Locked Up by a Lockup: Valuing Liquidity as a Real Option

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Abstract

Hedge funds often impose lockups and notice periods to limit the ability of investors to withdraw capital. We model the investor's decision to withdraw capital as a real option and treat lockups and notice periods as exercise restrictions. Our methodology incorporates time-varying probabilities of hedge fund failure and optimal early exercise. We estimate a two-year lockup with a three-month notice period costs approximately 1% of the initial investment for an investor with CRRA utility and risk aversion of 3. The magnitude is sensitive to a fund's age, expected return, volatility, and the liquidation cost upon failure. The cost of illiquidity can easily exceed 10% if the hedge fund manager suspends withdrawals.

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Locked Up by a Lockup: Valuing Liquidity as a Real Option

1. Introduction

Hedge funds and funds-of-funds, along with many other alternative investment vehicles, place a variety of restrictions on the ability of investors to redeem their capital. A lockup requires an investor to wait a specified length of time after the initial deposit of capital, typically one to three years, before requesting a redemption. A notice period requires an investor to wait a specified length of time, typically one to three months, before a redemption request is processed. In addition, fund managers often have the authority to process only a fraction of a redemption request, known as a gate, or even to suspend redemptions altogether. As argued by Aragon (2007), the advantage of redemption restrictions is that they allow fund managers to invest in illiquid assets and earn an associated return premium. Redemption restrictions can levy an important cost, however, if they prevent investors from withdrawing capital before anticipated losses are realized.[1] Our goal is to develop a methodology to estimate the implied cost of redemption restrictions, thereby allowing investors to more accurately tabulate hedge fund fees.

We model the ability of a risk-averse investor to withdraw capital as a real option. Upon exercise, the investor gives up ownership in the fund and receives a cash payoff per share equal to the fund's net asset value (hereafter "NAV"). The investor exercises the option when the investor's own valuation of a share of ownership in the fund, expressed as a certainty equivalent, falls below the NAV. We assume investors value the fund taking into account the probability of fund failure, liquidation costs, and the impact of future exercise decisions. Redemption restrictions, such as lockups and notice periods, constrain the investor's ability to exercise, and their cost can be measured by the resulting reduction in value of the "liquidity option."[2]

[1] The case of Amaranth Advisors is a good example, as described in "At Hedge Funds, Study Exit Guidelines," *Wall Street Journal* 10/23/06.

[2] See Longstaff (1995), Scholes (2000), Bollen, Smith, and Whaley (2004), and Chacko, Jurek and Stafford (2008) for examples of liquidity options in other contexts.

Our approach has three key elements. First, for the liquidity option to have value there must be a difference between the NAV and an investor's valuation of ownership in the fund. The NAV is the value of the fund's portfolio reported by the fund manager on a given date. If the fund is invested in illiquid assets for which market prices are not readily available, the fund manager may employ subjective marking to model when computing the fund's NAV, and the NAV may be susceptible to managerial misreporting.[3] We abstract from differences of opinion regarding the value of a fund's assets. Instead, in our model, an investor's valuation of ownership in the fund differs from the NAV because the latter reflects neither the capitalization of future managerial performance nor the probability of fund failure and the associated liquidation cost.

Second, we employ a data generating process (hereafter "DGP") for hedge fund returns that includes a normal regime with a constant expected return and an absorbing failure state in which investors are forced to accept a payout per share equal to a fraction of the fund's NAV. Motivated by Gregoriou (2002) and Grecu, Malkiel and Saha (2006), who find that most hedge funds stop reporting to databases because of failures following low returns, we use a log-logistic duration function to predict hedge fund failure, and allow hazard rates to depend on realized performance. Specifically, the probability of fund failure, and hence the value of a liquidity option, changes over time as a function of fund age and performance, so that a fund with poor cumulative performance relative to its peers is more likely to fail.

Third, we model the payoff of investing in a hedge fund with and without a liquidity option using a binomial lattice that embeds time-varying probabilities of fund failure and, most importantly, allows for early exercise. We measure the cost of a lockup as the difference between the value of a liquidity option that allows exercise at any time and another that does not permit exercise during the lockup. Similarly, we measure the cost of a notice period as the difference between the values of two liquidity options. The first liquidity option generates a payoff equal to the fund NAV immediately upon exercise. The second liquidity option generates an uncertain payoff because the

[3] See, for example, Asness, Liew and Krail (2001), Getmansky, Lo, Makarov (2004), and Bollen and Pool (2008).

redemption is not processed until the notice period has elapsed. During the notice period, the NAV can rise or fall and the fund may fail, hence the two liquidity options can have different payoffs. We also compute the combined cost of lockups and notice periods.

We estimate the cost of lockups and notice periods by calibrating our model to a large sample of hedge funds using the CISDM database. Parameters for the log-logistic duration function indicate that there is a 50% chance a fund will fail by age 78 months, though failures cluster in the first few years. A fund with a cumulative return that falls one standard deviation below the cross-sectional mean has a 38% increased risk of failure. We estimate that the combined cost of a two-year lockup and a three-month notice period is approximately 1% of the initial investment for a CRRA investor with risk aversion level of 3. More risk-averse investors assign higher costs to the restrictions because they tend to want to liquidate sooner, hence exercise restrictions are binding more often. Furthermore, we show that a manager's discretion to block redemption requests using gate restrictions or suspension clauses generates an implied cost that can easily exceed 10% of the initial investment, even for low levels of risk aversion.

The rest of the paper is organized as follows. Section 2 discusses the relations between our paper and existing research. Section 3 presents the DGP of hedge fund returns and shows how we value lockups and notice periods. The data and the calibrated DGP are described in Section 4. Section 5 computes the cost of lockups and notice periods over a range of inputs. Section 6 concludes.

2. Related literature

Our study of the ability of hedge fund investors to withdraw capital is related to the literature on closed-end mutual funds (hereafter "CEF") as well as existing work on hedge fund share restrictions.

Investors in our model exercise their redemption option when their own valuation of ownership in the fund falls below a hedge fund's NAV. In contrast, investors in CEFs can never redeem capital from a fund and instead trade shares of ownership on the secondary market, where they typically observe a difference between NAV and the market price. The price is usually below the NAV; hence the difference is labeled the CEF discount. Berk and Stanton (2007) model the CEF discount as managerial ability to deliver abnormal returns net of fees, whereas Cherkes et al. (2008) model the CEF discount as a liquidity benefit net of fees. The liquidity benefit arises when the fund is invested in underlying illiquid securities that are costly to trade. Similar to these authors, we model a difference between the hedge fund NAV and the investor's value of the hedge fund, with the investor's valuation incorporating the net of after-fee abnormal returns and the cost of fund failure.

Our paper differs fundamentally from the literature on CEF pricing on two dimensions. First, unlike CEF investors, hedge fund investors do have the ability to exchange shares for NAV, although that ability is often restricted. Second, we model the investor's decision to redeem capital in the presence of time-varying probabilities of fund failure and risk aversion. In our model an investor's decision to redeem is an optimal exercise of a real option which is affected by a fund failure process dependent on past performance.

Our paper is also related to existing studies of the relation between redemption restrictions and hedge fund returns. Ding et al. (2007) show that redemption restrictions affect the empirical cross-sectional relation between aggregate capital flow and returns. Aragon (2007) documents that hedge funds with lockups have expected returns that are 4% – 7% per annum higher than hedge funds without lockups. Aragon interprets this difference as an illiquidity premium: lockups allow managers to invest in more illiquid securities and earn higher returns as a result. However, Aragon does not explicitly compute the cost of a lockup and cannot determine if the illiquidity premium is fair compensation for bearing the liquidity restriction. A related literature explores whether lockup provisions are a component of an optimal incentive contract for a fund manager, especially one investing in illiquid assets, as in Lerner and Schoar (2004).

Our paper is most closely related to Derman (2007), who models hedge fund returns using a three-state model in which hedge funds are good, sick, or dead. Derman, Park and Whitt (2007) extend this approach to allow for more complex Markov chain models. In both approaches, lockups prevent an investor from withdrawing capital from a sick fund and investing the proceeds in a good fund. Our valuation strategy differs from Derman (2007) in four ways. First, Derman assumes investors swap capital invested in a poorly performing hedge fund for capital invested in a superior hedge fund, whereas we assume investors withdraw capital as cash. Thus, our approach explicitly models the actual decision that investors face. Second, we compute the costs of illiquidity born by a risk-averse CRRA investor. Third, we differentiate between lockups and notice periods, and develop a methodology that can estimate the cost of the two restrictions separately, or in combination. Finally, we model fund failure using a hazard rate that can depend on fund age and performance, as described next.

3. Methodology

In Section 3.1, we use a binomial lattice to model the stochastic evolution of fund NAVs conditional on a fund surviving. In Section 3.2, we augment the binomial lattice to incorporate default probabilities. Section 3.3 explains how to use the augmented binomial lattice to value hedge funds with and without liquidity options, and how to estimate the cost of illiquidity by comparing hedge fund values when the liquidity option is restricted by a lockup, a notice period, or both.

