Financial Equilibrium with Career Concerns

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Abstract

What are the equilibrium features of a financial market where a sizeable proportion of traders face reputational concerns? This question is central to our understanding of financial markets that are increasingly dominated by institutional investors. We construct a model of delegated portfolio management that captures key features of the US mutual fund industry and embed it in an asset pricing framework. We thus provide a formal model of financial equilibrium with career concerned agents. Fund managers differ in their ability to understand market fundamentals, and in every period investors choose a fund. In equilibrium, the presence of career concerns induces uninformed fund managers to *churn*, i.e. to engage in trading even when they face a negative expected return. As churning plays the role of noise trading, the asset market displays non-fully informative prices and positive (and high) trading volume. The equilibrium relationship between fund return and net fund flows displays a skewed shape that is consistent with stylized facts. The robustness of our core results is probed from several angles.

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1 Introduction

There has been a substantial increase in the institutional ownership of corporate equity around the world in recent decades. On the New York Stock Exchange, for example, the percentage of outstanding corporate equity held by institutional investors has increased from 7.2% in 1950 to 49.8% in 2002 (NYSE Factbook 2003). For OECD countries as a whole, institutional ownership now accounts for around 30% of corporate equity (Nielsen [28]).

Institutional traders may be guided by incentives that are not fully captured by standard models in finance (Lakonishok, Shleifer, and Vishny [26]).¹ Consider, for example, the case of US mutual funds, which make up a significant proportion of institutional investors in equity markets.² Because of SEC regulations, most mutual funds charge fees that are independent of performance.³ Their revenue depends only on the amount of assets that investors choose to entrust to them. At the same time, there is a substantial amount of empirical evidence that investors shift their money towards funds that have performed well in the recent past creating a "flow-performance" relationship (Ippolito [22] and Chevalier and Ellison [6]). Funds have implicit incentives to "impress investors" in the current period in order to retain their investor base and attract new business in future periods. Indeed, Chevalier and Ellison present evidence that funds alter their behavior in response to such implicit incentives.⁴ Given the size of the mutual fund industry, such behavior is likely to affect prices and quantities in financial markets. In this paper, we consider the theoretical implications of the situation just described. In particular, we ask: What are the equilibrium features of a financial market in which a sizeable proportion of traders face implicit incentives of the kind that drive US mutual funds?

As a starting point, we draw upon the burgeoning theoretical literature on career concerns for experts (e.g. Holmstrom and Ricart i Costa [20], Scharfstein and Stein [32], Prendergast and Stole [31], and Ottaviani and Sørensen

¹Allen [1] presents persuasive arguments for the importance of financial institutions to asset pricing.

 $^{^{2}}$ According to the NYSE Factbook 2003, around 37% of equity held by institutions in 2003 was held by mutual funds; this number does not include state and private pension funds.

 $^{^{3}\}mathrm{See}$ Elton, Gruber, and Blake [12] for details. Only 1.7% of US mutual funds charged incentive fees in 1999.

⁴Lakonishok, Shleifer, Thaler, and Vishny [25] present evidence of related "window dressing" behaviour by pension fund managers.

[29]). An expert is an agent whose type determines his ability to understand the state of the world. This differs from "classical career concerns" (Holmstrom [19]) in which the agent's type determines his ability to exert effort. Expert models are particularly suited to analyze agency relationships in financial setups, in which the key driver appears to be the ability to pick the right portfolio rather than pure effort exertion. Some expert models have been used to analyze precisely such settings. For example, Scharfstein and Stein [32] develop an agency theoretic model in which experts mimic the decisions of other experts due to career concerns. However, that paper, and other such applications, consider only partial equilibrium analysis in which prices are fixed.

The main contribution of this paper is to develop a financial equilibrium model in which prices are determined endogenously by the behavior of experts with reputational concerns.⁵ This enables us to examine the effects of career concerned behavior on financial market variables.⁶

In our model there are three classes of agents: investors who cannot trade directly (we refer to them as *investors*), traders who trade on behalf of investors (*fund managers*), and other investors who trade directly (we refer to them as *traders*). It is a dynamic model: in every period the investors select among available fund managers. Fund managers face career concerns, which are the driving force behind our results.⁷

In the baseline model, the form of the payment from the investor to the fund manager is exogenously given and does not depend directly on performance. As discussed above, this assumption applies by and large to US mutual funds. This allows us to make our main points in a simple, tractable model. Later in the paper we show that the results are still valid in an environment with endogenous contracting, as long as only short-term contracts are feasible.

There are two periods. In each period there is a market for a risky asset, which is liquidated at the end of the period. In the beginning of the first period, investors entrust a fund manager with a sum of money. The fund

⁵We survey the related literature on asset pricing and agency problems below.

⁶It also enables us to examine whether career concerns can persist in a setting where prices play both informational and allocational roles.

⁷For tractability, we abstract from agency problems between the fund company and the fund manager, such as those documented by Chevalier and Ellison [7]. In our set-up, the fund manager and the fund company are the same entity: the terms fund manager and mutual fund can be (and are) used interchangeably.

manager trades on their behalf, and at the end of the period the investors observe the return obtained by the manager. At the beginning of the second period the investors can choose to retain the current fund manager or to replace him with a new one. Again, the fund manager trades on behalf of the investors.

Fund managers are characterized by their ability to observe market fundamentals. A good fund manager is more likely to learn the liquidation value of the asset before the asset is liquidated. In equilibrium investors can attempt to infer the ability of their fund manager from the outcome of trading.

The rest of the market is made of a large number of uninformed traders. Traders acts as *market makers* and post bid and ask prices. As traders are rational, there may be an endogenous bid-ask spread to account for adverse selection. In the baseline case, we make a simplifying assumption: each trader is short-lived and does not know what happened in the past (in particular, they do not know if they find themselves in the first or in the second period). Later in the paper, we show that this assumption is not necessary if one considers a more complex model with overlapping generations of fund managers.

The main findings are:

- 1. Without career concerns, prices are fully informative, and trading volume is zero. As a benchmark case, suppose that the fund manager has no career concerns (because the decision to replace him or retain him is exogenous). Then there is no equilibrium in which trade occurs. Absent career concerns, we fall back on a familiar no-trade result: in the absence of exogenous shocks and risk-sharing motives, there can be no trade in markets with asymmetric information.
- 2. With career concerns, prices are not fully informative, and trading volume is positive. If the decision to replace or retain the fund manager is endogenous, there exists a *churning equilibrium* in which a young manager always trades. If he is informed, he trades correctly. If not, he churns, that is, he buys or sells at random. If an uninformed young manager does not churn, he signals his lack of information and gets replaced in the following period.

From the viewpoint of the rest of the asset market, churners play the role of noise traders because their orders are not correlated with fundamentals. They lessen the adverse selection problem for informed traders, who now have opportunities for profitable trade. This closes the circle because ex post the investor has a strict incentive to retain a successful trader.

- 3. Trade volume is high. In the churning equilibrium, trading volume is not only positive but also large: all young fund managers and all informed old managers engage in trading. Below, we examine the existing literature and interpret this finding as a potential solution to the trade volume puzzle.
- 4. The endogenous compensation function is skewed against average performers. In the churning equilibrium, achieving an average return (as a result of non-trading) is as bad as achieving a negative return (as a result of a wrong trade). Both outcomes signal poor information and lead to the manager being replaced. Instead a positive outcome ensures that the manager is kept. The endogenous incentive structure is such the implicit compensation of an average performer is closer to that of an under-performer than to that of an over-performer.

While our model may be special, the intuition behind this result is general. If bad agents have less useful private information than good agents, their expectations of fundamentals are less likely to deviate from the market expectation (the technical conditions for this to be true are examined in Section 3). Hence, bad agents are less likely to benefit by trading, and in equilibrium lack of trade must carry a reputational cost. This in turn creates an incentive for fund managers to take excessive risk. An uninformed manager prefers randomizing over performance rather than getting stuck with an average outcome. Both the shape of the implicit compensation function and the risktaking behavior are consistent with the findings of Chevalier and Ellison [6]. Their empirical flow-performance relationship displays convexity, and, as a consequence, recently established funds face a measurable incentive to increase the variance of performance.

It is useful to place the present contribution in the context of the literature on the "trade volume puzzle". Any attempt to model financial trading faces the mighty obstacle of no-trade theorems.⁸ Under general conditions, the arrival of new private information cannot generate trade among rational

⁸See Brunnermeier [5] for an overview of the topic and for further references.

traders. The intuition is related to Akerlof's lemons problem. A trader who shows willingness to buy (sell) a given asset signals that he has private information indicating that the asset is worth more (less) than its market price. In equilibrium, this adverse selection problem results in zero trading. To get around the no-trade issue, the finance literature, beginning with Grossman and Stiglitz [16], has assumed the presence of noise trading. Noise traders are agents who must sell or buy because something has changed in their personal situation. For instance they are compelled by unforeseen circumstances to generate or utilize liquidity (hence, noise traders are sometimes referred to as liquidity traders) or they need to buy particular securities to hedge against new risks. The presence of noise traders reduces adverse selection and it allows for trade by informed speculators.

However, noise trading theories have come under increasing attack for their perceived inability to explain the order of magnitude of financial trade. While no conclusive evidence is available, many scholars are reluctant to accept that the trading volumes observed on modern stock markets (over \$10 trillion in 2002 on the New York Stock Exchange) can be explained by the kind of exogenous events that drive noise trading (Glaser and Weber [14]). De Bondt and Thaler [9, p. 392] go as far as to say that the high trade volume observed in financial markets "is perhaps the single most embarassing fact to the standard finance paradigm." One solution to the trading volume puzzle is to abandon the rational paradigm, for instance by allowing for overconfidence (e.g. Kyle and Wang [24] and Glaser and Weber [14]). Our paper provides an alternative solution. The presence of implicit incentives similar to those faced by US mutual funds is enough to generate a substantial amount of trade. Moreover, the model yields testable implications, which are discussed in the conclusion.

