Intertemporal Welfare Economics and the Environment[1]

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[1]Author's contact data: gmh1@columbia.edu, www.gsb.columbia.edu/faculty/gheal/ I am grateful to Karl-Göran Mäler and Jeff Vincent for valuable coments. This chapter contains in condensed form several chapters of my book Valuing the Future: Economic theory and Sustainability, Columbia University Press, 1998.

Abstract

This is an entry for the Handbook of Environmental Economics, edited by Karl-Göran Mäler and Jeffrey Vincent. I review the complex welfare economic issues that arise in environmental decision-making over very long periods, as in cases relating to climate change and biodiversity loss. I also consider the issues that arise in choosing a discount rate to apply to very long-run projects and indicate how such rates should be chosen.

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Contents

1 Fundamental Dilemmas of Dynamic Welfare Economics

Intertemporal issues play a particularly central role in environmental problems because the time scale of many environmental processes is radically longer than conventional economic time scales. Global warming and loss of biodiversity provide perfect illustrations: global warming may well have its main impacts on human societies one hundred or more years hence, and likewise the costs of loss of species diversity in terms of simplification of ecosystems and loss of genetic variability are likely to be felt most strongly by generations quite remote from us. This is not to say that there will be no short-run impacts from these phenomena: there will, but they are likely to be dwarfed by consequences that will become apparent only over very long periods. So to assess and evaluate properly these anthropogenic changes in the biosphere we need to look relatively far into the future - possibly a century at least. More generally, environmental assets such as watersheds, species diversity, rangelands, marine ecosystems, and climate regimes, are assets that are in principle very long-lived. They have functioned as they do today for millennia, and if well managed will continue to do so equally far into the future. In this they are rather different from the assets that humans construct, and that we are used to valuing. These typically have life spans measured in years or decades. So to appreciate fully the contributions that environmental assets can make to human welfare we need a very long view. This does not sit easily with the economist's standard practice of discounting the future at a real rate of at least 3% or 4%. After all, if we discount at 3% per year, then $100 one hundred years from now is worth only $5, so that events so far into the future will be of little consequence in a cost-benefit analysis. In an obvious and intuitive sense, discounting seems to tip the scales against the future. Is this indeed the case? What are the arguments for discounting over such a long period? What are the alternatives, if any?

The debate to which these questions give rise is not a new one. It runs right back to the origins of dynamic welfare economics. As most of us are aware, Ramsey, who wrote a paper on optimal growth from which we can still learn today, commented that "discounting of future utilities is ethically indefensible and arises purely from a weakness of the imagination" [43]. And his contemporary Harrod was equally outspoken, remarking that "discounting is a polite expression for rapacity and the conquest of reason by passion." [20] More recently even *The Economist*, normally a repository of mainstream thinking on economics, was driven to remark that "There is something awkward about discounting benefits that arise a century hence. For, even at a moderate discount rate, no investment will look worthwhile." (the Economist [18] cited in Heal [27] p 59) Yet Ramsey's pointed remark, though intuitively appealing, misses some deep technical points relating to the ranking of alternative consumption streams over time. These are important issues conceptually and also complex and indeed treacherous ones from a technical perspective.

In working through these issues, we need to start by understanding our options for formalizing rankings of intertemporal utility or consumption streams. Consider for the purposes of introductory discussions a model in which time is discrete. Each generation lives for a finite number of periods and generations may overlap in time. Many people - though not all - would agree with the idea that we ought to give equal weight to the welfare levels at all points in time. It is after all difficult to make a really strong case for treating some generations better or worse than others. Of course if we are utilitarians we may place less weight on a marginal increment of consumption to rich generations than to poor ones, but this is not an intertemporal judgement but rather an interpersonal one. It arises from diminishing marginal utility and not from differential treatment of generations. Suppose then that we want to treat all generations equally, and that we are utilitarians at least in the limited sense that we assume at each date t that welfare is a concave increasing function of consumption, $u(c_t)$. We want not unreasonably to maximize the total welfare, $\sum_{t=1}^{t=T} u(c_t)$.

This is easy when we have a finite number T of periods: we just give equal weight to the utility of each, as we are doing here. But economists have typically wanted to work with infinite horizons. This is mainly because of a reluctance to specify a date beyond which nothing matters, as one does when one chooses a terminal date T that is finite. Also influential is a concern about the impact of end effects, by which I mean sensitivity of the ranking of paths to the precise end date specified. So we tend to look instead at $\sum_{t=1}^{t=\infty} u(c_t)$.

This is now quite different. If all periods have utility levels bounded away from zero, then this sum will be infinite. In fact it will be infinite for all consumption sequences on which utility levels are bounded away from zero, and on many others as well. This means that we cannot represent choosing the best consumption sequence as maximizing a real-valued function on the set of feasible consumption sequences. The only way of making sure that the sum converges is to treat generations unequally, and in particular to give little weight to "most" of them, which of course is what discounting does. So there is apparently a practical reason for discounting. It is a way of ensuring that we have a well-defined preference order over the set of alternative consumption sequences between which we must choose, and that this ordering can be represented by a real-valued function [25].

Is it the only way or are there others? Ramsey was clearly aware of the difficulty and had an ingenious alternative. He assumed that utility levels are bounded above and then sought to minimize the total shortfall over time of actual utility levels from their maximum level: $\sum_{t=1}^{t=\infty}[b-u(c_t)]$. Here b is the upper bound of the utility function: think of it as "bliss". Ramsey made some rather special assumptions that ensured that this sum converged in his case. His approach certainly ensures that we give equal weight to all generations. Others have tried to develop more general approaches. For example, von Weizäcker and other have tried to develop the overtaking approach. This ranks as best the consumption sequence, if any, whose cumulative utility sum eventually exceeds that on any other path. Formally this means that path c_t ranks above path c_t^1 if and only if there exists a time $\widehat{T}$ such that whenever $T>\widehat{T}$, then $\sum_{t=1}^{t=T}u(c_t)>\sum_{t=1}^{t=T}u(c_t^1)$. The idea behind this approach is that we can avoid discounting future utility levels and thus treat all generations equally. As we shall see below, it is not completely successful in this. And once again it does not provide us with a ranking that can be represented by a real-valued function defined on the set of alternative consumption sequences.

The conclusion of this discussion is that it is not in general possible to evaluate consumption sequences over time in a manner that gives equal weight to all generations if at the same time we insist on working with infinite horizons and a roughly utilitarian framework. This observation lies at the heart of one of the main problems in intertemporal welfare economics.

A second problem, less fundamental but nonetheless demanding and only recently moving to the center of the stage, is that of dynamic consistency. A choice of a consumption path is dynamically consistent if it has the following property. If at some date during the execution of the chosen path we stop and ask what path we would now choose, given what we have done to date, then (provided that no parameters have changed) the answer is that we continue with the original choice. In other words, we see no reason to revise our choice merely because of the passage of time.

Not all algorithms for making choices over time are dynamically consistent. Indeed, relatively few are. If we feel the need for choices over time that are dynamically consistent, then this again constrains how we can approach this area.

Finally, a third issue is that a path of consumption over time should generally be intertemporally efficient. This means that it is a path with the property that no variation about it will make some generation better off and none worse off. Obviously, if such a variation were possible, we would normally want to take advantage of it.

Finding a way to make choices that attain or come close to these three desiderata - equal treatment of all generations, and consistent and efficient choices - is difficult. It is the subject matter of dynamic welfare economics. As noted above, it is particularly relevant in the environmental context because of the unusually long time horizons implied by the unfolding of anthropogenically-induced modifications to the biosphere.

In this chapter I review the literature on making intertemporal choices that are fair between generations, efficient and consistent. Obviously, I place emphasis on the environmental aspects and implications of this, although the general issues raised are important for economics as a whole and indeed were frequently raised initially as general theoretical issues rather than specifically in an environmental context. Neither Ramsey nor von Weizäcker were motivated by environmental issues. After reviewing the general issues associated with intertemporal welfare judgements and their representation in an objective function, I spend some time examining the implications of the different judgements that one can make, and the alternative philosophical approaches, in the context of some simple and standard economic growth models. I conclude by venturing into controversial territory and offering some constructive observations on the choice of a social discount rate in long-run environmental analyses.

2 Dynamic Utilitarianism

The framework most widely used in dynamic welfare economics is that provided by discounted utilitarianism. This is the framework alluded to in the previous section. We take it that the benefits from consuming at rate c_t at date t are represented by a smooth function $u(c_t)$. We then rank alternative consumption sequences $c_t, t = 1, 2, 3, 4, ...$ by their discounted utility sums: c_t^1 is better than c_t^2 if and only if $\sum_{t=1}^{t=\infty} u(c_t^1)\,\delta^{t-1} > \sum_{t=1}^{t=\infty} u(c_t^2)\,\delta^{t-1}$ where δ is a constant utility discount factor.

What is the justification for such an approach? What assumptions are we making when we adopt such a framework?

Two authors have provided axiomatic justifications for this approach, Tjalling Koopmans and John Harsanyi. Their approaches differ and I shall consider each separately. In analyzing them I shall continue to use the framework of the previous section, time being discrete.

2.1 Koopmans' Axioms

In a widely-cited and elegant paper Koopmans [30] established the following. If we rank utility sequences in a fashion that satisfies two key axioms - plus three other more technical conditions - then it follows that we can represent our ranking by the sum $\sum_{t=1}^{t=\infty} u(c_t)\,\delta^{t-1}$ where $0 < \delta < 1$. In other words, if our preferences satisfy Koopmans' axioms, then choosing the most preferred utility sequence is the same as choosing the one with the highest geometrically discounted sum of utilities.

What are the implications of the key axioms in Koopmans' framework? These Koopmans named stationarity and independence. Stationarity means that

$$\{c_1, c_2^a, c_3^a, c_4^a, ...\} > \left\{c_1, c_2^b, c_3^b, c_4^b, ...\right\} \text{ if and only if } \{c_2^a, c_3^a, c_4^a, ...\} > \left\{c_2^b, c_3^b, c_4^b, ...\right\}.$$

In words, if two utility sequences have a common first element and one ranks above the other, then the elimination of this common element and the advancement of the remains of the sequences does not change their ranking.

Independence means that the rate at which we trade off consumption in period i against that in period j depends only on c_i and c_j and not at all on consumption levels in any other periods $t \neq i, j$. Mathematically independence in this sense has the following implications. Suppose - and this does not follow from independence but from other axioms - that we can represent our preferences over sequences by a real-valued function defined on them: $u = u(c_1, c_2, c_3, c_4...)$. Then independence implies that the ratio of the derivatives of w with respect to any two of its arguments is independent of the values of its other arguments:

$$\frac{\partial u/\partial c_i}{\partial u/\partial c_j} \text{ is independent of } c_t \text{ for any } t \neq i, j$$

Koopmans formalized this in rather different terms, but the implications are the same. He used the following formalization:

$$u(x_1, x_2, x_3, x_4, ..) > u\left(x_1', x_2', x_3, x_4, ..\right) \Rightarrow u\left(x_1, x_2, x_3', x_4', ..\right) > u\left(x_1', x_2', x_3', x_4', ..\right)$$

and

$$u(x_1, x_2, x_3, x_4, ..) > u\left(x_1^{'}, x_2, x_3^{'}, x_4^{'}, ..\right) \Rightarrow u\left(x_1, x_2^{'}, x_3, x_4, ..\right) > u\left(x_1^{'}, x_2^{'}, x_3^{'}, x_4^{'}, ..\right)$$

These are not easy conditions to evaluate, and many people have taken them to be innocuous. I think that this is incorrect: they are restrictive. They may or may not be reasonable, but they are surely not innocuous. To understand this, it helps to think about them geometrically. First take the independence condition, as this is the easier to visualize. Consider a three period problem with consumption levels c_1, c_2 and c_3 in the three periods. We have preferences that rank all possible sequences of the form $\{c_1, c_2, c_3\}$. Figure one shows what it means for these preferences to satisfy independence. The point a has $c_2 = 0$: the vertical line through it joins all points whose $c_1 - c_2$ values are the same as at a. Independence requires that the slope in the horizontal plane of an indifference curve through this line be the same as that at a. So at a point such as b, with c_1 and c_3 values equal to those at a, the slope of the tangent in the horizontal plane is equal to that at a. The same argument applies at point c : the horizontal line from c joins all points with the same c_2 and c_3 coordinates as c. Again independence requires that at all points on this line the slope of the tangent in the vertical plane $c_2 - c_3$ be the same as at c. Finally, independence implies a third such condition on lines perpendicular to the $c_1 c_2$ plane.