3.1. Modeling fund NAVs

We assume that continuously compounded fund NAV returns are initially normally distributed and that this "normal regime" continues as long as the fund survives. We use a binomial lattice to model the evolution of a hedge fund's NAV. Let $S_{t,j}$ denote the NAV, where t denotes the time step, running from 0 to T, and j denotes the level in the lattice, running from 1 to $t+1$ at time step t, with 1 being the highest, as depicted in Figure 1. We refer to the combination of time step t and level j as node (t, j). The time between nodes is denoted by Δt. Date T represents the end of the hedge fund's life if failure never occurs. This can be interpreted as either the investor's investment horizon, the retirement of the hedge fund manager, or the feasible horizon of the fund's investment strategy, and the purposeful unwinding of the hedge fund's positions.

The geometry of the lattice is defined by the step size *u* and branch probability *p*, which are determined setting the mean and variance implied by the lattice equal to those of the hedge fund's normal regime. With probability *p* the NAV increases from $S_{t,j}$ to $S_{t+1,j}$ where

$$S_{t+1,j} = S_{t,j} u \,, \tag{1}$$

with the multiplicative increase *u* and the probability *p* given by

$$\begin{aligned} u &= e^{\sigma\sqrt{\Delta t}} \\ p &= \frac{e^{\mu\Delta t} - u^{-1}}{u - u^{-1}} \,, \end{aligned} \tag{2}$$

where μ and σ are the mean and standard deviation of hedge fund returns in the normal regime. With probability $1 - p$ the NAV decreases from $S_{t,j}$ to $S_{t+1,j+1}$ where

$$S_{t+1,j+1} = S_{t,j} u^{-1} \,. \tag{3}$$

The parameters in (2) ensure that the distribution of NAV returns implied by the lattice converge to the assumed normal distribution as $\Delta t \to 0$.

The NAV is of central concern because this is the quantity received by the investor upon redemption contingent on the fund not failing. Furthermore, the NAV is empirically convenient because hedge funds report NAV returns to hedge fund databases. As described next, we model failure to be a function of fund age and performance, as measured by cumulative NAV returns relative to competing funds.

3.2. Failure process

Let *D* be the duration of a hedge fund, which we define as the random time that a fund fails, and at which point the manager liquidates the fund's remaining assets.[4] Empirically, we measure duration as the time that a hedge fund manager stops reporting returns. While some hedge fund managers may stop reporting for good performance, the

[4] We do not model the strategic interactions in a liquidating event, as in Cherkes et al. (2008), but our empirically estimated failure rate's sensitivity to performance may capture some of this effect.

majority of funds cease reporting due to failure as argued by Ackermann, McEnally and Ravenscraft (1999) and Grecu, Malkiel and Saha (2006). In practice, a failing fund could continue to survive for some period of time after the manager stops reporting, hence our measure likely understates durations. If the fund fails at node (t, j), we assume that the fund NAV drops to a level $S_{t,j}l$ where l represents the proportion of pre-failure NAV that the manager is able to raise through liquidating asset sales, with $0 < l < 1$. The investor receives the liquidating dividend at time $t+1$. Our model of the hedge fund failure process extends the Markov chain model employed by Derman, Park and Whitt (2007) to shift the baseline default intensity up and down by a performance covariate, as discussed below.

Denote the baseline density of durations as $f_b(t)$ with cumulative density function $F_b(t) = \int_0^t f_b(s)\,ds$. The baseline survival function $1-F_b(t)$ is the unconditional probability of surviving up to at least time *t*, evaluated at time 0. The baseline hazard rate at time *t* is the probability of failure per time increment conditioned on surviving until *t*:

$$\lambda_b(t) = \lim_{\Delta t \to 0} \frac{\Pr\left(t \le D \le t + \Delta t \,\middle|\, D \ge t\right)}{\Delta t} = \frac{f_b(t)}{1-F_b(t)}. \tag{4}$$

Grecu, Malkiel and Saha (2006) find the log-logistic distribution fits the empirical density of hedge fund durations better than other distributions; hence we use a log-logistic hazard rate function to model hedge fund failures. In Section 4, we present evidence that the log-logistic hazard rate function provides a tight fit to the empirical distribution of hedge fund failures in our sample. The log-logistic distribution is defined by two parameters λ and *q* with density, survival function, and hazard rate given by:

$$\begin{aligned} f_b(t) &= \lambda q (\lambda t)^{q-1} \Big/ \left[1 + (\lambda t)^q\right]^2 \\ 1 - F_b(t) &= 1 \Big/ \left[1 + (\lambda t)^q\right] \\ \lambda_b(t) &= \lambda q (\lambda t)^{q-1} \Big/ \left[1 + (\lambda t)^q\right]. \end{aligned} \tag{5}$$

We estimate λ and *q* by maximum likelihood. We separate hedge fund durations into *n* uncensored observations, for which the hedge fund manager stopped reporting prior to the end of the database, and m censored observations from "live" funds with observations through the end of the database. The likelihood of the data is then given by:

$$\prod_{i=1}^{n} f\left(t_i\right)\prod_{j=1}^{m}\left(1-F_b\left(t_j\right)\right) \tag{6}$$

with durations t_i for the n funds leaving the database and durations t_j for the m funds surviving until the end of the sample. This yields a log-likelihood of

$$n\ln\left(\lambda q\right)+\left(q-1\right)\sum_{i=1}^{n}\ln\left(\lambda t_i\right)-2\sum_{i=1}^{n}\ln\left(1+\left(\lambda t_i\right)^q\right)-\sum_{j=1}^{m}\ln\left(1+\left(\lambda t_j\right)^q\right). \tag{7}$$

We assume that failure rates also depend on hedge fund cumulative performance relative to other funds. Specifically, the hazard rate of an individual hedge fund equals the baseline hazard rate scaled up or down depending on the value of covariate z as follows:

$$\lambda\left(t;z\right)=\lambda_b\left(t\right)e^{z\beta}. \tag{8}$$

We follow standard practice and demean the covariate so that values above or below zero increase or decrease the hazard rate. We choose a performance-based covariate equal to the difference between the cumulative return of fund i at time t and the cross-sectional mean return. The difference is then scaled by the cross-sectional standard deviation. The cross-sectional mean and standard deviation are computed using the cumulative returns of each fund when they are the same age as fund i at time t.

The sensitivity of the failure rate to performance can be motivated in at least three ways. First, and most importantly, Liang (2000), Brown, Goetzmann and Park (2001), and Jagannathan, Malakhov and Novikov (2006), among others, empirically document that liquidated hedge funds are more likely to be funds with poor past performance. Second, managers of funds with low cumulative returns are less likely to capture performance fees, since the NAV must recover to previously set high-water marks before the fees accrue. This provides managers of poorly performing funds a strong incentive to close those funds. Third, investors are more likely to withdraw capital from poorly performing funds, forcing the manager to liquidate assets, possibly leading to further reductions in NAV, again leading the manager to close those funds.[5]

The form of the proportional hazard rate in (8) is convenient because the coefficient β can be estimated independently from the baseline hazard rate as noted by Kalbfleisch and Prentice (2002). In particular, the relevant partial likelihood can be expressed as

$$\prod_{i=1}^{n}\left[e^{z_i\beta}\left[\sum_{k=1}^{N_i}e^{z_k\beta}\right]^{-1}\right], \tag{9}$$

where n is the number of uncensored observations, or fund failures, in the sample, z_i is the value of the performance covariate at failure of fund i, N_i is the number of funds in the sample with durations at least as long as that of fund i, and z_k is the value of the performance covariate of fund k evaluated at age equal to that of fund i at the time of failure of fund i.

The hazard rate in (8) allows for the probability of failure to depend non-linearly on age and realized performance of the fund. In many option applications state-dependent payoffs lead to path dependence and cause the number of nodes in a lattice to explode. We avoid this by specifying the performance covariate so that it can be computed at each node in the lattice without knowledge of the path taken.

Let $\pi_{t,j}$ denote the probability of failure at node (t, j), with the failure occurring prior to the return of the fund being realized between time t and $t+1$. At node (t, j) we numerically evaluate this by setting $\pi_{t,j} = \lambda_b\left(\text{age}_t + 0.5\right)\Delta t$ in the case of the base hazard rate and $\pi_{t,j} = \lambda_b\left(\text{age}_t + 0.5\right)e^{z_{t,j}\beta}\Delta t$ in the case of the proportional hazard rate, where Δt is the increment of time in the lattice, $z_{t,j}$ is the value of the performance covariate at node (t, j), and age_t is the age of the fund at time t. The hazard rates over t to $t+\Delta t$ depend

[5] As reported in Ding et al. (2007) academic evidence on the flow-performance relation in hedge funds is mixed, with prior research finding linear, convex, and concave relations between fund flow and performance. Ding et al. argue that the presence of redemption restrictions can explain these results.

on the value of the covariate at t. Evaluating the baseline hazard rate at the midpoint amounts to taking the integral over t to $t+\Delta t$. Note that time $t=0$ corresponds to the investor's initial subscription to the fund rather than the fund's age. If the fund has already been in existence for some period of time, its cumulative return from inception of the fund impacts the value of $\pi_{0,1}$, since both the cumulative relative performance z as well as the baseline hazard function λ_b depend on the fund's age.

