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# Optimal Debt Structure and the Number of Creditors

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Within an optimal contracting framework, we analyze the optimal number of creditors a company borrows from. We also analyze the optimal allocation of security interests among creditors and inter-creditor voting rules that govern renegotiation of debt contracts. The key to our analysis is the idea that these aspects of the debt structure affect the outcome of debt renegotiation following a default. Debt structures that lead to inefficient renegotiation are beneficial in that they deter default, but they are also costly if default is beyond a manager's control. The optimal debt structure balances these effects. We characterize how the optimal debt structure depends on firm characteristics such as its technology, its credit rating, and the market for its assets.

## I. Introduction

Much of corporate finance theory is devoted to understanding the trade-off between debt and equity financing. Yet it is striking that firms almost always resolve this trade-off by choosing debt over eq-

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uity. From 1946 to 1987, debt issues accounted for 85 percent of all external financing and equity only 7 percent. Thus if one wants a theory of the composition of external financing, it may be more important to understand the structure of debt financing than the choice between debt and equity.

The goal of this paper is to analyze debt structure using an optimal (but incomplete) contracting framework. We focus on three aspects of debt structure. First, what determines the number of creditors a company should borrow from? Second, how should security interests be allocated among creditors? Third, what sort of voting rules should govern changes in the debt contract?

In our model, an optimal debt structure tries to meet two objectives. First, there should be little loss in value when the firm is liquidated: the costs of financial distress should be low. Second, it should discourage firms from defaulting. As we shall see, these objectives sometimes conflict, and an optimal debt structure must balance the two.

We make this point in a model similar to those of Hart and Moore (1989) and Bolton and Scharfstein (1990). In these models, there are two types of defaults: *liquidity* defaults, in which a firm does not have the cash to make debt payments; and *strategic* defaults, in which a firm defaults because managers want to divert cash to themselves. In the absence of default penalties, firms would always choose to default and creditors would be unwilling to lend. Debt contracts reduce the incentive for strategic defaults by giving creditors the right to liquidate the company's assets following a default. But the right to liquidate results in inefficiencies following liquidity defaults. An optimal contract balances the benefits of deterring strategic defaults against the costs of realizing a low liquidation value in a liquidity default.

Debt structure affects this trade-off because it affects the price at which creditors can sell the firm's assets following a default. Suppose that the firm's assets are worth more combined than separated: the whole is worth more than the sum of the parts. Then if there are many creditors, each of whom is secured by a different asset, a buyer would have to get a large number of them to agree to a sale to maximize the combined value of the assets. But once the buyer has struck a deal with one creditor, his marginal valuation of the other assets rises. So he will end up paying more for the remaining assets than if those assets were sold on a stand-alone basis.

By influencing the price at which creditors sell the firm's assets, debt structure affects both the manager's incentive to default strategically and the expected liquidation value of the firm. Following a strategic default, the manager will have to pay more to stop the creditors from liquidating the assets when there are many creditors than when there is only one creditor. Therefore, borrowing from many creditors

disciplines managers by lowering their payoffs from a strategic default; they have less incentive to divert cash flow to themselves.

However, there is a cost of having multiple creditors. When the firm defaults for liquidity reasons, creditors will try to sell their assets to another firm—the second-best user of the assets. This firm will have to pay more for the assets if there are multiple creditors. But while the high price of the assets benefits creditors if there is a buyer willing to bid, there is a potential downside: the buyer may not have the incentive to sink the costs of becoming informed about the firm's assets. Thus, from an *ex ante* perspective, the liquidation value can actually be lower when there are multiple creditors. While borrowing from multiple creditors provides managerial discipline, doing so can also reduce efficiency when the firm defaults because of liquidity problems.

If a firm chooses to borrow from more than one creditor, then the allocation of security interests among creditors and the voting rules that govern the sale of assets also affect efficiency. We show that security interests and voting rules can be chosen to balance the benefits of deterring strategic defaults against the cost of liquidity defaults.

In addition, we try to link each of these aspects of debt structure—the number of creditors, the allocation of security interests, and voting covenants—to observable firm characteristics. Our principal finding is that it is optimal for firms with low credit quality to maximize liquidation values: by borrowing from just one creditor, by giving only one creditor a security interest, and by adopting voting rules that make it easier to complete an asset sale or debt restructuring.

By contrast, it is optimal for firms with high credit quality to have debt structures that make strategic default less attractive: by borrowing from multiple creditors, by giving each equal security interests, and by adopting voting rules that allow some creditors to block asset sales.

Finally, we show how optimal debt structure depends on how efficiently assets can be redeployed to other users, the cyclicity of the industry, and the degree to which the assets are worth more together than apart (what we call asset complementarity).

Our results on the number of creditors could be interpreted as suggesting a trade-off between bank debt and public debt since firms often have many public debt holders but few banks. The model would therefore suggest that firms with low credit quality, those in noncyclical industries, and those with highly complementary assets will issue bank debt, whereas firms with the opposite characteristics will issue public debt. However, it would be misleading to put too much weight on this interpretation. Bank debt can be syndicated to many banks, and some public debt instruments are held by only a few investors.

Thus we would prefer to interpret the results as suggesting when bank debt is syndicated and when public debt is widely held.

Of course, we are not the first to consider debt structure issues. There are a number of strands of the literature. One strand, which includes Bulow and Shoven (1978), White (1980), and Gertner and Scharfstein (1991), analyzes how debt structure affects liquidation values when firms are in financial distress. But there is no consideration of how this, in turn, affects optimal debt structure. Another strand, including papers by Smith and Warner (1979), Berkovitch and Kim (1990), and Hart and Moore (1990), considers how priority and security can be allocated to mitigate over- and underinvestment problems. Diamond (1991) and Hoshi, Kashyap, and Scharfstein (1992) consider the trade-off between bank debt and public debt. The line of work closest in spirit to ours includes Hart and Moore (1989), Diamond (1992, 1993), Rajan (1992), and Dewatripont and Maskin (1995), who examine how optimal debt structure is determined by ex post bargaining considerations.

This paper is also related to the literature on corporate governance. While that literature explores optimal voting rules for shareholders (e.g., whether one share—one vote is optimal as in Grossman and Hart [1988], Harris and Raviv [1988], and Gromb [1993]) and the optimality of having large shareholders (as in Zingales [1991]), our paper is the first to analyze similar governance issues for debt holders.

The paper is organized as follows: Section II outlines the basic model. Section III presents the trade-off between one and two creditors. Section IV discusses the optimal allocation of security interests, and Section V takes up the issue of voting rules. Concluding comments are in Section VI.

## II. The Basic Model

We study a two-period investment project that requires an outlay of  $K$  at an initial date 0 for the purchase of two physical assets,  $A$  and  $B$ . The project is run by a manager who has no wealth. At date 1, the project produces a random cash flow of  $x$  with probability  $\theta$  or zero with probability  $1 - \theta$ .