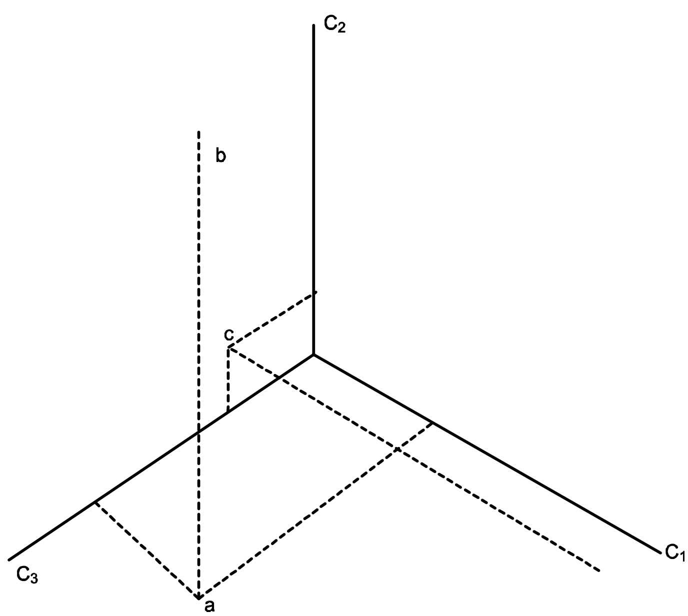

Figure 1: A geometric interpretation of the independence condition.

Clearly we have here quite tight restrictions on the shapes of indifference curves, and with infinitely many periods there are infinitely many such restrictions. Arbitrary convex increasing preferences over utility sequences will certainly not satisfy independence. Nor do Cobb-Douglas or most homothetic preferences. Intuitive arguments also indicate that independence is restrictive. The reason is that the way we trade off utility today versus that say fifty years hence will normally depend on what happens in the interim. We may be more tolerant of a low welfare level in period fifty if prior welfare levels have been high but less tolerant if they have been low. In ranking consumption sequences the equivalent statement is that my trade-off between breakfast and dinner depends on what is for lunch. It is a matter of intertemporal complementarities. Formally independence can be shown to imply that

$$u(c_1, c_2, c_3, c_4...) = u_1(c_1) + u_2(c_2) + u_3(c_3) + u_4(c_4) + ... \tag{1}$$

Clearly this is a sufficient condition for independence: it can also be shown, along with additional technical conditions, to be necessary.

Next look at stationarity. We can again get some feel for the implications of this geometrically, as shown in figure two. Consider the plane parallel to the $c_2 - c_3$ plane through point a. This contains all points that have the same first period consumption level as a. Interpreting stationarity a little loosely as we must in a finite horizon model, it tells us that if from all points in this plane we delete their common first component and view them as two period sequences then their ranking is unchanged. In other words, if we project from this plane into the $c_2 - c_3$ plane then the ranking of consumption sequences is the same as in the plane through a. This same argument holds for any initial value of c_1, so the ranking must be the same in all planes orthogonal to the c_1 axis and parallel to the $c_2 - c_3$ plane. Again, this is a very strong condition in terms of the shapes of indifference curves. Clearly arbitrary smooth convex increasing preferences will not satisfy this.

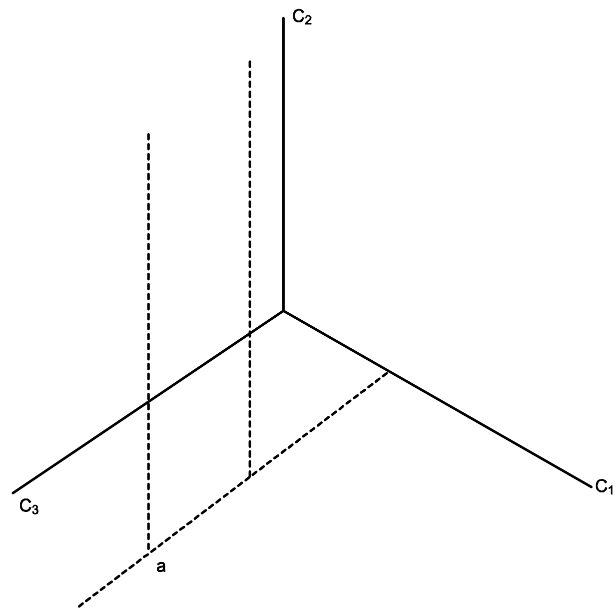

Figure 2: A geometric interpretation of the stationarity condition.

Intuitively why might preferences not satisfy the stationarity condition? Why might the deletion of an initial segment common to two sequences change the ranking of the remainders of the sequences? The answer hinges on intertemporal complementarities once again. Deleting an initial segment common to sequence a and b might reverse their ranking if the common initial sequence were complementary to something in a but not in b and so enhanced the value of a but not that of b. Deletion of the common a-enhancing segment reduces the ranking of a relative to b.

We now have a reasonable grasp of what stationarity and independence mean. How do they work to produce Koopmans' theorem? Independence, as we have noted, means that

$$u(c_1, c_2, c_3, c_4 ...) = u_1(c_1) + u_2(c_2) + u_3(c_3) + u_4(c_4) + ... \tag{2}$$

Suppose that the sequence u_t^a is preferred to the sequence u_t^b :

$$u_1(c_1^a) + u_2(c_2^a) + u_3(c_3^a) + u_4(c_4^a) + .. > u_1\left(c_1^b\right) + u_2\left(c_2^b\right) + u_3\left(c_3^b\right) + u_4\left(c_4^b\right) + ..$$

Now suppose that $c_1^a = c_1^b$. By stationarity we have

$$u_1(c_2^a) + u_2(c_3^a) + u_3(c_4^a) + .. > u_1\left(c_2^b\right) + u_2\left(c_3^b\right) + u_3\left(c_4^b\right) + ..$$

And if in addition we picked sequences such that $c_1^a = c_1^b$ and $c_2^a = c_2^b$ then

$$u_1(c_3^a) + u_2(c_4^a) + .. > u_1\left(c_3^b\right) + u_2\left(c_4^b\right) + ..$$

Intuitively u being a discounted sum is a sufficient condition for these inequalities to hold, and it is also intuitively clear that there must be some simple relationship between the functions u_1, u_2 etc. for these inequalities to hold. The real contribution of Koopmans was to show that in some regions of the space of consumption sequences, though not necessarily in all, only functions that are discounted sums meet these conditions.[1]

Now you have a good intuitive grasp of Koopmans' result. It is striking. It does provide an axiomatization of discounted utilitarianism. But it is not self-evident that the axioms are acceptable. They rule out all other than the most trivial patterns of intertemporal complementarities. However, the Koopmans axiomatization is not the only one.

2.2 Harsanyi's Axioms

Harsanyi [21] has produced a different justification for discounted utilitarianism, based on arguments very close to those subsequently used by Rawls. Rawls of course argued that a just society is one that people would choose if they had to choose from behind a "veil of ignorance", to use his famous phrase. Rawls assumed that people would be infinitely risk averse and would focus entirely on the worst that could occur to them, and thus reached his famous conclusion that they would choose the society in which the position of the least well off person was as advantageous as possible. They would, he assumed, maximin, i.e., maximize over all scenarios the position of the minimally advantaged person. Applied over time, in a context in which people are uncertain when they will be alive, this produces the recommendation that we seek the utility sequence that makes the least-favored generation as advantaged as possible. One can describe the difference between Harsanyi's work and that of Rawls in the following terms. Rawls assumed an extreme form of risk aversion in the choice behind a veil of ignorance, as represented by the maximin solution. Harsanyi asked what the outcome would be if choosers instead displayed a finite level of risk aversion. He showed that this leads to seeking to maximize a weighted sum of utilities, which looks very much like a utilitarian solution, rather than to maximizing the lowest utility level. The argument is a relatively straightforward application of the theory of

[1] Koopmans in addition assumes that the utility functions are bounded and that they are *uniformly* continuous in the sup norm.

choice under uncertainty, and is developed in the intertemporal context in Dasgupta and Heal [16]. The weights that play the roles of discount factors are now derived from agents' risk aversion and the subjective probabilities that they assign to being alive at various dates.

2.3 Utilitarianism – a Summary

There are two known justifications for ranking utility sequences by their discounted sums. One relies on Koopmans' axioms, the other on an argument due to Harsanyi that has elements in common with Rawls' subsequent theory of justice, but invokes a finite but positive level of risk aversion. Neither is totally compelling, yet the practical attractions of Utilitarianism, as I noted in the first section, are compelling.

As a matter of interest, how does Ramsey's approach fit here? Minimizing the sum $\sum_{t=1}^{t=\infty} [b - u(c_t)]$ is of course the same as maximizing $\sum_{t=1}^{t=\infty} [u(c_t) - b]$. So Ramsey is making his infinite sum converge, not by discounting, but by subtracting a common number from all of its terms. This does not always work: when it does, however, it does not require that we adopt the stationarity assumption. Of course, in working with a sum of utilities, Ramsey is implicitly adopting the independence assumption. Exactly the same comments apply to von Weizäcker's overtaking criterion, to which I turn next.

3 Overtaking

In an attempt to avoid the problems of zero discount rates, and yet give equal weight to present and future, von Weizäcker [51] introduced the overtaking criterion. In order to formalize this and derive results relating it to other criteria I shall switch now to using continuous time, and will follow this path for the remainder of the chapter. Discrete time is more intuitive for presenting concepts but continuous time lets one use more and neater mathematics.

Definition 1 *A path c^1 is said to weakly overtake a path c^2 if there exists a time T^* such that for all $T > T^*$, we have*

$$\int_0^T u\left(c_t^1\right) dt \geq \int_0^T u\left(c_t^2\right) dt$$

c^1 is said to strictly overtake c^2 if the inequality is strict.

Like Ramsey's, this is an ingenious approach: it replaces infinite integrals by finite ones, and says that one path is better than another if from some date on cumulative utility on that path is greater. This is a relationship that can be checked even if both cumulative utility totals go to infinity as $T \to \infty$, so this approach does to some degree extend the applicability of an approach based on a zero discount rate.

Unfortunately, this way of ranking paths is again like Ramsey's incomplete: it is easy to construct pairs of paths, say c_t^1 and c_t^2, such that c_t^1 does not overtake c_t^2 and vice versa, see figure 3. A more detailed analysis is given in Dasgupta and Heal [16].

Another limitation of the overtaking criterion, noted by Lauwers [33], is that in spite of having a zero discount rate, it is not neutral with respect to timing, but clearly displays impatience. An example given by Lauwers is $\{1, 0, 1, 0, 1, 0, ..\}$ versus $\{0, 1, 0, 1, 0, 1, ..\}$: the former weakly overtakes the latter, and the converse is not true, although each is a permutation of the other, and they differ only in that one is lagged a single period behind the other. Another example can reinforce this point. Consider the sequences $\{1, 1, 1, 1, ..\}$, $\{0, 1, 1, 1, 1, ..\}$, $\{0, 0, 1, 1, 1, 1, ..\}$, $\{0, 0, 0, 1, 1, 1, 1, ..\}$: clearly these have the same total consumption and differ only in that this is postponed further and further into the future. Equally clearly, the first strictly overtakes the second which strictly overtakes the third, etc... Once again, it is clear that the overtaking criterion with a zero discount rate is not neutral with respect to timing, but does display impatience.[2]

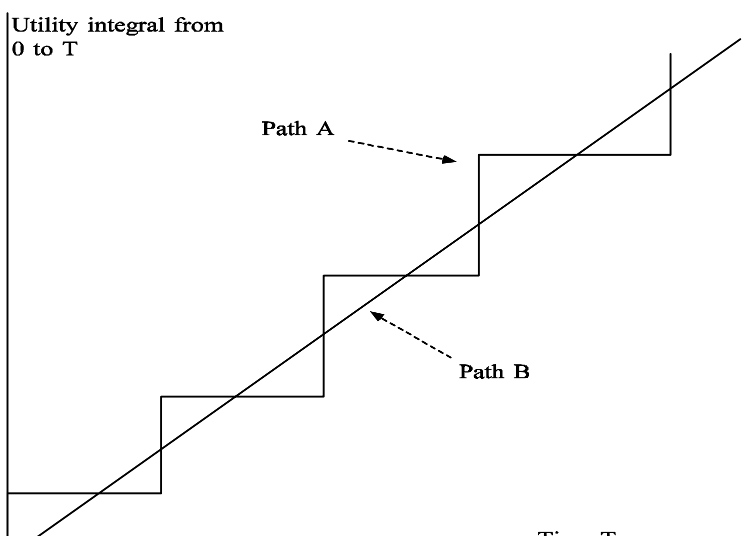

Figure 3: The overtaking criterion cannot rank paths u' and u".

[2]I am grateful to Yuliy Barishnikov for this example.

