

Endogenous Liquidity and the Business Cycle

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Abstract

I present a model in which asymmetric information in capital quality endogenously determines the amount liquidity in an economy. Liquid funds are key to relax financial constraints that affect investment and employment decisions. These funds are obtained by selling capital or using capital as collateral. Liquidity is determined by balancing the costs of obtaining funding under asymmetric information and the benefits of relaxing financial constraints. Aggregate fluctuations can be attributed to increases in the dispersion of capital quality which increment the cost of obtaining liquidity. The model can generate patterns for quantities and credit conditions similar to the Great Recession.

JEL: D82, E32,E44, G01, G21

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1 Introduction

The recent financial crisis began with an abrupt collapse in liquidity. A common view is that the crisis arose when lenders found it difficult to distinguish the value of collateral assets.

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This shortfall in liquidity would have spread to the real economy as firms were restrained from accessing funds to hire workers and finance investment. The overall outcome was the deepest recession of the post-war era.

This paper develops a theory to formalize and quantify this chain of events. The theory builds on the interaction of two frictions: *limited enforcement* in contractual agreements and *asymmetric information* about the quality of capital. Limited enforcement prevents agents from carrying out transactions by promising repayment. This market imperfection imposes constraints on the possible contractual agreements that can be reached. These constraints are only relaxed if transactions are paid on the spot. To make spot payments, the firm can sell or collateralize its capital. Asymmetric information induces a shadow cost on selling or collateralizing these assets. These costs arise because highly valued capital is transacted at a low price as it cannot be distinguished from capital of lower quality. The first contribution of the paper is to characterize, in general equilibrium, the firm's strategic decision to use assets of privately known quality to obtain liquidity in order to relax enforcement constraints. In particular, the paper shows how, at the margin, the cost of selling (or collateralizing) assets must equal the marginal benefit of relaxing enforcement constraints. Through the lens of the theory, recessions occur after an exogenous increase in the dispersion of asset qualities. The increase in dispersion translates into real effects because dispersion increases the cost of obtaining liquidity and, consequently, enforcement constraints are tighter.

The second contribution is to provide a quantitative assessment of this theory. In particular, a calibrated version of the model is fed with a sequence of dispersion shocks that is capable of generating a pattern for output, consumption, investment and hours losses comparable to the ones observed during the 2008-2009 recession. Interestingly, these asymmetric-information driven recessions occur despite the fact that dispersion shocks have no effect on the production possibility frontier or the distribution of wealth. Beyond replicating patterns observed during the Great Recession, the quantitative analysis also shows that increases in the dispersion of asset quality can generate economic fluctuations consistent with several other business-cycle features. [1] The model explains sizeable liquidity-driven recessions which operate primarily through movements in the labor wedge. This friction is a salient feature of the business-cycle decomposition of [Chari, Kehoe and McGrattan \(2007\)](#). [2] Liquidity-driven recessions are characterized by increases in labor productivity. This feature cannot be generated through solely total factor productivity (TFP) shocks but was characteristic of the 2008-2009 crisis (see [Ohanian \(2010\)](#)). [3] The model accounts for a negative correlation between aggregate investment and labor wedges, supporting the view in [Justiniano, Primiceri and Tambalotti \(2010b\)](#) that financial factors are responsible for this co-movement. [4] The model is also in line with evidence on counter-cyclical capital

reallocation documented by [Eisfeldt and Rampini \(2006\)](#). [5] The model produces two forces that counterbalance the relation between Tobin's Q and investment: TFP shocks induce a positive correlation (as in standard Q-theory) but dispersion shocks reverse the correlation. This second force explains how capital reallocation slows down in periods where the benefits seem greater as described in [Eisfeldt and Rampini \(2006\)](#). [6] Finally, the model can explain why the risk-free interest rate falls whereas the cost of borrowing by firms increases during recessions.

The heart of the paper is an endogenous liquidity mechanism by which increases in asset-quality dispersion translate into negative impacts on real activity. To explain the intuition, it is useful to draw an analogy with [Akerlof \(1970\)](#)'s lemons problem. In the classic lemons problem, a seller owning an asset with private information sells it only if its price is above his valuation. Instead, an uninformed buyer buys it only if its price is below his valuation of the quality he expects to receive. Equilibria are summarized by a marginal asset such that all assets of inferior quality are sold. The spread between the quality of the marginal asset and the expected quality sold equals the spread between valuations of buyers and sellers. Thus, shocks to quality dispersion reduce the average of sold qualities, the price and the volume of trade.

In the paper, the source of asymmetric information is the depreciation rate (the quality) of different capital units in the portfolio of entrepreneurs. Knowing their quality, entrepreneurs select and sell units to financial intermediaries who ignore their quality. The fundamental difference with is that the relative valuation by buyers and sellers is an endogenous outcome of the limited enforcement problems faced by entrepreneurs.

Limited enforcement induces entrepreneurs to value capital units less than intermediaries which is essential for trade under asymmetric information. The reason is that entrepreneurs receive an additional benefit from liquid funds. Liquidity allows entrepreneurs to relax their enforcement constraints and scale up their operations. Thus, like in [Akerlof \(1970\)](#), an entrepreneur sells a capital unit if the value of obtaining liquidity is greater than his valuation of that unit. Intermediaries pay a pooling price equal to the full-information price of the average quality sold. Thus, on the threshold asset, the entrepreneur receives a pooling price which is necessarily lower than the full-information price it would have to pay to repurchase that unit. The only reason why the entrepreneur is willing to take that loss is the additional benefit obtained by relaxing his enforcement constraints. Hence, the endogenous spread in valuations that stem from limited enforcement is what supports trade under asymmetric information. Again, dispersion shocks that worsen adverse selection, lead firms to sell capital of lower quality and face tighter constraints which bears aggregate consequences.

In the paper I also study the effects of dispersion shocks when entrepreneurs issue collat-

eralized debt (or repo) contracts following [DeMarzo and Duffie \(1999\)](#).¹ Allowing for these richer type of contracts alleviates the asymmetric-information problem but does not alter the essence of the mechanism. Adverse selection prevents high-quality capital from being sold because the pooling price is too low compared to the replacement cost. If high-quality assets can be repurchased, paying a premium, entrepreneurs are better off. This form of collateralized debt allows entrepreneurs to obtain more liquidity by providing incentives to transact better qualities, improving allocations. However, the essence of the problem does not change. In fact, allocations obtained by collateralizing capital are the same as those where only selling is permitted if the dispersion of asset qualities is sufficiently worse. Thus, collateralized debt and asset sales are indistinguishable if dispersion is unobserved.

The model requires enforcement constraints to operate on labor contracts to generate quantitatively relevant effects. Limited enforcement in labor contracts induce entrepreneurs to finance a portion of their payroll with liquid funds. Dispersion shocks cause an endogenous reduction in liquid funds which tightens these constraints, generating a contraction in labor demand. This feature is crucial to explain substantial movements in output and has an empirical support in the work of [Chodorow-Reich \(2013\)](#). This feature distinguishes this model from other models with financial frictions, most of which focus on distortions on investment. This distinction is important because it is known by now that investment frictions alone cannot generate strong output responses to fundamental shocks.²

I also show that without additional frictions that distort investment decisions, the model cannot deliver pro-cyclical investment. As labor demand contracts there is wealth shift from workers to entrepreneurs via a monopsony effect. Since wealth shifts towards the entrepreneurial sector, investment increases in response to this wealth transfer causing an unrealistic increase in investment. The enforcement problem in investment counteracts this wealth effect.

The paper is novel in several dimensions. Until recently, there were only a few models that incorporated asymmetric information into general equilibrium. A notable exception is [Eisfeldt \(2004\)](#), who studies a stationary model where agents sell assets under private information for self-insurance motives. The present paper builds on ideas in [Kiyotaki and Moore \(2008\)](#) (henceforth KM) who argue that the combination of asymmetric information and

¹I thank an anonymous referee for suggesting this section of the paper. will use the term collateralized debt and repo contracts interchangeably as both contracts are isomorphic in the environment studied.

²The reason is that big changes in investment flows have only a minor impact on the capital stock. Since, capital is ultimately what determines potential output, the overall effect is small absent other frictions. Shocks that exacerbate financial frictions on investment resemble investment shocks (see [Chari, Kehoe and McGrattan \(2007\)](#)). [Barro and King \(1984\)](#) found that investments shocks are not an important source of business cycle fluctuations. This result has been rediscovered in various forms of financial frictions that affect investment.

liquidity constraints could explain aggregate fluctuations by inducing endogenous changes in liquidity.³ However, liquidity in KM is determined by exogenous shocks whereas here, asymmetric information endogenously determines liquidity. The flipside idea that relaxing limited enforcement can support trade is found in [DeMarzo and Duffie \(1999\)](#). However, limited enforcement is also not modeled explicitly in that paper.

The contribution of this paper and a closely related paper, [Kurlat \(2009\)](#), are to formalize the ideas in KM and [DeMarzo and Duffie \(1999\)](#). In the case of Kurlat, gains from trade under asymmetric information come from a combination of heterogeneous technology and a form of incompleteness. Here, gains from trade occur because liquidity relaxes financial constraints. Another distinction is that the endogenous liquidity mechanism here operates through the labor market and this is important to obtain quantitatively significant output fluctuations. In this dimension, this paper also connects to the recent work of [Jermann and Quadrini \(2012\)](#) (henceforth JQ) who also stress the importance of financial factors for labor demand. JQ studies a business-cycle model where entrepreneurs face shocks to an enforcement coefficient that limits their debt holdings. Both papers share the common feature entrepreneurs need to obtain liquid funds to finance working capital.⁴ The key distinction between JQ and this paper is that fluctuations in this model are caused by shocks that aggravate adverse selection.

Why should one model asymmetric information to explain fluctuations in liquidity? First, asymmetric information relates to the literature on financial frictions to the growing literature on counter-cyclical dispersion and reallocation. [Eisfeldt and Rampini \(2006\)](#) and [Bloom \(2009\)](#) provide evidence that the dispersion of profits and revenues increases during recessions and capital reallocation moves in opposite direction. Thus, the paper provides a link between these literatures via asymmetric information.⁵ A second reason is that asymmetric information imposes restrictions on the time-series properties of allocations and liquidity. For example, adverse selection is less severe when the return to capital is high. Likewise, the analysis uncovers an inefficiency result: if efficient allocations require some liquidity, allocations are never efficient. These features are testable implications which impose moment restrictions. Finally, a formal model can teach us whether asymmetric information is a quantitatively relevant friction for financial markets.

The paper also has some technical contributions. I introduce asymmetric information

³Asymmetric information is the natural market imperfection that motivates liquidity constraints because, as known at least since the work of [Stiglitz and Weiss \(1981\)](#), it can induce credit rationing outcomes.

⁴Both papers share the insights of work on working-capital constraints. The early work of [Christiano and Eichenbaum \(1992\)](#) shows that monetary policy can induce increases in financial costs which carry real effects via working capital constraints.

⁵A similar link motivates the work of [Arellano, Bai and Kehoe \(2010\)](#) and [Christiano, Motto and Rostagno \(2012\)](#).

into a dynamic general equilibrium model with aggregate shocks. I show how to solve for the full dynamics of the model without keeping track of trade histories. I also show how to obtain global solutions to the model allowing for collateralized debt with default. This allows me to provide a rich description of loan sizes, interest rates, default rates and the fraction of collateralized assets throughout the state-space which I then use to evaluate the model.

The rest of the paper is organized as follows. Section 2 describes a static model of a firm that needs to raise liquid funds by selling capital under asymmetric information and relax enforcement constraints. This exercise describes the key tradeoffs in the determination of liquidity and how this affects labor demand and output. That section also describes a similar problem that distorts investment. Section 3 shows the relationship between selling under asymmetric information and collateralized debt under asymmetric information. Section 4 presents the dynamic model. Section 5 provides some further characterizations using the solutions to the problems of Section 2. Section 6 presents some quantitative exercises and Section 7 concludes. A detailed discussion of the literature is contained in the Online Appendix. Proofs omitted from the text are included in the Appendix.

2 Forces at Play

This section presents two static models to illustrate the key forces in the dynamic model. Both models are subcomponents of the dynamic model and, hence, serve as intermediate steps in the analysis of the dynamic model presented later.

2.1 Endogenous Liquidity, Output and Hours

Consider a static economy in partial equilibrium. The economy is populated by workers that only supply labor, financial firms that buy and sell capital and entrepreneurs. An entrepreneur maximizes the value of his firm which is the sum of current profits and the value of his capital stock. The entrepreneur holds k units of capital.

Production. Production is carried out via k , combined with labor, l , using a Cobb-Douglas technology $F(k, l) \equiv k^\alpha l^{1-\alpha}$ to produce output. The entrepreneur's profits are $AF(k, l) - wl$. The entrepreneur hires workers from an elastic supply schedule $w = l^\nu$. Wages are given.

Limited enforcement in labor contracts. Before production, an entrepreneur hires an amount of labor promising to pay wl . It is possible that the entrepreneur reneges on this promise and defaults on his payroll. In that case, workers are able to seize a fraction θ^L of

production and the entrepreneur diverts $(1 - \theta^L)$ for himself.

To relax this problem, the entrepreneur can pay a fraction $(1 - \sigma)$ of the wage bill upfront. Of course, he has to obtain working capital to make this payment before production although he has no output yet. He obtains this working capital by selling some capital units. Sold capital units are reallocated after production takes place so they are used for production. Thus, capital serves two purposes. It is a production input and it is also used to obtain working capital. Due to asymmetric information about its quality, selling capital will induce a cost.

Heterogeneous Capital. The capital stock held by the entrepreneur is composed of a continuum of pieces. Pieces are identified by their quality $\omega \in [0, 1]$. Qualities determine the depreciation of each unit. In particular, there is an increasing, bounded and continuous function $\lambda(\omega) : [0, 1] \rightarrow R_+$ that determines the efficiency units that will remain from a given piece of quality ω .⁶ The distribution of ω in that continuum is given by some $f_\phi(\omega)$ with c.d.f. denoted by F_ϕ . For now, ϕ is a parameter.

Pieces can be sold separately. I use the indicator function $\iota(\omega) : [0, 1] \rightarrow \{0, 1\}$ to indicate the decision of selling a unit of quality ω .⁷ That is, the entrepreneur transfers

$$k \int \lambda(\omega) \iota(\omega) f_\phi(\omega) d\omega$$

efficiency units to the buyer. The efficiency units that remain with his capital stock are

$$k \int \lambda(\omega) [1 - \iota(\omega)] f_\phi(\omega) d\omega.$$

Information. When a given piece is sold, its ω (quality) cannot be identified by a buyer. This implies that only the entrepreneur knows the efficiency units that will remain from a particular sold unit. The buyers of these units are financial intermediary firms. Intermediaries only know the quantity of units they buy, $k \int_0^1 \iota(\omega) f_\phi(\omega) d\omega$. However, since they ignore ω behind a sale, they also ignore how many efficiency units will remain from this portfolio, $k \int_0^1 \lambda(\omega) \iota(\omega) f_\phi(\omega) d\omega$. Since they ignore this, they also ignore the market value of the capital bought.

For now, I assume that units sold to financial intermediaries are sold at the same price p . Later, I impose conditions such that there is a unique pooling price. Thus, when units are sold, the entrepreneur obtains liquidity $p k \int_0^1 \iota(\omega) f_\phi(\omega) d\omega$. Let $x = p \int_0^1 \iota(\omega) f_\phi(\omega) d\omega$ be

⁶One can think of $\lambda(\omega)$ as $1 - \delta(\omega)$, where δ is a ω -specific depreciation that occurs after production takes place. Using the notation $\lambda(\omega)$ is convenient. Shocks to the efficiency units are commonly used in continuous time settings as for example Brunnermeier and Sannikov (2009).

⁷Each quality has zero measure so focusing on all-or-nothing sales is without loss of generality.

the liquidity per unit of capital k . I assume that financial firms sell efficiency units at an exogenous price q .⁸ A non-profit condition for the financial firms requires them to equate the value of efficiency units bought to the amount of liquidity given to the entrepreneur. Thus, equilibrium requires,

$$pk \int_0^1 \iota(\omega) f_\phi(\omega) d\omega = qk \int_0^1 \lambda(\omega) \iota(\omega) f_\phi(\omega) d\omega.$$

This expression yields a relationship between the price under asymmetric information and the perfect information price of efficiency units q :

$$p = q \mathbb{E}_\phi [\lambda(\omega) | \iota(\omega) = 1]$$

where \mathbb{E}_ϕ is the conditional expectation under f_ϕ . This relationship states that the pooling price equals the value of the expected quality sold. Formally, the entrepreneur's problem is defined as follows:

Problem 1 (Producer) *The entrepreneur solves:*

$$W^p(k; p, q, w) = \max_{\sigma, \iota(\omega), l} [Ak^\alpha l^{1-\alpha} - \sigma wl] + (xk - (1 - \sigma) wl) + q \int_0^1 (1 - \iota(\omega)) \lambda(\omega) k f_\phi(\omega) d\omega$$

subject to:

$$Ak^\alpha l^{1-\alpha} - \sigma wl \geq (1 - \theta^L) Ak^\alpha l^{1-\alpha} \quad (1)$$

$$(1 - \sigma) wl \leq xk \quad (2)$$

$$x = p \int_0^1 \iota(\omega) d\omega. \quad (3)$$

Recall that σ is the fraction of the wage bill that is paid after production. The first constraint in this problem, (1), is an incentive compatibility constraint that states that the output that remains with the entrepreneur after it pays the σ -fraction of the wage bill must exceed the amount of funds he can divert. Rational workers require this incentive compatibility because they could otherwise provide work to other entrepreneurs at the market wage without risking a default. The second constraint, (2), is a working-capital constraint and it says that the fraction of the wage bill payed in advance, $(1 - \sigma) wl$, cannot exceed the liquid funds, xk , held by the entrepreneur.

To solve this problem, I employ a version of the envelope theorem and exploit the fact that this problem is homogeneous in capital. The strategy consists of breaking the problem into

⁸This price is an equilibrium object in the following section.

two subproblems. The first subproblem is an optimal labor choice subject to the enforcement and working-capital constraints for given an amount of liquidity. The value of this problem yields an indirect profit function of liquidity. The second subproblem determines what qualities are sold given this indirect profit function.

Hence, let's hold $\iota(\omega)$, and therefore x at its optimal. Once x is fixed, the objective of the entrepreneurs must be to choose employment subject to the enforcement constraint (1) and the working-capital constraint (3). I solve this problem for $k = 1$ because the objective and constraints are linear in k .

Problem 2 (Optimal Labor) *Given x , the entrepreneur solves*

$$r(x; w) = \max_{l, \sigma} [Al^{1-\alpha} - \sigma wl] + x - (1 - \sigma) wl$$

subject to

$$Al^{1-\alpha} - \sigma wl \geq (1 - \theta^L) Al^{1-\alpha}$$

and

$$(1 - \sigma) wl \leq x.$$

The optimal employment decision is given by:

Proposition 1 (Optimal Labor) *The solution to Problem 2 is $l^*(x) = \min \{l^{cons}(x), l^{unc}\}$ where $l^{cons}(x) = \max \{l : \theta^L Al^{1-\alpha} + x = l\}$ and l^{unc} is the unconstrained labor choice. Constraints are always slack if $\theta^L \geq (1 - \alpha)$. If $\theta^L < (1 - \alpha)$, $x > 0$ is needed to have the unconstrained labor choice.*

This proposition states that the entrepreneur is constrained to hire less labor than the unconstrained optimal if liquidity does not reach a certain level. When this is the case, the enforcement and the working-capital constraints bind. The entrepreneur is bound to choose employment so that his wage bill equals his liquid funds plus the pledgeable fraction of income. An immediate corollary from Proposition 1 is that if the pledgeable amount of output is less than the efficient labor share, $\theta^L < (1 - \alpha)$, efficient employment requires a positive amount of liquid funds. The condition is intuitive: θ^L is the fraction of output that can be fully pledged to workers and since $(1 - \alpha)$ is the efficient labor share of output, liquid funds must fill the gap. I return to this observation when I argue that the enforcement constraint will always be active.

Using the envelope theorem, the problem of choosing $\iota(\omega)$ can be solved using the indirect profit of liquidity $r(x; w)$, the objective of Problem 2:

Lemma 1 (Producer's Problem II) *Problem 1 is equivalent to:*

$$W^p(k; p, q, w) = \max_{\iota(\omega) \geq 0} r(x; w) k + xk + qk \int \lambda(\omega) (1 - \iota(\omega)) f_\phi(\omega) d\omega$$

$$x = p \int \iota(\omega) d\omega$$

where $r(x; w)$ is the value of Problem 2.

By reducing the problem I can solve for the optimal selling decision $\iota(\omega)$ directly and obtain an equilibrium expression for the pooling price p .

Proposition 2 (Producer's Equilibrium Liquidity) *An equilibrium is characterized by a threshold quality function ω^* . All qualities under ω^* are sold by the producer. The equilibrium liquidity x and the pooling price p are given by*

$$x = pF_\phi(\omega^*) \quad \text{and} \quad p = q\mathbb{E}_\phi[\lambda(\omega) | \omega \leq \omega^*].$$

In addition, ω^* belongs to one of the following cases. [1] Interior solution: $\omega^* \in (0, 1)$ and solves,

$$(1 + r_x(x)) \mathbb{E}_\phi[\lambda(\omega) | \omega \leq \omega^*] = \lambda(\omega^*), \quad (4)$$

[2] Fully liquid: $\omega^* = 1$ if $r_x(q\mathbb{E}_\phi[\lambda(\omega)]) \geq 0$, or [3] Market Shutdown: $\omega^* = \emptyset$ with $p = 0$.