Our core results are derived in the simplest possible setting. We probe the robustness of our conclusions by extending the analysis in several directions. First, we note that our game can have other perfect Bayesian equilibria in which churning does not occur. We can, however, show that such equilibria are perverse: in these equilibria, investors must punish managers for obtaining high returns on their behalf.

Second, we extend our analysis to incorporate a richer information structure. We show that a necessary and sufficient condition for churning to occur is that a good fund manager is more likely than a bad fund manager to receive a signal about the liquidation value of the asset. If that mild condition is satisfied, there is a strict reputational cost of not trading. In equilibrium, a fund manager who has received an uninformative signal prefers churning to not trading.

Third, we allow for endogenous contracts and we show that the churning equilibrium exists as long as the contractual environment is not sufficiently rich to avoid career concerns. If investors and fund managers can sign longterm (multi-period) contracts, churning disappears and trading volume is zero. This is because the investor can commit not to replace a bad manager, which eliminates career concerns. However, the churning equilibrium still exists when contracts are endogenous but only short-term contracting is available. Short-term contracts allow for payment contingent on current performance but not payments contingent on future performance or on the choice to retain the manager. The fund manager is replaced if he underperforms, and this is sufficient to create career concerns and hence churning equilibria.

Fourth, we deal with the awkward assumption that market makers do not know in which period they live. This assumption was made to eliminate a final-period effect and is unnecessary once we consider an infinite horizon. We prove that the infinite-horizon game has a churning equilibrium in which the fund manager always trades.

Finally, we examine the issue of investors' returns. In our baseline model, the investor must use a fund manager. However, the investor's expected net return of using a fund manager is strictly negative. This is a straightforward consequence of the fact that the rational uninformed traders must make a zero expected profit and that there are transaction costs. If we interpret a "zero return" as the normalized return of holding the market portfolio, our result should be taken to mean that investing in an actively managed fund generates a lower net return than buying an index fund. If we remove the initial assumption that investors have no choice, why would they choose active fund management? An obvious way to deal with the issue of negative net returns is to introduce noise traders. We show that a sufficiently large probability of exogenously-driven trade generates positive expected net returns for the investor. The amount of noise trade needed to ensure a positive return goes to zero as the transaction cost goes to zero. Note that the amount of churning is unrelated to the amount of noise trading: if transaction cost is negligible, our model predicts a constant (and high) level of trade volume for any positive amount of noise trade.⁹

⁹One could also postulate that investors are irrational: for unmodeled psychological

Our paper is closely linked to the pioneering work of Dow and Gorton [10]. They embed an agency problem between investors and their fund managers into an asset pricing model and they show that under the optimal contract fund managers have an incentive to trade even when they have no private information, that is, they churn. In their model, some uninformed traders are motivated by the desire to hedge against risks that arise with exogenous probability. In equilibrium, the presence of an agency problem may generate a high trade volume even in the presence of a small hedging component.¹⁰ While Dow and Gorton consider a static model with complete contracts, we focus on a dynamic model with incomplete contracts, which is perhaps a better representation of the mutual fund industry. As a consequence, we are able to study phenomena of practical interest, such as the endogenous incentive structure faced by fund managers (the equilibrium flow/performance relationship) and the relationship between churning and seniority. We are also able to show the optimality of non-contingent pay for fund management companies, which is a common feature of the mutual fund industry in the US and elsewhere.

In a related paper, Allen and Gorton [2] present a model in which prices can diverge from fundamentals due to churning by portfolio managers. In their model, bad fund managers buy bubble stocks at prices above their known liquidation value in the hope of reselling them before they die — at even higher prices — to other bad fund managers. Their behavior is the result of an option-like payoff structure under which profits are shared with managers but losses are not. Churning thus creates the possibility of shortterm speculative profits. In contrast, in our setting, churning occurs under a flat fee structure due to reputational concerns over the life of the fund.

An earlier paper by Trueman [33] considers a delegated portfolio management model in which the fund manager's ability is unknown. Compensation depends on performance and on the posterior belief on the fund's manager ability. Trueman shows that there is a churning equilibrium in which un-

reasons, they prefer to buy actively managed funds even if there is enough evidence to conclude that index funds provide a better performance. While this behavioral explanation is not entirely satisfactory at a conceptual level, it may be consistent with available evidence. Despite the observation that actively managed funds underperform index funds after accounting for expenses (see, for example, Gruber [17] or Wermers [34]), U.S. investors put only 12% of their delegated funds in index funds, and Europeans put just 2% (The Economist [11])

¹⁰For further discussion of Dow and Gorton's results, see Bhattacharya [4].

informed fund managers trade. Our paper differs in two respects. First, Trueman assumes that the fund manager's future compensation depends on his posterior in an exogenously given way. Instead in our model, future compensation depends on the investor's retention decision, which is endogenous. Second, Trueman considers a partial equilibrium model (and therefore he cannot discuss trade volume) while we also take into account the feedback that the fund manager's trade has on the asset market.

In a recent study of active management, Berk and Green [3] present a model with symmetric information, learning, and dimishing returns to fund size that replicates several features of the data, including the flowperformance relationship. Like Trueman [33], Berk and Green consider a partial equilibrium framework. Hence, unlike us, they do not focus on prices and volume. While our goals are different, our results overlap with theirs in providing an alternative explanation for the flow-performance relationship (based on career concerns). It is worth noting that, in examining the financial equilibrium impact of institutional trading, we do not attempt to match another aspect of the data — the lack of performance persistence — that is captured by Berk and Green. It would be interesting, though beyond the scope of the current exercise, to incorporate dimishing returns to scale (as in Berk and Green [3]) in a general equilibrium framework (such as ours) to match the data along a further dimension.

Finally, our paper bears a conection to Cuoco and Kaniel [8]. They show that exogenously specified performance fees can have significant asset pricing implications. Symmetric and asymmetric fees induce different effects. While they consider a richer environment in continuous time with two risky assets, we endogenize the manager's compensation.

The plan of the paper is as follows. The following section develops the simplest model which is sufficient to generate our main results. Section 3 extends the baseline model in various directions: existence of other equilibria besides the churning equilibrium; effects of endogenous contracting; infinite-horizon model; and positive net returns. Section 4 concludes.

2 The Baseline Case

To present the essence of our results, we begin by discussing a simple baseline model. The main assumptions are: (1) investors must use active management; (2) contracts are exogenously given; (3) there are only two periods

and market makers do not know in which period they live; and (4) a good manager has perfect information while a bad manager has no private information. All these assumptions are made for analytical convenience and are relaxed in Section 3.

2.1 Model

Consider an economy with two periods, t = 1, 2. There is a single risk-neutral principal (investor) and a large pool of ex-ante identical risk-neutral agents (fund-managers). One of these is hired at random at t = 1 to trade for the principal. At the end of the period, the principal may retain the agent or hire a new one of average quality from the pool. The agent can be of two types: $\theta \in \{b, g\}$ with probabilities $1 - \gamma$ and γ respectively. The type of the agent is unknown to both the principal and the agent.

At each time period t, there is exactly one risky asset with liquidation value $v \in \{0, 1\}$ which occur with equal probability. The payoff v is realized at the end of each period and it is independent across periods. The agent's type θ and the asset payoff v are independent.

At the end of each period, the principal can observe the net return obtained by the agent. After such observation, the principal decides whether to retain or fire the fund manager.¹¹

There are a large number of risk-neutral short-lived uninformed rational traders who act as market-makers. Half of them operate in t = 1, the other half operate in t = 2. In each period t, the fund manager submits a market order $a \in A = \{0, \emptyset, 1\}$, where 0 stands for "sell one unit at highest available price", 1 stands for "buy one unit at lowest available price", and \emptyset , the empty set, represents lack of activity. The traders observe the order and each of them announces a price. Thus, the action of a trader consists of setting two prices: an ask price $p_A \in \left[\frac{1}{2}, 1\right]$ for a = 1 and a bid price $p_B \in \left[0, \frac{1}{2}\right]$ for a = 0. The bid-ask spread $p_A - p_B$ may be positive. The fund manager is free to trade with any rational trader, and, when indifferent between them, chooses one at random. The traders are thus subject to Bertrand competition (as is now standard in the literature, following Glosten and Milgrom [15] and Kyle

¹¹The principal observes the return but not the portfolio choice or the liquidation value. As it will become apparent later, this assumption is immaterial to our results. This property is a feature of the simple setting used here. See Prat [30] for a more general analysis of the effect of the structure of the principal's information on the agent's behavior.

[23]) and each sets prices equal to the expected value of the asset conditional on the order.

We make one important simplifying assumption. Traders do not know whether they are in period 1 or 2. Therefore, they are unable to condition their action on the fund manager's seniority. In section 3 we show that this assumption is not necessary if we consider an infinite-horizon model with overlapping generations of fund managers.

The fund manager's information structure is common to both periods and it depends on the fund manager's type. A good fund manager receives a signal conveying the true liquidation value v, while a bad fund manager receives no signal. The signal s can take three values, 0, 1, and \emptyset , and it is determined as follows:

$$s(\theta, v) = \begin{cases} v & \text{if } \theta = g \\ \emptyset & \text{if } \theta = b \end{cases}$$

In the present setup, s reveals θ . When the fund manager learns his signal he also learns his type. The investor does not observe either the signal or the type.