Finally, it is important to note a connection between the overtaking criterion and the limiting behavior of utility along a path. Intuitively, it is clear that if one path has a higher limiting utility value than another, then it overtakes the latter. This intuition is formalized in the following result. Consider two paths $U' = \{u'_1, u'_2, u'_3, ...\}$ and $U'' = \{u''_1, u''_2, u''_3, ...\}$ (we use the capital letter U to denote an infinite utility sequence: $U = \{u_1, u_2, u_3, ...\}$) and let both paths have limits: $\lim_{t\to\infty} u'_t = \overset{\star}{u}'$ and $\lim_{t\to\infty} u''_t = \overset{\star}{u}''$. Then:

Proposition 1 *(1) A path with a higher limiting utility value always strictly overtakes one with a lower limiting utility value. Formally, the path U' strictly overtakes the path U'' if $\lim_{t\to\infty} u'_t = \overset{\star}{u}' > \lim_{t\to\infty} u''_t = \overset{\star}{u}''$. (2) If the path U' strictly overtakes the path U'', then its limiting utility value is not lower, i.e., $\lim_{t\to\infty} u'_t = \overset{\star}{u}' \geq \lim_{t\to\infty} u''_t = \overset{\star}{u}''$.*

Proof. See Heal [27].

In summary, when we are dealing with utility paths which have limits, which is generally the case in optimal growth problems, then the overtaking criterion ranks paths with different limits by their limiting utility values. To check whether one path overtakes another, if they have different limits, one need only inspect their limits. If two paths have the same limit, then this observation is of no value, yet the overtaking criterion still ranks them. So on the space of utility paths which have limits, the overtaking criterion in effect acts lexicographically: first it ranks paths by their limiting values, and if these are equal, it then ranks them by their partial utility sums. One final comment on the overtaking criterion: unlike any of the other criteria we shall consider, it gives an ordering which cannot be represented by a real-valued function: there is no numerical function defined on alternative paths such that the path which overtakes all others, is that giving the highest value of this function. (If we restrict attention to the space of paths with different limits, there is such a function: it is just the limiting payoff. This representation cannot be extended to the space of all paths.) This leads naturally to the next criterion that we shall consider.

4 Limiting Payoffs

An approach to evaluating intertemporal payoff streams which is widely used in the theory of repeated games, is to rank paths by their long-run average payoffs, defined as

$$\lim_{m,n\to\infty}\left[\frac{1}{n}\sum_{m}^{m+n}u\left(c_t,s_t\right)\right]$$

This expression takes the average payoff over the period from m to $m+n$, and then takes the limit of this as m and n increase without limit. In this expression I have assumed, as I shall from now on, that the current utility level depends not only on the current consumption level c_t but also on the current stock of environmental assets, s_t.(For more discussion see section 10.3 below.) These could be forest, biodiversity, etc.: for a discussion see Heal [27], chapter 1. A similar approach is to rank according to limiting payoffs,[3] as in the case of the green golden rule considered below. As we have noted, this is in effect what the overtaking criterion does for paths which have limits. Of course, not all payoff sequences will have limits, so that the approach based on long-run averages is more general (see Heal [27]). A criterion focussed on evaluating paths according to their limiting behavior has nothing to say about almost all of the paths. It ranks as the same two paths whose utility levels differ in every period, provided that they have the same limits. For example, consider the two utility sequences $\frac{1}{2},\frac{3}{4},\frac{7}{8},\frac{15}{16},\frac{31}{32},....$ and $0,0,0,0,\frac{1}{2},\frac{3}{4},\frac{7}{8},\frac{15}{16},\frac{31}{32},.....$ Both have the same limit: one. So ranking them according to their limits will rank them equally for any finite number of zeros at the start. Clearly this is not satisfactory. The "green golden rule" criterion of Beltratti Chichilnisky and Heal [9] is an example of such a criterion: it defines as best the path that leads to the highest sustainable welfare level (see section 10.3 below).

5 Chichilnisky's Criterion

Chichilnisky [10] proposes that we replace the discounted integral of utilities, or the long-run utility level, by the following maximand:

$$\alpha\int_0^\infty u\left(c_t,s_t\right)\Delta\left(t\right)dt+\left(1-\alpha\right)\lim_{t\to\infty}u\left(c_t,s_t\right) \tag{3}$$

In this, $\alpha\in(0,1)$ and $\Delta(t)$ is any measure (i.e., $\int_0^\infty\Delta(t)\,dt=1$) (it has to be countably additive[4]), and in particular it could be a conventional exponential discount

[3] Other functions of the limiting behavior of the sequence are possble, such as $\limsup$.

[4] A countably additive measure is one which gives to any *countable* union of disjoint sets a measure which is the sum of the measures of the individual sets.

factor: $\Delta(t) = e^{-\delta t}$. The term lim could also be replaced by an alternative function which depends only on the limiting behavior of utility over time, such as the long-run average. Intuitively, this second term reflects the *sustainable utility level* attained by a policy. (Technically, any purely finitely additive measure will do.[5]) So Chichilnisky is in effect recommending a mixture of two approaches that have have been used so far: a generalization of the discounted utilitarian approach (to allow for any countably additive measure instead of just exponential discount factors with constant discount rates), mixed with the approach which ranks paths according to their very long run characteristics or sustainable utility levels.

Chichilnisky does not pull this result out of a hat: she has a very precise rationalization, and shows that if we accept certain axioms about the ranking of alternative utility paths, then they *must* be ranked according to (3). Chichilnisky adopts two main axioms, which require that the ranking of alternative consumption paths be sensitive both to what happens in the present and immediate future, and also to what happens in the very long run.[6]

Sensitivity to the long-run future is defined as follows: given any pair of consumption paths U' and U'', we can find no date T (which may depend on U' and U'') such that the ranking is insensitive to changes in the paths after the date T. In other words, there is no date such that changes after that date do not matter, in the sense of affecting the ranking. If we could find such a date, the ranking would be in an obvious sense insensitive to the long-run future, i.e., to the future beyond T. Hence the definition of sensitivity to the long-run future.

If this condition fails, then we can find a date T such that whatever changes are made in the two paths beyond T, their ranking will not be reversed. As an example, consider two utility sequences U' and U'', with U' ranked above U''. Suppose that no matter how much we improve U'' and worsen U' after the date $T(U', U'')$, we will never be able to change their ranking: U' will always rank higher. Such a situation is in an obvious sense insensitive to the long-run future.

Sensitivity to the present is defined symmetrically, as follows: given any two paths U' and U'' we can find no date T (which may depend on U' and U'') such that

[5] A finitely additive measure is one which gives to any *finite* union of disjoint sets a measure which is the sum of the measures of the individual sets.

[6] Chichilnisky's axioms are inspired by the framework used in social choice theory. For an earlier approch to this problem also rooted in social choice, see Frerejohn and Page [19].

whatever changes are made in the two paths U' and U'' before T, their ranking will not be reversed. In other words, there is no date such that what happens before that date does not affect the ranking. If we could find such a date, the ranking would be in an obvious sense insensitive to the present and near future, i.e., the period up to T.

The function $u(.)$ is assumed the same for all dates t, so that generations are assumed to be the same in the way in which they rank alternatives.[7] Chichilnisky proves the following

Proposition 2 *If the ranking of alternative utility sequences satisfies axioms 1 and two, plus additional technical conditions, to 5, then it must be represented by the following functional (3):*

$$\alpha \int_0^\infty u(c_t, s_t) \Delta(t) dt + (1-\alpha) \lim_{t\to\infty} u(c_t, s_t), 0 < \alpha < 1$$

where $0 < \alpha < 1$, $\Delta(t)$ *is any countably additive measure and the term* lim *could be replaced by any other purely finitely additive measure. There is no other way of ranking paths that meets all of the axioms.*

Although the derivation of this ranking requires technical arguments, there is in fact an intuitive explanation. Observe first that the first element in this expression is just a discounted sum of utilities: this is precisely the standard approach. However,

[7] In addition Chichilnisky makes the following more technical assumptions, analagous to those used by Koopmans.

Axiom 1 *Continuity: the total measure of welfare of a utility sequence varies continuously (in the sup norm) with changes in the sequence.*

Axiom 2 *The Pareto condition, i.e., if a change in a utility stream makes one generation better off and no other worse off, then it is ranked higher than before.*

Axiom 3 *Linearity: the total welfare measure is linear in the welfares of generations.*

Chichilnisky makes the additional assumption that the utility function $u(c_t, s_t)$ is bounded above and below:

$$\exists b_1, b_2 : b_2 \geq u(c_t, s_t) \geq b_1 \forall c_t, s_t$$

the criterion as a whole is a generalization of this by the addition of a term which values the very long run (or limiting) behavior of the economy. Of course, if only short periods of time are at stake, this will make no difference. Similarly, if the economy's limiting behavior is totally determined by technological and resource constraints, the presence of the additional term will have no impact. It counts only if the time horizon is long and there are several alternative behavior patterns available over that period. So it is a generalization of the standard approach, and differs only in precisely the case in which we are dissatisfied with the discounted utilitarian approach—the valuation of the very long run. It relates to that approach as general relativity relates to Newtonian mechanics: it supplements it without replacing it in routine applications.

Another way of understanding this approach is to observe that we would probably like to weight consumption in all periods—present and future—equally. However, this is not possible: one cannot weight all elements of an arbitrarily long stream equally and positively, for if one did, the weights would sum to infinity. A natural response is therefore to concentrate some weight on "the present" ($\int_0^\infty u(c_t, s_t)\,\Delta(t)\,dt$) and some on "the future" ($\lim_{t\to\infty} u(c_t, s_t)$). This is precisely what the expression (3) does.

5.1 Comparison with Overtaking

Our earlier analysis of the overtaking criterion suggests that it has some features in common with Chichilnisky's criterion. They both give weight to limiting utility values, and they both base a ranking of utility streams on the properties of the entire stream, and not just of its limiting properties. What are the differences?

One which we have already seen is that overtaking gives only a partial ordering of the set of possible utility sequences: Chichilnisky's criterion gives a complete ordering. Like Chichilnisky's criterion, overtaking uses as information both the limiting behavior of a path and its finite utility sums, but it uses them differently: it uses them lexicographically, first checking to see if two sequences have different limits and in that case ranking by the limit, and otherwise looking at finite utility sums. As an illustration of the differences which follow from this, consider two sequences with different limits, $U' = \{u'_1, u'_2, u'_3, ...\}$ and $U'' = \{u''_1, u''_2, u''_3, ...\}$ with $\lim_{t\to\infty} u'_t = \overset{\star}{u}{}' > \lim_{t\to\infty} u''_t = \overset{\star}{u}{}''$. Then U' is preferred to U'' according to the overtaking criterion. However, Chichilnisky's criterion ranks them according as $\alpha \sum_0^\infty \delta^t u'_t + (1-\alpha)\,\overset{\star}{u}{}' \lesseqgtr \alpha \sum_0^\infty \delta^t u''_t + (1-\alpha)\,\overset{\star}{u}{}''$, so that their ranking may be in the reverse order of their limits if the difference of their discounted utility sums is sufficiently large in the opposite direction ("sufficiently large" depends on α). Figure 4 illustrates such a case: it requires that the sequence with the smaller limit have a larger sum of utilities in the near future, which offsets the difference in long-run behavior. Chichilnisky's criterion therefore puts less weight on the very long run than does the overtaking criterion: it does permit differences over the near future to compensate for limiting differences.

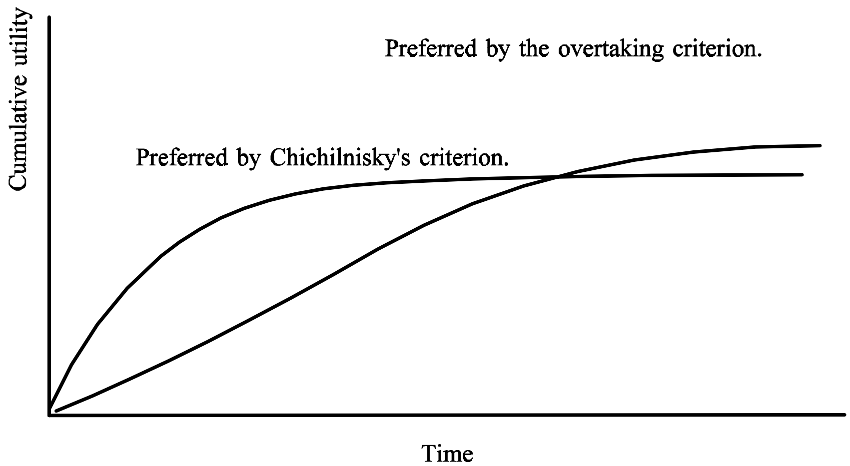

Figure 4: A case in which the overtaking criterion is defined and differs from Chichilnisky's.

