EXPO\(\text{NENTIAL INDIVIDUALS, HYPERBOLIC HOUSEHOLDS}\)

ANDREW HERTZBERG*

July 2012

Abstract

This paper introduces a model of household consumption and savings in which each household member values their own consumption more than their partner’s. In equilibrium the household consumes a higher fraction of wealth each period than under the agreed full commitment Pareto optimum. Despite both members individually having the same time consistent exponential discount rate, the representative agent for the household has a hyperbolic discount factor that is microfounded in the degree of internal preference misalignment. The model rationalizes savings commitment technologies such as assets that require joint approval for withdrawal, a feature that is mandated for most retirement plans.

JEL Codes: D13, H31, D91

---

*Columbia Business School. E-mail: ah2692@columbia.edu. I thank Patrick Bolton, Pierre-Andre Chiappori, Daniel Gottlieb, Ulrike Malmendier, Daniel Paravisini, Bryan Routledge, Tano Santos, Daniel Wolfenzon, and seminar participants at Anderson School of Management, Columbia Business School, Carlson School of Management, Dartmouth University, Fuqua School of Business, Rady School of Management, Stockholm School of Economics, as well as conference participants at the American Economic Association Meetings, Corporate Finance NBER Fall Session, Household Finance NBER Summer Institute, Western Finance Association Conference, and World Finance Conference. All remaining errors are my own.
I Introduction

The standard theory of household financial decision making is based upon the optimization of a single unified maximizing agent. However a considerable body of empirical work strongly rejects the unitary model as an adequate description of household decision making (see for example Lundberg, Pollak, and Wales 1997; Browning and Chiappori 1998; Phipps and Burton 1998; and, Ashraf 2009). Taking up the challenge posed by these empirical findings has led to considerable theoretical work that reconsiders the theory of household demand for goods and labor supply (see for example Chiappori 1988; Browning et al. 1994; Browning and Chiappori 1998; Chiappori Fortin and Lacroix 2002). Little attention has been directed to the question of how financial decision making will be affected by the misalignment of preferences within the household. ¹ This paper provides a new framework for addressing this question. In particular I focus on the most basic household financial decision and ask: are households with multiple members able to carry out optimal consumption and savings plans?

In the model I propose the household is comprised of two members who each choose how much of the combined household wealth to spend on their own private consumption each period. The crucial assumption I make is that household members have imperfectly aligned altruistic preferences. Specifically, member A cares more about the utility from his consumption than B cares about A’s consumption and vice versa. To be stark I assume that both members have the same exponential time-consistent preferences and therefore agree on the optimal savings rate for the household. I characterize the household’s equilibrium consumption path without commitment as a subgame perfect Nash equilibrium in consumption choices. This is the equilibrium that obtains when household members are unable to enforce contracts conditional on their consumption choices. The household systematically overconsumes relative to the optimal consumption plan because both members wish to unilaterally deviate and increase their own consumption. This intuition is closely related to the theoretical liter-

¹Recent exceptions include Mazzocco 2005; Mazzocco 2007; Schaner 2011.
ature on dynamic commons problems that has been used to study national underinvestment (Lancaster 1973, Tornell and Velasco 1992), overexploitation of natural resources (Levhari Mirman 1980), and sovereign debt (Amador 2008). In this context, savings is a shared public good that is underprovided in equilibrium.

The household is willing to pay for a technology that allows them to achieve any Pareto optimal consumption plan. The model allows me to numerically calculate the value of commitment. I show this is monotonically increasing in the degree of preference misalignment within the household. The model rationalizes the use of commitment technologies that limit the ability of individual members to unilaterally deviate from agreed consumption and savings plans. Empirical evidence indicates that such commitment technologies are widely used and are effective in increasing savings rates for multiple person households. As a prime example, in the US the Retirement Equity Act 1984 mandates that all retirement plans covered by the ERISA 1974 laws (this includes all defined benefit plans, IRA accounts, and all 401(K) plans) require joint approval by both spouses before money can be withdrawn or loans can be taken against such savings. Aura (2005) shows that the introduction of this law increased household saving. In the context of developing economies savings commitment technologies such as ROSCAS are motivated by the ability of one spouse to limit the ability of a partner to overconsume out of shared wealth (Andersen and Balland 2002; Collins et al. 2009). Thus, the theory in this paper rationalizes a large set of technologies that are not directed at solving time inconsistency problems stemming from individual time preferences.

Next I find the preferences of a single representative agent that would achieve the same time path of consumption as the multiperson household. This representative agent is shown to have time preferences with the same exponential discount factor as the household members as well as a hyperbolic discount factor. This is despite both household members individually having the same time consistent exponential discount rates and not being hyperbolic discounters. The hyperbolic discount factor is microfounded in the misalignment of preferences between the two household members. It is decreasing (that is, “more hyperbolic”) when
household members have more divergent interests.

I extend the basic model in several ways. First, I generalize the results beyond the case of log utility functions to allow individual members of the household to have HARA preferences which nests most commonly used utility functions including CRRA, CARA and quadratic utility and show that the representative agent result holds in this generalized setting as well. I use this generalization to show that the distortion to consumption caused by the intra-household commitment problem as well as the value of commitment are increasing in the elasticity of intertemporal substitution. Next, I introduce a public non-rival consumption good such as housing or children that directly produces utility for both members of the household. I show that public consumption aligns the preferences of both members and as a result the intertemporal inefficiency and the value of commitment are strictly decreasing in the importance of these public goods in the household.

This paper is closely related to the literature that studies individuals with hyperbolic discount factors or self control problems that render optimal savings plans time inconsistent (for example: Thaler Shefrin 1981; Laibson 1997; Harris and Laibson 2001; Laibson, Repetto, and Tobacman 2003). That literature is based on considerable evidence of time preferences at the individual level (see for example Ainslie 1992; Frederick Lowenstein and O’Donoghue 2002; Shapiro 2005). The goal of this paper is not to replace that explanation of time-inconsistency but rather to stress that misaligned preferences within the household can lead to further savings distortions and will rationalize commitment technologies not previously considered by the literature focusing on individual time preferences or self control (see for example Thaler and Benartzi 2004; Ashraf, Karlan, and Yin 2006; Beshears et al. 2011). Moreover, I argue that the two problems have an important interaction. To show this I extend the base model to allow the individual members of the household to have hyperbolic time preferences. Not surprisingly, this exacerbates the inefficiency of the household consumption path and further increases the value of commitment technologies. More interestingly, I show that hyperbolic individual preferences amplify the household problem and that the value of commitment
when both problems are combined is roughly twice its value from summing up each problem in isolation. As such, one contribution of this paper is to show that intrahousehold decision making by individuals who themselves are time inconsistent can produce sizeable distortions to optimal savings plans and create large demands for commitment technologies.

In a final extension I consider household consumption of private durable goods. I show that misaligned preferences lead the household to overconsume these goods. By comparison no such distortion exists if household members preferences are aligned even if they have hyperbolic time preferences. This result is important because it demonstrates that one way in which the intra-household framework I present here can be distinguished from models based purely on hyperbolic time preferences is by the ability to explain the overconsumption of consumer durables such as jewelry and sports cars.

A large theoretical literature studies household decision making when members have misaligned preferences (see Lundberg and Pollack 2007; Browning, Chiappori, and Lechene 2006 for comprehensive surveys of the literature). In these papers, household decision making is typically modeled as the outcome of an efficient bargaining process and the focus is directed to determining what determines the threat points and bargaining weights of each household member.\(^2\) Evidence on the question of whether households are able to enforce Pareto efficient allocations is mixed. Chiappori and Donni (2009) point out that tests for static efficiency (see for example Bobonis 2009, Browning and Chiappori 1998, and Chiappori Fotin and Lacroix 2002) find in the affirmative whereas tests for dynamic efficiency find the opposite (see for example de Mel et al. 2009, Duflo and Udry 2004, Mazzocco 2007, Robinson 2011, Udry 1996). Since this paper is interested in the intertemporal decision making of the household I study the equilibrium that obtains when commitment is not possible.\(^3\) However, it is not my objective to argue that households suffer the time inconsistency problem without taking ac-

\(^2\)As an exception non-cooperative decision making within the household has been considered by Lundberg and Pollack 1993; Chen Woolley 2001; and, Lundberg and Pollack 2003.

\(^3\)Lundberg and Pollack (2003) argue that pareto efficient bargaining may breakdown in a dynamic context because current decisions may affect future bargaining power.
tions to mitigate it. Rather the goal is to show that households have an inherent tendency to undersave and to provide a framework for assessing the types of technologies that households may employ to achieve savings levels closer to the optimum.

The paper proceeds as follows. Section II sets up the base model. Section III characterizes the equilibrium consumption choices of the household, compares them to the full commitment solution, and computes the value of commitment for the household. Section IV characterizes the preferences of the household’s representative agent. Section V generalizes the basic model in several ways, by allowing household member to have HARA preferences, to consume a shared public consumption good, to individually have hyperbolic time preferences, and to consume durable goods. Section VI discusses empirical implications of the model including strategies that the household might adopt to mitigate the undersaving problem.

II Model of Household Consumption

The household has two members indexed by $i$ labeled $A$ and $B$. Time is discrete and indexed by $t$. The household is formed at the beginning of period $t = 1$. Both household members live for $T$ years. Each year contains $N \geq 1$ periods so that there are $NT$ periods in total. For the bulk of the analysis it is sufficient to think of $N = 1$ however in Section ?? I will consider the limiting case as consumption decisions are made in continuous time by letting $N \rightarrow \infty$. I assume that the household remains together for their entire lives with certainty and abstract from endogenous household formation.

A Preferences

Each period member $i$ consumes a single consumption good. Let $C_{i,t}$ denote the amount of this good consumed by member $i$ in period $t$. The utility derived by member $i$ from their own consumption in period $t$ is

$$u_{i,t} = \ln C_{i,t}. \quad (1)$$
Note that member $i$ does not directly derive utility from member $j$’s consumption. Later in Section V I extend the model to also allow for a non-rival public consumption good which is consumed jointly by both household members.

Both household members discount utility from future consumption using exponential discount factor $\delta^{\frac{1}{N}} \in (0, 1)$. The individual discounted utility of household member $i$ in period $t$ is

$$U_{i,t} = \sum_{x=0}^{T-t} \delta^{\frac{x}{N}} u_{i,t+x}. \quad (2)$$

Thus $U_{i,t}$ is the discounted utility of household member $i$ absent any concern for the other household member. Note that these are standard time preferences so that when considered on their own the optimal consumption plan for each household member will be time consistent. Only in Section V do I extend the model to also allow the individual members of the household to have time inconsistent preferences.

One of the defining characteristics of the household is that its members are altruistic. I capture this by supposing that member $i$ places weight $\gamma_i \in (0, 1)$ on their own utility and weight $1 - \gamma_i$ on the utility of the other member. I focus on the case where the altruism between household members is imperfect by assuming that

$$\Delta \equiv \gamma_A - (1 - \gamma_B) \geq 0. \quad (3)$$

In words, $\Delta$ measures the degree to which member $i$ places more weight on her own discounted utility $U_{i,t}$ than member $j \neq i$ places on $U_{i,t}$. When $\Delta = 0$, both members agree on the weights to place on their own individual utility with the simplest case being $\gamma_A = \gamma_B = \frac{1}{2}$. The framework can also be used to study the case where members care more about each other than themselves ($\Delta < 0$) however since the evidence on household consumption decisions suggests this is generally not the case I will not focus on this scenario.
The total discounted utility of member $i$ at $t$ is

$$V_{i,t} = \gamma_i U_{i,t} + (1 - \gamma_i) U_{j,t}. \quad (4)$$

This is the objective function that each household member will maximize when taking actions at $t$.

**B Household Budget Constraint**

The present value of all combined household wealth at the beginning of $t = 1$ is $W_1$. For simplicity I assume the household starts life with wealth $W_1$ which is taken as given and has no income.\(^4\) The second defining characteristic of the household is that all wealth is combined so that both household members have full access to the remaining combined wealth in each period. This assumption is made to keep the framework as close to the standard unitary model of intertemporal decision making. Moreover the 2002 General Social Survey (Smith et al. 2011) finds that 53 per cent of all married households in the US share all financial wealth suggesting that this is the most empirically relevant characterization of the household budget constraint. Consideration of commitment technologies that involve household members having separate accounts is left for future work. For simplicity I normalize the price of the consumption goods consumed by both household member to one. Any wealth not consumed by the household is saved between periods at a gross risk free interest rate of $R^{\frac{1}{2}}$. Household wealth evolves according to the following

$$W_{t+1} = R^{\frac{1}{2}} (W_t - X_t) \quad (5)$$

where

$$X_t = C_{A,t} + C_{B,t} \quad (6)$$

\(^4\)This is identical to assuming that household labor supply decisions are fixed in which case $W_t$ is the present value of all future income plus (minus) any savings (debt) that the household has at period $t$. 

is total household expenditure in period $t$.

C Decision Making

Household members cannot commit to a path of consumption. As a result, household members are unable to enforce mutually agreed levels of consumption, either in the present or the future. Household members non-cooperatively simultaneously decide how much of the household wealth $W_t$ to spend on their own private consumption $C_{i,t} \geq 0$ each period. The dynamic equilibrium path of consumption will be the Nash subgame perfect solution to the consumption game between these two members. Let a single “*” denote the non-cooperative equilibrium quantities $C_{i,t,*}$.

Since both members make consumption decisions simultaneously it is possible that both members could attempt to spend more than total household wealth. To avoid this problem I assume that both members are able to consume at most half the total household wealth in any single period\(^5\):

$$C_{i,t} \leq \frac{W_t}{2}. \quad (7)$$

This condition can be made arbitrarily weak by making $N$ large. For example, \(7\) implies that within a year one member can withdraw up to $W_t \left(1 - \frac{1}{2^N}\right)$. As $N \rightarrow \infty$ this implies that all wealth can be withdrawn in any finite period of time. By imposing \(7\) I ensure $C_{A,t} + C_{B,t} \leq W_t$ and hence have a well defined budget constraint for each household member’s consumption problem each period. I show in the Appendix that \(7\) does not bind in any period $t < NT$ if and only if

$$|\gamma_A - \gamma_B| \leq \delta^{\frac{1}{N}}. \quad (8)$$

I assume that \(8\) holds and so, in equilibrium, condition \(7\) will only bind in the final period of the household’s life and will ensure that in that period $C_{i,NT} = \frac{W_{NT}}{2}$. Note that when $N$

\(^5\)If the model were extended to allow household income then this constraint would impose the condition that credit markets will not allow the household to raise debt in excess of the present value of all future income.
is large, $\delta^{\frac{1}{N}} \to 1$ and thus (8) places almost no limit on parameters.

I have chosen to require (7) in order to avoid imposing arbitrary tie breaking rules to deal with situations where members attempt, in total, to spend more than $W_t$. These problems arise because of the assumption that consumption occurs simultaneously each period. In the internet appendix I revisit this problem by otherwise resolving an identical model in which household members make consecutive consumption decisions. In that setting a standard one person budget constraint in which members are able to spend up to the full present value of remaining household wealth in any period is imposed. In that Appendix I show that the equilibrium studied here is arbitrarily close to the unique equilibrium from the consecutive move model as $N \to \infty$ thus demonstrating that (7) does not drive the results studied below.