Proposition 2 establishes that all equilibria are characterized by a threshold quality ω^* . All qualities below this are sold. The interesting cases are the interior solutions. Equation (4) resembles the equilibrium condition in Akerlof (1970)'s classical lemons example where a marginal quality valued by a seller equals the expected quality valued by the buyer. However, there is a key distinction. Whereas in Akerlof (1970) valuations by buyers and sellers are exogenously given, here those valuations depend on the shadow value of an extra unit of liquidity.

The shadow value of additional liquidity is $(1 + r_x(x))$. By selling a given unit, the entrepreneur obtains p liquid funds. Those liquid funds are used to pay for the entrepreneur's pay-roll in advance. Those funds are eventually returned to the entrepreneur in the form of less wages after production plus the benefit of additional workers $r_x(x)$. Hence, the overall, marginal benefit of a given quality of capital is $p(1 + r(x))$. Naturally, costs and benefits must be equal at the margin. When the entrepreneur sells the threshold unit $\lambda(\omega^*)$, he loses this in efficiency units. Those units are worth to him their opportunity cost, $q\lambda(\omega^*)$. Substituting market clearing condition and clearing out q from both sides gives us the corresponding expression for the interior solutions.

Homotheticity. A corollary to Proposition 2 shows that the entrepreneur's problem is linear in his capital stock. This corollary will be used when we introduce the dynamic model.

Proposition 3 (Value of the Firm) $W^p(k; p, q, w) = \tilde{W}^p(q, w)k$ where

$$\tilde{W}^p(q, w) \equiv r(x; w) + q\bar{\lambda}. \quad (5)$$

Here, $r(x; w)$ is the solution to Problem 1 and x, p and ω^* are given by Proposition 2.

W^p is the sum of profits per unit of capital given x and the value of his initial capital stock. The entrepreneur's financial wealth is $xk + qk \int_{\omega^*}^1 \lambda(\omega) f_\phi(\omega) d\omega$ but the zero-profit condition for intermediaries implies $x = q \int_0^{\omega^*} \lambda(\omega) f_\phi(\omega) d\omega$. Hence, his financial wealth equals $q\bar{\lambda}$. It is worth discussing some features of this model before proceeding to the model with the investment friction.

Comparative Statics about ϕ . Assume the following about the advantage rate of f_ϕ :

Assumption 1. f_ϕ satisfies that $\frac{\lambda(\omega^*)}{\mathbb{E}_\phi[\lambda(\omega)|\omega \leq \omega^*]}$ is increasing in ω^* .

This assumption is useful to guarantee:

Proposition 4 (Interior Solutions) *Assume 1 and that $\lambda(0) > 0$. Then, there always exists a single positive ω^* in Proposition 2.*

Comparative statics about f_ϕ provide the intuition behind the endogenous liquidity mechanism. Consider a family of distributions $\{f_\phi\}$ indexed by ϕ . I impose some structure on the quality distributions $\{f_\phi\}$ to provide an interpretation to ϕ :

Assumption 2. The set $\{f_\phi\}$ satisfies:

1. *Mean preserving:* $\int \lambda(\omega) f_\phi(\omega) d\omega = \bar{\lambda}$ for any $\phi \in \Phi$.
2. *Monotone adverse selection:* $\mathbb{E}_\phi[\lambda(\omega) | \omega \leq \omega^*]$ is weakly decreasing in ϕ for any ω^* .

The first condition states that for any ϕ , the mean of f_ϕ is always $\bar{\lambda}$. The implication of this condition is that the aggregate amount of capital does not change with ϕ . The second condition provides an order to Φ because adverse selection is necessarily worse with a greater ϕ . Since the second property can be often obtained by an increase in the variance of f_ϕ , from now I refer to an increase in ϕ as an increase in dispersion or on a mean-preserving spread.

In the dynamic model, a Markov process will draw a values for ϕ along the business cycle. Periods with higher ϕ will be periods of worse adverse selection although the production possibility will remain unchanged. This will have dramatic effects on output whose

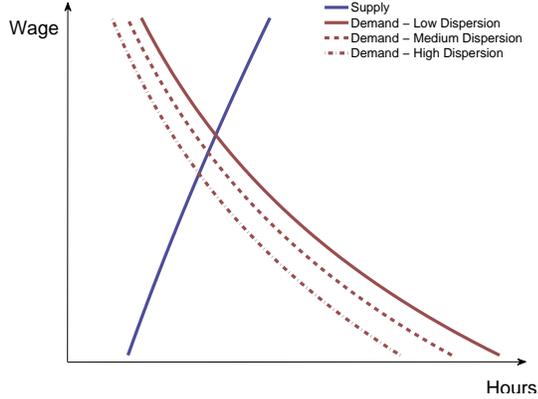


Figure 1: Labor Demand and Supply.

intuition is captured by the static forces discussed here. Take a given value of ϕ . Equilibrium requires to solve equation (4) which combines the 0 profit condition of intermediaries with the entrepreneur's incentive to sell capital under asymmetric information:

$$\underbrace{(1 + r_x(x))}_{\text{Marginal Value of Liquidity}} = \underbrace{\frac{\lambda(\omega^*)}{\mathbb{E}_\phi[\lambda(\omega) | \omega \leq \omega^*]}}_{\text{Marginal Cost of Liquidity}}. \quad (6)$$

Now consider an increase in ϕ . Since by assumption, $\mathbb{E}_\phi[\lambda(\omega) | \omega \leq \omega^*]$ must fall with ϕ for any ω^* , the marginal benefit of liquidity, $(1 + r_x(x))$ must increase and the threshold quality ω^* must fall to restore equilibrium. The intuition is that for any given ω^* , after an increase in ϕ , financial firms will pay a lower price because they expect a worse average quality sold below ω^* . If the entrepreneur does not choose a lower cut-off quality, he will face marginal loss because losing $\lambda(\omega^*)$ is not compensated enough at the new price. The entrepreneur reduces ω^* to the point where the shadow value of relaxing his enforcement constraint compensates for the loss of a new marginal quality. This means that increases in ϕ causes a reduction in the equilibrium amount of liquidity. By Proposition 1, we know this translates into a contraction in labor demand.

Figure 1 plots the increasing labor-supply schedule against three labor demands corresponding to different values of ϕ in this static model.⁹ For any wage, an increase ϕ reduces the labor demand because the cost of obtaining liquidity is higher. Figure 2 describes the effects of ϕ on the rest aggregate outcomes. As ϕ induces worse adverse selection, ω^* , p and x decline. Hours fall in response to less liquidity. The contraction in hours explains the contraction in output. Moreover, wages fall as labor is moving downwards along the supply

⁹The figure corresponds to the actual demand and supply schedules at the mean of the invariant distribution of the state space in the dynamic model.

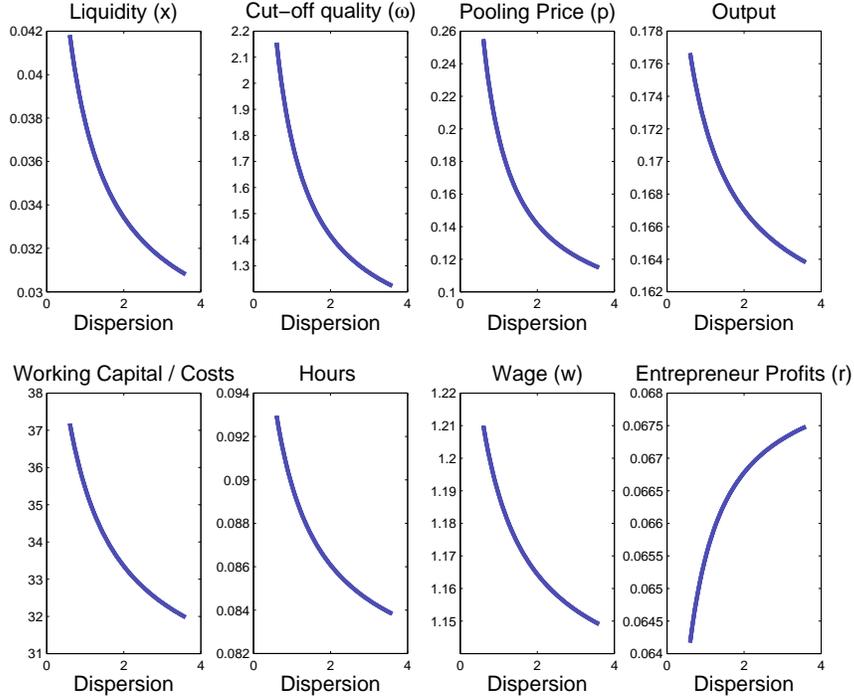


Figure 2: Comparative Statics about ϕ

schedule. A final observation is that the entrepreneur's profits increase. This effect depends on the initial condition for ϕ because the sign of the response of profits depends on whether hours are lower than the monopsonic quantity.

Figure 3 illustrates how liquidity is determined for different levels of dispersion. The top panels show the value loss on the marginal quality sold and the marginal return to liquidity. These amounts satisfy equation (6). The marginal value of liquidity has two components, the additional increase in the labor force with more liquidity and the additional profits obtained by hiring an additional worker which are depicted in the bottom.

2.1.1 Discussion

Limited Enforcement on Labor Contracts. The option to default on labor contracts imposes a constraint on the entrepreneur's employment decision that depends on his liquid funds. This form of limited enforcement has a similar effect to the working-capital constraint that requires the wage bill to be payed up-front. In fact, it corresponds to the limiting case where θ^L to 0. Working-capital constraints, originally introduced by [Christiano and Eichenbaum \(1992\)](#), relate labor demand responses to borrowing costs. Quantitative work by [Christiano, Eichenbaum and Evans \(2005\)](#) or [Jermann and Quadrini \(2012\)](#) has shown that these are important features of business cycle models. A similar force operates under limited enforce-

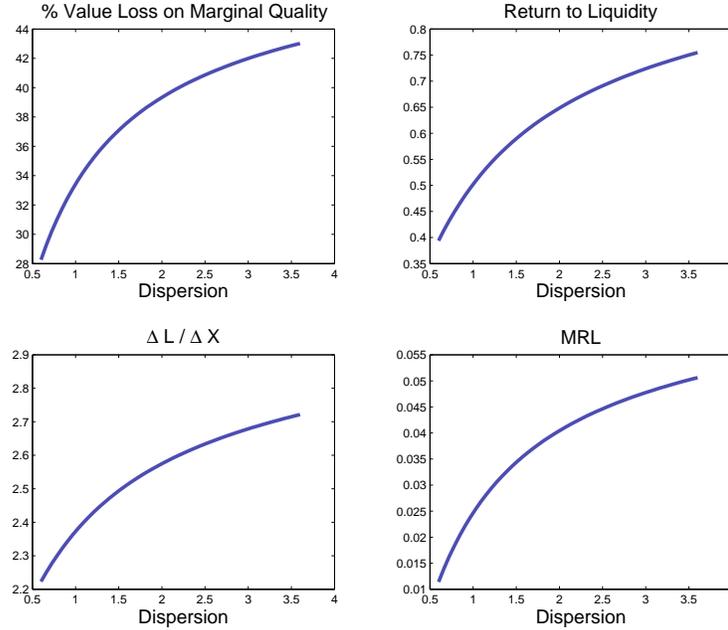


Figure 3: Trade Offs in Choice of Liquidity.

ment. However, under limited enforcement, the fraction of the wage bill payed up front, $(1 - \sigma)$ here, is not a constant.

Under decreasing returns to scale, production costs become a higher proportion of output as more labor is used. Since the fraction of output that can be pledged is constant and but costs are an increasing proportion, the entrepreneur needs a higher proportion of liquid funds to operate at a bigger scale. This feature has the observable implication that working capital over total costs must be increasing in output. Hence, in the model, the working capital to costs ratio falls as ϕ induces higher costs of obtaining liquidity. In the empirical assessment of the model, I present evidence that the working capital over costs ratio fell during the Great Recession.

Figure 4 replicates partial equilibrium exercise of Figure 2 when σ is constant (as with a standard working-capital constraints). The main takeaway from the figure is that a standard working-capital constraint would amplify the impact of ϕ despite that liquidity holdings are higher as entrepreneurs have more incentives to obtain liquidity. The overall qualitative implications are similar.

Wage Rigidity. The model abstracts from any form of wage rigidity. The empirical assessment of the dynamic model shows that wages are more responsive in the model than in the data (see Shimer (2013)). Figure 2 presents the behavior of the model holding real wages fixed to their equilibrium level at the lowest value of dispersion. The figure shows that wage rigidity amplifies the effects of ϕ for several reasons. First, as liquidity falls with more

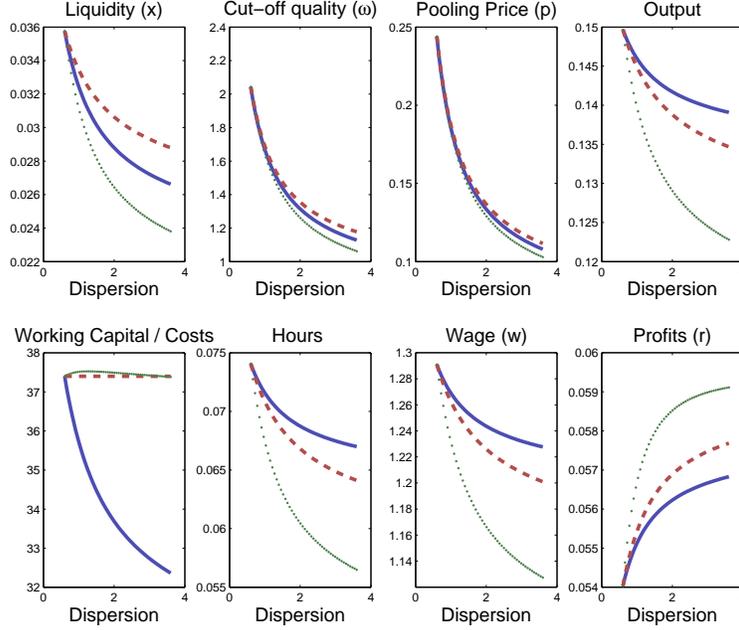


Figure 4: Comparative Statics about ϕ with wage rigidity and constant working-capital constraints.

dispersion, entrepreneurs are constrained to hire less workers than if wages could adjust. Second, if wages do not adjust, total costs over output do not decrease with less liquidity. Hence, the enforcement problem becomes worse. This will reduce the benefits of obtaining liquidity, so liquidity will be less than otherwise. These forces make output more sensible to ϕ under wage rigidities. In addition, wage rigidities also lead to a reduction in profits after an increase in ϕ as the monopsony effect vanishes.

2.2 Endogenous Liquidity and Investment

In this section, we study the problem of an entrepreneur who produces capital. He also lacks the input for his production and faces a similar enforcement problem to the one studied before. I call this entrepreneur the i-entrepreneur to distinguish him from the p-entrepreneur of the previous section. Everything else is the same and the essence of his problem is the same as in KM.

Production of investment goods. The i-entrepreneur has a constant-returns-to-scale technology that transforms a unit of consumption into a unit of capital.

Limited enforcement in investment claims. The i-entrepreneur can sell claims on capital goods in exchange for consumption goods. Following KM, an i-entrepreneur has access to a technology to divert a fraction $(1 - \theta^I)$ of his investment projects for personal use. This imposes a constraint on the issuance of claims. A similar restrictions is obtained in models

of hidden-effort (e.g., [Holmstrom and Tirole \(1997\)](#)).

Information. The entrepreneur holds a capital stock which he only uses to obtain liquid funds. The i-entrepreneur has the same private information about ω as before. In contrast, investment projects are homogeneous so there is no asymmetric information problem for newly produced capital. Financial intermediaries buy capital under asymmetric information and resell them at an exogenous price q . They earn zero profits.

An i-entrepreneur's problem is similar to the p-entrepreneurs except that he chooses an optimal financial structure for investment projects. He also maximizes the value of his assets. Note that like his p-counterpart, he also lacks the input to his production technology, consumption goods. To finance his production the i-entrepreneur obtains inputs by either selling capital under asymmetric information or issuing claims against his output.

Problem 3 (Investor) *The i-entrepreneur solves:*

$$W^i(k; p, q) = \max_{k^b, i^d, i^s, \iota(\omega)} i - i^s + k^b + \int_0^1 (1 - \iota(\omega)) \lambda(\omega) k f_\phi(\omega) d\omega$$

subject to:

$$i = i^d + qi^s$$

$$i - i^s \geq (1 - \theta^I) i \tag{7}$$

$$qk^b + i^d \leq xk \tag{8}$$

$$x = p \int_0^1 \iota(\omega) f_\phi(\omega) d\omega.$$

Liquid funds available to the i-entrepreneur xk are obtained by selling capital $\int \iota^s(\omega) f_\phi(\omega) d\omega$ at a price p^i . The use of liquid funds is captured by equation (8). These funds are used to buy k^b at q or to internally invest i^d . In addition, the i-entrepreneur obtains production inputs issuing i^s claims at the market price q . Thus, his output is $i = i^d + qi^s$ since production is linear in inputs. Thus, since $i^d = i - qi^s$, i^d is the portion of investment financed internally. This down payment plays a similar role as the portion of the wage bill paid upfront. Finally, (7) is an incentive compatibility condition that prevents the entrepreneur from defaulting on his claims. The constraint states that investment, net of claims, should be larger than the capital kept upon default $(1 - \theta^I) i$. This constraint is introduced in KM. I follow the same steps as for p-entrepreneurs solving the i-entrepreneur's financial decision given x (and $\iota(\omega)$):

Proposition 5 (Optimal Financing) *When $q > 1$, any solution to Problem 3 requires $i^s = \theta^I i$, $k^b = 0$ and $i^d = xk$. When $q = 1$, the solution for i^s , i^d and k^b is indeterminate. If $q < 1$,*

$$k^b = xk \text{ and } i^d = i^s = 0.$$

The interesting case is when $q > 1$. The proposition says that if $q > 1$, the entrepreneur will issue as many claims as he can. The reason is that he is exploiting an arbitrage opportunity: creating capital costs one unit of consumption but by selling claims he obtains q . Thus, for any unit of investment, he finances the $(1 - \theta^I q)$ the fraction of his output but he owns the $(1 - \theta^I)$ fraction. Thus, his replacement cost is $q^R = \frac{(1 - \theta^I q)}{(1 - \theta^I)}$. Replacement costs determine his choice of liquid funds. Proposition 6 is the analogue of Proposition 2 which describes the equilibrium liquidity chosen by the i-entrepreneur:

Proposition 6 (Investors Equilibrium Liquidity) *An equilibrium is characterized by a threshold quality function ω^i such that all qualities under ω^i are sold by the i-entrepreneur. The equilibrium liquidity and price for i-entrepreneurs are given by:*

$$x^i = p^i F(\omega^i) \text{ and } p^i = q \mathbb{E}_\phi [\lambda(\omega) | \omega \leq \omega^i].$$

In addition ω^i is either: [1] Interior solution: $\omega^i \in (0, 1)$ and solves,

$$\frac{q}{q^R} \mathbb{E}_\phi [\lambda(\omega) | \omega \leq \omega^i] = \lambda(\omega^i), \quad (9)$$

[2] Fully liquid: $\omega^i = 1$ if $\frac{q}{q^R} \geq \lambda(1) / \bar{\lambda}$ or [3] Market Shutdown: $\omega^i = \emptyset$ with $p^i = 0$.

As with producers, Proposition 6 states that the solution is also characterized by a threshold quality. However, in this case, the exogenous valuations in the lemons problem are replaced by Tobin's Q, the market price of capital q over the replacement cost q^R . Thus this entrepreneur equates the marginal cost of liquidity to the marginal benefit of obtaining liquidity, which is given by his arbitrage opportunity in the creation and financing of capital:

$$\underbrace{\frac{q}{q^R}}_{\substack{\text{Marginal Value of Liquidity} \\ \text{(Tobin's Q)}}} = \underbrace{\frac{\lambda(\omega^i)}{\mathbb{E}_\phi [\lambda(\omega) | \omega \leq \omega^i]}}_{\text{Marginal Cost of Liquidity}}$$

As with the p-entrepreneur, an increase in ϕ will increase the marginal cost of obtaining liquid funds, leading to a fall in the threshold quality of capital sold, ω^i . The consequent reduction in liquidity leads to a contraction in the supply of investment claims. Aggregate investment falls together with an increase in the price of these claims.

Homotheticity. I final result is that we also have linearity for the i-entrepreneur's problem:

Proposition 7 (Value of the Firm) $W^i(k; p, q, w) = \tilde{W}^i(q)k$ where

$$\tilde{W}^i(q) \equiv \frac{1}{q^R} \left[q \int_0^{\omega^i} \lambda(\omega) k f_\phi(\omega) d\omega + q^R \int_{\omega^i}^1 \lambda(\omega) k f_\phi(\omega) d\omega \right] \quad (10)$$

where ω^i is given by Proposition 6.