3.3. Valuing liquidity options

We have described the evolution of a hedge fund's NAV. But what is the fund worth to an investor? An investor's own valuation may differ from the NAV for a number of reasons. The investor's valuation incorporates risk aversion, the mean and variance of fund returns, as well as the probability that the hedge fund will fail in the future and the associated liquidation cost.[6] When there are no binding liquidity restrictions, hedge fund investors have a real option to give up their ownership and receive the NAV. Investors exercise when the current NAV exceeds the certainty equivalent of holding the hedge fund.[7]

When an investor redeems, he is exchanging a share of ownership in the fund for its NAV, so one might think the redemption is the exercise of an exchange option, as in Margrabe (1978). Indeed, Derman (2007) explicitly models the exchange of an ownership in a bad fund for ownership in a good fund and computes the cost of a lockup as the inability to exchange a good fund for a bad fund during a fixed time period. In Margrabe's model, the option holder exchanges one risky asset for another, and both assets are governed by a distinct, but correlated, stochastic process. In our model, however, the investor's valuation of the fund is an explicit function of the NAV, so only

[6] Our pricing methodology does not assign any premium to an investor requiring immediate access to invested capital during the lockup period for exogenous reasons. Incorporating exogenous liquidity demands would increase the cost of lockups and notice periods.

[7] If investors were identical they would all decide to withdraw simultaneously, forcing the manager to liquidate the fund so all investors would bear the liquidation cost. In practice, investors are not identical because some will be subject to lockups and others will not, depending on when they entered the fund. Some investors may also be better informed than other investors, or may possess different priors regarding the DGP of returns.

the NAV's stochastic process is necessary. In addition, since the investor exchanges ownership for cash, a more appropriate analog is a put option, in which the investor can sell the share of ownership back to the fund for the NAV. The NAV is of course constantly changing, so the liquidity option can be viewed as a put option with a variable exercise price.

We assume that investors are risk-averse with a CRRA utility function for time T wealth, W_T, given by

$$U\left(W_T\right) = W_T^{1-\gamma} \big/ \left(1-\gamma\right), \tag{10}$$

where γ is the agent's risk aversion. We expect, and confirm in Section 5, that more risk-averse investors assign a higher cost to lockups and redemption periods than less risk-averse investors. This is similar to the literature on executive stock options (ESOs), which shows that the value of an ESO to a manager who cannot short the underlying stock is affected by risk aversion.[8] Higher risk aversion coincides with a higher likelihood that constraints bind and this reduces the ESO value. Similarly, higher risk aversion causes the restrictions on exercise imposed by lockups and notice periods to bind more often, and this increases the cost of lockups and notice periods.

3.3.1. Hedge fund value with no liquidity option

Let $H_{t,j}$ denote the value of a hedge fund per share conditional on survival at node $\left(t, j\right)$ from the perspective of a "passive" investor who does not possess a liquidity option. The passive investor is an artifice necessary to compute the value of liquidity options, as shown below. In addition, note that a fund manager can unilaterally impose restrictions on exercise that reduce the value of an investor's liquidity option, and in the extreme case can eliminate the liquidity option completely. Here is an excerpt from an actual partnership agreement that is typical of many suspension clauses employed by hedge funds:

[8] See, among others, Kulatilaka and Marcus (1994), Detemple and Sundaresan (1999), and Murphy (1999).

> *The Fund may suspend redemptions and defer payment of redemption proceeds during any period in which disposal of all or part of the Fund's assets, or the determination of Net Asset Value, would not be reasonable or practical or would be prejudicial to the Fund or the Shareholders.*

During times of high demand for the withdrawal of capital, for example, managers may temporarily suspend redemptions to avoid high transaction costs, such as price impact or fire sales, which would be incurred when selling fund assets. Thus, $H_{t,j}$ represents the lower bound on the value of the hedge fund to an investor who possesses a liquidity option. This bound would be reached when the fund manager always suspends redemptions when it is optimal for the investor to withdraw. In other words, we can interpret $H_{t,j}$ as the extreme case when a hedge fund manager disables an investor's liquidity option.

A passive investor will receive a payoff at the time of fund failure or at date T if the fund survives until the end of the investment horizon. For simplicity, we assume that if the fund fails, the investor places the proceeds in a risk-free account until time T. Assume that at the terminal set of nodes investors receive the NAV, $S_{T,j}$. This assumption could easily be relaxed to allow for the cost of unwinding existing positions in the fund at the terminal date.

Prior to the terminal set of nodes, the value of the investment conditional on survival is given by its certainty equivalent, which we define as the current value of a risk-free security that matures at time T and provides a guaranteed level of utility equal to the expected utility of holding the hedge fund. At node $(T-1, j)$, the value of the fund, $H_{T-1,j}$, is the solution to

$$\frac{1}{1-\gamma}\left(R_f H_{T-1,j}\right)^{1-\gamma} = \frac{1}{1-\gamma}\left[\pi_{T-1,j}\left(S_{T-1,j}l\right)^{1-\gamma} + \left(1-\pi_{T-1,j}\right)\left(pS_{T,j}^{1-\gamma} + \left(1-p\right)S_{T,j+1}^{1-\gamma}\right)\right], \qquad (11)$$

where R_f is the gross risk-free return for a period of length Δt.

The RHS of equation (11) is the expected utility of random time T wealth computed at node $(T-1, j)$ when the NAV of the fund is $S_{T-1,j}$. The first term in square brackets allows for the probability that the fund fails between $T-1$ and T, which results in a loss due to the liquidation cost and a payoff at T equal to $S_{T-1,j}l$. The second term captures the expected utility of an investment in the fund if failure does not occur. The LHS of equation (11) is the utility of a certain time T wealth level $R_f H_{T-1,j}$, which gives the same expected utility of holding the hedge fund. We use $H_{T-1,j}$ to express the certainty equivalent of the hedge fund investment at node $(T-1, j)$, so that the investor is indifferent between staying in the fund at node $(T-1, j)$ and exchanging for cash equal to $H_{T-1,j}$, which is then deposited in a risk-free account.

We can solve explicitly for the investor's valuation of the fund at node $(T-1, j)$ by rewriting (11) as

$$H_{T-1,j} = R_f^{-1}\left[\pi_{T-1,j}\left(S_{T-1,j}l\right)^{1-\gamma} + \left(1-\pi_{T-1,j}\right)\left(pS_{T,j}^{1-\gamma} + \left(1-p\right)S_{T,j+1}^{1-\gamma}\right)\right]^{1/1-\gamma}. \qquad (12)$$

Prior to $T-1$, the computation is similar to (12) but involves next-period certainty equivalents:

$$H_{t,j} = R_f^{-1}\left[\pi_{t,j}\left(S_{t,j}l\right)^{1-\gamma} + \left(1-\pi_{t,j}\right)\left(pH_{t+1,j}^{1-\gamma} + \left(1-p\right)H_{t+1,j+1}^{1-\gamma}\right)\right]^{1/1-\gamma}. \qquad (13)$$

Under CRRA utility, the indirect utility is also of the power form, which permits using the certainty equivalent in (12) in the computation of equation (13). The recursion in (13) is repeated until the initial node in the lattice is reached, where the value of the hedge fund to the investor is $H_{0,1}$.

Note that the approach outlined above nests the case of risk neutrality, which can be represented by setting $\gamma = 0$. For a risk-neutral investor, the calculations in (12) and (13) simply compute the discounted, expected payoff, where the discounting occurs at the risk-free rate.

3.3.2. Hedge fund value with unrestricted liquidity option

Let $O_{t,j}$ denote the value of the hedge fund per share at node (t, j), again conditional on survival, but this time from the perspective of an "active" investor who possesses an unrestricted liquidity option. We estimate the cost of lockups and notice periods, which impose restrictions on exercise, from the basis of this unrestricted liquidity option. The unrestricted liquidity option is defined as the ability to exchange a share in the hedge fund for the NAV at any time prior to the terminal set of nodes. The investor will exercise the option at a given node (t, j) if an immediate payoff equal to the fund NAV, $S_{t,j}$, provides greater utility than the expected utility of remaining in the fund.

