Columbia Business School

Continuous-Time Finance

B9304–Spring 2019 **Professor Neng Wang**

Syllabus

1 Contact Information

- Office Hours: 812 Uris Hall; open-door or by email
- Email: neng.wang@columbia.edu
- TA: Ran Liu, RLiu21@gsb.columbia.edu
- Course materials: I will distribute course materials in class as we move forward. Please email Leticia Jerman at lj2192@columbia.edu. She is super nice and will be extremely helpful.
- Class-room and Time: URIS 327, Wednesday 9:00AM-12:15PM.
  But we need to reschedule several sessions to Fridays.

2 Course Description

This course covers topics on no-arbitrage-based asset pricing (e.g. option pricing, term structure, credit risk), optimal consumption and portfolio choice, general equilibrium/asset pricing theory, dynamic contracting, and dynamic corporate finance theory using continuous-time methods. We cover both the classics and frontier research papers. The specific coverage for the course also depends on your background and preferences. I will not delve into the mathematical details of Stochastic Calculus but rather focus on economic applications and insights. It is very important for you to understand the economic mechanism and insights in these models. Most of you will not work with continuous-time models and hence my philosophy is to prepare you as an intelligent consumer of models and research that use continuous-time methods. I use continuous-time methods to teach economics of finance, rather than force this method onto economic and financial applications.

3 Readings and References

- The "textbook" (broadly interpreted) for the asset pricing part of the course is
  - Duffie, D. [2001]. **Dynamic Asset Pricing Theory**

  Lectures will be based primarily on my notes and sometimes research articles. We will have a tentative course schedule. The exact progression of the course will depend on you. We will adapt our speed and coverage as we move forward.

- A standard reference book is
  - Merton, R. C. [1990]. **Continuous Time Finance**

  which includes a few overview chapters and a collection of Merton's classic articles.

- Additionally, I also recommend the following books:
  - Dumas and Luciano's "The Economics of Continuous-Time Finance"
  - Kerry Back's "Asset pricing and portfolio choice theory"
  - George Pennachi's "Theory of asset pricing"
  - Steve Shreve's "Stochastic Calculus for Finance II: Continuous-Time Models."

4 Course Requirements and Grading

Course requirements include (1) individual homework assignments, (2) a final exam, and (3) a referee report. It is possible that I may decide to do away with either (2) or (3), depending on how the course evolves. If I ask you to write a referee report, you will choose a paper within a subset of papers related to the course theme and approved by me. Writing high-quality referee reports is an important academic skill. Of course, class participation (broadly interpreted) is also part of your course evaluation.

Grading will be based on your overall performance on individual homework assignments, the final exam and the referee report.

Ive intentionally chosen to provide a flexible course requirement. This is one of the few occasions in teaching that the value of ex post flexibility is greater than the value of ex ante commitment.

5 Course Outline and Readings (Tentative)

Below is a tentative schedule of topics and readings.

5.1 Brownian Motion and Stochastic Calculus

- Brownian motion and stochastic integration
- Stochastic discount factor (SDF)
- Equivalent Martingale Measure (EMM)
- SDF, EMM, and No Arbitrage

Readings:

- Harrison, J.M., and D. Kreps. [1979]. Martingales and Arbitrage in Multiperiod Securities Markets, Journal of Economic Theory 20, 381-408.
- Duffie, D. [2001]. Dynamic Asset Pricing Theory, 3th Ed. Princeton: University Press(Chap 5-6).

5.2 Risk Neutral Pricing and Option Pricing

- Redundant Securities
- Complete Markets
- The Black and Scholes economy
- Pricing: The Martingale Approach
- Pricing: The PDE Approach
- Replication
- Put-Call Parity
- Stochastic Volatility and Jump Models

Readings:

- Duffie, D. [2001]. Dynamic Asset Pricing Theory, 3th Ed. Princeton: University Press
- Merton, R. [1973]. Theory of Rational Option Pricing, Bell Journal of Economics and Management Science 4, 141-183.
- Black, F. and M. Scholes. [1973]. The Pricing of Options and Corporate Liabilities, Journal of Political Economy 81, 637-654.
- Merton, R. C. [1990]. Continuous Time Finance, Oxford, UK: Basil Blackwell

- Cox, J. and S. Ross [1976]. The Valuation of Options for Alternative Stochastic Processes, Journal of Financial Economics 3, 145-166.
- Ross, S. [1978]. A Simple Approach to the Valuation of Risky Streams, Journal of Business 51, No. 3, 453-475.
- Merton, R. [1977]. Option Pricing When the Underlying Stock Returns are Discontinuous, Journal of Financial Economics 5, 125-144.