3 Collateralized Debt with Default

I now allow p-entrepreneur's to issue collateralized debt (CD) contracts like the ones obtained in DeMarzo and Duffie (1999). These contracts are equivalent to repurchase contracts (repos) so for the rest of the paper I use the terms interchangeably. A CD contract is characterized by a pair of prices (p^S, p^F) and a unit of capital. When an entrepreneur enters a contract, it transfers the property of the capital unit to the intermediary. The intermediary provides the entrepreneur p^S liquid funds: p^S is a spot purchase price. The entrepreneur then agrees to repurchase the unit paying a pre-specified future price p^F . The entrepreneur may choose not to repurchase the capital unit, defaulting on the contract and in that case, the intermediary retains the unit. If the entrepreneur pays p^F , he is returned the unit. I assume that financial intermediaries can commit to return the asset if the entrepreneur pays p^F .

One may reinterpret this contract as a CD contract with default. Let p^S represent the loan size, $R \equiv \frac{p^F}{p^S}$ the gross interest rate and the unit of capital represents the collateral. Thus, collateral is pledged to obtain p^S and is returned only if Rp^S is repaid.

Equilibria. For simplicity of exhibition, I focus on equilibria with a single contract (p^S, p^F) with zero-profits for intermediaries. I only solve the p-entrepreneur's problem because by symmetry, outcomes should be expected to be similar for i-entrepreneurs.

Recall that Lemma 1 shows that the p-entrepreneur's problem (Problem 1) can be summarized without reference to the labor-working capital choice. Once liquidity is determined, the value of Problem 2 is the indirect value of liquidity which is a microfoundation for the reduced form for internal funds in DeMarzo and Duffie (1999). Thus, to summarize the entrepreneur's problem, allowing for CD contracts, one uses this indirect value of liquidity and an analogue problem to Lemma 1 determines his optimal policies:

Problem 4 (Producer with repo) *The p-entrepreneur maximizes:*

$$W^p(k; p^S, p^F, q, w) = \max_{I(\omega), \iota(\omega)} r(x)k + xk + \dots \\ k \int_0^1 (1 - I(\omega)) \iota(\omega) (q\lambda(\omega) - p^F) + (1 - \iota(\omega)) q\lambda(\omega) f(\omega) d\omega \quad (11)$$

subject to:

$$x = p^S \int_0^1 \iota(\omega) f(\omega) d\omega.$$

In this problem, $r(x)$ is again the value of Problem 2. As before, $\iota(\omega)$ is the indicator that is 1 when a unit of quality ω participates in a contract. The indicator function $I(\omega)$ takes the value 1 when a repo contract with collateral quality ω is defaulted. Note that when this indicator is 0, the entrepreneur retains $\lambda(\omega)$ efficiency units, which he values at a price q , but pays p^F . When he defaults, this indicator becomes 1, and the unit is no longer part of his capital stock. In this case, he saves p^F .

Profits for the intermediary are:

$$\Pi(p^F, p^S, \iota(\omega), I(\omega)) = \int_0^1 \iota(\omega) [((1 - I(\omega))p^F + I(\omega)q\lambda(\omega)) - p^S] f(\omega) d\omega.$$

Profits are the sum of profits from all contracts (hence the $\iota(\omega)$ at the outset of the integral). The intermediary earns $(p^F - p^S)$ on contracts that are not defaulted, indicated by $(1 - I(\omega))$. The intermediary keeps the collateral from defaulted contracts and resells them at q . Thus, he earns $q\lambda(\omega)$ minus p^S on defaulted contracts which are indicated by $I(\omega)$.

Equilibrium with CD. The equilibrium liquidity with CD contracts is given by a pair of prices (p^S, p^F) and policy functions $(I(\omega), \iota(\omega))$ such that (1) $(I(\omega), \iota(\omega))$ are solutions to Problem 4 given prices, and (2) intermediaries are competitive, $\Pi(p^F, p^S, \iota(\omega), I(\omega)) = 0$. Equilibria with CD contracts are summarized by:

Proposition 8 (CD Equilibria) *An equilibrium with a single CD contract is characterized by a pair of prices (p^S, p^F) and a pair of threshold qualities $(\omega^p, \bar{\omega}^p)$. These satisfy the following conditions:*

$$p^S \int_0^{\bar{\omega}^p} f(\omega) d\omega = \int_0^{\omega^p} q\lambda(\omega) f(\omega) d\omega + p^F \int_{\omega^p}^{\bar{\omega}^p} f(\omega) d\omega \quad (12)$$

and

$$q\lambda(\omega^p) = p^F \quad (13)$$

and

$$r_x \left(p^S \int_0^{\bar{\omega}^p} f(\omega) d\omega \right) p^S = (p^F - p^S). \quad (14)$$

In addition, $\iota(\omega)$ equals 1 for $\omega < \bar{\omega}^p$ and $I(\omega)$ equals 1 for $\omega < \omega^p$.

The proof is relegated to the appendix but its idea is intuitive. If an agent defaults on a given contract, he will also default on all contracts with collateral of inferior quality. Otherwise, the agent can be better off keeping that quality and defaulting on lower qualities while not affecting his liquidity. This implies that there is a threshold ω^p such that all

contracts with inferior collateral are defaulted. This explains the form of the zero profit condition (12). The threshold quality for defaults must be such that the entrepreneur is indifferent between default and repurchase, which yields (13). Since some qualities may be defaulted, the repurchase price is (weakly) higher than the selling price: $(p^F - p^S) \geq 0$. Hence, the agent pledging a given quality to a repo contract will experience a financial loss. Thus, it must be that the value of additional liquidity compensates that loss. This intuition motivates equation (14).

According to the proposition, equilibrium liquidity is characterized by 3 equations and 4 unknowns. Thus, there is a continuum of solutions. For a particular example, Figure 5 depicts the entire set of equilibria. Each equilibrium is indexed by some ω^* corresponding to a participation threshold $\bar{\omega}^p$. The figure depicts the properties of the set of equilibria. The upper panels display equilibrium liquidity and the implied interest rate for a participation cutoff ω^* . The bottom panels show the implied default rate, $F(\omega^p)/F(\bar{\omega}^p)$ and the loan size p^S for each equilibria. There are three equilibria of particular interest: the one for which, $\omega^p = \bar{\omega}^p$ (wide circle), the equilibrium where $\bar{\omega}^p = 1$ (square) which corresponds to the optimal liquidity contract in DeMarzo and Duffie (1999), and the equilibrium with the largest loan size, p^S (diamond).

Properties of CD contracts. The first property is that the CD for which $\bar{\omega}^p = \omega^p$, corresponds to the selling contracts of Section 2. This is the case because, in equilibrium, defaulting or selling is isomorphic. This is also the equilibrium with the lowest participation because contracts. Second, liquidity is increasing in the participation cutoff ω^* . The more collateralization, the higher the quality collateral pool and the lower the default rate. Third, because higher participation rates require greater incentives to participate, p^S may be decreasing in ω^* . As a consequence, p^S is possibly non-monotone in ω^* . In the quantitative section I focus on the contract with the highest loan size p^S for reasons left out of the discussion here.¹⁰ However, there is an observational equivalence by which all equilibria are indistinguishable through the lense of an econometrician that does not observe the terms of these contracts directly.

Observational Equivalence. If we substitute the zero-profit condition for the intermediary into the entrepreneur's budget constraint, the value for the entrepreneur is:

$$W^p(k; p^S, p^F, q, w) = (r(x) + q\bar{\lambda}) k.$$

¹⁰This equilibrium is of particular interest. I conjecture that it would arise in an environment where intermediaries compete for costumers and where contracts are non-exclusive like in Section 2. The intuition behind is that if an agent is defaulting on a given quality, he would sign the contract with the highest price p^S . Since he knows he will default anyway, he is better off taking the highest price. Competition drives all contracts to this price. This assumption differs from the implicit commitment to a contract in DeMarzo and Duffie (1999) that leads to the highest liquidity provision.

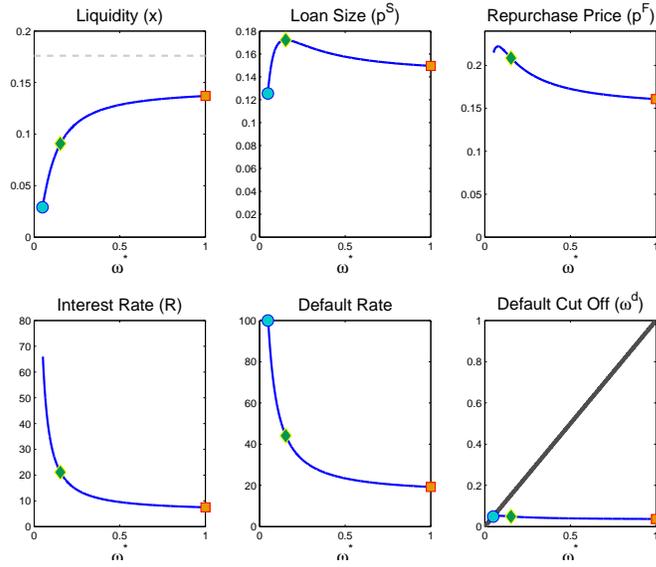


Figure 5: Set of Equilibria CD Contracts.

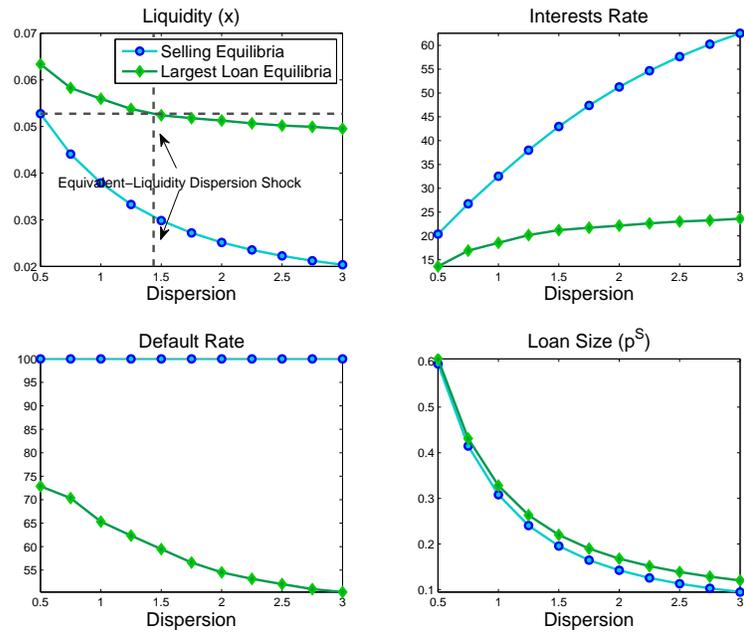


Figure 6: Observational Equivalence between CD contracts and sales contracts.

This is the same value function as in Proposition 3. Thus, this property states that as long as the sales contract of Section 2 and the CD contract yield the same amount of liquidity, allocations will be the same. A corollary of this result is that for a given an equilibrium with sales and given a shock ϕ , if one can find another shock ϕ' such that equilibrium with CD contracts yield the same x . It follows that all allocations must be the same. Figure 6 follows the procedures to compute equilibria in Figure 5 and computes the highest loan size contracts for different values of dispersion. In the top panel, one can observe that given an initial value of liquidity with sales, one can increase the dispersion in the equilibria with CD to obtain the same amount of liquidity. Thus, if f_ϕ is unobservable to an econometrician, both contracts are indistinguishable from aggregate data on x . This procedure provides an algorithm to compute equilibria with CD departing from equilibria with sales. I use this observational equivalence to compute equilibria with CD in the dynamic model of the next section and analyze its predictions about credit market conditions.

4 Dynamic Model

4.1 Environment

The dynamic model is formulated in discrete time and infinite horizon. There are two goods: a perishable consumption good (the *numeraire*) and capital. Every period there are two aggregate shocks: a TFP shock $A_t \in \mathbb{A}$ and a shock $\phi_t \in \{\phi_1, \phi_2, \dots, \phi_N\}$ that selects a member among the family of capital quality distributions $\{f_\phi\}$. A Markov process for (A_t, ϕ_t) evolves according to a transition probability Π .

4.2 Demography and Preferences

The agents in this economy are as before: workers, entrepreneurs, and financial firms. The measure of entrepreneurs is normalized to a unity, the mass of workers to ϖ and the measure of financial firms is irrelevant.

Workers. Workers choose consumption and labor only. They don't have access to savings technologies. Their period utility is given by:

$$U^w(c, l) \equiv \max_{c \geq 0, l \geq 0} c - \frac{(l)^{1+\nu}}{(1+\nu)}$$

where l is the labor supply and c consumption and ν is the inverse of the Frisch elasticity. Workers satisfy a static budget constraint in every period: $c_t = w_t l_t^w$ where w_t is the wage.

The only role for workers is to provide the elastic labor-supply schedule of Section 2.¹¹

Financial Firms. Financial firms are intermediaries that purchase capital under asymmetric information and resell it with full disclosure. These firms are competitive profit-maximizers. As in Prescott and Townsend (1984) and Bisin and Gottardi (1999), they simplify the definition of equilibria and can be replaced by a market clearing condition.

Entrepreneurs. An entrepreneur is identified by a number $z \in [0, 1]$. Every period, entrepreneurs are randomly assigned one of two possible types: investors and producers. I refer to these types as i-entrepreneurs and p-entrepreneurs because they will face the same problems as in Section 2. At the beginning of each period, entrepreneurs draw a type where the probability of becoming an i-entrepreneur is always equal to π . Thus, every period there is a mass π of i-entrepreneurs and $1 - \pi$ of p-entrepreneurs.¹²

The entrepreneur's preferences over consumption streams are given by an expected utility criterion:

$$\mathbb{E} \left[\sum_{t \geq 0} \beta^t U(c_t) \right]$$

where $U(c) \equiv \frac{c^{1-\gamma}}{1-\gamma}$ and c_t is the entrepreneur's consumption at date t .

4.3 Technology

Technology of p-entrepreneurs. A p-entrepreneur produces consumption goods with the same technology of Section 2. Thus his profits are $A_t F(k_t(z), l_t) - w_t l_t$. Again, he has the technology to divert θ^L of their output for his personal benefit and so can default on his workers.

Technology of i-entrepreneurs. The i-entrepreneur has access to the same constant-returns-to-scale technology that transforms consumption goods into capital as in Section 2. In his case, he can issue investment claims and divert θ^I of the capital he creates.

Thus, the economy operates like a two-sector economy with sectors producing according to the technologies of the static models presented before.

Capital. At the beginning of every period, capital is divisible into a continuum of pieces. Each piece is identified by a quality ω . The differentiable function $\lambda(\omega)$ determines the corresponding efficiency units that remain from a quality ω by the end of the period. Efficiency units can also be interpreted as random depreciation shocks. Thus, ω and λ correspond to

¹¹The model could be modified to allow workers to save. With GHH preferences, this would not alter the labor supply. However, the model would require an additional state variable but the substance of the model would not change. GHH are commonly used to prevent counterfactual contractions in the labor supply.

¹²This randomization is convenient to avoid keeping track of wealth distributions across groups. This feature reduces the dimension of the state space.

the same objects as in Section 2.

The distribution among qualities assigned to each piece is randomly changing over time. In particular, at a given point in time, the distribution of capital qualities is determined by a density function f_ϕ , which, in turn, depends on the current realization of ϕ_t . The distribution is the same for all entrepreneurs although it differs across time. Therefore, the measure of units of quality ω out of a capital stock k is $k(\omega) = k f_\phi(\omega)$. Between periods, each piece is transformed into future efficiency units by scaling qualities by their corresponding $\lambda(\omega)$. Thus, $\lambda(\omega) k(\omega)$ efficiency units remain from the ω -qualities. Once capital units are scaled, they form homogeneous quantities of capital that can be merged or divided for form larger or smaller pieces. Thus, by the end of the period, the capital stock that remains from k is,

$$\tilde{k} = \int \lambda(\omega) k(\omega) d\omega = k \int \lambda(\omega) f_\phi(\omega) d\omega. \quad (15)$$

In the following period, the capital held by every entrepreneur is again divided in the same way and the process is repeated indefinitely. This does not mean that an entrepreneur with k capital at t will necessarily hold all of \tilde{k} at $t+1$. On the contrary, entrepreneurs can choose to sell particular qualities. Using the earlier notation, $\iota^s(\omega)$ is indicator for the decision on selling the capital units of quality ω .

In equilibrium, financial firms purchase all the units sold by entrepreneurs. Thus, an entrepreneur transfers $k \int \iota^s(\omega) f_\phi(\omega) d\omega$ units of capital to the financial sector. Accounting shows that the efficiency units that remain with the entrepreneur after his sale are $k \int \lambda(\omega) (1 - \iota^s(\omega)) f_\phi(\omega) d\omega$. Including investments and purchases of capital, the entrepreneur's capital stock evolves according to:

$$k' = i - i^s + k^b + k \int \lambda(\omega) (1 - \iota^s(\omega)) f_\phi(\omega) d\omega, \quad (16)$$

where $i - i^s$ is the total investment i carried out by the entrepreneur net of issued claims i^s . Finally, this stock is augmented by capital purchases, k^b .

I impose the same assumptions on the family $\{f_\phi\}$ as before. The implications of these assumptions is that the production possibility of the economy is invariant so if ϕ has any effects on allocations, it is because it affects the actual but not the possible set of allocations.

4.4 Timing, Information and Markets

Information. Aggregate capital, $K_t \in \mathbb{K} \equiv [0, \bar{K}]$, is the only endogenous aggregate state variable. The aggregate state of the economy is therefore summarized by the vector $X_t = \{A_t, \phi_t, K_t\} \in \mathbb{X} \equiv \mathbb{A} \times \Phi \times \mathbb{K}$. At the beginning of each period, X_t and the entrepreneurs'

types become common knowledge. This means that financial firms can discriminate between an entrepreneur's activity.

The ω -qualities are only known to the entrepreneur. In turn, financial firms can observe the amount of capital being transferred to them, $k \int \iota^s(\omega) f_\phi(\omega) d\omega$. However, they ignore the efficiency units that remain from that purchase, $k \int \lambda(\omega) \iota^s(\omega) f_\phi(\omega) d\omega$. The remaining efficiency units \tilde{k} from a capital stock k are independent of the entrepreneur's choice of $\iota^s(\omega)$. Using equation (15) we have:

$$\tilde{k} = k \underbrace{\int \lambda(\omega) (1 - \iota^s(\omega)) k(\omega) d\omega}_{\text{fraction kept}} + k \underbrace{\int \lambda(\omega) \iota^s(\omega) k(\omega) d\omega}_{\text{fraction sold}}.$$

Hence, the choice of $\iota^s(\omega)$ only affects the distribution of $t+1$ capital between entrepreneurs and intermediaries. Since in the following period f_ϕ affects every entrepreneur no matter how they obtain k' , entrepreneurs only care about maximizing the fraction of capital that remains in their possession. This modeling choice is essential to solve the model without keeping track of the history of trades or the distribution of capital qualities. It also makes the lemons problem static.

Timing. The sequence of actions taken by the agents in this economy is as follows: at the beginning of each period, all the relevant information is revealed. Then p-entrepreneurs choose which qualities to transfer to financial firms in exchange for claims to consumption goods (liquid funds) denoted by x . Financial firms credibly guarantee to deliver these goods to their owners by the end of the period. Entrepreneurs transfer consumption claims to workers as an upfront payment of the endogenous fraction $(1 - \sigma)$ of payroll. Workers then provide labor, and production is carried out. The consumption goods are then used by p-entrepreneurs to: (i) pay for the remaining fraction of the wage bill $(1 - \sigma)w_t$, (ii) consume, and (iii) purchase (or repurchase) capital to be delivered by financial firms. In return for capital transferred to p-entrepreneurs, financial firms obtain consumption goods which are used partially to settle claims on x and partially to transact with i-entrepreneurs. In exchange for consumption goods, i-entrepreneurs sell capital qualities and claims to new investment projects, i^s , to financial firms. After the production of capital, all capital claims are settled.

This sequence of events is consistent with the physical requirement that consumption goods must be created before capital goods. For the rest of the paper, I treat these actions as occurring simultaneously.

Markets. Labor markets are competitive. I impose the following :

Assumption 3. *Financial firms are **competitive** and the capital market is **anonymous** and **non-exclusive**.*

Competition ensures financial firms earn zero profits. Anonymity and non-exclusiveness guarantees that the market for capital has pooling price. Without anonymity and exclusivity, financial firms pay a different price depending on the quantity of capital traded by each entrepreneur. For example, they can recover the full information outcomes if they offer a price schedule proportional to the cumulative distribution of f_ϕ .

Notation: For the remainder of the paper, I append terms like $v^j(k, X)$ to indicate the policy function of an entrepreneur of type j in state (k, X) . I use $v^j(\omega, k, X)$ to refer to the decision to sell a quality ω in that state. I denote by \mathbb{E}_ϕ the expectations over the quality distribution f_ϕ and \mathbb{E} the expectations about future states.

