

# **ASSESSING FINANCIAL PRODUCT SOLUTIONS TO AMERICA'S RETIREMENT INCOME CHALLENGE:<sup>1,2</sup>**

*A closer look at annuity lifetime withdrawal guarantees and target date mutual funds*

By<sup>3</sup>

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## 1. EXECUTIVE SUMMARY

One of the main challenges facing current and future retirees is trying to ensure that sufficient funds are available to sustain them throughout their retirement years. The obstacles that stand between the investor and this objective have been exacerbated by the steady increase in longevity and the gradual deferral of pension investment risks from employers to employees. Future retirees often find themselves caught between Scylla and Charybdis: supporting sufficient future consumption streams essentially requires them to seek relatively aggressive investments in equities (or other high yield riskier assets); in doing so, they face severe downside risks which should be carefully controlled as retirement age approaches. Target Retirement Date Funds (TDFs) are among the most common instruments in the market which implement a risk mitigation strategy by limiting the investor's allocation in equities, shortly after reaching retirement age, to approximately 33%, and maintaining the rest in more conservative fixed income components. The investor's predicament stems from the fact that such conservative strategies, while limiting the volatility of the returns, increase the likelihood of the retirement fund running out (i.e., the investor outliving their nest egg), unless the monthly withdrawals from the account are suitably restricted. Curtailing withdrawals obviously has a negative impact on the investor's quality of life.

The purpose of this document is to evaluate a relatively recent alternative to traditional TDF instruments, specifically, a variable annuity (VA) that ensures the investor a certain income throughout the lifetime of the contract. In particular, the most salient feature of this product is that it *guarantees* the investor annual withdrawals up to 5% of the highest value his/her account has reached since its initiation<sup>4</sup>. This insurance mechanism essentially provides a safety net that enables the investor to maintain a more aggressive position in equities, and hence enjoy the opportunity for a more significant yield on his/her investment, while concomitantly limiting the downside risk. The objective of the new instrument is therefore to limit the downside risk without having to resort to the

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<sup>4</sup> The highest achieved account balance is adjusted, approximately on an annual basis; the precise rules governing the adjustment epochs are explained in the next section.

typical conservative investment strategies adopted by, for example, traditional TDFs, with the above mentioned drawbacks of limiting the growth potential and hence the income potential resulting from the account.

In this paper, we evaluate the relative merits of a hypothetical, low cost variable annuity product<sup>5</sup> with the lifetime income guarantee -- which we will refer to as LGWA (Lifetime Guaranteed Withdrawal Annuity) -- and compare it with traditional TDFs.<sup>6</sup> For both types of instruments, we assume that, under continuous rebalancing, a given percentage of the account is invested in equities, with the remaining percentage allocated to a particular mix of fixed income components comprised of corporate and treasury bonds. Our results are based on an extensive Monte Carlo simulation study which examines the performance of the two types of instruments. The study involves both commonly used stochastic models whose parameters are estimated from the equity and fixed income returns over 3 (partially overlapping) 35 year intervals in the period 1926-2006, as well as historical bootstrapping procedures which do not rely on any assumptions with regard to the dynamics of the underlying equity and fixed income processes. As will be reported later in this document, we have evaluated the performance of each instrument (the LGWA and several TDFs) using a variety of measures that characterize the return and risk profile the investor faces with each fund type. A fairly extensive sensitivity analysis was carried out as well to ensure that the results we report on are robust to relatively large perturbations in the values of the parameters inferred from historical data.

To illustrate the key insights that arise from our study, we focus next on the performance of a particular variant of the hypothetical LGWA product that has an allocation of 80% to equities, (referred to as the L80 product), in comparison with the average load TDF (referred to as TM33 since it allocates 33% to equities). The rules governing withdrawals

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<sup>5</sup> The asset-based fees (base insurance and fund) for the hypothetical VA discussed herein total 1.15%, while the average fees for an advisor sold VA would typically be in excess of 2.0%. The relative performance of a hypothetical VA product with lifetime income guarantees and a typical TDF vary significantly depending on the asset-based fees for each.