6 The Rawlsian Criterion

The Rawlsian criterion, namely choosing the path which maximizes the welfare of the least well-off generation, is often discussed in the context of sustainability (see for example Solow [48], Asheim [5], [**?**]). Where does it stand in weighing the present against the future? How does it compare in this important respect with the other criteria which have been reviewed?

On the key issue of present versus future, Rawls is ambivalent: Rawls is about rich versus poor, and will be pro-future if the future is relatively poor, and vice versa (see Heal [27]). For the pure depletion model, it is strongly conservationist and strongly pro-future: it prescribes no consumption. But for a model with growth possibilities, as with the renewable resource model studied in Section 10 below, Rawls' approach prescribes a policy which neglects the possibility of building up stocks and consumption over time at the cost of lower initial consumption. The reason is clear:

Rawlsianism sanctions no sacrifice by the present for the future if this would make the present the poorest generation. Likewise, it sanctions no sacrifice by the future for the present if this would make the future the poorest period. It is neutral with respect to timing, but it will always rule out the most extreme exploitation of the future because of the strong egalitarianism inherent in the approach. In Chichilnisky's terms, Rawlsianism is insensitive not to both present or future, but to whatever time interval does not contain the poorest generation. It is sensitive only to the poorest generation, wherever that may be.

7 Discounting Utility or Consumption?

An important distinction is between the rate at which utility is discounted—this is just the normal discount rate in the discounted utilitarian approach, also known as the social rate of time preference or pure rate of time preference—and the rate at which consumption is discounted. This latter is often called the social discount rate [17]. The distinction is simple: both concepts arise within the discounted utilitarian framework, with utility discounted at a rate δ in the objective $\int_0^\infty u(c_t, s_t) e^{-\delta t}$. Within this framework, one can ask the following question:

Suppose the economy follows a time path which is optimal for this criterion, and we consider adding an increment of consumption at some date t. What is the value of this increment in terms of its contribution to the objective function, and how does this value change as the date t is changed?

This contribution to the objective function is the value that should be assigned to an increment of consumption. The rate at which this contribution changes over time is the consumption rate of discount: it is the rate at which the weight of an increment of consumption, in terms of its contribution to the objective function, changes over time.

Clearly the value of an increment of consumption Δc at date t is[8]

$$u_c(c_t, s_t) e^{-\delta t} \Delta c$$

[8] u_c and u_s are the first partial derivatives of the function u with respect to its arguments c and s. u_{cc} etc. are likewise the second partial derivatives, using obvious notation.

and the rate at which this value changes with t is

$$\frac{1}{u_c(c_t, s_t)\, e^{-\delta t}} \frac{du_c(c_t, s_t)\, e^{-\delta t}}{dt}$$

which we can easily compute to be

$$-\delta - \eta_{c,c} \frac{\dot{c}}{c} - \eta_{c,s} \frac{\dot{s}}{s}$$

where $\eta_{c,c} = -c u_{c,c}/u_c > 0$ is the elasticity of the marginal utility of consumption with respect to the level of consumption and $\eta_{c,s} = -s u_{c,s}/u_c$ is the elasticity of the same quantity with respect to the level of the stock of the environmental asset. Consider for simplicity the case in which the utility function is additively separable, so that the cross derivative is zero and $\eta_{c,s} = 0$. Then the consumption rate of discount is $\delta + \eta_{c,c} \frac{\dot{c}}{c}$. For a linear utility function, or for variations in the level of consumption small enough that a linear approximation to the utility function suffices, this reduces to δ, the utility rate of discount: the two concepts are the same. In this case, we have a positive consumption rate of discount if and only if we have a positive utility rate of discount. As Dasgupta Mäler and Barrett [17] note, the consumption discount rate is in general not constant over time so that consumption plans chosen using this rate may not be dynamically consistent - see section 10.6 below.

More generally, the two discount rates differ: and if consumption falls over time so that $\frac{\dot{c}}{c} < 0$, then the consumption discount rate may be negative, so that the weight given to a increment of consumption actually rises over time (a point also emphasized by Dasgupta Mäler and Barrett). In fact, if an economy is following an optimal path in the utilitarian sense, then the first order conditions for optimality give us further information about the consumption discount rate.

One obvious and general proposition is the following. If the utility function has an elasticity of the marginal utility of consumption which is bounded, and if the utilitarian optimal path tends in the limit to a stationary solution, then *at this stationary solution the consumption discount rate is equal to the utility discount rate.* Another way of saying this is that the social discount rate equals the pure rate of time preference. This is immediate from the expression $\delta + \eta_{c,c} \frac{\dot{c}}{c}$ for the consumption discount rate: if $\dot{c}$ is zero at a stationary solution, then this expression is just δ, the utility discount rate. So in general the utility and consumption discount rates converge in the limit along utilitarian optimal paths, or indeed any paths that have limits.

Consider some particular cases in more detail, firstly the pure depletion model of Hotelling: then first order condition is that

$$\frac{\dot{c}}{c} = -\frac{\delta}{\eta}$$

so that along an optimal path the consumption discount rate is always zero, whatever the utility discount rate. In fact this is obvious: the first order condition for optimality is just that the marginal contribution of an increment of consumption to the objective should be the same at all times, which is precisely that the consumption discount rate be zero. So on an optimal path in the Hotelling model *whatever the utility discount rate, and however uneven the distribution of consumption between generations, the consumption discount rate is zero.* This shows that a *zero consumption discount rate does not imply any degree of equality of consumption over time.* Note that in this case there is no stationary solution to the first order conditions for optimality so that the consumption and utility discount rates do not converge.

For the renewable resource model analyzed below (section 10) the consumption rate of discount along a utilitarian optimal path is

$$\frac{u_2^{'}}{u_1^{'}} + r^{'}$$

which is generally positive and again goes to the utility discount rate δ in the limit. In both of these cases the first order conditions admit a stationary solution so that the utility and consumption discount rates - pure rate of time preference and social rate of discount - converge in the limit. A similar convergence holds for models with capital accumulation and production: see Heal [26] and chapters 9 and 10 of Heal [27].

In summary, the consumption and utility discount rates, or social discount and time preference rates, are not independent concepts: when there is a stationary solution to the utilitarian problem, they are equal asymptotically along a utilitarian optimal path, and are always linked by the first order conditions. Furthermore, the fact that the consumption discount rate is zero, does not imply any degree of intergenerational equity, as the Hotelling example shows clearly. So having a zero consumption discount rate is not a solution to the ethical problem that led Ramsey and Harrod to decry the discounting of future utilities.

In principle we could define and measure the consumption discount rate along a path which is not a utilitarian optimum: we could consider the impact on a discounted utility sum of marginal variations about any reference time path of consumption. But the result would still depend on the utility discount rate chosen, and would be fundamentally arbitrary.

The central point is that we cannot avoid the need to choose a utility discount rate by focussing instead on consumption discount rates. Nor can we justify a positive utility discount rate by invoking an argument that the consumption discount rate may be quite different. The consumption discount rate is driven by the utility discount rate, the form of the utility function and the technology of the economy. Which of these two rate should be used for cost-benefit analysis? The answer should be clear from the analysis so far. The utility discount rate is a general equilibrium concept, used in models of the evolution of an entire economy over time. So if we have a planning model of the economy as a whole and are using this to assess how much to devote to preventing climate change, the general equilibrium concept is appropriate: we should use the utility discount rate. If however we are evaluating a small project that will have no economy-wide implications - say the conservation of a regional forest or fish stock - then this is a partial equilibrium exercise and the consumption discount rate is appropriate. I return to this issues in the concluding section of this chapter.

8 Empirical Evidence

There is interesting evidence that in making choices over time people use a framework different in certain salient respects from the standard discounted utilitarian approach. Of course, even if we have a clear picture of how individuals form their judgements about the relative weights of present and future, this does not necessarily have normative implications: we might still feel that relative to some appropriate set of ethical standards they give too little (or too much) weight to the future, and so are an imperfect guide to social policy. However, in a democratic society, individual attitudes towards the present-future trade-off presumably have some informative value about the appropriate social trade-off and have at least an element of normative significance.

There is a growing body of empirical evidence (see for example Lowenstein and Thaler [39], Lowenstein and Prelec [36], the papers in the volume by Lowenstein and Elster [35], Thaler [50] and Cropper et al. [12]) which suggests that the discount rate which people apply to future projects depends upon, and declines with, the futurity of the project. Over relatively short periods up to perhaps five years, they use discount rates which are higher even than many commercial rates - in the region of 15% or in some cases very much more. For projects extending about ten years, the implied discount rates are closer to standard rates - perhaps 10%. As the horizon extends the implied discount rates drops, to in the region of 5% for thirty to fifty years and down to of the order of 2% for one hundred years. It also appears from the empirical evidence that the discount rate used by individuals, and the way in which it changes over time, depends on the magnitude of the change in income involved.

8.1 Logarithmic Discounting and the Weber-Fechner Law

This empirically-identified behavior has been termed "hyperbolic discounting" (see Lowenstein and Prelec [36] or Ainslie and Haslam [1]) and is consistent with a very general set of results from natural sciences which find that human responses to a change in a stimulus are non-linear, and are inversely proportional to the existing level of the stimulus (as mentioned in [1]).

As an example, the human response to a change in the intensity of a sound is inversely proportional to the initial sound level: the louder the sound initially, the less we respond to a given increase. The same is true of responses to an increase in light intensity. These are illustrations of the Weber-Fechner law, which is formalized by the statement that human response to a change in a stimulus is inversely proportional to the pre-existing stimulus. In symbols,

$$\frac{dr}{ds} = \frac{K}{s} \text{ or, integrating, } r = K \log s$$

where r is a response, s a stimulus and K a constant.

The empirical results on discounting cited above suggest that something similar is happening in human responses to changes in the futurity of an event: a given change in futurity (e.g., postponement by one year) leads to a smaller response in terms of the decrease in weighting, the further the event already is in the future. This is quite natural: postponement by one year from next year to the year after, is clearly quite a different phenomenon from postponement from fifty to fifty one years hence. The former obviously represents a major change: the latter, a small one. If we accept that the human reaction to postponement of a payoff or cost by a given period of time is indeed inversely proportional to its initial distance in the future, then this suggests that the Weber-Fechner law can be applied to responses to distance in time, as well as to sound and light intensity. The result of this is that the discount rate is inversely proportional to distance into the future. Another way of saying this, is that we react to proportional rather than absolute increases in the time distance. Denote the discount factor at time t by $\Delta(t)$, so that this represents the weight placed on benefits at date t relative to a weight of unity at time zero. In this case the discount rate $q(t)$, which is minus the rate of change of this weight over time[9], is $q(t) = -\dot{\Delta}(t)/\Delta(t)$. We can formalize the idea that a given increase in the number of years into the future has an impact on the weight given to the event which is inversely proportional to the initial distance in the future as

$$q(t) = -\frac{1}{\Delta}\frac{d\Delta}{dt} = -\frac{K}{t} \text{ or } \Delta(t) = e^{-K\log t} = t^{-K}$$

for K a positive constant. Such a formulation has several attractive properties: the discount rate q goes to zero in the limit,[10] the discount factor $\Delta(t)$ also goes to zero and the integral $\int_1^\infty \Delta(t)\,dt = \int_1^\infty e^{-K\log t}dt = \int_1^\infty t^{-K}\,dt$ is finite for K positive and greater than unity.[11]

A discount factor $\Delta(t) = e^{-K\log t}$ has an interesting interpretation: the replacement of t by $\log t$ implies that we are measuring time differently: we are measuring it by equal proportional increments rather than by equal absolute increments. We react in the same way to a given percentage increase in the number of years hence of an event, rather than to a given absolute increase in its number of years hence. We shall call this "logarithmic discounting": this is quite consistent with the approach taken in for example acoustics, where in response to the Weber-Fechner law sound intensity is measured in decibels which respond to the logarithm of the energy content of the sound waves, and not to energy content itself. In general, non-constant discount rates can be interpreted as a non-linear transformation of the time axis.

[9] The inclusion of a minus sign is required becuse it is conventional to report the discount rate as a positive number, whereas the rate of change of the discount factor Δ is normally negative.

[10] This is called "slow discounting" by Harvey, [22].

[11] The lower limit of integration in this example is one not zero as t^{-K} is ill-defined for $t = 0$ and $K > 0$.