As before, we assume that the investor receives the NAV at the terminal set of nodes. Prior to the terminal set of nodes, the hedge fund value is the maximum of immediate exercise of the redemption option and the certainty equivalent. At node $(T-1, j)$, the value of the fund, $O_{T-1,j}$, is the solution to

$$O_{T-1,j} = \max\left(S_{T-1,j}, R_f^{-1}\left[\pi_{T-1,j}\left(S_{T-1,j}l\right)^{1-\gamma} + \left(1-\pi_{T-1,j}\right)\left(pS_{T,j}^{1-\gamma} + (1-p)S_{T,j+1}^{1-\gamma} \right) \right]^{1/1-\gamma} \right). \quad (14)$$

The first element in the RHS of equation (14) is the value received by the investor upon immediate exercise, which is the NAV of the fund at the beginning of the node, $S_{T-1,j}$. The second term is the value of the fund to the investor expressed as a certainty equivalent computed at node $(T-1, j)$. If the investor obtains a higher utility by receiving the NAV, the investor exercises his option. Prior to time $T-1$, the option value is computed as

$$O_{t,j} = \max\left(S_{t,j}, R_f^{-1}\left[\pi_{t,j}\left(S_{t,j}l\right)^{1-\gamma} + \left(1-\pi_{t,j}\right)\left(pO_{t+1,j}^{1-\gamma} + (1-p)O_{t+1,j+1}^{1-\gamma} \right) \right]^{1/1-\gamma} \right). \quad (15)$$

The initial value of the hedge fund to the investor with an unrestricted liquidity option is therefore $O_{0,1}$ and the initial value of the liquidity option is $O_{0,1} - H_{0,1}$.

3.3.3. Valuing a notice period

Let $O_{NP,t,j}$ denote the value of the hedge fund per share at node (t,j), again conditional on survival of the fund, when the liquidity option is subject to a notice period. The hedge fund value is generally reduced by the notice period because when the option is exercised at node (t,j) the investor does not immediately receive a payoff, but rather must wait some period of time before receiving the prevailing NAV at that future date. The payoff may drop substantially if the fund fails while the investor waits, and indeed this is relatively likely since the investor chooses to exercise the option when the failure probability is high.

Consider a one-period delay between option exercise and receipt of the NAV. As before, we assume that the investor receives the NAV at the terminal set of nodes. With a one-period notice, the option to redeem is irrelevant at the penultimate set of nodes, hence

$$O_{NP,T-1,j} = R_f^{-1}\left[\pi_{T-1,j}\left(S_{T-1,j}l\right)^{1-\gamma} + \left(1-\pi_{T-1,j}\right)\left(pS_{T,j}^{1-\gamma} + \left(1-p\right)S_{T,j+1}^{1-\gamma}\right)\right]^{1/1-\gamma}. \quad (16)$$

Prior to the penultimate set of nodes, however, the hedge fund value is given by the maximum of the certainty equivalent of exercising the liquidity option and the certainty equivalent of staying in the fund. If the investor exercises at node (t,j) the certainty equivalent is a function of the NAV at time $t+1$ since there is a one-period delay. We allow for the possibility that the fund fails while the investor is waiting, so that the certainty equivalent at node (t,j) of exercising the liquidity option, denoted *CE*, is

$$CE_{t,j} = R_f^{-1}\left[\pi_{t,j}\left(S_{t,j}l\right)^{1-\gamma} + \left(1-\pi_{t,j}\right)\left(pS_{t+1,j}^{1-\gamma} + \left(1-p\right)S_{t+1,j+1}^{1-\gamma}\right)\right]^{1/1-\gamma}. \quad (17)$$

Thus, the hedge fund value at node (t,j) is

$$O_{NP,t,j} = \max\left\{CE_{t,j}, R_f^{-1}\left[\pi_{t,j}\left(S_{t,j}l\right)^{1-\gamma} + \left(1-\pi_{t,j}\right)\left(pO_{NP,t+1,j}^{1-\gamma} + \left(1-p\right)O_{NP,t+1,j+1}^{1-\gamma}\right)\right]^{1/1-\gamma}\right\}. \quad (18)$$

If there is no notice period, $CE_{t,j} = S_{t,j}$ and (18) reduces to (15).

Longer notice periods can be incorporated by replacing the expression in (17) with a sub-lattice that accounts for the possibility of failure at each time step. Consider an *m*-period delay in processing a redemption request that is made at node (t, j). To evaluate the hedge fund value with an *m*-period notice at node (t, j), we compute the certainty equivalent taking into account the possibility the fund fails over the next *m* periods. The *CE* with an *m*-period delay is computed in a recursive fashion, beginning with the set of $m+1$ NAVs that are possible in *m* periods assuming that the fund survives the notice period. In *m* periods, assuming the fund does not fail, the possible NAVs are $S_{t+m,j}$ through $S_{t+m,j+m}$. Moving back one time step, the *CE* is computed at each node as follows:

$$CE_{t+m-1,k} = R_f^{-1}\left[\pi_{t+m-1,k}\left(S_{t+m-1,k}l\right)^{1-\gamma} + \left(1-\pi_{t+m-1,k}\right)\left(pS_{t+m,k}^{1-\gamma} + \left(1-p\right)S_{t+m,k+1}^{1-\gamma}\right)\right]^{1/1-\gamma}, \quad (19)$$

where *k* runs from *j* to $j+m-1$. Moving back one more time step, *CE* is computed at each node in a similar fashion:

$$CE_{t+m-2,k} = R_f^{-1}\begin{bmatrix}\pi_{t+m-2,k}\left(S_{t+m-2,k}l\right)^{1-\gamma} + \\ \left(1-\pi_{t+m-2,k}\right)\left(pCE_{t+m-1,k}^{1-\gamma} + \left(1-p\right)CE_{t+m-1,k+1}^{1-\gamma}\right)\end{bmatrix}^{1/1-\gamma}, \quad (20)$$

where *k* runs from *j* to $j+m-2$. The computation in (20) is then repeated for all remaining time steps in the notice period.

As before, we assume that the investor receives the NAV at the terminal set of nodes. When the notice requires a wait of *m* periods, the option to file a notice does not exist in the *m* steps prior to the terminal date T. That is, the hedge fund value is computed as in (16) for date $T-1$ with

$$O_{NP,t,j} = R_f^{-1}\left[\pi_{t,j}\left(S_{t,j}l\right)^{1-\gamma} + \left(1-\pi_{t,j}\right)\left(pO_{NP,t+1,j}^{1-\gamma} + \left(1-p\right)O_{NP,t+1,j+1}^{1-\gamma}\right)\right]^{1/1-\gamma}. \quad (21)$$

for $T-m \le t \le T-2$. Prior to this date, the investor decides whether or not to file a notice using the decision rule in (18).

The initial value of the hedge fund to the investor with a liquidity option restricted by a notice period is $O_{NP,0,1}$ and the initial value of the restricted liquidity option is $O_{NP,0,1} - H_{0,1}$. Recall the initial value of the liquidity option with no redemption restriction is $O_{0,1} - H_{0,1}$. Thus, the cost imposed on the investor by the notice period is the difference between the unrestricted and unrestricted option, i.e., $O_{0,1} - O_{NP,0,1}$. Hence, the cost of the redemption notice is the difference between two liquidity options. The first provides a payoff equal to the NAV immediately upon exercise. The second provides a payoff equal to the prevailing NAV at the end of the notice period if the fund does not fail between the exercise of the option and the end of the notice period, or a payoff equal to a fraction of the prevailing NAV if the fund fails after the investor has given notice.

3.3.4. Valuing a lockup

A lockup prevents an investor from exercising his liquidity option prior to date *L*. Let $O_{L,t,j}$ denote the value of a hedge fund, at node (t, j), with a liquidity option restricted by a lockup, and $O_{LNP,t,j}$ denote the value subject to both a lockup and a notice period. After the lockup expires, there is no restriction, hence $O_{L,t,j} = O_{t,j}$ and $O_{LNP,t,j} = O_{NP,t,j}$ for $t > L$, and hedge fund values can be computed as described in Sections 3.3.2 and 3.3.3. For $t \le L$, however, the investor cannot exercise the liquidity option, hence

$$O_{L,t,j} = R_f^{-1}\left[\pi_{t,j}\left(S_{t,j}l\right)^{1-\gamma} + \left(1-\pi_{t,j}\right)\left(pO_{L,t+1,j}^{1-\gamma} + \left(1-p\right)O_{L,t+1,j+1}^{1-\gamma}\right)\right]^{1/1-\gamma}$$

and (22)

$$O_{LNP,t,j} = R_f^{-1}\left[\pi_{t,j}\left(S_{t,j}l\right)^{1-\gamma} + \left(1-\pi_{t,j}\right)\left(pO_{LNP,t+1,j}^{1-\gamma} + \left(1-p\right)O_{LNP,t+1,j+1}^{1-\gamma}\right)\right]^{1/1-\gamma}$$

for $t \le L$. The lockup restricts exercise of the liquidity option in the same way that a vesting period prevents exercise of ESOs, as in Hull and White (2004).