4.5 Entrepreneur Problems and Equilibria

I begin with the description of the p-entrepreneur's dynamic problem:

Problem 5 (Producer's Problem) *The p-entrepreneur solves*

$$V^p(k, X) = \max_{c \geq 0, k^b \geq 0, \iota(\omega), l, \sigma \in [0, 1]} U(c) + \beta \mathbb{E} [V^j(k', X') | X], \quad j \in \{i, p\}$$

subject to

$$(Budget\ constraint) \quad c + q(X) k^b = AF(k, l) - \sigma wl + xk - (1 - \sigma)wl \quad (17)$$

$$(Capital\ accumulation) \quad k' = k^b + k \int \lambda(\omega) (1 - \iota(\omega)) f_\phi(\omega) d\omega \quad (18)$$

$$(Incentive\ compatibility) \quad AF(k, l) - \sigma wl \geq (1 - \theta^L) AF(k, l) \quad (19)$$

$$(Liquid\ funds) \quad x = p^p(X) \int \iota^s(\omega) f_\phi(\omega) d\omega \quad (20)$$

$$(Working-capital\ constraint) \quad (1 - \sigma) wl \leq xk \quad (21)$$

The first constraint is his budget constraint. The right-hand side of the budget constraint corresponds to the entrepreneur's profits minus the amount of liquid funds he holds after paying for the σ fraction of the wage bill. The entrepreneur uses these funds to consume c , and to purchase k^b at the full-information price $q(X)$. The second constraint corresponds to the evolution of the entrepreneur's capital stock with the restriction that p-entrepreneurs cannot produce capital or issue claims. The last three constraints are the constraints described in Section 2: the incentive compatibility constraint (19), the accounting of liquid funds per unit of capital (20), and the use of liquidity as working capital (21).

An i-entrepreneur's problem is similar.

Problem 6 (Investor's Problem) *The i -entrepreneur solves*

$$V^i(k, X) = \max_{c \geq 0, i, i^s \geq 0, k^b \geq 0, \iota(\omega) \geq 0} U(c) + \beta \mathbb{E} [V^j(k', X') | X], j \in \{i, p\}$$

subject to

$$(Budget\ constraint) \quad c + k' = \tilde{k} \quad (22)$$

$$(Capital\ accumulation) \quad \tilde{k} = k^b + i - i^s + k \int \lambda(\omega)(1 - \iota^s(\omega)) f_\phi(\omega) d\omega \quad (23)$$

$$(Incentive\ compatibility) \quad i - i^s \geq (1 - \theta^I) i \quad (24)$$

$$(Working\ Capital) \quad q(X)k^b + i^d \leq xk \quad (25)$$

$$(Investment\ Funds) \quad i = q(X)i^s + i^d \quad (26)$$

$$(Liquid\ funds) \quad x = p^i(X) \int \iota^s(\omega) f_\phi(\omega) d\omega \quad (27)$$

The right-hand side of the i -entrepreneur's budget constraint is the entrepreneurs output capital stock. He comes to build this capital stock sum by selling capital under asymmetric information and by issuing i^s claims to investment at price $q(X)$. The constraints in this problem have the same interpretation as in Problem 3. Since capital is reversible, this capital stock can be used to consume c or to transfer capital to subsequent periods.

Financial firms. Financial firms purchase capital units of different qualities from both entrepreneur types at corresponding pooling prices p^p and p^i . They also purchase claims to investment projects at the full information price q . Units are resold as homogeneous capital. Competition in financial markets ensures zero profits from trading with either entrepreneur type since there is no risk in this operation. I assume and later verify that the decision to sell a unit of quality ω is a function only of the entrepreneur's type and the aggregate state X and is independent of the size of his capital stock. Hence, we have the same zero-expected-profit conditions as before:

$$p^p(X) = q(X) \mathbb{E}_\phi [\lambda(\omega) | \iota^{s,p}(\omega, X) = 1] \quad (28)$$

and

$$p^i(X) = q(X) \mathbb{E}_\phi [\lambda(\omega) | \iota^{s,i}(\omega, X) = 1]. \quad (29)$$

The measure over capital holdings and entrepreneur types at a given period is denoted by $\Gamma(k, j)$ for $j \in \{i, p\}$. By independence,

$$\int \Gamma(dk, i) = \pi K \quad \text{and} \quad \int \Gamma(dk, p) = (1 - \pi) K. \quad (30)$$

The total aggregate demand for efficiency units and the supply of investment claims are respectively:

$$D(X) \equiv \underbrace{\int k^{b,p}(k, X) \Gamma(dk, p)}_{\text{Capital demand of p-types}} + \underbrace{\int k^{b,i}(k, X) \Gamma(dk, i)}_{\text{Capital demand of i-types}} \quad \text{and} \quad I^s(X) \equiv \underbrace{\int i^s(k, X) \Gamma(dk, i)}_{\text{Supply of new units by i-types}} .$$

Finally, transfers of efficiency units from both groups to the financial sector are obtained by integrating over the corresponding qualities and capital stocks:

$$S(X) \equiv \underbrace{\int k \left[\int \iota^{s,i}(k, X, \omega) \lambda(\omega) f_\phi(\omega) d\omega \right] \Gamma(dk, i)}_{\text{Efficiency units supplied by i-entrepreneurs}} \dots \\ + \underbrace{\int k \left[\int \iota^{s,p}(k, X, \omega) \lambda(\omega) f_\phi(\omega) d\omega \right] \Gamma(dk, p)}_{\text{Efficiency units supplied by p-entrepreneurs}} .$$

Capital market clearing is given by $D(X) = I^s(X) + S(X)$. Finally, the labor market clearing requires: $\int l(k, X) \Gamma(dk, p) = \varpi l^w(X)$. The definition of equilibria does not depend on the distribution of capital because this economy admits aggregation, as shown later.

Definition (Recursive Competitive Equilibrium). *A recursive competitive equilibrium is (1) a set of price functions, $q(X)$, $p^i(X)$, $p^p(X)$, $w(X)$, (2) a set of policy functions, $\{c^j(k, X), k^{b,j}(k, X), \iota^{s,j}(\omega, k, X)\}_{j=p,i}$, $c^w(X)$, $l^w(X)$, $i(k, X)$, $i^s(k; X)$, $l(k, X)$, $\sigma(k, X)$, (3) a pair of value functions, $\{V^j(k, X)\}_{j=p,i}$, and (4) a law of motion for the aggregate state X such that for any distribution of capital holdings Γ satisfying (30), the following hold: (1) taking price functions as given, the policy functions solve the entrepreneurs' and worker's problem and V^j is the value of the j -entrepreneur's problem. (2) $p^p(X)$ and $p^i(X)$ satisfy the zero-profit conditions (28) and (29). (3) The labor market clears. (4) The capital market clears. (5) Capital evolves according to $K' = \int i(k, X) \Gamma(dk, i) + \bar{\lambda}K$. (6) The law of motion for the aggregate state is consistent with the individual's policy functions and the transition function Π .*

5 Characterization

Producer's dynamic problem. I begin the characterization of equilibria by describing the solution to the p-entrepreneur's problem. The strategy consists of breaking his problem into two subproblems. The first subproblem corresponds to the problem solved in Section 2. Then, the dynamic problem is collapsed into a standard consumption-savings problem with

stochastic linear returns.

To observe how this is done, notice that once k^b is substituted from the capital accumulation equation (18) into the p-entrepreneurs budget equation (17), we obtain a budget constraint in terms of capital:

$$c + q(X)k' = AF(k, l) - wl + xk + q(X)k \int \lambda(\omega) (1 - \iota^s(\omega)) f_\phi(\omega) d\omega.$$

The choices of $\iota^s(\omega)$, l , and σ only affect the right-hand side of this budget constraint and not the objective function. The entrepreneur's constraint set only affects $\iota(\omega)$, l , and σ , but not the consumption or savings decision. Hence, this independence implies that the entrepreneur's problem can be broken into two subproblems. First one solves for the $\iota^s(\omega)$, l , and σ maximizing the right-hand side of the p-entrepreneur's budget constraint. Then one solves for c , and k^b given the solution to the maximal budget. The first subproblem corresponds to Problem 1 in Section 2.

The solutions to $l(X)$ and $\sigma(X)$ are given by Proposition 1. Moreover, the equilibrium qualities sold are given by Proposition 2. Hence, any recursive competitive equilibrium is characterized by a threshold quality function $\omega^p(X)$ such that in state X , all qualities under $\omega^p(X)$ are sold by all p-entrepreneurs. The amount of liquid funds available to the p-entrepreneur, $x^p(X)$, is also determined.

Once we substitute the optimal policy for $\iota^p(k, X, \omega)$ into the p-entrepreneur's problem, we collapse his decisions into a standard consumption-savings problem where the return and price of capital depend on equilibrium liquidity.

Problem 7 (Producer's Reduced Problem)

$$V^p(k, X) = \max_{c \geq 0, k' \geq 0} U(c) + \beta \mathbb{E} [V^j(k', X') | X], \quad j \in \{i, p\} \quad (31)$$

$$\text{subject to } c + q(X)k' = W^p(X)k \quad (32)$$

$$\text{where } W^p(X) \equiv [r(x^p(X), X) + q(X)\bar{\lambda}] \quad (33)$$

$W^p(X)$ is the entrepreneur's virtual wealth per unit of capital described in Proposition 3. Since all decisions are linear in the capital stock, the economy admits aggregation. This property delivers the tractability of the entrepreneurs problem.

Investor's dynamic problem. The investor's problem can be solved in the same way as the p-entrepreneur's problem. The idea is to break their problem into a consumption-savings problem contingent on the solution to their financing and quality sales decisions. A recursive equilibrium is characterized also by a threshold quality function $\omega^i(X)$ such that all qualities

below $\omega^i(X)$ are sold by all i-entrepreneurs and this threshold is given by Proposition 6. Their financing decisions are the same as in Proposition 6. Substituting these results, the i-entrepreneur's problem simplifies it to:

Problem 8 (Investor's Reduced Problem)

$$V^i(k, X) = \max_{c \geq 0, k' \geq 0} U(c) + \beta \mathbb{E} [V^j(k', X') | X], \quad j \in \{i, p\}$$

$$\text{subject to } c + k' = W^i(X) k$$

$$\text{where } W^i(X) \equiv \frac{1}{q^R(X)} \left[q(X) \int_{\omega \leq \omega^i(X)} \lambda(\omega) f_\phi(\omega) d\omega + q^R(X) \int_{\omega > \omega^i(X)} \lambda(\omega) f_\phi(\omega) d\omega \right] \quad (34)$$

For investors, virtual wealth per unit of capital $W^i(X)$ takes a different form than for p-entrepreneurs. This quantity is a weighted sum over the entrepreneur's capital qualities. The first term is the value of liquid funds: liquid funds correspond to the price of capital sold by i-entrepreneurs, $p^i(X)$, times the volume of capital sold, $F_\phi(\omega^i(X))$ which yield $q(X) \int_{\omega \leq \omega^i(X)} \lambda(\omega) f_\phi(\omega) d\omega$ over the replacement cost of capital. Illiquid funds are valued at one because capital is reversible.

Optimal consumption-savings decisions. So far I have shown that the entrepreneurs' problems can be summarized by two standard consumption-savings problems, Problem 7 and Problem 8. These problems are standard consumption-savings problems with homogeneous preferences and constant returns to scale. It is straightforward to show that the policy functions are linear functions of the capital stocks and therefore, invoking Gorman's aggregation result, we have the necessary conditions for the existence of a representative agent. This result guarantees the internal consistency of the definition of competitive recursive equilibrium without any reference to distributions. The optimal consumption-savings decisions are given by:

Proposition 9 (Optimal Policies) *The policy functions for p-entrepreneurs are $c^p(k, X) = (1 - \zeta^p(X)) W^p(X) k$ and $k'^p(k, X) = \frac{\zeta^p(X) W^p(X)}{q(X)} k$. For i-entrepreneurs these are $c^i(k, X) = (1 - \zeta^i(X)) W^i(X) k$ and $k'^i(k, X) = \zeta^i(X) W^i(X) k$.*

The functions $\zeta^p(X)$ and $\zeta^i(X)$ are marginal propensities to save for p-entrepreneurs and i-entrepreneurs. These functions solve a pair of functional equations. The proof in the appendix shows that marginal propensities to save of both types solve a non-linear functional equation which can be easily solved by repeated iteration.¹³ When $\gamma = 1$, one can verify that $\zeta^p = \zeta^i = \beta$.

¹³A similar operator is found in Angeletos (2007). I cannot provide a direct proof for a theorem that

Full-information price of capital. The last objects to be characterized is q and aggregate investment. One can arrange the i-entrepreneur's capital accumulation equation, substitute in the capital policy functions obtained from Proposition 9, and integrate across individuals to obtain their aggregate demand for investment net of claims:

$$I(X) - I^s(X) = \left[\zeta^i(X) W^i(X) - \int_{\omega > \omega^i(X)} \lambda(\omega) f_\phi(\omega) d\omega \right] \pi K. \quad (35)$$

Considering that in equilibrium only producers purchase capital, similar steps lead to an expression for the aggregate demand for capital purchases:

$$D(X) = \left[\frac{\zeta^p(X) W^p(X)}{q(X)} - \int_{\omega > \omega^p(X)} \lambda(\omega) f_\phi(\omega) d\omega \right] (1 - \pi) K. \quad (36)$$

Total sales of used capital under asymmetric information is obtained by aggregating over the capital sales of both types:

$$S(X) = \underbrace{\left[\int_{\omega \leq \omega^p(X)} \lambda(\omega) f_\phi(\omega) d\omega \right]}_{\text{Capital sales by p-types}} (1 - \pi) K + \underbrace{\left[\int_{\omega \leq \omega^i(X)} \lambda(\omega) f_\phi(\omega) d\omega \right]}_{\text{Capital sales by i-types}} \pi K. \quad (37)$$

Market clearing requires $D(X) = S(X) + I^s(X)$. To satisfy this expression, producers must repurchase all the units they sold in the period. In addition, it must be the case that investors satisfy their constraints on the issuance of investment claims (7). Since these constraints are linear in the entrepreneur's capital stock, an aggregate version of this condition, $I(X) - I^s(X) \leq (1 - \theta^I) I(X)$, holds if and only if there exists an allocation such that all the individual constraints are satisfied. Thus, any equilibrium must be characterized by a $q(X)$ such that $D(X) = S(X) + I^s(X)$ and $\theta^I I(X) \leq I^s(X)$.

Solving for $q(X)$ is not immediate from market clearing since enforcement constraints must also be met. The solution for $q(X)$ and the rest of the equilibrium conditions are described in the Appendix.

Economic properties. A distinguishing feature of this environment is that, in equilibrium, despite they can do so, entrepreneurs will not choose to acquire the amount of liquidity that would allow them to entirely relax their enforcement constraints. This result follows from an existing tension between the enforcement constraints and the incentives to sell capital under asymmetric information. On one hand, selling a marginal unit of capital under asymmetric

guarantees that the repeated iteration of this operator converges for $\gamma > 1$. Nevertheless, [Alvarez and Stokey \(1998\)](#) show that the standard dynamic programming properties of this problem are guaranteed. Thus, if the operator converges, it converges to its unique fixed point.

information is costly to the entrepreneur because he receives a pooling price for an object that he values above that price. On the other hand, when financial frictions are active, they provide the incentives that support trade under asymmetric information because relaxing these constraints is valued by the entrepreneur. When constraints are entirely relaxed by acquiring sufficient funds, liquidity has no value on the margin because there is no point in having additional funds. Nevertheless, to obtain this amount, the entrepreneur must incur a loss from selling a marginal asset in a pooling market.

Take for example a p-entrepreneur. If employment in his firm is efficient, the marginal loss of reducing liquidity is negligible because the marginal profit from labor is 0. Because selling the marginal quality asset is costly, the entrepreneur is better-off if he reduces part of his capital sales. A similar consideration is true for i-entrepreneurs.¹⁴

Thus, the takeaway is that financial frictions must be active in order to support trade under asymmetric information. In other words, when liquidity is needed to enforce efficient employment or investment, the economy will feature under-employment and under-investment. The conditions on parameters that generate these inefficiencies are summarized by:

Proposition 10 *Employment is sub-efficient $(l^w)^\nu < A_t F_l(l^w, K_t)$ if and only if $\theta^L < (1 - \alpha)$. Investment is sub-efficient in the sense that $I_t > 0$ implies $q_t > 1$.*

The summary of the parameter conditions that yield inefficiencies is the following. When $\theta^L < (1 - \alpha)$, the financial frictions that affect labor markets are active so dispersion shocks impact the labor wedge. $\theta^L = 0$ corresponds to the working-capital constraints in [Christiano, Eichenbaum and Evans \(2005\)](#) or [Jermann and Quadrini \(2012\)](#). Investment frictions must be active ($q_t > 1$) if investment is to be positive because otherwise the i-entrepreneur has no incentives to obtain liquidity. Without liquidity, his enforcement constraint is so severe that $I_t = 0$.

6 Quantitative Results

6.1 Calibration

The model period is a quarter. The calibration of preference parameters is standard. I use log-utility for the calibration. Numerically, for any choice of (γ, β) one can find a corresponding value for β such that marginal propensities to consume under log-preferences are roughly

¹⁴If investment is efficient, then the physical cost of creating capital units equal its price $q(X) = 1$. Nevertheless, if $q(X) = 1$, trade under asymmetric information cannot be supported because liquidity has no value for the entrepreneur.

the same as with the original parameters.¹⁵ I set $\beta = 0.97$ and $\gamma = 1$ to approximate policy functions corresponding to CRRA preferences with a coefficient of relative risk aversion of 2 and a discount factor of 0.991. Log-utility is a convenient choice because the stochastic process that determines the quality distributions does not affect intertemporal decisions. I calibrate the model to obtain a Frisch elasticity of 2. This elasticity is within the range used in macro models.

Technology shocks follow a log-AR(1) process. The parameters for this process are obtained by estimating TFP using quarterly data since 1970. $\bar{\lambda}$ is set to obtain an annualized depreciation rate of 10%. The fraction of investors, π , is set to match the plant investment frequencies in [Cooper, Haltiwanger and Power \(1999\)](#).¹⁶

The rest of the parameters must be calibrated jointly. This set of parameters is composed of the capital share, α , the family of quality distributions $\{f_\phi\}$, and the parameters that govern the extent of limited enforcement, θ^L and θ^I . With log-utility, only the current realization ϕ matters for determining allocations and not the underlying stochastic process. Since the experiments I analyze do not address any serially-correlated moments, I assume an IID process for ϕ . I choose the family $\{f_\phi\}$ to be a set of log-normal distributions, so ϕ represents the variance of the log distribution. The particular choice of log-normals is immaterial for the results.¹⁷ In models where the labor market is distorted, the labor share is no longer equal to α . In this model in particular, the choice of $\{f_\phi\}$ and θ^L also affect the labor market share. I calibrate (α, θ^L) and the average variance of f_ϕ to match a three moments related to the labor market. First, I target a labor wedge of 0.18. This number is close to the estimates of [Shimer \(2009\)](#) and [Chari, Kehoe and McGrattan \(2007\)](#). Then I target an average working capital-to-costs ratio, (σ in the model) of 0.4.¹⁸ As an outcome of this calibration, the fraction of the wage bill secured in advance is on average close to 60%.¹⁹ Finally, I target a labor share of output to 2/3. I obtained values of θ^L equal to 0.4 and a mean standard deviation of f_ϕ set to 1.5. The distribution Π and values of Φ are chosen to match the turnover rates of capital reallocation in [Eisfeldt and Rampini \(2006\)](#).²⁰ To get

¹⁵I perform these numerical experiments in [Bigio \(2009\)](#). This result is also related to findings in [Tallarini \(2000\)](#) that show that under CRRA preferences, risk aversion does not affect allocations in a standard growth model.

¹⁶The data suggests that around 20% to 40% plants augment a considerable part of their physical capital stock in a given year. These figures vary depending on plant age. Setting π to 0.1, the arrival of investment opportunities is such that 30% of firms invest in a year.

¹⁷I performed a robustness check for the choice of $\{f_\phi\}$. I calculated the impulse response analysis for families of Beta, Gamma and exponential distributions. Only minor changes in the quantitative results are found. A log-normal family is chosen because it is used in many papers in continuous-time with stochastic volatility and on dispersion shocks.

¹⁸See the Data Appendix for details.

¹⁹This figure is also consistent with a production cycle of a quarter and wages paid at a monthly basis.

²⁰See Table 2 in [Eisfeldt and Rampini \(2006\)](#) for estimates of sales of property, plant and equipment

Parameter	Value	Notes
Preferences		
γ	1	2.5% risk-free rate and crra of 2.
β	0.97	2.5% risk-free rate and crra of 2.
ν	1/2	Frisch elasticity of 1/2.
Technology		
α	1/3	Jointly matched.
π	0.1	Investment freq. in Cooper, Haltiwanger and Power (1999) .
λ	0.9781	10% annual depreciation.
θ^L	0.375	Jointly matched.
θ^I	0.1	Jointly matched.
Aggregate Shocks		
μ_A	0	Normalized Constant.
ρ_A	0.95	Estimated AR(1).
σ_A	0.016	Estimated AR(1)
f_ϕ		Log-Normal family.
Φ		Reallocation in Eisfeldt and Rampini (2006) .
Π		i.i.d process.
Matched Moments		
Moments	Model	Data Target
Working Capital over Costs	0.4127	0.41
Labor Wedge	0.1843	0.18
Labor Share	0.6345	0.66
Reallocation Turnover	1.0965	1.086

Table 1: Calibration Summary Table.

a rough idea of what this figure means, the calibration requires a 3-fold increase in capital quality dispersion from the average value to generate a reduction of 25% in the liquidity of p-entrepreneurs and i-entrepreneurs. This increase is a low probability event that occurs every 30 years in the model. This shock is similar in magnitude to the dispersion shocks in [Bloom \(2009\)](#).