8.2 Constancy of Discount Rates in Normative Models

To attain constancy of the discount rate as a conclusion in an axiomatic approach, one has to invoke the set of axioms due to Koopmans [30] and introduced above, or something equivalent such as Harsanyi's formulation. Recall that Koopmans' axioms require preferences to satisfy *independence* and *stationarity*: it is these two conditions which lead to a discount factor implying a constant discount rate. The conditions implying a constant discount rate are quite distinct from those used in Chichilnisky's axiomatization: by imposing Koopmans' axioms in addition to Chichilnisky's, one would have a ranking of the form

$$\alpha \int_0^\infty u\left(c_t, s_t\right) e^{-\delta t} dt + (1-\alpha) \lim_{t \to \infty} u\left(c_t, s_t\right), 0 < \alpha < 1$$

with an exponential discount factor and constant discount rate. Chichilnisky's formulation is compatible with constant discount rates, logarithmic discounting or general non-constant discounting, as is the general formulation of utilitarianism.

9 Final Comments on Utilitarianism et al.

A criterion that some writers have taken as defining, or at least as a key element of, sustainability, is that welfare levels be non-decreasing over time. While it has some intuitive appeal, this criterion does not seem satisfactory. To see this point, note that in the case of pure depletion *a la* Hotelling there is only one path which satisfies this condition: this is the path with zero consumption for ever. On any other feasible path consumption and so utility must fall. By contrast, in the case of a renewable resource, this condition is satisfied by any utilitarian path with a positive discount rate provided that the initial resource stock is less than the utilitarian stationary stock, and is satisfied by no utilitarian path if the initial stock exceeds this value. Yet in the renewable case, utilitarian optima are always in some loose sense sustainable, in that they involve maintaining a positive stock of the resource for ever, indeed they involve maintaining a stock in excess of that which gives the maximum sustainable yield.

What this observation illustrates is part of a more general proposition, which is the following: it is difficult to judge the appropriateness of an optimality criterion defining intertemporal welfare without some awareness of its implications in the context of specific models of the economy. Of course, there are certain minimal requirements that any criterion must satisfy—logical consistency, completeness, representability—and certain candidates can be eliminated by reference to these requirements. Once we are over these basic hurdles, we have to investigate how the criteria perform and whether they lead to choices which are intuitively in accordance with our beliefs. In this process, we may be forced to clarify or even revise the way we formalize our beliefs, or even the beliefs themselves: there can be a two-way iteration between formal optimality criteria and the value judgements to which we subscribe. In keeping with the implications of this observation I now switch from the study of optimizing criteria in the abstract to the study of their implications in a specific growth model. Time and space permit the use of only one such model: a wider range is available in Heal [27].

10 Implications for the Choice of Growth Paths

The next step is to consider the implications of alternative ways of specifying the intertemporal objective for the growth path selected as optimal. Obviously this depends on the precise model used, and here shortage of space requires that I focus on one specific model, which will be an optimal growth model with a renewable resource as the economy's main source of utility. This resource is valued both as a source of consumption and also as a stock. A good interpretation would be forests, which contribute to welfare both as sources of timber, which involves consuming the forest, and also as stocks of forest, in which capacity they sequester carbon, support biodiversity, provide recreational opportunities, act as watersheds, and provide many other ecosystem services to human societies. (See Heal [27] for extensions and Daily [14] and Heal [28] for more on ecosystem services).

The maximand is the discounted integral of utilities from consumption and from the existence of a stock, $\int_0^\infty u(c,s)\, e^{-\delta t} dt$, where $\delta > 0$ is a discount rate. As the resource is renewable, its dynamics are described by

$$\dot{s}_t = r(s_t) - c_t$$

Here r is the growth rate of the resource, assumed to depend only on its current stock. More complex models are of course possible, in which several such systems interact: a well-known example is the predator-prey system. In general, r is a concave function which attains a maximum at a finite value of s, and declines thereafter. This formulation has a long and classical history, which is reviewed in Dasgupta and Heal [16]. In the field of population biology, $r(s_t)$ is often taken to be quadratic, in which case an unexploited population (i.e., $c_t = 0 \forall t$) grows logistically. Here we assume that $r(0) = 0$, that there exists a positive stock level $\overline{s}$ at which $r(\overline{s}) = 0 \forall s \geq \overline{s}$, and that $r(s)$ is strictly concave and twice continuously differentiable for $s \in (0, \overline{s})$. The overall problem can now be specified as

$$\max \int_0^\infty u(c,s)\, e^{-\delta t} dt \text{ s.t. } \dot{s}_t = r(s_t) - c_t,\ s_0 \text{ given.} \tag{4}$$

The Hamiltonian in this case is

$$H = u(c_t, s_t)\, e^{-\delta t} + \lambda_t e^{-\delta t}\left[r(s_t) - c_t\right]$$

Maximization with respect to consumption gives as usual the equality of the marginal utility of consumption to the shadow price for positive consumption levels:

$$u_c(c_t, s_t) = \lambda_t$$

and the rate of change of the shadow price is determined by[12]

$$\frac{d}{dt}\left(\lambda_t e^{-\delta t}\right) = -\left[u_s(c_t, s_t)\, e^{-\delta t} + \lambda_t e^{-\delta t} r'(s_t)\right]$$

To simplify matters we shall take the utility function to be separable in c and s: $u(c,s) = u_1(c) + u_2(s)$, each taken to be strictly concave and twice differentiable. In this case a solution to the problem (4) is characterized by

$$\left.\begin{aligned} u_1'(c_t) &= \lambda_t \\ \dot{s}_t &= r(s_t) - c_t \\ \dot{\lambda}_t - \delta\lambda &= -u_2'(s_t) - \lambda_t r'(s_t) \end{aligned}\right\} \tag{5}$$

In studying these equations, we first analyze their stationary solution, and then examine the dynamics of this system away from the stationary solution.

[12] I denote the first derivative of a function of a single variable by a prime, the second by two primes, etc.

10.1 Stationary Solutions

At a stationary solution, by definition s is constant so that $r(s_t) = c_t$: in addition the shadow price is constant so that

$$\delta u_1'(c_t) = u_2'(s_t) + u_1'(c_t)\, r'(s_t)$$

Hence:

Proposition 3 *A stationary solution to the utilitarian optimal use pattern (5) satisfies*

$$\left.\begin{array}{r} r(s_t) = c_t \\ \frac{u_2'(s_t)}{u_1'(c_t)} = \delta - r'(s_t) \end{array}\right\} \tag{6}$$

The first equation in (6) just tells us that a stationary solution must lie on the curve on which consumption of the resource equals its renewal rate: this is obviously a prerequisite for a stationary stock. The second gives us a relationship between the slope of an indifference curve in the $c - s$ plane and the slope of the renewal function at a stationary solution: the indifference curve cuts the renewal function from above. Such a configuration is shown in figure 5. This is just the result that the slope of an indifference curve should equal the discount rate if $r'(s) = 0 \forall s$, i.e., if the resource is non-renewable (see Heal [27]).

There is a straightforward intuitive interpretation to the second equation in (6). Consider reducing consumption by an amount Δc and increasing the stock by the same amount. The welfare loss is $\Delta c u_1'$: there is a gain from increasing the stock of $\Delta c u_2'$, which continues for ever, so that we have to compute its present value. But we also have to recognize that the increment to the stock will grow at the rate r': hence the gain from the increase in stock is the present value of an increment which compounds at rate r'. Hence the total gain is

$$\Delta c \int_0^\infty u_2' e^{r't} e^{-\delta t} dt = u_2' \Delta c / (r' - \delta)$$

When gains and losses just balance out, we have

$$u_1' + u_2' / (r' - \delta) = 0$$

which is just the second equation of (6). So (6) is a very natural and intuitive characterization of optimality.

10.2 Dynamic Behavior

What are the dynamics of this system outside of a stationary solution? These are also shown in figure 5. They are derived by noting the following facts:

1. beneath the curve $r(s) = c$, s is rising as consumption is less than the growth of the resource.

2. above the curve $r(s) = c$, s is falling as consumption is greater than the growth of the resource.

3. on the curve $r(s) = c$, s is constant.

4. from (5), the rate of change of c is given by

$$u_1''(c)\overset{\bullet}{c} = u_1'(c)\left[\delta - r'(s)\right] - u_2'(s)$$

   The first term here is negative for small s and vice versa: the second is negative and large for small s and negative and small for large s. Hence c is rising for small s and vice versa: its rate of change is zero precisely when the rate of change of the shadow price is zero, which is on a line of positive slope containing the stationary solution.

5. by linearizing the system

$$\left.\begin{array}{r} u_1''(c)\overset{\bullet}{c} = u_1'(c)\left[\delta - r'(s)\right] - u_2'(s) \\ \overset{\bullet}{s}_t = r(s_t) - c_t \end{array}\right\}$$

around the stationary solution, one can show that this solution is a saddle point. The matrix of the linearized system is

$$\begin{bmatrix} r'(s) & -1 \\ -r''\frac{u_1'}{u_1''} & \delta - r'(s) \end{bmatrix}$$

and the eigenvalues of this are

$$\frac{\delta}{2} \pm \frac{1}{2u_1''}\sqrt{\left((u_1'')^2\delta^2 - 4(u_1'')^2 r'\delta + 4(u_1'')^2(r')^2 + 4u_1''r''u_1'\right)}$$

This shows that the stationary solution (6) is a saddlepoint locally for small values of the discount rate δ, and for large values of r', r'' or u_1'.

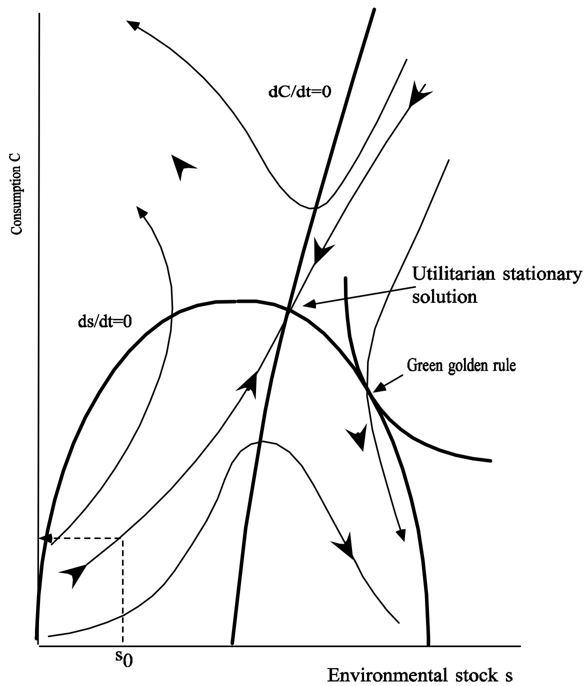

Figure 5: dynamics of the utilitarian solution.

Hence the dynamics of paths satisfying the necessary conditions for optimality are as shown in figure 5, and we can establish the following result (for a proof see Heal [27]):

Proposition 4 *For small values of the discount rate δ or large values of the derivatives r', r'' or u_1', all optimal paths for the utilitarian problem (4) tend to the stationary solution (6). They do so along a path satisfying the first order conditions (5), and follow one of the two branches of the stable path in figure 1 leading to the stationary solution. Given any initial value of the stock s_0, there is a corresponding value of c_0 which will place the system on one of the stable branches leading to the stationary solution. The position of the stationary solution depends on the discount rate, and moves to higher values of the stationary stock as this decreases. As $\delta \to 0$, the stationary solution tends to a point satisfying $u_2'/u_1' = r'$, which means in geometric terms that an indifference curve of $u(c, s)$ is tangent to the curve $c = r(s)$ given by the graph of the renewal function.*

Note that if the initial resource stock is low, *the optimal policy requires that consumption, stock and utility all rise monotonically over time.* The point is that because the resource is renewable, both stocks and flows can be built up over time provided that consumption is less than the rate of regeneration, i.e., the system is inside the curve given by the graph of the renewal function $r(s)$. In practice, unfortunately, many renewable resources are being consumed at a rate greatly in excess of their rates of regeneration: in terms of figure 5, the current consumption rate c_t is much greater than $r(s_t)$. So taking advantage of the regeneration possibilities of these resources would in many cases require sharp limitation of current consumption. Fisheries are a widely-publicized example: another is tropical hardwoods and tropical forests in general. Soil is a more subtle example: there are processes which renew soil, so that even if it suffers a certain amount of erosion or of depletion of its valuable components, it can be replaced. But typically human use of soils is depleting them at rates far in excess of their replenishment rates. Note that as the discount rate approaches zero the utilitarian stationary solution approaches the green golden rule. And for the overtaking criterion with a zero discount rate the stationary solution is the green golden rule. As we shall see in the next sections, the green golden rule is an interesting and important configuration of the economy.

10.3 Renewable Resources and the Green Golden Rule

We have used the simple model of equation (4) to analyze the implications of a conventional utilitarian approach. Next we use the same model with a range of other optimization criteria. In this section we focus on long-run payoffs and ask: what configuration of the economy gives the maximum sustainable utility level? There is a simple answer.