The initial value of a hedge fund with a liquidity option subject to a lockup is therefore $O_{L,0,1}$ and the initial value of the restricted liquidity option is $O_{L,0,1} - H_{0,1}$. The ex-ante cost of the lockup itself is the difference between the unrestricted and restricted options, i.e., $O_{0,1} - O_{L,0,1}$. Similarly, the initial value of a hedge fund with a liquidity option subject to both a lockup and a notice period is $O_{LNP,0,1}$ and the initial value of the restricted liquidity option is $O_{LNP,0,1} - H_{0,1}$. The combined ex-ante cost of the lockup and notice period is the difference between the unrestricted and restricted options, i.e., $O_{0,1} - O_{LNP,0,1}$.

3.3.5. Summary

Table 1 contains a summary of notation describing model parameters, including expressions for the value of liquidity options and the cost of notice periods and lockups. The value of a liquidity option is computed as the difference between hedge fund value with the liquidity option and hedge fund value from the perspective of a passive investor with no liquidity option. The cost of a notice period or a lockup is computed as the difference between the hedge fund value with an unrestricted liquidity option and the hedge fund value with a liquidity option subject to the restriction.

4. Data and Parameter Estimates

The hedge fund data used in our empirical analysis are from the Center for International Securities and Derivatives Markets (CISDM) database. The sample period runs through December 2005. The CISDM database includes live and defunct hedge funds, funds of funds, CTAs, commodity pool operators, and indices. We eliminate indices, since they have no partners and hence no lockup feature. There are 4,260 defunct funds and 4,272 live funds in the sample, with a total of 504,979 monthly observations.

Figure 2 shows the percentage of defunct funds at each possible duration, as well as the percentage of live funds at each possible history length. The most common durations for defunct funds lie in the two to four year range, suggesting that many funds fail early in their lives. The most common history length for live funds is one year or less, a result of the tremendous growth in the industry. Table 2 lists the interquartile range of the history lengths of live funds, defunct funds, and the full sample. The ranges are similar with medians of 44 months for live funds and 47 months for defunct funds. The hedge fund data are right-tailed censored because we know only the history of returns up until a fund stops reporting, or until the end of the database is reached. Hence the expected duration of a fund will likely be much longer than these medians, and this is verified by maximum likelihood estimates of the hazard rate function as described below.

Cross-sectional averages of annualized summary statistics of the monthly returns are listed in Table 3. We require at least 24 observations for a fund to be included – a total of 3,287 defunct funds and 3,023 live funds, covering 476,178 monthly returns, are represented. The performance of defunct funds is clearly inferior, as is to be expected if poor performance is a predictor of fund failure. For example, defunct hedge funds have an annualized average return of 11.86% compared to 13.87% for live hedge funds. Sharpe ratios of defunct hedge funds average 0.63 versus 1.27 for live hedge funds. Note there is also substantial variation across fund types. In Panel C, for example, hedge funds have annualized volatility of 15.74%, compared to 22.57% for CTAs. In unreported analysis, wide variation also exists across subsets of hedge funds formed by strategy. Thus, when computing the value of liquidity options, and the cost of lockups and notice periods, it will be important to consider a wide range of parameters since there is large heterogeneity across hedge funds. As a base case, we use expected return of 12% and volatility of 15% for the "normal regime."

Table 4 lists parameter estimates of the hazard rate function. Parameters of the baseline log-logistic function in (5) are estimated very precisely, with $\lambda = 0.0129$ and $q = 1.6517$. The duration at which the unconditional probability of survival is 50% is given by λ^{-1}, which equals approximately 78 months. Thus, taking into account the large number of censored observations in the sample, the expected duration of a fund is indeed much longer than the median 44-month duration of live funds reported in Table 2. Figure 3 compares the actual number of hedge funds with uncensored duration t to the predicted number based on the parameter estimates of λ and q. The predicted number equals the hazard rate evaluated at duration t times the total number of funds, both live and defunct, with history length at least t. The fit is good, with the sharp peak for short durations consistent with the empirical distribution of durations of defunct funds in Panel A of

Figure 2. This result suggests that fund age is a significant determinant of the probability that a manager will stop reporting to the database. The other determinant in our specification is the relative cumulative fund performance which shifts the baseline hazard rate up or down. Table 4 reports the maximum likelihood estimate of β is -0.3237 with standard error 0.0059. Thus, a cumulative return that is one standard deviation below the mean increases the hazard rate by about 38%.

Upon failure, we assume as a base case that investors receive a payoff of $l = 75\%$ of the prevailing NAV of the fund, reflecting additional loss of asset value during liquidation. The 25% liquidation cost is based on results reported in Ramadorai (2008), who analyzes a sample of transactions on a secondary market for hedge fund investments conducted on Hedgebay. During 66 "disaster" transactions, involving fraud or collapse, the average discount of transaction price to NAV is 49.6%. It is likely that not all hedge fund liquidations incur losses this extreme, since some managers are able to unwind positions in a deliberate manner. To be conservative, we halve Ramadorai's estimate, but examine sensitivity of lockup costs with respect to this parameter.[9]

5. Value of liquidity options and the cost of restrictions

We estimate hedge fund values with and without liquidity options, and compute costs of redemption restrictions, over a wide range of parameter values.

5.1. Cost of Lockups and Notice Periods

Table 5 lists the combined costs of lockups and notice periods when initial fund NAV is \$100, returns are normally distributed with annual expected return of 12% and volatility of 15%, the fund has a ten-year horizon, and fund failures incur a 25% loss upon liquidation. Panel A shows results for new funds, whereas Panel B shows results

[9] In conversations with fund-of-funds managers, our value of the NAV dropping 25% contingent upon failure was remarked to be conservative, with their (informal) estimates closer to 50%. Some failing funds have a drop of nearly 100% in value contingent upon default, even in cases not involving fraud. For example, investors in Citigroup's Corporate Special Opportunities Fund received 3 cents on the dollar when the fund was wound up, as reported in "Investors Hammered by Citi Fund Setback," *Financial Times* 1/15/09.

when funds have age equal to 24 months at $t=0$, both with risk aversion $\gamma=3$. Panel A shows that costs are increasing in the length of the lockup and notice period. For a notice period of three months, the combined cost of restrictions ranges from $0.04 for a one-year lockup to $2.13 for a five-year lockup. For a lockup of three years, the combined cost of restrictions ranges from $0.68 for a one-month notice period to $0.92 for a five-month notice period. These results indicate that the length of the lockup has much more impact on the combined cost of restrictions than the length of the notice period. Introducing notice periods has only a small effect on hedge fund value because the probability of failure during the notice period is small.

Panel B of Table 5 shows that slightly older funds generally have higher restriction costs – the intuition for this is that failure probabilities peak for funds that are a few years old, as indicated by Figure 2. For a lockup of three years, for example, the combined costs range from $1.21 to $1.39 over the notice periods when the fund is two years old, substantially more than the cost when funds are new.

5.2. Comparative Statics

We display in Figure 4 the value of liquidity options and the cost of restrictions as a function of expected return, hazard rate, loss upon liquidation, volatility, and risk aversion. We report costs with a two-year lockup and a three-month notice period. The base case in each figure is a hazard rate equal to the empirical estimate, loss upon liquidation equal to 25% of the prevailing NAV, annual volatility of 15%, and risk aversion of 3.

In all cases, liquidity option values and restriction costs are decreasing in expected return. The reason is that higher expected returns increase the expected utility of remaining in the fund, ceteris paribus.

Panel A shows results for different hazard rates. The empirical estimate of the parameter λ is scaled by a factor ranging from 0.5 to 2.5. Liquidity option value and restriction costs are increasing in λ because when the baseline hazard rate increases, the failure probability increases at every node, and this decreases the expected utility of staying in the fund. When $\mu = 12\%$ and λ is 1.5 times its empirical estimate, for example, the combined restriction costs are \$2.81, more than ten times the \$0.23 reported in Table 5 when the hazard rate equals its empirical estimate. Note that the cost of restrictions is essentially zero at all levels of expected return when the λ is low. The reason for this is that at very low hazard rates, the probability that exercise will be optimal during the lockup period is negligible.

Panel B shows results when *l*, the proportion of assets retained upon fund failure, is varied. Liquidity option value and the cost of restrictions are both decreasing in *l* since the benefit of exercise is lower when losses due to failure are lower. When $\mu = 12\%$ and investors only recover 60% of NAV, for example, the restrictions cost \$5.17 compared to the \$0.23 reported in Table 5 when investors recover 75% of NAV. For recovery rates greater than 80%, the combined restriction costs are zero. The high sensitivity of the lockup costs to *l* is intuitive: if there is only a small loss in the NAV contingent upon default, the investor loses little in bearing the liquidity restriction. As *l* increases, the utility costs can become very large, especially for highly risk-averse investors.

Panel C shows results when annual volatility in the normal regime ranges from 5% to 25%. Both liquidity option value and the cost of restrictions are increasing in volatility. Higher volatility decreases expected utility for a risk-averse investor. In addition, higher volatility raises the failure probability following down steps, since the fund return is further away from the cross-sectional mean. When $\mu = 12\%$ and annual volatility is 20%, for example, the combined restriction costs are \$2.92, more than ten times the \$0.23 reported in Table 5 when volatility equals 15%.