I calibrate θ^I using investment data. I set θ^I to 0.1 to obtain a fall investment that generates a drop comparable to the drop during the Great Recession (see next section). A summary of the calibration is reported in [Table 1](#). I use global methods in the computation of equilibria and impulse responses. All the exercises use a grid of 6 elements for both \mathbb{A} and Φ and 120 for the aggregate capital state. Increasing the grid size does not affect results.

6.2 A Glance at Equilibria

Endogenous liquidity. [Figure 7](#) presents four equilibrium objects in each panel. Within each panel, the four curves correspond to combinations A (high and low) and ϕ (high and low). The x-axis of each panel is the aggregate capital stock, the endogenous state.

reallocation turnover during the business cycle.

The top panels describe the equilibrium liquid funds per unit of capital, x , for both entrepreneur types. Given a combination of TFP and dispersion shocks, liquidity per unit of capital decreases with the aggregate capital stock (although its total value increases) for both types. For p-entrepreneurs, this negative relation follows from decreasing marginal profits in the aggregate capital stock. With lower marginal benefits from increasing liquidity, p-entrepreneurs have less incentives to sell capital under asymmetric information. Comparing the curves that correspond to low and high dispersion shocks, we observe that liquidity falls with dispersion. As explained in Section 2, increases in the quality dispersion increases the shadow cost of selling capital under asymmetric information. In contrast, TFP has the opposite effect. These results are clear from equation (6) which captures the tradeoffs in the choice of liquidity. An analogous pattern is found for i-entrepreneur’s liquidity. The reason is that the demand for investment is weaker when the capital stock is greater or TFP is low.

Hours, consumption, investment and output. As dispersion reduces the liquidity of producers, their effective demand for hours falls, causing a reduction in output. When TFP or the capital stock are high, hours and output are higher, as in any business cycle model. The figure also shows the perverse effects of dispersion shocks on investment. With less liquidity available, the supply of investment claims shrinks. The reduction in the liquidity of p-entrepreneurs has ambiguous effects on their profits because this reduces the amount of labor hired but wages also fall. This ambiguous wealth effect implies that the demand for capital may increase after liquidity shortages. Also, the ambiguous wealth effect could also increase consumption because of the increase in the cost of investment. For the calibration, the overall effect involves a strong reduction in investment, consumption and hours together with an increase in the price of capital q , as we should expect in a recession. The subsequent section discusses the ingredients that are needed for this result.

The analysis shows how the low correlation between Tobin’s Q and investment is determined by two counterbalancing forces as in [Lorenzoni and Walentin \(2009\)](#). The first is TFP, which produces a positive correlation between Q and investment. The second is dispersion, which causes an increase in Tobin’s Q together with a reduction in investment. This shows the connection among the six business cycle facts presented discussed in the introduction.

6.3 Impulse Response Analysis

Figure 8 reports the impulse response analysis to a shock ϕ that induces a reduction in output comparable to the fall in output at the trough of the Great Recession. The idea of this exercise is to explain the dynamics of the model. In response to the shock, the value of liquid funds of both entrepreneurs contracts immediately. In the background, the shock induces

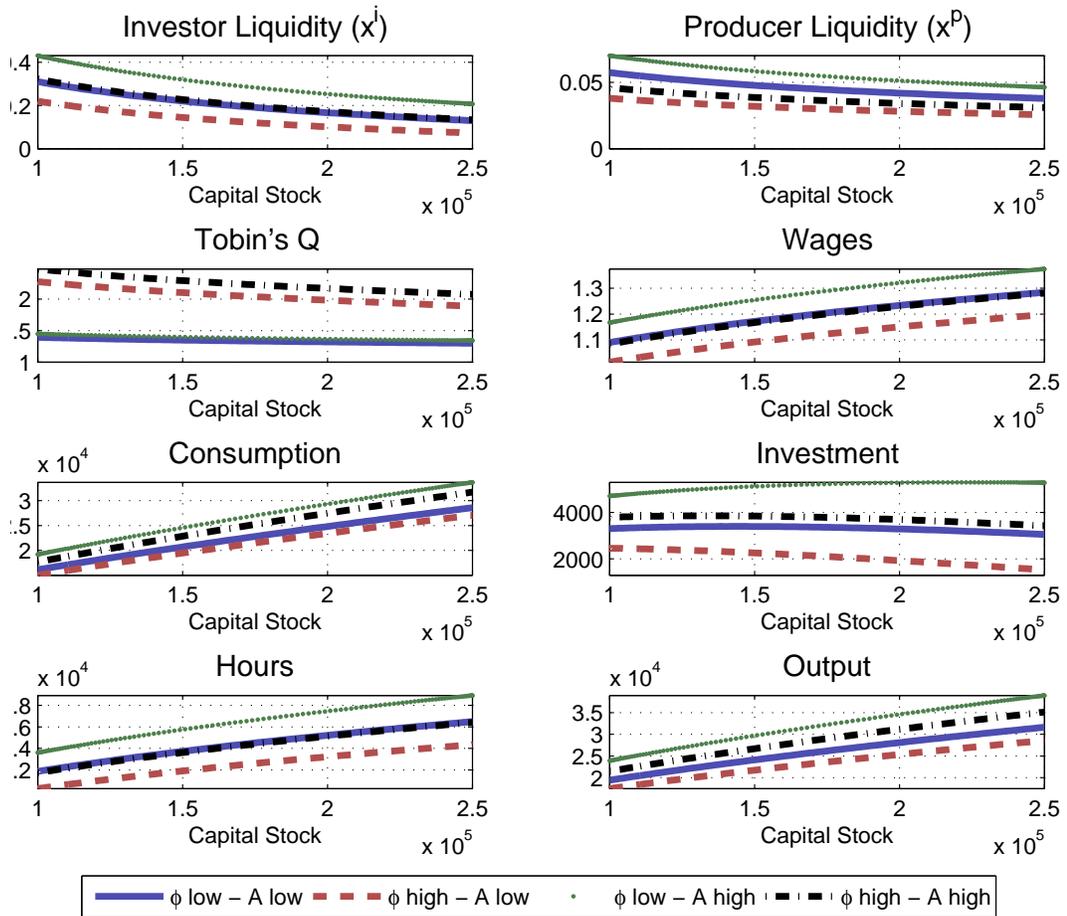


Figure 7: Equilibrium Variables across State-Space.

a drop in the volume of reallocated capital, as occur in the data (Eisfeldt and Rampini (2006)). This reaction is induced by adverse selection effects that explain larger premia between the pooling and full information price of capital. Large premia imply a greater cost of selling capital as explained in Section 2. As a consequence, both entrepreneurs opt to scale down. The collapse in liquid funds has real effects on output. Aggregate output falls by 5.5% on impact due to the reduction in hours. In response to the lack of working-capital funds, hours and wages fall by 6.5% and 4% respectively given the contraction in effective labor demand. The contraction in labor demand following reductions in liquidity has found support in the work of Chodorow-Reich (2013), for example.

The contraction in liquidity affects investors by tightening their capacity to finance investment projects. Issued investment claims fall to meet incentives, causing aggregate investment to fall by 21%. With less investment projects, the post-impact capital stock shrinks. In subsequent periods, the capital stock dynamics drive the response of the entire system.

The plots at the bottom present the responses of different measures of the financial frictions in the model. There are important responses in the investment wedges q and q^R (an increase in 0.4 and -0.4 respectively). The heightened response contrasts with the less dramatic response in the investment wedge computed from the residual of the aggregate consumption-Euler equation (henceforth CKM-wedge, following Chari, Kehoe and McGrattan (2007), see Buera and Moll (2012) for a discussion). The labor wedge, however, increases 7%. A higher labor wedge follows from the increase in the marginal product of labor (MPL) and the decrease in hours.

The experiment shows that large output contractions can be explained by dispersion shocks that reduce labor productivity along with increases in MPL. Labor wedge increases are common to many of the modern recessions documented by Chari, Kehoe and McGrattan (2007) and Shimer (2009). Moreover, as shown by Ohanian (2010) or Hall (2010), the 2008-2009 crisis was an episode in which hours fell in parallel with increases in MPL. Note also that this happens without a response in the CKM-wedge. This last feature is incorrectly interpreted as evidence against the presence of financial frictions when, in fact, investment is being distorted dramatically. As discussed earlier, these features provide a link between the investment and labor market frictions as suggested by Justiniano, Primiceri and Tambalotti (2010a) which occur in tandem with procyclical reallocation in business cycles (Eisfeldt and Rampini (2006)).

Which frictions matter? Enforcement constraints for investment and labor have differential impacts on aggregate outcomes. Figure 9 presents responses to once-and-for-all dispersion shocks when (a) only the investment friction is active, (b) only the labor friction is active, and (c) both frictions are active. The point of the exercise is to show that to obtain

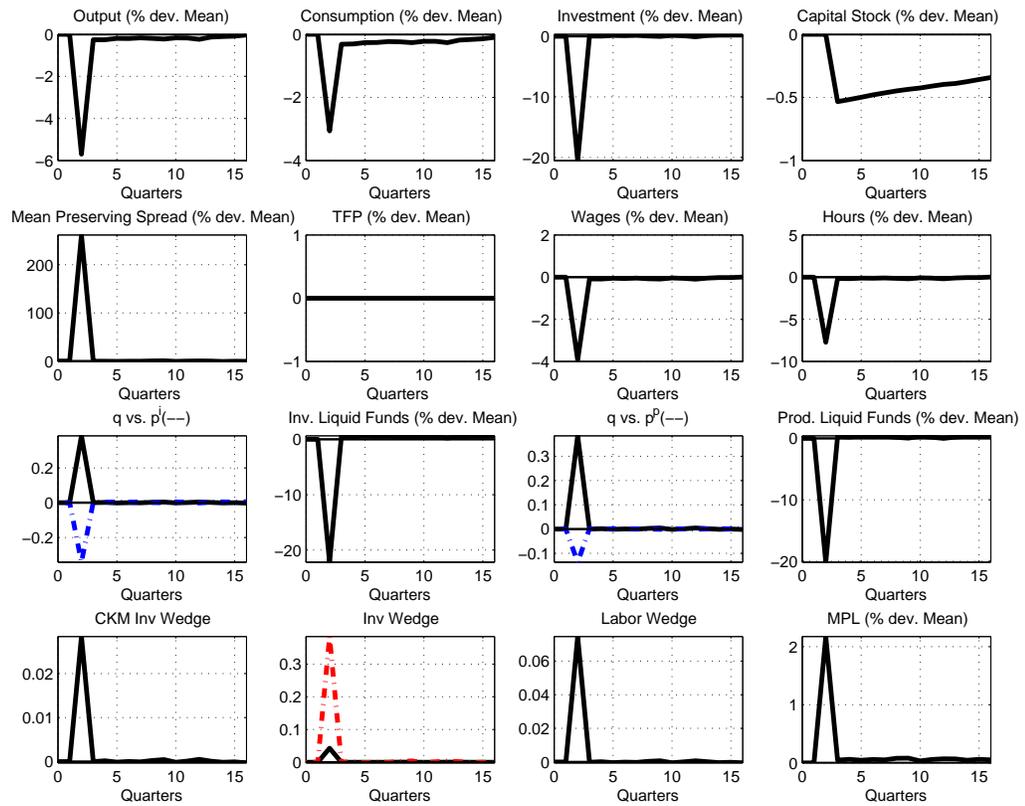


Figure 8: Impulse Response to increase in Quality Dispersion.

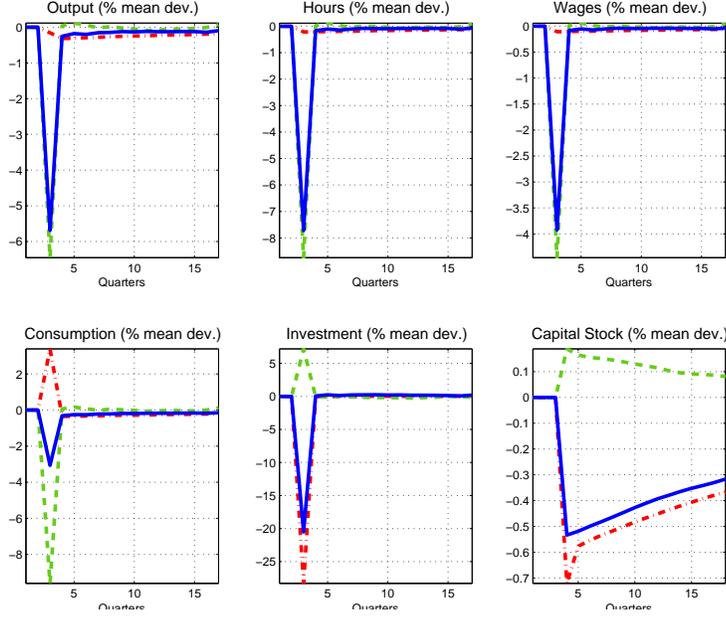


Figure 9: Response to an increase in Capital Quality Dispersion with Alternated Frictions.

quantitative success both frictions are needed.

The exercise shows that the enforcement constraint on labor is key to generate a strong output response. The contemporaneous output response is the same with and without the investment friction but vanishes once the labor friction is turned off. The reason for this discrepancy is that output fluctuations are explained by variation in hours and not investment since the capital stock is fixed in the short run. In contrast, the investment friction distorts the accumulation of capital, so it is responsible for the dynamics output after the shock is gone. Nevertheless, the investment friction is needed to explain the dynamics of aggregate investment. In the absence of investment frictions, investment can react positively to a dispersion shock because profits can increase following the reduction in wages via a monopsony effect discussed in Section 2.

Labor-supply elasticity. The key parameter for the magnitude of the output response is the labor-supply elasticity. Figure 10 presents the response to ϕ for different values of the Frisch elasticity. As explained before, reductions in producers' liquidity affects labor demand. This is met, in equilibrium, via a reduction in hours and wages. The relative response of either margin depends on the labor-supply elasticity. Not surprisingly, the response of hours, and consequently output, is stronger as the Frisch elasticity is increased. The overall magnitude of the response of output varies from -5.5% (when the Frisch elasticity is 2) to -2% (when it is equal to 1/2). Although a Frisch elasticity of 2 falls within the common values in the macroeconomics literature, one can also argue in favor of a large Frisch elasticity since the model abstracts from features such as wage rigidities or worker savings that would magnify

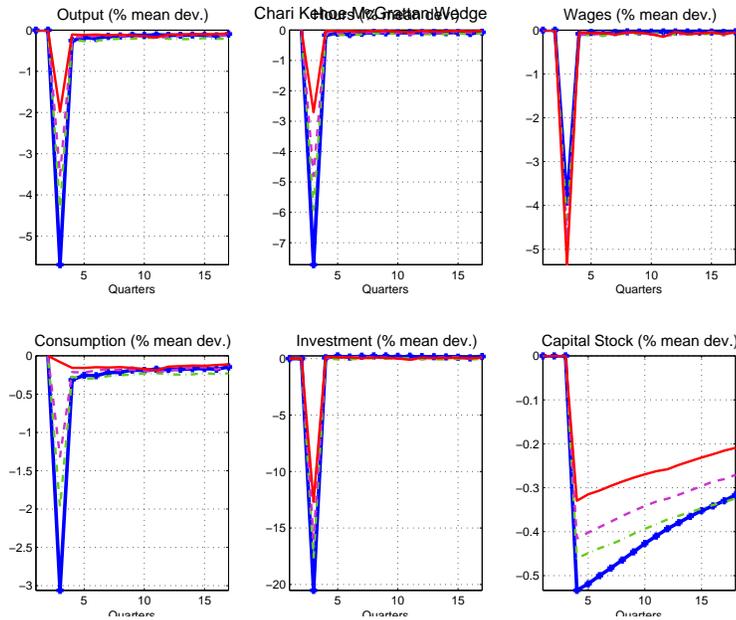


Figure 10: Response to an increase in Capital Quality Dispersion with alternative Frisch elasticities

the responses. Naturally, the drop in consumption is weaker as the elasticity is reduced.

6.4 Evaluating the Model

I now evaluate the capacity of dispersion shocks to explain the patterns of the Great Recession. The mechanics of the model should be clear from the previous section. The strategy here consists of fitting a sequence of dispersion shocks into the model and compare the outcomes to data counterparts. The sequence of dispersion shocks is constructed by tracking a measure of dispersion used in the literature, the firm-level sales dispersion for publicly traded firms (see Data Appendix for more details).²¹ Thus, I feed the model the sequence of values for sales dispersion and use a sequence of ϕ shocks replicates the pattern but whose scale is chosen to induce the drop in output in the model close to the drop in the data.

The model is initiated from a point in the state space randomly drawn from its invariant distribution. This delivers different times series for each initial condition which are averaged to obtain the average behavior of outcomes in the model. Outcomes are reported as percent deviations from initial conditions. The data analogues are reported in percent deviation

²¹Although this measure of dispersion is not evidence of worse asymmetric information, the working assumption is that this measure of dispersion is correlated with unobserved dispersion of assets qualities during the cycle. Thus, this is an exercise in the same spirit of other recent empirical work on uncertainty shocks like Bloom (2009), Schaal (2012) and Vavra (2013). It is important to acknowledge that these exercises are not designed to address causality.

Variable	Model at Trough	US Data - 2008I-2009I
Output	-5.16%	-4.72%
Hours	-6.87%	-9.14%
Wage	-3.84%	2.12%
MPL	1.92%	5.35%
Investment	-16.31%	-20.12%
Consumption	-3.45%	-3.34%

Table 2: Summary Table of Model Fit to Great Recession Data.

from levels during the third quarter of 2007. I report the behavior of aggregate output, consumption, investment, hours and the marginal product of labor. I subtract the estimate of the growth rate of potential US output from the data analogues to correct for the lack of a trend in the model. The model’s fit compared to the data is described in Figure 11. The model tracks the magnitude and pattern of consumption, investment, hours and the marginal product of labor of the Great Recession very closely. A detailed inspection shows that the model reacts faster than the data during the first periods of the recession. However, at the trough of the recession, the model is remarkably close to the data considering that I am only using one shock. Table 2 reports model and data deviations at these points. The model fails only by showing a decrease in wages which does not occur in the data but as explained earlier, features left out of the analysis like the wage rigidities discussed in Section 2 could amend this defect.

The aftermath of the Great Recession cannot be explained by the model. In particular, output recovers whereas hours and investment remain at depressed levels in the data. It follows that consumption and the marginal product of labor in the data feature an increase that the model lacks. These patterns are characteristics of the jobless recovery experienced by the US economy. This observation represents a puzzle which the model is not designed to address, but that is worth pointing out.²²

Implied credit conditions. What about the model’s predictions about credit markets? To compare the model’s predictions about credit conditions with those of the Great Recession, I draw on the observational equivalence between sales of capital and collateralized debt explained in Section 3. In that section, I show how to reconstruct an equilibrium with collateralized debt that replicates equilibrium allocations when only selling capital is possible. I do this by computing an equilibrium with sales only and then backing out the dispersion shock that delivers the same amount of liquidity with collateralized debt. The results of this procedure are values for interest rates, default rates, collateralization rates, loan sizes,

²²In fact, it is also a challenge for other business cycle models as neither factor of production, labor or capital, is recovering. If labor productivity is explaining the pattern, how come hours are not recovering?

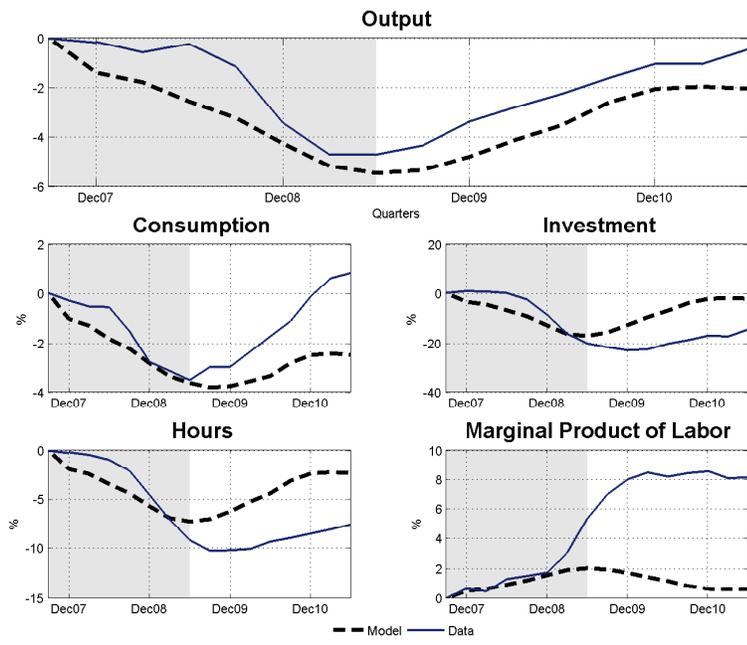


Figure 11: Model Fit to Great Recession Data for Macroeconomic Variables.

liquidity and working capital-over-costs (σ in the model) for each point in the state space. I use these equilibrium objects to reconstruct the model’s expected response for these variables. The resulting paths are reported in Figure 12 together with data analogues.

The upper panel presents the path for sales dispersion. The dashed line is the implied dispersion shock used as input in the simulations of this figure and Figure 11. Dispersion in the model is higher than the data counterpart (solid line) by a factor of two, but on the other hand, is lower than the increase in, for example, the VIX index of stock volatility.