The project also produces cash flow at date 2, which depends on how the assets are deployed at date 1. If the manager continues to manage the assets, the date 2 cash flow is  $y$ . This alternative generates the highest possible cash flow. The project can also be liquidated (separated from the manager) in one of two ways. The project's investors can manage the assets, in which case they generate no cash flow at date 2. Or the assets can be managed by another manager, generating a date 2 cash flow of  $\alpha y$ , where  $\alpha \leq 1$ . One might think

of this other manager as another firm in the industry. These assumptions reflect the idea that other managers can get more out of the assets than the investors, but perhaps not as much as the original manager. The assumption that investors get zero is just a normalization. Everyone is risk neutral, and the riskless interest rate is zero.

If one were able to write a complete financial contract, it would specify date 1 and date 2 payments from the firm to the investors conditional on the uncertain first-period cash flow. The project would never be liquidated, and investors would receive payments with an expected value of  $K$ . There is a continuum of contracts that satisfy these conditions.

We assume, however, that contracts are incomplete in that they cannot be made contingent on cash flow. Although cash flow is assumed to be observable to both the manager and the investors, it is not observable to outside parties. Thus a court could not enforce a contract contingent on realized cash flows. In the language of Grossman and Hart (1986), cash flow is “observable” but not “verifiable.”

This assumption is meant to capture the idea that managers have some ability to divert corporate resources to themselves at the expense of outside investors. We do not mean that managers actually “take the money and run,” but rather that they may choose to spend it on perquisites or unprofitable new projects that they value more than investors. Such perk consumption and investment may be difficult to distinguish from appropriate business decisions and thus impossible to control through contracts.

While one cannot write contracts contingent on cash flows, we do assume that contracts contingent on the firm’s physical assets are feasible. This assumption is based on the idea that it is harder to divert physical assets than to divert cash flow. This distinction between cash flows and assets is emphasized by Hart and Moore (1989).

In our model, the most general type of contract specifies that if the firm makes a payment  $R^t$  at date  $t$ , creditors have the right to liquidate some fraction of the assets,  $z^t(R^t) \leq 1$ , with probability  $\beta^t(R^t) \leq 1$ . A special case of this contract is a standard debt contract with the principal and interest due at date 1. The contract specifies that if the firm repays some amount, say  $D$ , at date 1, then it has the right to continue without intervention by creditors. If, however, the firm does not make the payment, the creditor has the right to seize the company’s assets. This amounts to setting  $z^1 = \beta^1 = 1$  if  $R^1 < D$ ,  $z^1 = \beta^1 = 0$  if  $R^1 \geq D$ , and  $z^2 = \beta^2 = 0$ .

In our model, however, the standard debt contract is not, in general, optimal. Such a contract is generally too harsh: it leads to too much liquidation if the firm does not make the payment  $D$ . It may be better not to liquidate all the firm’s assets, and only with probability

less than one. (We shall show that partial liquidation is not optimal, though we shall see that probabilistic liquidations are optimal.)

The contract therefore specifies that if the manager makes the payment  $R_x$  when cash flow is  $x$ , investors have the right to liquidate the assets with probability  $\beta_x$ . If the manager makes the payment  $R_0$  when cash flow is zero, investors have the right to liquidate the assets with probability  $\beta_0$ . Note that investors cannot induce the manager to pay anything at date 2 because there are no assets to liquidate.

Given this contract, the firm's expected payoffs are

$$\theta[x - R_x + (1 - \beta_x)y] + (1 - \theta)[-R_0 + (1 - \beta_0)y]. \quad (1)$$

The investor's expected profits are

$$\theta(R_x + \beta_x L_x) + (1 - \theta)(R_0 + \beta_0 L_0) - K, \quad (2)$$

where  $L_x$  and  $L_0$  refer to the liquidation value of the assets in the hands of the creditor when cash flow is  $x$  and zero, respectively. Although investors get no direct value from owning the assets, we shall see later that they can sell the assets. For the moment, we write the proceeds of this asset sale as  $L_x$  and  $L_0$ .

Given that the manager has no wealth of his own, payments cannot exceed the available funds at date 1, that is,  $R_0 \leq 0$  and  $R_x \leq x$ . These payments must also satisfy an incentive constraint that ensures that the manager has an incentive to pay  $R_x$  when cash flow is  $x$ . One can write this incentive constraint as

$$x - R_x + (1 - \beta_x)y \geq x + \beta_0 S + (1 - \beta_0)y. \quad (3)$$

Two points are worth noting about this constraint. First,  $S$  denotes the utility the manager receives from paying out  $R_0$  when cash flow is really  $x$  and the creditor is entitled to liquidate the asset. One might be tempted to think that this is zero. However, this assumes that it is optimal to follow through with the liquidation with probability  $\beta_0$  when the firm pays out zero. We shall see that this is not the case, so for the moment we write the payoff as  $S$ .

Second, in principle, there should also be another constraint stating that the manager has an incentive to pay out  $R_0$  when cash flow is zero rather than  $R_x$ . However, since  $R_x$  will be greater than zero, it is never feasible for the manager to pay out  $R_x$  when he has no cash flow. So this constraint is not binding.

The optimal contract maximizes the firm's expected payoffs (1) subject to the incentive constraint (3) and the constraint that the creditor's profits (2) are nonnegative. It is straightforward to establish two preliminary results: (i)  $\beta_x = 0$ : it is never optimal to liquidate when the firm makes the payment  $R_x$ ; and (ii)  $R_0 = 0$ .

Setting  $\beta_x = 0$  is optimal because it achieves the two goals of max-

imizing the surplus from the contract and providing the manager with incentives to pay out cash flow. If  $\beta_x$  were strictly positive, then the creditor could be made no worse off by reducing  $\beta_x$  by some infinitesimal amount  $\epsilon$  and increasing  $R_x$  by  $\epsilon L_x$ . Such a change would increase the manager's expected payoff by  $\theta\epsilon(y - L_x)$  and relax the incentive constraint.

Setting  $R_0 = 0$  also maximizes the surplus from the contract and provides the manager with incentives. The only alternative is to set  $R_0 < 0$ , that is, pay the manager something in the bad state. But it would be better to increase  $R_0$  and reduce  $\beta_0$ . As before, given that liquidation is inefficient, this increases the manager's expected payoff while making investors no worse off.

Given these results, the incentive constraint reduces to

$$R_x \leq \beta_0(y - S). \quad (4)$$

This constraint must be binding at an optimum; otherwise it would be optimal to set  $\beta_0 = 0$  and the manager would have no incentive to pay out any cash flow. The nonnegative profit constraint must also be binding; otherwise  $R_x$  could be reduced while making the manager better off and still satisfying the constraint. Thus substituting the incentive constraint into the nonnegative profit constraint implies that

$$\beta_0[\theta(y - S) + (1 - \theta)L_0] - K = 0. \quad (5)$$

Using (5), we can write the manager's expected payoff as

$$\theta x + y - K - (1 - \theta)\beta_0(y - L_0). \quad (6)$$

The first three terms in (6) sum to the net present value of the project in the first-best case of no liquidation. The last term is the expected efficiency loss from contractual incompleteness; it is the probability of a liquidation,  $(1 - \theta)\beta_0$ , times the loss in value from a liquidation,  $y - L_0$ . Given that the manager's expected payoff is decreasing in  $\beta_0$  and the creditor's profits are increasing in  $\beta_0$ , the optimal solution is to set  $\beta_0$  equal to its minimum feasible level, that is, the solution to (5). Thus

$$\beta_0 = \frac{K}{\theta(y - S) + (1 - \theta)L_0}. \quad (7)$$

There is a feasible solution provided that the  $\beta_0$  that solves (7) is no greater than one. That is, there will be a solution if  $K$  is less than the maximum feasible gross profit of the creditors,  $\pi \equiv \theta(y - S) + (1 - \theta)L_0$ .