D Full Commitment Problem and the Value of Commitment

To evaluate the optimality of the non-cooperative equilibrium consumption path, I compare it to the consumption path that would be achieved if the household was able to fully commit to consumption choices at the start of $t = 1$. Consider the problem the household would face in setting a full commitment path. Whenever $\gamma \neq \frac{1}{2}$ household members will disagree over the optimal allocation. However any allocation that they would choose must be Pareto optimal and hence I characterize the solution to the following full commitment Pareto problem:

$$
\max_{\{C_{i,t}\}_{t=1}^{NT}} \{\text{subject to } W_1 - \sum_{x=1}^{NT} R^{-\frac{x-1}{N}} [C_{A,x} + C_{B,x}] \geq 0 \text{ and } \{C_{A,t}, C_{B,t}\}_{t=1}^{NT} \geq 0. \}
$$

where $\eta \in [0, 1]$ is the Pareto weight placed on the objective of member $A$. Let a double "**" denote the full commitment Pareto optimal consumption quantities $C_{i,t}^{**}$ that solve this
problem.

To quantify the welfare loss incurred by the household under the non-cooperative equilibrium I calculate how much the household would be willing to pay at \( t = 1 \) for a technology that allowed them to commit to an optimal consumption path. Let \( V_{i,1}^* (W_1) \) be the discounted lifetime utility that will be achieved by household member \( i \) absent commitment as a function of initial household wealth. Let \( V_{i,1}^{**} (W_1 (1 - \phi), \eta) \) be the counterpart for the case where the household has spent a fraction \( \phi \) of their initial wealth \( W_1 \) to achieve the full commitment plan that places weight \( \eta \) on the preferences of member \( A \). The value of commitment \( \phi^{**} \) is defined as the most that the household will pay while ensuring that there exists a weight \( \eta \) so that the purchase is a Pareto improvement for both members. Formally \( \phi^{**} \) solves:

\[
\phi^{**} = \max_{\phi, \eta} \phi \\
\text{subject to } V_{i,1}^{**} (W_1 (1 - \phi), \eta) \geq V_{i,1}^* (W_1) \text{ for } i \in \{A, B\}, \text{ and } \eta \in [0, 1].
\]

An analytical solution for \( \phi^{**} \) is intractable in most cases so this will be solved for numerically.

III Consumption Choices and Value of Commitment

A Non-Cooperative Equilibrium Consumption Choices

The equilibrium consumption path is solved in the Appendix. The equilibrium level of consumption by member \( i \) in period \( t < NT \) is\(^6\)

\[
C_{i,t}^* = \frac{\gamma_i}{1 + \Delta + \sum_{x=1}^{NT-t} \delta^x} W_t.
\]

\(^6\)By assumption the equilibrium consumption in period \( t = NT \) is \( C_{i,t}^* = \frac{W_2}{2} \).
This is the unique interior equilibrium. The allocation of consumption within any period is determined the weights each member place on their own utility

\[
\frac{C_{i,t}^*}{C_{j,t}^*} = \frac{\gamma_i}{\gamma_j}.
\]  

(14)

If member A places more weight on his own utility than B places on hers then A will have a larger share of consumption in each period. Total equilibrium household expenditure in period \( t \) is

\[
X_t^* = \frac{1}{1 + \frac{1}{1+\Delta} \sum_{x=1}^{NT-t} \delta^x} W_t.
\]

(15)

The share of wealth that is spent in any period \( t < NT \) is strictly increasing in \( \Delta \), the degree of preference misalignment. The dynamics of equilibrium consumption between periods is

\[
\frac{X_{t+1}^*}{X_t^*} = (R\delta)^{\frac{1}{\Delta}} \left( \frac{\sum_{x=0}^{NT-(t+1)} \delta^x}{\Delta + \sum_{x=0}^{NT-(t+1)} \delta^x} \right).
\]

(16)

The higher is \( \Delta \) the more downward sloping is the equilibrium consumption path. The dynamics of household consumption is determined only by \( \Delta \) and not the particular values of \( \gamma_A \) and \( \gamma_B \) that give rise to that degree of misalignment. So two households in which both household members are care slightly more for themselves with \( \gamma_A = \gamma_B = 0.6 \) will have an identical path of total consumption as one in which one member cares more for himself and the other cares equally for both with \( \gamma_A = 0.7 \) and \( \gamma_B = 0.5 \). The only difference in these household will be the consumption share of each member within a period.

\[\text{7}\]There is another set of trivial equilibria in which both members set \( C_{i,t}^* = 0 \). This corner solution exists only because of the log utility assumption. If instead I assume the period utility function to be \( u_{i,t} = \ln (C_{i,t} + \varepsilon) \) for any arbitrarily small \( \varepsilon > 0 \) this equilibrium would not exist. No equivalent of this equilibria exists in the consecutive move version of the model in Section V. Hence I ignore this for the rest of the paper.
B Comparison to Full Commitment Consumption Path

To assess the optimality of the equilibrium consumption path that the household will achieve without commitment, I compare it to the set of Pareto optimal consumption paths, one of which would be chosen if the household had access to perfect commitment at $t = 1$. The total level of consumption that the household would commit to in any period is

$$X_t^{**} = \frac{1}{1 + \sum_{x=1}^{NT-t} \delta^x} W_t.$$  \hfill (17)

The optimal total level of consumption is not affected by the Pareto weight $\eta$ given to each member in the planning problem. Comparing the full commitment solution to the equilibrium level of consumption leads directly to the following proposition:

**Proposition 1**: If $\Delta > 0$ then in any period $t < NT$ the non-cooperative equilibrium level of consumption is higher than the amount that the household would commit to conditional on entering the period with wealth $W_t$.

The proof of Proposition 1 comes directly by comparing (15) with (17). The intuition for this result is as follows. Household savings is a shared public good. Household savings and consumption is therefore a dynamic commons problem. Whenever household members place more weight on their own utility than their partner does ($\Delta > 0$) then this public good will be underprovided in equilibrium. Put differently, the full commitment solution is not subgame perfect because at least one household member will wish to unilaterally deviate from this allocation by spending slightly more on themselves.\(^8\)

The dynamics of consumption under the full commitment consumption path are given by:

$$\frac{X_{t+1}^{**}}{X_t^{**}} = (R\delta)^{\frac{1}{\eta}}.$$  \hfill (18)

\(^8\)If $\Delta < 0$ then equilibrium household consumption path depends on whether or not member $i$ is able to buy consumption goods for his partner. If so then Proposition 1 holds and overconsumption occurs whenever $\Delta \neq 0$. If members are only able to purchase consumption goods for themselves then the household will underconsume in each period.
Notice that it is determined only by the time preferences and the gross rate of return and is unaffected by $\Delta$. A direct corollary of Proposition 1 is that the slope of the consumption path in the non-cooperative equilibrium is strictly below the slope of the full commitment consumption path whenever $\Delta > 0$. This is seen immediately by comparing (16) and (18).

The equilibrium consumption path is compared to the full commitment solution in Figure 1 assuming $T = 50$, $\delta = 0.95$, $R = \frac{1}{0.95}$, and $N = 1$. The figure illustrates that early in the life of the household the equilibrium level of consumption is higher than under the full commitment solution. When both members place 60% weight on their own utility ($\gamma_A = \gamma_B = 0.6$) in total the household spends over 18% more than it would under the full commitment solution in the first year. If this altruism is reduced so that $\gamma_A = \gamma_B = 0.7$ then the household overspends by more than 37% in the first year of its life. The under-provision of savings means that later in the households life they consume much less than under the full commitment optimum. If $\gamma_A = \gamma_B = 0.6$ then the household consumption is less than 60% of the level that the household would like to commit to for each of the last five years of the households life.

In the full commitment solution, only the allocation of consumption within each period is determined by the Pareto weight assigned to each household member in the planning problem. For a given Pareto weight $\eta$ the ratio of each members consumption in any period $t < NT$ is

$$\frac{C_{A,t}^{**}}{C_{B,t}^{**}} = \frac{\eta \gamma_A + (1 - \eta)(1 - \gamma_B)}{(1 - \eta) \gamma_B + \eta (1 - \gamma_A)}.$$ 

From this expression we see that if member $i$ was given full control to chose the consumption path of both members then the ratio of her consumption to her partner’s would be $\frac{\gamma_i}{1 - \gamma_i}$ in each period.
C The Value of Commitment

Having shown that the allocation of consumption achieved in the non-cooperative solution is inefficient I now turn to quantifying this inefficiency. To do this I ask what fraction $\phi^{**}$ of the household’s initial wealth would both household members agree to spend in order to achieve a Pareto efficient allocation. I defer the discussion of the particular commitment technologies that the household may employ to achieve efficiency to Section VI. Due to the assumption of log utility $\phi^{**}$ will be independent of the level of initial household wealth. Despite this, an analytical solution is in most cases intractable. Instead I solve for this fraction numerically. Figure 2 shows that the value of commitment increases monotonically with the weight that household members place on their own utility relative to the utility of the other. A household in which both members place weight $\gamma_i = 0.6$ on their own utility will be prepared to pay up to 1.61% of the present value of their total wealth to achieve full commitment. If both members place weight $\gamma_i = 0.7$ on their own utility the undersaving problem is more severe and they would be willing to pay up to 5.62% of total household wealth to eliminate this inefficiency.\(^9\)

IV Representative Agent

Typically household savings and consumption decisions are modeled as if they are made by a single optimizing representative agent. If the interests of household members are perfectly aligned ($\Delta = 0$) then this assumption involves no loss of generality since both members have the same objective function. In this case the representative agent will have the same time preferences as the individual household members. In this section I find the representative agent for a household in which the interests of its members are not perfectly aligned. In particular, I ask whether it is possible to find a representative agent that would achieve the

\(^9\)Unreported numerical simulations show that the value of commitment is (i) non-monotonic in $\delta$ peaking when $\delta = \frac{1}{2}$, and (ii) varies only a trivial amount with the distribution of $\gamma_A$ and $\gamma_B$ for a given level of $\Delta$.\(^9\)
same consumption path and what time preferences would this agent have. We know already that the time preferences of the representative agent must be different to that of its individual members since those preferences are time consistent and would give rise a consumption path identical to the full commitment solution.

Since the primary focus of this paper is the intertemporal choices of the household I consider a representative agent with preferences over the level of total household consumption $X_t$. Matching the allocation of consumption within each period between $C_{A,t}$ and $C_{B,t}$ is not interesting because in equilibrium these are consumed in a constant ratio.\(^{10}\) Consider the problem of a single representative agent who chooses the level of $X_t$, each period. The period utility of the representative agent is

$$u_{r,t} = \ln X_t. \quad (19)$$

The discounted utility of the representative agent at time $t$ is

$$U_{r,t} = u_{r,t} + \beta_r \sum_{x=1}^{NT-t} \delta^x u_{r,t+x} \quad (20)$$

where $\delta_r \in (0, 1]$ is a standard exponential discount factor and $\beta_r \in (0, 1]$ is a quasi-hyperbolic discount factor of the type introduced by Laibson (1997). While more general utility functions and discount functions could be considered the results below show that this form is sufficiently flexible to represent the household. The representative agent faces the same intertemporal budget constraint as the household (5).

As stressed by Laibson (1997), when $\beta_r < 1$ any optimal path of consumption from the perspective of the representative agent at $t$ will be time inconsistent. When considering the representative agent without commitment, I study the problem where the agent is aware of this time inconsistency and takes it into account when making consumption choices each

\(^{10}\)This can be achieved by using a more general period utility function over $C_{A,t}$ and $C_{B,t}$ of the form $u_{r,t} = \mu_{A,r} \ln C_{A,t} + (1 - \mu_{A,r}) \ln C_{B,t}$ does not alter the time preferences found in Proposition 2.
period. As a result the consumption path chosen by the representative agent will be found by backward induction where consumption choices are subgame perfect best responses given the resulting choices that they will lead to in the future. The consumption path of the representative agent with and without commitment is solved in the Appendix.

**Proposition 2:** The representative agent without commitment has an identical path of total consumption as the household without commitment if:

\[
\begin{align*}
i. \ & \delta_r = \delta, \text{ and} \\
\text{ii.} \ & \beta_r = \frac{1}{1 + \Delta}. 
\end{align*}
\]

Whenever \( NT > 2 \) and is finite this is the unique set of time preferences that replicate the consumption path of the household.

Proposition 2 is proved in the Appendix. The key result in Proposition 2 is that \( \Delta > 0 \) implies \( \beta_r < 1 \). Thus, when household members care more about their own utility than their partner does, the representative agent for the household must have hyperbolic time preferences that are microfounded in the degree of preference misalignment within the household. The intuition for why the representative agent is exactly quasi-hyperbolic is as follows. When comparing the value of a dollar three or four years from now each member compares the shared value of a dollar in each period. Thus the misalignment of preferences does not alter this tradeoff and it is determined only by the time preferences \( \delta \) and interest rate \( R \). However this comparison changes when members tradeoff consumption in the present with saving for the future. The difference stems from the fact that in the present each member’s decision is a tradeoff between her own individual consumption and the shared valued of a dollar for the household in the future. The consumption decision of each member equalizes their individual private marginal utility today weighted by \( \gamma_i \) with the marginal utility of the household in the future weighted by unity. So for example, if both members place weight...
60% on their own private utility ($\gamma = 0.6$) then collectively the household acts as though it is worth 120% today relative the 100% combined future entity. Thus, as (22) indicates the household acts as though it discounts the future relative to the present with a factor of $\beta_r = \frac{1}{120\%} = 0.833$. The tradeoff between $t$ and $t + 1$ is transformed because this is the only point at which household members are able to tradeoff between themselves and the combined future welfare of the household.\(^{11}\)

This result is significant because the household euler equation is the cornerstone for understanding savings and investment decisions in finance and macroeconomics. Presumably one reason why intra-household interactions have not been widely incorporated into this analysis is due to tractability. However the representative agent result here shows that standard approach whereby a household is represented as if it were a single entity can easily be adapted to incorporate the savings commitment problem studied in this paper. Moreover, by providing a microfoundation for the quasi-hyperbolic parameter itself, Proposition 2 shows how the euler equation can be calibrated based on the degree of preference misalignment within the household. The usefulness of this result extends well beyond the special case of log utility studied here. This is shown in the next section where Proposition 2 is extended to the case where household members have HARA period utility functions.

V Generalizing Household Preferences

This section studies several extensions to the preferences assumed for household members in the model studied so far. First, I allow household members to have HARA period utility functions and show that the representative agent result applies to the broad class of utility functions nested with the HARA specification. I also use this setting to study how the results vary with the elasticity of intertemporal substitution of the household members. Next, I study\(^{11}\) This intuition indicates that while time inconsistency will remain the strict quasi-hyperbolic representation will not obtain if household members are able to purchase durable goods. This is explored further is Section V.
how the presence of public consumption goods within the household impacts equilibrium savings. I then allow the individual members of the household to have time inconsistent hyperbolic time preferences of the type emphasized by Laibson (1997). Finally, I demonstrate how misaligned preferences will affect household demand for durable goods. I will consider each extension one at a time and in isolation so as to highlight the differences from the base model results presented above.