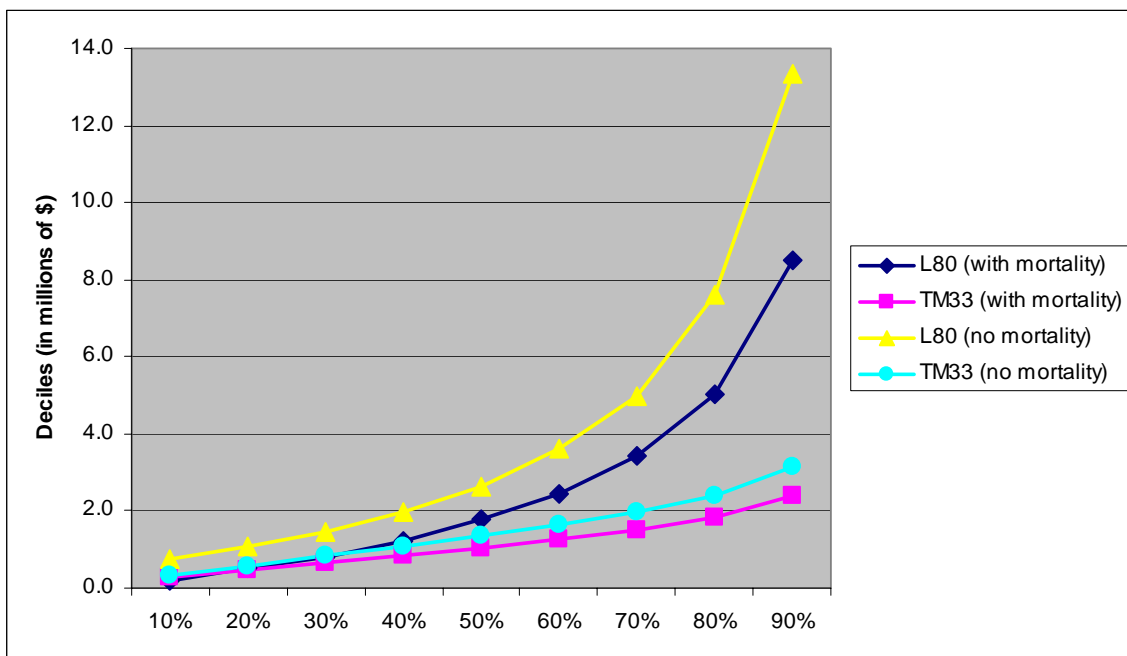
<sup>6</sup> TDF instruments and variable annuities may serve purposes other than providing retirement income and wealth, ( such as funding catastrophic health care events or long term care. An analysis of such alternative uses is beyond the scope of this paper.

in the case of the LGWA product have been described above, and we assume the same apply to the TDF, with the important caveat that at the time of withdrawal the amount of money in the TDF *may not allow* the investor to receive the annual equivalent of 5% of the running maximum value of the account (a simple consequence of the fact that the account may run out and hence the withdrawal rate is not guaranteed).

