

# Tipping Climate Negotiations

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## Abstract

Thinking about tipping provides a novel perspective on finding a way forward in climate negotiations and suggests an alternative to the current framework of negotiating a global agreement on reductions in greenhouse gas emissions. Recent work on non-cooperative games shows games with increasing differences have multiple equilibria and have a “tipping set,” a subset of agents who by changing from the inefficient to the efficient equilibrium can induce all others to do the same. We argue that international climate negotiations may form such a game and so have a tipping set. This set is a small group of countries who by adopting climate control measures can make in the interests of all others to do likewise.

Key words: climate change, greenhouse gases, tipping, strategic complementarity, supermodularity, increasing differences, leakage

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## 1. Tipping

Tipping is a trendy concept: almost everything is at or near its tipping point these days. We are even told that the climate system might be about to tip! Malcolm Gladwell is largely responsible for this, with his book *The Tipping Point* (2000). Although he popularized the idea of tipping, he did not invent it: the idea appears to have originated with Tom Schelling's analysis of changes in the racial composition of a neighborhood (1971) and was expanded upon in his book *Micro Motives and Economic Behavior* (Schelling 1978).

In his dynamic model of segregation, people decide to reside in a neighborhood if there are enough others like themselves who are already there. Thus the rate at which a minority population moves into a region previously dominated by a majority increases with the number of minorities who have already located there. Similarly the rate at which the majority group moves from a neighborhood increases with the number who have already done so. You can see intuitively that this describes a process that will start slowly and then speed up rather rapidly. Hence the idea of a tipping point.

Here we treat tipping as a game theoretic phenomenon (Heal and Kunreuther 2010) in spirit of Dixit (2003) and not as a dynamic process or a network phenomenon. We apply that framework to the question: Is there a *tipping point* for the adoption of climate policies by the international community? Our conclusion is that under plausible assumptions about the interactions between participants in climate negotiations, there is indeed a tipping point, in that once enough countries have joined a climate agreement, others will all follow suit.

Granovetter (1978), Watts (2002) and others have utilized essentially the same basic concepts by modeling tipping in terms of crossing of certain thresholds in the behavior of a group, using network theory. More specifically they look for cascades of behavior patterns through social networks. One of the most widely-known results to emerge from this study of connection in networks is that of six degrees of separation (Watts 2003). In these network models, each agent has a threshold for adopting a new behavior pattern that is expressed in terms of the number of other people she must see following the new pattern before she also adopts it. Reasonable distributions of thresholds lead to tipping-like outcomes in the adoption of new behavior patterns.

## **2. Motivation**

A principal reason for asking this question is that the existing framework for discussing climate policies, the United Nations Framework Convention on Climate Change (UNFCCC), has not proven to be effective in gaining consensus on adopting specific measures for reducing GHG emissions. Those that have a real interest in stopping agreement on climate issues, such as major oil and coal producers, can block agreements by withholding their assent. More generally, it seems that 192 countries have too many different interests to expect consensus on something so far reaching as a radical reduction of GHG emissions. To date, the focus of the UNFCCC negotiations has been on procedural issues, on Targets and Timetables, and not on substantive ones.

### 3. Substantive Issues

What are the key issues that need to be addressed? They involve stopping deforestation, which generates between 15 and 20% of all greenhouse gases (IPCC, 2007), and decarbonizing the world's electricity supplies. If we can solve these problems we have a good chance of containing the increase in global temperatures within reasonable limits. The international community has ideas about stopping deforestation: one of the few areas of progress and agreement at the UNFCCC has been the need for a mechanism to reduce deforestation and the emergence of general agreement on the possible role for the REDD+ mechanism (REDD = Reduction of Emissions from Degradation and Deforestation, introduced by the Coalition for Rainforest Nations, [www.rainforestcoalition.org](http://www.rainforestcoalition.org)) as a way of providing financial incentives for maintaining tropical forests intact. If this is implemented, it will greatly reduce rates of tropical deforestation.

So that leaves decarbonization as the real substantive issue. With carbon-free electricity we can go on to decarbonize ground transportation by moving to electric vehicles, and to do likewise with heating and cooling, switching from fossil fuels to electricity. These measures will control the climate problem.

### 4. What should we change?

There have been many suggestions about improving the UNFCCC process, which currently is based on annual meetings of all members of the Convention at which decisions are made by consensus. A common element to these suggestions is that fewer countries be involved, such as only the major emitters of greenhouse gases. The objection to this, of course, is that many of the countries most affected by

climate change, are not major emitters themselves. They nevertheless have a right to be represented and participate in these discussions because the outcomes of any agreement will have major implications for them.