A HARA Utility

So far the analysis has focused on the case where household members have log period utility function. This section generalizes the analysis by replacing (1) with the assumption that both household members have HARA period utility functions of the form:

$$u_{i,t} = \frac{\rho}{1-\rho} \left( \frac{C_{i,t}}{\rho} - \varphi \right)^{1-\rho}. \quad (23)$$

Note that this significantly generalizes the preferences of the household members since it nests most common period utility functions as special cases: linear ($\rho = 0$), quadratic ($\rho = -1$), CARA ($\rho = \infty, \varphi = -1$), and CRRA ($\rho > 0, \varphi = 0$). I limit the analysis in this section to the case of a symmetric household: $\gamma_A = \gamma_B$. Otherwise, all assumptions are identical to the base model studied in Section II.\textsuperscript{12}

I now show that the representative agent results of Section IV generalize to the case where individual household members have HARA utility. To do this consider the same representative agent as before except now replace the log period utility function in (19) with

$$u_{r,t} = \frac{\rho}{1-\rho} \left( \frac{X^r_t}{\rho} - 2\varphi \right)^{1-\rho} \quad (24)$$

\textsuperscript{12}The representative agent results do not obtain for all parameterizations of HARA utility when altruism in the household is not symmetric. Unreported numerical simulations suggest that the representative agent proposed here obtains a consumption very close to that obtained by the non-cooperative household in this asymmetric case.
**Proposition 3:** Assume the household is symmetric so that \( \gamma_A = \gamma_B \). Assume also that household members and the representative agent have HARA utility as per (23) and (24). The representative agent without commitment has an identical path of total consumption as the household without commitment if:

\[
\begin{align*}
  i. \ & \delta_r = \delta, \text{ and} \\
  ii. \ & \beta_r = \frac{1}{1 + \Delta}.
\end{align*}
\]

Whenever \( NT > 2 \) and is finite this is the unique set of time preferences that replicate the consumption path of the household.

Proposition 3 establishes that the standard euler equation can be adapted to incorporate intra-household savings distortions by the inclusion of a quasi-hyperbolic discount factor that is microfounded in the degree of preference misalignment between household members. This shows that the usefulness of Proposition 2 for log utility extends to the far more general case of HARA period utility.

**A-1 The Value of Commitment and the Elasticity of Substitution**

Due to the assumption of log utility, the results presented so far on the value of commitment are limited to the case where the elasticity of intertemporal substitution (EIS) for both household members is unity. The literature which has sought to estimate the EIS has produced mixed results. Estimates range between being close to zero (Hall 1988, Dynan 1993) to being as high as two (Blundell, Browning and Meghir 1994; Mulligan 2002; and Gruber 2006). I study how the household saving problem changes with different values of the EIS by focusing on a special case of (23) where \( \rho > 0 \) and \( \varphi = 0 \). This gives the CRRA utility specification where \( \zeta = \frac{1}{\rho} \) is the constant EIS.

An analytical solution to the non-cooperative equilibrium with CRRA utility is characterized in the Appendix. In this setting the solutions for equilibrium consumption choices are
generally intractable. To avoid this I focus on numerical examples illustrating the resulting equilibrium household consumption path. These are presented in Figure 3. Panel A shows how the equilibrium consumption path of the household varies with the EIS and compares it to the full commitment consumption path. In each case I assume that both members place weight $\gamma_i = 0.6$ on their own utility. Note first that since these are drawn for $\delta = \frac{1}{R}$, the full commitment consumption path is flat and identical for each value of $\zeta$. The panel shows that degree of undersavings is increasing in the EIS. For these parameters, when $\zeta = 0.5$, the household spends over 8% more than the full commitment level of consumption in the first year of its life. If instead, $\zeta = 1.5$ then household consumption is more than 30% higher than optimal level. Of course when the EIS is higher the utility cost from an intertemporal inefficiency of a fixed size is also lower. So as we increase the EIS the size of intertemporal inefficiency increases but the utility cost of a given distortion falls.

Panel B shows which of these countervailing forces dominates by showing how the value of commitment varies with the EIS. As in Panel A, this is drawn assuming $\gamma_i = 0.6$ for both household members. The clear comparative static result from Panel B is that the value of commitment increases with the EIS. Despite the fact that the utility cost of a given distortion is lower, the increased size of the intertemporal inefficiency dominates this effect. For the parameters assumed in Figure 3, the household will be willing to pay 0.77% of household wealth to achieve full commitment if $\zeta = 0.5$. If instead $\zeta = 1.5$ the household will be willing to pay 2.54% of household wealth to achieve full commitment.

**B Public Consumption Good**

**B-1 Setup with Public Consumption**

So far I have assumed that all consumption goods are consumed individually by one member or the other. As such, $C_{A,t}$ only contributes utility to household member $B$ in so far as $B$ cares about the utility of $A$. However one advantage of being in a household is
that it allows the household members to share non-rival public consumption goods such as housing, children, and consumer durables.\textsuperscript{13} To study how this impacts the intertemporal consumption that the household will achieve suppose that there is a second good, $H_t$, that provides utility directly to both household members. The total level of public consumption is the sum of the amount purchased in each period by both household members

$$H_t = H_{A,t} + H_{B,t}$$

where $H_{i,t}$ is the amount of the public consumption good purchased by member $i$ in period $t$. Assume now that the period utility of member $i$ is

$$u_{i,t} = \mu \ln C_{i,t} + (1 - \mu) \ln H_t$$

where $\mu \in [0, 1]$ captures the relative weight that household members place on private consumption relative to public consumption.\textsuperscript{14} This period utility function replaces the simple period utility function in (1) which is just a special case were $\mu = 1$. Apart from this change the preferences of the household members remain the same as described for the base model in (2), (3), and (4).

I assume that public consumption is also continuous and decided non-cooperatively. Each period both members simultaneously chose how much of the remaining household wealth to spend on $C_{i,t} \geq 0$ and $H_{i,t} \geq 0$. As before, consumption choices are chosen non-cooperatively as Nash equilibrium subgame perfect best responses to each other. To avoid the possibility

\textsuperscript{13}The economies of scale from public consumption goods will provide one rationale for the household to endogenously form despite the intertemporal inefficiencies documented here.

\textsuperscript{14}The model can be extended to allow each member to place different weights on public versus private consumption. If we assume that members are unable to reverse the consumption decision of the other ($H_{i,t} \geq 0$) then the level of public consumption will be determined by the level desired by the member with the highest weight on public consumption (lowest value of $\mu_i$).
that household members spend more than total household wealth I adapt (7) to assume that

$$C_{i,t} + H_{i,t} \leq \frac{W_t}{2}.$$  \hspace{1cm} (25)

To ensure (25) never binds outside of $t = NT$ I adapt (8) to now assume

$$|\gamma_A - \gamma_B| \leq \frac{1 - \mu + \delta^\frac{T}{T}}{\mu}$$  \hspace{1cm} (26)

which, since $\mu < 1$, is less restrictive than (8). Total expenditure in period $t$ is now

$$X_t = C_{A,t} + C_{B,t} + H_{A,t} + H_{B,t}.$$  \hspace{1cm} (27)

The intertemporal budget constraint (5) remains the same as before. The benchmark full commitment planning problem is amended in the same way to incorporate the public consumption good.

**B-2 Non-Cooperative Equilibrium Consumption Choices with Public Consumption**

The model with public household consumption is solved in the Appendix. The primary focus is to study how the presence of this shared consumption goods affects the intertemporal decisions of the household. The equilibrium level of consumption by member $i$ in period $t < T$ is

$$C^*_{i,t} = \frac{\gamma_i \mu}{1 + \mu \Delta + \sum_{x=1}^{NT-t} \delta^x} W_t.$$  \hspace{1cm} (28)

Since it doesn’t matter who buys a given unit of the public consumption good the individual choices of $H_{A,t}$ and $H_{B,t}$ are not uniquely determined in equilibrium. However the total level of public consumption is uniquely determined in equilibrium and is

$$H^*_t = \frac{1 - \mu}{1 + \mu \Delta + \sum_{x=1}^{NT-t} \delta^x} W_t.$$  \hspace{1cm} (29)
Total equilibrium consumption in each period is

\[ X^*_t = \frac{1}{1 + \frac{1}{1+\mu\Delta} \sum_{x=1}^{NT-t} \delta^x} W_t. \]

The full commitment optimal level of total consumption is the same as before as described in (17).

**Proposition 4:** In the model with public consumption, if \( \mu\Delta > 0 \) then in any period \( t < NT \) the non-cooperative equilibrium level of consumption is higher than the amount that the household would commit to conditional on entering the period with wealth \( W_t \).

This highlights the importance of private consumption in the intertemporal distortion to household savings. This is the decision which is distorted because it requires each member to trade off between their own individual utility and the combined interests of the household. This is the tradeoff at the heart of the dynamic commons problem that leads to undersavings saving. When deciding on the level of public consumption each member trades off the combined interest of the household today versus the future combined interest. This trade-off is not distorted by the self interest of the individual household members. In the extreme, if all consumption were public (\( \mu = 0 \)) then both members would have the same objective and would choose an intertemporally efficient consumption path even if they cared very little for their partner (i.e. if \( \Delta \) was large). This intuition is captured in the intertemporal preferences of the representative agent for household with public consumption.

**Proposition 5:** In the model with public consumption, the representative agent without commitment has an identical path of total consumption as the household without commitment if:

1. \( \delta_r = \delta \), and
2. \( \beta_r = \frac{1}{1 + \mu\Delta} \).

24
Whenever $NT > 2$ and is finite and $\mu \Delta > 0$, this is the unique set of time preferences that replicate the consumption path of the household.

Thus the representative agent for the household remains a single agent with a hyperbolic discount factor. The size of the hyperbolic discount factor is now microfounded in the degree to which household members disagree over the relative weight they assign to each others private consumption. The larger the fraction of household consumption that is private, the smaller will be $\beta_r$, and hence the further will the household be from the time consistent consumption path it would like to commit to. This intuition is captured in Figure 4 which plots the value of commitment as a function of $\mu$. The value of commitment is strictly increasing in the weight that both household members place on private versus public consumption. This demonstrates that the savings problem is less severe in a household where members draw a larger fraction of their utility directly from the same things. This suggests that increases in the importance of shared consumption, say through having children, may also reduce the savings distortion. In addition, it implies that one reason why households are more likely to form amongst people with more shared consumption is that this helps alleviate the undersaving problem.

C Hyperbolic Household Members

The central message of this paper is that households whose members have misaligned preferences will be unable to carry out optimal consumption plans without a commitment technology. To emphasize that divergent preferences within the household are sufficient to render optimal consumption and savings plans time inconsistent I have made the stark assumption that individuals have standard exponential time preferences. Hence the members of the household, left to themselves, are able to carry out optimal consumption and savings plans. However considerable evidence in both psychology and economics suggests that individuals have hyperbolic time preferences (for example Ainslie 1992; Frederick Lowenstein and
O’Donoghue 2002; Shapiro 2005). For this reason, I now study how time inconsistency in the individual time preferences of the household members interacts with the time inconsistency exhibited by the combined household.

To do this I re-examine the base model introduced in Section II. The only change to that setup is to the time preferences of both household members so that (2) is replaced with

$$U_{i,t} = u_{i,t} + \beta \sum_{x=1}^{NT-t} \delta^x u_{i,t+x}$$

(30)

where $\beta \leq 1$ is the quasi-hyperbolic discount factor used by both household members to discount future utility relative to the present.\textsuperscript{15} Both household members are assumed to be sophisticated hyperbolic agents who recognize that their time preferences will change in the future.

The equilibrium of the model is solved in the Appendix along with the full commitment optimal allocation. The equilibrium level of consumption by member $i$ in period $t < NT$ is

$$C_{i,t}^* = \frac{\gamma_i}{1 + \Delta + \beta \sum_{x=1}^{NT-t} \delta^x} W_t$$

(31)

and total consumption

$$X_t^* = \frac{1}{1 + \frac{\beta}{1 + \Delta} \sum_{x=1}^{NT-t} \delta^x} W_t.$$  

(32)

For a given level of wealth $W_t$ the household will consume more in a period if the members are more hyperbolic ($\beta$ lower) and have more misaligned preferences ($\Delta$ larger). For all periods after $t = 1$ the optimal level of household consumption is still given by (17). Direct comparison shows that the household consumes more than the full commitment fraction of wealth in every period $t > 1$ whenever $\frac{\beta}{1 + \Delta} < 1$. This is the analog to Proposition 1 but the commitment problem is now exacerbated by the inconsistent time preferences of the household members.

\textsuperscript{15}I also amend the assumption in (8) with $|\gamma_A - \gamma_B| \leq \beta \delta^\frac{1}{N}$ to ensure (7) does not bind in any period $t < NT$.  

26
By comparing (32) to the case where $\beta = 1$ it is clear that the representative agent result of Section IV can be extended directly to the case where household members have hyperbolic time preferences.

**Proposition 6:** In the model where household members discount future utility with a quasi-hyperbolic discount factor $\beta \leq 1$, the representative agent without commitment has an identical path of total consumption as the household without commitment if:

i. $\delta_r = \delta$, and

ii. $\beta_r = \frac{\beta}{1 + \Delta}$.

Whenever $NT > 2$ and is finite this is the unique set of time preferences that replicate the consumption path of the household.

The hyperbolic time preferences of the household are now microfounded in the degree of preference misalignment and the time preferences of the members. The time inconsistency of the households overall consumption path is amplified when the individual members are themselves hyperbolic. This point is made clearer by considering how the value of commitment is effected when the individual members of the household are hyperbolic. This is shown in Figure 5 which plots the value of commitment for a household in which both members place symmetric weight on their own utility versus their partners (i.e. $\gamma = \gamma_A = \gamma_B$). The relationship between the value of commitment and $\gamma$ is shown for for $\beta = 0.85$ and $\beta = 1$.

When household members place the same weight on each other’s utility ($\gamma_A = \gamma_B = \frac{1}{2}$) the value of commitment with $\beta = 0.85$ is $1.19\%$ of household wealth. This is the force that Laibson (1997) documents showing that hyperbolic individuals will value commitment to prevent themselves from deviating from the ex-ante optimal consumption plans in the future. Conversely, when $\gamma_A = \gamma_B = 0.5852$ and $\beta = 1$ the household is also willing to spend $1.19\%$ of its lifetime wealth to achieve full commitment and overcome the inefficiency due purely to

---

16I choose $\beta = 0.85$ to match the calibration parameters used in Laibson 1997.
the divergence in both member’s objectives. A household which faces both problems, so that 
\( \gamma_A = \gamma_B = 0.5852 \) and \( \beta = 0.85 \) will pay 4.82\% of household wealth for commitment. This is more than double (2.44 percentage points higher) than the sum of the value of commitment (1.19\% + 1.19\%) from considering both of these forces in isolation. The striking result here is that misaligned preferences within the household significantly amplifies the welfare cost of household members’ inconsistent time preferences.