The middle and bottom panels present the credit conditions in the model (dashed) against their data counterparts (solid lines). The middle-left panel shows the path for loan size, p^S , against the pre-recession normalized average size of syndicated loans in the US. The loan size in the model falls by 20% during the trough of the recession. This occurs as financial intermediaries lend less per unit of collateral. A pattern of similar magnitude is observed for syndicated loans which fell by about 25%. Similar magnitudes are documented by [Ivashina and Scharfstein \(2010\)](#). There is a debate about how important was the contraction in loans during the crisis. Syndicated loans (or data on outstanding commercial paper) may overrepresent the overall decline in lending during the Great Recession. However, the decline in lending observed from the Survey of Terms of Business Lending shows that the decline in

lending after the recession is very similar in magnitude.²³ Furthermore, to favor the model, one can argue that the model also misses features that could magnify the response such as interfirm trade and wage rigidities.

The middle panel describes the behavior of interest rates in the model. Although not reported, compared to US corporate-bond yields or the rates on syndicated loans, show patterns similar in timing though the magnitudes in the model are several times larger. Interest rates in the model increase because higher dispersion leads to a high number of defaults. With more dispersion, the average value of the defaulted collateral is also lower. Naturally, this heightened default risk leads financial intermediaries to charge a higher interest rate and lend less per unit of collateral.

The model is perhaps missing additional ingredients such as disruptions in the intermediary sector or high transactions costs in dealing with corporate defaults that would explain a drop in liquidity without an increase in interest rates. Moreover, any model that attempts to generate a reduction in hours that generates a substantial drop in output will have to generate an important increase in interest rates absent other frictions. The reason is that interest rates have to be proportional to the increase in the marginal product of labor. [Bigio and La'O \(2013\)](#) shows that if one allows for interfirm trade, network effects may explain a substantial drop in hours with only a mild increase in funding costs.

As a consequence of the fall in the average loan size, the model explains a drop in investor and producer liquidity (bottom left). The drop in liquidity reaches 20% in the model. The dollar-value of all syndicated loans fell by approximately 25% at the peak of the Great Recession. Both series for total loans have a similar pattern as the fall in loan sizes. This suggests that both, in the model and the data, most of the drop in liquidity responds to reductions in loan size rather than in number of loans.²⁴ A similar magnitude is observed for the investor's liquidity, both in the model and in the data until the end of the Great Recession.

Finally, the model delivers predictions about $1 - \sigma$, the fraction of working capital over total costs. As producers hold less liquidity, limited enforcement is aggravated, although the relative importance of the effect diminishes with less liquidity. That is, the model predicts that the fraction of trade credit over total costs (from workers to entrepreneurs) increases as entrepreneurs have less liquidity. This means that $(1 - \sigma)$ decreases after liquidity shocks, a pattern shared with firm-level data during the Great Recession. Models with a fixed working

²³The Survey of Terms of Business Lending may show a misleading picture as it features an initial increase in lending during the Great Recession followed by a substantial decline. The initial increase is often attributed to previously-agreed credit lines. See [Ivashina and Scharfstein \(2010\)](#) or the Data Appendix for discussions.

²⁴I also report the change in the stock of debt from COMPUSTAT firms which initially increases during the crisis but eventually declines.

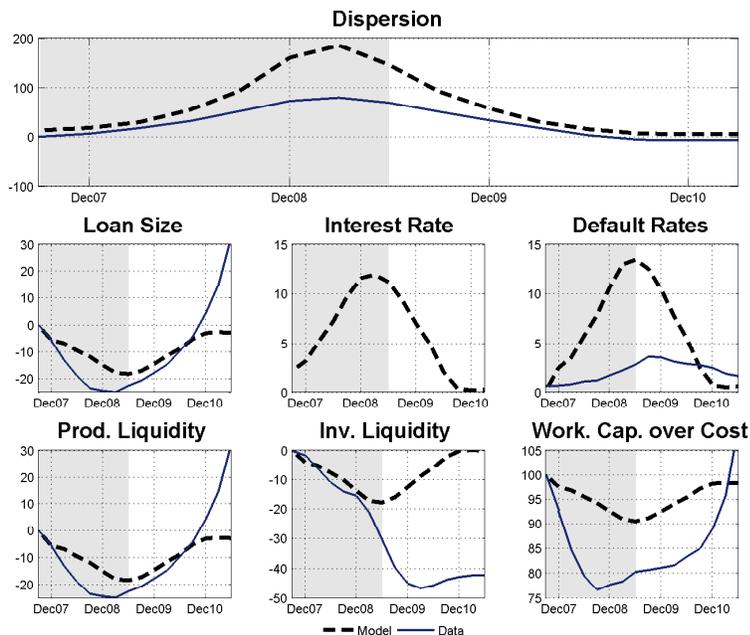


Figure 12: Model Fit to Great Recession Data for Macroeconomic Variables.

capital constraint cannot account for this pattern.

Risk-free rate. The pattern for the response in the risk-free rate to dispersion shocks depends on the participation of workers in asset markets. If workers and entrepreneurs can trade risk-free assets and are similarly important in this market, the risk-free rate increases after a drop in liquidity is ambiguous because workers wish to borrow against future consumption. Entrepreneurs would supply more risk-free bonds to smooth consumption in this case. If workers are excluded from this market, as from the capital market in this model, the model predicts a fall in the real risk-free rate like we see in recessions. Numerical exercises show that the movements in the model are substantially more dramatic than in the data. This excess sensitivity stems follows from the low level of risk aversion in the calibration.

The next section conducts an impulse response analysis of the model to explain its mechanics and the role of different parameters.

7 Conclusions

This paper describes how asymmetric information about capital quality endogenously determines the amount of liquid funds in capital markets when liquidity is used to relax limited enforcement constraints. The paper shows that the dispersion of capital quality increases

the cost of using capital as collateral. This increased cost of collateralization carries real effects by exacerbating financial frictions.

The main lessons are the following. [1] In equilibrium, liquidity is determined by equating the marginal benefit of relaxing financial constraints to the marginal cost of obtaining liquidity under asymmetric information. This means that liquidity is always insufficient to relax enforcement constraints completely. [2] Dispersion shocks to capital quality can cause collapses in liquidity that deliver patterns observed during the Great Recession, both for quantities and variables that reflect credit market conditions. [3] The key friction to explain this large impact is limited enforcement in labor markets and the key parameter is the labor-supply elasticity.

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8 Equilibrium Conditions

Labor Demand: Aggregate labor demand is obtained by aggregating across p-entrepreneurs. Since labor/capital ratios are constant, aggregate labor demand is,

$$L^d(X) = l^*(x^p(X), X) (1 - \pi) K. \quad (38)$$

Equilibrium employment. Worker's consumption is $c = w(X) l^w(X)$ where $l^w(X)$ is the labor supply. In equilibrium, the leisure-consumption defines the aggregate labor supply: $w(X)^{\frac{1}{\nu}} = l^w(X)$. Hence, the equilibrium wage solves $w(X) = (l^*(x^p(X), X) (1 - \pi) K / \bar{\omega})^\nu$.

Aggregate output. Aggregate output, $Y(X)$, is the sum of the return to labor and capital:

$$Y(X) = r(x, X) (1 - \pi) K + (l^*(x^p(X), X) (1 - \pi) K)^{\nu+1}. \quad (39)$$

Aggregate variables. Using the results from Proposition 9, one can aggregate across entrepreneurs to obtain aggregate consumption and capital holdings:

$$\begin{aligned} C^p(X) &= (1 - \zeta^p(X)) W^p(X) (1 - \pi) K \text{ and } C^i(X) = (1 - \zeta^i(X)) W^i(X) \pi K \\ K'^p(X) &= \zeta^p(X) W^p(X) (1 - \pi) K / q(X) \text{ and } K'^i(X) = \zeta^i(X) W^i(X) \pi K / q^R(X) \end{aligned}$$

Aggregate capital evolves according to $K'(X) = K'^i(X) + K'^p(X)$.

8.1 Solving for $q(X)$

A thought experiment can clarify how market clearing and enforcement constraints are satisfied together. A first observation is that $q(X) < 1$ can never be part of an equilibrium. To see this, observe that if $q(X) < 1$ there would be no supply of investment claims by i-entrepreneurs. The reason is that they would find it cheaper to purchase capital rather than to investing. Since they would not invest at all, if $q(X) < 1$ only disinvestment ($I(X) < 0$) could occur in equilibrium for that state. But if this is the case, since capital is reversible, $q(X) = 1$ because the technical rate of transformation for all agents is 1.

Hence, $q(X) \geq 1$. Given prices and policy functions, $I(X) - I^s(X)$ can be solved for from (35) whereas $I^s(X)$ equals $D(X) - S(X)$. Given that $I^s(X)$ and $I(X) - I^s(X)$ are known quantities, one can check whether they satisfy $\theta^I I(X) \leq I^s(X)$. When this condition is satisfied, then $q(X) = 1$ is an equilibrium. When it is violated, $q(X)$ must be greater than 1 to satisfy incentive compatibility.

Proposition 5 ensures that when $q(X) > 1$, enforcement constraints bind so $I^s(X) = \theta^I I(X)$. Substituting this equality into (35) yields a supply schedule for claims that is

increasing in $q(X)$. In addition, the supply of capital $S(X)$ is increasing and demand $D(X)$ decreasing in $q(X)$. Thus, $q(X)$ is found by solving for the market clearing condition when enforcement constraints are binding. Proposition 11 describes the solution to $q(X)$:

Proposition 11 (Market Clearing) *The equilibrium full information price of capital is given by:*

$$q(X) = \begin{cases} q^o(X) & \text{if } q^o(X) > 1 \\ 1 & \text{if otherwise} \end{cases} \quad (40)$$

where $q^o(X)$ is a function of $(W^p(X), W^i(X), \varsigma^s(X), \varsigma^i(X))$.

The proof is presented in the Online Appendix but is similar to the one found in Bigio (2009).

8.2 Optimal Policies in Proposition 9

Define the virtual return to capital conditional on the entrepreneur's type as:

$$\begin{aligned} R^{pp}(X', X) &\equiv \frac{W^p(X')}{q(X)} \text{ and } R^{pi}(X', X) \equiv W^p(X') \\ R^{ii}(X', X) &\equiv W^i(X') \text{ and } R^{ip}(X', X) \equiv \frac{W^i(X')}{q(X)} \end{aligned}$$

where $R^{jm}(X, X')$ is the X' virtual contingent return of capital from to an entrepreneur m who becomes j at the current state X . These virtual returns are used to obtain the marginal propensities to save, $\varsigma^i(X)$ and $\varsigma^p(X)$, according to:

Proposition 12 (Recursion) *Marginal propensities to save, ς^i and ς^s satisfy:*

$$(1 - \varsigma^i(X))^{-1} = 1 + \beta^{1/\gamma} \Omega^i((1 - \varsigma^p(X')), (1 - \varsigma^i(X'))) \quad (41)$$

$$(1 - \varsigma^p(X))^{-1} = 1 + \beta^{1/\gamma} \Omega^p((1 - \varsigma^p(X')), (1 - \varsigma^i(X'))) \quad (42)$$

where

$$\Omega^i(a(X'), b(X')) \equiv \mathbb{E} \left[(1 - \pi) (a(X'))^\gamma R^{pi}(X')^{1-\gamma} + \pi (b(X'))^\gamma R^{ii}(X')^{1-\gamma} \right]^{1/\gamma}$$

$$\Omega^s(a(X'), b(X')) \equiv \mathbb{E} \left[(1 - \pi) (a(X'))^\gamma R^{pp}(X')^{1-\gamma} + \pi (b(X'))^\gamma R^{ip}(X')^{1-\gamma} \right]^{1/\gamma}$$

In addition, $\varsigma^p, \varsigma^i \in (0, 1)$ and equal (β, β) if $\gamma = 1$.

8.3 Remaining Equilibrium Equations

An equilibrium can be entirely characterized by a fixed point problem in the functions $q(X)$, $\omega^p(X)$, $\omega^i(X)$, $\varsigma^p(X)$ and $\varsigma^i(X)$. Once this fixed point is found, the rest of the equilibrium objects is obtained by equilibrium conditions. The following set of functional equations summarizes the equilibrium conditions. For presentation purposes, I present these in three blocks:

Capital Market-Clearing Block.

$$q^R(X) = \frac{1 - \theta q(X)}{1 - \theta}$$

$$I(X) - I^s(X) = \left[\varsigma^i(X) W^i(X) - \int_{\omega > \omega^i(X)} \lambda(\omega) f_\phi(\omega) d\omega \right] \pi K.$$

$$D(X) = \left[\frac{\varsigma^p(X) W^p(X)}{q(X)} - \int_{\omega > \omega^p(X)} \lambda(\omega) f_\phi(\omega) d\omega \right] (1 - \pi) K.$$

$$S(X) = \underbrace{\left[\int_{\omega \leq \omega^p(X)} \lambda(\omega) f_\phi(\omega) d\omega \right]}_{\text{Capital sales by p-types}} (1 - \pi) K + \underbrace{\left[\int_{\omega \leq \omega^i(X)} \lambda(\omega) f_\phi(\omega) d\omega \right]}_{\text{Capital sales by i-types}} \pi K$$

$$D(X) = S(X) + I(X)$$

$$I^s(X) (1 - \theta) \leq \theta (I(X) - I^s(X))$$

Marginal Propensities Block.

$$R^{pp}(X', X) \equiv \frac{W^p(X')}{q(X)} \text{ and } R^{ip}(X', X) \equiv W^p(X')$$

$$R^{ii}(X', X) \equiv W^i(X') \text{ and } R^{ip}(X', X) \equiv \frac{W^i(X')}{q(X)}$$

$$(1 - \varsigma^i(X))^{-1} = 1 + \beta^{1/\gamma} \Omega^i((1 - \varsigma^p(X')), (1 - \varsigma^i(X')))$$

$$(1 - \varsigma^p(X))^{-1} = 1 + \beta^{1/\gamma} \Omega^p((1 - \varsigma^p(X')), (1 - \varsigma^i(X')))$$

$$\begin{aligned}\Omega^i(a(X'), b(X')) &\equiv \mathbb{E} \left[(1 - \pi) (a(X'))^\gamma R^{pi}(X')^{1-\gamma} + \pi (b(X'))^\gamma R^{ii}(X')^{1-\gamma} \right]^{1/\gamma} \\ \Omega^p(a(X'), b(X')) &\equiv \mathbb{E} \left[(1 - \pi) (a(X'))^\gamma R^{pp}(X')^{1-\gamma} + \pi (b(X'))^\gamma R^{ip}(X')^{1-\gamma} \right]^{1/\gamma}\end{aligned}$$

$$W^i(X) \equiv \frac{1}{q^R(X)} \left[q(X) \int_{\omega \leq \omega^i(X)} \lambda(\omega) f_\phi(\omega) d\omega + q^R(X) \int_{\omega > \omega^i(X)} \lambda(\omega) f_\phi(\omega) d\omega \right]$$

$$W^p(X) \equiv r(x^p(X), X) + x^p(X) + q(X) \int_{\omega > \omega^p(X)} \lambda(\omega) f_\phi(\omega) d\omega$$

Liquidity Block.

$$\begin{aligned}\frac{q(X)}{q^R(X)} \mathbb{E}_\phi [\lambda(\omega) | \omega \leq \omega^i(X), X] &= \lambda(\omega^i(X)) \\ (1 + r_x(x^p, X)) \mathbb{E}_\phi [\lambda(\omega) | \omega \leq \omega^p(X), X] &= \lambda(\omega^p(X))\end{aligned}$$

$$x^i(X) = q(X) \mathbb{E}_\phi [\lambda(\omega) | \omega \leq \omega^i(X), X]$$

$$x^p(X) = q(X) \mathbb{E}_\phi [\lambda(\omega) | \omega \leq \omega^p(X), X]$$

$$w(X) = (l^*(x^p, X) K)^\nu$$

$$l^*(x^p, X) = \min \left\{ \arg \max_l \theta^L A l^{1-\alpha} - w l = x^p, l^{unc} \right\}$$

$$r(x^p, X) = A l^{*1-\alpha} - (l^*(x^p, X) K)^{\nu+1}$$

and $q(X)$ given by Proposition 11.

9 Proofs Appendix (not for publication)

9.1 Proof of Proposition 1

Rearranging the incentive compatibility constraints in the problem consists of solving:

$$\begin{aligned} r(x) &= \max_{l \geq 0, \sigma \in [0,1]} Al^{1-\alpha} - wl \text{ subject to} \\ \sigma wl &\leq \theta^L Al^{1-\alpha} \text{ and } (1 - \sigma) wl \leq x. \end{aligned}$$

Denote the solutions to this problem by (l^*, σ^*) . The unconstrained labor demand is $l^{unc} \equiv \left[\frac{A(1-\alpha)}{w} \right]^{\frac{1}{\alpha}}$. A simple manipulation of the constraints yields a pair of equations that characterize the constraint set:

$$l \leq \left[A \frac{\theta^L}{\sigma w} \right]^{\frac{1}{\alpha}} \equiv l^1(\sigma) \quad (43)$$

$$l \leq \frac{x}{(1-\sigma)w} \equiv l^2(\sigma) \quad (44)$$

$$\sigma \in [0, 1].$$

As long as l^{unc} is not in the constraint set, at least one of the constraints will be active since the objective is increasing in l for $l \leq l^{unc}$. In particular, the tighter constraint will bind as long as $l \leq l^{unc}$. Thus, $l^* = \min \{l^1(\sigma^*), l^2(\sigma^*)\}$ if $\min \{l^1(\sigma^*), l^2(\sigma^*)\} \leq l^{unc}$ and $l^* = l^{unc}$ otherwise. Therefore, note that (43) and (44) impose a cap on l depending on the choice of σ . Hence, in order to solve for l^* , we need to know σ^* first. Observe that (43) is a decreasing function of σ . The following properties can be verified immediately:

$$\lim_{\sigma \rightarrow 0} l^1(\sigma) = \infty \text{ and } l^1(1) = \left(\frac{\theta^L}{(1-\alpha)} \right)^{\frac{1}{\alpha}} \left[\frac{A}{w} (1-\alpha) \right]^{\frac{1}{\alpha}} = \left(\frac{\theta^L}{(1-\alpha)} \right)^{\frac{1}{\alpha}} l^{unc}. \quad (45)$$

The second constraint curve (44) presents the opposite behavior. It is increasing and has the following limits,

$$l^2(0) = \frac{x}{\omega} \text{ and } \lim_{\sigma \rightarrow 1} l^2(\sigma) = \infty.$$

These properties imply that $l^1(\sigma)$ and $l^2(\sigma)$ will cross at most once if $x > 0$. Because the objective is independent of σ , the entrepreneur is free to choose σ that makes l the largest value possible. Since $l^1(\sigma)$ is decreasing and $l^2(\sigma)$ increasing, the optimal choice of σ^* solves $l^1(\sigma^*) = l^2(\sigma^*)$ to make l as large as possible. This implies that both constraints will bind

if one of them binds. Adding them up, we find that $l^{cons}(x)$ is the largest solution to

$$\theta^L A l^{1-\alpha} - w l = -x. \quad (46)$$

This equation defines $l^{cons}(x)$ as the largest solution of this implicit function. If $x = 0$, this function has two zeros. Restricting the solution to the largest root prevents us from picking $l = 0$. Thus, if $x = 0$, then $\sigma = 1$ and l solves $w l = \theta^L A l^{1-\alpha}$. This is the largest l within the constraint set of the problem.

Thus, we have that,

$$l^*(x) = \min \{l^{cons}(x), l^{unc}\}.$$

Since $l^1(\sigma)$ is monotone decreasing, if $\theta^L \geq (1 - \alpha)$, then, $l^1(1) \geq l^{unc}$, by (45). Because for $x > 0$, $l^1(\sigma)$ and $l^2(\sigma)$ cross at some $\sigma < 1$, then, $l^{cons} > l^{unc}$ and $l^* = l^{unc}$. Moreover, if $x = 0$, then the only possibility implied by the constraints of the problem is to set $\sigma = 1$. But since, $l^1(1) \geq l^{unc}$, then $l^* = l^{unc}$. Thus, we have shown that $\theta^L \geq (1 - \alpha)$ is sufficient to guarantee that labor is efficient for any x . This proves the second claim in the proposition.

Assume now that $l^{unc} \leq \frac{x}{w}$. Then, the wage bill corresponding to the efficient employment can be guaranteed upfront by the entrepreneur. Obviously, $x \geq w l^{unc}$ is sufficient for optimal employment.

To pin down the necessary condition for the constraint to bind, observe that the profit function in (46) is concave with a positive interior maximum. Thus, at $l^{cons}(x)$, the left hand side of (46) is decreasing. Therefore, if $l^{cons}(x) < l^{unc}$, then it should be the case that $\theta^L A (l^{unc})^{1-\alpha} - w l^{unc} < -x$. Substituting the formula for l^{unc} yields the necessary condition for the constraints to be binding:

$$x < w^{1-\frac{1}{\alpha}} [A(1-\alpha)]^{\frac{1}{\alpha}} \left(1 - \frac{\theta^L}{(1-\alpha)}\right).$$

This shows that if $\theta^L < (1 - \alpha)$, the amount of liquidity needed to have efficient employment is positive.

Figure 13 provides a graphical description of the arguments in this proof. The left panel plots l^1 and l^2 as functions of σ . It is clear from the figure that the constraint set is largest at the point where both curves meet. If l^{unc} is larger than the point where both curves meet, then, the optima is constrained. A necessary condition for constraints to be binding is that l^{unc} is above $l^2(1)$, otherwise l^{unc} will lie above. A sufficient condition for constraints to be binding is described in the right panel. The dashed line represents the left hand side of (46) as a function of labor. The figure shows that when the function is evaluated at l^{unc} , and the result is below $-x$, then the constraints are binding.