The fund manager incurs a cost of trading $\epsilon > 0$ every time he buys or sells. The cost is introduced in order to break the indifference between trading and not trading in favor of the latter. Most of the results we present are obtained for the asymptotic case $\epsilon \to 0$.

In each period, the net return on investment is

$$\chi(a, p_A, p_B, v, \epsilon) = \begin{cases} 0 & if \quad a = \emptyset \\ v - p_A - \epsilon & if \quad a = 1 \\ p_B - v - \epsilon & if \quad a = 0 \end{cases}$$

Write a time-t mixed strategy for an agent as the mapping $a_t: S \to \Delta A$.

In this baseline version of the model, the contractual arrangement between the investor and the fund manager is exogenously determined. Note that we abstract throughout from agency problems that may arise within fund management companies. Thus, mutual fund managers can be identified with mutual funds companies. Accordingly, we model payoffs to managers along the lines of the fees charged by mutual fund companies. Given return χ , the payment from the investor to the manager is

$$t = \alpha \chi + \beta,$$

where $\alpha \in (0, 1)$ and $\beta \in (0, \infty)$. In most of the results of the present section we focus on the case $\alpha \to 0$. However, the fact that $\alpha > 0$ guarantees that when career concerns are absent, the interests of the fund manager are aligned with those of the investor.

For simplicity there is no discounting. The investor's payoff is given by $\chi_1 - t_1 + \chi_2 - t_2$. The fund manager's payoff is $t_1 + t_2$.

To summarize, timing is:

- t = 1 The fund manager learns s_1 and chooses a_1 ;
 - Traders observe a_1 and set prices;
 - The investor observes the net return obtained by the manager; All other traders observe v; Payments to the fund manager are made.
- t = 2 The investor retains the incumbent or hires the challenger.
 - The fund manager learns s_2 and chooses a_2 ;
 - Traders observe a_2 and set prices;
 - The investor observes the net return obtained by the manager; All other traders observe v; Payments to the fund manager are made.

2.2 No Trade without Career Concerns

We first establish the benchmark case without career concerns, in order to reassure readers that our core results are indeed due to the presence of implicit incentives and not to other features of our dynamic model.

Career concerns arise when the fund manager knows that his chance of being replaced depends on his behavior. To eliminate the implicit incentive component, assume that the probability that the first period fund manager is retained is exogenously given by $r \in [0, 1]$ and it is stochastically independent from any other variable in the model.

Proposition 1 For any exogenous $r \in [0, 1]$, there is no trade in equilibrium.

The proof of this result, and that of all results other than the main result (Proposition 2, below), are detailed in the appendix. Proposition 1 is akin to a no-trade theorem. In the absence of career concerns, our model does not support positive trade volumes. When fund managers are not careermotivated, they trade optimally. Uninformed traders realize that because of adverse selection they can only lose from trading with fund managers. Trade cannot occur in equilibrium.

If the exogenous retention rate r is set to one, our model can be interpreted as a situation in which the investor and the fund manager are the same person: an informed investor. Unsurprisingly, there cannot be trade in such a setting.

2.3 Churning with Career Concerns

We now return to the initial model and let the retention choice be made by the investor. What follows is our core result on the existence and characterization of the churning equilibrium:

Proposition 2 For α and ϵ low enough, there exists an equilibrium in which.¹²

- 1. The investor retains the fund manager if he trades correctly and replaces him if he makes a wrong trade or no trade.
- 2. A good fund manager always trades. A bad fund manager churns if t = 1 and he does not trade if t = 2. Formally,

$$a_t(s_t) = s_t \text{ for } t = 1, 2, \ s_t \neq \emptyset$$

$$a_1(\emptyset) = \begin{cases} 0 \text{ with probability } \frac{1}{2} \\ 1 \text{ with probability } \frac{1}{2} \end{cases}$$

$$a_2(\emptyset) = \emptyset$$

3. Traders set prices:

$$\hat{p}_A = \frac{1}{2} (1 + \hat{\gamma}) \text{ and } \hat{p}_B = \frac{1}{2} (1 - \hat{\gamma})$$

where

$$\hat{\gamma} = \gamma \frac{5 - \gamma}{2 + 3\gamma - \gamma^2}.$$

¹²Namely, the equilibrium exists if $\alpha \leq \beta$ and $\epsilon < \frac{1}{2}(1-\hat{\gamma})$.

Proof. Fund manager's strategy at t = 2: At t = 2, a bad manager never trades because $\hat{p}_A > \frac{1}{2}$ and $\hat{p}_B < \frac{1}{2}$. A good fund manager with signal s = 1 is strictly better off buying if $1 - \hat{p}_A - \epsilon > 0$, which is satisfied if

$$\epsilon < 1 - \hat{p}_A = \frac{1}{2} \left(1 - \hat{\gamma} \right) \equiv \hat{\epsilon} \tag{1}$$

A good fund manager with s = 0 is better off selling if $\hat{p}_B - \epsilon > 0$, which is also satisfied under (1).

Investor's belief: Given the price structure, only three possible firstperiod realizations of the gross return are possible: the one corresponding to a succesful purchase or sale:

$$\chi_1 = 1 - \hat{p}_A = \hat{p}_B = \frac{1}{2} (1 - \hat{\gamma}) - \epsilon;$$

the one corresponding to an wrong purchase or sale:

$$\chi_1 = -\hat{p}_A = \hat{p}_B - 1 = -\frac{1}{2}(1+\hat{\gamma}) - \epsilon;$$

and the one corresponding to no trade: $\chi_1 = 0$.

In the present equilibrium only the former two are observed. The requirement that investors beliefs are consistent with equilibrium play implies that

$$\Pr\left(\theta = g | \chi_1\right) \begin{cases} = 0 & \text{if } \chi_1 = -\frac{1}{2} \left(1 + \hat{\gamma}\right) - \epsilon \\ = \frac{\gamma}{\gamma + \frac{1}{2}(1 - \gamma)} = \frac{2\gamma}{\gamma + 1} & \text{if } \chi_1 = \frac{1}{2} \left(1 - \hat{\gamma}\right) - \epsilon \\ \in [0, 1] & \text{if } \chi_1 = 0 \end{cases}$$

The return $\chi_1 = 0$, which corresponds to the action $a = \emptyset$, is off the equilibrium path at t = 1. Perfect Bayesian equilibrium imposes no restriction. We choose to set:¹³

$$\Pr\left(\theta = g | \chi_1 = 0\right) = 0.$$

Investor's retaining strategy: Suppose (1) holds. A good fund manager generates a positive net return while a bad fund manager generates a zero net return. It is a best response for the investor to retain the fund manager if and only if

$$\Pr\left(\theta = g | \chi_1\right) \ge \gamma.$$

¹³The proof also works with any $\Pr(\theta = g | \chi_1 = 0) \in (0, \gamma)$.

Combined with the posteriors above, this condition implies that the investor retains the fund manager if and only if the fund manager trades successfully.

Good fund manager's strategy at t = 1: A good fund manager who plays a = s produces net return $\frac{1}{2}(1 - \hat{\gamma}) - \epsilon$ in t = 1. He is retained and he produces again the same net return in t = 2. His total payoff is

$$\pi_g \left(a = s \right) = 2 \left(\alpha \left(\frac{1}{2} \left(1 - \hat{\gamma} \right) - \epsilon \right) + \beta \right).$$

If ϵ is small enough to satisfy (1), it is easy to see that $\pi_g (a = s)$ is higher than the payoff that the fund manager would get if he plays $a = \emptyset$. It is also obvious that $\pi_g (a = s)$ is higher than the payoff the manager would get if he plays a = 1 - s.

Bad fund manager's strategy at t = 1: A bad fund manager who does not trade generates a zero net return in t = 1 and he is not retained. Therefore, his total payoff is

$$\pi_b \left(a = \emptyset \right) = \beta.$$

If instead the bad manager plays either a = 1 or a = 0 in t = 1, he is successful with probability $\frac{1}{2}$. His expected net return at t = 1 is

$$\frac{1}{2} - \hat{p}_A - \epsilon = -\frac{1}{2} + \hat{p}_B - \epsilon = -\frac{\hat{\gamma}}{2} - \epsilon$$

If the churner is successful, he is retained and he does not trade at t = 2. His total expected payoff is

$$\pi_b \left(a \in \{0, 1\} \right) = \alpha \left(-\frac{\hat{\gamma}}{2} - \epsilon \right) + \beta + \frac{1}{2}\beta.$$

Then, $\pi_b (a \in \{0, 1\}) \ge \pi_b (a = \emptyset)$ if

$$\alpha \le \frac{\frac{1}{2}\beta}{\frac{\hat{\gamma}}{2} + \epsilon}.$$

As ϵ is bounded above by (1), a sufficient condition for churning to occur is

$$\alpha \le \frac{\frac{1}{2}\beta}{\frac{\hat{\gamma}}{2} + \hat{\epsilon}} = \beta.$$

Traders' pricing strategy: The probability that the second-period fund manager is good is equal to the probability that the manager is good in t = 1 (because he is retained for sure) plus the probability that the manager is bad and he is replaced with a good one:

Pr
$$(\theta = g | t = 2) = \gamma + \frac{1}{2} (1 - \gamma) \gamma.$$

Second-period managers trade only if they are good. First-period managers always trade, and churners randomize with equal probability between buying and selling. Thus, by symmetry,

$$\Pr(\theta = g | a = 1) = \Pr(\theta = g | a = 0) = \Pr(\theta = g | a \in \{0, 1\}).$$