The main feature of the optimal contract is that there is liquidation when the firm's first-period performance is poor. This is not because poor early performance signals poor future performance, that is,



they are uncorrelated. Rather, the threat of liquidation serves as a way of inducing the manager to pay out cash flow instead of diverting it to himself. If he were to keep the first-period cash flow, the creditor would be given the right to liquidate the assets with probability  $\beta_0$ , which generates a second-period payoff of  $S < y$ . When the first-period cash flow is in fact  $x$ , the manager would (weakly) prefer to pay  $R_x = \beta_0(y - S)$  than to keep the cash and forfeit  $\beta_0(y - S)$ . Unfortunately, if first-period cash flow turns out to be zero, the manager has no choice but to have the assets liquidated with probability  $\beta_0$  since he lacks the cash to prevent such a liquidation.

Recall that we have ruled out contracts in which creditors limit themselves to liquidating only one of the assets. We show in the Appendix that it is never optimal for creditors to limit themselves in this way. There are two reasons why such contracts are not optimal. First, liquidating only part of the asset means that  $\beta_0$  must be higher than if all of the asset is liquidated. Given asset complementarity, it is better to liquidate all of the asset less often than to liquidate only one of the assets more often. Second, when only one of the assets is liquidated, an outside buyer has less to gain from incurring a fixed transaction cost of becoming informed. Therefore, the asset is sold with lower probability, and the expected liquidation value is lower under partial liquidation.

### III. One Creditor or Two?

The model above is incomplete because it is not specific about the liquidation value  $L_0$  and the manager's payoff following default,  $S$ . In this section, we fill in these important details and begin to address the question of when it is optimal to borrow from one or two creditors. The trade-offs that emerge in this model capture all the trade-offs that would emerge in a model of more than two creditors. In fact, we have analyzed such a model, and all the basic results carry through.

To emphasize the dependence of the variables on the number of creditors, we write  $L_0(n)$ ,  $S(n)$ , and  $\beta_0(n)$ , where  $n$  is the number of creditors. We first consider the case of one creditor.

#### A. One Creditor

We have assumed that creditors get no value from managing the asset. Therefore, they may choose to sell the asset to an outside buyer—another firm in the industry or some other investor who can manage the asset. As noted, the outside buyer's value of the asset is  $\alpha y$ , where  $0 < \alpha \leq 1$ . Suppose that the buyer incurs a cost,  $c$ , to

bargain for control of the assets. This cost  $c$  is unknown at date 0 and is distributed uniformly along the interval  $[0, \bar{c}]$ . Thus the buyer will bargain provided that  $c$  is less than his share of the surplus from bargaining.

Now suppose that cash flow is zero and the firm defaults. Control of the assets passes to the creditor. We assume Nash bargaining so that the buyer and the creditor share the surplus  $\alpha y$  equally provided that the buyer incurs the transaction cost. Thus the buyer gets  $1/2\alpha y$  and the creditor receives the remaining  $1/2\alpha y$ . It pays for the buyer to bargain provided that  $c \leq 1/2\alpha y$ ; the assets are therefore sold with probability  $(1/2\alpha y)/\bar{c}$  assuming, as we do, that  $1/2\alpha y < \bar{c}$ . So, at date 0, the expected value of the liquidation payoff when cash flow is zero is given by

$$L_0(1) = \frac{\alpha^2 y^2}{4\bar{c}}. \quad (8)$$

One should keep in mind that we have assumed that the buyer has the up-front cash to pay for the asset at date 1. Given that cash flow is not verifiable, the buyer would not be able to borrow the purchase price, promising to repay the loan out of date 2 cash flows. Thus, as Shleifer and Vishny (1992) have argued, assets may get sold to the most liquid bidder rather than to the highest-value user.

What happens following a strategic default? There are now three potential parties to the negotiations: the outside buyer, the creditor, and the original manager. Since the original manager is the most efficient user of the assets, he will buy them back from the creditor (assuming as we do that  $x$  is large enough). To analyze this case, we would need a model of three-person bargaining such as the Shapley value generalization of Nash bargaining. In such a model, even though the outside buyer never ends up buying the assets, the manager would end up paying the outsider something because of his positive Shapley value. (One can show that this is  $1/6\alpha y$ .)

However, for several reasons, we assume that the outside buyer does not enter the bargaining following a strategic default. First, because the Shapley value model implies that the outsider gets payments without getting control of the assets, "bogus" buyers could claim to value the assets at  $\alpha y$  in order to extract some payment from the manager. So it may be hard to distinguish between genuine outside buyers and bogus ones. Second, in an alternative model in which the manager and the outside buyer bid against each other, the outsider would always lose against the more efficient manager. Therefore, he would have no incentive to incur the transaction cost of becoming informed about the assets. Finally, we have solved the model under

the alternative assumption that the outside buyer does bargain. Nothing of substance changes.

Without the outside buyer, there are two parties to the bargain, so the manager and the creditor split the surplus of  $y$  equally according to Nash bargaining. Thus

$$S(1) = \frac{y}{2}. \quad (9)$$

Note that the manager does not incur a cost to bargain. The assumption that the cost is zero is just a normalization and reflects the idea that the manager is more informed about the firm and therefore has a lower cost of preparing a bid.

We can now use (8) and (9) to write  $\beta_0$  in terms of the exogenous parameters,  $\alpha$ ,  $y$ , and  $\bar{c}$ . Specifically,  $\beta_0(1) = K/\pi(1)$ , where

$$\pi(1) \equiv \theta \frac{y}{2} + (1 - \theta) \left( \frac{\alpha^2 y^2}{4\bar{c}} \right). \quad (10)$$

It is straightforward to establish that  $\beta_0(1)$  is decreasing in  $\theta$  so that the inefficiency,  $(1 - \theta)\beta_0(1)[y - L_0(1)]$ , is lower for high- $\theta$  (low-default-risk) firms. Also,  $\beta_0(1)$  is decreasing in  $\alpha$  and  $L_0(1)$  is increasing in  $\alpha$ , so the inefficiency is also decreasing in  $\alpha$ .

### B. Two Creditors

Now suppose that the firm raises capital from two creditors to purchase the two assets,  $A$  and  $B$ . In order to analyze this case, we have to specify what happens in a default. Suppose, for the moment, that creditor  $a$  is "secured" by asset  $A$  and gets to liquidate that asset only, and creditor  $b$  is secured by asset  $B$  and gets to liquidate that asset only. Each creditor makes his own decision about whether to take possession of the asset, renegotiate with the manager, or sell it to an outside buyer. In Section V, we shall consider alternative mechanisms by which creditors decide when to sell the assets.