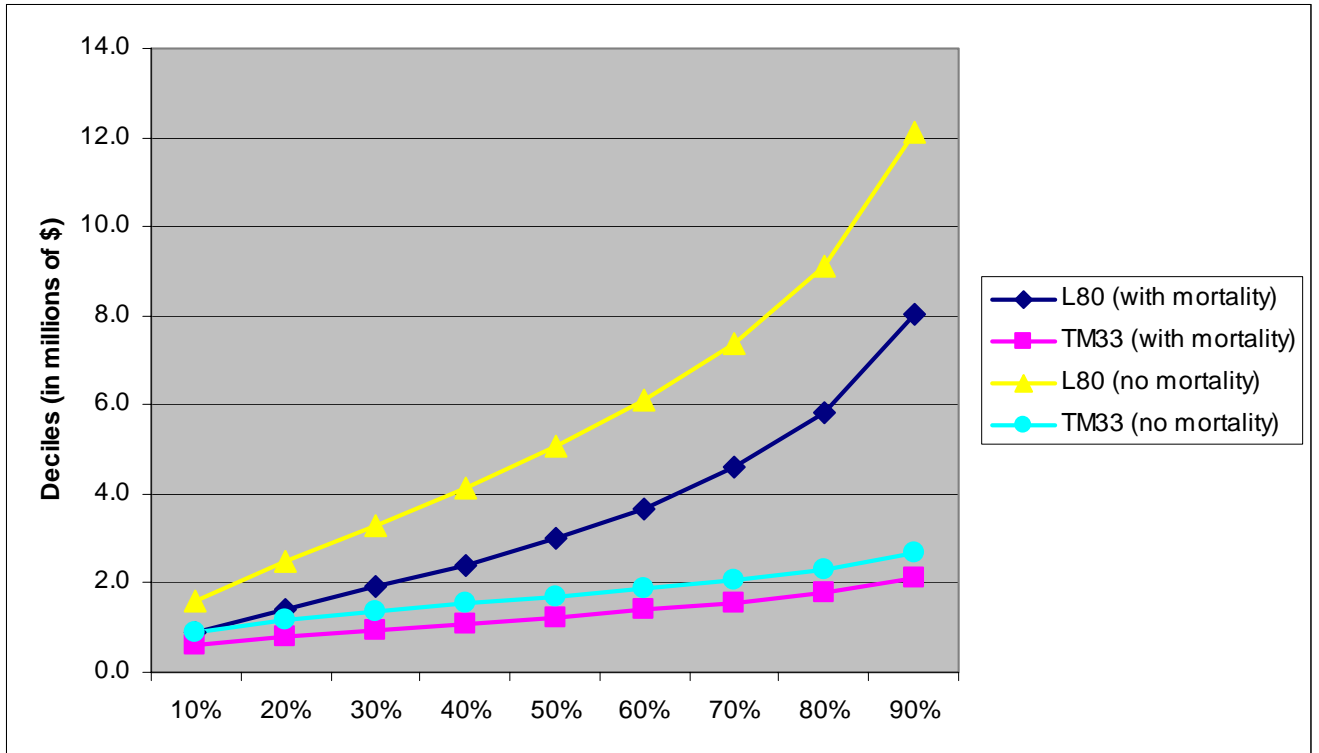
We observe, on a consistent basis, that the L80 product offers a significantly higher rate of return and a more substantive withdrawal stream compared to the TM33 product. Yet, the L80 alternative is able to mitigate downside risk in a manner that is on par with the conservative TM33 product. In particular, the L80 product provides an increased mean internal rate of return (IRR), approximately 50 to 330 basis points higher than that characterizing the TM33 annuity. The average monthly income generated by the L80 product is on average anywhere from 17%-90% higher than that corresponding to the TM33 product. Another measure of importance is the total cash flows, i.e., the sum total of the withdrawals plus the terminal value of the account at the end of the contract term, minus the initial investment. The median of this total cash flow value is always larger (up to 142%) under L80, as compared to the traditional TM33 fund. Similarly, the ninetieth percentile of the total cash flow under L80 exceeds that of the TM33 instrument by anywhere between 60% and 275%. The absolute magnitude of these values depend, as one would expect, on the particulars of the economic environment in which one is testing performance; in our case this is dictated by the particular 35 year interval chosen within the 1926-2006 time frame, as well as the specifics of the stochastic models describing the returns on the equity and fixed income markets. One should add that these margins were obtained assuming actuarial lifetime distributions for the investor and spouse, and turn out to be even significantly higher if one assumes the investor survives the entire 35 year horizon, and hence faces the challenge of managing his/her retirement income over the full 35 years.

Figures 1-4 display the deciles of the total cash flow measure for the two instruments, both when assuming that the lifetimes of the investor and the spouse follow the actuarial probability distributions, and when the investor is assumed to outlive the full 35 years of

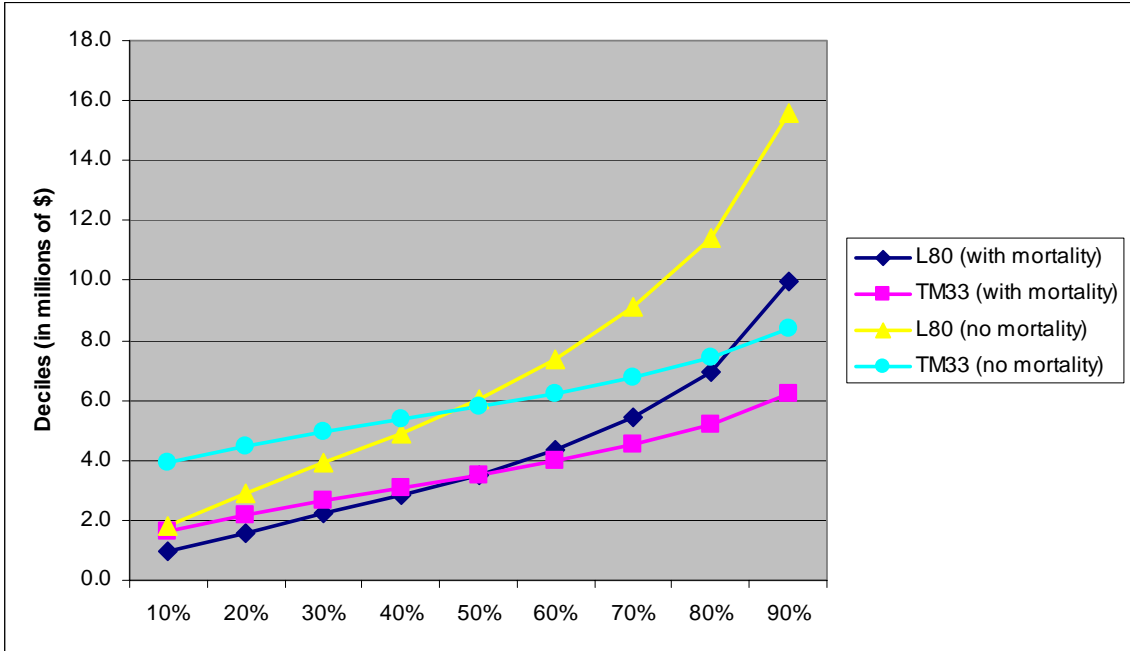
the contract term. The curves reflect the *basic* stochastic model describing the monthly returns of the equity and fixed income pools in which the account is invested, as explained in Section 3 (see Approach 1 there). Figure 1( 2,3) corresponds with the case where the parameters of this stochastic returns model have been estimated to match the historical data pertaining to the 35 year interval from January 1926-December 1960 (August 1948-July 1983, June 1971-May 2006). Finally, Figure 4 provides the same comparisons, when the parameters of the basic stochastic model are chosen to match the historical data over the full 80 year horizon from 1926-2006.



