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A Theory of Trickle-Down Growth and Development

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This paper develops a model of growth and income inequalities in the presence of imperfect capital markets, and it analyses the trickle-down effect of capital accumulation. Moral hazard with limited wealth constraints on the part of the borrowers is the source of both capital market imperfections and the emergence of persistent income inequalities. Three main conclusions are obtained from this model.

First, when the rate of capital accumulation is sufficiently high, the economy converges to a unique invariant wealth distribution. Second, even though the trickle-down mechanism can lead to a unique steady-state distribution under laissez-faire, there is room for government intervention: in particular, redistribution of wealth from rich lenders to poor and middle-class borrowers improves the production efficiency of the economy both because it brings about greater equality of opportunity and also because it accelerates the trickle-down process. Third, the process of capital accumulation initially has the effect of widening inequalities but in later stages it reduces them: in other words, this model can generate a Kuznets curve.

1. INTRODUCTION

It is widely believed that the accumulation of wealth by the rich is good for the poor since some of the increased wealth of the rich trickles down to the poor. This paper formalizes an important mechanism through which wealth may trickle down from the rich to the poor.

The mechanism we focus on is borrowing and lending in the capital market: as more capital is accumulated in the economy more funds may be available to the poor for investment purposes. This in turn enables them to grow richer.[1] In our model, persistent wealth inequalities arise because investment projects generate random returns and entrepreneurs do not insure themselves perfectly against this income risk.[2] While in the existing literature on endogenous income distribution the supply side of the credit market is not explicitly modelled and the interest rate is given exogenously, in our model the equilibrium

1. What we call a "loan contract" can also be reinterpreted as an "employment contract" and our capital market could be reinterpreted as a labour market. It is important to highlight this reinterpretation of our model since empirically trickle-down may be as (or even more) important in the labour market. In a stripped-down model like ours it is not surprising that we cannot give a satisfactory answer to the question of what distinguishes a credit relationship from an employment relationship, besides the difference in the timing of monetary transfers.

2. We thus follow a distinguished transition in modelling inequalities initiated by Champernowne (1953) and successively refined by Loury (1981) and Banerjee and Newman (1991), among others. See Cowell (1977), Aghion and Bolton (1992) for brief overviews of this literature.

interest rate schedule is determined endogenously by the interplay between the supply and demand for investment funds. Endogeneizing the interest rate schedule is a necessary modelling step in order to fully address the question of the effects of capital accumulation on the income distribution.

When the interest rate schedule is determined endogenously it is no longer possible to simply trace the wealth of a single individual in isolation from the rest of the economy, since the stochastic evolution of her wealth depends on the evolution of the state of the economy through the equilibrium interest rate schedule. As a result, the dynamics of an individual's wealth are now nonlinear. It turns out that there are no general mathematical methods for dealing with nonlinear Markov processes. However, we show that with sufficiently fast capital accumulation the equilibrium interest rate schedule converges to a fixed schedule. Once this schedule is attained standard linear methods again apply to the stochastic evolution of an individual's wealth. We are thus able to derive sufficient conditions for the convergence of our complicated Markov process to a unique invariant distribution. This proposition is by no means obvious or general. Indeed, in related work Banerjee and Newman (1993) and Piketty (1993) provide examples of nonlinear Markov processes which have multiple invariant distributions.[3]

The main economic insight which comes out of our analysis in this paper is that, even though wealth does trickle-down from the rich to the poor and leads to a unique steady-state distribution of wealth under sufficiently high rates of capital accumulation, there is still room for wealth redistribution policies to improve the long-run efficiency of the economy. In other words, the trickle down mechanism is not sufficient to eventually reach an efficient distribution of resources, even in the best possible scenario. The reason why redistribution improves production efficiency is that with redistribution the poor need to borrow less to invest and therefore their incentives to maximize profits are distorted less. Thus, redistribution improves the efficiency of the economy because it brings about greater equality of opportunity and because it accelerates the trickle-down process.

However, one-shot redistributions in our model only have temporary effects. In order to improve the efficiency of the economy permanently, permanent redistribution policies must be set up. This observation is a direct consequence of our convergence result to a unique invariant distribution. In contrast, one-shot redistribution policies may have permanent effects in the models considered in Banerjee and Newman (1993) and Piketty (1993).

It is worth pointing out that the need for redistribution arises from the existence of an incentive problem. In the absence of incentive considerations there is no need for redistribution in our model. Our justification should be contrasted with most other motivations for redistribution in the literature which emphasize insurance or fairness considerations that must be weighed against incentive efficiency considerations (see e.g. Mirrless (1971)).

The paper is organized as follows: Section 2 outlines the model; Section 3 characterizes the equilibrium in the capital market. Section 4 analyses the interaction between growth and redistribution. Section 5 establishes the convergence to a unique invariant

3. The set up considered by Piketty is most similar to ours, with the interest rate also being endogenously determined by the supply and demand for investment funds. However, the focus of Piketty's analysis is on situations where the rate of capital accumulation is not sufficiently high for the economy to converge to a unique wealth distribution. Rather, there exist multiple invariant distributions. When there are multiple steady-state distributions, a one-shot redistribution of wealth may push the economy from one steady-state to another and therefore have long-run effects. However, since little is known about the *dynamics* of wealth distribution in these models, it is not always clear *a priori* what the long-run effects of a one-shot redistribution might be.

distribution. Finally, Section 6 discusses redistribution and explains why the positive incentive effects of redistribution for the borrowers outweigh the negative incentive effects for the wealthy entrepreneurs, who may see some of their accumulated wealth taxed away.

2. THE MODEL

We consider a closed economy with a continuum of identical agents of total mass 1. Each agent lives for one period during which he or she works and invests. The resulting income is divided between consumption and bequests. Each agent has one offspring and generations succeed other each *ad infinitum*.

The only source of heterogeneity among individuals is assumed to be their initial wealth endowments. We denote by $G_t(w)$ the distribution of wealth endowments at the beginning of period t which results from the previous generation's bequests (where $w \geqq 0$). All the agents in the economy are endowed with one unit of labour which they supply (inelastically) at no disutility cost.

Our detailed description of this economy begins with the technological assumptions. At the beginning of each period t an agent has the following options:

(a) she can use her unit of labour to work on a "backyard" (or "routine") activity which requires no capital investment. The return to this activity is assumed to be deterministic and small, equal to $n > 0$.

(b) alternatively, she may choose to be self-employed and invest her unit of labour in a high yield "entrepreneurial" activity which requires a fixed initial capital outlay of $\hat{k} = 1$. The uncertain revenue from investment in this high yield project is given by[4]

$$F(k, l) = \left\{ \begin{matrix} r \text{ with probability } p \\ 0 \text{ with probability } 1-p \end{matrix} \right\} \quad \text{if } k \geqq \hat{k} = 1 \text{ and } l \geqq 1; \tag{2.1}$$

$$F(k, l) = 0 \text{ otherwise;}$$

The probability of success p can be influenced by the individual's effect. We denote by $C(p)$ the effort cost of reaching probability p, and for computational simplicity we only

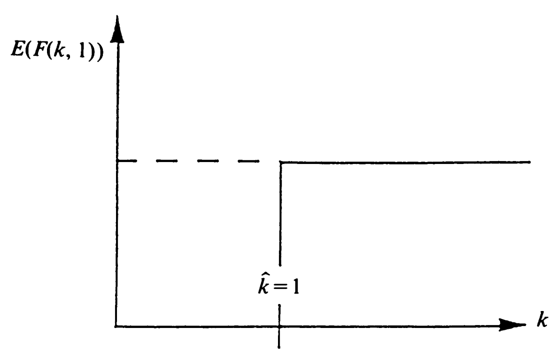

FIGURE 1

4. The high-yield technology defined by (2.1) corresponds to an extreme form of U-shaped average cost curve with respect to capital outlays. What is important for the analysis that follows is that the cost function is U-shaped; and especially that the production technology exhibit diminishing returns. The specific form chosen in the text is otherwise not important.

consider the quadratic cost function

$$C(p)=\frac{rp^2}{2a}, \qquad \text{where } a\in(0, 1]. \tag{2.2}$$

(c) the agent's initial endowment of capital w_t can be used in the entrepreneurial activity or it can be invested in an economy-wide mutual fund. The equilibrium unit (gross) return of this mutual fund, A_t, is determined endogenously by equating aggregate savings with aggregate investment. As in Diamond (1984) or Green (1987) we suppose that there is free-entry into the mutual fund market so that all extra-normal profits from the intermediation activity are competed away and borrowers obtain funding in exchange for an expected unit repayment on the loan of A_t.