The critical assumption that drives the analysis is that assets  $A$  and  $B$  are worth more together than apart. Let  $y^A$  be the date 2 cash flow from asset  $A$  if it is used without asset  $B$ , and let  $y^B$  be the date 2 cash flow from asset  $B$  if it is used without asset  $A$ . Our assumption amounts to the condition that  $y - y^A - y^B > 0$ . We label this difference  $\Delta$ .

There are two rationales behind the assumption that  $\Delta$  is strictly positive. The first is that the two assets are essentially the same and there are increasing returns to scale. For example, asset  $A$  may be

one plant and asset  $B$  another, and there are benefits to having a large market share.

The second rationale—and the one we shall stress—is that there are project-specific complementarities between the assets. For example, a building designed to manufacture aircraft equipment is more valuable when used together with the machines designed to build aircraft. The building could be used to manufacture something else, or even other aircraft, but it is most efficient to use it for its original purpose. Certainly not all assets exhibit this sort of complementarity. Farmland and farm equipment can both be separated from each other without any real diminution in value: a tractor can be used just as efficiently on other farms, and farmland can be used just as efficiently with other tractors.<sup>1</sup>

Following a liquidity default, there are three parties to the bargaining: the outside buyer and the two creditors. As before, the manager is not in a position to buy the assets from the creditors because he has no cash to offer. Since the most efficient use of the assets is to keep the assets together, the outside buyer will buy both of them if it pays for him to incur the transaction cost  $c$ .

In this case, we have to calculate the creditors' Shapley values. The basic idea of the Shapley value is that the bargaining power of each agent is related to his marginal value to the various coalitions that could form in the process of bargaining. In particular, the Shapley value gives each agent his expected marginal value to a coalition, where the expectation is taken over all coalitions to which the agent might belong.

Creditor  $a$ 's Shapley value is  $\frac{1}{2}\alpha y^A + \frac{1}{3}\alpha\Delta$ . This was calculated as follows. With probability  $\frac{1}{3}$ , creditor  $a$  is part of a coalition with creditor  $b$  and the buyer. His marginal value to this coalition is  $\alpha(y - y^B)$  since the buyer and  $b$  can get  $\alpha y^B$  without  $a$ , but with  $a$  they can get  $\alpha y$ . With probability  $\frac{1}{6}$ , creditor  $a$  is in a coalition with only the buyer, and his marginal value is  $\alpha y^A$ . With probability  $\frac{1}{6}$ , creditor  $a$  is in a coalition only with creditor  $b$ , and his marginal value is zero since they need the buyer to get any value from the assets. Finally, with probability  $\frac{1}{3}$ , creditor  $a$  is in a coalition by himself, which has no value. The weighted average of these marginal values is creditor  $a$ 's Shapley value,  $\frac{1}{3}\alpha(y - y^B) + \frac{1}{6}\alpha y^A$ . This can be written as  $\frac{1}{2}\alpha y^A + \frac{1}{3}\alpha\Delta$ .

Analogously, creditor  $b$ 's Shapley value is  $\frac{1}{2}\alpha y^B + \frac{1}{3}\alpha\Delta$ . The sum of the two creditors' Shapley values is  $\frac{1}{2}\alpha y + \frac{1}{6}\Delta$ . Thus the two

<sup>1</sup> It might also be possible for  $\Delta$  to be negative if there are decreasing returns to scale. We do not consider this possibility because we think that the case of asset complementarity is more relevant.

creditors together get more than a single creditor would if he controlled both assets  $A$  and  $B$ .

This result clearly depends on the assumption of asset complementarity ( $\Delta > 0$ ). Recall that an agent's Shapley value is higher to the extent that he adds value to each possible coalition of agents. For example, creditor  $a$  adds a value of  $\alpha(y^A + \Delta)$  to the coalition of creditor  $b$  and the outside buyer, but adds a value of  $\alpha y^A$  only if creditor  $b$  is not in the coalition. So creditor  $a$  is able to capture some of the benefits of asset complementarity.

Although the Shapley value is not based on an explicit model of multiperson bargaining, it captures some of the holdup problems arising in multiperson bargaining. These effects are modeled, for example, by Gertner (1990) in an extensive-form bargaining game with three players, asymmetric information, and asset complementarity.

It follows then that the outside buyer's Shapley value is  $1/2\alpha y - 1/6\alpha\Delta$  in the event he bargains, and therefore he will bargain with probability  $(1/\bar{c})(1/2\alpha y - 1/6\alpha\Delta)$ . So the expected payoff to the two creditors is

$$\begin{aligned} L_0(2) &= \frac{1}{\bar{c}} \left( \frac{\alpha y}{2} + \frac{\alpha \Delta}{6} \right) \left( \frac{\alpha y}{2} - \frac{\alpha \Delta}{6} \right) \\ &= \frac{\alpha^2 y^2}{4\bar{c}} - \frac{\alpha^2 \Delta^2}{36\bar{c}} \\ &< \frac{\alpha^2 y^2}{4\bar{c}} \\ &= L_0(1). \end{aligned} \tag{11}$$

Thus two creditors receive more than one if the outside buyer enters the bargain, but they receive less from an ex ante perspective since they are less likely to find a buyer of the assets. The reason is that the low payoff of making the bid discourages the outside buyer from incurring the cost  $c$  of bidding. That the latter effect is strong enough to outweigh the former follows from our assumption that the transaction costs are uniformly distributed. We discuss the importance of these assumptions later in this section.

The calculation of  $S(2)$  follows similarly. Again, there are three parties to the bargain: the outside buyer and the two creditors. Since the manager is the most efficient user of the assets, he will end up buying them back from the creditors. The manager's Shapley value is given by

$$S(2) = \frac{y}{2} - \frac{\Delta}{6}. \tag{12}$$

Given  $\Delta > 0$ , this value is clearly lower than the original manager's Shapley value when there is only one creditor,  $1/2y$ . As in the case of a liquidity default, asset complementarity forces the buyer (in this case the manager) to pay more because control of an asset is worth more if the buyer already owns the complementary asset.<sup>2</sup>

Given (7), we know that  $\beta_0(2) = K/\pi(2)$ , where

$$\begin{aligned}\pi(2) &= \theta \left( \frac{y}{2} + \frac{\Delta}{6} \right) + (1 - \theta) \left( \frac{\alpha^2 y^2}{4\bar{c}} - \frac{\alpha^2 \Delta^2}{36\bar{c}} \right) \\ &= \pi(1) + \theta \frac{\Delta}{6} - (1 - \theta) \frac{\alpha^2 \Delta^2}{36\bar{c}}.\end{aligned}\tag{13}$$

Thus the two creditors' maximum gross profit could be greater or less than the single creditor's gross profit.

Borrowing from two creditors is beneficial in that the manager has to pay them more than one creditor following a strategic default (thus disciplining the manager), but it is costly in that two creditors are paid less than one creditor following a liquidity default (thus leading to more costly financial distress). The result that two creditors receive more following a strategic default but less following a liquidity default may seem inconsistent, but there is an important idea underlying it. In a liquidity default, the high price of the assets required when there are two creditors is costly in that it discourages outside buyers from bidding. But when there is a strategic default, the original manager always buys back the assets, so there is no cost of borrowing from two creditors.