Panel D shows results for different levels of risk aversion as measured by the parameter γ. Liquidity option values and restriction costs are substantially larger when risk aversion is high. The intuition is that, as is the case with ESOs, higher levels of risk aversion tend to accelerate the exercise decision, and hence increase the likelihood that liquidity restrictions are binding. When $\mu = 12\%$ and risk aversion is 6, for example, the combined restriction costs are \$6.77, compared to the \$0.23 reported in Table 5 when risk aversion is 3.

5.3. Underlying hedge fund values

The costs of hedge fund lockups and notice period restrictions reported so far are differences between the values of hedge funds with and without exercise restrictions. We now examine the underlying hedge fund values to gain further insight. The underlying hedge fund values under various liquidity restrictions allow investors to produce fair values for illiquid assets, such as those required under accounting standards like "Fair Value Measurements" (FAS No. 157) of the Financial Accounting Standards Board (FASB).

Table 6 reports hedge fund values with and without restrictions, over a range of expected returns in the normal regime. Panel A lists hedge fund values when the fund has age = 0 at time *t* = 0. The passive value of the hedge fund with no liquidity option, $H_{0,1}$, ranges from \$81.40 when $\mu = 8\%$ to \$121.40 when $\mu = 16\%$. Discounts to NAV at lower levels of expected return reflect the probability of failure and the subsequent loss upon liquidation. Premiums to NAV at higher levels of expected return occur because the risk premium is more than enough compensation for the volatility and probability of failure. For an investor with an unrestricted liquidity option, the hedge fund value, $O_{0,1}$, has no discount to NAV. The investor only remains invested when the expected holding period return is satisfactory, taking into account the expected return in the normal regime and the probability of fund failure. As with the case of no liquidity option, the value of the hedge fund increases with expected return, with $O_{0,1}$ ranging from \$100.02 when $\mu = 8\%$ to \$121.40 when $\mu = 16\%$.

When expected returns are high, hedge fund value with a lockup, $O_{L,0,1}$, is not substantially different from the unrestricted hedge fund value $O_{0,1}$. The reason is that when returns are high, the probability that the investor optimally exercises and redeems during the lockup period is extremely low. This is also true for the value of the hedge fund when the investor possesses a liquidity option subject to both lockup and notice period restrictions, $O_{LNP,0,1}$. For example, in Panel A when $\mu = 12\%$, the restricted liquidity option with a lockup and notice period is $O_{LNP,0,1} = \$103.32$ whereas, for an unrestricted liquidity option, $O_{0,1} = \$103.55$, making the cost of the lockup and notice period small, at $O_{0,1} - O_{LNP,0,1} = 0.23$. In contrast, the restriction imposed by a lockup on the hedge fund value causes a noticeable drop from the unrestricted liquidity option value for low expected returns because the probability that exercise is optimal during the lockup period is much higher. For $\mu = 10\%$, the value of the fund with a lockup and a notice period is $O_{LNP,0,1} = \$98.95$, compared to \$100.81 without the restrictions, hence the restrictions cost a combined \$1.86.

Panel B lists values when the fund is 24 months old at time t = 0 and has cumulative return equal to the cross-sectional mean at that time, so that the performance covariate has no impact on failure probability initially. In all cases, the values are lower than in Panel A because the initial failure probability is higher. Panel B shows that investors with an unrestricted liquidity option would in fact optimally exercise immediately, so that the value of the fund equals the NAV, for all levels of expected return less than 12%.

The underlying hedge fund value in the case where there is no liquidity option, $H_{0,1}$, warrants further discussion. As discussed in Section 3, a common restriction managers can impose, separate from lockups and notice periods, is a gate, which limits the quantity of redemption requests that are accommodated in any period. Gate restrictions are typically invoked when there are an unexpected large number of redemptions requested by investors and the fund restricts, usually on a pro-rata basis, the amount of money each investor can receive. In some cases, outright suspensions of redemption ability can disable an investor's real option to withdraw capital.[12,13] In the limit, gates and suspensions ultimately eliminate the investor's liquidity option, which is especially costly during times when investment performance is extremely bad and the

[12] See, for example, "Ore Hill Closes Fund to Client Withdrawals," *Wall Street Journal* 8/23/08.

[13] Gates could be accommodated in our lattice by modifying the payoffs that occur when redemptions are processed after a notice period has elapsed – instead of the payoff occurring all at once, they occur over a sequence of nodes following the gate. Similarly, suspensions of redemption ability could be represented by extending the lockup beyond the stated horizon.

investor most wishes to redeem. The value of the fund when the redemption option is always suspended when exercised is precisely equal to $H_{0,1}$.

Panel A of Table 6 shows that for relatively low expected returns, such as $\mu = 9\%$, the potential cost of redemption suspensions can be enormous, with $H_{0,1} = \$85.84$ and $O_{LNP,0,1} = \$96.89$. This implies that the investor is receiving an asset worth about 14% less than NAV if future withdrawals are always blocked, rather than one worth about par when the liquidity option is honored.

In summary, standard lockup and notice period restrictions on an investor's redemption option pose only modest costs of roughly 1% of the initial NAV for our base case parameters. In some instances, such as high risk aversion or extremely low recovery rates on failure, lockups and notice periods can be more onerous as depicted in Figure 4. The cost to investors of the suspension of future redemption requests, however, is substantial even under base case parameters, exceeding 10% of an investor's original deposit.

6. Conclusion

We model the investor's decision to redeem capital from a hedge fund as a real option, and develop a methodology to value the cost of lockups and notice periods to a risk-averse CRRA investor. An investor who is always able to redeem from the fund and receive the prevailing NAV has an unrestricted liquidity option. Lockups and notice periods are exercise restrictions that reduce the value of the liquidity option. The cost of the restrictions is estimated by the resulting reduction in the value of the liquidity option that investors possess.

We value the liquidity options using a lattice which accounts for the possibility of early exercise and incorporates time-varying probabilities of fund failure that vary with fund age and fund performance. We find that typical parameter values can generate costs of 1% of initial NAV for a two-year lockup and a three-month notice period for a risk aversion level of 3. For this level of risk aversion, this cost is well below the liquidity premium that hedge fund investors gain, as reported by Aragon (2007), so hedge funds are able to earn returns that more than compensate investors for the cost of lockups and notice periods. For more risk-averse investors, however, the cost of restrictions can be much higher so that the tradeoff between the cost and benefit of illiquidity is less clear. We show that the cost of restrictions substantially increases when the failure rate increases, when the proportion of assets retained upon fund failure declines, and when investors become more risk averse. We leave for future research the exercise of estimating the cost of restrictions for each fund in our sample, as well as an empirical study of the relation between strategy, the presence of restrictions, and their cost.

When fund managers can unilaterally suspend an investor's real option to redeem, we show that the cost of illiquidity can exceed 10% of initial fund NAV. This result suggests that hedge fund investors should be more concerned about the discretion asserted by fund managers in their partnership agreement, and conditions under which redemption suspensions can be imposed, rather than by the standard terms of lockup and notice periods.

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Table 1. Nomenclature

Listed are the symbols used to denote hedge fund value with and without a liquidity option, the value of liquidity options, and the cost of redemption restrictions.

H	=	Fund value with no liquidity option
O	=	Fund value with unrestricted liquidity option
O_L	=	Fund value subject to lockup
O_{NP}	=	Fund value subject to notice period
O_{LNP}	=	Fund value subject to lockup and notice period
$O-H$	=	Value of unrestricted liquidity option
O_L-H	=	Value of liquidity option subject to lockup
$O_{NP}-H$	=	Value of liquidity option subject to notice period
$O_{LNP}-H$	=	Value of liquidity option subject to lockup and notice period
$O-O_L$	=	Cost of lockup
$O-O_{NP}$	=	Cost of notice period
$O-O_{LNP}$	=	Combined cost of lockup and notice period

Table 2. Durations.

Listed is the interquartile range of durations in months of hedge funds in the 2005 CISDM hedge fund database.

	No. of Funds	25^{th}	50^{th}	75^{th}
Live	4,272	20	44	84
Defunct	4,260	25	47	80
All	8,532	23	45	82

Table 3. Summary statistics.

Listed are annualized summary statistics of monthly returns of funds in the 2005 CISDM database. Only funds with at least 24 observations are included. Fund types are hedge funds, HF, funds of funds, FOF, commodity trading advisors, CTA, and commodity pool operators, CPO. Summary statistics include mean, μ, standard deviation, σ, Sharpe ratio, *SR*, skewness, *Skew*, and excess kurtosis, *Kurt*. Monthly means are annualized by multiplying by 12 and monthly standard deviations are annualized by multiplying by $\sqrt{12}$.