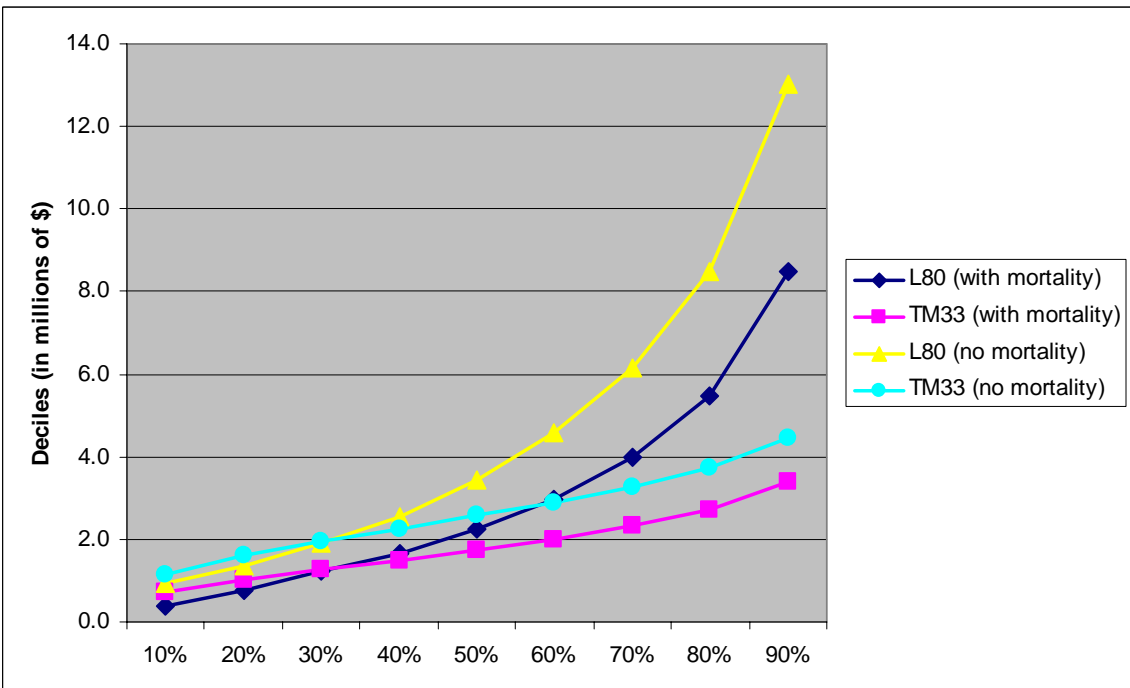
**Figure 1: Total NET CASH FLOW for the calibration period of Jan-26 -- Dec-60. The y-axis represents the deciles (in millions of dollars) of the total net cash flow distribution; for each point on the curve, the likelihood of the total net cash flow being at or below the y-coordinate is given by the value of the x-coordinate.**



**Figure 2: Total NET CASH FLOW for the calibration period of Aug-48 -- Jul-83.**  
**The y-axis represents the deciles (in millions of dollars) of the total net cash flow distribution; for each point on the curve, the likelihood of the total net cash flow being at or below the y-coordinate is given by the value of the x-coordinate.**



**Figure 3: Total NET CASH FLOW for the calibration period of Jun-71 -- May-06. The y-axis represents the deciles (in millions of dollars) of the total net cash flow distribution; for each point on the curve, the likelihood of the total net cash flow being at or below the y-coordinate is given by the value of the x-coordinate.**





