Fall 2019 PhD course: Statistical Physics, Markets and Algorithms

http://www.columbia.edu/cu/bulletin/uwb/#/cu/bulletin/uwb/subj/DROM/B9144-20193-001
Fridays 1:15 to 4:15 pm. Full semester.
Instructor: Yash Kanoria

Statistical physics studies the macroscopic phenomena that emerge when a large number of simple units (like atoms or molecules) come together. Deep understanding developed in such study, especially the study of so-called "spin glasses" by Parisi, Mezard and others, has been leveraged to great effect in diverse areas including image processing, machine learning, computer science, and theoretical biology. This research-oriented PhD class will begin with the fundamentals of spin glasses. We will then proceed to the cavity method and message passing algorithms, including exciting recent developments in obtaining (1-\epsilon)-approximation algorithms for a range of problems satisfying "full replica symmetry breaking". Lastly, we will cover some recent work by Peski on matching markets which has a distinct statistical physics flavor, with agents replacing atoms/molecules. More broadly, we will ask if statistical physics can tell us something about market equilibria, and perhaps about how to design better markets.

The course is expected to cover the following.
Statistical Physics (spin glasses) and Algorithms:

- Intro to Markov random fields and spin glasses via the Ising model and maybe the Potts model.
- *Maybe:* Sherrington Kirkpatrick model, replica symmetry breaking, random overlap structures and ultrametricity. D. Panchenko, The Sherrington-Kirkpatrick model, Springer, 2013
- Random Constraint Satisfaction Problems (CSPs) including the satisfiability and clustering phase transitions, and connections to algorithms.
  J Ding, A Sly, N. Sun, Satisfiability Threshold for Random Regular NAE-SAT
  The set of solutions of random XORSAT formulae, M Ibrahimi, Y Kanoria, M Kraning, and A Montanari, 2015. Also Molloy and Achlioptas, 2015.
  Frozen variables in random boolean constraint satisfaction problems, M Molloy, R Restrepo, 2013.
- Cavity method and message passing algorithms, sum-product and max product belief propagation. Algorithms for maximum matching, decoding of error correcting codes. Belief propagation as first order conditions for minimization of free energy. Density evolution and the correlation decay.
- Recent advances in using message passing to get (1-\epsilon)-optimality for problems satisfying full replica symmetry breaking:
  Following the ground-states of full-RSB spherical spin glasses, Eliran Subag (2018).
  The algorithmic hardness threshold for continuous random energy models, Louigi Addario-Berry, Pascal Maillard (2018).
  Optimization of the Sherrington-Kirkpatrick Hamiltonian, A. Montanari (2019).
- Principle of local optimality/correlation decay. Can the principle extend to online decision making, such as online matching?
  Maximum weight independent sets and matchings in sparse random graphs. Exact results using the local weak convergence method, David Gamarnik, Tomasz Nowicki, Grzegorz Swirszcz (2006).

Connections to Markets:

- Do matching markets and related markets minimize free energy? Papers by Marcin Peski:
  Large roommate problem with non-transferrable random utility, accepted at Journal of Economic Theory, 2017
  Utility and entropy in social interactions, July 2017.
- Informed versus uninformed equilibria in markets. Grossman-Stiglitz (1980) and descendants. Is there a phase transition lurking? E.g., is it that when information acquisition costs exceed some threshold, this results in an uninformed equilibrium as a result of “information deadlock” analogous to the clustering phase transition in random CSPs (appearance of a giant 2-core). How to design/operationalize the market to produce good equilibria/release a market from information deadlock?
  Challet, Damien, Matteo Marsili, and Yi-Cheng Zhang. "Minority games: interacting agents in financial markets." *OUP Catalogue* (2013).

More on the Statistical Physics and math background is available here:
https://www.amazon.com/Information-Physics-Computation-Oxford-Graduate/dp/019857083X
https://web.stanford.edu/~montanar/TEACHING/Stat316/stat316.html
http://web.stanford.edu/class/stats369/refs.html

Some recent related papers:
Extremal cuts of sparse random graphs, A Dembo, A Montanari, S Sen, Ann. Prob. 2017.
Capacity lower bound for the Ising perceptron, J Ding, N Sun, STOC 2019.
Mean-field approximation, convex hierarchies, and the optimality of correlation rounding: a unified perspective, Vishesh Jain, Frederic Koehler, Andrej Risteski, STOC 2019.
Persuasion in Networks: Public Signals and k-Cores, O. Candogan, 2019.