The intuition for this amplification result is as follows. If \( \beta = 1 \) and \( \gamma_A = \gamma_B = \frac{1}{2} \) then the equilibrium household consumption path will be optimal and thus will equate the discounted marginal utility of consumption at each point in time. Therefore, by virtue of the envelope theorem, when \( \gamma_A, \gamma_B = \frac{1}{2} \) slight decreases in \( \beta \) below 1 will distort this intertemporal consumption path but will have only a second order effect on welfare. However if the individual household members have misaligned preferences (\( \Delta > 0 \)) then even with \( \beta = 1 \) the household consumption path will be suboptimal in the sense that the household will consume too much early in its life. Put differently, the discounted marginal utility of consumption in the near future will be below the marginal utility of consumption later in life. Starting from this position decreases in \( \beta \) below 1 will result in more consumption earlier in the household’s life and, since marginal utilities are unequal already, will have a first order impact on household welfare. By this logic, the presence of misaligned preferences within the household magnifies the impact of the inconsistent time preferences that have already been studied extensively in the literature.

\( D \) Durable Goods

As is standard for most benchmark models of consumption and saving the analysis so far has considered only household consumption of non-durable goods. However the framework has novel predictions about the fraction of wealth that household members will direct to the consumption of durable goods. These are particularly interesting because, as I now show, they set the intra-household framework apart from a model where the only distortion to
consumption decisions comes from hyperbolic time preferences.

To show this I adapt the benchmark model introduced in Section II by now assuming that household members also derive utility from a private durable good that is purchased at the start of the first period. Label this expenditure $D_i$ for member $i$. I assume that the durable good does not depreciate over time, can only be purchased at the start of $t = 1$ (and in no other period), and that the purchase is irreversible so that $D_i$ cannot be sold in the future to fund non-durable consumption. These assumptions allow me to abstract from many interest features of durable goods consumption that are not directly related to the intra-household problem I study here. The stock of durable goods $D_i$ delivers consumption services to member $i$ each period of $\alpha D_i$ where $\alpha$ is a positive constant. To capture the utility from durable consumption, extend the period utility function in (1) to

$$ u_{i,t} = \ln C_{i,t} + \ln \alpha D_i. \quad (33) $$

To facilitate the comparison between the effect of intra-household preferences and hyperbolic discounting I allow individual members to discount the utility from future consumption with quasi-hyperbolic time preferences of the type described in (30). In the first period, prior to the choice of $C_{i,1}$ both members simultaneously select an expenditure on durable goods $D_i \geq 0$ as Nash best responses. As in the previous analysis I restrict these choices to be no more than half household wealth:\footnote{As before, this assumption could be removed if household members were instead allowed to choose the level of durable consumption consecutively. In the appendix I show that this condition does not bind as long as $\gamma_A, \gamma_B \leq 1$.}

$$ D_i \leq \frac{W_1}{2}. \quad (34) $$

After these are chosen the household then plays the non-cooperative consumption game studied in Section II where the starting value of wealth is now $\bar{W}_1 = W_1 - D_A - D_B$. Since (33) is additive, this ensures that the resulting equilibrium with non-durable consumption is identical
to that studied before taking $\widehat{W}_1$ as given. The equilibrium level of consumption is characterized in the Appendix. In equilibrium member $i$ will choose a level of durable expenditure of

$$D_i^* = \frac{\gamma_i}{2 + \Delta} W_1$$  \hspace{1cm} (35)

which implies that total household expenditure on durable goods (label this $D_{Tot}^* = D_A^* + D_B^*$) will be

$$D_{Tot}^* = \frac{1 + \Delta}{2 + \Delta} W_1.$$  \hspace{1cm} (36)

The total amount of durable consumption is strictly increasing in the level of preference misalignment $\Delta$.

Next I compare this to the full commitment Pareto efficient allocation. With full commitment, the total level of durable consumption is $D_{Tot}^{**}$

$$D_{Tot}^{**} = \frac{W_1}{2}.$$  \hspace{1cm} (37)

This leads directly to the following Proposition.

**Proposition 7:** If $\Delta > 0$ then the household without commitment will overconsume durable goods relative to the full commitment allocation.

The intuition for this result is identical to the logic behind Proposition 1. Neither member fully internalize the cost of their durable good expenditure on the future combined welfare of the household and hence durable goods are overconsumed. Note however that the distortion in durable consumption is driven only by the competition for resources within the household. Observe that the equilibrium level of durable consumption is unaffected by hyperbolic time preferences of both members $\beta$. And hence even with $\beta < 1$ this choice will be efficient as long as $\Delta = 0$. Hyperbolic time preferences do not distort the tradeoff between the consumption of durable goods and the benefit of saving since both are long lived and hence affected in the same proportion by whatever discount rate the household uses. Thus the model’s prediction
of overconsumption of durable goods distinguishes it from a model of household consumption with only time inconsistent preferences. I discuss the empirical implications of this in the next section.

VI Discussion and Empirical Implications

This paper introduces a model of household consumption and savings in which household members have imperfectly aligned altruistic preferences. The central empirical prediction of the model is that, left to their own devices, multiperson households will overconsume relative to their ex-ante desired consumption path. In anticipation of this problem these households will be prepared to adopt or pay for technologies which bring consumption closer to its optimal level. There are several challenges to identifying this empirically. First, much of the evidence that households demand savings commitment technologies such as saving in illiquid assets could be fully explained by household members having hyperbolic time preferences (see for example Beshears et al. 2011; Laibson 1997; Laibson Repetto and Tobacman 1998). In principal one way to distinguish these mechanisms is to identify a demand for savings commitment in multiperson households that is not present in single person households. However naive empirical comparisons of savings rates or commitment products between single and married households are likely to be confounded by selection into marriage.18 A sharper empirical distinction between these competing theories comes from looking at commitment technologies designed to prevent unilateral consumption deviations by individual household members. A chief example is saving in the form of assets that require the approval of both spouses to draw against. If the household members source of time inconsistency comes only from time preferences then this will not alter consumption and savings decisions at all. However joint approval for withdrawals will force the household members to bargain over the level

---

18 As an example, if selection in the marriage market implies that individuals with more hyperbolic time preferences remain unmarried then we may observe the same savings patterns and demand for commitment products for single and married households despite the fact that married households demand comes from misaligned preferences within the household.
of total consumption and any efficient bargaining solution will fully internalize the benefits of savings. Several of the most important household saving assets require joint approval to withdraw or borrow against. As a primary example, the 1984 Retirement Equity Act revised the rules governing all retirement plans covered by the 1974 Employee Retirement Income Security Act to require exactly this form of joint approval. This covers all assets held by married households in 401(k) plans, IRA accounts, and defined benefit plans and thus accounts for the bulk of US retirement savings outside of housing. The presence of these laws suggest that law makers were trying to remedy a savings problem due to suboptimal unilateral withdrawals. Aura (2005) shows that the passing of these laws did in fact increase savings for households affected by this law change. Similarly, joint ownership of a house prevents a household member borrowing against home equity savings without the approval of his spouse.\textsuperscript{19}

Time inconsistency within the household may also arise from each household member having different discount rates.\textsuperscript{20} Jackson and Yariv (2011) study this problem in a generalized setting and show that the time inconsistency problem persists even if household members are forced to bargain efficiently each period. Thus savings in assets with joint approval would not solve this problem and thus is unlikely to explain the prevalence of this commitment feature either.

The theory also suggests that household members will have a different propensity to consume out of wealth depending on whether it is shared with their spouse or not. If one member secretly receives income that the other is unaware of (and they are able to consume privately) then the permanent income hypothesis will apply and the effect on consumption will be smoothed over time. However wealth shocks that are known by all household members will exhibit a higher marginal propensity to consume as each member strategically tries

\textsuperscript{19}In community property states within the US joint approval is required even when the deed is held in the name of one spouse.

\textsuperscript{20}Schaner (2011) provides evidence that the difference of time preferences within the household can affect the decision to save in separate or joint accounts.
to claim a higher fraction of the shared resource. Evidence for this variation in marginal propensity to consume is provided by Goldberg (2011) who conducts an experiment in rural Malawi in which she studies the consumption decision of individuals who win a raffle. She randomly varies whether a raffle winnings are received publicly or privately and shows that individuals for whom the raffle is publicly observed have a marginal propensity to consume out of wealth in the first week that is 35% percent higher. The random assignment of information condition rules out any explanation due to time preferences. Similarly, Asraf (2009) conducts an experiment on households in the Philippines and shows that the decision to consume or save out an income shock depends on whether the shock is privately observed by one household member or common knowledge to both.

Section V demonstrates that the theory in this paper can also be distinguished from time inconsistency due to hyperbolic time preferences with respect to the over consumption of durable goods. Evidence for hyperbolic time preferences suggests that utility as close as one month away or less is heavily discounted (see for example Frederick Lowenstein O’Donoghue 2002; and Shapiro 2005). Thus these preferences explain deviations towards goods and services that provide instant gratification. The tendency to over consumption of durable goods such as sports cars, jewelry, sports equipment, and luxury clothes is therefore inconsistent with this hyperbolic individual discount rates. Such purchases are however consistent with a theory of excess consumption due to misaligned preferences within the household. By extension of the same argument we should expect to see commitment mechanisms, such as intrahousehold punishments directed to limiting these purchases.

The model also predicts that the way assets are held within the household can be used to address the tendency to over consume. For example, in the base model the household may be able to achieve an optimal consumption path if wealth is held separately.\(^\text{21}\) The

\(^{21}\)This solution may breakdown if household members cannot commit ex-ante not to make intra-household transfers. Once public consumption is introduced separate accounts can also be rendered ineffective if members can free ride on the provision of the public good from each other. Formal theoretical study of separate accounts within the household is left for future work.
division of accounts within the household cannot be understood as a solution to hyperbolic
time preferences of household members. Survey evidence suggests that separate accounts
are used by many US household Grose (2011). They are also common outside of the US
as well. For example a 2006 survey of Japanese wives found that fifty percent held secret
savings (referred to in Japan as "hesokuri") (see Alexy 2007). Anderson and Baland (2002)
show that participation in rotating savings and credit associations within Kenya is largely
accounted for by married women, suggesting it is a strategy to protect savings from her
husbands consumption demands.

Finally the model suggests that household members may use punishment strategies to
mitigate the temptation to over consume. If the model were extended to an infinite horizon
such strategies could be enforced. Evidence that households do not exhibit dynamic efficiency
(see for example de Mel Mckenzie and Woodruff 2009, Duflo and Udry 2004, Mazzocco
2007, Robinson 2011, Udry 1996) suggest that in practice these strategies are of limited
effectiveness. This may be because shocks to marginal utility are unobserved or because
households are unable to credible commit not to renegotiate planned punishment strategies.\footnote{A more thorough examination of the effectiveness of punishment strategies within the household is left for future work.}
The model suggests that increased risk of household dissolution may weaken the power of
such punishment strategies and thereby exacerbate the overconsumption problem.

\section*{References}


Anderson, Siwan and Jean-Marie Baland, “The Economics of Roscas and Intrahousehold

Amador, Manuel, “Sovereign Debt and the Tragedy of the Commons,” Mimeo, Stanford
University, 2008.


Smith, Tom W, Peter Marsden, Michael Hout, and Jibum Kim. General social surveys, 1972-2010, Principal Investigator, Tom W. Smith; Co-Principal Investigator, Peter V. Marsden;


A Non-Cooperative Equilibrium Household Consumption with Log Utility

This section of the Appendix solves for subgame perfect equilibrium non-cooperative household consumption decisions. I solve a generalized model in which both members have a period utility function that places weight $\mu$ on the utility from private consumption and $1-\mu$ on the utility from public consumption $H_t$ as introduced in Section V. I also allow individual members to have a hyperbolic discount factor $\beta$ (as introduced in Section V). The results for the rest of the paper will be special cases of the results I find here where $\mu = 1$ and/or $\beta = 1$.

A-1 Equilibrium at $t = NT$

In the final period $t = NT$ member $i$ takes $C_{j,NT}$ and $H_{j,NT}$ as given and solves the following problem:

$$\max_{C_{i,NT}, H_{i,NT}} \left[ \gamma_i \left[ \mu \ln C_{i,NT} + (1 - \mu) \ln (H_{i,NT} + H_{j,NT}) \right] + (1 - \gamma_i) \left[ \mu \ln C_{j,NT} + (1 - \mu) \ln (H_{i,NT} + H_{j,NT}) \right] \right]$$
subject to
$$\frac{W_{NT}}{2} - C_{i,NT} - H_{i,NT} \geq 0 \quad \text{and} \quad C_{i,NT}, H_{i,NT} \geq 0.$$  

Since (38) is strictly increasing in $C_{i,NT}$ and $H_{i,NT}$ it follows that (39) will bind with equality and hence can be substituted into the objective. Ignoring terms which $i$ takes as given we can rewrite her problem as

$$\max_{H_{i,NT}} \gamma_i \mu \ln \left( \frac{W_{NT}}{2} - H_{i,NT} \right) + (1 - \mu) \ln (H_{i,NT} + H_{j,NT})$$
subject to
$$\frac{W_{NT}}{2} - H_{i,NT} \geq 0 \quad \text{and} \quad H_{i,NT} \geq 0.$$  

Start by ignoring the boundary conditions (42) and (43) on $H_{i,NT}$. The first order condition for the unconstrained problem rearranges to give:

$$H_{i,NT} = \frac{(1 - \mu) \frac{W_{NT}}{2} - \gamma_i \mu H_{j,NT}}{1 - \mu (1 - \gamma_i)}.$$  

40
Since the objective is strictly concave in $H_{i,NT}$, using the boundary conditions (42) and (43) on $H_{i,NT}$ gives that $i$’s unique best response to any possible choice of $H_{j,NT} \geq 0$ is

$$H_{i,NT}^{BR}(H_{j,NT}) = \begin{cases} b_{i,NT} \frac{W_{NT}}{2} - m_{i,NT}H_{j,NT} & \text{if } H_{j,NT} \leq \frac{1-\mu}{\gamma_{i,\mu}} \frac{W_{NT}}{2} \\ 0 & \text{if } H_{j,NT} > \frac{1-\mu}{\gamma_{i,\mu}} \frac{W_{NT}}{2} \end{cases}$$

(45)

where $b_{i,NT} = \frac{1 - \mu}{1 - \mu (1 - \gamma_{i})} > 0$, and

$$m_{i,NT} = \frac{\gamma_{i,\mu}}{1 - \mu (1 - \gamma_{i})} \in (0, 1).$$

Note that $H_{i,NT}^{BR}(H_{j,NT})$ is weakly decreasing and hence the most that $i$ will spend on public consumption is

$$H_{i,NT}^{BR}(0) = \frac{1 - \mu}{1 - \mu (1 - \gamma_{i})} \frac{W_{NT}}{2}$$

which is strictly less than the upper bound $\frac{W_{NT}}{2}$ since $\gamma_{i} > 0$. Thus ((42)) can be ignored. Note that $H_{i,NT}^{BR}(0) > 0$ and hence $H_{A,T} = H_{B,T} = 0$ cannot be a Nash equilibrium. If $b_{i,NT} \geq b_{j,NT} m_{j,NT}$ then $H_{i,NT}^{*} = b_{i,NT} \frac{W_{NT}}{2}$ and $H_{j,NT}^{*} = 0$ is a Nash equilibrium. In this case equilibrium, private consumption will be

$$C_{i,NT}^{*} = (1 - b_{i,NT}) \frac{W_{NT}}{2} \text{ and } C_{j,NT}^{*} = \frac{W_{NT}}{2}.$$