A trader who receives a buy or sell order computes the following posterior probability:

$$\begin{split} \hat{\gamma} &\equiv \Pr\left(\theta = g | a \in \{0, 1\}\right) \\ &= \frac{\Pr\left(a \in \{0, 1\}, \theta = g\right)}{\Pr\left(a \in \{0, 1\}\right)} \\ &= \frac{\Pr\left(a \in \{0, 1\}, \theta = g, t = 1\right) + \Pr\left(a \in \{0, 1\}, \theta = g, t = 2\right)}{\Pr\left(a \in \{0, 1\}, t = 1\right) + \Pr\left(a \in \{0, 1\}, t = 2\right)} \\ &= \frac{\left[\Pr\left(a \in \{0, 1\} | \theta = g, t = 1\right) \Pr\left(\theta = g | t = 1\right) + \right]}{\Pr\left(a \in \{0, 1\} | \theta = g, t = 2\right) \Pr\left(\theta = g | t = 2\right)} \\ &= \frac{\Pr\left(\theta = g | t = 1\right) + \Pr\left(\theta = g | t = 2\right)}{1 + \Pr\left(\theta = g | t = 2\right)} \\ &= \frac{\gamma + \gamma + \frac{1}{2}(1 - \gamma)\gamma}{1 + \gamma + \frac{1}{2}(1 - \gamma)\gamma} \\ &= \gamma \frac{5 - \gamma}{2 + 3\gamma - \gamma^2}. \end{split}$$

The ask price is

$$\hat{p}_A = \Pr(\theta = g | a \in \{0, 1\}) 1 + \Pr(\theta = b | a \in \{0, 1\}) \frac{1}{2}$$

$$= \frac{1}{2} + \frac{1}{2} \Pr(\theta = g | a \in \{0, 1\})$$

$$= \frac{1}{2} (1 + \hat{\gamma}),$$

and the bid price is

$$\hat{p}_B = \Pr(\theta = g | a \in \{0, 1\}) 0 + \Pr(\theta = b | a \in \{0, 1\}) \frac{1}{2}$$
$$= \frac{1}{2} (1 - \Pr(\theta = b | a \in \{0, 1\}))$$
$$= \frac{1}{2} (1 - \hat{\gamma}).$$

Proposition 2 identifies a churning equilibrium. All first-period fund managers trade. The good ones make correct trades by following their private information. The bad ones randomize between buying and selling.

The investor realizes that a successful trade may come from a lucky churner. Still, she knows that a good manager is more likely to be right and she revises her posterior upwards if she observes a successful trade. She also knows that a wrong trade can only come from a bad manager, and she believes that no-trade (an off-equilibrium event) is more likely to come from a bad manager. Given this set of beliefs, the investor retains the first-period manager if and only if he has traded successfully.

A good manager makes positive returns in both periods (provided the trading cost is low enough). He knows the liquidation value and he buys or sells at prices that are strictly between 0 and 1. He is also certain to be retained.

A bad manager faces a depressing choice between churning and nontrading. If he churns, he makes negative expected return $-\frac{\hat{\gamma}}{2} - \epsilon$ but he has a 50% of being retained. If he does not trade, he makes a zero return and he is fired for sure. If the direct-stake parameter α is low enough (in particular, lower than the fixed payment), the bad manager prefers to churn.

Traders know that a market order may come from a good manager who knows the liquidation value or a first-period bad manager who is churning. The price will be based on the probability that the order comes from a good manager conditional on the presence of an order, which is

$$\hat{\gamma} = \Pr\left(\theta = g | a \in \{0, 1\}\right) = \gamma \frac{5 - \gamma}{2 + 3\gamma - \gamma^2} \in (0, 1) \,.$$

One can check that the posterior $\hat{\gamma}$ is greater than the prior γ . This is due to two factors. First, a good manager is more likely than a bad manager to be retained. Second, a bad manager does not trade in the second period.

However, $\hat{\gamma} \to \gamma$ when either $\gamma \to 0$ or $\gamma \to 1$. Given the posterior $\hat{\gamma}$, traders compute the bid price and the ask price. The bid-ask spread is simply $\hat{\gamma}$ and it is increasing in γ . It tends to 1 when $\gamma \to 1$ and it tends to zero when $\gamma \to 0$.

We now discuss in detail two important implications of Proposition 2: the comparative statics of trading volume and the equilibrium incentives faced by fund managers.

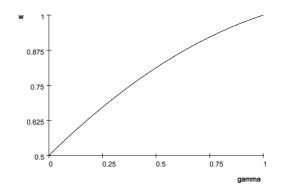
2.4 Comparative Statics of Trading Volume

Trading volume is the expected number of assets traded in an average period. It can be defined as the average of the probability that a trade occurs at t = 1 and the probability that a trade occurs at t = 2. From Proposition 2, we can easily compute trading volume in the churning equilibrium:

Corollary 3 The average trading volume is

$$w = \frac{1 + \Pr(\theta = g|t = 2)}{2} = \frac{2 + 3\gamma - \gamma^2}{4}$$

In the first period, there is always trade. In the second period, trade occurs only if the manager is good. Trading volume w is graphed below



Trading is at its lowest when almost all managers are bad. Still, the presence of churning guarantees that trading volume is always above $\frac{1}{2}$.

What are expected payoffs in this equilibrium? As the expected payoff of traders is zero, the gross expected return over the two periods must be zero as well. The expected net return, which is negative, is thus the expected number

of trades 2w times the trading cost: $E(\chi_1 + \chi_2) = -2\epsilon w$. The expected payment from the investor to the fund manager over the two periods is:

$$E(t_1 + t_2) = E(\alpha \chi_1 + \beta + \alpha \chi_2 + \beta) = 2(-\alpha \epsilon w + \beta).$$

The investor's expected payoff is:

$$E(\chi_1 - t_1 + \chi_2 - t_2) = -2((1 - \alpha) \epsilon w + \beta).$$

In this baseline model, the investor's expected net payoff is negative (but it becomes arbitrarily close to zero if α , β , and ϵ tend to zero). In section 3, we will see that the net return becomes positive if a small proportion of traders are noise traders.¹⁴

2.5 Asymmetric Flow-Performance Relationship

A further consequence of Proposition 2 is that the implicit incentives of young fund managers are skewed. The reputational reward for good performance is higher in absolute terms than the reputational cost of bad performance. The existence of such a flow-performance relationship has been documented empirically by Chevalier and Ellison [6]. In a seminal empirical study of a large sample of income and growth funds from 1982 to 1992, they find that fund companies face an asymmetric flow-performance relationship. Funds with better past performance receive larger net in-flows. However, starting from avarage performance, the absolute effect of an increase in performance is greater than the absolute effect of a decrease of the same size. This form of convexity is much more evident for young funds. As a consequence, Chevalier and Ellison also find that young funds face incentives for excessive risk taking.

Equilibrium behavior in our model generates a similar, albeit more stylized, picture. A fund manager who trades successfully (i.e. generates a positive return) has an investment in-flow of zero. A fund manager who makes either a wrong trade (negative return) or no trade (average return) has an investment in-flow of -1. This translates into skewed implicit incentives. To fix ideas, let the explicit incentive component go to zero: $\alpha \to 0$.

¹⁴The welfare properties of this model are uninteresting. As all market participants are risk-neutral and have no hedging motive, trading is a zero-sum game. The only nondistributional effect or market activity is the deadweight imposed by transaction costs. A Utilitarian planner would just want to minimize trade. More insightful normative implications could be obtained by including a microfounded hedging motive (Dow and Gorton [10]).

The manager's payoff depend only on whether he is retained and it can be written as

$$t_1 + t_2 = \begin{cases} \beta & \text{if } \chi_1 < 0 \\ \beta & \text{if } \chi_1 = 0 \\ 2\beta & \text{if } \chi_1 > 0 \end{cases}$$

Like in Chevalier and Ellison, our young fund manager may have an incentive to increase the variability of the first-period return. If he is uninformed, he prefers a lottery over χ_1 , which he achieves by churning, rather than $\chi_1 = 0$ for sure, which he could obtain by not trading.

The intuition behind the asymmetry of the flow-performance relationship goes beyond our simple set-up. In any model of career concerns for experts, the value of an expert must depend on his ability to obtain information that is not publicly available (see Zitzewitz [35]). If the expert is a fund manager, his usefulness corresponds to his ability to form an opinion that differs from the market opinion. This leads a manager to identify assets that are over- or under-priced. A better manager holds a portfolio that is farther away from the market portfolio and, as consequence, generates a return that is less correlated to the market return (but on average higher). Such a manager should garner reputational rewards. Conversely, a fund manager who systematically tracks the market must have limited private information and should be less sought after by rational investors. In equilibrium, managers who obtain returns that are close to the market return must face some reputational cost.

3 Discussion and Extensions

It is important to probe the robustness of the findings of the previous section. In this section, we discuss: (1) the robustness of churning; (2) endogenous contracts; (3) infinite horizon; and (4) noise trading.

3.1 How robust is churning?

The core analysis focused on one particular equilibrium (the churning equilibrium) given one particular information structure (good agents know everything, bad agents know nothing). We now show that the churning equilibrium is the most plausible one and that our results do not depend on the particular information structure we chose for expositional purposes. In order to abstract from pathological equilibria supported by carefully chosen out-of-equilibrium beliefs, we carry out both of these exercises under the assumption that an arbitrarily small proportion μ of managers are "naive", i.e., always play according to their signal. For pedagogical purposes, it is useful to hold prices fixed, interior, and symmetric about $\frac{1}{2}$, and consider the partial equilibrium incentives for managers for arbitrary $p_B = 1 - p_A \in (0, \frac{1}{2}]$. We make these two assumptions for the remainder of this subsection. All results obtained in partial equilibrium hold *a fortiori* in general equilibrium.