**Figure 4: Total NET CASH FLOW for the calibration period of Jan-26 -- May-06. The y-axis represents the deciles (in millions of dollars) of the total net cash flow distribution; for each point on the curve, the likelihood of the total net cash flow being at or below the y-coordinate is given by the value of the x-coordinate.**

In two of the three market environments, the L80 product shows a compelling and unequivocal advantage over the TM33 product, while in the third the median and mean results for the two products are very nearly equal. (The third 35 year time interval exhibited an unusually high average fixed income return rate of 9% per annum.) When the parameters of the stochastic model for equity and fixed income returns are chosen to match the full historical experience over the past 80 years (Figure 4), the L80 product dominates TM33 for all but the first and second deciles; thus, only in the 20% worst possible market scenarios, does the total cash flow obtained by the investor under the traditional target date fund exceed that received under L80, and this by a mere \$300,000 approximately. (The median value under L80 is, approximately, \$500,000 or \$900,000 *larger*, depending upon whether the investor's lifetime follows the actuarial probability distribution, or exceeds the 35 year contract horizon; the 90-th percentile for L80 exceeds that of TM33 by approximately 4.5 and 8 million \$ respectively.)

In terms of downside risk, measured by the probability of the account running out of money and the mean and standard deviation of the time at which it runs out of money, the L80 product provides (even with its greater allocation to equities) an almost identical profile to that of the TM33 target date fund. The key to achieving this is of course the lifetime income guarantees provided in the L80 product. Indeed we observe that even under extremely favorable market conditions this insurance guarantee is activated with a likelihood that ranges from 3% - 8%. The probability that the insurance guarantee is activated can be as high as 33% under less favorable conditions observed in one of the three tranches of our historical data. (If one considers a fixed 35-year horizon as opposed to actuarial lifetime distributions, this probability increases even further.)

Viewed through the lens of the aforementioned summary statistics, and as will be shown in far greater detail in section 5, it is possible to conclude with a high level of confidence that the hypothetical, LGWA-type product allows the investor to benefit from a significantly higher mean IRR and increased withdrawal streams relative to traditional Target Date Funds, yet by virtue of the lifetime income guarantee provisions, this form of variable annuity is able to mitigate the downside risks to almost identical levels as those found in the traditional conservative funds of the TM33 variety. Given the range of scenarios, models, and sensitivity analyses reported in this paper, we believe that the above statements paint an accurate and representative picture.

## **2. PROBLEM DESCRIPTION**

Twenty-five years ago, more than 60% of employed Americans enjoyed defined benefit retirement plans, where the employer provides a fixed monthly payment throughout the employee's retirement. Today, more and more companies have eliminated traditional pension plans and have shifted the investment risk to their employees, often foregoing retirement benefits altogether. As a consequence, a rapidly declining minority of future retirees continue to enjoy these defined benefit plans.

Many studies<sup>7</sup> have concluded that most households save too little and, in addition, invest too conservatively. In 2005, the national savings rate dipped into negative territory. Analyses of 401(k) asset allocations show that Americans are placing between 55% and 67% of their account into lower-return instruments, such as bond and money market funds, as opposed to equities. Almost all experts agree that this is overly conservative. The Securities and Exchange Commission phrased this as follows:” The public has a ‘play it safe’ approach to investment. People seem so concerned with avoiding investment disasters that they make do with overly conservative investments. Much of the public is intimidated by the stock market and frightened of its volatility.”<sup>8</sup>

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<sup>7</sup> These studies have been conducted by private consultants like Hewitt Associates, government arms like The SEC and think tanks like the Employee Benefit Research Institute.

<sup>8</sup> See, for example, D. Mastio, “Lessons Our 401(k)s Taught Us”, Policy Review 95 (1999), Heritage Foundation.

Many face the possibility of depleting their savings or having too little to live on, in their retirement years. This problem is particularly prevalent among retirees and is compounded by the increasing longevity of the population. The tendency to invest too conservatively is based on most investors' averseness to face the risks of equity markets for a large percentage of their investment portfolio, in particular as they face the retirement stage of their life. Indeed, most financial advisors recommend that at the age of 65, investors should limit their holdings in equity funds or individual stocks to approximately 33%<sup>9</sup>, with further downward adjustments as the investor advances in age. Recently, so-called Target Retirement Date Funds (TDFs) have emerged as a popular vehicle to implement this strategy by automatically shifting assets over time to more conservative allocations.