Since $m_{i,T}, m_{j,T} < 1$ then this equilibrium is unique. A symmetric argument, applies when $b_{i,NT} \leq m_{i,NT} b_{j,NT}$. Finally, if $b_{i,NT} \in \left(m_{i,NT}b_{j,NT}, \frac{b_{j,NT} m_{j,NT}}{m_{i,NT}}\right)$ then there is an interior nash equilibrium. This is found by substituting the interior portion of $j$’s reaction function into the reaction function of $i$:

$$H_{i,NT}^{*} = \frac{b_{i,NT} - m_{i,NT} b_{j,NT} W_{NT}}{1 - m_{i,NT} m_{j,NT}} \frac{W_{NT}}{2}.$$

To total expenditure on public consumption in this interior solution is

$$H_{NT}^{*} = \left(\frac{b_{i,NT} (1 - m_{j,NT}) + b_{j,NT} (1 - m_{i,NT})}{1 - m_{i,NT} m_{j,NT}}\right) \frac{W_{NT}}{2}.$$

The equilibrium level of private consumption in this interior solution is

$$C_{i,T}^{*} = \left(1 - \frac{b_{i,NT} - m_{i,NT} b_{j,NT}}{1 - m_{i,NT} m_{j,NT}}\right) \frac{W_{T}}{2}.$$

Thus the equilibrium value of member $i$’s objective function is

$$V_{i,NT} = \ln W_{NT} + k_{i,NT}.$$
where $k_{i,NT}$ is a constant term that depends on parameters in the following way

$$
\begin{align*}
    k_{i,NT} & \equiv \begin{cases} 
    \gamma_i \mu \ln (1 - b_{i,NT}) + (1 - \mu) \ln (b_{i,NT}) - \ln 2, & \text{if } b_{i,NT} \leq m_{i,NT} b_{j,NT} \\
    \gamma_i \mu \ln \left( 1 - \frac{b_{i,NT} - m_{i,NT} b_{i,NT}}{1 - m_{i,NT} m_{j,NT}} \right) + (1 - \gamma_i) \mu \ln \left( 1 - \frac{b_{j,NT} - m_{i,NT} b_{i,NT}}{1 - m_{i,NT} m_{j,NT}} \right) - \ln 2, & \text{if } b_{i,NT} \in \left( m_{i,NT} b_{j,NT}, \frac{b_{i,NT}}{m_{j,NT}} \right) \\
    (1 - \gamma_i) \mu \ln (1 - b_{j,T}) + (1 - \mu) \ln (b_{j,T}) - 2, & \text{if } b_{i,NT} \geq \frac{b_{j,NT}}{m_{j,NT}} 
\end{cases}
\end{align*}
$$

A-2 Solve for Subgame Perfect Consumption Path by Induction

I conjecture the following form for the subgame perfect household allocation.

**Conjecture 1** The subgame perfect equilibrium household allocation from $t$ until $NT$ is proportional to $W_t$. That is, for any period $t \in \{1, \ldots, NT\}$ the subgame perfect equilibrium levels of private and public consumption can be written as $C_{i,t+x}^* = g_{i,t+x} W_t$ and $H_{i,t+x}^* = h_{i,t+x} W_t$ for $x \in \{0, 1, \ldots, NT - t\}$ where $g_{i,t+x}$ and $h_{i,t+x}$ are strictly positive constants independent of $W_t$.

I will establish this conjecture by induction below. Consider the problem that each household member faces in period $t < NT$. Member $i$ takes $C_{i,t}$ and $H_{i,t}$ as given and solves the following:

$$
\max_{C_{i,t}, H_{i,t}} \gamma_i \mu \ln C_{i,t} + (1 - \mu) \ln (H_{i,t} + H_{j,t}) + (1 - \gamma_i) \mu \ln C_{j,t}
+ \beta \sum_{x=1}^{NT-t} \delta_x \left[ \gamma_i \mu \ln C_{i,t+x}^* + (1 - \gamma_i) \mu \ln C_{j,t+x}^* + (1 - \mu) \ln (H_{t+x}^*) \right]
$$

subject to

$$
\begin{align*}
    W_{t+1} & = R_{t+1} W_t - C_{i,t} - C_{j,t} - H_{i,t} - H_{j,t}, \\
    W_{t+1} & \geq C_{i,t} + H_{i,t}, \\
    W_{t+1} & \geq H_{j,t} + 0, \\
    C_{i,t} & \geq 0, \text{ and} \\
    H_{i,t} & \geq 0.
\end{align*}
$$

Conjecture 1 implies that

$$
\beta \sum_{x=1}^{NT-t} \delta_x \left[ \gamma_i \mu \ln C_{i,t+x}^* + (1 - \gamma_i) \mu \ln C_{j,t+x}^* + (1 - \mu) \ln (H_{t+x}^*) \right] = Y_{t+1} \ln W_{t+1} + k_{i,t}
$$

where $Y_{t+1} \equiv \beta \sum_{x=1}^{NT-t} \delta_x$.
and $k_{i,t}$ is a constant. In equilibrium the budget constraint will bind. Log utility will ensure $C_{i,t}^* > 0$ in equilibrium and hence (49) can be ignored for now and verified later. Ignoring terms that $i$ takes as given in $t$ and substituting (47) into the objective, $i$'s problem can be rewritten as

$$\max_{C_{i,t}, H_{i,t}} \gamma_i \mu \ln C_{i,t} + (1 - \mu) \ln (H_{i,t} + H_{j,t})$$

$$+ Y_{t+1} \ln (W_t - C_{i,t} - C_{j,t} - H_{i,t} - H_{j,t})$$

subject to

$$\frac{W_t}{2} - C_{i,t} - H_{i,t} \geq 0 \text{ and}$$

$$H_{i,t} \geq 0.$$  \hspace{1cm} (52) (53)

Start by ignoring (52) and (53). The first order conditions for the unconstrained problem are

$$C_{i,t} : \frac{\gamma_i \mu}{C_{i,t}} - \frac{Y_{t+1}}{W_t - C_{i,t} - C_{j,t} - H_{i,t} - H_{j,t}} = 0$$

$$H_{i,t} : \frac{1 - \mu}{H_{i,t} + H_{j,t}} - \frac{Y_{t+1}}{W_t - C_{i,t} - C_{j,t} - H_{i,t} - H_{j,t}} = 0$$

The first order condition for $H_{i,t}$ implies that

$$H_t = H_{i,t} + H_{j,t} = \frac{1 - \mu}{1 - \mu + Y_{t+1}} [W_t - C_{i,t} - C_{j,t}].$$ \hspace{1cm} (56)

Hence for any given level of $W_t$, $C_{i,t}$, and $C_{j,t}$ both members agree on the optimal level of $H_t$. Since it is funded jointly they are indifferent as to who pays for it. Equation (54) implies

$$C_{i,t} = g_{i,t} [W_t - C_{j,t} - H_t].$$ \hspace{1cm} (57)

where $g_{i,t} \equiv \frac{\gamma_i \mu}{Y_{t+1} + \gamma_i \mu} \in (0, 1).$

Substituting $j$'s analog of (57) into ((57)) gives

$$C_{i,t} = \frac{g_{i,t} (1 - g_{j,t})}{1 - g_{i,t} g_{j,t}} [W_t - H_t].$$ \hspace{1cm} (58)

Combining (56) and (58) gives the equilibrium level of public consumption

$$H_t^* = \left( \frac{1 - \mu}{1 + \mu \Delta + \beta \sum_{x=1}^{N_{T-t}} \delta^x} \right) W_t.$$ \hspace{1cm} (59)

Combining (58) and (59) gives the equilibrium level of private consumption for each member

$$C_{i,t}^* = \left( \frac{\gamma_i \mu}{1 + \mu \Delta + \beta \sum_{x=1}^{N_{T-t}} \delta^x} \right) W_t.$$ \hspace{1cm} (60)
Equilibrium total expenditure is thus
\[ X_t^* = H_t^* + C_{A,t}^* + C_{B,t}^* = \left( \frac{1}{1 + \frac{\beta}{1 + \mu \Delta} \sum_{x=1}^{NT-t} \delta_x^i} \right) W_t. \] (61)

These solutions were derived for the unconstrained problem ignoring (52) and (49). The expression for \( C_{i,t}^* \) in (60) demonstrates that (49) is slack. It just remains to show that (52) is not violated for either household member. First note that \( X_t^* < W_t \) for any \( t < NT \) and hence the expenditure limit can at most be violated for one household member. Since both members agree on the level of public consumption and are indifferent who pays for it then (52) will be satisfied if and only if \( C_{i,t}^* \leq \frac{W_t}{2} \) for both members. This requires
\[ \gamma_i \leq \frac{1 + \mu \Delta + \beta \sum_{x=1}^{NT-t} \delta_x^i}{2 \mu}. \]

This constraint is more restrictive the higher is \( t \) and hence holds in every period if it is true for the household member with the largest \( \gamma_i \) in period \( t = NT - 1 \). This requires
\[ \max \{ \gamma_A, \gamma_B \} \leq \frac{1 + \mu \Delta + \beta \delta_{NT}^i}{2 \mu}. \]

Using the fact that \( \Delta = \gamma_A + \gamma_B - 1 \) this re-arranges to
\[ |\gamma_A - \gamma_B| \leq \frac{1 - \mu + \beta \delta_{NT}^i}{\mu}. \] (62)

I assume (62) holds and hence (52) is also slack. Thus, conditional on Conjecture 1 being true (59), (60) and (61) are the unique subgame perfect equilibrium consumption choices.

The final step of the derivation is to prove Conjecture 1 by induction. As the first step, note that Conjecture 1 is verified for \( t = NT \) above. Next observe that (59) and (60) give equilibrium consumption levels that are proportional to \( W_t \). Observe also that using (61) we can compute \( W_{t+1} \) as
\[ W_{t+1} = R^{\frac{1}{2}} \left( \frac{\beta}{1 + \mu \Delta} \sum_{x=1}^{NT-t} \delta_x^i \right) W_t \]
which is also proportional to \( W_t \). By extension of (59) and (60) this implies that \( H_{t+1}^*, C_{A,t+1}^*, C_{B,t+1}^* \) are also proportional to \( W_t \). The same argument applies for any period \( x > t \). Hence this establishes Conjecture 1 by induction.

**B Solution to Household Allocation with Full Commitment**

This section of the Appendix solves for the full commitment Pareto optimal household allocation. I solve a generalized model in which both members have a period utility function that places weight \( \mu \) on the utility from private consumption and \( 1 - \mu \) on the utility from public consumption \( H_t \) as introduced in Section B. Also, I allow individual members to have
a hyperbolic discount factor \( \beta \) as introduced in Section C. The results for the rest of the paper will be special cases of the results I find here where \( \mu = 1 \) and/or \( \beta = 1 \).

The problem is to solve

\[
\max_{\{C_{A,t},C_{B,t},H_t\}_{t=1}^{NT}} \Pi = \eta V_{A,1} + (1 - \eta) V_{B,1} \tag{63}
\]

subject to

\[
W_1 - \sum_{x=0}^{NT-1} R^{-\frac{t}{T}} [C_{A,1+x} + C_{B,1+x} + H_{1+x}] \geq 0 \text{ and} \tag{64}
\]

\[
\{C_{A,t},C_{B,t},H_{A,t},H_{B,t}\}_{t=1}^{NT} \geq 0. \tag{65}
\]

The objective of this problem can be re-written as

\[
\Pi = (1 - \theta) U_{A,1} + \theta U_{B,1} \tag{66}
\]

where \( \theta \equiv \gamma_B + \eta (1 - \gamma_A - \gamma_B) \tag{67} \)

using the expressions for \( U_{A,1} \) and \( U_{B,1} \) (66) becomes

\[
\Pi = (1 - \theta) \mu \left[ \ln C_{A,1} + \beta \sum_{x=1}^{NT-1} \delta^\frac{x}{T} \ln C_{A,1+x} \right] \tag{68}
\]

\[+ \theta \mu \left[ \ln C_{B,1} + \beta \sum_{x=1}^{NT-1} \delta^\frac{x}{T} \ln C_{B,1+x} \right]
\]

\[+ (1 - \mu) \left[ \ln H_1 + \beta \sum_{x=1}^{NT-1} \delta^\frac{x}{T} \ln H_{1+x} \right].
\]

I will start by ignoring the non-negativity constraints in (65) and verify that these hold later.

Writing the Lagrangian for the remaining problem with \( \Gamma \geq 0 \) being the multiplier on the resource constraint we have

\[
\max_{\{C_{A,t},C_{B,t},H_t\}_{t=1}^{NT}} (1 - \theta) \mu \left[ \ln C_{A,1} + \beta \sum_{x=1}^{NT-1} \delta^\frac{x}{T} \ln C_{A,1+x} \right] \tag{69}
\]

\[+ \theta \mu \left[ \ln C_{B,1} + \beta \sum_{x=1}^{NT-1} \delta^\frac{x}{T} \ln C_{B,1+x} \right]
\]

\[+ (1 - \mu) \left[ \ln H_1 + \beta \sum_{x=1}^{NT-1} \delta^\frac{x}{T} \ln H_{1+x} \right]
\]

\[+ \Gamma \left[ W_1 - \sum_{x=0}^{NT-1} R^{-\frac{t}{T}} [C_{A,1+x} + C_{B,1+x} + H_{1+x}] \right].
\]

The first order conditions give the optimal level of expenditure on each type of consumption
in every period as a function of $\Gamma$:

\[
C_{A,1} : C_{A,1}^{**} = \frac{(1 - \theta) \mu}{\Gamma} \tag{70}
\]

\[
C_{A,1+x} : C_{A,1+x}^{**} = \frac{(1 - \theta) \mu \beta \delta^{\frac{x}{N}}}{\Gamma R^{-\frac{x}{N}}} \tag{71}
\]

\[
C_{B,1} : C_{B,1}^{**} = \frac{\theta \mu}{\Gamma} \tag{72}
\]

\[
C_{B,1+x} : C_{B,1+x}^{**} = \frac{\theta \mu \beta \delta^{\frac{x}{N}}}{\Gamma R^{-\frac{x}{N}}} \tag{73}
\]

\[
H_1 : H_1^{**} = \frac{1 - \mu}{\Gamma} \tag{74}
\]

\[
H_{1+x} : H_{1+x}^{**} = \frac{(1 - \mu) \beta \delta^{\frac{x}{N}}}{\Gamma R^{-\frac{x}{N}}} \tag{75}
\]

where $x \in \{1, 2, ..., NT - 1\}$ and “**” indicates solution to the full commitment problem. In the first period, the optimal level of total expenditure is

\[
X_1^{**} = C_{A,1}^{**} + C_{B,1}^{**} + H_1^{**} = \frac{1}{\Gamma}. \tag{76}
\]

For any period after the first, the optimal level of total expenditure is

\[
X_{1+x}^{**} = \frac{\beta \delta^{\frac{x}{N}}}{\Gamma R^{-\frac{x}{N}}} \tag{77}
\]