3.1.1 Non-churning equilibria

While churning need not be an essential property of all equilibria of our baseline game, we shall show here that equilibria without churning must be "peculiar": in these equilibria, over some ranges of returns, investors must *punish* a manager for *doing well*. To be precise, denote the equilibrium retention probabilities used by the investor by r(1), r(0), and r(-1) for the the cases where the manager generates positive, zero, and negative returns respectively. Now we can state:

Proposition 4 When α and ϵ are sufficiently small, in any equilibrium in which managers do not churn, it cannot be the case that $r(1) \ge r(0) \ge r(-1)$.

If $r(1) \ge r(0) \ge r(-1)$, it is clear that informed fund managers would trade according to their signals. Then it must be the case that $r(0) = 0.^{15}$ In addition, for the investor's strategy to be optimal, it must be the case that r(1) > 0. Thus, r(1) > r(0) = r(-1) = 0. But then it follows that $r(0) < \frac{1}{2}[r(1) + r(-1)]$, and then (for small α) the uninformed manager will churn.

The relative values of the equilibrium retention probabilities determine the slope of the flow-performance relationship discussed above. The interpretation of this result is that any equilibrium where fund managers' strategies are different from those identified in Proposition 2, must be characterized by a flow-performance relationship that is strictly decreasing over some range. In other words, fund managers must be explicitly punished for doing better. This is at odds with empirical findings.

 $^{^{15}}$ This is because beliefs are no longer arbitrary in the presence of a small measure of naive managers.

3.1.2 A more general information structure

The core result was obtained under the assumption that the agent has an extremely simple information structure: a good agent receives a perfectly informative signal, a bad agent receives no signal. We now generalize the information structure to allow a good agent to receive no signal and a bad agent to receive an informative signal.

Suppose that the distribution of private signal s can be written as

$$\Pr\left(s|v,\theta\right) = \begin{cases} \rho_{\theta} & \text{if } s = v\\ 1 - \rho_{\theta} - \tau_{\theta} & \text{if } s = \emptyset\\ \tau_{\theta} & \text{if } s = -v \end{cases},$$

where ρ_g , τ_g , ρ_b , and τ_b are parameters with values in the interval [0, 1] such that $\rho_g + \tau_g \leq 1$ and $\rho_b + \tau_b \leq 1$. The information structure is based on the implicit assumption that the signal is symmetric in v = 1 and v = 0. Our core results were obtained under the assumption that $\rho_g = 1$ and $\tau_g = \rho_b = \tau_b = 0$. The agent does not observe his own type θ .

We assume that the signal is useful, in the sense that it provides some information on the valuation v (even for a bad agent). Moreover, a good agent gets a more useful signal than a bad agent:

$$1 \ge \rho_g > \rho_b > \tau_b > \tau_g \ge 0. \tag{2}$$

We now identify an important condition which implies that receiving informative signals is "good news" about a manager:

$$\Pr\left(\theta = g | s \neq \emptyset\right) > \Pr\left(\theta = g | s = \emptyset\right). \tag{3}$$

Stated in terms of primitives, this is equivalent to $\rho_g + \tau_g > \rho_b + \tau_b$. We now show that this condition is necessary and sufficient for churning.

Proposition 5 For simplicity, suppose that there are no transaction costs and no direct benefit for the agent: $\epsilon = 0$, $\alpha = 0$. Then:

1. There exists a churning equilibrium in which

$$a_t (s_t) = s_t \text{ for } t = 1, 2, \ s \neq \emptyset$$

$$a_1 (\emptyset) = \begin{cases} 0 & \text{with probability } \frac{1}{2} \\ 1 & \text{with probability } \frac{1}{2} \end{cases}$$

$$a_2 (\emptyset) = \emptyset$$

if and only if $\rho_g + \tau_g > \rho_b + \tau_b$.

2. Further, when $\rho_g + \tau_g > \rho_b + \tau_b$, there exists no equilibrium in which the agent follows his signal in both periods: $a_t(s) = s$ for t = 1, 2 and all s.

We conclude this section with a discussion of the second part of Proposition 5. If inequality (3) is satisfied, there cannot exist a partial equilibrium in which the fund manager follows his signal. The condition requires that observing an informative signal is better news for the manager's type than observing an uninformative signal: a good expert is more likely than a bad expert to get some kind of information, however flawed, rather than no information at all. If this condition is satisfied and the fund manager follows his signal, then not trading is bad news about the manager's type. The investor should fire a non-trader. If explicit incentives are not too large, an uninformed fund manager would then prefer to churn and there exists no equilibrium in which the manager follows his signal. In general equilibrium, it is precisely this lack of sincerity that supports interior bid and ask prices, which in turn provides a basis for churning as outlined above.

3.2 Endogenous Contracts

The baseline model postulates exogenously given linear contracts. We now remove this assumption and we consider endogenous contracting between the investor and the fund manager(s). First, we show that if long-term contracts are available, in equilibrium there is no churning (and therefore no trade). Second, we show that if only short-term contracts are available the results of the baseline model are still valid.

To begin, consider the following form of *long-term contracting*. A contract specifies the payment from the investor to the agent and a rule for retaining or replacing the fund manager. The payment can be contingent on observables, i.e., on the realized return χ_t at t = 1, 2. If the investor replaces the manager, she and the new manager agree on a new contract on the observables at t = 2.

The investor has all the bargaining power: he makes a take-it-or-leave-it offer to the fund manager. To make things interesting, it is useful to assume that the fund manager must receive a minimum non-negative payment \bar{w} if he is employed (for every period he is employed). If this were not the case, the investor would just offer a zero payment in both periods and the fund manager would be entirely indifferent (and therefore he might as well behave

optimally). As $\bar{w} > 0$, we can disregard the fund manager's participation constraint.

Traders do not observe the contract signed by the investor and the fund manager. We can now prove the following:

Proposition 6 With long-term contracting, trading volume is zero.

The result relies on the investor's ability to commit to retain the current fund manager (or to replace him for sure). This kills off career concerns and therefore churning. Positive trading volumes cannot be supported in equilibrium.

We now move on to short-term contracting. The environment is the one above except that there are two short-lived investors. Investor 1 offers a contract b_1 to the fund manager in the beginning of the first-period and she receives payoff $\chi_1 - t_1$. The payment between investor 1 and the manager may depend on all the observables at t = 1 (but it cannot depend on what happens in t = 2). Investor 2 is born in the beginning of the second period and she observes the return obtained by the manager in the previous period (she does not observe the contract used in the previous period). She then chooses between the incumbent manager and the challenger and he selects a contract b_2 . We can then write a contract as a triple (corresponding to positive, negative, and zero net returns respectively):

 $b_t = (b_t \text{ (success)}, b_t \text{ (failure)}, b_t \text{ (no trade)}),$

under the contraint – discussed above – that all three values are not below \bar{w} . The time line is:

t = 1 – Investor 1 specifies contract b_1 ;

- The fund manager learns s_1 and chooses a_1 ;
- Traders observe a_1 and set price;
- Investor 1 observes the net return; All other traders observe v; Payments to the fund manager are made.
- t = 2 Investor 2 observes the net first period return. She retains the incumbent or hires the challenger. She specifies contract b_2 ;
 - The fund manager learns s_2 and chooses a_2 ;
 - Traders observe a_2 and set price;

- Investor 2 observes the net second period return; All other traders observe v. Payments to the fund manager are made.

We show that, under certain conditions there exists an equilibrium which is essentially identical to the churning equilibrium which we found in the baseline model:

Proposition 7 For any $\bar{w} > 0$, if the proportion of good traders γ and trading cost ϵ are low enough, there exists a churning equilibrium in which:

- 1. Investor 1 selects a flat contract $b_1 = (\bar{w}, \bar{w}, \bar{w})$.
- 2. Investor 2 retains the fund manager if and only if he traded successfully. In either case, the investor selects a flat contract $b_2 = (\bar{w}, \bar{w}, \bar{w})$.
- 3. A good fund manager always trades. A bad fund manager churns if t = 1 and he does not trade if t = 2.
- 4. Traders set prices as in Proposition 2.

Even with endogeneous contracting, the churning equilibrium of Proposition 2 is still an equilibrium if: (i) only short-term contracts are possible; and (ii) the proportion of good managers is low. Such equilibrium has the same high levels of trading volume identified in Corollary 3.

Churning hurts the first-period investor, who faces a negative expected return (plus trading cost). If churning stops, the investor makes an expected gain

$$R_{\max} = (1 - \gamma) \frac{1}{2} (\hat{\gamma} + \epsilon).$$

The investor can eradicate churning by offering an appropriate contract. The benefit of churning to a bad manager is given by a 50% chance of being hired again in the next period: $\frac{1}{2}\bar{w}$. To persuade him to stop trading, the investor needs to set

$$b_1$$
 (no trade) > $\bar{w} + \frac{1}{2}\bar{w}$.

