Columbia University

Graduate School of Business

Introduction to Continuous-time Finance

Business 9336-001

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Lectures: TBA
Room: TBA
TA: TBA

Course Description

This course is an introduction to continuous-time finance. The course starts by covering the basic mathematical tools: stochastic processes in continuous-time, stochastic differential equations, Ito's lemma, connections to partial differential equations, etc. This will require some digging through some more advanced mathematical material, and is at the level of an advanced calculus course. We are approaching these tools from a user's perspective, and not from a formal mathematics perspective. We will then cover the main pricing tools: arbitrage and equilibrium pricing in continuous-time asset markets, with a focus on both the mechanics and intuition.

Course Requirements

The course requires weekly lectures, problem sets for each class, class participation, and a 3-hour final exam.

Problem Sets

i. There are problem sets for every class during the semester, each will be graded on a scale of 0-10 points. The lowest one will be dropped from your grade.

ii. You can work on the homework with others, but you need to turn in your own work. I take the hone code seriously, so be sure to turn in only your own work. Homework should be uploaded to canvas at the beginning of class on the day it is due. Late homework will not be accepted under *any* circumstances. Since your lowest homework is dropped, you have a cushion to work with.

Exams

i. The will be an in-class, closed book final exam.
ii. Exam re-grade policy: (1) If points do not add up correctly, please submit your exam to the TA for correction. (2) I won't discuss informally the number of points given on a question. If you feel that are inaccuracies, then you must (i) submit a written explanation detailing what the problem is; (ii) submit the explanation and your exam within ten days of receiving the graded exam back. Your entire exam will be re-graded and recall that there is no guarantee that the initial grade will not be lowered.

Course Grade

Your overall course score is given by: 10% class participation, 30% homework and 60% final exam.

Recommended Text/Reference Books

- Required textbook:
  - Duffie, Darrell. 2001. Dynamic Asset Pricing Theory. Third Edition, Princeton.

- Recommended probably textbook for background:
  - Billingsley, Patrick. 1995. *Probability and Measure*. Wiley.
  - Williams, David. *Probability with Martingales*. Cambridge

- Stochastic Calculus
  - Readable introductions (in order of my recommendations)
    - Shreve, Steven. 2000. *Stochastic Calculus for Finance II: Continuous-time models.*
    - Steele, Michael, 2010, Stochastic Calculus and Financial Applications (Stochastic Modelling and Applied Probability).
    - Klebaner, Fima. 2005. *Introduction to Stochastic Calculus with Applications*, Imperial College Press. PDFs available online. Very useful for background in calculus, readable introductory coverage of stochastic calculus. https://univer30t.com/content/files/594dc8fa7e130%25-%25[Fima_C._Klebaner]_Introduction_to_Stochastic_Calc(BookFi.org).pdf
  - The real deal (for those highly mathematically inclined)
    - Karatzas, I. and Shreve, S. E., *Brownian Motion and Stochastic Calculus*. New York, Springer-Verlag 1988.

- Philip Protter, *Stochastic integration and differential equations : a new approach*, Springer-Verlag, 2005.

Course Outline

Weeks 1-2

- Introduction and the original Black-Scholes Derivation: the famous PDE.
- Probability Spaces and Building Blocks of Continuous Time
- Information, filtrations and martingales
- Ito's formula
- Existence and Uniqueness of Solutions
- Simulating SDE's: The Euler Scheme
- Solving SDE's: Geometric Brownian motion and a continuous-time AR(1).

Week 3

- Stochastic integrals in financial markets and the Black-Scholes analysis
    - Merton's argument for Black-Scholes
    - The general PDE and Feynman-Kac
    - Option hedging
- Self-Financing trading strategies and numeraire invariance

Week 4

- Equivalent martingale measures and the absence of arbitrage: P vs. Q.
    - Girsanov's theorem
- Complete Markets and Uniqueness of EMM's.
    - Invertibility
    - Stochastic volatility examples and incomplete markets

Week 5

- Stochastic differential equations with jumps
- Pricing assets with jumps: augmenting the Black-Scholes model with crash risk

Week 6

- Topics