Panel A. Defunct Funds

Type	No. of Funds	μ	σ	*SR*	*Skew*	*Kurt*
HF	1,685	11.86%	18.59%	0.63	-0.02	4.14
FOF	388	7.22%	9.65%	0.58	-0.31	4.13
CTA	548	12.83%	23.64%	0.31	0.64	3.87
CPO	666	7.29%	19.46%	0.15	0.34	3.60
All	3,287	10.55%	18.56%	0.47	0.13	3.99

Panel B. Live Funds

Type	No. of Funds	μ	σ	*SR*	*Skew*	*Kurt*
HF	1,450	13.87%	12.43%	1.27	0.15	3.55
FOF	1,025	8.54%	5.91%	1.26	-0.24	2.50
CTA	303	15.11%	20.62%	0.60	0.54	2.88
CPO	245	10.76%	18.54%	0.47	0.46	2.01
All	3,023	11.93%	11.53%	1.14	0.08	3.00

Panel C. All Funds

Type	No. of Funds	μ	σ	*SR*	*Skew*	*Kurt*
HF	3,135	12.79%	15.74%	0.92	0.06	3.87
FOF	1,413	8.18%	6.94%	1.08	-0.26	2.94
CTA	851	13.64%	22.57%	0.41	0.60	3.52
CPO	911	8.22%	19.21%	0.23	0.37	3.17
All	6,310	11.21%	15.19%	0.79	0.11	3.51

Table 4. Duration parameters.

Listed are parameter estimates for the following hazard rate to model the probability of hedge fund failure:

$$\lambda(t;z) = \lambda q(\lambda t)^{q-1} \Big/ \left[1 + (\lambda t)^q\right] \exp(z\beta)$$

where $\lambda, q,$ and β are parameters, t is the age of the fund, and z is the value of a performance score which equals the number of cross-sectional standard deviations the fund's cumulative return is from the cross-sectional mean. Parameters are estimated by maximum likelihood using the 2005 CISDM hedge fund database.

	λ	q	β
Estimate	0.0129	1.6517	-0.3237
Std Error	0.0002	0.0200	0.0059

Table 5. Combined Cost of Lockup and Notice Period.

Listed are the combined costs of lockups and notice periods of different lengths, in months, per share of a hedge fund with initial NAV of $100. Returns are normally distributed with annual volatility 15% and expected return of 12%. Fund failures arrive randomly following a log-logistic distribution. Upon failure, NAV drops 25% and the investor receives the remaining assets as a liquidating dividend. Panel A shows results for new funds and Panel B shows results for funds with initial age of 24 months, both with risk aversion $\gamma = 3$.

Panel A. Age = 0

	Lockup				
Notice	12	24	36	48	60
1	0.01	0.15	0.68	1.39	2.03
2	0.03	0.19	0.74	1.45	2.08
3	0.04	0.23	0.80	1.51	2.13
4	0.05	0.27	0.86	1.57	2.18
5	0.07	0.31	0.92	1.62	2.22

Panel B. Age = 24

	Lockup				
Notice	12	24	36	48	60
1	0.15	0.64	1.21	1.69	2.06
2	0.19	0.69	1.26	1.73	2.09
3	0.23	0.74	1.30	1.76	2.11
4	0.27	0.80	1.35	1.80	2.14
5	0.31	0.85	1.39	1.83	2.16

Table 6. Hedge Fund Values.

Listed in Panel A are values per share of a new hedge fund with initial NAV of $100 and a ten-year life. Returns are normally distributed with annual volatility of 15% and expected return as listed. Fund failures arrive randomly following a log-logistic distribution. Upon failure, NAV drops 25% and the investor receives the remaining assets as a liquidating dividend. The investor has risk aversion parameter $\gamma = 3$. The five columns are: the value to an investor with no liquidity option, $H_{0,1}$; the value to an investor with a liquidity option subject to a two-year lockup and a three-month notice period, $O_{LNP,0,1}$; the value subject only to a lockup, $O_{L,0,1}$; the value subject only to a notice period, $O_{NP,0,1}$; and the value when no restrictions are in place, $O_{0,1}$. Panel B lists values when the fund is 24 months old at the time of investment and has cumulative return at that time equal to the cross-sectional mean.

Panel A. Age = 0

μ	$H_{0,1}$	$O_{LNP,0,1}$	$O_{L,0,1}$	$O_{NP,0,1}$	$O_{0,1}$
8%	81.40	94.88	95.70	99.96	100.02
9%	85.84	96.89	97.53	100.24	100.25
10%	90.45	98.95	99.40	100.79	100.81
11%	95.22	101.06	101.32	101.81	101.84
12%	100.15	103.32	103.43	103.52	103.55
13%	105.24	106.34	106.39	106.36	106.41
14%	110.48	110.68	110.71	110.68	110.71
15%	115.87	115.89	115.89	115.89	115.89
16%	121.40	121.40	121.40	121.40	121.40

Panel B. Age = 24

μ	$H_{0,1}$	$O_{LNP,0,1}$	$O_{L,0,1}$	$O_{NP,0,1}$	$O_{0,1}$
8%	81.30	92.29	93.06	99.12	100.00
9%	85.29	94.05	94.67	99.37	100.00
10%	89.34	95.84	96.30	99.61	100.00
11%	93.45	97.66	97.97	99.86	100.00
12%	97.61	99.55	99.70	100.25	100.30
13%	101.81	102.18	102.24	102.25	102.30
14%	106.03	106.06	106.06	106.06	106.06
15%	110.28	110.28	110.28	110.28	110.28
16%	114.53	114.53	114.53	114.53	114.53

Figure 1. The Lattice.

The figure depicts the binomial lattice structure representing the normal regime of a hedge fund. The passage of time is represented by moving from left to right and denoted by an increase in the variable *t*. Horizontal movement represents a positive return. Diagonal downward movement represents a negative return. We refer to the combination of time step t and level *j* as node (*t*,*j*).

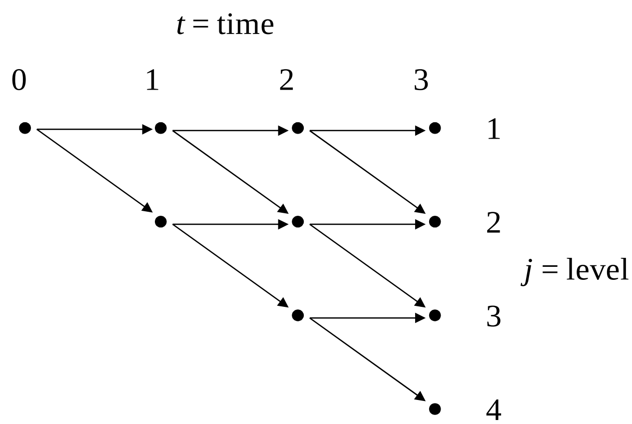
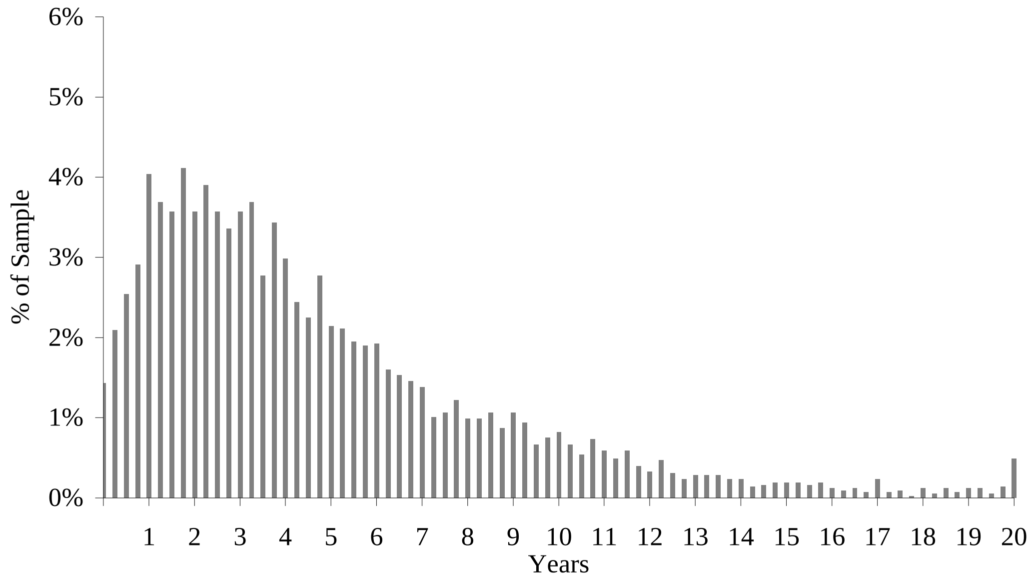


Figure 2. Durations of Defunct Funds and History Lengths of Live Funds.

Panel A shows the percentage of the 4,260 defunct funds in the 2005 CISDM database with duration equal to the value on the horizontal axis. Panel B shows the percentage of the 4,272 live funds in the 2005 CISDM database with history lengths equal to the value on the horizontal axis.