A broader interpretation of who should be officially involved in UNFCCC process is to include major stakeholders who are impacted by a change in climate policy. Would this include the OPEC countries, who are neither major emitters nor greatly threatened physically by changes in the climate? As major providers of fossil fuels, OPEC countries clearly have an economic stake in the outcome, though one that is different from the concerns of most other countries.

Another suggestion is that we drop or weaken the consensus requirement in the UNFCCC process. Both this proposal and the one to reduce the size of the participating group face the following problem: ultimately we need most countries to adopt a treaty on greenhouse gases, which presumably requires that they have some say in its drafting. Will countries agree to adopt a treaty on something so central as energy use that they did not play a role in constructing?

## **5. Economic Incentives Leading to Tipping**

Ultimately, the solution seems to lie in getting the economic incentives right and structuring a treaty so that it is in all emitting countries' interests to adopt it (see Barrett (2003)). If we can do this, we can rely on self-interest to bring about the right outcome. This goal runs into the classic problem that each country bears its costs of cutting emissions while the benefits are shared by all countries. Even if the benefits of action exceed the costs globally, there is not necessarily an incentive for any country to act on its own. Given the apparently intractable nature of the

challenge, it is worth pausing to look at some cases in which a problem of this type has in fact been resolved.

**5.1. Illustration 1: Unleaded Gasoline<sup>2</sup>** In moving from leaded to unleaded gasoline, there are three fixed costs: one is the research to find an alternative to lead as an additive, the second is adapting refinery capacity to deal with the new additive and the third is modifying automobile engines to burn the reformulated fuel. The U.S. moved first to adopt unleaded gasoline, and in doing so solved the first and third problems: it found a new additive and worked out how engines needed to be modified for this.

Once this change was in place, the incremental costs of adoption for other countries were only the costs of modifying refinery capacity. In fact, adoption was made even easier because the Japanese and European motor industries both export extensively to the U.S. Their manufacturers had to start making engines for lead-free fuel as soon as the U.S. made its move. So for the second and subsequent movers, the incremental costs of going lead-free were negligible. An interesting detail is that in Europe, Germany went lead-free before Italy: many affluent German tourists visit Italy by car, so Italian refineries and gas stations were adapted to unleaded gasoline several years before Italy formally adopted unleaded fuel. Though there are no formal studies of this, it is reasonable to assume that the incremental cost of to Italy of formally adopting lead-free fuel was very low indeed. In this story we have an example of the global economic system being tipped from leaded to unleaded gasoline by the actions of the United States.

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<sup>2</sup> For more details on this case see Heal (1994)

### 5.2. Illustration 2: Phasing out Chlorofluorocarbons (CFCs)<sup>3</sup>

The Montreal Protocol on Substances that Deplete the Ozone Layer, signed in September 1987, is the most successful example of international cooperation. Twenty three countries agreed to cut their consumption of CFCs and other ozone-destroying chemicals by half before the end of the 20<sup>th</sup> century. Yet it faced many of the same obstacles we face today with greenhouse gases. The critical issue here was development of a new technology by Du Pont, the world's largest producer of CFCs, which made it profitable for them to agree to elimination of CFCs, coupled with an agreement to make compensatory payments to poor countries to cover the costs of replacing CFCs. As in the unleaded gas example, a change of attitude by one company led the United States to agree to sign the Montreal Protocol and many other countries to follow suit with further adjustments in the phase-out schedule over time. As of August 24 2010, 196 countries had signed the Protocol. (For more details see [http://ozone.unep.org/Ratification\\_status/](http://ozone.unep.org/Ratification_status/).)

## 6. Tipping Sets

In determining the conditions under which tipping can occur, one needs to determine how many agents are required to change their behavior to ensure that others will want to follow suit. More specifically, a *tipping set* is a set of agents who can, by changing their strategies, tip the rest of the agents from one equilibrium to another. If there is a small tipping set, then we do not have to get everyone to agree change their behavior: agreement by a small subset will suffice.

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<sup>3</sup> See Barrett (2003) for more details on the Montreal Protocol and the reasons for its success.

Heal and Kunreuther (2010) show that tipping is induced by a pattern of positive reinforcement, also known more formally as supermodularity or strategic complementarity. More specifically, they show that tipping occurs if there are increasing differences with respect to the net benefits of taking a certain action as more agents follow suit. In other words, a choice of the efficient strategy by one agent raises the net return to the choice of this strategy for other agents. This was clearly the case in the unleaded gasoline example above.

Below, we determine the size of the tipping set with two illustrative examples taken from Heal and Kunreuther (2010). A formal structure and general framework for determining tipping sets is given in the Appendix.