### C. Which Is Better?

We are now in a position to compare the two alternatives. Recall that the inefficiency is proportional to  $\beta_0(n)[y - L_0(n)]$ . Thus there are two ways in which the number of creditors affects this measure. First, it affects the liquidation value. From (11), we know that the liquidation value is always higher when there is only one creditor. So, on this score, the manager would always choose to borrow from one creditor.

<sup>2</sup> When there are two creditors, there is the possibility that the manager could default on one creditor and not on the other. However, one can show that when  $\Delta > 0$ , the manager has no incentive to do so. That is, the manager is better off making no payment and renegotiating with both creditors than making the payment to one creditor and renegotiating with the other creditor. In the case in which  $\Delta < 0$ , the manager *would* have an incentive to default on only one creditor. To avoid such an outcome, the contract could specify a cross-default provision (as is common in practice) giving each creditor the right to seize all the firm's assets in the event of a partial default. There would then be no gain from defaulting on only one of the creditors.

Second, the number of creditors affects the liquidation probability; low values of  $\beta_0$  reduce the inefficiency. This liquidation probability, in turn, is low when creditors can require high payments in the good state (because  $S$  is low) or when the liquidation value,  $L_0$ , is high (see eq. [7]). While  $S$  is lower with the two creditors,  $L_0$  is higher with one creditor. So  $\beta_0(2)$  could be greater or less than  $\beta_0(1)$ .

The following three propositions characterize the conditions under which the two-creditor arrangement is more or less efficient than the one-creditor arrangement.

**PROPOSITION 1.** The firm borrows from two creditors when default risk is low ( $\theta$  high) and from one creditor when default risk is high ( $\theta$  low).

The rationale is simple. When default risk is high, the best way to reduce  $\beta_0$  is to have a high liquidation value. This is best achieved with one creditor. By contrast, when default risk is low, the more effective way to reduce  $\beta_0$  is to limit the manager's renegotiation rents, thereby allowing a larger payment in the good state. This is best achieved with two creditors.

The proposition assumes that it is feasible for the firm to borrow from one or from two creditors. However, for low enough  $\theta$ , neither may be feasible or only one of the alternatives may be feasible. Indeed, there are parameters for which the optimal number of creditors is nonmonotonic in  $\theta$ . For high values of  $\theta$ , the firm borrows from two creditors; for intermediate values of  $\theta$ , the firm borrows from one creditor; and for low values of  $\theta$ , the firm borrows from two creditors. This last region is possible because at low  $\theta$  it may not be feasible to borrow from one creditor, but it may be feasible to borrow from two creditors. In particular, this occurs when

$$\begin{aligned} & \theta \frac{y}{2} + (1 - \theta) \left( \frac{\alpha^2 y^2}{4\bar{c}} \right) < K \\ & < \theta \frac{y}{2} + (1 - \theta) \left( \frac{\alpha^2 y^2}{4\bar{c}} \right) + \theta \frac{\Delta}{6} - (1 - \theta) \frac{\alpha^2 \Delta^2}{36\bar{c}}. \end{aligned} \tag{14}$$

**PROPOSITION 2.** The firm borrows from two creditors when asset complementarity is low ( $\Delta$  low) and from one creditor when asset complementarity is high ( $\Delta$  high).

On the one hand, a large  $\Delta$  is beneficial in that it reduces the manager's renegotiation rent; on the other hand, it is detrimental in that it lowers liquidation payoffs. However, as one can see from (11) and (12),  $\Delta$  has a first-order effect on renegotiation rents but only a second-order effect on liquidation payoffs. Thus small increases in  $\Delta$

from zero increase efficiency. Beyond a certain point, the liquidation costs outweigh the renegotiation benefits.

**PROPOSITION 3.** The firm borrows from two creditors when outside buyers place low valuations on the assets ( $\alpha$  small) and from one creditor when their valuations are high ( $\alpha$  high).

As  $\alpha$  increases, liquidation values rise. But, more important, the one-creditor liquidation value rises faster than the two-creditor liquidation value. This effect makes it more attractive for firms to borrow from one creditor when  $\alpha$  is high.

One way to interpret the results is to view  $\alpha$  as a measure of the ease with which assets can be redeployed. Firms with easily redeployable assets will borrow from one creditor. For example, an office building is likely to be easily redeployable: an outside buyer can get just as much value from the office building as the manager. However, highly redeployable assets probably also do not exhibit much complementarity. Indeed, one of the reasons they can be easily redeployed is that they can be easily combined with other assets. The office building is highly redeployable because it can just as easily be combined with the office equipment of an accounting firm as with the office equipment of a law firm. Thus, while firms with highly redeployable assets might want to borrow from one creditor, those assets may exhibit low asset complementarity, suggesting that the firm should borrow from two creditors. Which effect is more important is an open question.

Another interpretation of  $\alpha$  is that it reflects the extent to which outside buyers have liquid resources to purchase the assets. Loosely speaking, when  $\alpha$  is low, buyers do not have the resources to pay for the assets up front, even if they have high fundamental valuations of the assets.

To make this idea more precise, we need to introduce some notation. Let  $p$  be the probability that the buyer has enough cash to buy the asset;  $1 - p$  is the probability that the buyer has no cash at all. If cash flows are correlated in an industry, and we think of the outside buyer as another firm in the industry, then  $p$  should be lower in the low-cash flow state than in the high-cash flow state. Let  $p_0$  be the probability that the outside buyer has the cash when the firm's cash flow is low, and let  $p_x$  be the corresponding probability when cash flow is high. Low values of  $p_0$  relative to  $p_x$  would be characteristic of cyclical industries (such as durables) or those in which firms are subject mainly to common shocks (such as oil).<sup>3</sup> For simplicity, let  $p_x = 1$ .

<sup>3</sup> This point is made by Shleifer and Vishny (1992), who go on to argue that firms in these industries will choose low leverage as a result. Asquith, Gertner, and Scharfstein (1994) present evidence that financially distressed firms are less likely to sell assets when the other firms in the industry are also distressed.



In this setup, the expected liquidation values can be written as

$$L_0(1) = p_0 \frac{\alpha^2 y^2}{4\bar{c}} \quad (15)$$

and

$$L_0(2) = p_0 \left( \frac{\alpha^2 y^2}{4\bar{c}} - \frac{\alpha^2 \Delta^2}{36\bar{c}} \right). \quad (16)$$

These are just the standard liquidation values, given in (8) and (11), times the probability that buyers have enough liquidity in the low-cash flow state to buy the assets.

As  $p_0$  falls, expected liquidation values fall and, more important, the difference between the one-creditor and two-creditor expected liquidation values falls. Thus, when  $p_0$  is small, the cost of borrowing from two creditors (a lower liquidation value) is small. The firm, therefore, borrows from two creditors.

Thus one might argue that firms in cyclical industries or in industries exposed to common shocks will choose to borrow from two creditors. These firms are likely to realize low liquidation values for their assets regardless of the number of creditors, so they are better off taking advantage of the discipline provided by two creditors. By contrast, firms in noncyclical industries or those in industries subject to idiosyncratic shocks—industries in which liquidation values can be high—should borrow from one creditor to maximize the liquidation value.