To complete our description of the model, we must specify individual preferences and the chronology of the main events of an agent's life. We assume that agents are risk-neutral and their utility depends only on consumption and bequests. We assume that parents have "warm-glow" preferences over bequests (see Andreoni (1989)). That is, parents derive utility by giving to their children, independently of the extent to which their children actually benefit from the bequest.

The chronology of an agent's main events and decisions in her life is as shown in Figure 2.

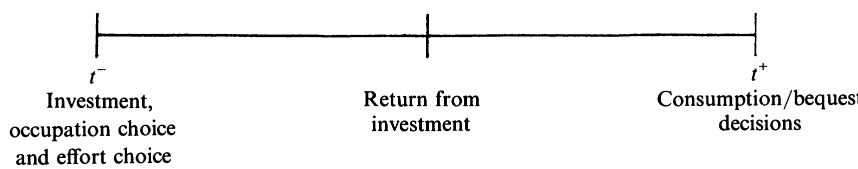

FIGURE 2

At the beginning of each period each individual decides to invest her unit of labour and her inherited wealth w_t in one (or two) of the above activities. At the end of her lifetime, the individual allocates her net final wealth between consumption and bequests. Agents are assumed to have Leontieff preferences over consumption and bequests,[5] so that optimal bequests are a linear function of end of period wealth. Let $w(t^+)$ denote wealth at the end of period t, then $b_{t+1}=w_{t+1}=(1-\delta)w(t^+)$.

3. STATIC EQUILIBRIUM IN THE CAPITAL MARKET

We shall assume that, initially at least, the aggregate wealth in the economy is not high enough for all individuals to be able to invest in their high yield entrepreneurial projects.

Assumption 1

$$\int_{w\geq 0} w\,dG_0(w)=W_0<1.$$

The economy should comprise three classes of agents, at least in the early periods[6]:

5. Ex-ante, preferences have the form $U=\min\{(1-\delta)c;\ \delta b\}-C(p)$, where c, b, $C(p)$ denote respectively the agent's consumption, bequests and effort cost; ($C(p)=0$ whenever the individual does not engage in entrepreneurial activities).

6. This heuristic claim will be formally established in the remaining part of the section.

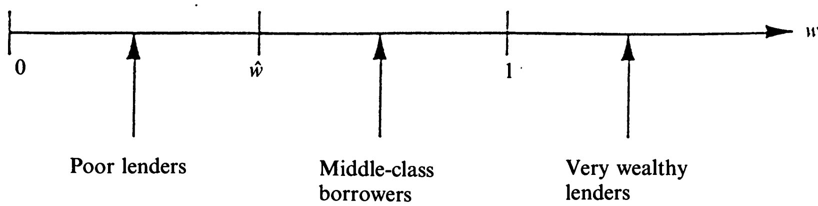

FIGURE 3

the very wealthy (with initial wealth $w_t > 1$) who have enough funds to invest both in their high-yield project and in the projects of other agents via the capital market; the middle-class composed of agents investing only in their own high-yield project but who need to complement their initial wealth $w_t < 1$ with a loan of $(1 - w_t)$ to cover the set up cost $\hat{k} = 1$; finally the poor who do not invest in their own project. This is illustrated in Figure 3.

The equilibrium terms at which the middle-class will borrow are determined by equalizing the aggregate demand for funds (emanating from this class) with aggregate supply (from the very wealthy and the poor).

Second, we assume that the (effort-driven) probability of success p is not observable, and that

Assumption 2

A borrower's repayment to her lenders cannot exceed her end of period wealth.

Thus, the incentive problem here is a moral hazard problem with limited wealth constraints, as in Sappington (1983).

3.1. *The optimal lending contract*

An optimal investment contract between a borrower with initial wealth $w < 1$ and his lender(s) will then specify a repayment schedule $R(w)$ such that:

$$R(w) = \begin{cases} (1-w)\rho(w) & \text{if the project succeeds;} \\ 0 & \text{if the project fails,} \end{cases}$$

where $(1 - w)$ is the amount borrowed and $\rho(w)$ is the unit repayment rate.

Given this contract, a borrower chooses p to maximize her expected revenue net of both repayment and effort costs

$$\max_p \{pr - p(1-w)\rho(w) - C(p)\}. \tag{3.1}$$

The solution is given by

$$p(w) = a\left(1 - (1-w)\frac{\rho(w)}{r}\right). \tag{3.2}$$

We see from equation (3.2) that when $\rho(w)$ is fixed and independent of w the lower the borrower's initial wealth the less effort she devotes to increasing the probability of success of her project. The more individuals need to borrow in order to invest, the less incentives they have to supply effort since they must share a larger fraction of the marginal returns from effort with lenders.

The very wealthy, who do not need to borrow in order to invest in their own project, supply the first-best level of effort since they remain residual claimants on all returns from such effort

$$w \geqq 1 \Rightarrow p(w) = \arg\max\,(pr - C(p)) = p^{fb}(w),$$
$$= a. \tag{3.3}$$

This implies that only those individuals who need to borrow funds to invest in their project actually choose to do so; the very wealthy prefer not to borrow in order to achieve first-best efficiency.[7]

So far we have treated the repayment schedule ρ as independent of the initial wealth w. However, since the risk of default on a loan may vary with the size of the loan, the unit repayment rate ρ must vary with w to reflect the change in default risk.

In equilibrium, all loans must yield the same expected return, so that the following condition must hold[8]

$$p(w)\rho(w) = A_t, \tag{3.4}$$

where A_t is the equilibrium unit rate of return of the mutual fund. We assume that A_t is a certain return.[9] Combining (3.2) and (3.4), the required repayment rate $\rho(w)$ must satisfy

$$a\rho(w)\left[1-(1-w)\frac{\rho(w)}{r}\right] = A_t. \tag{3.5}$$

From equations (3.2), (3.4) and (3.5) one derives the important proposition, that even when $\rho(w)$ varies with w, effort supply is increasing in w.

Proposition 0. *In equilibrium effort supply is increasing in* $w: p'(w) > 0$.

Proof. From equation (3.4) we know that $p(w) = A_t/\rho(w)$. Thus the sign of $p'(w)$ is given by the sign of $\{-\rho'(w)\}$. Define the function

$$f(\rho, w) = a\left[\rho - \rho^2\frac{1-w}{r}\right].$$

The left-hand intersection point of $f(p, w)$ with the horizontal line A_t defines the solution $\rho(w)$ to equation (3.5), see figure 4.

This solution is given by

$$\rho(w) = \left[1-\sqrt{1-\frac{4A_t(1-w)}{ar}}\right]\frac{r}{2(1-w)}.$$

It is easy to see that $\rho'(w) < 0$. ||

7. The existence of a fixed-cost indivisibility in the entrepreneurial technology might create incentives for our risk-neutral individuals to buy *lotteries*. For example, an agent with initial wealth $w < \hat{k}$ would take the lottery: (invest 0 with probability $1 - w/\hat{k}$; invest k with probability $w/\hat{k}$). For the sake of expositional clarity we shall rule out lotteries in the analysis below.

8. Because they are all risk-neutral, lenders will never invest in a high-yield project whose expected repayment revenue is strictly less than the market rate of return A_t.

9. Since a continuum (number) of i.i.d. projects are being financed by the mutual fund, one can appeal to the law of large numbers and assume without loss of generality that the expected rate of return A_t accruing to investors in the mutual fund is a safe rate of return. Introducing even a little amount of risk-aversion into individual preferences would suffice to rule out random rates of return in equilibrium.