While these conservative investment strategies protect the investor against the volatility risks associated with equity markets, they provide little opportunity for pre-consumption growth and, hence, have increased potential for the savings to be depleted, prematurely. Alternatively, under traditional investment strategies, the investor's consumption potential is drastically reduced. Thus, under traditional TDFs, the investor faces a significant risk of outliving his or her assets, precisely because the conservative allocation strategy prevents the funds from growing at a rate approximating the expected rate associated with equity markets. For those investors with somewhat larger savings balances at retirement age, the traditional investment strategies deprive their heirs of the normal growth potential, associated with a "typical" early or mid-career diversification rule.<sup>10</sup>

Variable annuity contracts provide one or several performance guarantees. Typically, the guarantees have been of one of the following two types or a combination thereof:

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<sup>9</sup> See, for example, Table 1 in J. Poterba, D. Wise, J. Rauh and S. Venti. "Lifecycle Asset Allocation Strategies and the Distribution of 401(k) Retirement Wealth", (2006), National Bureau of Economic Research (NBER) Working Paper W 11974.

<sup>10</sup> A simple average of current target asset allocations of three major providers of life cycle funds (Fidelity, Vanguard and T. Rowe Price) exhibits a gradual decline of the allocation percentage to equity funds from 91.5% at age 25 to 27.7% at age 70.

- (a) A Guaranteed Minimum Withdrawal Benefit (GMWB): the investor is given a guaranteed return on his investment, with some potential for higher returns based on market performance of the underlying funds..
- (b) A Guaranteed Minimum Death Benefit (GMDB): here, the investor's beneficiaries are guaranteed to receive a specific minimum sum upon the death of the investor during the contract term.

The United States market for variable annuities has rapidly grown to the level of approximately \$130 billion in annual sales of newly initiated contracts. While the traditional guarantees succeed in limiting the downside risk associated with the *terminal* value of the fund at the end of the contract term or at the time of death, they do not (directly) address the investor's concern of providing sufficient funds to support ongoing expenditures *during his or her life time and that of the surviving spouse (if applicable)*.

A Lifetime Guaranteed Withdrawal Annuity (LGWA) is able to ensure the retiring investor a sufficient guaranteed income stream during his or her lifetime, as well as that of the surviving spouse, while permitting him to participate aggressively in the growth potential associated with equities. Under this vehicle, the investor is, during his or her entire lifetime, guaranteed to be able to withdraw in any given year an amount equal to 5%<sup>11</sup> of the *highest* anniversary value his account has achieved<sup>12</sup>.

This Lifetime Income Guarantee is available even if the investor chooses to (continuously) allocate as much as 80% of the variable annuity's value to equities. Indeed, the new Lifetime Income Guarantee is designed to permit the investor to participate aggressively in equity markets and hence to benefit from its growth potential, while ensuring him or her a minimum income stream for the complete duration of his life. By adjusting the guaranteed annual withdrawal amount (GAWA) upward, as a percentage of the highest previously observed account value (at specific potential adjustment or "step up" dates, see Footnote 3), the GAWA is expected to grow at a rate given by a weighted

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<sup>11</sup> This corresponds with a *monthly* withdrawal amount of 1/12 of 5%.

<sup>12</sup> This highest value is reassessed, periodically, in accordance with the prevailing account balance. During the first 10 years of the contract, the reassessment, or so-called *step up*-dates, coincide with the anniversary dates of the fund. Thereafter, the investor has the ability to choose the reassessment dates to his or her benefit, subject to the provision that at least one year must elapse between two consecutive "step ups".

average of the growth rates in the equity and fixed income pools<sup>13</sup>. It is therefore likely to grow at a rate well in excess of the inflation rate: *real* long term interest rates<sup>14</sup> have averaged 1.5% since 1880 and have never been negative over the past 35 years<sup>15</sup>; equity markets consistently exhibit a sizable *equity premium*, defined as the difference between its average growth rate and the interest rate of long term fixed income instruments.<sup>16</sup> The expected growth rate of equity funds therefore exceeds the rate of inflation by the sum total of the equity premium and the expected real interest rate.

Finally, since the GAWA is never reduced, even when the account balance or the underlying equity or fixed income markets decline, the investor never needs to scale down his or her consumption level, at least in nominal terms.

The above annual withdrawal guarantee is in place for as long as the investor is alive. Thereafter, the surviving spouse, if applicable, continues to benefit from a restricted version of the withdrawal guarantee: the potential monthly withdrawal amount continues to be determined in the same way it is during the lifetime of the primary investor, but the total withdrawal amount is limited to the so-called *Benefit Base*. The latter is initiated at the level of the initial account balance and is reduced, each month, by the prevailing withdrawal amount; the balance is adjusted to the level of the prevailing account balance, at the above mentioned “step up dates”, see Footnote 3, but only if this adjustment results in an increase of the Benefit Base. While the surviving spouse continues to have the opportunity for “step ups” to the Benefit Base, he or she does not receive a new “lifetime guarantee” of payments. Payments continue only until both the Benefit Base and the Account Value are reduced to zero.