5.3 Term Structure of Interest Rates

- 'Equilibrium' models
- Short-rate models
- Arbitrage-free models
- Affine models

Readings:

- Duffie, D. [2001]. Dynamic Asset Pricing Theory, 3th Ed. Princeton: University Press
- Vasicek. [1977]. An equilibrium characterization of the term structure, Journal of Financial Economics, 5:177-188.
- Cox, J. C., J. E. Ingersoll Jr., and S. A. Ross. [1985]. A theory of the term structure of interest rates, Econometrica, 53:385-407.
- Richard, S. F. [1978]. An arbitrage model of the term structure of interest rates, Journal of Financial Economics, 6:33-57.
- Cox, J. C., J. E. Ingersoll Jr., and S. A. Ross. [1981]. A reexamination of the traditional hypotheses about the term structure of interest rates, Journal of Finance, 36:769-799.
- T. Björk. [1998]. Arbitrage Theory in Continuous Time. Oxford University Press(Chap 15-20).
- Heath, D., R. Jarrow, and A. Morton. [1992]. Bond pricing and the term structure of interest rates: A new methodology for contingent claims evaluation, Econometrica, 60:77-105.
- Karoui, N. E., C. Lepage, R. Myneni, N. Roseau, and R. Viswanathan. [1991].The valuation and hedging of contingent claims with gaussian markov interest rates. Laboratoire de Probabilities, University Paris VI.

- Duffie, D., and R. Kan. [1996].A yield-factor model of interest rates, Mathematical Finance, 6:379-406.
- Duffie, D., J. Pan, and K. Singleton. [2000]. Transform analysis and option pricing for affine jump- diffusions, Econometrica, 68:1343-1376.
- Collin-Dufresne, P., and R. S. Goldstein. [2002]. Do bonds span the fixed-income markets? theory and evidence for unspanned stochastic volatility, Journal of Finance,4.
- Collin-Dufresne , P.,and R. S. Goldstein. [2002]. Generalizing the affine framework to HJM and random field models, Carnegie Mellon University Working Paper.
- Piazzesi, M. [1999]. An econometric model of the yield curve with macroeconomic jump effects, Working paper, Stanford University.

5.4 Stochastic Volatility and Jump Models

- Heston model
- Affine jump diffusion

Readings:

- Duffie, D. [2001]. Dynamic Asset Pricing Theory, 3th Ed. Princeton: University Press
- Heston S. [1993]. A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options, Review of Financial Studies 6, 327-344.
- Duffie, D., J. Pan, and K. Singleton. [2000]. Transform analysis and option pricing for affine jump- diffusions, Econometrica, 68:1343-1376.

5.5 Credit Risk Models

- "Structural" models, e.g. Merton, Leland, and etc.
- Reduced form models, affine models, doubly stochastic (Cox) models

Readings:

- Duffie, D. [2001]. Dynamic Asset Pricing Theory, 3th Ed. Princeton: University Press

- Leland, H. 1994. Corporate debt value, bond covenants, and optimal capital structure. Journal of Finance 49:1213–52.
- Duffie and Singleton, Credit Risk, Princeton University Press
- Duffie, D., J. Pan, and K. Singleton. [2000]. Transform analysis and option pricing for affine jump- diffusions, Econometrica, 68:1343-1376.
- Merton, R. C. [1990]. Continuous Time Finance, Oxford, UK: Basil Blackwell

5.6 Optimal Portfolio and Consumption Choices

- Merton's Problem
- The Hamilton Jacobi Bellman approach (Dynamic Programming)
- The Martingale approach
- Example: Portfolio choice with margin constraints, predictability, or labor income shocks