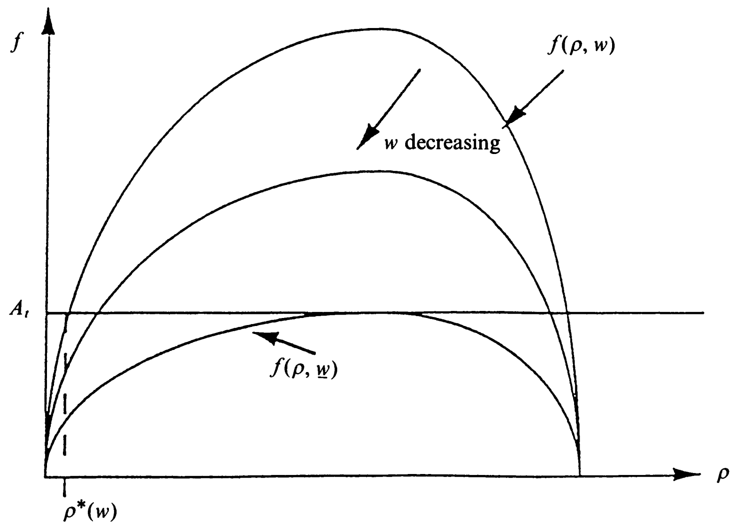

FIGURE 4

Because effort supply is decreasing when the agent borrows more, the unit repayment $\rho(w)$ must be commensurately increased to ensure that the lender obtains the same expected repayment. In other words, the poorer the borrower, the higher is her unit repayment—$\rho(w)$—to compensate for a lower probability of repayment, $p(w)$.

When

$$w < \underline{w}(A_t) \equiv 1 - \frac{ar}{4A_t},$$

there is no solution to equation (3.5). This means that the maximum expected return on a loan $1-w$, with $w < \underline{w}$, is strictly less than A_t. Thus all agents with wealth $w \in [0, \underline{w}(A_t)]$ won't be able to borrow, even if they want to borrow.

Definition. We say that there may be *credit rationing* in the capital market when $\underline{w}(A_t) > 0$. All agents with wealth $w \in [0, \underline{w}(A_t)]$ are said to be *credit rationed* if they would like to borrow but cannot since they cannot guarantee a return of A_t.

As will become clear in section 5, Proposition 0 is of fundamental importance for the wealth dynamics. Indeed, a consequence of this proposition is that an individual's bequest is stochastically increasing with her inherited wealth, so that the iterated map representing the dynamics of a lineage's wealth distribution is monotonic in the lineage's initial wealth.

3.2. *Equilibrium credit rationing*

When $\underline{w}(A_t) > 0$ it does not follow that all agents with wealth $w \in [0, w(A_t)]$ are credit rationed, since all these agents may actually prefer not to undertake the high-yield project and borrow. The higher the cost of capital, A_t, the less attractive it is to borrow and the more attractive it is to lend. Thus, we should expect that no credit-rationing takes place in equilibrium when A_t is high.

More formally, let $\tilde{w}=\tilde{w}(A_t)$ denote the initial wealth endowment of an individual of generation t who is *indifferent between borrowing and lending*. We have

$$p(\tilde{w})r-A_t(1-\tilde{w})-C(p(\tilde{w}))-1=A_t\tilde{w}+n. \tag{3.6}$$

The L.H.S. of (3.6) is the expected utility from being a borrower {using the fact that $\rho(\tilde{w})p(\tilde{w})=A_t$}, the R.H.S. of (3.6) is the expected utility from being a lender.

We can rewrite the above indifference condition as

$$p(\tilde{w})r-C(p(\tilde{w}))-1=A_t+n. \tag{3.7}$$

Lemma 1. *All individuals with initial wealth $w<\tilde{w}$ strictly prefer to be lenders; all individuals with initial wealth $w\in(\tilde{w}, 1)$ strictly prefer to be borrowers. Moreover, $\tilde{w}=\tilde{w}(A)$ is increasing with the market rate of return A.*

Proof. Let $\theta(p)=pr-C(p)-1$. By assumption $\theta(p)$ is concave, which implies that θ is increasing on $[0, a]$ since $a=p^{fb}=\arg\max\theta(p)$. The lemma then follows from the fact that $p(w)<p^{fb}=a$ for all $w<1$ and that $p(w)$ is increasing in w on that same wealth interval. ||

Now we can both determine the equilibrium rate of return A_t on the capital market and characterize the situations in which credit-rationing occurs in equilibrium. The net suppliers of funds are first the wealthy, with initial wealth $w>1$; they put $(w-1)$ in the mutual fund; second, the poor who either are denied access to borrowing ($w<\underline{w}(A_t)=1-ar/4A_t$) or prefer to lend rather than borrow on the capital market ($w<\tilde{w}(A_t)$); they put w in the mutual fund. The aggregate supply of funds on the capital market, for given A_t, is then equal to

$$S(A_t)=\int_0^{\hat{w}(A_t)} w\,dG_t(w)+\int_1^{\infty}(w-1)dG_t(w). \tag{3.8}$$

where $\hat{w}(A_t)=\max(\underline{w}(A_t), \tilde{w}(A_t))$ is increasing in A_t.

By complementarity, the aggregate demand for funds at the same market rate of return A_t is equal to

$$D(A_t)=\int_{\hat{w}(A_t)}^{1}(1-w)dG_t(w). \tag{3.9}$$

Clearly, $S(A_t)$ is increasing in A_t and $D(A_t)$ is decreasing in A_t. Thus, an equilibrium rate of return corresponding to the wealth distribution $G_t(w)$, whenever it exists, is uniquely defined by

$$S(A_t)=D(A_t). \tag{3.10}$$

The description of the capital-market equilibrium at each period t is now completed. Credit-rationing occurs in equilibrium whenever the set of poor individuals who would prefer to borrow but are denied access to credit is non-empty, i.e. whenever

$$\underline{w}(A_t)>\max\{0, \tilde{w}(A_t)\}. \tag{3.11}$$

Then all individuals with wealth

$$w\in[\max\{0, \tilde{w}(A_t)\}, \underline{w}(A_t)],$$

are credit-rationed.

The following proposition characterizes the situations in which condition (3.11) is satisfied.

Proposition 1. *There is credit-rationing in equilibrium whenever*

$$\frac{ar}{4}<A_t<\frac{3}{8}ar-n-1.$$

Proof. See Appendix. ||

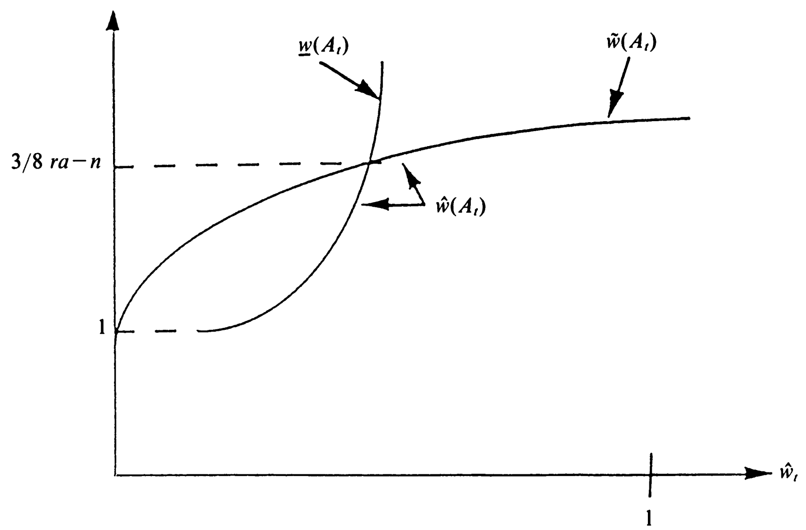

FIGURE 5

This result, depicted in Figure 5, may appear somehow surprising, especially in light of the existing literature on credit-rationing (see Stiglitz and Weiss (1981) and Bernanke and Gertler (1989)).

Indeed, this literature predicts that credit-rationing is more likely when the cost of capital is high while here we predict the opposite. The main reason why our prediction differs from the existing literature is that the identity of lenders and borrowers is fixed exogenously in all existing models of credit-rationing, while in our model agents can choose to be borrowers or lenders. Their choice depends on the size of inherited wealth and the rate of return on the capital market.[10] When the cost of capital (or market return), A_t, is high, poor agents prefer to lend at rate A_t; this rate is highly favourable to lenders and highly unfavorable to borrowers, especially those who need to borrow large amounts.