**6.1. Example 1** Consider a game with ten agents, each of whom has two strategies which we designate  $0$  and  $1$ . We can think of  $0$  as not joining and  $1$  as joining a climate change treaty. The payoffs to the agents  $i$  ( $U_i$ ) in this game are  $U_i = 0.5$  if  $S_i = 0$  and  $U_i = \#(1)$  if  $S_i = 1$  where  $\#(1)$  is the number of other agents choosing  $1$ . Thus, your utility for choosing  $0$  is  $0.5$  and for choosing  $1$  is the number of others who also choose  $1$ .

This game has two Nash equilibria: all choose  $0$  and all choose  $1$ . The latter clearly is Pareto dominant. If none of the others choose  $0$ , you are also better off making the same choice since  $U_i$  is  $.5$  with a choice of  $0$  rather than  $0$  with a choice of  $1$ . If at least one person chooses  $1$  you will also want to choose  $1$ . The key point for our purposes is the following: if the system starts with all choosing  $0$ , and one agent – any agent – changes to  $1$ , then every agent will respond by changing to  $1$ . The best



response when one agent chooses 1, is 1. So if the system is at the inefficient equilibrium, and one agent can be persuaded – or perhaps paid – to change to strategy 1, then the system moves in response to the efficient equilibrium. The smallest *tipping set* for this problem is any one agent.

**6.2. Example 2** Now we move to a more complex illustration of tipping. There are now ten agents with numbers  $i=1$  to  $10$ , each of whose strategies are again to choose 0 or 1, and now the payoff ( $U_i$ ) for each agent  $i$  is given by  $U_i = 0.91i$  if  $S_i = 0$  and  $U_i = \#(1)$  if  $S_i = 1$ . So the only difference in the payoff from Example 1 is that the returns to strategy 0 depend on which agent chooses 0: they are greater, the greater the number of the agent. Again there are two Nash equilibria, all zeros and all ones.

In this case, agent ten can tip the equilibrium with all zeros to that with all ones: if agent ten changes from zero to one then agent one's best response is now : as  $1 > 0.91$ ; agent one will change too. But now the payoff to another agent from choosing 1 is 2; and as  $2 > 1.82$ , agent two will change also. This logic continues until all agents have changed, so that the only Nash equilibrium consistent with ten choosing 1 is all ones. Agent ten starts a cascade. Note that no agent other than ten can tip the equilibrium of zeros.

In this example agent ten can also tip the equilibrium of all ones to that of all zeros. If all are choosing 1 and then agent ten changes to 0, the payoff to agent nine from choosing 1 is 8 and the payoff to choosing 0 is 8.19. Now there are two agents choosing zero, so for agent eight, the payoff to 1 is 7 and to zero is 7.28. Again, the change by agent ten initiates a cascade from one equilibrium to another. The change

by agent ten starts what Dixit (2003) calls a cascade. In this example, the tipping set is agent ten.

In the present context, our question is: Is there a tipping set for the climate negotiations game? If so, what can we say about it? A relevant and related question is: Will one country adopting climate policies make these more attractive to others? We argue below that this is the case.

## 7. Climate Change

We can now apply these concepts and results to our main question: Is there a tipping set for climate negotiations, and if so, what is it likely to look like? What are the factors that might generate increasing differences – i.e. positive reinforcement of climate control choices? First we need to assess whether the assumption of increasing differences – necessary for characterizing a tipping set – is reasonable in this context.

There are several reasons to think that the net benefits from joining a climate agreement rises as the number of those already a party to the agreement increases. Reducing GHG emissions is likely to require the development of new technologies. The two illustrative examples in Section 5 – introducing unleaded gasoline and phasing out CFCs – illustrate this point. In those two examples a first mover was in effect a single-member tipping set.

A reasonable assumption is that the fixed costs of developing new technologies are shared amongst those who commit to reducing GHG emissions. The more countries are party to the agreement, the less the cost per party. This leads to

an increase in net benefits to all other countries that are already part of the agreement. So this is a reason for reinforcement.

A second point is that the costs often associated with reducing GHG emissions are a loss of competitiveness and the risk of carbon leakage. Competitiveness, it is argued, may be reduced by the requirement that domestic industries use carbon-free energy sources, which may be more costly than fossil fuels: but clearly the more a country's competitors are also incurring such costs, then the less risk there is of a loss of competitiveness vis-a-vis trading partners. Similarly, the fewer countries are outside an agreement to reduce GHG emissions, the less is the risk of carbon leakage by carbon-intensive industries migrating to non-member states. So it is reasonable for the costs of joining a GHG-abatement agreement in terms of possible loss of competitiveness drop as more other countries join. This is a second reason for thinking that the increasing difference condition is satisfied.