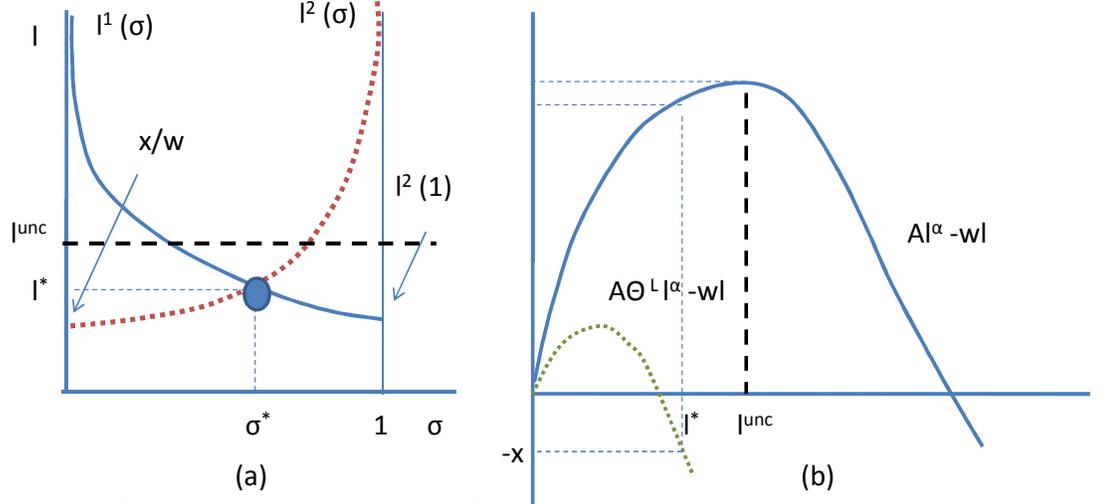


Figure 13: Derivation of Optimal Labor Contract.

9.2 Proof of Lemma 1

This Lemma is an application of the Principle of Optimality. By homogeneity, given a labor-capital ratio l/k , p-entrepreneur profits are linear in capital stock:

$$[A(l/k)^{1-\alpha} - w(l/k) + x] k. \quad (47)$$

Observe that once x is determined by the choice of $\iota(\omega)$, the incentive compatibility constraint (1) and the working capital constraint (3) can be expressed in terms of the labor-capital ratio only:

$$A(l/k)^{1-\alpha} - \sigma w(l/k) \geq (1 - \theta^L) A(l/k)^{1-\alpha} \quad (48)$$

and

$$(1 - \sigma) w(l/k) \leq x. \quad (49)$$

l and σ don't enter the entrepreneur's problem anywhere else. Thus, optimally, the entrepreneur will maximize expected profits per unit of capital in (47) subject to (48) and (49). This problem is identical to the to Problem 2. Thus, the value of profits for the entrepreneur considering the optimal labor to capital ratio is $r(x; w) k$.

Substituting this value into the objective of Problem 1 yields the following objective

$$W^p(k; p, q, w) = \max_{\iota(\omega) \geq 0} r(x; w) k + xk + qk \int \lambda(\omega) (1 - \iota(\omega)) f_\phi(\omega) d\omega \quad (50)$$

subject to:

$$x = p \int \iota(\omega) d\omega$$

where $r(x; w)$ is the value of Problem 2 which shows. Lemma 1.

9.3 Proof of Proposition 2

The proof requires some preliminary computations. Note that the choice of ι determines x . In addition, Lemma 1 shows that the entrepreneur's profits are linear in the entrepreneur's capital stock. Thus, the following computations are normalized to the case when $k = 1$.

Marginal labor of liquidity. For any x such that $l^*(x) = l^{unc}$, the constraints (2) and (3) are not binding. Therefore, when x is sufficiently large to guarantee the efficient amount of labor per unit of capital, an additional unit of liquidity does not increase $r(x)$. For x below the amount that implements the efficient level of labor both constraints are binding. Applying the Implicit Function Theorem, to the pseudo-profit function (46) yields an expression for the marginal increase in the labor with a marginal increase in liquidity,

$$\frac{\partial l^{cons}}{\partial x} = -\frac{1}{(1-\alpha)\theta^L A l(x)^{-\alpha} - w}.$$

Note that the denominator satisfies,

$$(1-\alpha)\theta^L A l^{-\alpha} - w \leq \frac{[\theta^L A l^{1-\alpha} - w l]}{l} = \frac{-x}{l} < 0.$$

which verifies that $\frac{\partial l^{cons}}{\partial x} > 0$.

Marginal profit of labor. Let $\Pi(l) = A l^{1-\alpha} - w l$. The marginal product of labor is,

$$\Pi_l(l) = A(1-\alpha)l^{-\alpha} - w > 0 \text{ for any } l < l^{unc}.$$

Marginal profit of liquidity. Using the chain rule, we have an expression for the marginal profit obtained from an additional unit of liquidity.

$$r_x(x) = \Pi_l(l^*(x)) l^{*'}(x) = -\frac{A(1-\alpha)l^*(x)^{-\alpha} - w}{(1-\alpha)\theta^L A l^*(x)^{-\alpha} - w}, \quad l^*(x) \in (l^{cons}(0), l^{unc})$$

and 0 otherwise.

Thus, liquidity has a marginal value for the entrepreneur whenever constraints are binding. Since $l^*(x)$ is the optimal labor choice, $\Pi(l^*(x)) = r(x)$, which explains the first equality $r_x(x) = \Pi_l(l^*(x)) l^{*'}(x)$. Since $A(1-\alpha)l(x)^{-\alpha} - w$ approaches 0 as $l(x) \rightarrow l^{unc}$, $r_x(x) \rightarrow 0$, as x approaches its efficient level. Hence, $r_x(x)$ is continuous and $r(x)$ is everywhere differentiable. The marginal value of liquidity, $r_x(x)$, is decreasing in x ($r_{xx}(x) < 0$) since the numerator is decreasing and the denominator is increasing in x .

Equilibrium liquidity. To establish the result in Proposition 2, observe that as in the standard lemons problem in Akerlof (1970), if any capital unit of quality ω is sold in equilibrium, all the units of lower quality must be sold. Otherwise, the entrepreneur would be better-off by substituting high-quality units and selling low-quality units instead. A formal argument requires dealing with measure-zero non-monotonicities but the essence does not change.

Thus a cutoff rule defines a threshold quality ω^* for which all qualities below ω will be sold. Choosing the qualities to be sold is equivalent to choosing a threshold quality ω^* to sell. The entrepreneur chooses that threshold to maximize his objective function. Thus, ω^p solves:

$$\omega^p = \arg \max_{\omega^*} r(x)k + x + qk \int_{\omega^*}^1 \lambda(\omega) f_\phi(\omega) d\omega$$

where

$$x = p^p \int_0^{\omega^*} \iota(\omega) f_\phi(\omega) d\omega.$$

The objective function is continuous and differentiable, as long as $f_\phi(\omega)$ is absolutely continuous. Thus, interior solutions are characterized by first order conditions. Substituting x , in $r(x)$ and taking derivatives yields the following first order condition:

$$(1 + r_x(x)) p f_\phi(\omega^*) - q \lambda(\omega^*) f_\phi(\omega^*) \geq 0 \text{ with equality if } \omega^* \in (0, 1). \quad (51)$$

Qualities where $f_\phi(\omega^*) = 0$ are saddle points of the objective function, so without loss of generality $f_\phi(\omega^*)$ is canceled from both sides. There are three possibilities for equilibria: $\omega^* = 1$, $\omega^* \in (0, 1)$, or $\omega^* \neq \emptyset$, where the latter case is interpreted as no qualities are sold. Thus, substituting the zero-profit condition for financial intermediaries, $pF(\omega^*) = q\mathbb{E}_\phi[\lambda(\omega) | \omega \leq \omega^*] F(\omega^*)$, we obtain that 51 becomes

$$(1 + r_x(x)) \mathbb{E}_\phi[\lambda(\omega) | \omega \leq \omega^*] > \lambda(\omega^*).$$

In equilibrium, ω^* must belong to one of the following cases:

Full liquidity. If $\omega^* = 1$, then it must be the case that

$$(1 + r_x(q\bar{\lambda})) \bar{\lambda} \geq \lambda(1). \quad (52)$$

This condition is obtained by substituting $\omega^* = 1$ into 51. If this condition is violated, by continuity of r_x , the entrepreneur could find a lower threshold ω^* that maximizes the value of his wealth.

Interior solutions. For an interior solution $\omega^* \in [0, 1)$, it must be the case that

$$(1 + r_x(x)) \mathbb{E}_\phi[\lambda(\omega) | \omega \leq \omega^*] = \lambda(\omega^*) \quad (53)$$

for $x = q\mathbb{E}_\phi[\lambda(\omega) | \omega \leq \omega^*] F(\omega^*)$. Since $r_x(x)$ is continuous and decreasing, if the condition does not hold, the entrepreneur can be better off with a different cutoff.

Market Shutdowns. Finally, as in any lemons problem, there exists a trivial market shutdown equilibrium with $\omega^* = \emptyset$, and $p^p = 0$.

9.4 Proof of Proposition 3

Since, we can factor k from the objective in (50) to obtain

$$W^p(k; p, q, w) = k \left(\max_{\iota(\omega) \geq 0} r(x; w) + x + q \int \lambda(\omega) (1 - \iota(\omega)) f_\phi(\omega) d\omega \right). \quad (54)$$

For the optimal choice of $\iota(\omega)$, call it $\iota^*(\omega)$, zero profits for the intermediary require:

$$p \int_0^1 \iota^*(\omega) f_\phi(\omega) d\omega = q \int_0^1 \lambda(\omega) \iota^*(\omega) f_\phi(\omega) d\omega.$$

Substituting this condition into (54) the objective yields:

$$\begin{aligned} W^p(k; p, q, w) &= k \left(r(x; w) + q \int_0^1 \lambda(\omega) \iota^*(\omega) f_\phi(\omega) d\omega + q \int \lambda(\omega) (1 - \iota^*(\omega)) f_\phi(\omega) d\omega \right) \\ &= k (r(x; w) + q\bar{\lambda}). \end{aligned}$$

This shows that $W^p(k; p, q, w)$ can be written as $W^p(k; p, q, w) = \tilde{W}^p(p, q, w)k$ if

$$\tilde{W}^p(p, q, w) \equiv r(x; w) + q\bar{\lambda}.$$

Here, $r(x; w)$ is the solution to Problem 1 and x, p and ω^* are given by Proposition 2.

9.5 Proof of Proposition 4

Note that $\frac{\lambda(\omega^*)}{\mathbb{E}_\phi[\lambda(\omega) | \omega \leq \omega^*]}$ is increasing. Under the assumptions, the advantage rate is 1 when $\omega^* = 0$. At $\omega^* = 1$, the advantage rate is greater than 1. In contrast, $1 + r_x(q\mathbb{E}_\phi[\lambda(\omega) | \omega \leq \omega^*])$ is decreasing in ω^* , starts at a number greater than 1. Thus, if the two curves cross, they must cross at a single point. Otherwise, or if they don't cross $\omega^* = 1$ is an admissible solution.

9.6 Proofs of Proposition 5

The proof of Proposition 5 is similar to one that appears in Bigio (2009) and relies on linear programming. Once $\iota(\omega)$ and x are determined, the problem of the i -entrepreneur becomes:

$$\hat{k}(x) = \max_{i^d, i^s} i - i^s + k^b$$

subject to:

$$i = i^d + qi^s$$

$$\theta^I i \geq i^s$$

$$qk^b + i^d \leq xk.$$

To solve this linear program we substitute for i . To obtain an objective equal to:

$$\hat{k}(x) = \max_{k^b, i^d, i^s} i^d + (q-1)i^s + k^b$$

$$\theta^I i^d \geq (1 - q\theta^I) i^s$$

$$qk^b + i^d \leq xk.$$

There are several cases. (i) When $q = 1$ the objective becomes: $i^d + k^b$ and the working capital constraint becomes $k^b + i^d \leq xk$. Since i^s reduces the objective, $i^s = 0$. Hence, the value of the problem is $\hat{k}(x) = xk$, and policies are indeterminate. (ii) When $q > 1/\theta^I$. The value of the problem is indeterminate since $i^s \rightarrow \infty$ is feasible. This clearly is a solution that cannot be part of an equilibrium. (iii) If $q \in [0, 1)$, $i^s = 0$, $i^d = 0$ and $k^b = xk/q$. The value of the problem is $\hat{k}(x) = xk/q$. Finally, when $q \in (1, 1/\theta^I)$, we obtain that $i^d = xk$, $k^b = 0$ and $\theta^I i^d = (1 - q\theta^I) i^s$. Substituting for i^s , into the objective of the problem yields becomes: $i^d + \frac{(q-1)\theta^I}{(1-q\theta^I)} i^d = \frac{(1-\theta^I)}{(1-q\theta^I)} i^d$. Hence, $\hat{k}(x) = \frac{(1-\theta^I)}{(1-q\theta^I)} xk$. Using the definition in the text we obtain: $\hat{k}(x) = (q^R)^{-1} xk$. Thus, if $q \in [1, 1/\theta^I)$, $\hat{k}(x) = (q^R)^{-1} xk$.

9.7 Proof of Proposition 6

The proof of Proposition 6 is the similar to the proof of Proposition 2. Thus, I skip minor details. There is only one distinction. Due to the linearity in the production of capital and the constraints, in this case, the marginal value of an additional unit of liquidity is constant and equal to $\frac{q(x)}{q^R(x)}$, or Tobin's q . From Proposition 5 we know that for values of $q \in [1, 1/\theta)$ the value of the optimal financing problem is $\hat{k}(x) = (q^R)^{-1} xk$. Thus, the value of Problem 3 becomes:

$$W^i(k; p, q) = \max_{\iota(\omega)} (q^R)^{-1} xk + \int_0^1 (1 - \iota(\omega)) \lambda(\omega) k f_\phi(\omega) d\omega$$

subject to:

$$x = p \int_0^1 \iota(\omega) f_\phi(\omega) d\omega.$$

Following the same steps as in the proof of steps of Proposition 2, we can argue that the equilibrium is determined by a threshold quality, ω^i . Substituting x:

$$W^i(k; p, q) = \max_{\omega^i} (q^R)^{-1} p \left(\int_0^{\omega^i} f_\phi(\omega) d\omega \right) k + \left(\int_{\omega^i}^1 \lambda(\omega) f_\phi(\omega) d\omega \right) k. \quad (55)$$

Taking first order conditions yields:

$$(q^R)^{-1} p f_\phi(\omega^i) k \geq \lambda(\omega^i) f_\phi(\omega^i) k$$

and by substituting the zero-profit condition for intermediaries yields:

$$(q^R)^{-1} q \mathbb{E}_\phi [\lambda(\omega) | \omega \leq \omega^i] \geq \lambda(\omega^i)$$

which is the desired condition. The three cases in the statement of the proposition also follow from the proof of Proposition 2.

9.8 Proof of Proposition 7

From equation (55), the objective of the entrepreneur can be written as:

$$\begin{aligned} & \left[(q^R)^{-1} p F(\omega^i) + \int_{\omega^i}^1 \lambda(\omega) k f_\phi(\omega) d\omega \right] k \\ &= \left[(q^R)^{-1} q \mathbb{E}_\phi [\lambda(\omega) | \omega \leq \omega^i] F(\omega^i) + \int_{\omega^i}^1 \lambda(\omega) k f_\phi(\omega) d\omega \right] k \\ &= \frac{1}{q^R} \left[q \int_0^{\omega^i} \lambda(\omega) k f_\phi(\omega) d\omega + q^R \int_{\omega^i}^1 \lambda(\omega) k f_\phi(\omega) d\omega \right] k \\ &\equiv \tilde{W}^i(q) k. \end{aligned}$$

where the second line follows from the zero-profit condition for intermediaries.

9.9 Proof of Proposition of 8

Given a set of prices (p^S, p^F, q) a p-entrepreneur maximizes,

$$W^p(k) = \max_{I(\omega), \iota(\omega)} r(x)k + xk + \dots$$

$$k \int_0^1 (1 - I(\omega)) \iota(\omega) (q\lambda(\omega) - p^F) + (1 - \iota(\omega)) q\lambda(\omega) f(\omega) d\omega$$

subject to:

$$x = p^S \int_0^1 \iota(\omega) f(\omega) d\omega.$$

Let $\Omega^D \equiv \{\omega : I(\omega) = 1, \iota(\omega) = 1\}$ be the set of qualities that feature a default in a Repo market equilibrium. Let $\Omega^{ND} \equiv \{\omega : I(\omega) = 0, \iota(\omega) = 1\}$. Finally, let $\Omega \equiv \Omega^D \cup \Omega^{ND}$. The first step is to show that if a given quality is defaulted, all lower qualities will feature participation and default. This means that $I(\cdot)$ is almost everywhere decreasing. The second is to show that without loss of generality we can treat $\iota(\cdot)$ as almost everywhere decreasing. By an almost everywhere decreasing function I mean that there exists two intervals $[0, \omega^o]$ and $[\omega^o, 1]$ such that the function is values 1 almost everywhere in $[0, \omega^o]$ and $I = 0$ in $(\omega^o, 1]$.

The value of objective of the entrepreneur can be expressed in terms for these sets:

$$V = x + r(x, X) + \int_{\Omega^{ND}} (q(X)\lambda(\omega) - p^F) f(\omega) d\omega + \int_{[0,1] \setminus \Omega} q\lambda(\omega) f(\omega) d\omega$$

with

$$x = \int_{\Omega^{ND}} p^S d\omega + \int_{\Omega^D} p^S d\omega.$$

Suppose $I(\cdot)$ is not decreasing almost everywhere. Then, we can find two intervals: $(\omega_{N_1}, \omega_{N_2})$ and $(\omega_{D_1}, \omega_{D_2})$ such that $I = 0$ almost everywhere in $(\omega_{N_1}, \omega_{N_2})$ and $I = 1$ almost everywhere in $(\omega_{D_1}, \omega_{D_2})$. Moreover, since $f(\omega)$ is continuous, we can find intervals of same measure. We want to show that if $I(\cdot)$ is non-monotone, the p-entrepreneur the entrepreneur is not optimizing. The strategy consists on setting $I = 1$ in $(\omega_{D_1}, \omega_{D_2})$ and viceversa in $(\omega_{N_1}, \omega_{N_2})$ and to show that this improves his value. Since both sets have the same measure, x remains invariant then only the first integral in the objective changes with the policy perturbation. The value of the integral terms in the objective is then:

$$\begin{aligned}
& \int_{\Omega^{ND} \setminus (\omega_{D_1}, \omega_{D_2})} (q(X) \lambda(\omega) - p^F(\omega)) f(\omega) d\omega + \int_{(\omega_{N_1}, \omega_{N_2})} (q(X) \lambda(\omega) - p^F) f(\omega) d\omega \\
= & \int_{\Omega^{ND} \setminus (\omega_{D_1}, \omega_{D_2})} (q(X) \lambda(\omega) - p^F(\omega)) f(\omega) d\omega + \int_{(\omega_{N_1}, \omega_{N_2})} q(X) \lambda(\omega) f(\omega) d\omega \dots \\
& + p^F [F(\omega_{N_2}) - F(\omega_{N_1})] \\
> & \int_{\Omega^{ND} \setminus (\omega_{D_1}, \omega_{D_2})} (q(X) \lambda(\omega) - p^F(\omega)) f(\omega) d\omega + \int_{(\omega_{D_1}, \omega_{D_2})} q(X) \lambda(\omega) f(\omega) d\omega + \dots \\
& p^F [F(\omega_{N_2}) - F(\omega_{N_1})] \\
= & \int_{\Omega^{ND} \setminus (\omega_{D_1}, \omega_{D_2})} (q(X) \lambda(\omega) - p^F(\omega)) f(\omega) d\omega + \int_{(\omega_{D_1}, \omega_{D_2})} q(X) \lambda(\omega) f(\omega) d\omega + \dots \\
& p^F [F(\omega_{D_2}) - F(\omega_{D_1})]
\end{aligned}$$

The first line is the value of the alternative strategy for the entrepreneur. The second line is an algebraic manipulation of the integral. The third follows from the monotonicity of λ , which holds by assumption. The third follows from the equivalence in the lengths of both intervals. The inequality shows that a non-monotone default strategy violates optimality.