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<sup>13</sup> If, for example, the investor allocates 80% of his account to the equity fund, the annual withdrawal amount can be expected to grow at a rate given by  $0.8 * \text{the expected growth rate in the equity fund} + 0.2 * \text{the expected growth rate in the fixed income fund}$ .

<sup>14</sup> The real interest rate is defined as the difference between the nominal interest rate and the rate of inflation.

<sup>15</sup> See, e.g., F. Breedon, B. Henry and G. Williams, *Long Term Real Interest Rates: Evidence on the Global Market*, Oxford Review of Economic Policy 15(1999)

<sup>16</sup> Mehra, R. and E. Prescott, “The Equity Premium in Retrospect”, NBER Working Paper W9525( 2003) estimate the average size of the equity premium for US equity markets, over the past 110 years, at no less than 6.9%, annually. Prescott, the 2004 Nobel Prize Winner in Economics and his co-author had identified this phenomenon in their much cited and analyzed 1985 paper “The Equity Premium: A Puzzle”, Journal of Monetary Economics.

The objective of this paper is to assess the *relative* performance of a hypothetical, low cost variable annuity with a Lifetime Income Guarantee vis-à-vis that of traditional Target Date funds. Our assessments are based on extensive Monte Carlo simulations of the alternative product trajectories under a variety of plausible stochastic models for the monthly co-movements of the returns of the underlying equity and fixed income pools over a 35 year horizon. To do so, we have used four types of stochastic models, frequently employed to describe the monthly co-movements of equity-and fixed income returns. Within each type of model we have estimated parameters to match 3 different 35 year intervals in the period 1926-2006, while performing sensitivity analyses with respect to the main parameters in each model. The different stochastic models- twenty in total- are described in Section 3. All simulations are evaluated along a variety of measures to characterize the return and risk profiles the investor faces under different investment vehicles. These are described in Section 4.

When evaluating the traditional Target Date funds, we assume either that, *for as long as there is sufficient money in the account,*

- (i) the investor withdraws the same amount from these funds, as would be the case if the money were invested in the LGWA, or
- (ii) the investor withdraws each month, 1/12 of 5 % of the periodically determined highest value the account balance in the Target Date Fund has reached up until the current month.<sup>17</sup>

Under the first withdrawal rule (i), the investor's cash flows remain identical under the LGWA and the Target Date Fund, for as long as there is money in the *latter's* account. This allows for a meaningful comparison of terminal account balances, internal rates of return and run-out times and probabilities, while holding the investor's consumption pattern constant, for as long as possible. The second rule is more practically plausible, and, hence, allows for a comparison with a traditional investment strategy.

In particular since the new annuity is designed for retirees or those approaching retirement age, we have modeled the investor's and the spouse's mortality in accordance

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<sup>17</sup> In this context, potential step-ups of the withdrawal amounts are assessed at the anniversary dates of the fund

with actuarial survival rates. All simulations are carried out by generating a pair of remaining life times for the investor and spouse, in accordance with these survival probabilities.<sup>18</sup> However, in parallel each set of simulation scenarios has been evaluated under the assumption that the investor continues to live for the full 35 years, i.e. until the end of the contract term. The first version provides a statistically realistic representation of the return and risk profile which a typical investor faces. The second set of simulations exhibits, however, how the investor would fare under the different investment vehicles if he faced what, in terms of the onus to finance retirement income, may be viewed as a *worst case* scenario.

Much of our discussion in Section 5 is devoted to a comparison of

- (a) the LGWA with an 80% allocation of the account balance to the equities, and
- (b) the Target Date Fund with a 33% participation in the equities' pool, both under withdrawal strategies (i) and (ii), above.

We consider these comparisons particularly relevant, since, as discussed above, most financial analysts would, in the absence of income guarantees such as those provided by the LGWA, advise their clients at (or after) retirement age to limit their exposure to equity markets to approximately the 33% level. However, we have systematically evaluated this low cost LGWA and the Target Date Fund with the above two withdrawal strategies under 4 alternative portfolio allocations, as follows:

- (1) 33% equity and 67% fixed income;
- (2) 40% equity and 60% fixed income;
- (3) 60% equity and 40% fixed income;
- (4) 80% equity and 20% fixed income.