We shall assume that a country's benefits are the same whether it joins a climate agreement or not, as the benefits from the agreement are a pure public good and accrue to all, joiners or not. In reality there may be some benefits that are in fact specific to joining, perhaps preferential access to technologies, better relations with important countries, etc.

We shall also assume that there is a cost to not joining the agreement, perhaps one that starts at zero but increases with the number of countries that have already joined. This could be an explicit cost imposed by treaty members on non-

members, such as a carbon tariff on imports (so-called border adjustment tariffs, something that has been seriously discussed in both the US and the EU), or it could be a more implicit cost associated with not being party to important negotiations and not being a member of an influential group of countries.

We formalize these ideas as follows. Each country has a benefit function and a cost function, with net benefit  $B_i(S_i, S_{-i}) - C_i(S_i, S_{-i})$  where  $S_i$  is the strategy chosen by country  $i$  and  $S_{-i}$  is a vector of all the strategies chosen by all agents except  $i$ . The gain from switching from not joining to joining, i.e. from strategy 0 to strategy 1, is

$$\Delta = \{B_i(1, S_{-i}) - C_i(1, S_{-i})\} - \{B_i(0, S_{-i}) - C_i(0, S_{-i})\}$$

which we can rewrite as

$$\Delta = \{B_i(1, S_{-i}) - B_i(0, S_{-i})\} - \{C_i(1, S_{-i}) - C_i(0, S_{-i})\}$$

We need to understand how this changes with  $S_{-i}$ , and in particular if there are increasing differences, i.e. if  $\Delta$  increases as another country switches from 0 to 1. The first parenthesis here is non-negative but by assumption independent of  $S_{-i}$ . This is because we have assumed that the gains from reducing emissions are a pure public good and are equally available to all countries whether joiners or not. So the first term does not increase as another country joins, and neither does it decrease.

In the second parentheses of the expression for  $\Delta$ , the first term, the cost of joining, is decreasing as the number of other joiners rises and the second term, the cost of not joining, is constant or increasing. If the cost of not joining – the second

term – is large enough, the second parenthesis becomes negative and the overall expression is positive. Hence, the overall term is positive. So if, as we have argued, it is plausible the cost of joining falls and that of not joining rises, then we have the increasing difference property.

Formally, we can show that the climate negotiation game has a tipping set if (i) the cost of joining a treaty drops as the number of joiners rises, and (ii) there is a cost to not joining that is constant or rising with the number who have joined, and (iii) if once a sufficient number have joined this cost exceeds the cost of joining for those who have not yet joined (see Appendix Proposition 1).

It can also be shown that the tipping set will contain the countries that by joining have the largest reinforcing effects on others.<sup>4</sup> It is a set of countries that by adopting climate policies will make all others follow suit. In some intuitive sense these are strategically important countries.

As the European Union has both adopted a GHG abatement policy and also actively recruited others to the cause, we know that they do not form a tipping set, though they may be part of one. Note however that the EU has played a major role in the move to reduce greenhouse gas emissions: its aggressive clean energy policies have been responsible for driving down the costs of wind and solar power, generating learning and economics of scale. Policies that have led to this include the generous feed-in tariffs offered for clean energy in Germany and Spain, and of course the introduction of a price on the emission of greenhouse gases through the

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<sup>4</sup> This is an application of Proposition 2 in the Appendix.

EU's Emission Trading Scheme, which requires that energy-intensive industries purchase of tradable greenhouse gas emission permits to cover their emissions.<sup>5</sup> China has also been an important part of this equation, as the opening of large new silicon panel fabrication plants there in 2009 and 2010 has been responsible for a massive drop in the cost of solar panels. Almost 95% of China's production of solar panels is exported, with a large fraction sold to Germany, which itself has over 40% of global installed capacity.<sup>6</sup> So China and the EU between them have been instrumental in making carbon-free energy considerably less expensive, suggesting that they could be part of a tipping set, or even form one. By way of illustrating the importance of these effects, note that the cost of photovoltaic solar panels has fallen in the last two years from \$8 per Watt of capacity to under \$2, bringing solar PV power close to being competitive with new coal plants.<sup>7</sup>

Concerns about leakage, about the movement of carbon-intensive industries from countries with GHG abatement policies, also suggests that China could be an important part of a tipping set: most leakage is envisaged as movement from industrial countries to China, so if China were also to restrict GHG emissions that concern would be allayed, as would a large part of the concern about loss of competitiveness.