We now turn to the non-monotonicity of $\iota(\omega)$. Observe that if $\iota(\omega) = 1$ and $I(\omega) = 0$, then the entrepreneur and the intermediary are indifferent between which qualities are brought to the contract. Collateral will be repurchased. Thus, without loss in generality, we can restrict attention to a decreasing $\iota(\omega)$. Thus, there are two threshold qualities: ω^p and $\bar{\omega}^p$. The first, defines a cutoff under which all qualities are defaulted. The second a participation cutoff. An equilibrium for which $\omega^p = \bar{\omega}^p$ is identical to the sales-only contract of Section 2. Hence, we assume that $\omega^p < \bar{\omega}^p$. The objective for the entrepreneur thus becomes:

$$V = x + r(x) + \int_{\omega^p}^{\bar{\omega}^p} (q\lambda(\omega) - p^F) d\omega + \int_{\bar{\omega}^p}^1 q\lambda(\omega) d\omega$$

subject to

$$x = \int_0^{\bar{\omega}^p} p^S d\omega.$$

The first-order conditions for ω^p is

$$q(X) \lambda(\omega^p) - p^F \geq 0, \tag{56}$$

but since λ is continuous and ω^p interior, the equation holds with equality. The first order condition for $\bar{\omega}^p$ is:

$$\begin{aligned}
(1 + r_x(x)) p^S &\geq (p^F - q\lambda(\bar{\omega}^p)) + q\lambda(\bar{\omega}^p) \rightarrow \\
r_x(x) p^S &\geq (p^F - p^S).
\end{aligned} \tag{57}$$

Finally, the zero-profit condition written in terms of ω^p and $\bar{\omega}^p$ yields:

$$p^F = \int_0^{\omega^p} q\lambda(\omega, \phi) d\omega + p^S \int_{\omega^p}^{\omega^*} d\omega. \tag{58}$$

Equations (56), (57) and (58) correspond to the equations that characterize equilibria.

9.10 Obtaining Equivalent Problems 7 and 8

The substituting the capital accumulation equation into the p-entrepreneur's budget constraint to obtain the following equivalent problem:

$$V^p(k, X) = \max_{c \geq 0, k' \geq 0, \iota(\omega), l, \sigma \in [0, 1]} U(c) + \beta \mathbb{E} [V^j(k', X') | X], \quad j \in \{i, p\}$$

subject to

$$c + q(X) k' = AF(k, l) - \sigma w(X) l + xk - (1 - \sigma) w(X) l + q(X) \int_0^1 (1 - \iota(\omega)) \lambda(\omega) k f_\phi(\omega) d\omega$$

$$AF(k, l) - \sigma w l \geq (1 - \theta^L) A k^\alpha l^{1-\alpha}$$

$$(1 - \sigma) w l \leq xk$$

$$x = p^p(X) \int_0^1 \iota(\omega) d\omega.$$

His objective function is a function of c and k' and do not appear in the constraints below this budget constraint. In contrast, the choice of $\iota(\omega)$, l , σ only affects right hand side of the consolidated budget constraint and are constrained through the additional constraints. Thus, the entrepreneur maximizes his value function by choosing $\iota(\omega)$, l , σ to maximize the right hand side of his budget constraint. This problem is identical to Problem 1. Therefore, we can re-write the p-entrepreneur's problem as:

$$V^p(k, X) = \max_{c \geq 0, k' \geq 0, \iota(\omega), l, \sigma \in [0, 1]} U(c) + \beta \mathbb{E} [V^j(k', X') | X], \quad j \in \{i, p\}$$

subject to

$$c + q(X) k' = \tilde{W}^p(X) k$$

where $\tilde{W}^p(X)$ is the marginal value of capital in Proposition 3 for prices $p(X), q(X)$ are $w(X)$. This is consumption-savings problem with linear returns. Similar steps can be followed to obtain the value for i-entrepreneurs in Proposition 8.

9.11 Proof of Proposition 10

Both statements of Proposition 10 follow from previous Propositions. I first proof the statements about labor inefficiency for any arbitrary state X. From Proposition 1, we know that if $\theta^L \geq (1 - \alpha)$, then labor to capital ratio of the individual entrepreneur is efficient for any choice of x . This proves the only if part. Instead, if $\theta^L < (1 - \alpha)$, we know also from Proposition 1 that some positive amount of liquidity is needed to have the efficient labor to capital ratio. It is sufficient to show that amount is not obtained in equilibrium. From Proposition 2 we know that ω^p must satisfy

$$(1 + r_x(x)) \mathbb{E}_\phi [\lambda(\omega) | \omega \leq \omega^p] \geq \lambda(\omega^p).$$

However, also from From Proposition 1 we know that efficient employment implies that $r_x(x) = 0$. Thus, the above condition becomes $\mathbb{E}_\phi [\lambda(\omega) | \omega \leq \omega^p] \geq \lambda(\omega^p)$ which by Assumption 1 implies that this is true only for $\omega^p = 0$. This in turn implies that $x = q(X) \mathbb{E}_\phi [\lambda(\omega) | \omega \leq 0] F(0) = 0$. By Proposition 1 employment cannot be efficient as it requires some positive amount of liquidity.

I now prove the result for investment. Assume that $q(X) = 1$ and, thus, $q^R(X) = 1$. Therefore, by Proposition 6 we have that,

$$\mathbb{E}_\phi [\lambda(\omega) | \omega \leq \omega^i] = \lambda(\omega^i)$$

which implies that $\omega^i = 0$. This in turn implies $x^i = 0$ and, consequently, $i^d = 0$ from Proposition 3. Since $i^d = 0 \rightarrow i = 0$, we have that aggregate investment cannot be positive.

9.12 Proof of Proposition 11

Substitute the optimal policies described in Proposition 9 into the expression for $D(X)$ and $S(X)$ to obtain $I^s(X) = D(X) - S(X)$. Then uses (35) and to clear out expressions for $I^s(X)$ and $I(X)$. In the proof the state X is fixed so I drop the arguments from the functions. Performing these substitutions, the aggregate version of the incentive compatibility condition becomes:

$$\frac{(1 - \pi) (\zeta^p (r + q\psi^p) / q - \psi^p) K - (1 - \pi) \varphi^p K - \pi \varphi^i K}{\theta} \leq \frac{\pi [\zeta^i (W^i) K - \psi^i K]}{(1 - \theta)}$$

I have introduced the following variables:

$$\begin{aligned} \varphi^p &= \int_{\omega \leq \omega^p} \lambda(\omega) f_\phi(\omega) d\omega & \varphi^i &= \int_{\omega \leq \omega^i} \lambda(\omega) f_\phi(\omega) d\omega \\ \psi^p &= \int_{\omega > \omega^p} \lambda(\omega) f_\phi(\omega) d\omega & \psi^i &= \int_{\omega > \omega^i} \lambda(\omega) f_\phi(\omega) d\omega \end{aligned}$$

that correspond to the expectations over the sold and unsold qualities of both groups. K clears out from both sides. I then use the definition of q^i and rearrange the expression to obtain:

$$\begin{aligned} \frac{(1 - \pi)\zeta^p r - ((1 - \pi)(1 - \zeta^p)\psi^p + (1 - \pi)\varphi^p + \pi\varphi^i)q}{\theta q} &\leq \frac{\pi [\zeta^i q \varphi^i - (1 - \zeta^i)\psi^i q^R]}{(1 - \theta) q^R} \\ &\leq \frac{q\pi\zeta^i\varphi^i}{(1 - \theta q)} - \frac{\pi(1 - \zeta^i)\psi^i}{(1 - \theta)} \end{aligned}$$

I get rid of q from the denominators, rearrange terms and obtain,

$$\begin{aligned} &(1 - \pi)\zeta^p r (1 - \theta q) - ((1 - \pi)((1 - \zeta^p)\psi^p + \varphi^p) + \pi\varphi^i) q (1 - \theta q) \\ &\leq \theta q^2 \pi \zeta^i \varphi^i - \theta q (1 - \theta q) \pi \frac{(1 - \zeta^i)\psi^i}{(1 - \theta)} \end{aligned}$$

By arranging terms, the inequality includes linear and quadratic terms for q . This expression takes the form:

$$(q^*)^2 A + q^* B + C \geq 0 \tag{59}$$

where the coefficients are:

$$\begin{aligned} A &= -\theta \left((1 - \pi)((1 - \zeta^p)\psi^p + \varphi^p) + \pi(1 - \zeta^i)\varphi^i - \pi\theta \frac{(1 - \zeta^i)\psi^i}{(1 - \theta)} \right) \\ B &= \theta(1 - \pi)\zeta^p r + \left((1 - \pi)((1 - \zeta^p)\psi^p + \varphi^p) + \pi\varphi^i - \pi\theta \frac{(1 - \zeta^i)\psi^i}{(1 - \theta)} \right) \\ C &= -(1 - \pi)\zeta^p r \end{aligned}$$

C is negative. Observe that

$$\begin{aligned}
& (1 - \pi) ((1 - \varsigma^p) \psi^p + \varphi^p) + \pi \varphi^i - \pi \frac{(1 - \varsigma^i)}{(1 - \theta)} \psi^i \theta \\
\geq & (1 - \pi) ((1 - \varsigma^p) \psi^p + \varphi^p) + \pi (1 - \varsigma^i) \varphi^i - \pi \frac{(1 - \varsigma^i)}{(1 - \theta)} \psi^i \theta \\
\geq & (1 - \pi) ((1 - \varsigma^p) \psi^p + \varphi^p) + \pi (1 - \varsigma^i) \varphi^i - (1 - \pi) (1 - \varsigma^i) \psi^i \\
\geq & (1 - \pi) \bar{\lambda} - (1 - \pi) \varsigma^p \psi^p + \pi (1 - \varsigma^i) \bar{\lambda} - \pi (1 - \varsigma^i) \psi^i - (1 - \pi) (1 - \varsigma^i) \psi^i \\
\geq & \bar{\lambda} - (1 - \pi) \varsigma^p \psi^p - \pi \psi^i \\
\geq & 0
\end{aligned}$$

where the second line follows from the assumption that $(1 - \theta) \geq \pi$. The third line uses the identity $\bar{\lambda} = \psi^p + \varphi^p = \psi^i + \varphi^i$. The fourth line uses the fact that $(1 - \varsigma^i) < 1$ and the last line uses the fact that ψ^p and ψ^i are less than $\bar{\lambda}$. This shows that A is negative and B is positive. Evaluated at 0, (59) is negative. It reaches a maximum at $-\frac{B}{2A} > 0$. Thus, both roots of (59) are positive. Let the roots be (q_1, q_2) where q_2 is the largest. There are three possible cases: *Case 1:* If $1 \in (q_1, q_2)$, then $q = 1$ satisfies the constraint.

Case 2: If $1 < q_1$, then $q = q_1$, since it is the lowest price such that the constraints bind with equality.

Case 3: If $q_2 < 1$, then there exists no incentive compatible price. Thus, $I = 0$ and i -entrepreneurs consume part of their capital stock.

9.13 Proof of Proposition 12

An identical proposition is shown in Bigio (2009). The proof is standard for consumption-savings problems with stochastic linear returns and homothetic preferences. The proof also has the implication that the economy admits an aggregation.

10 Data Appendix (not for publication)

10.1 Macroeconomic Variables

All the aggregate macroeconomic variables are obtained from the Federal Reserve Bank of St. Louis Economic Research Database, FRED©. The data is downloaded directly into Matlab© using the Datafeed Toolbox©. The sources of this data are the Bureau of Economic Analysis (BEA) and the Bureau of Labor Statistics (BLS). The Matlab© code *FRED_RealVariables.m* downloads the time series for macroeconomic variables used in Section 6.

The data used in the construction of Figure 11 is quarterly, begins in the fourth quarter of 2007 and ends in the second quarter of 2011. I use the following strategy to compute the Output, Consumption and Investment analogue deviations in the data. I compute the per-cent deviations from the fourth quarter of 2007 levels and subtract the growth rate of US potential GDP to account for the lack of a trend in the model. Deviations in the marginal product of labor are computed by dividing real GDP by total hours.²⁵ The following table summarizes the data analogues.

Variable in Model	Data Analogue Used	FRED Acronym	Source
Output (Y_t)	Real Gross Domestic Product, 3 Decimal	<i>GDPC96</i>	<i>BEA</i>
Investment (I_t)	Real Private Nonresidential Fixed Investment	<i>PNFIC96</i>	<i>BEA</i>
Consumption (C_t)	Real Personal Consumption Expenditures	<i>PCECC96</i>	<i>BEA</i>
-	Real Potential Gross Domestic Product	<i>GDPPOT</i>	<i>BEA</i>
Wages (w_t)	Non-farm Business Sector: Unit Labor Cost	<i>ULCNFB</i>	<i>BLS</i>
Labor (l_t)	Non-farm Business Sector: Hours of All Persons	<i>HOANBS</i>	<i>BLS</i>

10.2 Credit Market Data

Credit Market Data. The credit market data is obtained from two sources. I build time series using data from syndicated loans and the cross section of public firms in the US.

Firm Cross-Section Data: I use data from COMPUSTAT - North America - Fundamentals Quarterly. The data is downloaded from the Wharton Research Database Site (WRDS). I use quarterly data from 01/01/2000 to 06/30/2012. The Stata© do-files *create-CCCdata2.do* and *data_analysis_TS2.do* aggregates across firms to generate time series. The

²⁵Since with a Cobb-Douglas production function, GDP per hour is proportional to the marginal product of labor, the normalized series of average output per hour is equal to the normalized series of the marginal product. The following table summarizes the macroeconomic data used in the model.

data features many outliers so I drop observations with sales less than 5 million dollars. I use two definitions of working capital:

- $wc \equiv \text{accounts receivable} + \text{inventory} - \text{accounts payable}$

and

- $wc2 \equiv \text{accounts receivable} + \text{inventory} - \text{accounts payable} - \text{accrued expenses} - \text{tax liabilities}$.

The calibration of the model targets the ratio of working capital over costs. Thus, I use $wc_costs2 \equiv wc2/costs$ and $wc_costs \equiv wc/costs$. I drop observations that are below 1st percentile or above 90th percentile because the data is extremely skewed. The values of this two variables range from 0.5 to 0.3, hence, we use the target of 0.41 for $(1 - \sigma)$.

I also use the cross-sectional standard deviation of sales for all firms as an indirect measure of dispersion. I use the entire sample for the computation of the dispersion of sales. Using the 1st percentile-90th percentile window does not alter the pattern of the time series. The following table summarizes the firm-level data used:

Variable in Model	Data Analogue Used	COMPUSTAT Variable
Sales ($r_t K_t$)	Firm-level pre-tax sales	<i>saleq</i>
Costs ($w_t L_t$)	Firm-level costs	<i>cogsq</i>
Working Capital (x_t^p)	<i>wc, wc2</i> (constructed)	
Working Capital over Costs ($1 - \sigma$)	(constructed)	
Dispersion (ϕ_t)	Cross-section dispersion of sales	

Syndicated Loans Data: The variables on loans are obtained from the Thomson Reuters LPC DealScan dataset. This dataset covers almost the entire universe of syndicated bank loans world-wide. DealScan sources include regulatory filings, bank submissions and journalist contributions. The data is downloaded from WRDS. I use quarterly data from 01/01/2000 to 06/30/2012. The data format is a cross section of loans which include several characteristics. The Stata© do-file *DealScanBuild.do* creates time series for aggregate total amounts of loans, the number of loans and their interest rate. To construct the aggregate total amounts of loans, I sum across all loans the variable *dealamount* which is the descriptor for loan size. I count the number of loans across time to obtain the volume of loans and use an arithmetic mean to obtain a time series for the interest rate spread *spreadoverdefaultbase*.

DealScan includes information on the purpose of each loan which are encoded in the variable *purpose*. The DealScanBuild.do saves these time series into a .csv file labeled *SyndicatedLoans.csv*. The Matlab© code *DealScanBuild.m* loads the data from the .csv file

and generates quarterly sum and average variables to that are used in the paper. The code reconstructs the time series described above for two subsets of loans: those with an investment (INV) purpose and those from a working capital (WC) purpose. Time series for loans where the value of *purpose* is *Working Capital* (the time series ending in 40 in the .csv file) are used to construct the series for working capital. For the investment-purpose time series, I use the time series where the purpose variable takes values *Acquisitions line*, *Levered Buyout (LBO)*, *Project finance*, or *Takeover* (the time series ending in 1,18,25,36 .csv file). The code *DealScanBuild.m* saves a .mat file titled *DealScan.Loans.mat* which is used in the quantitative evaluation of the model. The following table summarizes the loans data used.

Variable in Model	Data Analogue Used	Dealscan <i>purpose</i> value
Producer Liquidity (x_t^p)	Syndicated loans for WC use	40
Loan Size (p_t^S)	Average loan size for WC use	40
Producer Liquidity (x_t^i)	Syndicated loans for INV use	1 + 18 + 25 + 36

Working Capital over Costs: The calculation of working capital over costs involves simultaneous use of data from DealScan and Compustat. I separately calculate the growth rates of costs from Compustat and the growth rates of working capital loans from the DealScan data, the measure of producer liquidity. I then obtain the changes of working capital over costs by subtracting the working capital growth rates from the costs growth rates. This yields a series for the change in $(1 - \sigma)$ corresponding to a model analogue.

10.3 Alternative Data

Below is a list of variables from other data that I have examined in the course of writing this paper:

Survey of Terms of Business Lending (STL). The Data from the Survey of Terms of Lending can be obtained directly from FRED© and downloaded directly into Matlab© using the code supplied by the author, *FRED_RealVariables.m*. This data collects information concerning both price and certain nonprice terms of loans made to businesses the first full business week of the mid-month of each quarter (February, May, August, and November). The information from the reports include average rates, average maturity in days, average loan size, and total amount loan separately for all maturities, short maturity(1-30 days) and long maturity(31 - 365 days) and for different assessments of risk level assessments. Figure 14 plots the time series for total loans, interest rates and average loan size. There is clear pattern in the series that includes all loans although the timing and magnitudes differ across risk ratings and maturities. The pattern shows a 25% drop in overall lending that occurs after the Great Recession is over. The magnitude is in line with the magnitudes observed in

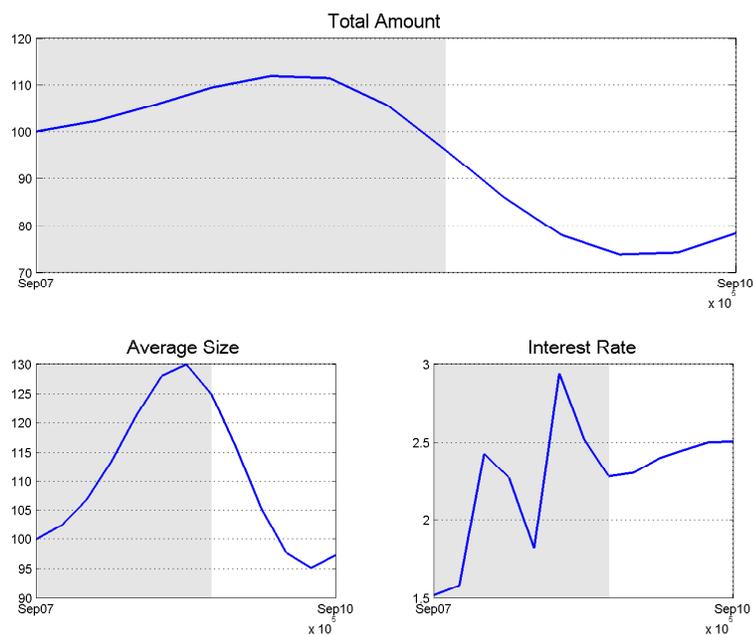


Figure 14: Evolution of Commercial and Industrial Loans (SLT) 2007-2011.

Syndicated loans and, off course, the model. Also, one observes an increase in interest rate spreads during the recession in loan size during the Great Recession. Many researchers argue that the initial increase in lending had to do with firms drawing on previously contracted credit lines. The advantage of using the data from syndicated loans is the availability of information about the use of the loan and that it coincides with the timing of the recession.

Commercial Paper. Data on Commercial non-financial and non-asset-backed paper can be also obtained from FRED[©]. The data includes information about outstanding amounts and spreads. Since commercial paper is issued for no longer than 89 days, flows and stocks are very close to each other after 2 quarters. Figure 15 shows the behavior for spreads and outstanding volumes for all types of commercial paper. The figure shows that the pattern for non-abcp issuances and spreads. The overall decline by the end of the recession is close to 25% but reaches 40% post recession. The data shows a very similar behavior than the information from loan surveys and syndicated lending. Commercial paper is typically used to fund working capital needs.

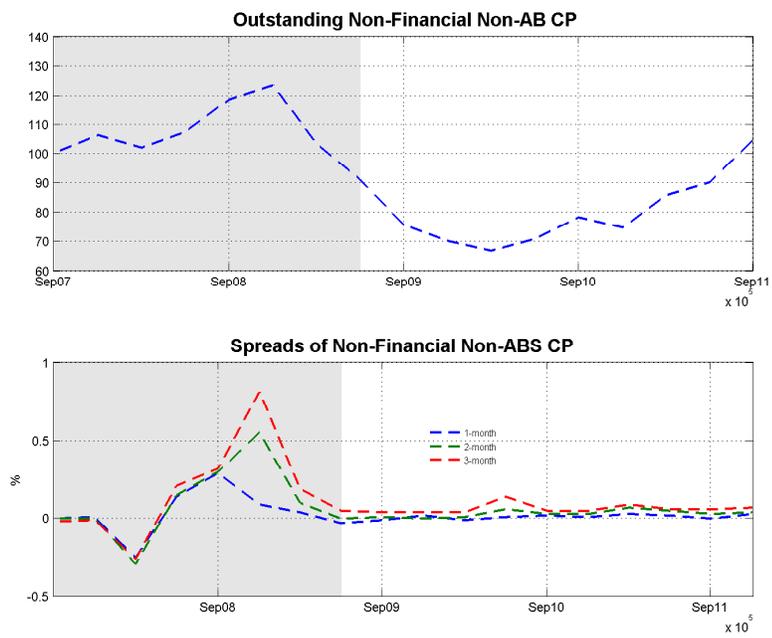


Figure 15: Evolution of Commercial Paper Market 2007-2011.