#### IV. Security Allocation

Our framework also has implications for how security should be allocated between creditors. Should creditor  $a$  be secured by asset  $A$  and creditor  $b$  secured by asset  $B$ , as we have assumed? Or should creditor  $a$  be secured by both assets while creditor  $b$  has no security?

Let  $R_x^a$  be the payment to creditor  $a$  and  $R_x^b$  be the payment to creditor  $b$ . If only  $a$  is secured, then there is only one creditor with whom to negotiate. The liquidation value is therefore equal to  $L_0(1)$  and the renegotiation rent is  $S(1)$ . Each creditor makes zero profits in equilibrium:

$$\theta R_x^a + (1 - \theta)\beta_0 L_0(1) = K^A, \quad (17)$$

$$\theta R_x^b = K^B, \quad (18)$$

where  $K^A$  is the capital invested by creditor  $a$  to purchase asset  $A$ , and  $K^B$  is analogous for creditor  $b$ .

Equation (18) pins down  $R_x^b$ . Combining this with the binding incentive constraint pins down  $R_x^a$ :

$$R_x^a + R_x^b = \beta_0[y - S(1)]. \quad (19)$$

Substituting (18) and (19) into (17), we see that  $\beta_0$  is solved by

$$\beta_0\{\theta[y - S(1)] + (1 - \theta)L_0(1)\} = K, \quad (20)$$

which is precisely the condition determining  $\beta_0(1)$ . Thus giving one creditor all of the security replicates the one-creditor solution. It generates a relatively high liquidation value, but also high renegotiation rents. Giving both creditors security generates a low liquidation value, but also low renegotiation rents.

It follows then that one can use propositions 1, 2, and 3 to characterize when both creditors will be secured and when only one of them will be secured. Firms with high  $\theta$ , low  $\alpha$ , and low  $\Delta$  will give equal security interests to both creditors, whereas firms with low  $\theta$ , high  $\alpha$ , and high  $\Delta$  will grant security to only one creditor.

## V. Voting Rules

Although we do not have an explicit model of bargaining, we have implicitly assumed that each creditor negotiates on an individual basis with the manager or the outside buyer. However, indentures in credit agreements typically specify voting rules that determine how creditors make collective decisions regarding debt renegotiation, asset sales, and many other terms of the debt contract. For example, an indenture to a bank loan might specify that the terms of the agreement can be changed only if a majority of the creditors agree to do so.

In this section, we take up the question of the optimal voting rule to govern renegotiation. To do this in the simplest possible way, we assume that there is only one asset and that it generates a date 2 value of  $y$ . We also assume that there are  $n > 1$  creditors, each of whom is secured by an equal amount of the asset.

A voting rule specifies that if  $m$  creditors agree to sell the asset to the outsider (in the case of a liquidity default) or the manager (in the case of a strategic default), then the asset gets sold and the proceeds are paid out pro rata. A *majority voting* rule occurs when  $m/n = 1/2$ , a *supermajority voting* rule occurs when  $m/n > 1/2$ , and a *submajority voting* rule occurs when  $m/n < 1/2$ .

The goal here is to solve for the optimal voting rule,  $m/n$ . To do this, we need to first calculate the creditors' payoffs in a liquidity default and the manager's payoff in a strategic default, which we denote by  $L_0(m, n)$  and  $S(m, n)$ , respectively.

First, consider a strategic default. If the voting rule requires  $m$  of the  $n$  creditors to agree, then the manager will contribute a positive value of  $y$  only to those coalitions that have at least  $m$  creditors. Since there are  $n + 1$  parties in the bargaining, this occurs with probability  $1 - [m/(n + 1)]$ . Thus

$$S(m, n) = \left(1 - \frac{m}{n + 1}\right)y. \quad (21)$$

Note that the higher  $m$  is—the more stringent the voting rule—the lower the manager's surplus from renegotiation.

Similarly, following a liquidity default, the outside buyer will add value only to coalitions with  $m$  creditors. So his Shapley value is  $\{1 - [m/(n + 1)]\}\alpha y$  and the probability that he bargains is therefore  $\{1 - [m/(n + 1)]\}(\alpha y/\bar{c})$ . The creditors get the remaining surplus of  $[m/(n + 1)]\alpha y$ . Thus

$$L_0(m, n) = \frac{m}{n + 1} \left(1 - \frac{m}{n + 1}\right) \frac{\alpha^2 y}{\bar{c}}. \quad (22)$$

In this case, an increase in  $m$  has ambiguous effects on  $L_0$ . On the one hand, it increases creditors' payoffs in liquidation because it increases the probability that they will have a positive value to a coalition. On the other hand, it reduces the buyer's Shapley value and discourages him from becoming informed. Differentiating (22) with respect to  $m/(n + 1)$ , we see that the value of  $m/(n + 1)$  that maximizes  $L_0(m, n)$  is  $1/2$ . Thus the maximal value of  $L_0(m, n)$  is  $\alpha^2 y^2/4\bar{c}$  (which is exactly the payoff in the one-creditor case under the alternative mechanism considered in Sec. III).

A couple of points are worth noting before we continue. First,  $m/(n + 1) = 1/2$  is not quite majority rule since  $m/n = 1/2(n + 1)/n > 1/2$ . However, as  $n \rightarrow \infty$ ,  $m/n \rightarrow 1/2$ . So we shall consider this majority rule. Second, majority rule maximizes the expected liquidation value because we assumed a uniform distribution of bidding costs. This is by no means a general result.

Given these values of  $L_0$  and  $S$ , the expected efficiency loss given  $m$  and  $n$ ,  $EL(m, n)$ , can be written as

$$\begin{aligned} EL(m, n) &= (1 - \theta)\beta_0(m, n)[y - L_0(m, n)] \\ &= (1 - \theta)K \frac{1 - v(1 - v)(\alpha^2 y/\bar{c})}{v[\theta + (1 - \theta)v(1 - v)(\alpha^2 y/\bar{c})]}, \end{aligned} \quad (23)$$

where  $v \equiv m/(n + 1)$ .

If integer problems are ignored, the optimal  $v$  minimizes (23). Thus

differentiating with respect to  $v$  gives the following first-order condition:

$$\theta v^2 \frac{\alpha^2 y}{\bar{c}} - \theta - (1 - \theta)(1 - 2v) \frac{\alpha^2 y}{\bar{c}} = 0. \quad (24)$$

The second-order condition is always satisfied.

First, note that the optimal  $v$ ,  $v^*$ , is greater than  $1/2$ , the voting rule that maximizes the liquidation value. At  $v = 1/2$ , (24) is negative, indicating that the efficiency loss could be reduced by increasing  $v$  above  $1/2$ . The reason is that a small increase in  $v$  has a second-order effect in lowering the liquidation value but a first-order effect in lowering the renegotiation rent. This in turn lowers  $\beta_0$ .

Thus, in our model, supermajority voting is always optimal. However, this result is not general; it is driven by the assumption of uniform distribution of  $c$ . What is general is the idea that the optimal voting rule is more stringent than the rule that maximizes liquidation values.

Two results follow immediately from (23).

**PROPOSITION 4.** (i) The optimal voting rule is more stringent for firms with lower default risk ( $v^*$  is increasing in  $\theta$ ). (ii) The optimal voting rule is more stringent when outside buyers have a low valuation of the firm's assets ( $v^*$  is decreasing in  $\alpha$ ).