Another important characteristic of a tipping set would be the ability to impose costs on non-joiners, costs such as border adjustment tariffs to compensate

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<sup>5</sup> For a review of Europe's policies supporting clean energy, see World Bank 2010

<sup>6</sup> See International Energy Agency 2010

<sup>7</sup> Personal communication between Geoff Heal and Toni Volpe, CEO of ENEL North America and Mark Fulton of Deutsche Bank Climate Change Advisors.

for the lack of a price on carbon. A threat by China and the EU to do this would be taken seriously, as indeed would be a threat by a group containing the U.S.

## 8. Conclusions

Negotiating a climate treaty can be seen as a non-cooperative game with several Nash equilibria, only one of which is efficient. Due to the opportunity to free ride by not incurring the costs associated with joining a climate treaty, the natural equilibrium is the inefficient one. But if there are positive reinforcing effects or strategic complementarities between agents' choices of strategies to reduce emissions, then it may be possible to tip the system from the inefficient to the efficient equilibrium. This suggests that there may be a subset, ideally a small one, of countries who, by adopting GHG abatement policies, can trigger a movement by the rest in the same direction. If so, working with these countries is strategically the best direction for the international community.

## Appendix

### Formal Structure for Determining Tipping Sets

Consider a game with  $N$  players each choosing a strategy from the set  $\{0,1\}$ .  $S_i$  is agent  $i$ 's strategy and  $S_{-i}$  is the vector of all other agents' strategies. Agents have payoff functions that satisfy the condition of *uniform strict increasing differences* (Heal and Kunreuther 2010):

$$U_i : \{0,1\}^N \rightarrow R, \text{ and for some } \varepsilon > 0, \quad (1)$$

$$S'_{-i} > S_{-i} \text{ implies } U_i(1, S'_{-i}) - U_i(0, S'_{-i}) \geq \varepsilon + U_i(1, S_{-i}) - U_i(0, S_{-i})$$

Here  $S'_{-i} > S_{-i}$  implies that the former is at least as great in all components and greater in at least one. This condition is a formalization of the positive reinforcement property mentioned above. It implies that the payoff to agent  $i$  from changing from zero to one increases by at least  $\varepsilon$  when one more agent changes from zero to one.

Let  $T$  be a subset of players: we shall talk of the  $T$ -game as the above game where all agents in the subset  $T$  choose strategy 1. Then  $T$  is a tipping set if  $\{1,1,\dots,1\}$  is the only NE of the  $T$ -game. It is a minimal  $T$ -set if no subset is a  $T$ -set.

**Proposition 1: With enough agents, there is a tipping set with less than  $N-1$  members that tips the Nash equilibrium with all zeros to that with all ones.**

**Proof of Proposition 1.**

Let  $(0^{N-2}, 1_i, 1_j)$  be a vector with zeros in  $N-2$  positions and ones in the  $i$ -th and  $j$ -th positions. Define

$$\Delta_{i,j} = [U_j(0^{N-2}, 1_i, 1_j) - U_j(0^{N-2}, 1_i, 0_j)] - [U_j(0^{N-2}, 0_i, 1_j) - U_j(0^{N-2}, 0_i, 0_j)] = \Delta_i$$



This is the change in the return to  $j$  from switching strategy as a result of  $i$  switching strategy, and we are going to assume that it is independent of the index  $j$ : we will therefore denote it  $\Delta_i$ . This is a measure of the external benefits that agent  $i$  confers on others by changing from 0 to 1. With this definition, we can characterize a tipping set: not surprisingly, it consists of the agents who generate large external benefits.

From (1) it follows that

$$U_i(0^{N-k}, 1_1, 1_2, 1_i) - U_i(0^{N-k}, 1_1, 1_2, 0_i) > (k-1)\varepsilon + U_i(0^{N-1}, 1_i) - U_i(0^{N-1}, 0_i) \quad (2)$$

Clearly

$$U_i(0^{N-1}, 1_i) - U_i(0^{N-1}, 0_i) < 0$$

as all zeros is a Nash equilibrium, and likewise

$$U_1(1_1, 1_2, \dots, 1_{N-1}, 1_1) - U_1(1_1, 1_2, \dots, 1_{N-1}, 0_1) > 0$$

as all ones is also a Nash equilibrium. As the sequence of differences (2) starts negative and ends positive it changes sign there is a  $k$  such that

$$(k-1)\varepsilon - U_i(0^{N-1}, 1_i) + U_i(0^{N-1}, 0_i) > 0$$

and the first  $k$  agents form a T-set.

**Proposition 2: If a minimal T-set exists then it consists of the first  $F$  agents ranked by  $\Delta_i$ .**

For a proof see Heal and Kunreuther (2010).

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