate risk by holding the optimal international portfolio.

However, when the policymaker lacks the ability to commit, there is a tension between pooling idiosyncratic risk and holding a diversified international portfolio. If a large proportion of government debt is held by domestic investors, the government, who prevails local interests over foreign ones, will be able to commit to a higher return on its debt. On the other hand, whenever foreign investors hold government bonds, the fiscal policy will be suboptimal, and the domestic idiosyncratic risk will not be perfectly diversified.

This result provides a rationality for restricting the portfolio choice of pension funds, with minimum requirements of government bonds. This restriction results in a superior fiscal policy and better diversification of the idiosyncratic risk. However, as a downside, it implies a suboptimal international risk sharing. Moreover, these restrictions reduce the effectiveness of reputation as a commitment device for implementing the ex-ante best policy.