Readings:

- Duffie, D. [2001]. Dynamic Asset Pricing Theory, 3th Ed. Princeton: University Press(Chap 5-6).
- Merton, R. [1973]. Theory of Rational Option Pricing, Bell Journal of Economics and Management Science 4, 141-183.
- Cox and Huang (1989, JET)
- Black, F. and M. Scholes. [1973]. The Pricing of Options and Corporate Liabilities, Journal of Political Economy 81, 637-654.
- Merton, R. C. [1990]. Continuous Time Finance, Oxford, UK: Basil Blackwell(Chap 8).
- Cox, J. and S. Ross [1976]. The Valuation of Options for Alternative Stochastic Processes, Journal of Financial Economics 3, 145-166.
- Ross, S. [1978]. A Simple Approach to the Valuation of Risky Streams, Journal of Business 51, No. 3, 453-475.
- Merton, R. [1977]. Option Pricing When the Underlying Stock Returns are Discontinuous, Journal of Financial Economics 5, 125-144.
- Heston S. [1993]. A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options, Review of Financial Studies 6, 327-344.

- Davis and Norman
- Grossman and Laroque (1990, Econometrica)

5.7 Equilibrium

- The Lucas-Breeden Model and Consumption CAPM
- Production-based Asset Pricing
- Intertemporal CAPM
- Long-Run Risk Models (Bansal and Yaron)
- Habit Models (Campbell and Cochrane)
- Heterogeneous Agents Models

Readings:

- Duffie, D. [2001]. Dynamic Asset Pricing Theory, 3th Ed. Princeton: University Press.
- Merton, R. [1973]. An Intertemporal Capital Asset Pricing Model, Econometrica 41, 867-888.
- Breeden, D. [1979]. An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities, Journal of Financial Economics 7, 265-296.
- Grossman, S. and R. Shiller [1982] Consumption Correlatedness and Risk Measurement in Economies with Non-Traded Assets and Heterogeneous Information, Journal of Financial Economics 10, 195-210.
- Huang, C.-F. [1987]. An Intertemporal General Equilibrium Asset Pricing Model: The Case of Diffusion Information, Econometrica 55, No. 1, 117-142.
- Duffie, D. and C-F. Huang. [1985]. Implementing Arrow-Debreu Equilibria by Continuous Trading of Few Long-Lived Securities, Econometrica 53, No. 6, 1337-1356.
- Lucas (1978, Econometrica)
- Bansal and Yaron (2004, JF)
- Campbell and Cochrane (1999, JPE)

5.8 Dynamic Corporate Investment under MM

Readings:

- Hayashi (1982, Econometrica) and the q theory of investment
- McDonald and Siegel (1986, QJE), Dixit and Pindyck (1994) and Real options
- Abel and Eberly (1994, AER)

5.9 Contingent-Claims Capital Structure Models

Readings:

- Leland (1994, JF), Goldstein, Ju, and Leland (2001, JB)
- Leland and Toft (1996, JF)
- More recent work on banking

5.10 Dynamic Models with Financial Constraints

Readings:

- Bolton, Chen, and Wang (2011 JF, 2013 JFE)
- DeCamps, Mariotti, Rochet, and Villeneuve (2011 JF)

5.11 Dynamic Contracting

Readings:

- Dynamic Agency: DeMarzo and Sannikov (2006, JF), DeMarzo, Fishman, He, and Wang (2012, JF)
- Inalienable Human Capital and Corporate Finance: Bolton, Wang, and Yang (2018, JF forthcoming)

5.12 Financial Frictions and Macro

Readings:

- Brunnermeier and Sannikov (AER)
- Krishnamurthy and He (REStud)

5.13 Dynamic Learning Models

- Kalman filter and applications in Economics and Finance
- nonlinear filtering and applications in Economics and Finance

5.14 Other Topics and Applications in Finance and Economics

Readings:

- Macro: household savings and self insurance models
- Wealth distribution and inequality
- International Finance: sovereign debt