The expected cost of eliminating churning is thus

$$C_{\min} = (1 - \gamma) \frac{1}{2} \bar{w}.$$

If $\frac{\hat{\gamma}}{2} + \epsilon$ is small enough, the difference $R_{\text{max}} - C_{\text{min}}$ is negative, and the investor is not willing to incur the cost necessary to eradicate churning.¹⁶

The damage of churning *per churner* on investor 1 is $\frac{1}{2}(\hat{\gamma} + 2\epsilon)$. It is lowest when the proportion of good types γ is low because the bid-ask spread is narrow. Churning is least costly when there are many churners. Therefore, if γ is low enough, the benefit of stopping churning is lower than the cost of reimbursing the bad manager for the lost career opportunity.

At a deeper level, the result may also be understood in terms of inefficiencies generated by bilateral contracting in an environment with more than two agents. Churning increases the probability that the fund manager is retained in the second period. This creates an additional rent to the incumbent which in part is paid for by investor 2 (who cannot tell for certain between a good and a bad incumbent) and by the challenger (who is hired with a lower probability). As investor 1 and the incumbent do not internalize the cost that churning imposes on the other two parties, they find it optimal to sign a contract that does not prevent churing. That is why full bilateral contracting can still lead to socially inefficient outcomes.

3.3 Infinite Horizon

Churning cannot occur if there is an end-game effect. In the last period, there are no career concerns and the no-trade theorem holds. But then the ability of a manager does not matter in the last period and there are no career concerns in the period before the last. Then, the no trade theorem applies to the second-last period as well, and so on. In the baseline model we employed a trick to overcome the end-game effect: we assumed the uninformed traders do not know in which period they live. In this section we consider an infinite-horizon version of the baseline model and show, in the absence of this assumption, that there exists an equilibrium with churning. A by-product of the analysis of the infinite-horizon case is a richer characterization of the dynamics of fund managers' reputation.

At each period t, there are: one incumbent fund manager and one challenger; one short-lived investor; and a large number of short-lived rational traders. As before, the type of a fund manager is $\theta \in \{b, g\}$ and the prior

¹⁶The formal condition is $\bar{w} > \hat{\gamma} + 2\epsilon$. This is a plausible requirement in practice in liquid markets. The percentage fee for fund managers (\bar{w}) is of the order of 0.5-2%, while the bid-ask spread ($\hat{\gamma}$) for a liquid stock and the exogenous transaction costs (ϵ) paid by institutional traders are probably of a smaller order of magnitude.

is γ . In each period a potentially immortal fund manager is born. If the fund manager is not hired or he is replaced, he dies. In every period, there is a probability $\delta \in \left(0, \frac{1-\gamma}{2}\right)$ that a good fund manager becomes bad. A bad fund manager stays bad. The fund manager maximizes the expected sum of future payments (because of δ there is no need for further discounting).¹⁷

In every period t there is a short-lived investor who observes all the past returns and hiring decisions. The investor chooses whether to retain the incumbent from t - 1 or hire the challenger who is born at t. As in the baseline model, the contract between the investor and the fund manager is exogenously given. At the end of t the investor pays the fund manager:

$$x_t = \alpha \chi_t + \beta,$$

where the net return χ_t is as in the baseline model.

In every period t, there are a large number of short-lived rational traders who have the same information as the investor. As before, the fund manager submits a market order and each trader offers an ask price p_A and a bid price p_B .

To summarize the stage game at t is:

- a. Investor t observes χ_{t-1} . She retains the incumbent or she hires a challenger with prior γ .
- b. The fund manager observes s_t and he selects a_t .
- c. The valuation v_t is realized. With probability δ a good incumbent becomes bad. A bad incumbent stays bad.

We prove the existence of a churning equilibrium:

Proposition 8 (Infinite horizon) For t = 1, 2, ..., define the variables $\tilde{\gamma}$ and G recursively as follows:

$$\begin{array}{rcl} G_1 &=& \gamma;\\ \tilde{\gamma}_t &=& \max\left(G_t, \gamma\right);\\ G_{t+1} &=& \left\{ \begin{array}{cc} (1-\delta) \frac{2\tilde{\gamma}_t}{\tilde{\gamma}_t+1} & \text{if } \chi_t > 0\\ 0 & & \text{otherwise} \end{array} \right. \end{array}$$

¹⁷A career concern model in which the agent's type varies over time is discussed in Holmstrom [19].

The event that a good manager turns bad proxies for unmodeled changes inside the fund, e.g., the loss of a talented trader. It is easy to see that our assumption that a bad manager cannot become good again is made just to simplify analysis.

For α and ϵ low enough, there is a perfect Bayesian equilibrium in which:

1. At t, the belief of traders and investors about the current incumbent's type is:

$$\Pr\left(\theta_t = g | \{\chi_s\}_{s=1,\dots,t-1}\right) = G_t;$$

- 2. The investor at t retains the incumbent if and only if $G_t \geq \gamma$;
- 3. At t a good fund manager trades correctly $(a_t = v_t)$ and a bad fund manager churns $(a_t \in \{0, 1\}$ with equal probability);
- 4. Traders post prices

$$\hat{p}_{A,t} = \frac{1}{2} (1 + \tilde{\gamma}_t) \text{ and } \hat{p}_{B,t} = \frac{1}{2} (1 - \tilde{\gamma}_t).$$

The churning equilibrium of the infinite-horizon game has a simple structure. The fund manager in charge always trades. If he is good, he makes a correct trade; if he is bad, he churns. The investor evaluates the fund manager's quality from past performance. A fund manager who has just been hired has expected ability γ . A fund manager who has been hired last period and has performed a correct trade has expected ability

$$(1-\delta)\,\frac{2\gamma}{\gamma+1},$$

where the "good news" component $\frac{2\gamma}{\gamma+1}$ is discounted by the exogenous probability that the manager has turned bad. If the fund manager keeps trading correctly, the belief is updated in a similar fashion:

$$\tilde{\gamma}_{t+1} = (1-\delta) \frac{2\tilde{\gamma}_t}{\tilde{\gamma}+1}.$$

The belief is increasing in each successful trade but because $\delta > 0$ it does not converge to 1. One can show that it is bounded above by $\tilde{\gamma}_{\max} = 1 - 2\delta$. If, at some point, the fund manager makes a wrong trade (or no trade), the belief collapses to zero. The investor replaces the disgraced manager with an average manager, and the updating process restarts with $\tilde{\gamma}_t = \gamma$.

The uninformed traders have the same information as investors and they follow the same updating process. At time t, they know they face a fund

manager who knows the true liquidation value with probability $\tilde{\gamma}_t$. The bidask prices they quote reflect their belief: the higher $\tilde{\gamma}_t$, the wider the bid-ask spread. A fund manager with a long streak of successes will be confronted by cautious traders. Still, the existence of the upper bound $\tilde{\gamma}_{\text{max}} = 1 - 2\delta$ guarantees that prices are always bounded away from zero and one. If the transaction cost is low enough, trade occurs.¹⁸

3.4 Positive Net Returns for Investors

In all versions of our model, delegation to active fund managers leads to negative ex ante expected net return for investors. This is because there are no exogenous traders on the market who are willing to lose money. If the transaction cost ϵ and exogenous compensation parameters α and β tend to zero, the net return goes to zero as well.

It is not difficult to ensure positive returns from delegation by introducing a small amount of exogenous noise trade. It is more interesting (and algebraically simpler) to perform this exercise in the infinite-horizon model. Specifically, re-consider the model in the previous subsection but assume that in every period t a noise trader is present with probability $\eta > 0$ (independent across time and from other events). For unmodeled reasons, the noise trader must buy one unit of the asset with probability $\frac{1}{2}$ and must sell one unit with probability $\frac{1}{2}$ (See Glosten and Milgrom [15]).

As before, there are a large number of uninformed traders at every period t. Each of them can buy at most one unit and sell at most one unit. Each of them posts an ask price p_A and a bid price p_B . The noise trader and the fund manager look for the best prices. Trades are anonymous: a trader who receives an order does not know who made the order.¹⁹ The bid-ask prices are then equal to the expected liquidation value of the asset conditional on that trader receiving a buy order or a sell order. For the rest, the game is exactly as in the baseline case.

It is easy to check that Proposition 8 holds as stated, except that trading occurs at different prices. If the trader is faced with a buyer, he faces a good

¹⁸Like in the baseline model, the expected profit of an investor is still strictly negative at every t. As before, uninformed traders must make a zero expected payoff given $\tilde{\gamma}_t$. The fund's expected gross return is zero and the investor is left to pay the manager's fee and the transaction cost.

¹⁹We assume that the number of uninformed traders is large enough that the probability that the noise trader and the fund manager go to the same trader is negligible.

fund manager with probability $\frac{\tilde{\gamma}_t}{1+\eta}$, a bad manager with probability $\frac{1-\tilde{\gamma}_t}{1+\eta}$, and a noise trader with probability $\frac{\eta}{1+\eta}$. The ask price equals the expected liquidation value and can be written as:

$$\hat{p}'_{A,t} = \frac{\tilde{\gamma}_t}{1+\eta} + \frac{1}{2} \frac{1-\tilde{\gamma}_t + \eta}{1+\eta} = \frac{1}{2} \left(\frac{1+\tilde{\gamma}_t + \eta}{1+\eta} \right).$$

Similarly, the bid price is

$$\hat{p}_{B,t}' = \frac{1}{2} \left(\frac{1 - \tilde{\gamma}_t + \eta}{1 + \eta} \right).$$

It is easy to see that $\hat{p}'_{A,t} < \hat{p}_{A,t}$ and $\hat{p}'_{B,t} > \hat{p}_{B,t}$, and thus prices are (not surprisingly) less informative in the potential presence of a noise trader. In addition, the prices with noise trading converge smoothly to the original prices as the probability of noise trade vanishes.