Conversely, when the cost of capital A_t is low (that is sufficiently close to 1) and the lending terms are shifted in favour of the borrowers, the same poor agents prefer to borrow at such a low rate but may be denied access to credit. This explains why credit-rationing is more likely to occur when the cost of capital is low. Finally, when $A_t \leqq ar/4$, the cost of capital is sufficiently low that it is worth lending to all borrowers with wealth $w \geqq 0$.

10. We obtain the same conclusion as Stiglitz–Weiss and Bernanke–Gertler if, as they implicitly do, we assume $\tilde{w}(A_t)=0$.

4. THE EVOLUTION OF EQUILIBRIUM INTEREST RATES

Once the equilibrium unit cost of borrowing in period t, A_t, has been determined one can specify the stochastic process of lineage wealth as follows.

Let $\hat{w}_t$ denote the cut-off wealth level below which individuals are either unwilling or unable to invest in the risky project (that is, $\hat{w}_t = \min(\tilde{w}(A_t), \underline{w}(A_t))$). Then

$$\begin{aligned} &\text{For } w_t \in [0, \hat{w}_t], \quad w_{t+1} = (1-\delta)(A_t w_t + n) \text{ with probability one;} \\ &\text{For } w_t \in [\hat{w}_t, \infty], \; w_{t+1} = f_t(w_t, \theta_t), \end{aligned} \tag{4.1}$$

where

(a) $\theta_t \in \{0, 1\}$ is an i.i.d. variable that refers to the high yield project being successful ($\theta_t = 1$) or unsuccessful ($\theta_t = 0$). Recall from Section 3 that the probability distribution for θ_t when $w_t = w$ is given by

$$\text{prob}(\theta_t = 1/w) = h(1, w) \begin{cases} = a \text{ for } w \geqq 1; \\ = p(w) \text{ for } w < 1, \end{cases}$$

and

$$\text{prob}(\theta_t = 0/w) = h(0, w) = 1 - h(1, w).$$

(b)

$$\begin{aligned} f_t(w, 1) &= (1-\delta)(r - (1-w)A_t) && \text{for } w \geqq 1; \\ f_t(w, 1) &= (1-\delta)(r - (1-w)\rho(w)) && \text{for } w < 1, \end{aligned}$$

and

$$f_t(w, 0) = \max\{0, (1-\delta)(w-1)A_t\}.$$

Given that the economy comprises a continuum of agents and that the random returns on each risky project (with equity participation by the entrepreneur of w) are independently identically distributed, the aggregate wealth level and its distribution function can be interpreted as deterministic variables, by the law of large numbers. The wealth distribution G_{t+1} in period $t+1$ is then obtained from the distribution in period t as follows.

Consider an individual with an inherited wealth of w_{t+1}. This wealth can come from a parent who successfully invested in the risky project, in which case the parent's initial wealth was

$$w_t = \psi_t(w_{t+1}, 1) \quad \text{where } \psi_t(\cdot, 1) = f_t^{-1}(\cdot, 1).$$

Or it can come from a parent who invested unsuccessfully in the risky project, in which case the parent's initial wealth was

$$w_t = \psi_t(w_{t+1}, 0) \quad \text{where } \psi_t(\cdot, 1) = f_t^{-1}(\cdot, 0).$$

Finally, it can come from a parent who did not invest in the risky project, who had initial wealth

$$w_t = \frac{1}{A_t}\left(\frac{w_{t+1}}{1-\delta} - n\right).$$

This latter possibility only arises if $w_t < \hat{w}_t$.

Then the wealth distribution in period $(t+1)$ is simply obtained by adding up the total mass of lineages who end up with wealth less than w_{t+1}. In the special case where $\hat{w}_t = 0$[11] one obtains

$$G_{t+1}(w) = \int_0^{\psi_t(w,1)} h(x,1)dG_t(x) + \int_0^{\psi_t(w,0)} h(x,0)dG_t(x), \tag{4.2}$$

where the fraction of successful and unsuccessful parents, respectively $h(x, 1)$ and $h(x, 0)$, have been specified above.

The dynamics of the wealth distribution, together with the determination of the equilibrium cost of capital A_t, fully describe the evolution of the capital market and the evolution of lineage wealth over time.

We shall mostly be interested in the evolution of this economy under conditions of rapid capital accumulation. Namely, we shall assume that the returns on individual projects (r) *and* the saving propensity of individuals $(1-\delta)$ are both sufficiently large that

Assumption 3

$$\tfrac{3}{8}ar(1-\delta) > 1+n.$$

Now, it is easy to see that, in the absence of credit-rationing, the economy will grow until *all* investment opportunities have been exploited. Indeed, take any period in which some investment projects have not been undertaken. This cannot be a long-run equilibrium: first, since $p(0)r(1-\delta) > (ar/2)(1-\delta) > 1$, by Assumption 3 all individuals who are undertaking a risky project will, as a group, leave more wealth than they started out with to their offspring. There will thus be more funds available to finance risky projects at the end of the period and thus, whenever the equilibrium cost of capital A_t is strictly greater than 1, A_{t+1} will necessarily be strictly lower than A_t and more risky projects will be financed in period $t+1$. If the cost of capital, A_t is equal to 1, Assumption 3 implies that all individuals choose to invest in the risky projects rather than in the safe activity[12]. Therefore, all the investment opportunities will end up being exploited. Once all investment opportunities have been exploited growth tapers off and the cost of capital stays at the lower bound $A = 1$.

There is an important proviso to this convergence argument: even if there is no credit-rationing with $A = 1$, there may be credit-rationing along the development path where $A_t > 1$. The persistence of credit-rationing along the development path may prevent further risky investments from being undertaken even when more funds become available for such investments. This, in turn, creates problems with our convergence argument and with the existence of a unique steady-state. We shall briefly return at the end of this section to these problems. The next assumption, which is again satisfied under joint conditions of high productivity and high saving propensity, that is under fast capital accumulation,

11. When aggregate wealth accumulation is sufficiently rapid, the economy ends up with $\hat{w}_t \leqq 0$ after a finite number of periods. See Proposition 2 below.

12. More precisely, the poorest individual will strictly prefer to invest in the risky project if and only if:

$$p(0)r - C(p(0)) - 1 > n.$$

But the L.H.S. of the above inequality is greater than:

$$\frac{a}{2}r - \frac{a}{8}r - 1 = \frac{3}{8}ar - 1$$

since $p(0) > a/2$. That $\frac{3}{8}ar - 1 > n$ is in turn clearly implied by Assumption 3.

removes the difficulties that might result from credit-rationing[13] and thus guarantees convergence of the economy to a no-growth/low interest rate steady-state equilibrium. In making such an assumption, we are de facto focusing our attention on the most favourable case for trickle-down.[14] Nevertheless, as we shall argue in Section 6 below, even in that case, laissez-faire trickle-down does not work optimally and therefore it leaves room for efficiency improving redistribution policies. So, let us assume

Assumption 4

$$\frac{ar}{4}(1-\delta)>1$$

One can then establish

Proposition 2. *Given Assumptions* 3 *and* 4 *the cost of capital* A_t *converges to* 1 *in finite time.*

Proof. See Appendix. ||

In words, as more capital is accumulated (and is accumulated quickly enough) there are more and more funds available in the economy to finance a smaller and smaller pool of borrowers. Thus the equilibrium lending terms are progressively shifted in favour of borrowers.

This effect of capital accumulation on the evolution of A_t can give rise to a Kuznets curve[15] type relation between growth and wealth inequality: indeed, to the extent that in the early phases of development the lending terms are favourable to the lenders (A_t is initially high if aggregate wealth is small) the wealth of rich lenders (with $w>1$) grows relatively faster.[16] In later stages of development we know from Proposition 2 that lending terms become more favourable to borrowers so that the wealth of the middle-class tends to catch up with that of the rich whilst an increasing fraction of the poor can borrow and thus invest in their own individual projects.[17] In other words, initial phases of growth tend to increase inequalities while later stages tend to reduce them. This Kuznets effect is reinforced by the existence of capital market imperfections, since the higher the cost of capital, A_t, the more rapidly the (second-best) probability of success $p(w)$ increases with the initial wealth of borrowers w. Whereas the first-best probability of success is the same for all investors and equal to a (see Aghion and Bolton (1993)).

