Columbia Business School

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B9118: Foundations of Optimization, Fall 2020

Time: Friday, 10:20am–1:35pm
First Day of Class: 9/11/2020
Location: 305 Uris Hall (map)

Course Description

Mathematical optimization provides a unifying framework for studying issues of rational decision-making, optimal design, effective resource allocation and economic efficiency. It is therefore a central methodology of many business-related disciplines, including operations research, economics, game theory, marketing, accounting, and finance. More broadly, techniques and ideas from large scale optimization theory are important engineering tools in areas as diverse as automatic control systems, data analysis and modeling, statistics and machine learning, communications and networks, and computer-aided design. In many of these disciplines, a solid background in optimization is essential for doing research.

This course provides a rigorous introduction to the **fundamental theory of optimization**. It examines optimization theory in continuous, deterministic settings, including optimization in $\mathbb{R}^n$ as well as in more general vector spaces. The course emphasizes the unifying themes (optimality conditions, Lagrange multipliers, convexity, duality) that are common to all of these areas of mathematical optimization. Particular emphasis will be placed on the geometric intuition underlying modern optimization theory. Applications across a range of problem areas will serve to illustrate and motivate the theory that is developed. Further, the class will highlight the importance of problem formulation in the effective application of large scale optimization techniques.

Review sessions will be offered in which students learn how to solve complex optimization problems using **CVX and Python**.

The goal of the course is to provide students with a foundation sufficient to use basic optimization in their own research work and/or to pursue more specialized studies involving optimization theory.

The course is open to all students, but it is designed for entering doctoral students. The prerequisites are calculus, linear algebra and some familiarity with real analysis (e.g., norms, convergence, basic point set topology). Other concepts (e.g., vector spaces) are developed as needed throughout the course.

Course Outline

1. Introduction
   (a) Basic problems and motivating examples
   (b) Review of real analysis and linear algebra
2. Local theory of optimization
   (a) Unconstrained optimization: Weierstrass' Theorem, first- and second-order conditions, gradient methods
   (b) Constrained optimization: Lagrangian, KKT conditions
3. Global theory of optimization
   (a) Convex sets and functions, implications of convexity for optimization
   (b) Duality: geometric interpretation, strong and weak duality, properties of dual problems, duality for LPs and QPs, conjugate duality
4. Problem formulation and applications
   (a) Standard forms: LPs, QPs, SOCPs, SDPs, GPs
   (b) Robust optimization
   (c) Applications: Approximation and fitting, estimation, geometric problems
5. Optimization in vector spaces
   (a) Vector spaces, compactness, dual spaces
   (b) Local theory of constrained optimization, calculus of variations
   (c) Hahn-Banach theorem, minimum norm duality
   (d) Convex duality in Banach spaces

Required Texts

- D. P. Bertsekas, *Nonlinear Programming*, 2nd Edition. Athena Scientific, 1999.
- S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004. Available online at `http://www.stanford.edu/~boyd/cvxbook`.
- D. G. Luenberger, *Optimization by Vector Space Methods*. Wiley, 1969.

Selected References

Real Analysis:

- W. Rudin, *Principles of Mathematical Analysis*, 3rd Edition. McGraw-Hill, 1976.

Linear Algebra:

- G. Strang, *Linear Algebra and Its Applications*, 3rd Edition. Brooks Cole, 1988.

Optimization:

- D. P. Bertsekas, *Convex Optimization Theory*. Athena Scientific, 2009.
- D. G. Luenberger and Y. Ye, *Linear and Nonlinear Programming*, 3rd Edition. Springer, 2008.

Coursework and Grading

Five homework assignments will be given out during the semester. There will be a 24-hour long take-home final exam. The course grade will be the weighted average of the homework (50%) and the final (50%).

Office Hours

Office hours will be offered online via Zoom.

Review Sessions

Coding optimization problems is a skill that is becoming increasingly more useful for students' doctoral research and future professional careers. The objective of review sessions is to teach students how to solve complex optimization problems using CVX (`http://cvxr.com`) and Python (`https://www.python.org`). CVX is a modeling system for convex optimization, allowing constraints and objectives to be specified using standard Python expression syntax. The review sessions are hands-on and interactive: students are expected to bring their laptops to class and learn how to solve problems with their computers.

Honor Code

All students are expected to comply with Columbia Business Schools's Honor Code. You may discuss homework assignments or share materials with students currently taking the course, but you should work individually when writing up assignments and be prepared to explain your work. You may not use materials from people who took this course in previous years or similar courses elsewhere. You must work individually on the take-home exam. If you are unclear about the instructions on any assignment, please ask for clarification. Failure to comply with the Honor Code will result in disciplinary action.