First, note that a sustainable utility level must be associated with a sustainable configuration of the economy, i.e., with sustainable values of consumption and of the stock. But these are precisely the values that satisfy the equation

$$c_t = r(s_t)$$

for these are the values which are feasible and at which the stock and the consumption levels are constant. Hence in figure 5, we are looking for values which lie on the curve $c_t = r(s_t)$. Of these values, we need the one which lies on the highest indifference curve of the utility function $u(c, s)$: this point of tangency is shown in figure 5. At this point, the slope of an indifference curve equals that of the renewal function, so that the marginal rate of substitution between stock and flow equals the marginal rate of transformation along the curve $r(s)$. Hence:

Proposition 5 *The maximum sustainable utility level (the Green Golden Rule) satisfies*

$$\frac{u_2'(s_t)}{u_1'(c_t)} = -r'(s_t)$$

Recall from (6) that as the discount rate goes to zero, the stationary solution to the utilitarian case tends to such a point. Note also that any path which approaches the tangency of an indifference curve with the reproduction function, is optimal according to the criterion of maximizing sustainable or long-run utility. In other words, this criterion of optimality only determines the limiting behavior of the economy: it does not determine how the limit is approached. This clearly is a weakness: of the many paths which approach the green golden rule, some will accumulate far more utility than others. One would like to know which of these is the best, or indeed whether there is such a best.

10.4 The Rawlsian Solution

Next we review the Rawlsian solution to our model. Consider the initial stock level s_1 in figure 5: the utilitarian optimum from this is to follow the path that leads to the saddle point. In this case, as noted, consumption, stock and utility are all increasing. So the generation which is least well off, is the first generation. What is the Rawlsian solution in the present model, with initial stock s_1? It is easy to verify that this involves setting $c = r(s_1)$ for ever: this gives a constant utility level, and gives the highest utility level for the first generation compatible with subsequent levels being no lower. This remains true for any initial stock no greater than that associated with the Green Golden Rule: for larger initial stocks, the Green Golden Rule is a Rawlsian optimum. Formally,

Proposition 6 *For an initial resource stock s_1 less than or equal to that associated with the Green Golden Rule, the Rawlsian optimum involves setting $c = r(s_1)$ for ever. For s_1 greater than the Green Golden Rule stock, the Green Golden Rule is a Rawlsian optimum.*

10.5 Chichilnisky's Criterion

Our final essay into alternative choice criteria takes us to Chichilnisky's criterion, reviewed in section 5 above. We ask how the Chichilnisky criterion alters matters when applied to an analysis of the optimal management of renewable resources. The problem now is to pick paths of consumption and resource accumulation over time to:

$$\left.\begin{array}{c} \max \alpha \int_0^\infty u\left(c_t, s_t\right) \Delta\left(t\right) dt + \left(1-\alpha\right) \lim\limits_{t \to \infty} u\left(c_t, s_t\right) \\ \text{s.t. } \dot{s}_t = r\left(s_t\right) - c_t,\ s_0 \text{ given.} \end{array}\right\} \tag{7}$$

where $\Delta\left(t\right)$ is a finite countably additive measure.

The change in optimal policy resulting from the change in the criterion of optimality is quite dramatic. With the Chichilnisky criterion and the measure $\Delta\left(t\right)$ given by an exponential discount factor, i.e., $\Delta\left(t\right) = e^{-\delta t}$, *there is no solution to the overall optimization problem.* There is a solution only if $\Delta\left(t\right)$ takes a different, non-exponential form, implying a non-constant discount rate which tends asymptotically to zero. Chichilnisky's criterion thus links in an unexpected way with recent discussions of individual attitudes towards the future (see section 8 above). Formally:

Proposition 7 *If $\Delta\left(t\right) = e^{-\delta t}$ then the problem (7) has no solution, i.e., there is no optimal pattern of use of a renewable resource using the Chichilnisky criterion with a constant discount rate.*

Proof. See Heal [27].

Intuitively, the non-existence problem arises here because it is always possible to postpone further into the future moving to the green golden rule, with no cost in terms of limiting utility values but with a gain in terms of the integral of utilities. This is possible because of the renewability of the resource. There is no equivalent phenomenon for an exhaustible resource.

10.5.1 Declining Discount Rates

With the Chichilnisky criterion formulated as

$$\alpha \int_0^\infty u\left(c_t, s_t\right) e^{-\delta t} dt + \left(1-\alpha\right) \lim_{t \to \infty} u\left(c_t, s_t\right)$$

there is no solution to the problem optimal management of a renewable resource. In fact as noted the discount factor does not have to be an exponential function of time.

We shall therefore consider a modified objective function

$$\alpha \int_0^\infty u(c_t, s_t)\,\Delta(t)\,dt + (1-\alpha)\lim_{t\to\infty} u(c_t, s_t)$$

where $\Delta(t)$ is the discount factor at time t, $\int_0^\infty \Delta(t)\,dt$ is finite, the discount rate $q(t)$ at time t is the proportional rate of change of the discount factor:

$$q(t) = \frac{\dot{\Delta}(t)}{\Delta(t)}$$

and we assume that the discount rate goes to zero with t in the limit:

$$\lim_{t\to\infty} q(t) = 0 \tag{8}$$

So the overall problem is now

$$\max \alpha \int_0^\infty u(c_t, s_t)\,\Delta(t)\,dt + (1-\alpha)\lim_{t\to\infty} u(c_t, s_t)$$

$$\text{s.t. } \dot{s}_t = r(s_t) - c_t,\ s_0 \text{ given.}$$

where the discount factor $\Delta(t)$ satisfies the condition (8) that the discount rate goes to zero in the limit. For this problem, there is a solution:[13] in fact, it is the solution to the utilitarian problem of maximizing just the first term in the above maximand, $\int_0^\infty u(c_t, s_t)\,\Delta(t)\,dt$. As before we take the utility function to be separable in its arguments: $u(c,s) = u_1(c) + u_2(s)$. Formally,

Proposition 8 *Consider the problem*

$$\max \alpha \int_0^\infty \{u_1(c) + u_2(s)\}\Delta(t)\,dt + (1-\alpha)\lim_{t\to\infty}\{u_1(c) + u_2(s)\}, 0 < \alpha < 1,$$

$$\textit{s.t. } \dot{s}_t = r(s_t) - c_t,\ s_0 \textit{ given.}$$

where $q(t) = \frac{\dot{\Delta}(t)}{\Delta(t)}$ *and* $\lim_{t\to\infty} q(t) = 0$*. For small values of the discount rate* δ *or large values of the derivatives* r', r'' *or* u_1'*, a solution to this problem is identical to the solution of "*$\max \int_0^\infty \{u_1(c) + u_2(s)\}\Delta(t)\,dt$ *subject to the same constraint". In words, the conditions characterizing a solution to the utilitarian problem with the variable discount rate which goes to zero also characterize a solution to the overall problem.*

[13] I am grateful to Harl Ryder for suggesting this result and outlining the intuition behind it.

Proof. See Heal [27].

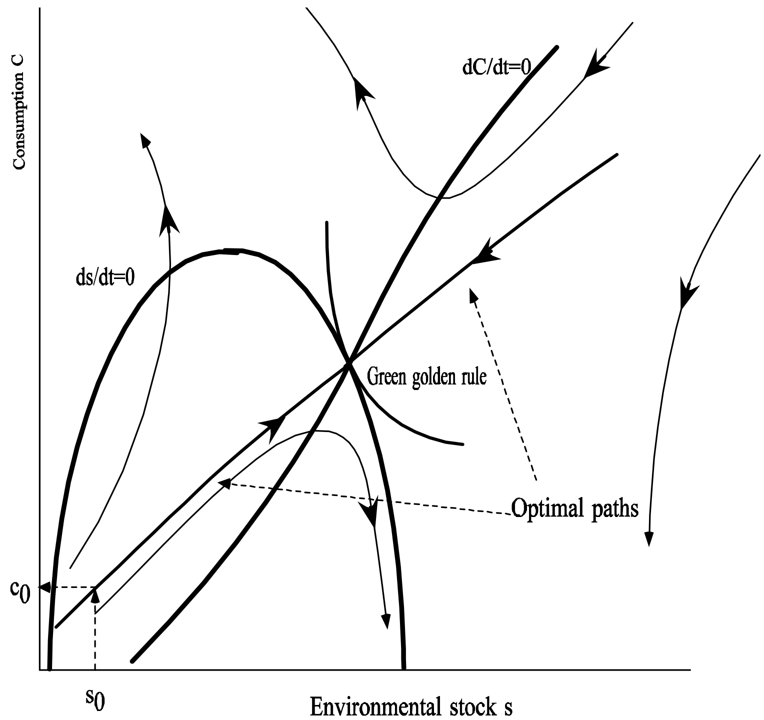

Figure 6: asymptotic dynamics of the utilitarian solution for the case in which the discount rate falls to zero.

Figure 6 shows the behavior of an optimal path in this case. Intuitively, one can see what drives this result. The non-existence of an optimal path with a constant discount rate arose from a conflict between the long-run behavior of the path that maximizes the integral of discounted utilities, and that of the path that maximizes the long-run utility level. When the discount rate goes to zero in the limit, that conflict is resolved. In fact, one can show that it is resolved only in this case.

10.5.2 Examples

To complete this discussion, we review some examples of discount factors which satisfy the condition that the limiting discount rate goes to zero. The most obvious is

$$\Delta(t) = e^{-\delta(t)t}, \text{ with } \lim_{t\to\infty} \delta(t) = 0$$

Another example[14] is

$$\Delta(t) = t^{-\alpha}, \alpha > 1$$

[14] Due to Harl Ryder.

Taking the starting date to be $t = 1$[15], we have

$$\int_1^{\infty} t^{-\alpha} dt = \frac{1}{\alpha - 1}$$

and

$$\frac{\dot{\Delta}}{\Delta} = \frac{-\alpha}{t} \to 0 \text{ as } t \to \infty$$

10.6 Time Consistency

An issue which is raised by the previous propositions is that of *time consistency*. Consider a solution to an intertemporal optimization problem which is computed today and is to be carried out over some future period of time starting today. Suppose that the agent formulating it—an individual or a society—may at a future date recompute an optimal plan, using the same objective and the same constraints as initially but with initial conditions and starting date corresponding to those obtaining when the recomputation is done. Then we say that the initial solution is *time consistent* if this leads the agent to continue with the implementation of the initial solution. Another way of saying this is that a plan is time consistent if the passage of time alone gives no reason to change it. The important point is that the solution to the problem of optimal management of the renewable resource with a time-varying discount rate, stated in proposition 8, is not time-consistent. A formal definition of time consistency is:[16]

Definition 2 *Let* $(c_t^*, s_t^*)_{t=0,\infty}$ *be the solution to the problem*

$$\max \alpha \int_0^{\infty} \{u_1(c) + u_2(s)\} \Delta(t)\, dt + (1-\alpha) \lim_{t\to\infty} \{u_1(c) + u_2(s)\}, 0 < \alpha < 1,$$

$$s.t. \dot{s}_t = r(s_t) - c_t, s_0 \text{ given.}$$

Let $(\widehat{c}_t, \widehat{s}_t)_{t=T,\infty}$ *be the solution to the problem of optimizing from* T *on, given that the path* $(c_t^*, s_t^*)_{t=0,\infty}$ *has been followed up to date* T*., i.e.,* $(\widehat{c}_t, \widehat{s}_t)_{t=T,\infty}$ *solves*

$$\max \alpha \int_T^{\infty} \{u_1(c) + u_2(s)\} \Delta(t - T)\, dt + (1-\alpha) \lim_{t\to\infty} \{u_1(c) + u_2(s)\}, 0 < \alpha < 1,$$

[15] This discount factor is infinite when $t = 0$: hence the need to start from $t = 1$.

[16] Further discussions of time consistency can be found in Heal [24].

$$s.t.\ \overset{\bullet}{s}_t = r(s_t) - c_t,\ s_T^* \text{ given.}$$

Then the original problem solved at $t = 0$ *is time consistent if and only if* $(\widehat{c}_t, \widehat{s}_t)_{t=T,\infty} = (c_t^*, s_t^*)_{t=T,\infty}$*, i.e., if the original solution restricted to the period* $[T, \infty]$ *is also a solution to the problem with initial time* T *and initial stock* s_T^**, for any* T*.*

It is shown in Heal [24] that the solutions to dynamic optimization problems are in general time consistent only if the discount factor is exponential. The following result is an illustration of this fact.

Proposition 9 *The solution to the problem of optimal management of a renewable resource described in proposition 8 above is not time consistent.*

Proof. See [27].