13. This assumption does not rule out credit rationing everywhere along the adjustment path, but it rules out credit rationing for low enough interest rates: that is, assumption A4 implies that $\underline{w}(A_t)\leqq 0$ for A_t sufficiently close to 1.

14. Indeed, in less favourable cases where there is a multiplicity of steady-state equilibria, some of these steady-states are necessarily inefficient (from the point of view of surplus maximization) and therefore reflect immediately a failure of the trickle-down mechanism.

15. See Glomm–Ravikumar (1994) for a good survey of recent theories of the Kuznets curve.

16. This is all the more true since the less favourable lending terms for the middle-class borrowers has a negative effect on their effort supply relative to the first best effort.

17. The emergence of a Kuznets curve in our model depends on the entrepreneurial technology having *decreasing* returns for high values of k. The introduction of increasing returns or of technological progress opening new investment opportunities to the rich, tend to counter the Kuznets effect obtained in this model.

5. THE LIMIT WEALTH DISTRIBUTION

In this section we determine the long run behaviour of the wealth distribution in the general case where $0<a<1$, under Assumptions 1 and 3. We know from Proposition 2 that when these assumptions hold, $A_t \to 1$ in finite time. Accordingly, our analysis here starts at the earliest date for which $A_t = 1$. We denote that date T. We begin by considering the long run evolution of lineage wealth for a single lineage in this economy. We show that the probability distribution of lineage wealth converges to a unique stationary distribution. This stationary distribution can be interpreted as the long run wealth distribution for the economy since all lineage wealth processes are identically and independently distributed, and since there is a continuum of lineages. To establish the convergence of the probability distribution of lineage wealth to a unique stationary distribution we appeal to recent results of convergence for monotonic Markov processes in Hopenhayn and Prescott (1992). The evolution of lineage wealth when $A_t = 1$ is entirely described by

$$w_{t+1} = f(w_t, \theta_t), \tag{5.1}$$

where

$$f(w, 1)\begin{cases} = (1-\delta)(r-(1-w)) \text{ for } w \geqq 1; \\ = (1-\delta)(r-(1-w)\rho(w)) = (1-\delta)(r-(1-w)/p(w)) \quad \text{for } w<1; \end{cases}$$

$$f(w, 0) = \max\{0, (1-\delta)(w-1)\}.$$

[Recall that $\theta_t \in \{0, 1\}$ is an i.i.d. random variable that refers to the high yield project being successful ($\theta_t = 1$) or unsuccessful ($\theta_t = 0$); and that $\text{prob}(\theta_t = 1/w) = p(w) = 1 - \text{prob}(\theta_t = 0/w)$ for all w.]

Let $\bar{w}$ denote the highest sustainable wealth level, defined by: $f(\bar{w}, 1) = \bar{w}$, i.e. $\bar{w} = (r-1)(1-\delta)/\delta^{-1}$ (see Figure 6 below); let $W = [0, \bar{w}]$ and let Ω denote the set of Borel subsets of W.

Given that the random variable θ_t is i.i.d., the stochastic process of lineage wealth described by (5.1) is a stationary linear Markov process. The corresponding transition function: P: $W \times \Omega \to [0, 1]$ is simply defined by

$$P(w, A) = \text{prob}\{f(w, \theta) \in A\}, \text{ for all Borel subsets } A \in \Omega. \tag{5.2}$$

We shall describe the long run dynamic behaviour implied by $P(\cdot, \cdot)$ by determining the existence of a unique invariant distribution G.

Notation. For any wealth distribution $G(\cdot)$, let $T^*G(\cdot)$ be the Markov transformation of G defined by

$$T^*G(A) = \int P(w, A) dG(w), \tag{5.3}$$

for all Borel subsets $A \subset W$.

Definition. A wealth distribution G on W is invariant for P if for all Borel subsets $A \subset W$, one has the equality

$$T^*G(A) = G(A).$$

To see intuitively why an invariant distribution $G(\cdot)$ exists for our Markov process of lineage wealth it is helpful to look at Figure 6.

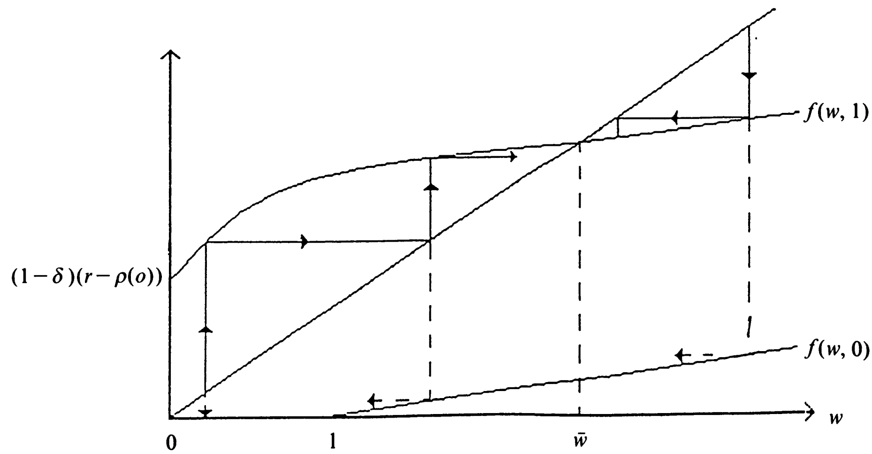

FIGURE 6

As can be seen from Figure 6, if lineage wealth at some date t is such that $w_t > \bar{w}$, then in finite time it can only shrink to a level less than or equal to $\bar{w}$, the highest sustainable wealth level. This wealth level can only be maintained if the high-yield project's returns are the highest possible. Once lineage wealth is less than or equal to $\bar{w}$ it can never exceed $\bar{w}$ but the Figure 6 suggests that it may take any value in the interval $[0, \bar{w}]$ with positive probability. Thus, any (measurable) subset of $[0, \bar{w}]$ may be visited an infinite number of times on average by the lineage wealth w_t. In the theory of Markov chains with a finite number of states existence of a unique invariant distribution is obtained only if the state variable can move from any recurrent state to any other with positive probability (see for example Cinlar (1975)). The above figure suggests that wealth lineages may move from any (measurable) subset $[0, \bar{w}]$ to any other (measurable) subset of $[0, \bar{w}]$ and thus one might expect this Markov process to have a unique invariant distribution.

As it turns out, one can indeed establish the following result.

Proposition 3. *There exists a unique invariant distribution G for the Markov process corresponding to $P(w, A)$. Moreover, for any initial wealth distribution G_0, the sequence $(T^*)^n(G_0)$ converges "weakly"*[18] *to G.*

Proof. See Appendix. ||

The proof is a straightforward application of Hopenhayn and Prescott's (1992) analysis of existence, uniqueness and convergence properties of monotonic stochastic processes. Indeed, an immediate consequence of Proposition 0 (that is, of the fact that effort supply is increasing in wealth) is that the transition function $P(w, A)$ is increasing in w in terms of first-order stochastic dominance.

The unique invariant distribution G derived in Proposition 3 satisfies the following equation

$$T^*G = G, \tag{5.4}$$

18. The term "weakly" refers to the weak-$*$ topology on the set of wealth distributions over the interval $[0, \bar{w}]$.

where:

$$T^*G(w)=\int_0^{\psi(w,1)} p(x)dG(x)+\int_0^{\psi(w,0)} (1-p(x))dG(x),$$

and

$$\psi(\cdot\,,1)=f^{-1}(\cdot\,,1), \qquad \psi(\cdot\,,0)=f^{-1}(\cdot\,,0).$$

In particular, when $0<a<1$, the unique stationary distribution G has full support on $[0, \bar{w}]$, so that some wealth inequality remains in the long run. Therefore, despite the trickle-down nature of growth in this model, wealth inequalities cannot be completely eliminated in the long run. One might be tempted to argue, however, that the long run wealth inequalities are unimportant since the Markov-operator T associated with the transition probability P is ergodic, which means that in the long run all lineages fare equally well on average. It is therefore unnecessary or even counter-productive, one would argue, to redistribute wealth in the long run. If all lineages are equally well off on average in the long run then on the basis of distributive justice alone there seems to be little reason to redistribute wealth. Moreover, a one-time distribution can only have temporary effects as the distribution of wealth tends to move back towards the stationary distribution.