Since the optimal allocation will exhaust the household budget constraint it must be that

\[
W_1 = X_1^{**} + \sum_{x=1}^{NT-1} \frac{X_{1+x}^{**}}{R^{\frac{x}{N}}} = \frac{1}{\Gamma} \left[ 1 + \beta \sum_{x=1}^{NT-1} \delta^{\frac{x}{N}} \right]
\]

which implies that

\[
\Gamma^{**} = \frac{1 + \beta \sum_{x=1}^{NT-1} \delta^{\frac{x}{N}}}{W_1}. \tag{78}
\]

Combining (78) with (76) and (77) gives

\[
X_1^{**} = \frac{1}{1 + \beta \sum_{x=1}^{NT-1} \delta^{\frac{x}{N}}} W_1 \tag{79}
\]

and for $t > 1$

\[
X_t^{**} = \frac{\beta \delta^{\frac{t-1}{N}}}{1 + \beta \sum_{x=1}^{NT-1} \delta^{\frac{x}{N}}} W_1. \tag{78}
\]
Note that under the full commitment allocation household wealth evolves as

\[ W_t = R^{t+1}_N W_1 - R^{t+1}_N X_1^{**} - \sum_{x=2}^{t-1} R^{t+1}_N X_x^{**} \]

\[ = R^{t+1}_N W_1 \beta \left[ \frac{\sum_{k=t-1}^{NT-1} \delta^k_N}{1 + \beta \sum_{k=t-1}^{NT-1} \delta^k_N} \right] \]

and so

\[ R^{t-1}_N W_1 = \frac{W_t}{\beta} \left( \frac{1 + \beta \sum_{k=1}^{NT-1} \delta^k_N}{\sum_{k=t-1}^{NT-1} \delta^k_N} \right). \]

Hence for \( t > 1 \), \( X_t^{**} \) can be re-written as

\[ X_t^{**} = \frac{\beta \delta^{t-1}_N}{1 + \beta \sum_{x=1}^{NT-1} \delta^x_N} \frac{W_t}{\beta} \left( 1 + \beta \sum_{k=1}^{NT-1} \delta^k_N \right) \cdot \sum_{k=t-1}^{NT-1} \delta^k_N \]

This simplifies to

\[ X_t^{**} = \frac{1}{1 + \sum_{k=t}^{NT-1} \delta^k_N} W_t. \quad (80) \]

This fully describes the total level of consumption each period under full commitment. The optimal levels of \( C_{A,t}^{**}, C_{B,t}^{**}, \) and \( H_t^{**} \) follow immediately by using (70) through (75) to get the following constant consumption shares within each period.

\[ \frac{C_{A,t}^{**}}{X_t^{**}} = (1 - \theta) \mu \]
\[ \frac{C_{B,t}^{**}}{X_t^{**}} = \theta \mu \]
\[ \frac{H_t^{**}}{X_t^{**}} = 1 - \mu. \]

Note that the optimal solution satisfies (65).

\[ C \quad \text{Representative Agent} \]

This section of the Appendix solves the problem of the representative agent without commitment. Since the representative agent is allowed to have hyperbolic time preferences I study for the subgame perfect equilibrium path \( X_t^{**} \) where the agent rationally anticipates the consumption choices she will make later in life (i.e. does not naively and incorrectly expect to follow the optimal consumption plan for the rest of her life). The goal is to find values for \( \delta_r \) and \( \beta_r \) that ensure \( X_t^{**} = X_t^* \) in every period.
C-1 Equilibrium Consumption

In the final period \( t = NT \) the representative agent will optimal consume all remaining wealth

\[
X^*_t = W_{NT}.
\]

In order to solve for equilibrium consumption choices for all \( t < NT \) I make the following conjecture.

**Conjecture 2**: The subgame perfect equilibrium household allocation of the representative agent from \( t \) until \( NT \) is proportional to \( W_t \). That is, for any period \( t \in \{1, ..., NT\} \) the subgame perfect equilibrium levels of \( X_t \) can be written as \( X^*_{t+x} = k_{t+x}W_t \) for \( x \in \{0, 1, ..., NT - t\} \) where \( k_{t+x} \) are strictly positive constants independent of \( W_t \).

I establish Conjecture 2 by induction. Consider the problem that the representative agent faces in period \( t < NT \)

\[
\max_{X_t} \ln X_t + \beta_r \sum_{x=1}^{NT-t} \delta_t^\frac{x}{N} \ln X^*_{t+x} \quad (81)
\]

subject to

\[
W_{t+1} = R^\frac{x}{N} (W_t - X_t), \quad (82)
\]

\[
X_t \leq W_t, \text{ and} \quad (83)
\]

\[
X_t \geq 0. \quad (84)
\]

I will solve this problem ignoring (83) and (84) and verify that these are satisfied at the end. Using Conjecture 2, substituting (82) into the objective function, and ignoring constant terms transforms the problem to

\[
\max_{X_t} \ln X_t + \beta_r \sum_{x=1}^{NT-t} \delta_t^\frac{x}{N} \ln (W_t - X_t). \quad (85)
\]

The first order condition for this problem is

\[
\frac{1}{X^*_t} - \frac{\beta_r \sum_{x=1}^{NT-t} \delta_t^\frac{x}{N}}{W_t - X^*_t} = 0.
\]

Which can be rearranged to give the equilibrium consumption choice of the representative agent in any period as

\[
X^*_t = \frac{1}{1 + \beta_r \sum_{x=1}^{NT-t} \delta_t^\frac{x}{N}} W_t. \quad (86)
\]

I can now prove Conjecture 2 by induction. First, observe that it is verified for \( t = NT \) above. Next, observe that (86) shows that \( X^*_t \) is proportional to \( W_t \). Moreover, since wealth
will evolve under these equilibrium choices as
\[ W_{t+1} = \frac{R^{\frac{1}{N}} \beta_r \sum_{x=1}^{NT-t} \delta_x^N}{1 + \beta_r \sum_{x=1}^{NT-t} \delta_x^N} W_t \]

then \( W_{t+1} \) is also proportional to \( W_t \). By extension of (86) this implies \( X_{t+1}^* \) is proportional to \( W_t \) and so on for all \( \{ X_{t+t}^* \}_{x=1}^{NT-t} \). This establishes Conjecture 2.

**C-2 Equivalence with Household Equilibrium**

Comparing (61) and (86) we see that \( X_t^* = X_t^* \) if
\[ \delta_r = \delta \text{ and } \beta_r = \frac{\beta}{1 + \mu \Delta}. \] (87)

To establish that (87) is a necessary condition consider what is required to achieve equivalence in \( t = NT - 1 \) and \( NT - 2 \). This requires
\[ t = NT - 1 : \beta_r \delta_r^N = \frac{\beta}{1 + \mu \Delta} \delta_r^N \text{ and } \]
\[ t = NT - 2 : \beta_r \left[ \delta_r^N + \delta_r^2 \right] = \frac{\beta}{1 + \mu \Delta} \left[ \delta_r^N + \delta_r^2 \right]. \] (89)

To satisfy (88) it must be that
\[ \beta_r = \frac{\beta}{1 + \mu \Delta} \left( \frac{\delta}{\delta_r} \right)^N. \] (90)

Substituting (90) into (89) gives
\[ \frac{\beta}{1 + \mu \Delta} \left( \frac{\delta}{\delta_r} \right)^N \left[ \delta_r^N + \delta_r^2 \right] = \frac{\beta}{1 + \mu \Delta} \left[ \delta_r^N + \delta_r^2 \right] \]

which upon simplification uniquely requires \( \delta_r = \delta \) and therefore implies that \( \beta_r = \frac{\beta}{1 + \mu \Delta} \) must also hold. Thus (87) is a necessary condition for equivalence if \( NT > 2 \) and finite. Note that if \( NT = 2 \) then any combination of \( \beta_r \) and \( \delta_r \) that satisfies (90) is sufficient and (87) is therefore not a necessary condition. This establishes Proposition 2.

**D HARA Utility**

In this section of the Appendix I analytically characterize the non-cooperative equilibrium levels of household consumption when members have HARA period utility functions. I characterize the solution for the representative agent with HARA utility and establish that under the parameter restrictions which ensure equivalence between the two consumption paths. The analysis focuses on the case of symmetric altruism \( (\gamma_A = \gamma_B = \gamma) \). Since these steps mirror
many of the proofs in the first three sections of the Appendix I keep derivations brief.

D-1 Definitions

Let \( \{ \tilde{C}_{A,x}^*(W_t), \tilde{C}_{B,x}^*(W_t) \}_{x=t}^{NT} \) be the series of subgame perfect equilibrium non-cooperative consumption choices made by each household member from \( t \) onwards as a function of the wealth \( W_t \) which they bring into period \( t \). Symmetry will ensure that \( \tilde{C}_{A,x}^*(W_t) = \tilde{C}_{B,x}^*(W_t) = \frac{1}{2} \tilde{X}_x^*(W_t) \) in each \( x \in \{t, t+1, ..., NT\} \). Define the value function for both household members in period \( t \) as

\[
V_t(W_t) = \sum_{x=t}^{NT} \delta^x \left( \gamma u \left( \tilde{C}_{i,x}^*(W_t) \right) + (1 - \gamma) u \left( \tilde{C}_{j,x}^*(W_t) \right) \right).
\]

Using the symmetry of the household and the HARA utility function this can be written as:

\[
V_t(W_t) = \frac{\rho}{1 - \rho} \left( \frac{1}{2} \right)^{1-\rho} \sum_{x=t}^{NT} \delta^x \left( \frac{\tilde{X}_x^*(W_t)}{\rho} - 2\varphi \right)^{1-\rho}.
\]

Similarly, for the representative agent let \( \{ \tilde{X}_x^{*r} (W_t) \}_{x=t}^{NT} \) be the series of subgame perfect equilibrium consumption choices she will make from \( t \) onwards as a function of \( W_t \). Define the following function:

\[
V_{t}^{r} (W_t) = \sum_{x=t}^{NT} \delta^x u_r \left( \tilde{X}_x^{*r} (W_t) \right)
\]

which using the HARA utility function can be written as

\[
V_{t}^{r} (W_t) = \frac{\rho}{1 - \rho} \sum_{x=t}^{NT} \delta^x \left( \frac{\tilde{X}_x^{*r} (W_t)}{\rho} - 2\varphi \right)^{1-\rho}.
\]

Observe that if \( \tilde{X}_x^{*r} (W_t) = \tilde{X}_x^* (W_t) \) for all \( x \in \{t, t+1, ..., NT\} \) then

\[
V_{t}^{r} (W_t) = \left( \frac{1}{2} \right)^{1-\rho} V_t(W_t).
\]

(91)

Since \( V_t(W_t) \) and \( V_{t}^{r} (W_t) \) are the sums of concave differentiable functions then it is easy to show that they are also concave and differentiable.

D-2 Final Period

By assumption

\[
\tilde{C}_{A,NT}^* (W_{NT}) = \tilde{C}_{B,NT}^* (W_{NT}) = \frac{W_{NT}}{2}.
\]
As a result

\[ V_{NT}(W_{NT}) = \frac{\rho}{1 - \rho} \left( \frac{1}{2} \right)^{1-\rho} \left( \frac{W_{NT}}{\rho} - 2\varphi \right)^{1-\rho}. \] (92)

Similarly, for the representative agent \( \tilde{X}^*_{NT}(W_{NT}) = W_{NT} \) and hence

\[ V^*_{NT}(W_{NT}) = \frac{\rho}{1 - \rho} \left( \frac{W_{NT}}{\rho} - 2\varphi \right)^{1-\rho}. \]

Notice that by construction \( \tilde{X}^*_{NT}(W_{NT}) = \tilde{C}^*_{A,NT}(W_{NT}) + \tilde{C}^*_{B,NT}(W_{NT}) = \tilde{X}^*_{NT}(W_{NT}) \) and hence

\[ V_{NT}(W_{NT}) = \left( \frac{1}{2} \right)^{1-\rho} V^*_{NT}(W_{NT}). \]

D-3 Equivalence for \( t < NT \)

Suppose that \( \tilde{X}^*_{x}(W_t) = \tilde{X}^*_{x}(W_t) \) for all \( x \in \{t + 1, ..., NT\} \). I will prove this is true by induction but observe that is has already been established for \( t = NT - 1 \). Now consider the problem faced by agent \( i \) in period \( t \). She will take \( C_{j,t} \) as given and choose \( C_{i,t} \) to solve

\[ \max_{C_{i,t}} \gamma \frac{\rho}{1 - \rho} \left( \frac{C_{i,t}}{\rho} - \varphi \right)^{1-\rho} + (1 - \gamma) \frac{\rho}{1 - \rho} \left( \frac{C_{j,t}}{\rho} - \varphi \right)^{1-\rho} + \delta^{\frac{1}{\rho}} V_{t+1}(W_{t+1}) \] (93)

subject to

\[ W_{t+1} = R^{\frac{1}{\rho}} (W_t - C_{i,t} - C_{j,t}) \] (94)

\[ C_{i,t} \leq \frac{W_t}{2}, \text{ and} \] (95)

\[ C_{i,t} \geq 0. \] (96)

The intertemporal budget constraint (94) can be substituted into (93). Let \( \lambda_{t,1} \) and \( \lambda_{t,2} \) be the Lagrangian multipliers associated with (95) and (96) respectively. Setting aside terms which \( i \) takes as given the Lagrangian for \( i \)'s problem can be written as

\[
\max_{C_{i,t}} \mathcal{L} = \gamma \frac{\rho}{1 - \rho} \left( \frac{C_{i,t}}{\rho} - \varphi \right)^{1-\rho} + \delta^{\frac{1}{\rho}} V_{t+1} \left( R^{\frac{1}{\rho}} (W_t - C_{i,t} - C_{j,t}) \right) \\
+ \lambda_{t,1} \left[ \frac{W_t}{2} - C_{i,t} \right] + \lambda_{t,2} C_{i,t}
\]

Taking first order conditions and invoking symmetry so that \( C^*_{A,t} = C^*_{B,t} = \frac{1}{2} X^*_t \). The equilibrium level of total household consumption in period \( t \) is fully characterized by the
following set of first order and Kuhn-Tucker conditions:

\[
\gamma \left( \frac{1}{2} \right)^{\rho} \left( \frac{X^*_t}{\rho} - 2\varphi \right)^{\rho} - \delta \frac{1}{\tau} R^\tau V_{t+1}' \left( R^\tau (W_t - X^*_t) \right) - \lambda_{t,1} + \lambda_{t,2} = 0 \quad (97)
\]

\[
\lambda_{t,1} [W_t - X^*_t] = 0 \quad (98)
\]

\[
\lambda_{t,2} X^*_t = 0 \quad (99)
\]

\[
\lambda_{t,1} \geq 0, \lambda_{t,2} \geq 0, C_{i,t} \geq 0, W_t \geq X^*_t. \quad (100)
\]

The conditions (97), (106), (99) and (100) combined with (92) iteratively define the entire set of equilibrium consumption choices.