These are interesting and surprising results: to ensure the existence of an optimal path which balances present and future "correctly" according to Chichilnisky's axioms, we have to accept paths which are not time consistent. Of course, the empirical evidence cited above implies that individual behavior must also be inconsistent, so society in this case is only replicating what individuals apparently do. Traditionally, welfare economists have always regarded time consistency as a very desirable property of intertemporal choice. More recently, this presumption has been questioned: philosophers and psychologists have noted that the same person at different stages of her or his life can reasonably be thought of as different people with different perspectives on life and different experiences.[17] The implications of working with inconsistent choices clearly need further research.

Understanding time consistency requires that we think hard about the discount factor $\Delta(t)$, which shows how the weight on utilities at different dates changes over time. In particular, we have to answer the following question: if we are presently at date $t = 0$, then does the weight we place on date t, $\Delta(t)$, depend on the date t itself, or on some measure of the difference between t and the present? Using time zero as the present it is easy to confuse these approaches. But consider replanning at a date $t = T$, as time consistency requires: then what is the discount factor to be applied then to date $T_1 > T$? Is this discount factor a function of T_1, or of a measure of the difference or distance between T_1 and T? Presumably it is the latter: we are

[17]For a further discussion, see Harvey [22] and references therein.

discounting for distance in time, not for some concept of absolute time. Given this fact, what is the appropriate measure of the difference between T_1 and T?

The conventional measure is $(T_1 - T)$, but this is clearly not the only possibility: T_1/T seems to have a claim to validity, as well. These two numbers measure respectively the absolute and proportional differences between T_1 and T. Depending on which of these is selected, we have different insights into time consistency.

Proposition 10 *(1) When the discount factor applied to a future date is a function of the difference between that date and the present, then the solutions to an optimization problem are time consistent if and only if the discount factor is of the form* $\Delta(t) = e^{-\delta t}$ *and in particular the discount factor applied to date* T_1 *at time* T *is* $e^{-\delta(T_1 - T)}$*. (2) When the discount factor applied to a future date is a function of the ratio of that date to the present, then the solutions are time consistent if and only if the discount factor is of the form* $\Delta(t) = e^{-\delta \log t}$ *and in particular the discount factor applied to date* T_1 *at time* T *is* $e^{-\delta \log T_1/T}$

Proof. The proof of point (1) is standard and can be found for example in Heal [24]. The proof of point (2) is rather simple. The key point is that for time consistency the ratio of the weights applied to two dates say T_1 and T_2 should not depend the date T at which the weights are calculated. But $e^{-\delta \log T_1/T}/e^{-\delta \log T_2/T} = e^{\delta \log T_2/T_1}$ which is independent of the date T. No other functional form for the dependence on time T will give this property. ■

So: if we discount with absolute distance into the future, we need conventional exponential discounting to be sure of time consistent solutions: if we discount with relative distance—as empirical evidence seems to suggest—then we need logarithmic discounting. If these conditions are not met, we have time inconsistent solutions.[18]

11 Conclusions on Models and Objectives

In the model we have studied, the green golden rule provides a configuration of the economy that is appealing in the context of long-term development. It gives the high-

[18] The first formal analysis of time consistency was provided by Strotz [49]. Phelps and Pollak [?] provided an early and innovative analysis of how society might react to dynamic inconsistencies. Recent work in this area includes interesting papers by Laibson [32] and Cropper and Laibson [13]. In spite of advances in our understanding of the issues relating to time inconsistency, we still lack a convincing normative model of behavior in the face of inconsistent preferences.

est level of utility that can be sustained for ever, and a path that converges to this will ultimately overtake any other that does not. The discounted utilitarian optimum stops short of this, providing a lower asymptotic utility level and a lower long-run resource stock. As the discount rate goes towards zero, the utilitarian optimum converges to the green golden rule. Paths that are optimal according to Chichilnisky's criterion will also asymptote to the green golden rule, as for such paths to exist the instantaneous discount rate must fall to zero. The green golden rule is therefore a focal point of long-run behavior of optimal paths for a wide range of optimality criteria. Of those mentioned above, only the Rawlsian has no connection with the green golden rule. This indicates that whatever the exact specification of the objective, a path that converges towards the green golden rule has much to recommend it. This result is more general than the specific model used here (see [27]). In the context of some uncertainty about the best specification of the economy's objective, this is encouraging. Uncertainty about the objective does not affect where we want to go in the long run, but affects only the route by which we arrive there. So in the absence of general agreement about an optimality criterion, investigating the nature of the green golden rule for a range of models might be a valuable approach.

12 Conclusions on the Choice of a Discount Rate

Economists are often asked, by international agency or government officials, or by members of environmental organizations, how in practice one should choose the discount rate to be used in project evaluation, or whether one should move away from discounting and use another approach. It should be clear from what has gone above that there are no simple answers to questions such as these. However, it is possible to give some general criteria that may be useful.

One can start with trying to clarify whether it is discounting utility or consumption that is at stake. Do we need a pure rate of time preference or a social discount rate? The former is appropriate when we are dealing with decisions that will affect the entire growth path of an economy or a region. Another way of saying this is that utility discounting is appropriate when we are working with a general equilibrium model and general equilibrium consequences will follow from the choices under consideration. By contrast, discounting consumption is appropriate when we are working in a partial equilibrium context and the underlying growth path and resource allocation of the economy can be taken as given. In such situation we are considering changes that will amount to marginal alterations of the initial situation. Some examples will help clarify this. Alterations in economic policy designed to reduce greenhouse gas emissions in an industrial country are probably in the first category: they could be sufficiently far-reaching to alter the general equilibrium of the economy. So could decision about the construction of a large dam in a small developing country. But with a purely local decision, such as the conservation of a local fishery or forest, it is clearly appropriate to view this in a partial equilibrium framework. We are considering marginal alterations about the economy's initial pattern of resource allocation.

As noted above, choosing utility and consumption discount rates involve different issues, and in general the former is a necessary but not sufficient condition for the latter: the consumption discount rate depends on but is not fully determined by the utility discount rate. So it is appropriate to start with a discussion of the choice of the utility discount rate. This of course presumes the choice of a utilitarian framework, or one like it - by which I mean a Ramseyesque, overtaking or Chichilnisky approach. All of these require the selection of either a single utility discount rate, which may be zero, or a schedule of time-varying discount rates as implied for example by the choice of a constant logarithmic discount rate. The only approach that lies outside this general utilitarian-style framework is the Rawlsian, which I believe ultimately has less than the others to recommend it, particularly in the intertemporal framework (see Dasgupta and Heal [16] and Heal [27]). Here it is important to distinguish between finite and infinite horizon models. In the latter, discounting may be logically and mathematically necessary: in the former it is not. So if the aim is to solve for an optimal path in a finite horizon general equilibrium model, a zero rate of pure time preference has to be considered as a possibility. This would probably still imply a positive social discount rate and return on capital for small projects within the economy.

Weitzman [52] recently conducted a survey of 1,720 professional economists, seeking their opinion on the appropriate choice of discount rate for long-term environmental problems such as global warming. The modal rate recommended by this group was 2%, the median 3% and the mean 4%. Unfortunately it was not clear from the survey whether the choice was to refer to a utility or a consumption discount rate, although it is perhaps reasonable to assume that respondents took the question to refer to utility discount rates as there was no information about growth rates and the other factors that would be necessary to select a consumption discount rate. These responses give us a clear picture that the majority of our profession are aware of some of the issues discussed above: in general the choice of a discount rate for projects whose life is a decade or less would be considerably above these rates. Clearly economists are selecting lower rates in these responses in recognition of the impact of "normal" rates over time horizons that are very long by conventional economic standards. This does not of course prove that any particular answer or approach is the correct one but it does provide a degree of reassurance that there is a general recognition of these problems.

Whatever one's ultimate aim, therefore, the first move has to be to choose a long-term utility discount rate. This could be in the context of a standard utilitarian approach where there is a single constant discount rate, or in the context of a non-exponential approach to discounting in which one picks an entire family or schedule of rates, on in the context of Chichilnisky's approach where one has in addition to a discount rate schedule to pick a weight to be placed on the long run or sustainable level of welfare. In this choice there are certain matters that are not relevant. We do not need to know risk premia or the riskiness of any projects that may be undertaken. Nor do we need to be aware of the nature of distortions in the economy and the impact of these on the relationship between market and shadow prices. Nor, finally, do we need to know the rate of return on capital investments or the cost of money. At an equilibrium or an optimum these are a function of the utility discount rate, and not the other way around. What is at issue is, quite simply, the relative weights to be placed on welfare levels occurring at different dates. Should these weights decline with futurity and if so according to what pattern? This judgement about intertemporal distribution is at the heart of the choice of a utility discount rate. This judgement has to be made first. Given such a judgement, we can work out the consumption discount rate from the formulae above.

Recall that the consumption discount rate or social rate of discount is

$$\delta + \eta_{cc}\frac{\dot{c}}{c} + \eta_{cs}\frac{\dot{s}}{s}$$

In the case of the renewable resource model of section 10, we can rewrite the first order conditions as

$$\delta + \eta_{cc}\frac{\dot{c}}{c} - \frac{u_2'}{u_1'} = r'$$

In the case when $\eta_{c,s} = 0$, this equates the social discount rate plus the marginal rate of substitution between stocks and flows to the rate of return on the natural capital stock. For a conventional Ramsey-type model without stock externalities this equation would be

$$\delta + \eta_{cc}\frac{\overset{\bullet}{c}}{c} = f'(k)$$

where f' is the marginal product of capital k. Note that here both the left and right hand sides —social discount rate and rate of return—are endogenous to the solution of the model and are driven by the utility discount rate and the preferences and technology. This reiterates a point made before, namely that in a general equilibrium framework we should not use the historical return on capital as a utility discount rate (in contrast with the positions of Weitzman [53] and Nordhaus [38]). In a partial equilibrium situation, however, and with no stock effects on welfare, we can take the social rate of discount to be the return on capital.

With a stock-dependent utility function, the social discount rate is $\delta + \eta_{cc}\frac{\overset{\bullet}{c}}{c} + \eta_{cs}\frac{\overset{\bullet}{s}}{s}$ and from the first order conditions (Heal, [27], chapter 10) on an optimal path $\delta + \eta_{cc}\frac{\overset{\bullet}{c}}{c} = f_k$. Hence in a model with both natural and produced capital the social rate of discount or utility discount rate is

$$f_k + \eta_{cs}\frac{\overset{\bullet}{s}}{s}$$

For a separable utility function, $\eta_{cs} = 0$, and we again have an equality between the utility discount rate and the return on capital, but not in general. In general we expect that $\eta_{cs} \neq 0$, as the utility of many goods will be affected by the state of the environment. If a better environment enhances the values of other goods—environmental stocks and other goods are complements—then $\eta_{c,s} = -su_{c,s}/u_c < 0$ so that the utility or social discount rate is greater or less than the return on capital according as the environmental stock is falling or rising. If environmental stocks and other goods are substitutes, these inequalities are reversed. In such cases, to compute the utility discount rate from the return on capital we need information about preferences, about complementarities or substitutabilities between environmental stocks and other goods, and about movements in environmental stocks.

In summary: we are working either with a general equilibrium or a partial equilibrium framework. In the former case we need a utility discount rate. This reflects an ethical judgement and is not obtained from economic data such as the return on capital, the risk premium, etc. If we are in a partial equilibrium situation, we use the consumption or social discount rate. We can use the return on capital as a starting point for calculating this. However, we have to modify this by the term $\eta_{cs}\frac{\dot{s}}{s}$ and the net result could be zero or negative if the environmental stock is changing. The result is also time-varying.

All of these comments on the consumption discount rate are of course premised on the assumption of a first best economy, i.e. an economy with no distortions and with a fully optimal allocation of resources. To understand this point, and begin to understand what would happen if we were to drop this assumption, recall that there is a standard duality between shadow prices, such as those arising from optimization problems like that in section 10, and competitive market prices. The shadow prices could also emerge as market clearing prices in a competitive economy with a complete set of markets. A complete set of markets in the present context means a complete set of futures markets, from the present to the infinite future. Given such a set of markets, a competitive economy would attain an intertemporal equilibrium at which prices were identical with the shadow prices of section 10 and would follow the path described by the conditions for optimality. An alternative to the assumption of a complete set of futures markets would be the assumption of fully rational expectations. This amounts to the same thing: the only difference is in the packaging. So under these admittedly rather strenuous assumptions we can think of f_k as the market return on capital and then talk about how to adjust this to calculate the social rate of discount to be used in a market economy.