However, we shall argue in the next section that permanent redistribution policies can improve the productive efficiency of the economy. By redistributing wealth the government can equalize opportunities of investment and thus improve productive efficiency. To illustrate this point we shall compare the stationary distribution obtained in our economy with the stationary distribution obtained in a first-best-economy.

6. THE CASE FOR A PERMANENT REDISTRIBUTION OF WEALTH

In a first-best world, a borrower can commit to choosing effort level $p=a$.[19] Thus, wealth accumulates faster over time than in a second-best economy where $p(w)<a$ for all $w<1$. Moreover, it is straightforward to establish a stronger result than Proposition 2 for the first-best economy; that is, under weaker assumptions than 3 and 4, there exists $T<\infty$ such that $\int_0^\infty wdG_t(w)>1$ and $A_t=1$ for all $t\geqq T$. As in the second-best economy, one can show that the stationary Markov process of lineage wealth for the first-best economy has a unique invariant probability measure. Thus, we can define the stationary wealth distribution for the first-best economy, as follows

$$T^*_{FB}G(w)=\int_0^{\psi(w,1)} adG(x)+\int_0^{\psi(w,0)} (1-a)dG(x). \tag{6.1}$$

We are now in a position to compare the stationary wealth distributions in the first-best and second-best economies, G_{FB} and G_{SB}, where G_{FB} is given by $T^*_{FB}G=G$ and G_{SB} is defined by $T^*G=G$.

19. How can a borrower commit to choosing $p=a$? In practice this is often impossible but one can imagine a world in which the borrower's choice of effort can be easily monitored at low cost; then, for example, the loan contract can specify that the borrower may not obtain the payoffs (net of repayments) from his/her investment if he/she is seen not to choose $p=a$. The threat of such a punishment is clearly more effective when the agent borrows less since then his/her payoffs (net of repayments) are higher. Thus, as long as monitoring of effort involves costs an agent wishing to borrow less is going to be served before an agent wishing to borrow more since the lender must incur smaller costs of monitoring for the agent borrowing less.

Proposition 4. *The second-best stationary distribution is dominated by the first-best stationary distribution*:

$$G_{SB}(w) \geqq G_{FB}(w) \quad \text{for all } w,$$

the inequality being strict for all $w < r(1-\delta)$.

Proof. See Appendix. ||

In steady-state equilibrium the second-best economy has both a lower output per capita and a wealth distribution which puts more weight on poorer individuals than the first-best economy (the latter observation follows from the fact that in steady state both wealth distributions have the same support). The source of the problem is, of course, moral hazard with limited wealth. Because borrowers cannot appropriate the full marginal return of their effort-investment, they tend to underinvest in effort and therefore they tend to get lower expected returns than wealthier agents who do not need to borrow to invest. There is a natural policy response available to correct this productive inefficiency: redistribute wealth permanently from the rich lenders to the poor and the middle-class who need to borrow funds to invest. The positive effect of such redistribution is to equalize opportunities by letting all agents have access to profitable activities on similar terms. Note that redistribution is desirable here not because it is an ex-post Pareto-improvement, but because it increases productive efficiency[20].

One might argue that redistribution is undesirable, because of the disincentive effect it might have on effort supply of the rich and because redistribution could be undone by a corresponding change in individuals' bequest decisions.

Concerning first the issue of effort supply, one can easily design a redistribution policy that increases the output and effort of the subsidized borrowers by more than it decreases the output and effort of the taxed rich, thereby increasing aggregate productive efficiency of the overall economy. For example, suppose that, starting from the second-best invariant distribution G_{SB}, the government imposes a profit tax on the returns r earned by a unit mass of rich individuals and uses the proceeds of this tax in order to increase the initial wealth of a unit mass of borrowers from w to say $w+\varepsilon$. Since only a fraction a of rich investors do generate positive returns r on their risky investments, each successful investor must pay a profit tax at least equal to ε/a if the government's budget balance is to be preserved. Let p_R and p'_R denote the effort of a rich individual respectively before and after the above redistribution policy has been introduced. Similarly, let p_P and p'_P denote the effort supplied by a borrower with initial wealth w respectively before and after the subsidy. We have[21] $C'(p_R)=r$ and $C'(p'_R)=r-\varepsilon/a$ hence

$$p_R - p'_R = \frac{r}{a}\frac{\varepsilon}{a}, \tag{6.2}$$

20. Pareto-improvements can always be achieved through private contracting. No state intervention is necessary for redistribution unless some agents are made worse off (ex-post) by the redistribution policy. This is typically what happens in this model. Note also that state invervention may be justified even when it does not achieve a Pareto-improvement. A good example of such state invervention is enforcement of property rights. Any tightening of enforcement of property rights benefits the owners but hurts the thieves. Thus, such a tightening is not Pareto-improving.

21. Note that future expected subsidies in the event that an agent does badly do not affect effort supply. The reason is once again that we do not assume a Ricardian model of bequests. We have assumed a "warm-glow" motive for bequests, so that an increase in future subsidies of poor children does not reduce the parents incentive to accumulate wealth.

and $C'(p_P)=r-p(w)(1-w)$, $C'(p'_P)=r-p(w+\varepsilon)(1-w+\varepsilon)$, hence

$$p'_P-p_P=\frac{r}{a}[h(w+\varepsilon)-h(w)]$$

$$\approx\frac{r}{a}h'(w)\varepsilon \text{ for } \varepsilon \text{ sufficiently small,} \tag{6.3}$$

where

$$h(w)=\rho(w)(1-w)=\left[\sqrt{1-\frac{4(1-w)}{ar}}\right]\frac{r}{2}.$$

That $p'_P-p_P>p_R-p'_R$ simply follows from the fact that $h'(w)=(2/ra)(1/h(w))>2/a$, so that

$$p'_P-p_P>2(p_R-p'_R). \tag{6.4}$$

We thus obtain a net gain in aggregate productive efficiency from a policy that would permanently implement the redistribution suggested in this example.[22] In addition, since individual preferences over bequests are assumed to be "warm-glow", a [proportional] tax on beginning of period wealth of rich individuals would reduce the disincentive effects of taxation on the rich relative to those given in the example.

This brings us to the second potential objection to redistribution, that subsidies and bequests may be perfect substitutes. However, when parents have "warm-glow" preferences for bequests, we know from Andreoni (1989) that bequests and subsidies are imperfect substitutes. Therefore the disincentive effect of subsidies on bequests will not be complete.[23] It is worth noting here that empirically the "warm-glow" model of bequests tends to perform better than the Ricardian model. For example one prediction of the Ricardian model is that in a growing economy (with technical progress) bequests should be negative on average. Similarly, parents should leave more to the worst-off children. None of these predictions are empirically verified. Note that the "warm-glow" model would predict positive bequests even in a growing economy and equal division among children even when the latter are not equally well off.

An important consequence of Proposition 3 is that one shot redistributions cannot have long-lasting effects in the sense that they do not affect the unique invariant distribution G_{SB}. This in turn follows from the convergence result. Permanent redistribution policies must be set up in order to durably improve the efficiency of the economy in steady state[24]. Temporary redistribution policies may, however, help the economy achieve its long-run steady-state faster; in other words, the government can improve productive efficiency along the growth-adjustment path.

22. This example may seem to rely heavily on the cost function $C(p)$ being quadratic. However its conclusion can easily be generalized to the case where C is sufficiently convex.

23. Even if agents had Ricardian preferences, subsidies and bequests are not necessarily perfect substitutes because parents may be wealth constrained and thus unable to borrow to bequeath to their children. In that case they won't cut back their bequests when their children receive a greater subsidy.

24. The idea that redistribution policies may have positive incentive effects has already been suggested by Bernanke and Gertler (1990), among others. However their discussion does not take into account the lending side of the capital market. In particular, it ignores the potential disincentive effects that redistribution might have on the lenders. Moreover, their analysis remains purely static and does not distinguish between a one-shot and a permanent redistribution policy.