Panel A. Durations of Defunct Funds



Panel B. History Lengths of Live Funds

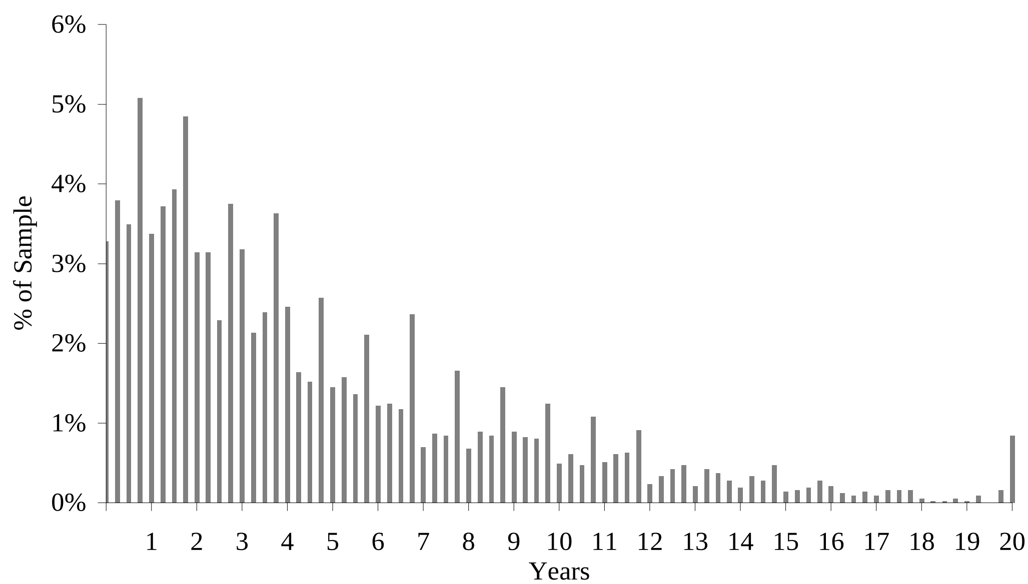


Figure 3. Log-logistic Function.

Depicted by hollow squares is the number of defunct hedge funds in the 2005 CISDM hedge fund database with lifespan equal to the values on the horizontal axis. Depicted in bold is the predicted number of hedge funds with lifespan equal to the values on the horizontal axis. Predicted number of hedge funds at lifespan *t* equals the hazard rate of the log-logistic function fitted to the data evaluated at *t* times the number of funds in the database with lifespan greater than or equal to *t*. Data include 8,532 funds with data through 2005.

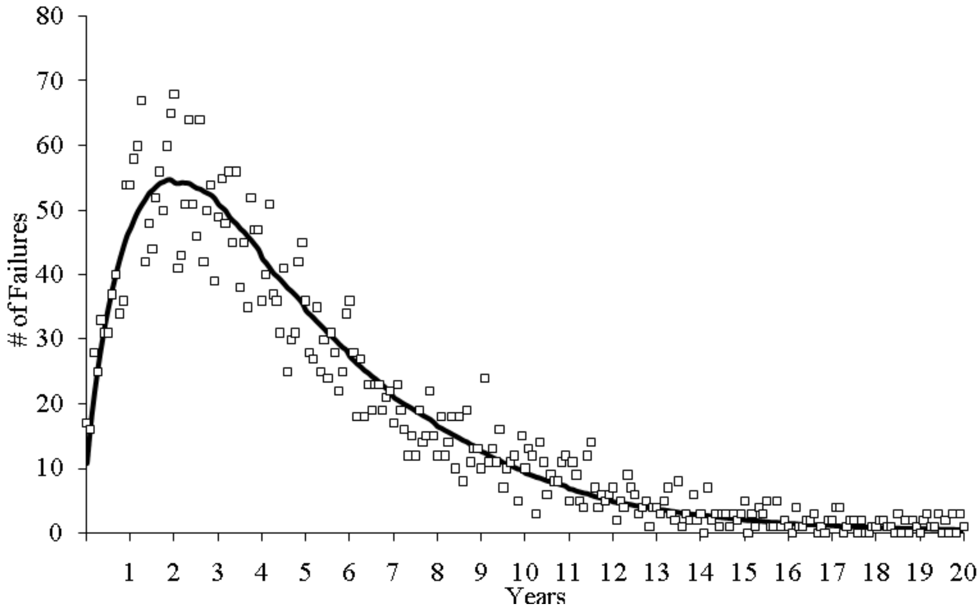
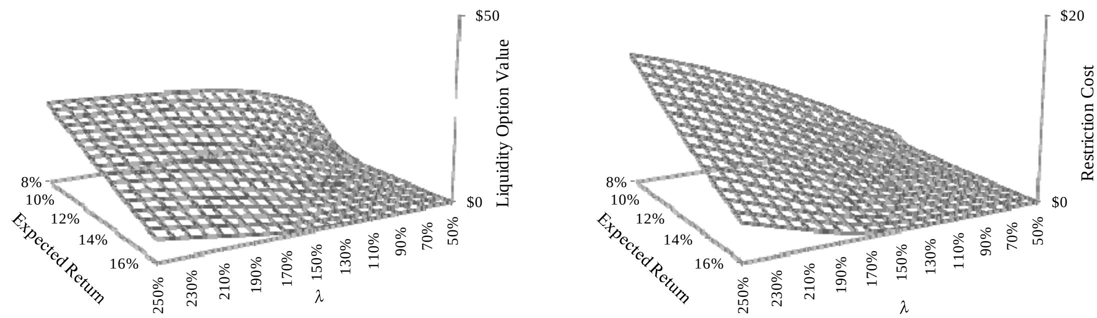


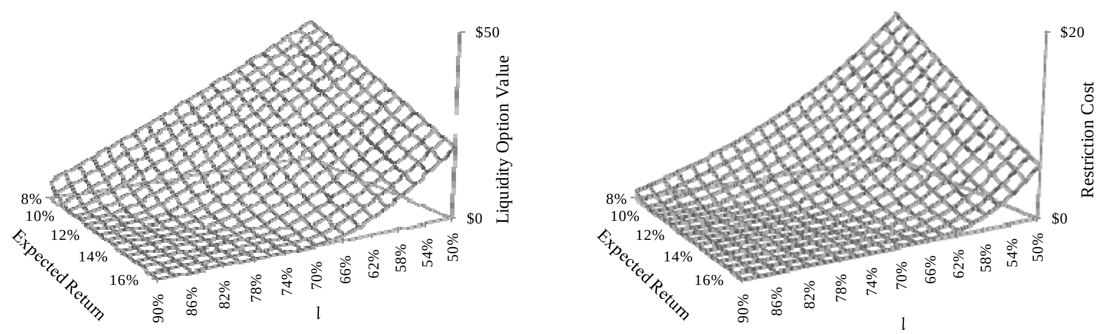
Figure 4. Liquidity Option Values and Restriction Costs.

Listed in Panel A are unrestricted liquidity option values (left graph) and the combined cost of a two-year lockup and a three-month notice period, for a risk-averse investor with CRRA preferences and risk aversion of 3 invested in a fund with initial NAV of \$100 and a ten-year life. Returns are normally distributed with annual volatility of 15% and expected return as listed. Fund failures arrive randomly following a log-logistic distribution. The λ parameter estimated from the data is scaled by the factor listed. Upon failure, the investor receives 75% of the remaining assets as a liquidating dividend. Panel B shows option values and restriction costs when the percentage of assets recovered upon failure, *l*, are varied as listed. Panel C shows option values and restriction costs when volatility is varied as listed. Panel D shows option values and restriction costs when risk aversion is varied as listed.

Panel A. Impact of Hazard Rate



Panel B. Impact of Loss



Panel C. Impact of Volatility

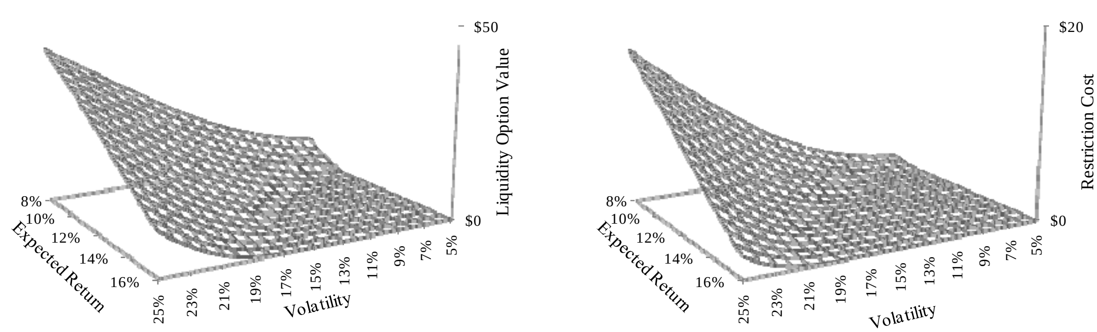


Figure 4. (continued)

Panel D. Impact of Risk Aversion