If the assumption of a fully first best economy is not met, then it becomes much harder to characterize the factors that determine the consumption rate of discount. The assumption of a fully first best economy may fail in several ways. One, as noted, is the lack of futures markets, or, equivalently, imperfections in capital markets. There is an equivalence here because futures markets are used for moving consumption and income over time and that is also what capital markets do. So we can think of futures markets as devices for borrowing and lending. Another possible source of departure from the first best is the presence of wedges between borrowing and lending rates: this could be a consequence of taxes on income, or of capital rationing - another aspect of capital market imperfections. In addition, there are reasons for differences from first best that are more specifically related to the environmental nature of the problems under consideration. For example, the environmental stocks considered in the model of section 10 are often public goods - forests, stocks of biodiversity, climate regimes - and of course the investor in these will often have difficulty in appropriating the returns that this investment generates for society as a whole. And likewise there may be external effects driving a wedge between the private and social returns to investment.

Given differences between borrowing and lending rates, between private and social returns, and other deviations from the first-best framework implicitly assumed above, what is the right choice of a consumption discount rate? Unfortunately this is an extremely complex subject, and one with no easy generalizations. Ideally we would modify the model to reflect the precise departures from first-best that are relevant in a particular situation, and then use this revised model to derive conclusions about the consumption discount rate, using the same mathematical methods as above. Consider as a crude illustration the model of section 10, and suppose that capital market imperfections made it impossible for agents in this model to save or dis-save at more than a specific rate, say $\epsilon > 0$. Then this would impose a constraint that $|c - r(s)| \leq \epsilon$, so that in graphical terms the system would be forced to stay in an ϵ-neighborhood of the curve $c = r(s)$ in figure 5. This constraint would in turn have a shadow price that would interact with other shadow prices and affects the consumption rate of discount. In practice there are too many different possible departures from first best for it to be practical to model each particular case. Some general points however are obvious: we should try to correct returns for the differences between private and social costs, and should impute to investments in public goods the full social benefits resulting.

There is one less obvious general point that is robust. Above we noted that in a first best situation with no stock externalities the consumption discount rate would equal the return on investment. The reason is obvious: a small increase in consumption will lead to a small (equal) decrease in investment and so the returns to the two must be equal on the margin on an optimal path. Suppose that there is a wedge between these two returns: which is the more appropriate as a discount rate - if either? In the presence of taxes on income, the return on investment is typically greater than the consumption rate of discount: the former is before tax and the latter after tax. In considering the rate at which the valuation of consumption changes over time it is important to note that a change in the time pattern of consumption will alter the time path of investment and so of capital and output, which will have further implications for future consumption. A reduction in future consumption will make society worse off directly at the date at which it occurs, but may make it better off by increasing output and consumption at a later date. At a first best allocation of resources all of this is reflected in shadow or market prices. At a second best, it is not. The rate of change of the marginal utility of consumption, which is the consumption rate of discount and is also the rate of change of the shadow price of investment (because $u_c(c, s) = \lambda$ in the first order conditions of section 10), does not reflect changes in the level and valuation of output if there is a difference between the consumption discount rate and the return on investment.

Some commentators have suggested that the appropriate response is to use as a consumption discount rate a weighted average of the rate of change of the marginal utility of consumption and the return on capital (Haveman [23]). A better approach is to assess the impact of consumption changes on the level of investment and to value these changes in investment by the shadow price of capital. This shadow price of capital at date t reflects the full contribution that extra capital available at date t makes to consumption from t onwards (Arrow [2] [3], Arrow and Kurz [4], Bradford [6], Lind [34]). We then value the sequence of consumption and investment changes resulting from a change in policy in terms of consumption (the shadow price of capital makes this conversion for changes in capital) and discount them to the present at the consumption rate of discount. So we are using the consumption rate of discount as a discount rate, but the return on investment is of course being used in calculating the shadow prices of capital. Lind ([34]) provides an excellent intridductory surveyt of these isues, and Bradford's paper ([6]) is a readable and authoritative original source.

References

[1] Ainslie, George and Nick Haslam. "Hyperbolic Discounting" in George Lowenstein and Jon Elster eds., *Choice Over Time*, New York: Russell Sage Foundation, 1992.

[2] Arrow, Kenneth J. "Dicounting and Public Investment Criteria," in A.V. Kneese and S.C. Smith, eds, *Water Resources Research*, Baltimore, 1966.

[3] Arrow, Kenneth J. "The Rate of Discount on Public Investments with Imperfect Capital Markets," in Robert C. Lind, ed, *Discounting for Time and Risk inenergy Policy*, RFF/Johns Hopkins, 1982.

[4] Arrow, Kenneth J. and Mordecai Kurz. *Public Investment, the Rate of Return and Optimal Fiscal Policy*. RFF/Johns Hopkins 1970.

[5] Asheim, Geir B. "Ethical preferences in the presence of resource constraints." *Nordic Journal of Political Economy* (1996), 23 (1): 55-67.

[6] Bradford, David A. "Constraints on Government Investment Opportunities and the Choice of Discount Rate." *The Americn Economic Review*, Vol. 65 Issue 5, December 1975, 887-899.

[7] Beltratti, Andrea, Graciela Chichilnisky, and Geoffrey M. Heal. "Sustainable growth and the green golden rule." In I. Goldin and L.A. Winters, eds. *Approaches to Sustainable Economic Development*, pp. 147-172, Paris: Cambridge University Press for the OECD, 1993.

[8] Beltratti, Andrea, Graciela Chichilnisky, and Geoffrey M. Heal. "The environment and the long run: a comparison of different criteria." *Ricerche Economiche (*1994), 48, 319-340.

[9] Beltratti, Andrea, Graciela Chichilnisky, and Geoffrey M. Heal. "The green golden rule." *Economics Letters,* (1995), 49: 175-179.

[10] Chichilnisky, Graciela. "What is sustainable development?" Paper presented at Stanford Institute for Theoretical Economics, 1993. Published as "An axiomatic approach to sustainable development," *Social Choice and Welfare* (1996), 13 (2): 219-248.

[11] Chichilnisky, Graciela, Geoffrey Heal, and Alesandro Vercelli eds. *Sustainability: Dynamics and Uncertainty*. Amsterdam: Kluwer Academic Publishers for Fondazione ENI Enrico Mattei, 1998.

[12] Cropper, Maureen L., Sema K. Aydede, and Paul R. Portney. "Preferences for life-saving programs: how the public discounts time and age." *Journal of Risk and Uncertainty* (1994), 8: 243-265.

[13]

[14] Daily, Gretchen C. ed. *Nature's Services: Societal Dependence on Natural Ecosystems*. Washington DC: Island Press, 1977.

[15] Dasgupta, Partha S. and Geoffrey M. Heal. "The optimal depletion of exhaustible resources." *Review of Economic Studies*, symposium (1974), pp. 3-28.

[16] Dasgupta, Partha S. and Geoffrey M. Heal. *Economic Theory and Exhaustible Resources*, Cambridge, England: Cambridge University Press, 1979.

[17] Dasgupta, Partha S., Karl-Göran Mäler and Scott Barrett. "Intergenerational Equity, Social Discount Rates, and Global Warming", chapter 7 of Portney, Paul and John Weyant (eds) *Discounting and Intergenerational Equity*, Resources for the Future, Washington DC, 1999.

[18] Economist, The. London, England: March 23rd. 1991.

[19] Frerejohn, John and Talbot Page. "On the foundations of intertemporal choice." *Journal of Agricultural Economics* (May 1978), pp. 15-21.

[20] Harrod, Roy. *Towards a Dynamic Economics.* Macmillan Press, London, 1948.

[21] Harsanyi, John C. "Cardinal Welfare, Individualist Ethics and Interpersonal Comparisons of Utility." *The Journal of Political Economy*, Vol 63, Issue 4, Aug. 1955, 309-321.

[22] Harvey, Charles. "The reasonableness of non-constant discounting," *Journal of Public Economics (*1994), 53: 31-51.

[23] Haveman, Robert H. "The Opportunity Cost of Displaced Private Spending and the Social Discount Rate." *Water Resources Research*, vol 5, no 5, 1969.

[24] Heal, Geoffrey M. *The Theory of Economic Planning*, Amsterdam: North Holland Publishing Company, 1973.

[25] Heal, Geoffrey M. "Depletion and discounting: a classical issue in resource economics." In R. McElvey ed. *Environmental and Natural Resource Mathematics* (1985), 32: 33-43, Proceedings of Symposia in Applied Mathematics, American Mathematical Society, Providence, RI.

[26] Heal, Geoffrey M. "The optimal use of exhaustible resources." In Alan.V. Kneese and Sweeney James L. eds. *Handbook of Natural Resource and Energy Economics*, ch. 18, pp. 855-880. Amsterdam: North Holland, 1993.

[27] Heal, Geoffrey. *Valuing the Future: Economic Theory and Sustainability.* Columbia University Press, 1998.

[28] Heal, Geoffrey. *Nature and the Marketplace: Capturing the Value of Ecosystem Services.* Island Press, Washington DC, 2000.

[29] Hotelling, Harold. "The economics of exhaustible resources." *Journal of Political Economy* (1931), 39, 137-75.

[30] Koopmans, Tjalling. "Stationary ordinal utility and impatience." *Econometrica* (1960), 28: 287-309.

[31] Krautkraemer, Jeffrey A. "Optimal growth, resource amenities and the preservation of natural environments." *Review of Economic Studies* (1985) 52: 153-170.

[32] Laibson, David I. "Life-Cycle consumption and Hyperbolic Discount Functions," *European Economic Review*, Vol 42 nos 3-5 May 1998 861-71.

[33] Lauwers, Luc. "Infinite Chichilnisky rules". Discussion paper, Katolik Universitaet Leuven, Belgium, 1992.

[34] Lind, Robert C. "A Primer on the Major Issues Relating to the Discount Rate for Evaluating National Energy Options. In Robert C. Lind, ed, *Discounting for Time and Risk inenergy Policy*, RFF/Johns Hopkins, 1982.

[35] Lowenstein, George and Jon Elster eds. *Choice Over Time.* New York: Russell Sage Foundation, 1992.

[36] Lowenstein, George and Drazen Prelec. "Anomalies in intertemporal choice: evidence and an interpretation." In George Lowenstein and Jon Elster eds. *Choice Over Time.* New York: Russell Sage Foundation, 1992.

[37] Meade, James E. "The effect of savings on consumption in a state of steady growth." *Review of Economic Studies* (June 1962), vol. 29.

[38] Nordhaus, William. "Discounting and Public Policies that Affect the Distant Future", chapter 15 of Portney, Paul and John Weyant (eds) *Discounting and Intergenerational Equity*, Resources for the Future, Washington DC, 1999.

[39] Lowenstein, George and Richard Thaler. "Intertemporal choice." *Journal of Economic Perspectives* (1989), 3, 181-193.

[40] Phelps, Edmund S. "The golden rule of accumulation: a fable for growthmen." *American Economic Review* (1961), 638-643.

[41] Phelps, Edmund S. and Robert Pollak. "On Second Best National Svaings and Game Equilibrium Growth." Review of Economc Studies Vol XXV (2) No. 102 1968.

[42] Rawls, John. *A Theory of Justice*. Oxford, England: Clarendon Press, 1972.

[43] Ramsey, Frank. "A mathematical theory of saving." *Economic Journal* (1928), 38: 543-559.

[44] Robinson, Joan. "A neoclassical theorem." *Review of Economic Studies* (June 1962) vol. 29.

[45] Ryder, Harl Jr. and Geoffrey Heal "Optimal growth with intertemporally dependent preferences." *Review of Economic Studies* (1973), V. 40 (121), 1-32.

[46] Solow, Robert M. "A contribution to the theory of economic growth." *Quarterly Journal of Economics* (1956), 70 (1): 65-94.

[47] Solow Robert M. "Intergenerational equity and exhaustible resources." *Review of Economic Studies*, Symposium, (1974), pp. 29-45.

[48] Solow Robert M. *An Almost Practical Step Towards Sustainability*. Invited lecture, fortieth anniversary of Resources for the Future, Resources and Conservation Center, Washington, DC, 1992.

[49] Strotz Robert H. "Myopia and Inconsistency in Dynamic Utility Maximization." *Review of Economic Studies* Vol XXII (3) No 62 1955-6.

[50] Thaler, Richard. "Some empirical evidence on dynamic inconsistency." *Economics Letters* (1981), 8: 201-207.

[51] von Weizäcker, Carl Christian. "Lemmas for a theory of approximately optimal growth." *Review of Economic Studies* (1967), pp. 143-151.

[52] Weitzman, Martin. "Gamma Discounting." Paper presented at the 1998 Summer Environmental Meetings of the NBER. See also Weitzman, Martin. "Discounting the Far Distant Future," *Journal of environmental Economics and Management*, Vol 36 No. 3 November 1998 pp 201-208.

[53] Weitzman, Martin. "Just Keep Discounting, but ... " Chapter 3 of Portney, Paul and John Weyant (eds) *Discounting and Intergenerational Equity*, Resources for the Future, Washington DC, 1999.