Now consider the problem of the representative agent in period \( t \). She will choose \( X^r_t \) to solve

\[
\max_{C_{i,t}} \frac{\rho}{1 - \rho} \left( \frac{X^r_t}{\rho} - 2\varphi \right)^{1-\rho} + \delta \frac{1}{\tau} \beta_r V_{t+1}' (W_t) \quad (101)
\]

subject to

\[
W_{t+1} = R^\tau (W_t - X^r_t) \quad (102)
\]

\[
X^r_t \leq W_t, \quad \text{and} \quad (103)
\]

\[
X^r_t \geq 0. \quad (104)
\]

The intertemporal budget constraint (102) can be substituted into (101). Let \( \lambda_{t,1}^r \) and \( \lambda_{t,2}^r \) be the Lagrangian multipliers associated with (103) and (104) respectively. Also using (91) the Lagrangian for the representative agent’s problem can be written as

\[
\max_{X^r_t} \mathcal{L} = \frac{\rho}{1 - \rho} \left( \frac{X^r_t}{\rho} - 2\varphi \right)^{1-\rho} + \delta \frac{1}{\tau} \beta_r \left( \frac{1}{2} \right)^{1-\rho} V_{t+1}' (R^\tau (W_t - X^r_t))
\]

\[
+ \lambda_{t,1}^r [W_t - X^r_t] + \lambda_{t,2}^r X^r_t
\]

The consumption choice of the representative agent is characterized by the following first order and Kuhn-Tucker conditions:

\[
\left( \frac{X^r_t}{\rho} - 2\varphi \right)^{\rho} - \delta \frac{1}{\tau} R^\tau \beta_r \left( \frac{1}{2} \right)^{\rho-1} V_{t+1}' (R^\tau (W_t - X^r_t)) - \lambda_{t,1}^r + \lambda_{t,2}^r = 0
\]

\[
\lambda_{t,1}^r [W_t - X^r_t] = 0
\]

\[
\lambda_{t,2}^r X^r_t = 0
\]

\[
\lambda_{t,1}^r \geq 0, \lambda_{t,2}^r \geq 0, C_{i,t} \geq 0, W_t \geq X^*_t.
\]
Now observe that if $\beta_r = \frac{1}{2}$ then this problem can be re-written as

$$\gamma \left( \frac{1}{2} \right)^{-\rho} \left( \frac{X_t^{*r}}{\rho} - 2 \varphi \right)^{-\rho} - \delta \frac{1}{\gamma} R^{\frac{1}{\gamma}} V_{t+1}^{r} \left( R^{\frac{1}{\gamma}} (W_t - X_t^{*r}) \right) - \lambda_{t,1}^r + \lambda_{t,2}^r = 0$$  \hspace{1cm} (105)

\[ \lambda_{t,1}^{r} [W_t - X_t^{*r}] = 0 \hspace{1cm} (106) \]

\[ \lambda_{t,2}^{r} X_t^{*r} = 0 \hspace{1cm} (107) \]

\[ \lambda_{t,1}^{r} \geq 0, \lambda_{t,2}^{r} \geq 0, C_{t,t} \geq 0, W_t \geq X_t^{*r}. \hspace{1cm} (108) \]

where $\lambda_{t,1}^{r} = \gamma \left( \frac{1}{2} \right)^{-\rho} \lambda_{t,1}^{r}$ and $\lambda_{t,2}^{r} = \gamma \left( \frac{1}{2} \right)^{-\rho} \lambda_{t,2}^{r}$. The conditions (105), (106), (107) and (108) combined with (92) iteratively define the entire set of equilibrium consumption choices of the representative agent.

Now observe that (97), (106), (99) and (100) are identical to (105), (106), (107) and (108). It follows automatically then that $X_t^{*r} = X_t^{*r}$. Notice that since this has been verified for $t = NT$ that $X_t^{*r} = X_t^{*r}$ then this implies that $X_t^{*r} = X_t^{*r}$ and hence by an iterated argument of induction that $X_t^{*r} = X_t^{*r}$ for all $t \in \{1, 2, ..., NT\}$. Hence this establishes Proposition 3, that $\beta_r = \frac{1}{2} = \frac{1}{1+\Delta}$ is sufficient to ensure the representative agent has the same consumption has the non-cooperative household. The proof of uniqueness result for $NT > 2$ is almost identical to the log case studied above and thus is omitted for brevity.

D-4 CRRA Utility ($\rho > 0, \varphi = 0$)

In the paper I discuss properties of analytical solutions associated with the special case of CRRA utility. I briefly sketch the derivation of that analytical solution here. I conjecture that in the case of CRRA utility the value function at $t + 1$ has the following form:

$$V_{t+1} (W_{t+1}) = \gamma_{t+1} \frac{\rho}{1 - \rho} \left( \frac{W_{t+1}}{\rho} \right)^{1-\rho}$$ \hspace{1cm} (109)

where $\gamma_{t+1} > 0$ is a constant independent of $W_t$. Applying (92) we see that this conjecture is true for $t = NT - 1$ with $\gamma_{NT} = (\frac{1}{2})^{1-\rho}$. Using (109) in (97) gives that the equilibrium consumption choice in period $t$ is characterized by

$$\gamma \left( \frac{1}{2} \right)^{-\rho} \left( \frac{X_t^{*}}{\rho} \right)^{-\rho} = \delta \frac{1}{\gamma} R^{\frac{1}{\gamma}} \gamma_{t+1} \left( R^{\frac{1}{\gamma}} (W_t - X_t^{*}) \right)^{-\rho}$$ \hspace{1cm} (110)

where I have disregarded (95) and (96). It is easy to verify that these are always satisfied in the CRRA case. Rearranging (110) gives

$$X_t^{*} = \tau_t W_t$$ \hspace{1cm} (111)

where

$$\tau_t = \frac{1}{1 + \frac{1}{2} R^{\frac{1}{\gamma}} \left( \frac{\delta \frac{1}{\gamma} R^{\frac{1}{\gamma}} \gamma_{t+1}}{\gamma} \right)^{1-\rho}} \hspace{1cm} (112)$$
Thus the value function in period $t$ can thus be written as

$$V_t(W_t) = \frac{\rho}{1-\rho} \Upsilon_t \left( \frac{W_t}{\rho} \right)^{1-\rho}$$

where

$$\Upsilon_t = \left[ \left( \frac{\tau_t}{2} \right)^{1-\rho} + \delta^{\frac{1}{\pi}} \Upsilon_{t+1} \left( R^{\frac{1}{\pi}} (1-\tau_t) \right)^{1-\rho} \right].$$

(113)

By an argument of induction this verifies the conjectured value function for the CRRA case in (109). More using $\Upsilon_{NT} = \left( \frac{1}{2} \right)^{1-\rho}$ and (113) iteratively defines the sequence of $\{\Upsilon_x\}_{x=1}^{NT}$ which when combined with (112) and (111) fully characterizes the household’s equilibrium consumption choices absent commitment.

**D-5 Full Commitment Consumption Path with CRRA Utility ($\rho > 0, \varphi = 0$)**

To establish the value of commitment I now characterize the full commitment consumption path in the case where both members have CRRA period utility functions. I do this by supposing that $W_1$ is divided between member $A$ and $B$ so that $A$ receives $(1-\theta)W_1$ and $B$ receives $\theta W_1$ where $\theta$ is defined in (67). For both members the optimal path of consumption will be characterized by the standard envelope condition:

$$\frac{\partial}{\partial \Delta} \left\{ \left( \frac{C_{i,t}^{**} + \Delta}{1-\rho} \right)^{1-\rho} + \delta^{\frac{1}{\pi}} \left( \frac{C_{i,t+1}^{**} - R^{\frac{1}{\pi}} \Delta}{1-\rho} \right)^{1-\rho} \right\}_{\Delta=0} = 0$$

(114)

which simplifies to give the standard Euler equation relating the optimal choice of consumption in one period to the next:

$$C_{i,t+1}^{**} = (R\delta)^{\frac{1}{\rho_N}} C_{i,t}^{**}.$$  

(115)

Equation (115) implies that

$$C_{i,t}^{**} = (R\delta)^{\frac{1-\varphi}{\rho_N}} C_{i,1}^{**}.$$  

(116)

Since the optimal allocation must exhaust the wealth allocated to $A$ it must be that

$$C_{A,1}^{**} + \frac{C_{A,2}^{**}}{R^{\frac{1}{\rho_N}}} + \frac{C_{A,3}^{**}}{R^{2\frac{1}{\rho_N}}} + ... + \frac{C_{A,NT}^{**}}{R^{(NT-1)\frac{1}{\rho_N}}} = (1-\theta)W_1$$

which in combination with (116) gives that

$$C_{A,1}^{**} = \frac{(1-\theta)W_1}{\sum_{x=0}^{NT-1} \vartheta^x} \text{ where } \vartheta \equiv (R^{1-\rho}\delta)^{\frac{1}{\rho_N}}.$$  

(117)

Combining (117) and (116) gives the optimal level of consumption for member $A$ in every period

$$C_{A,t}^{**} = (R\delta)^{\frac{1-\varphi}{\rho_N}} \frac{1 - (R^{1-\rho}\delta)^{\frac{1}{\rho_N}}}{1 - (R^{1-\rho}\delta)^{\frac{1}{\rho_N}}} (1-\theta)W_1.$$  

(118)
By symmetry, the full commitment solution for $B$ is

$$C_{B,t}^{**} = (R\delta)^{t-1} \left[ \frac{1 - (R^{1-\rho} \delta) \frac{1}{\rho N}}{1 - (R^{1-\rho} \delta)^{\frac{1}{p}}} \right] \theta W_1. \quad (119)$$

Adding (118) and (119) gives the optimal level of total household consumption in each period:

$$X_t^{**} = (R\delta)^{t-1} \left[ \frac{1 - (R^{1-\rho} \delta) \frac{1}{\rho N}}{1 - (R^{1-\rho} \delta)^{\frac{1}{p}}} \right] W_1. \quad (120)$$

This fully characterizes the optimal allocation for the household when both members have CRRA period utility functions.

### E Durable Goods

A direct implication of Conjecture 1 is that the value function relating to non-durable consumption (after $D_A$ and $D_B$) have been set can be written as

$$V_i = \left( 1 + \beta \left( \sum_{x=1}^{NT-t-1} \delta^x \right) \right) \ln (W_1 - D_A - D_B) + k_1$$

where $k_1$ is constant and $\beta$ is a quasi-hyperbolic discount factor. At the start of $t = 1$ member $i$ will take $D_j$ as given and solve

$$\max_{D_i} \gamma_i \left( 1 + \beta \left( \sum_{x=1}^{NT-t-1} \delta^x \right) \right) \ln (\alpha D_i) + \left( 1 + \beta \left( \sum_{x=1}^{NT-t-1} \delta^x \right) \right) \ln (W_1 - D_i - D_j) \quad (121)$$

subject to

$$D_i \geq 0 \quad (122)$$

$$D_i \leq \frac{W_1}{2} \quad (123)$$

where I have omitted constants and terms which $i$ takes as given from the problem. I solve this by first ignoring (122) and (123) and then verifying that they are satisfied by the solution. The first order condition from (121) is

$$\gamma_i \left( 1 + \beta \left( \sum_{x=1}^{NT-t-1} \delta^x \right) \right) \frac{1}{D_i} - \left( 1 + \beta \left( \sum_{x=1}^{NT-t-1} \delta^x \right) \right) \frac{1}{W_1 - D_i - D_j} = 0. \quad (124)$$

An analogous condition to (124) holds for member $j$. Solving these simultaneously gives (35) and (36). Note that (122) and (123) are both satisfied as long as $\gamma_i \in [0, 1]$.

Next consider the full commitment problem. By an analogous argument to the one used
to form (121), the planners problem can be written as

$$\max_{D_A, D_B} \quad (1 - \theta) \left( 1 + \beta \sum_{x=1}^{NT-t-1} \delta^x \right) \ln(\alpha D_A) + \theta \left( 1 + \beta \sum_{x=1}^{NT-t-1} \delta^x \right) \ln(\alpha D_B)$$

$$+ \left( 1 + \beta \sum_{x=1}^{NT-t-1} \delta^x \right) \ln(W_1 - D_A - D_B)$$

subject to

$$D_A, D_B \geq 0$$

$$D_A, D_B \leq \frac{W_1}{2}$$

where $\theta$ is defined in (67). Ignoring the constraints (126) and (127), the first order conditions characterizing the full commitment choice of $D_A$ and $D_B$ are

$$\left( 1 - \theta \right) \left( 1 + \beta \sum_{x=1}^{NT-t-1} \delta^x \right) \frac{1}{D_A} - \left( 1 + \beta \sum_{x=1}^{NT-t-1} \delta^x \right) \frac{1}{W_1 - D_A - D_B} = 0.$$  \hfill (128)

$$\theta \left( 1 + \beta \sum_{x=1}^{NT-t-1} \delta^x \right) \frac{1}{D_B} - \left( 1 + \beta \sum_{x=1}^{NT-t-1} \delta^x \right) \frac{1}{W_1 - D_A - D_B} = 0.$$  \hfill (129)

Solving (128) and (129) simultaneously gives

$$D_A^{**} = \frac{1 - \theta}{2} W_1 \text{ and } D_B^{**} = \frac{\theta}{2} W_1.$$  

Note that since $\theta \in [0, 1]$ then (126) and (127) are automatically satisfied. Summing these together gives the optimal level of total durable expenditure in (37).
VIII  Internet Appendix - Consecutive Consumption Choices

In the base model presented in Section II I assumed that household members simultaneously made consumption decisions each period. This assumption creates the theoretical possibility that household members could in any period attempt to spend more than the total amount of all household wealth. To avoid specifying arbitrary tie breaking rules to deal with such a scenario I assumed in (7) that each member is able to spend no more than half of the household’s wealth in any period. The purpose of this section is to show that this arbitrary assumption is not important for the results of that model. I do this by assuming that household members make consumption decisions consecutively within any period. This allows (7) to be replaced by a standard budget constraint whereby each member can spend up to the full amount of remaining household wealth each time they consume. I show that when \( N \) is large the simultaneous move equilibrium studied above is the limiting case of the unique equilibria reached in the consecutive move setup.

A  Consecutive Move Setup

Assume that the preferences of the household members is unchanged from the setup in Section II. The timing of decisions and the budget constraint facing each member is now as follows. The household starts the period with wealth of \( W_t \). Without loss of generality, assume that member A is able to decide her own level of consumption subject to

\[
C_{A,t} \leq W_t. \tag{130}
\]

Thus A is free to spend up to all of the household’s remaining wealth. After this decision is made, the interim level of household wealth is

\[
\tilde{W}_t = W_t - C_{A,t}. \tag{131}
\]

Member B learns how much wealth the household has remaining and chooses her own consumption level subject to

\[
C_{B,t} \leq \tilde{W}_t. \tag{132}
\]

Thus B is able to spend up to the full amount of remaining household wealth. From one period to the next wealth evolves in the same way as before as specified in (5). As before consumption choices are chosen non-cooperatively and are found as subgame perfect best responses at each point in time.
B Non-Cooperative Equilibrium Consumption Choices

The consecutive move version of the model is solved in the Appendix. The unique equilibrium consumption choice of member’s $A$ and $B$ as a function of $W_t$ are

$$C_{A,t}^{Con} = \frac{\gamma_A}{1 + \sum_{x=1}^{NT-t} \delta^x} W_t \quad \text{and} \quad C_{B,t}^{Con} = \left( \frac{\gamma_B}{\gamma_B + \sum_{x=1}^{NT-t} \delta^x} \right) \left( \frac{1 + \sum_{x=1}^{NT-t} \delta^x}{1 + \sum_{x=1}^{NT-t} \delta^x} \right) W_t. \quad (133)$$

The unique equilibrium level of total consumption in any period is

$$X_t^{Con} = \left( \frac{1}{1 + \sum_{x=1}^{NT-t} \delta^x} \right) \left( \frac{(1 + \Delta) \sum_{x=1}^{NT-t} \delta^x + \gamma_B}{\gamma_B + \sum_{x=1}^{NT-t} \delta^x} \right) W_t. \quad (134)$$

The equilibrium consumption choices are slightly complicated because of the stackelberg leader and follower dynamics within each period. This encourages $A$ to consume slightly more to strategically lower the amount of consumption from $B$. Apart from this within period strategic consumption motive the forces governing both consumption decisions are identical to before. As the length of each period becomes arbitrarily small (i.e. $N$ gets large) then the magnitude of these within period strategic incentives will diminish as well. This is established formally in the following Proposition which is proved in the Appendix.