Let us determine the investor's expected net return in a frictionless world:

Proposition 9 Fix $\eta > 0$. If the transaction cost ϵ and the explicit incentive parameters α and β tend to zero, the limit of the investor's expected net return in period t is

$$\frac{1}{2}\frac{\eta}{1+\eta}\tilde{\gamma}_t$$

As is well known, the presence of noise trade makes informed trade profitable. The expected net return in Proposition 9 is strictly positive for every probability of noise trade $\eta > 0$. As the amount of noise trade increases, the bid-ask prices move towards the unconditional value $\frac{1}{2}$: a good fund manager makes a larger profit and a bad fund manager makes a smaller loss.

The expected return is also increasing in the fund manager's perceived ability $\tilde{\gamma}_t$. The uninformed traders still adjust their bid-ask spread in proportion to $\tilde{\gamma}_t$, but the adjustment is now incomplete because some of the cost is borne out by noise traders.

Suppose that investors are fully rational: they are only willing to use a fund manager if the expected net return is positive. Proposistion 9 guarantees that, if transaction cost and exogenous compensation are small, any amount of noise trade is sufficient to guarantee positive returns to delegation in equilibrium. Thus, even a tiny amount of noise trade creates the necessary incentives for investors to delegate investment choices to portfolio managers. This, in turn, generates implicit incentives which lead to high volumes of trade via churning, thus vastly amplifying the initial amount of exogenous noise trade.

4 Conclusions

In this paper, we have studied the equilibrium features of a financial market in which a non-negligible share of the market participants are fund managers who face career concerns. These features differ markedly from the features of a standard market: prices are less informative and there is more trade. Uninformed fund manager engage in churning and in equilibrium they behave as if they were noise traders.

While we have examined several extensions of our model, many interesting avenues of research remain unexplored. First, it would be interesting to enlarge the set of assets and trades to make the analysis more comparable to standard financial models such as CAPM. It would also be important to introduce an element of risk-aversion for both investors and fund managers. Second, one could consider the presence of multiple fund managers. In particular, it would be important to study the general equilibrium effect of the conformistic drive identified by Scharfstein and Stein [32]. Third, our results were derived in a static trading game. Each security lasts for one period only and trade occurs at one specified instant. One should consider a richer setting in which both trading activity and career concerns display their effects over time. Gümbel [18] develops a multi-period model and shows that the presence of implicit incentives induces fund managers to take a suboptimal short-term perspective. Finally, as we discussed in the introduction, the present model may provide a solution to the trade volume puzzle. But then one should ask how the career concerns explanation fares compared to the over-confidence explanation (Kyle and Wang [24]). Our theory is falsifiable: if an increase in the share of institutional trading is, ceteris paribus, accompanied by a decrease in trading volume, we should reject the career concerns explanation. For the period 1955-1988 on the NYSE, Dow and Gorton [10] regress turnover on institutional ownership and real commissions, and find that the coefficient on institutional ownership is positive and significant.

5 Appendix: Omitted Proofs

Proof of Proposition 1:

For a fund manager with $\theta = b$, the expected value of the asset is $E[v] = \frac{1}{2}$. The bad fund manager never buys because

$$E\left[v\right] - \epsilon < \frac{1}{2} \le p_A.$$

The good manager is willing to buy if

$$p_A \leq v - \epsilon.$$

If the good fund manager buys, it means that he knows v = 1. But then the ask price should be $p_A = 1$, which is a contradiction. An analogous contradiction arises when we consider selling instead of buying.

Proof of Proposition 4:

Since there are no career concerns at t = 2, it is easy to see that optimal strategy at t = 2 are given by $a_2 = s_2$. Thus, investors have a clear incentive to retain fund managers for whom $\widehat{\gamma}(\chi_1) > \gamma$. Now consider t = 1 strategies. To begin, notice that in any equilibrium when α and ϵ are sufficiently small, if $\frac{r(1)+r(-1)}{2} > r(0)$ fund managers who do not receive information will churn. Suppose that $r(1) \ge r(0) \ge r(-1)$. Consider the payoffs to the informed (good) manager:

$$\pi (a_1 = s_1) = \beta + \alpha (p_B - \epsilon) + r(1)v_2$$

$$\pi (a_1 = 1 - s_1) = \beta + \alpha (p_B - 1 - \epsilon) + r(-1)v_2$$

$$\pi (a_1 = \emptyset) = \beta + r(0)v_2$$

where v_2 represents payoff from optimal behaviour at t = 2. Notice that $v_2 \ge \beta > 0$. For $\alpha > 0$ and ϵ sufficiently small, $r(1) \ge r(-1) \Rightarrow \pi(a_1 = s_1) > \pi(a_1 = 1 - s_1)$ and $r(1) \ge r(0) \Rightarrow \pi(a_1 = s_1) > \pi(a_1 = \emptyset)$. Thus, good managers will play $a_1 = s_1$. Given the small fringe of naive managers, the investor's beliefs upon seeing zero return are not arbitrary. They must (at least) be consistent with the equilibrium behaviour of good fund managers. Thus, upon observing $\chi_1 = 0$, the investor knows that the manager cannot be of type $\theta = g$, and thus optimally, r(0) = 0. Under the maintained hypothesis, this implies that r(-1) = 0. The two possible clases are r(1) =

r(0) = r(-1) = 0 (the manager is replaced for sure) and r(1) > r(0) = r(-1) = 0. But under any arbitrary mixed strategy chosen by the bad managers, it must be the case that $\widehat{\gamma}(\chi_1 > 0) > \gamma$. Thus, r(1) = 0 cannot be optimal for the investor. Thus, r(1) > 0, which implies that $\frac{r(1)+r(-1)}{2} > r(0)$, and thus non-naive managers of type $\theta = b$ will churn.

Proof of Proposition 5:

We use the following notation for unconditional probabilities:

$$\rho = \gamma \rho_g + (1 - \gamma) \rho_b = \Pr(s = v);$$

$$\tau = \gamma \tau_g + (1 - \gamma) \tau_b = \Pr(s = -v).$$

which implies

$$1 - \rho - \tau = \Pr\left(s = \emptyset\right).$$

Consider the first part of the proposition. Suppose that $\rho_g + \tau_g > \rho_b + \tau_b$. Now, under the (putative) equilibrium strategies, the principal's payoff in the second period is a linear and increasing function of the belief $\hat{\gamma}$. The agent is kept if and only if $\hat{\gamma} \ge \gamma$. Since everyone trades at t = 1, it is easy to see that $\hat{\gamma} (\chi_1 > 0) > \gamma$ and $\hat{\gamma} (\chi_1 < 0) < \gamma$, and thus r(1) = 1 and r(-1) = 0, reusing the notation introduced above. Consider $\hat{\gamma}(\chi_1 = 0)$. Since the only agents who do not trade at t = 1 are the naive managers who receive no information:

$$\hat{\gamma} \left(\chi_1 = 0 \right) = \Pr(\theta = g | \chi_1 = 0) = \frac{\gamma \Pr(\chi_1 = 0 | \theta = g)}{\Pr(\chi_1 = 0)} = \frac{\gamma \mu (1 - \rho_g - \tau_g)}{\mu (1 - \rho - \tau)} < \gamma$$

Thus, r(0) = 0. Therefore r(1) > r(0) = r(-1) = 0. It is then immediate that the managers' best response is as outlined above.

Now suppose that $\rho_g + \tau_g \leq \rho_b + \tau_b$. If the equilibrium strategies were as above, then as before, the agent is kept if and only if $\hat{\gamma} \geq \gamma$. Again, under the putative equilibrium strategies, $\hat{\gamma} (\chi_1 > 0) > \gamma$ and $\hat{\gamma} (\chi_1 < 0) < \gamma$, and thus r(1) = 1 and r(-1) = 0. However, when $\rho_g + \tau_g \leq \rho_b + \tau_b$, $\hat{\gamma} (\chi_1 = 0) \geq \gamma$, and thus r(0) = 1. Consider the payoffs to a trader who receives a period 1 signal $s = \emptyset$. Given the retention probabilities induced by equilibrium behaviour, trading without information leads to an expected payoff of $\beta + \frac{1}{2}\beta$, while not trading without information leads to an expected payoff of $\beta + \beta$. Clearly, then it is not optimal to churn, and the strategies outlined above cannot constitute an equilibrium. For the second part of the proposition, suppose $a_2 = s_2$. As in the first part, the agent is kept if and only if $\hat{\gamma} \ge \gamma$. Suppose $a_1 = s_1$. The principal's belief is

$$\hat{\gamma}\left(\chi_{1}\right) = \Pr\left(\theta = g | \chi_{1}\right) = \begin{cases} \gamma \frac{\rho_{g}}{\rho} & \text{if } \chi_{1} > 0\\ \gamma \frac{1 - \rho_{g} - \tau_{g}}{1 - \rho - \tau} & \text{if } \chi_{1} = 0\\ \gamma \frac{\tau_{g}}{\tau} & \text{if } \chi_{1} < 0 \end{cases}$$

If $\rho_g + \tau_g > \rho_b + \tau_b$,

$$\frac{1-\rho_g-\tau_g}{1-\rho-\tau} < 1,$$

and $\hat{\gamma}(\emptyset) < \gamma$: a fund manager who does not trade gets fired. Instead, by (2),

$$\frac{\rho_g}{\rho} > 1,$$

and $\hat{\gamma}(1) > \gamma$: a fund manager who trades correctly is retained. It is then clear that a fund manager who observes $s_1 = \emptyset$ prefers to play $a_1 = \{0, 1\}$ rather than $a_1 = \emptyset$.