The reasoning behind these results is simple. High- $\theta$  firms are less likely to default, so the higher liquidation value creditors receive under a lenient voting rule is not particularly valuable. But the lower renegotiation rents made possible by a more stringent rule allow the firm to pay out more cash flow in the good state, which is particularly valuable when  $\theta$  is high.

More stringent voting rules are also preferred by low- $\alpha$  firms. When  $\alpha$  is low, a stringent voting rule has little impact on liquidation values. So the cost of such a rule is small. As a result, low- $\alpha$  firms prefer more stringent voting rules.

Note that this version of the model does not pin down an optimal number of creditors. The trade-offs that emerge in the model of Section III do not arise here because the voting rule can be used to undo the undesirable aspect of having too many or too few creditors.

## VI. Conclusion

This paper analyzes some aspects of debt structure within an optimal contracting framework. Our basic point is that debt structure affects the negotiations that follow default. These negotiation outcomes can be affected by varying the number of creditors, by distributing security interests more or less widely, or by imposing more or less strin-

gent voting rules. An optimal debt structure balances two concerns: on the one hand, it should deter defaults; on the other hand, it should not make unavoidable defaults too costly. The model predicts that low–default risk firms, those with strong asset complementarities, and those in noncyclical businesses will tend to borrow from more creditors, spread out security interests, and adopt more stringent voting rules.

One issue that we have not considered is the role of bankruptcy law. We have unrealistically assumed that firms cannot file for bankruptcy and that creditors pursue their liquidation rights as specified in the debt contract. This issue is important in the context of our model because Chapter 11 specifies a detailed set of rules that govern debt renegotiation.

For example, Chapter 11 imposes a particular voting rule on debt renegotiation: one-half of the creditors in number or two-thirds of the creditors in the face value of their claims need to agree. This voting rule could be more or less stringent than the optimal voting rule,  $v^*$ . If it is less stringent, creditors might choose other aspects of debt structure that make it harder to renegotiate, for example, by borrowing from more creditors or spreading out security interests. Chapter 11 also includes an automatic stay and an exclusivity period, in which debtors can defer debt payments and remain in Chapter 11 for long periods. This lowers the creditors' liquidation value and increases the payoff from strategic default. In response, the optimal debt contract will specify a higher liquidation probability,  $\beta_0$ , to compensate for the lower payouts to creditors.

## Appendix

### *Proof That Partial Liquidation Is Not Optimal*

Let  $L_0^i$  be the liquidation value when asset set  $i = \{A\}, \{B\}, \{AB\}$  is liquidated following a liquidity default. Let  $S^i$  be the manager's payoff following a strategic default when creditors have the right to liquidate asset set  $i$ . In the text we consider the case in which  $i = \{AB\}$ . Here we show that  $i = \{AB\}$  is optimal.

The incentive constraint can now be written as

$$R_x \leq \beta_0(y - S^i). \quad (\text{A1})$$

Given the binding incentive constraint, the nonnegative profit constraint is

$$\beta_0[\theta(y - S^i) + (1 - \theta)L_0^i] - K \geq 0. \quad (\text{A2})$$

The manager's expected payoff can be written as

$$\theta x + y - K - \beta_0(1 - \theta)(y - y^{-i} - L_0^i), \quad (\text{A3})$$

where  $-i$  is the set of assets not in  $i$ .

Given that  $\beta_0$  solves (A2) with equality, the efficiency loss in the case in which  $i$  is liquidated can be written as

$$EL^i = \frac{y - y^{-i} - L_0^i}{\theta(y - S^i) + (1 - \theta)L_0^i} (1 - \theta)K. \quad (A4)$$

Total liquidation is preferred to partial liquidation provided that  $EL^{AB} < EL^A$  and  $EL^{AB} < EL^B$ . We shall compare  $EL^{AB}$  to  $EL^A$ . The comparison to  $EL^B$  is basically the same. The condition is met provided that

$$\frac{y - L_0^{AB}}{\theta(y - S^{AB}) + (1 - \theta)L_0^{AB}} < \frac{y - y^B - L_0^B}{\theta(y - S^A) + (1 - \theta)L_0^A}. \quad (A5)$$

Consider first the case of a single creditor. The values of  $S^{AB}(1)$  and  $L_0^{AB}(1)$  are given in the text. We need to calculate the payoffs in liquidation and following strategic default when there is partial liquidation. Following a strategic default, the surplus from renegotiating is  $y - y^B$  since the manager can get  $y^B$  without the involvement of the creditor. Thus

$$\begin{aligned} S^A(1) &= y^B + 1/2(y - y^B) \\ &= \frac{y + y^B}{2}. \end{aligned} \quad (A6)$$

This is greater than  $S^{AB}(1) = y/2$  given in equation (9).

The analysis of  $L_0^A(1)$  is a bit more complex. Implicit in (A2) is the assumption that the manager holds on to asset  $B$  and the creditor sells asset  $A$ . In order for this to be the case, it must be that

$$\alpha y^A + y^B > \alpha y, \quad (A7)$$

which can be written as

$$\alpha < \frac{y^B}{y^B + \Delta}. \quad (A8)$$

Thus if  $\alpha$  and  $\Delta$  are relatively small, this condition will be satisfied.

If this condition is met, the buyer just buys asset  $A$  from the creditor. It is straightforward to show that

$$L_0^A(1) = \frac{\alpha^2 (y^A)^2}{4\bar{c}}, \quad (A9)$$

which is clearly less than  $L_0^{AB}(1)$ .

Thus substituting these values of  $S^A(1)$  and  $L_0^A(1)$  into inequality (A5) yields

$$\frac{y - \frac{\alpha^2 y^2}{4\bar{c}}}{\theta \frac{y}{2} + (1 - \theta) \frac{\alpha^2 y^2}{4\bar{c}}} < \frac{y - y^B - \frac{\alpha^2 (y^A)^2}{4\bar{c}}}{\theta \frac{y - y^B}{2} + (1 - \theta) \frac{\alpha^2 (y^A)^2}{4\bar{c}}}. \quad (A10)$$

Dividing the numerator and denominator of the left-hand side by  $y$  and the numerator and the denominator of the right-hand side by  $y - y^B$  gives us

$$\frac{1 - \frac{\alpha^2 y}{4\bar{c}}}{\frac{\theta}{2} + (1 - \theta) \frac{\alpha^2 y}{4\bar{c}}} < \frac{1 - \frac{\alpha^2 (y^A)^2}{4\bar{c}(y - y^B)}}{\frac{\theta}{2} + (1 - \theta) \frac{\alpha^2 (y^A)^2}{4\bar{c}(y - y^B)}}. \quad (\text{A11})$$

Given that  $y^A < y - y^B < y$ , this inequality is satisfied.

If condition (A7) is violated, then the outside buyer buys asset  $A$  from the creditor and asset  $B$  from the manager. The outside buyer's Shapley value is

$$\frac{1}{2} \alpha y^A + \frac{1}{3} (\alpha y - \alpha y^A - y^B). \quad (\text{A12})$$

Thus the probability that the outside buyer buys the assets is (A12) divided by  $\bar{c}$ . The creditor's Shapley value is the same as (A12). Given that  $\alpha y - \alpha y^A - y^B > 0$  by (A7), this payoff is greater than  $\frac{1}{2} \alpha y^A$ . Let  $\frac{1}{3} (\alpha y - \alpha y^A - y^B) \equiv z$ .