**Proposition 7:** As $N \to \infty$ the equilibrium consumption choices of the consecutive move game become arbitrarily close to the simultaneous move equilibrium as defined in (13) and (15). Formally,

$$\lim_{N \to \infty} \frac{C_{i,t}^{Con}}{C_{i,t}} = 1 \quad \text{and} \quad \lim_{N \to \infty} \frac{X_t^{Con}}{X_t} = 1.$$

Proposition 7 establishes that the equilibrium studied in the simultaneous move model is not a by-product of the arbitrary expenditure limits assumed in (7).

IX Internet Appendix - Consecutive Consumption Choices (Mathematical Proofs)

This Appendix solves for the unique equilibrium consumption path in the consecutive move version of the model introduced in the Internet Appendix.

-1 Equilibrium at $t = NT$

In the final period member $B$ will optimally consume all remaining wealth:

$$C_{B,NT}^{Con} = \tilde{W}_{NT}. \quad (135)$$
Anticipating (135), member A will choose $C_{A,NT}$ to solve
\[
\max_{C_{A,NT}} \gamma_A \ln C_{A,NT} + (1 - \gamma_A) \ln (W_{NT} - C_{A,NT}) \quad (136)
\]
subject to
\[
C_{A,NT} \geq 0 \quad \text{and} \quad W_{NT} - C_{A,NT} \geq 0. \quad (137)
\]
\[
W_{NT} - C_{A,NT} \geq 0. \quad (138)
\]
Ignoring (137) and (138) since they will not bind at the optimal choice, A’s consumption choice is characterized by the first order condition
\[
\frac{\gamma_A}{C_{A,NT}} - \frac{1 - \gamma_A}{W_{NT} - C_{A,NT}} = 0. \quad (139)
\]
Rearranging (139) and combing with (135) gives the equilibrium consumption levels for A and B in $t = NT$:
\[
C_{A,NT}^* = \gamma_A W_{NT} \quad \text{and} \quad C_{B,NT}^* = (1 - \gamma_A) W_{NT}. \quad (140)
\]
And total equilibrium consumption in $t = NT$ is simply
\[
X_{NT}^* = W_{NT}. \quad (142)
\]

**2 Solve for the Subgame Perfect Consumption path by Induction**

I conjecture the following form for the subgame perfect household allocation.

**Conjecture 4:** The subgame perfect equilibrium household allocation from $t$ until $NT$ is proportional to $W_t$. That is, for any period $t \in \{1, ..., NT\}$ the subgame perfect equilibrium levels of $C_{A,t}^*$ and $C_{B,t}^*$ can be written as $C_{i,t+x}^* = g_{i,t+x} W_t$ for $x \in \{0, 1, ..., NT - t\}$ where $g_{i,t+x}$ are strictly positive constants independent of $W_t$.

I establish Conjecture 4 by induction. The problem that member B solves in any period $t$ taking $\tilde{W}_t$ as given is:
\[
\max_{C_{B,t}} \gamma_B \ln C_{B,t} + \sum_{x=1}^{TN-t} \delta_{NT}^x \left[ (1 - \gamma_B) \ln C_{A,t+x}^{Con*} + \gamma_B \ln C_{B,t+x}^{Con*} \right] \quad (143)
\]
subject to
\[
W_{t+1} = R_{NT}^\frac{1}{NT} \left( \tilde{W}_t - C_{B,t} \right), \quad (144)
\]
\[
C_{B,t} \geq 0, \quad \text{and} \quad \tilde{W}_t - C_{B,t} \geq 0. \quad (146)
\]
Conjecture 4 implies that
\[
\sum_{x=1}^{TN-t} \delta \tilde{\pi} \left[ (1 - \gamma_B) \ln C_{A,t+x}^{Con} + \gamma_B \ln C_{B,t+x}^{Con*} \right] = \Sigma_t \ln W_{t+1} + k_{i,t} \tag{147}
\]
where
\[
\Sigma_t \equiv \sum_{x=1}^{TN-t} \delta \tilde{\pi} \tag{148}
\]
and \( k_{i,t} \) is a constant. In equilibrium (145) and (146) will not bind and hence I ignore those constraints and verify this later. Using (147) in (143) and substituting in the intertemporal budget constraint (144) allows me to simplify B’s problem to:

\[
\max_{C_{B,t}} \gamma_B \ln C_{B,t} + \Sigma_t \ln \left( \tilde{W}_t - C_{B,t} \right). \tag{149}
\]

The first order condition is
\[
\frac{\gamma_B}{C_{B,t}} - \frac{\Sigma_t}{\tilde{W}_t - C_{B,t}} = 0
\]
which gives B’s best response for any given level of \( \tilde{W}_t \):

\[
\tilde{C}^*_{B,t} = \frac{\gamma_B}{\gamma_B + \Sigma_t} \tilde{W}_t. \tag{149}
\]

Note that (149) verifies that (145) and (146) are satisfied in equilibrium.

Member A will anticipate (149) and choose \( C_{A,t} \) to solve

\[
\max_{C_{A,t}} \gamma_A \ln C_{A,t} + (1 - \gamma_A) \ln \tilde{C}^*_{B,t} + \sum_{x=1}^{TN-t} \delta \tilde{\pi} \left[ \gamma_A \ln C_{A,t+x}^{Con} + (1 - \gamma_A) \ln C_{B,t+x}^{Con*} \right] \tag{150}
\]
subject to (149),

\[
W_{t+1} = R \tilde{\pi} \left( W_t - C_{A,t} - \tilde{C}^*_{B,t} \right), \tag{151}
\]

\( C_{A,t} \geq 0 \), and

\[
W_t - C_{A,t} \geq 0. \tag{153}
\]

I ignore (152) and (153) and verify that they are satisfied at the end. Using the analog of (145) for A and substituting (151) and (149) into (150) we rewrite A’s problem (ignoring constants) as:

\[
\max_{C_{A,t}} \gamma_A \ln C_{A,t} + ((1 - \gamma_A) + \Sigma_t) \ln (W_t - C_{A,t}). \tag{150}
\]

The first order condition is
\[
\frac{\gamma_A}{C_{A,t}} - \frac{(1 - \gamma_A) + \Sigma_t}{W_t - C_{A,t}} = 0.
\]
Which gives A’s optimal consumption choice as

\[
C_{A,t}^{Cons} = \frac{\gamma_A}{1 + \Sigma_t} W_t. \tag{154}
\]

Note that (154) demonstrates that (152) and (153) are satisfied as conjectured. Substituting (154) into (149) gives B’s equilibrium consumption choice as a function of \( W_t \):

\[
C_{B,t}^{Cons} = \frac{\gamma_B}{\gamma_B + \Sigma_t} \left( \frac{1 + \Sigma_t - \gamma_A}{1 + \Sigma_t} \right) W_t. \tag{155}
\]

Adding (154) and (155) gives the equilibrium level of total consumption in period \( t \):

\[
X_t^{Cons} = \left( \frac{1}{1 + \sum_{x=1}^{TN-t} \delta^x} \right) \left( \frac{\gamma_B + (\gamma_A + \gamma_B) \sum_{x=1}^{TN-t} \delta^x}{\gamma_B + \sum_{x=1}^{TN-t} \delta^x} \right) W_t. \tag{156}
\]

Note finally that Conjecture 4 was verified about for the case of \( t = NT \). Moreover (154) and (155) demonstrate that it is true for \( t = NT - 1 \) and so on by iteration. This establishes Conjecture 4 by induction.

### 3 Comparison of Consecutive and Simultaneous Move Equilibria

Comparing (15) to (156) gives

\[
\frac{X_t^*}{X_t^{Cons}} = \frac{\Sigma_t^2 + (1 + \gamma_B) \Sigma_t + \gamma_B}{\Sigma_t^2 + \left( \frac{\gamma_B}{\gamma_A + \gamma_B} + (\gamma_A + \gamma_B) \right) \Sigma_t + \gamma_B}.
\]

Taking the limit of this ratio as \( N \to \infty \) requires finding

\[
\lim_{N \to \infty} \frac{X_t^*}{X_t^{Cons}} = \lim_{N \to \infty} \frac{\Sigma_t^2 + (1 + \gamma_B) \Sigma_t + \gamma_B}{\Sigma_t^2 + \left( \frac{\gamma_B}{\gamma_A + \gamma_B} + (\gamma_A + \gamma_B) \right) \Sigma_t + \gamma_B}.
\]

Since both the numerator and denominator tend to infinity we can apply L’Hopital’s rule to get

\[
\lim_{N \to \infty} \frac{X_t^*}{X_t^{Cons}} = \lim_{N \to \infty} \frac{\frac{\partial \Sigma_t^2}{\partial N} \left( 2\Sigma_t + (1 + \gamma_B) \right)}{\frac{\partial \Sigma_t}{\partial N} \left( 2\Sigma_t + \frac{\gamma_B}{\gamma_A + \gamma_B} + (\gamma_A + \gamma_B) \right)}
\]

\[
= \lim_{N \to \infty} \frac{2\Sigma_t + (1 + \gamma_B)}{2\Sigma_t + \frac{\gamma_B}{\gamma_A + \gamma_B} + (\gamma_A + \gamma_B)}.
\]
Again both the numerator and denominator tend to infinity so we can re-apply L’Hopital’s rule to get

\[
\lim_{N \to \infty} \frac{X_t^S}{X_t^*} = \lim_{N \to \infty} \frac{2\Sigma_t + (1 + \gamma_B)}{2\Sigma_t + \frac{\gamma_B}{\gamma_A + \gamma_B} + (\gamma_A + \gamma_B)}
\]

\[
= \lim_{N \to \infty} \frac{2\Sigma_t}{2\Sigma_t} = 1.
\]

Comparing the ratio of consumption choices of A and B within any period gives

\[
\frac{C_{A,t}^{Con*}}{C_{B,t}^{Con*}} = \frac{\gamma_A (\gamma_B + \Sigma_t)}{\gamma_B (1 + \Sigma_t - \gamma_A)}.
\]

Taking the limit of this ratio as \( N \to \infty \) by applying L’Hopital’s rule gives

\[
\lim_{N \to \infty} \frac{C_{A,t}^{Con*}}{C_{B,t}^{Con*}} = \lim_{N \to \infty} \frac{\gamma_A (\gamma_B + \Sigma_t)}{\gamma_B (1 + \Sigma_t - \gamma_A)} = \frac{\gamma_A \frac{\partial \Sigma_t}{\partial N}}{\gamma_B \frac{\partial \Sigma_t}{\partial N}} = \frac{\gamma_A}{\gamma_B}
\]

(158)

which is the same as 14. The combination of (157) and (158) establishes Proposition 7.
Figure 1
Equilibrium and Full Commitment Consumption Path
This plot shows the equilibrium level of total household expenditure in every period without commitment $X_t$ and the optimal full commitment consumption path $X^{**}_t$. It is drawn using the following parameters: Initial household wealth is $W_t=3,000,000$, the exponential discount factor is $\delta=0.95$, the gross interest rate is $R=1/0.95$, the household exists for $T=50$ years and there are $N=1$ period within each year. The figure compares the scenario where household members place weight on their own utility of $\gamma_A=\gamma_B=0.6$ and $\gamma_A=\gamma_B=0.7$.

Consumption Paths with Different Weights on Own Utility
Figure 2
Comparative Statics: The Value of Commitment

This plot shows the fraction of $W_i$ that the household would be willing to pay at $t=1$ to achieve the full commitment consumption path. Due to log additive utility functions this fraction is invariant to the choice of $W_i$. The figure shows how the value of commitment varies with: $\gamma = \gamma_A = \gamma_B$ the weight household members place on their own utility. The plot is drawn using the following parameters: both household members have exponential discount factor of $\delta = 0.95$, the gross interest rate is $R = 1/0.95$ and the household exists for $T = 50$ years with $N = 1$ periods per year.

The Value of Commitment and the Weight on Own Utility $\gamma$
Figure 3
Household with CRRA Preferences

These plots study how changing the EIS varies the consumption path and value of commitment for the household. Each panel is drawn using the following parameters: Initial household wealth is $W_i=3,000,000$, $\gamma_A=\gamma_B=0.6$, exponential discount factor is $\delta = 0.95$, the gross interest rate is $R=1/0.95$, the household exists for $T=50$ years and there are $N=1$ period within each year. Panel A shows the equilibrium level of total household expenditure in every period without commitment $X^*_t$ for values of EIS of 0.5, 1, and 1.5 as well as the optimal full commitment consumption path $X^{**}_t$. (it is the same for all three parameters). Panel B shows how the value of commitment varies with the EIS.

Panel A: Consumption Paths for Different Values of EIS

Panel B: Value of Commitment for Different Values of EIS
Figure 4
The Value of Commitment with Public Consumption
This plot shows the fraction of $W_1$ that the household would be willing to pay at $t=1$ to achieve the full commitment consumption path for different values of $\mu$. Due to log additive utility functions this fraction is invariant to the choice of $W_1$. It is drawn using the following parameters: the weight both household members place on their own utility is $\gamma = 0.6$, their exponential discount factor is $\delta = 0.95$, the gross interest rate is $R = 1/0.95$ and the household exists for $T=50$ years with $N=1$ periods per year.

The Value of Commitment and the Weight on Private Consumption $\mu$
Figure 5

Comparative Statics: The Value of Commitment with Hyperbolic Individuals

These plots show the amount the household would be willing to pay at \( t=1 \) (as a fraction of \( W_1 \)) to achieve the full commitment consumption path. Due to log additive utility functions this fraction will be invariant to the choice of \( W_1 \).

The figure shows how the value of commitment varies with \( \gamma = \gamma_A = \gamma_B \) (i.e. varying both symmetrically). It is drawn for individual quasi-hyperbolic discount factors of \( \beta = 1 \) and \( \beta = 0.85 \) holding other parameters constant at, \( \delta = 0.95 \), \( R = 1/0.95 \), \( T = 50 \), and \( N = 1 \).

Intra-Household Preference Misalignment Amplifies the Value of Commitment from Individual Hyperbolic Discounting