To see this, let M_t be the aggregate wealth in period t. Then M_{t+1} in the second-best economy is given by

$$\frac{M_{t+1}}{(1-\delta)}=n\int_0^{\hat{w}_t} dG_t+r\left[\int_{\hat{w}_t}^1 p(w)dG_t+\int_1^{\infty} adG_t\right]+A_t\left[M_t-\int_{\hat{w}_t}^{\infty} dG_t\right]. \tag{6.5}$$

It is clear from this expression that the growth rate $[M_{t+1}-M_t/M_t]$ depends not only on the stock of wealth in period t but also on its distribution. Thus, the process of capital accumulation affects and is affected by the distribution of wealth. In the first-best economy, on the other hand, the growth rate is not affected by the distribution of wealth.[25] To see this, formally replace $\hat{w}_t$ by $v_t \leqq \hat{w}_t$, $p(w)$ by $a \geqq p(w)$ and A_t by $((ra/2)-n) \geqq A_t$ in (6.5), to obtain

$$\frac{M_{t+1}^{FB}}{1-\delta}=arM_t+\frac{r}{1-M_t}. \tag{6.6}$$

Comparing (6.5) and (6.6) one observes that growth in the second-best economy is always slower than in the first-best economy. However, through (costless) redistribution of wealth one can always achieve the first-best growth rate in the second-best economy. If *per capita* wealth $M_t<1$, the first-best growth rate is achieved by letting a mass M_t of agents start out with $w=1$ and the other agents have $w=0$. This form of redistribution may actually increase inequalities.[26] If $M_t>1$, the first-best is achieved by letting all agents have $w=M_t$. That is, the first-best is achieved by implementing perfect equality.

APPENDIX

Proof of Proposition 1

Let

$$V(p)=pr-\frac{rp^2}{2a}-1.$$

Using the formula for $\rho^*(w)$, we have for all A

$$\rho^*(\underline{w}(A))=\rho^*\left(1-\frac{ar}{4a}\right)=\frac{2A}{a}.$$

Hence

$$p(\underline{w}(A))=\frac{A}{\rho^*(\underline{w}(A))}=\frac{a}{2},$$

which in turn implies that

$$V(p(\underline{w}(A)))=\tfrac{3}{8}ar-1.$$

25. That the distribution of wealth does not affect the aggregate wealth M_{t+1} in the first-best is not surprising since all individual investors supply the same effort $p=a$ and achieve the same random return (r with probability a, 0 with probability $1-a$) independently of their initial wealth. This is no longer true in the second-best world where a borrower's probability of success $p(w)$ is strictly increasing with her wealth w.

26. The need for such (inequality increasing) redistribution within the wealth interval [0, 1] disappears if one allows for lotteries: in that case the unique invariant distribution should have mass points at 0 and 1, but the negative incentive effects of borrowing for those agents with wealth $w=0$ would still call for redistribution of wealth away from rich lenders (with $w>1$).

Now we know from Lemma 1 that the wealth level $\tilde{w}(A)$ at which individuals are indifferent between borrowing and lending, satisfies equation (3.7), or equivalently

$$V(p(\tilde{w})A)) = A_t + n.$$

The proof of Proposition 1 is now straightforward: first, the two wealth schedules $\underline{w}(A)$ and $\tilde{w}(A)$ intersect at a market rate A^* such that

$$V(p(\underline{w}(A^*))) = V(p(\tilde{w}(A^*))).$$

Using equations (A1) and (A2), we obtain the unique solution

$$A^* = \tfrac{3}{8}ar - n - 1.$$

Using equations (A1) and (A2), again, we have, for all $A_t > A^*$

$$V(p(\tilde{w}(A_t))) > V(p(\underline{w}(A_t))),$$

or: $\tilde{w}(A_t) > \underline{w}(A_t)$ since both V and p are increasing over the relevant range of variables. Similarly, whenever $A_t < A^*$ we must have $\tilde{w}(A_t) < \underline{w}(A_t)$ by the same reasoning. Finally, $A_t \leqq ar/4 \Rightarrow \underline{w}(A_t) \leqq 0$. This completes the proof of Proposition 1. ||

Proof of Proposition 2

The proof proceeds in three steps.

Step 1. *The cost of capital A_t cannot remain above $1/(1-\delta)$ for more than a finite number of periods.*

Suppose indeed that $A_t > 1/(1-\delta)$ for infinitely many t.

Then all investors in risky projects would see their expected lineage wealth increase over time, by an amount uniformly bounded away from zero.

More formally, if w_t denotes the initial wealth of an investor ($w_t \geqq \hat{w}_t$), we have, when $A_t > 1/(1-\delta)$:

$$\begin{aligned} E(w_{t+1}|w_t) - w_t &= [p(w_t)r - A_t(1-w_t)](1-\delta) - w_t \\ &\geqq p(0)r(1-\delta) - A_t(1-\delta) \\ &\geqq 2A_t(1-\delta) - A_t(1-\delta) \\ &= A_t(1-\delta) > 1 > 0. \end{aligned}$$

The last inequality follows from the fact that $p(0) = A/p(0) \geqq 2A/r$.

Also, whenever $A_t > 1/(1-\delta)$, the fraction of the population with wealth $w_t > \hat{w}_t$ is uniformly bounded away from zero. To see this, suppose instead that the fraction of investors was not bounded away from zero. One could then exhibit a subsequence (t_k) from the infinite set $\{t/A_t > 1/(1-\delta)\}$, such that the total mass of investors $\int_{\hat{w}_{t_k}}^{\infty} dG_{t_k} \to 0$ as $t_k \to \infty$. But then, as $t_k \to \infty$, there would eventually be an excess supply of funds unless $A_{t_k} \to 1$. However, having $A_{t_k} \to 1$ contradicts the fact that $A_{t_k} > 1/(1-\delta)$ for all t_k.

It follows from the above two remarks that whenever $A_t > 1/(1-\delta)$, the total wealth of investors will increase by an amount which is uniformly bounded away from zero. In all other periods where $A_t \leqq 1/(1-\delta)$ investors (i.e. with initial wealth $w_t > \hat{w}_t$) will still get richer on average, although by a smaller amount: $E(w_{t+1}|w_t > \hat{w}_t) - w_t \geqq A_t(1-\delta) \geqq 1-\delta > 0$, since A_t is bounded below by 1.

Thus if $A_t > 1/(1-\delta)$ for infinitely many t, the total wealth of investors would go to $(+\infty)$ over time, so for t sufficiently large there would necessarily be excess supply of funds in the economy; this in turn implies that A_t cannot remain indefinitely above $1/(1-\delta)$. ||

Step 2. *Under assumption A4, there cannot be credit-rationing when $A_t \in [1, 1/(1-\delta)]$*

Indeed, credit rationing can only occur at wealth levels less than $\underline{w}(A_t)$. But since the $\underline{w}(\cdot)$ function is increasing in A, we have $\underline{w}(A_t) \leqq \underline{w}(1/(1-\delta))$ for $A_t \leqq 1/(1-\delta)$. Finally, by Assumption 4, $\underline{w}(1/(1-\delta)) < 0$. This establishes Step 2.[27] ||

Step 3. $A_t = 1$ *for t sufficiently large.*

27. A weaker condition for avoiding credit-rationing is, from Proposition 1:

$$A4 \text{ or } \frac{1}{1-\delta} > \frac{3}{8}ar - n.$$

This step follows immediately from: (a) Assumption 3 which implies that, abstracting from credit-rationing considerations, the cost of capital A_t shall converge to 1 in finite time; (b) Assumption 4, which rules out the possibility of credit-rationing for $A_t \leqq 1/(1-\delta)$; (c) Step 1, which guarantees that A_t falls below $1/(1-\delta)$ in finite time. This establishes Proposition 2. ||

Proof of Proposition 3

Lemma 1. (*Monotonicity of P*) *The transition function $P(w, A)$ is increasing in its first argument w in the following (first-order stochastic dominance) sense: For all* $(w, w') \in W^2$, $w \leqq w' \Rightarrow \forall x \in W$, $P(w', [0, x]) \leqq P(w, [0, x])$.