The expression for the efficiency loss is slightly different in this case. It can be written as

$$\frac{y - y^B - \frac{1}{\bar{c}} (\frac{1}{2} \alpha y^A + z) (\frac{1}{2} \alpha y^A + 2z)}{\theta \frac{y - y^B}{2} + (1 - \theta) \frac{1}{\bar{c}} (\frac{1}{2} \alpha y^A + z)^2} (1 - \theta) K. \quad (\text{A13})$$

We compare the left-hand side of (A11) with (A13). First, divide the numerator and the denominator of (A13) by  $y - y^B$ . Expression (A13) will be greater than (A11) if

$$\frac{\alpha^2 y}{4\bar{c}} > \frac{(\frac{1}{2} \alpha y^A + z) (\frac{1}{2} \alpha y^A + 2z)}{\bar{c}(y - y^B)}. \quad (\text{A14})$$

Tedious but straightforward calculations establish that (A14) is indeed satisfied. Thus, in both cases, that is, when (A7) is satisfied and when it is violated, total liquidation is more efficient than partial liquidation.

Finally, it remains to show that total liquidation is preferred to partial liquidation when there are two creditors. Note, however, that partial liquidation with two creditors is the same as partial liquidation with one creditor since only one of the creditors is given the right to liquidate one of the assets. But we have already shown that one-creditor total liquidation is preferable to one-creditor partial liquidation. Therefore, the relevant choice is between total liquidation with two creditors and total liquidation with one creditor. This is the case we consider in the paper.

### *Proof of Proposition 1*

The firm is indifferent between borrowing from one and borrowing from two creditors when the efficiency losses under the two regimes are equal,

$EL(1) - EL(2) = 0$ , or

$$\frac{y - L_0(1)}{\theta[y - S(1)] + (1 - \theta)L_0(1)} - \frac{y - L_0(2)}{\theta[y - S(2)] + (1 - \theta)L_0(2)} = 0. \quad (A15)$$

This condition can be written as

$$\frac{\Delta y}{6} \left[ \theta \left( \frac{\alpha^2 \Delta}{12\bar{c}} + 1 - \frac{\alpha^2 y}{4\bar{c}} \right) - \frac{\alpha^2 \Delta}{6\bar{c}} \right] = 0. \quad (A16)$$

The condition is met when  $\Delta = 0$  or, for  $\Delta > 0$ , when

$$\begin{aligned} \theta &= \frac{\alpha^2 \Delta}{3\alpha^2 \Delta + 36\bar{c} - 9\alpha^2 y} \\ &\equiv \hat{\theta} \\ &< 1. \end{aligned} \quad (A17)$$

Since (A16) is increasing in  $\theta$ , the firm borrows from two creditors for all  $\theta > \hat{\theta}$ , and the firm borrows from one creditor for all  $\theta < \hat{\theta}$ .

#### *Proof of Proposition 2*

From (A16), it is clear that  $EL(1) = EL(2)$  at two values of  $\Delta$ : at  $\Delta = 0$  and at some positive  $\Delta$ . The positive value of  $\Delta$ ,  $\hat{\Delta}$ , is given by

$$\hat{\Delta} = \frac{\theta \left( 1 - \frac{\alpha^2 y}{4\bar{c}} \right)}{\left( 1 - \frac{\theta}{2} \right) \frac{\alpha^2}{6\bar{c}}}. \quad (A18)$$

This  $\hat{\Delta}$  exists provided that  $\hat{\Delta} < y$ . This condition can be written as

$$\frac{\theta}{1 + (\theta/2)} < \frac{\alpha^2 y}{4\bar{c}}, \quad (A19)$$

which is clearly satisfied for large enough  $\alpha$  and small enough  $\theta$ . Differentiating  $EL(2)$  with respect to  $\Delta$ , we get

$$\begin{aligned} \frac{dEL(2)}{d\Delta} &= -\frac{1}{\pi(2)^2} \left\{ [y - \theta S(2)] \frac{dL_2}{d\Delta} - [y - L(2)] \theta \frac{dS(2)}{d\Delta} \right\} \\ &= \frac{1}{\pi(2)^2} \left\{ \left[ \left( 1 - \frac{\theta}{2} \right) y + \theta \frac{\Delta}{6} \right] \frac{\alpha^2 \Delta}{18\bar{c}} - \left( y - \frac{\alpha^2 y^2}{4\bar{c}} + \frac{\alpha \Delta^2}{36\bar{c}} \right) \frac{\theta}{6} \right\}. \end{aligned} \quad (A20)$$

At  $\Delta = 0$ ,  $EL(2)$  is decreasing in  $\Delta$ , and for large enough  $\Delta$ ,  $EL(2)$  is increasing in  $\Delta$ .

It follows then that at  $\hat{\Delta}$  (if it exists),  $EL(2)$  is increasing in  $\Delta$ . Thus, for all  $\Delta > \hat{\Delta}$ ,  $EL(2) > EL(1)$ , and the firm borrows from one creditor; for all  $\Delta < \hat{\Delta}$ ,  $EL(1) > EL(2)$ , and the firm borrows from two creditors.



*Proof of Proposition 3*

Equation (A16) implies that the firm is indifferent between borrowing from one creditor and borrowing from two creditors for  $\alpha = \hat{\alpha}$ , where  $\hat{\alpha}$  is defined implicitly by

$$\hat{\alpha}^2 = \frac{12\theta\bar{c}}{\theta(3y - \Delta) + 2\Delta}. \quad (\text{A21})$$

This cutoff level of  $\hat{\alpha}$  exists provided that it is less than one, which will be true when  $\theta$  is small or  $y$  and  $\Delta$  are large.

Given that (A16) is decreasing in  $\alpha$ , it follows that the firm borrows from one creditor when  $\alpha > \hat{\alpha}$  and borrows from two creditors when  $\alpha < \hat{\alpha}$ .

*Proof of Proposition 4*

i) Totally differentiating (24) with respect to  $v$  yields

$$\frac{dv^*}{d\theta} = \frac{1 - (1 - 2v)\frac{\alpha^2 y}{\bar{c}} - v^2 \frac{\alpha^2 y}{\bar{c}}}{\frac{\alpha^2 y}{\bar{c}} [2\theta v + 2(1 - \theta)]}. \quad (\text{A22})$$

Substituting the first-order condition reduces the numerator of (A22) to  $-(1 - 2v^*)(\alpha^2 y / \theta \bar{c})$ , which is positive given that  $v^* > 1/2$ . Since the denominator is also positive,  $dv^*/d\theta > 0$ .

ii) Similarly, totally differentiating (24) with respect to  $\alpha$  shows that  $dv^*/d\alpha$  has the same sign as

$$\frac{2\alpha y}{\bar{c}} [(1 - \theta)(1 - 2v^*) - \theta v^{*2}]. \quad (\text{A23})$$

Given that  $v^* > 1/2$ , this expression is negative.

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