Proof of Proposition 6:

Suppose there is an equilibrium with trading volume t > 0. In such equilibrium, the expected net return is $-t\epsilon$. The investor's expected payoff is bounded above by $-2\bar{w}-t\epsilon$. But the investor can always deviate to a different contract in which she offers a fix payment \bar{w} to the fund manager in each period, a positive amount δ if the investor does not trade, and she commits not to replace him. Then, the trading volume is zero and the investor's expected payoff is $-2\bar{w}-2\delta$. As δ can be as small as we wish, this deviation is profitable.

Proof of Proposition 7:

Fund manager's trading strategy at t = 2. The fund manager has no career concerns. If offered a flat payment, she is indifferent among trading or not trading. Hence, we can assume that a good manager trades successfully and a bad manager does not trade.

Investor 2's contract choice. As the fund manager is indifferent, the investor can obtain optimal behavior by offering a flat contract $b_2 = (\bar{w}, \bar{w}, \bar{w})$, which is clearly optimal.²⁰

²⁰This is the unique continuation equilibrium of the second-period subgame. To see

Investor 2's hiring choice. The return at t = 2 is $\frac{1}{2}(1 - \hat{\gamma}) - \epsilon$ if the manager is good and 0 if the manager is bad. Given a belief $\Pr(\theta = g|\chi_1)$ on the incumbent's type, the expected net return from retaining the incumbent is

$$\Pr\left(\theta = g|\chi_1\right) \left(\frac{1}{2}\left(1 - \hat{\gamma}\right) - \epsilon\right),$$

while the expected net return from hiring the challenger is

$$\gamma\left(\frac{1}{2}\left(1-\hat{\gamma}\right)-\epsilon\right).$$

If ϵ is low, it is a best response for the investor to retain the incumbent if and only if $\Pr(\theta = g|\chi_1) \geq \gamma$.

Investor 2's belief. The belief is the same as in the churning equilibrium of Proposition 2, namely:

$$\Pr\left(\theta = g | \chi_1\right) = \begin{cases} \frac{2\gamma}{\gamma+1} & \text{if } \chi_1 > 0\\ 0 & \text{otherwise} \end{cases}$$

Fund manager's behavior at t = 1. Given contract b_1 and the continuation equilibrium at t = 2, the fund manager's expected payoff is:

$$\max_{a_1} E(b_1|a_1, s_1) + \Pr(\Pr(\theta = g|\chi_1) \ge \gamma | a_1, s_1) \bar{w}.$$

For a good manager $(s_1 \in \{0, 1\})$, the expected payoffs are:

$$\begin{cases} b_1 (\text{success}) + \bar{w} & \text{if } a_1 = s_1 \\ b_1 (\text{failure}) & \text{if } a_1 = 1 - s_1 \\ b_1 (\text{no trade}) & \text{if } a_1 = \emptyset \end{cases}$$

For a bad manager, expected payoffs are

$$\begin{cases} \frac{b_1(\text{success})+b_1(\text{failure})}{2} + \frac{1}{2}\bar{w} & \text{if } a_1 = \{0,1\}\\ b_1 \text{ (no trade)} & \text{if } a_1 = \emptyset \end{cases}$$

The fund manager chooses a_1 in order to maximize the payoffs above.

this, suppose that the fund manager were using a suboptimal strategy. The investor could make him strictly prefer the optimal strategy by offering a contract $b_2 = (\overline{w} + \lambda, \overline{w}, \overline{w} + \delta)$, where $\lambda > \delta > \frac{1}{2}\lambda > 0$ are two positive but infinitesimal numbers. A good fund manager would trade correctly. A bad fund manager would not trade.

Investor 1's contract choice. If $b_1 = (\bar{w}, \bar{w}, \bar{w})$, the good manager chooses $a_1 = s_1$ and the bad manager chooses $a_1 = \{0, 1\}$. Clearly, it is not in the interest of investor 1 to encourage the manager to choose $a_1 = 1 - v_1$. The only other possibility is to induce the good manager and/or the bad manager to play $a_1 = \emptyset$.

The minimal amount that the investor must pay in order to make the good manager play $a_1 = \emptyset$ is \overline{w} . The minimum amount she must pay to make the bad manager play $a_1 = \emptyset$ is $\frac{1}{2}\overline{w}$. Thus, the lower bound to the additional expected payment needed to induce *any* fund manager to play $a_1 = \emptyset$ is

$$C_{\min} = (1 - \gamma) \frac{\bar{w}}{2}.$$

The highest net return investor 1 can hope for is when the good manager trades correctly and the bad manager does not trade. This is:

$$\gamma\left(\frac{1}{2}\left(1-\hat{\gamma}\right)-\epsilon\right)$$

The equilibrium expected net return is instead:

$$\gamma\left(\frac{1}{2}\left(1-\hat{\gamma}\right)-\epsilon\right)-\left(1-\gamma\right)\left(\frac{1}{2}\hat{\gamma}+\epsilon\right).$$

Thus, the upper bound to the additional net return that the investor can get from using *any* contract that is different from $(\overline{w}, \overline{w}, \overline{w})$ is:

$$R_{\max} = (1 - \gamma) \frac{1}{2} \left(\hat{\gamma} + 2\epsilon \right)$$

An upper bound to the net benefit of inducing the manager to change his action is:

$$R_{\max} - C_{\min} = (1 - \gamma) \frac{1}{2} (\hat{\gamma} + 2\epsilon) - (1 - \gamma) \frac{\bar{w}}{2} = (1 - \gamma) \frac{1}{2} (\hat{\gamma} + 2\epsilon - \bar{w}),$$

which is negative if

$$\bar{w} > \hat{\gamma} + 2\epsilon$$

which is satisfied for γ low enough and ϵ low enough. Then, investor 1 prefers to offer $b_1 = (\overline{w}, \overline{w}, \overline{w})$.

Proof of Proposition 8:

Let us begin with the investor at time t. Given her belief, the retention strategy in (2) is clearly a best response. Given the fund manager's behavior in (3), the investor's belief is consistent with equilibrium play. It is also easy to check that the traders' price is the conditional expected value of liquidation given fund managers' play.

To analyze the fund manager's behavior, we first need to establish that the ask-bid prices are bounded away from zero and one. Let

$$\tau(\gamma) = (1 - \delta) \frac{2\gamma}{\gamma + 1}.$$

The limit of the sequence $(\gamma, \tau(\gamma), \tau(\tau(\gamma)), ...)$ can be computed by setting

$$\gamma = (1 - \delta) \frac{2\gamma}{\gamma + 1}.$$

The solution is

$$\tilde{\gamma}_{\max} = 1 - 2\delta.$$

As $\tilde{\gamma}_t < \tilde{\gamma}_{\max} < 1$, if ϵ is small enough, a good fund manager has an explicit incentive to trade (correctly). As the reputational incentive goes in the same direction, a good fund manager has no profitable deviation from equilibrium play.

Let $U_b(\tilde{\gamma}_t)$ denote the expected continuation payoff (sum of the expected payoffs in t, t + 1, t + 2, ...) on the equilibrium path for a fund manager of type b if the current belief is $\tilde{\gamma}_t$. We have

$$\begin{split} U_b(\tilde{\gamma}_t) &= \beta - \left(\frac{1}{2}\tilde{\gamma}_t + \epsilon\right)\alpha + \frac{1}{2}\left(\beta - \left(\frac{1}{2}\tau\left(\tilde{\gamma}_t\right) + \epsilon\right)\alpha\right) + \dots \\ &> \beta - \left(\frac{1}{2}\tilde{\gamma}_{\max} + \epsilon\right)\alpha + \frac{1}{2}\left(\beta - \left(\frac{1}{2}\tilde{\gamma}_{\max} + \epsilon\right)\alpha\right) + \dots \\ &= 2\left(\beta - \left(\frac{1}{2}\tilde{\gamma}_{\max} + \epsilon\right)\alpha\right) \\ &= 2\left(\beta - \left(\frac{1-2\delta}{2} + \epsilon\right)\alpha\right). \end{split}$$

Then,

$$\lim_{\alpha \to 0} U_b\left(\tilde{\gamma}_t\right) > 2\beta$$

The only possible deviation for a bad fund manager is to make no trade rather than churning. Given the investor's strategy a non-trader is replaced. Under this deviation, the expected continuation payoff at t is simply β . If α and ϵ are small enough, the deviation is not profitable.

Proof of Proposition 9:

The expected return at t given $\tilde{\gamma}_t$ is

$$E(\chi_t) = \tilde{\gamma}_t \left(1 - \hat{p}'_{A,t} - \epsilon\right) + (1 - \tilde{\gamma}_t) \left(\frac{1}{2} - \hat{p}'_{A,t} - \epsilon\right)$$
$$= \frac{1}{2} \tilde{\gamma}_t + \frac{1}{2} - \hat{p}'_{A,t} - \epsilon;$$

Thus,

$$\begin{split} \lim_{\epsilon,\alpha,\beta\to 0} E\left(\chi_1 - t_1\right) &= \lim_{\epsilon\to 0} E\left(\chi_1\right) \\ &= \frac{1}{2}\tilde{\gamma}_t + \frac{1}{2} - \hat{p}'_{A,t} \\ &= \frac{1}{2}\tilde{\gamma}_t + \frac{1}{2} - \frac{1}{2}\left(\frac{1 + \tilde{\gamma}_t + \eta}{1 + \eta}\right) \\ &= \frac{1}{2}\frac{\eta}{1 + \eta}\tilde{\gamma}_t \end{split}$$

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