Proof of Lemma 1. This follows immediately from Proposition 0. More specifically, we have for all $(x, w) \in W^2$

$$P(w, [0, x]) = p(w) 1_{f(w,1)<x} + (1-p(w)) 1_{f(w,0)<x},$$

where

$$1_{f(w,\theta)<x} \begin{cases} =1 & \text{if } f(w, \theta) < x \\ =0 & \text{otherwise.} \end{cases}$$

Thus, for all $(w, w') \in W^2$ such that $w' \geqq w$, we get

$$P(w', [0, x]) - P(w, [0, x]) = (p(w') - p(w))(1_{f(w,1)<x} - 1_{f(w,0)<x}) + p(w')[1_{f(w,'1)<x} - 1_{f(w,1)<x}] + (1-p(w'))[1_{f(w',0)<x} - 1_{f(w,0)<x}].$$

The first term is negative since $p(w') \geqq p(w)$ [Proposition 0] and $f(w, 1) > f(w, 0)$ (success is better than failure). The second and third terms are also negative since $f(w', \theta) \geqq f(w, \theta)$ for $w' \geqq w$.

This establishes Lemma 1. ||

Lemma 2. (*Monotone Mixing Condition*) *The monotonic transition function P satisfies the following property*:

For any $w^* \in (0, \bar{w})$ there exists an integer m such that:

$$P^m(0, [w^*, \bar{w}]) > 0 \quad \text{and} \quad P^m(\bar{w}, [0, w^*]) > 0^{28}.$$

In words, even the poorest individual will have his lineage wealth end up above w^* after m consequent successes; similarly, even the richest individual will have his lineage wealth end up below w^* after m consequent failures.

Proof of Lemma 2. Take any $w^* \in (0, \bar{w})$. Then, since $(f(w, 1) - w)$ is always strictly positive and continuous on $[0, w^*]$, it remains uniformly bounded below by some positive number α. Thus, there exists an integer n_1 such that: $n \geqq n_1 \Leftrightarrow f^{(n)}(0, 1) > w^*$, where $f^{(n)}(\cdot, 1)$ denotes the n-th iterate of $f(\cdot, 1)$.[29]

Similarly, since $[w - f(w, 0))]$ is strictly positive and continuous on $[w^*, \bar{w}]$, it remains uniformly bounded below by some positive number β. Thus, there exists an integer n_0 such that: $n \geqq n_0 \Leftrightarrow f^{(n)}(\bar{w}, 0) < w^*$, where $f^{(n)}(\cdot, 0)$ denotes the n-th iterate of $f(\cdot, 0)$.

Let $m = \max(n_0, n_1)$. We have

$$P^m(0, [w^*, \bar{w}]) \geqq p(0)^m > 0,$$

and

$$P^m(\bar{w}, [0, w^*]) \geqq (1 - p(\bar{w}))^m > 0.$$

This establishes Lemma 2. ||

From here on, the proof of Proposition 3 proceeds in two immediate steps.

28. where $P^m(w, A)$ denotes the probability of reaching A from w after m generations (i.e. after m iterations of the Markov Process defined by P).

29. n_1 is less than or equal to the smallest integer n such that $n \cdot \alpha > w^*$. Similarly n_0 is at most equal to the smaller integer n such that $\bar{w} - n\beta < w^*$.

Step 1. *Existence*

The existence of an invariant distribution G for the Markov process defined by $P(w, A)$ follows immediately from the monotonicity of P established in Lemma 1 and from Hopenhayn–Prescott's Corollary 4. ||

Step 2. *Uniqueness and Convergence*

This step follows from the monotonicity of P (Lemma 1), its monotone mixing property (Lemma 2), and Hopenhayn–Prescott's theorem 2 and Corollary 2.

Proposition 3 is now fully established. ||

Proof of Proposition 4 From (5.4) and (6.1) we have

$$T^*G_{SB}(w) - T^*_{FB}G_{SB}(w) = \int_{\psi(w,1)}^{\psi(w,0)} (a - p(x))dG_{SB}(w).$$

Since $\psi(w, 1) < \psi(w, 0)$[30] and $p(x) \leqq a$, we have

$$T^*G_{SB}(w) = G_{SB}(w) \geqq T^*_{FB}G_{SB}(w),$$

with a strict inequality if $\psi(w, 1) < 1$ or equivalently if $w < r(1-\delta)$.

Since T^*_{FB} is increasing we also have

$$T^*_{FB}G_{SB}(w) \geqq T^*_{FB}(T^*_{FB}G_{SB}(w)),$$

and more generally

$$T^*_{FB}G_{SB}(w) \geqq T^{*n}_{FB}G_{SB}(w), \text{ for all } n > 1.$$

(T^{*n} denotes the n-th iterate of T^*).

Since $T^{*n}_{FB}G_{SB}$ converges weakly to G_{FB} when $n \to +\infty$ (as in Proposition 3), we know that eventually $G_{SB}(w) \geqq G_{FB}(w)$ for all w, with a strict inequality for $w < r(1-\delta)$. This establishes Proposition 4. ||

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REFERENCES

AGHION, P. and BOLTON, P. (1992), "Distribution and Growth in Models of Imperfect Capital Markets," *European Economic Review*, **36**, 603–611.

AGHION, P. and BOLTON, P. (1993), "A Theory of Trickle-Down Growth and Development" (mimeo, Nuffield College, Oxford).

ANDREONI, J. (1989), "Giving with Impure Altruism: Applications to Charity and Ricardian Equivalence", *Journal of Political Economy*, **97**, 1447–1458.

BANERJEE, A. V. and NEWMAN, A. F. (1991), "Risk-Bearing and the Theory of Income Distribution", *Review of Economic Studies*, **58**, 211–255.

BANERJEE, A. and NEWMAN, A. (1993), "Occupational Choice and the Process of Development", *Journal of Political Economy*, **101**, 274–298.

BERNANKE, B. and GERTLER, M. (1990), "Financial Fragility and Economic Performance", *Quarterly Journal of Economics*, **105**, 87–114.

BERNANKE, B. and GERTLER, M. (1989), "Agency Costs, Collateral, and Business Fluctuations", *American Economic Review*, **79**, 14–31.

CHAMPERNOWNE, D. (1953), "A Model of Income Distribution", *Economic Journal*, **63**, 318–351.

CINLAR, E. (1975), *Introduction to Stochastic Processes* (Englewood Cliffs, New Jersey: Prentice Hall).

30. Indeed, we have

$$\psi(w, 1) = z \Leftrightarrow w = f(z, 1) \text{ and}$$

$$\psi(w, 0) = y \Leftrightarrow w = f(y, 0).$$

Thus $z < w < y$ be definition of $f(\cdot, 1)$ and $f(\cdot, 0)$.

COWELL, F. A. (1977), *Measuring Inequality*, (Oxford: Philip Allen).

DIAMOND, D. W. (1984), "Financial Intermediation and Delegated Monitoring, *Review of Economic Studies*, **51**, 393–414.

FIELDS, G. (1980), *Poverty, Inequality and Development* (Cambridge: Cambridge University Press).

GALOR, O. and ZEIRA, J. (1993), "Income Distribution and Macroeconomics", *Review of Economics Studies*, **60**, 35–52.

GLOMM, G. and RAVIKUMAR, B. (1994), "Equilibrium Theories of the Kuznets Curve" (University of Virginia).

GREEN, E. J. (1987), "Lending and the Smoothing of Uninsurable Income", in Prescott, E. and Wallace, N. (eds.), *Contractual Arrangements for Intertemporal Trade*, (Minnesota: University of Minnesota Press).

HOPENHAYN, H. and PRESCOTT, E. (1992), "Stochastic Monotonicity and Stationary Distributions for Dynamic Economies", *Econometrica*, **60**, 6.

LOURY, G. (1981), "Intergenerational Transfers and the Distribution of Earnings", *Econometrica*, **49**, 843–867.

MIRRLEES, J. (1971), "An Exploration in the Theory of Optimum Income Taxation", *Review of Economic Studies*, **38**, 175–208.

PIKETTY, T. (1993), "Imperfect Capital Markets and Persistence of Initial Wealth Inequalities", *Working Paper STICERD-LSE*, No. TE92255.

SAPPINGTON, D. (1983), "Limited Liability Contracts between Principal and Agent", *Journal of Economic Theory*, **29**, 1–21.

STIGLITZ, J. E. and WEISS, A. (1981), "Credit Rationing in Markets with Imperfect Information", *American Economic Review*, **71**, 393–410.