This allows us to assess, for example, the tradeoffs among alternative portfolio allocations for the LGWA compared to the performance of the Target Date funds<sup>19</sup> (for either of the two withdrawal strategies) under *identical* portfolio allocation rules. Since we consider a total of 12 investment vehicles, we use the following legend to distinguish

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<sup>18</sup> For this purpose, we have employed the most recent (2002) actuarial tables published by the Social Security Office, see <http://www.ssa.gov/OACT/STATS/table4c6.html>.

<sup>19</sup> For example, L80 represents the LGWA in which 80% of the account is invested in equities; TM33 denotes the Target date fund, with 33% of the money allocated to equities and with the modified withdrawal rule (ii).

between them: each investment vehicle is designated by the letter L (for the low cost variable annuity with the LGWA provisions), T (for a Target Date Fund, under the first withdrawal rule (i)), and TM (for a Target Date Fund under the second or modified withdrawal rule (ii)), followed by a double digit representing the percentage of the account balance that is invested in the equity pool.

The following additional assumptions underlie the performance evaluations:

- For the traditional funds, an annual management fee of 1.43% of the prevailing account balance is assumed<sup>20</sup>; for the Lifetime Guaranteed Withdrawal Annuity (LGWA), the combined annual asset based fee is set at 1.15%, plus a LGWA charge assessed against the Benefit Base, which is set at 0.90 %, per annum.<sup>21,22</sup>
- We assume that 80% of the equity portion of the account is allocated to the S&P 500 and 20% to Small Caps; the fixed income portion is spread equally between Long Term Corporate Bonds and Long Term Government Bonds.<sup>23</sup>
- The investor has the option to withdraw less than the guaranteed levels; however, in our simulations, we assume that the investor (and spouse) take full advantage of the withdrawal limits.
- As mentioned in Footnote 4, after the first 10 years, the step-up dates, at which the Benefit Base and Withdrawal Guarantee are re-determined, may be selected by the investor, with the proviso that at least 12 months must elapse between consecutive step-ups. In our simulations, we assume, that after the first 10 years, the investor always elects the *first* permitted step-up month, which results in an upward adjustment of the benefit base.

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<sup>20</sup> We assume that no front end expense load is assessed; these are typically waived if the initial investment is at least \$1 million.

<sup>21</sup> The combined asset based management fee consists of three components: (a) a Mortality and Expense Fee of 0.15%; (b) administrative expenses of 0.15%, and (c) Fund expenses of 0.85%. These fees are, in reality, to be charged, daily, in proportion to the prevailing account balance. In our simulations,, we assess these fees, monthly. The special LGWA charge is, in reality, to be charged, quarterly; in our simulations, the account is charged monthly.

<sup>22</sup> Variable Annuity issuers often reserve the right to increase fees on the LGWA after a specified period of time. We have, however, assumed that the fee rates remain constant.

<sup>23</sup> All returns data were obtained from Ibbotson.



- All simulations assume that the primary contract holder is male and married; he and his wife are assumed to be 65 and 62 years old, respectively.

Each set of simulations, for a given combination of an investment vehicle, stochastic model of equity and fixed income allocations, and a given assumption about the investor's (and the spouse's) remaining life time (actuarially based, or in excess of the full 35 year contract limit) is carried out 10,000 times to generate high precision estimators, see Section 4<sup>24</sup>. Since there are 480 sets of simulations, a grand total of 4,800,000 35-year trajectories (of 420 months each) have been evaluated.

### 3. MODEL DESCRIPTION

In this paper we take three different approaches to modeling the behavior of the equity and fixed income components of both the Target Date funds and the hypothetical Lifetime Guaranteed Withdrawal Annuity. The first two approaches use simple parametric models to capture the behavior of the fixed income and equity components of the various products; below we explain the details of both these models, noting that the simpler “basic” model is nested within the second one. The third approach is “model free” insofar as it does not assume any specific structure and is only driven by the historical data itself by means of a bootstrapping methodology.

**Approach 1 (the basic model):** Here we model the returns of the fixed income component as *constant* and model the behavior of the equity component using a standard log-normal hypothesis. That is, for the fixed income portfolio we assume that

$$R_t = r \quad \text{for all } t = 1, 2, \dots$$

where  $R_t$  denotes the rate of return during month  $t$ , and  $r$  is a constant.

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<sup>24</sup> For example, with a sample of 10000 instances, the standard error of the estimate of the internal rate of return IRR, see Section 4, is invariably less than 0.05%, resulting in a margin of error of 0.1%. This implies, for example, that if the estimate of the mean IRR is 6%, say, the 95% confidence interval for the mean is contained within the interval [5.9%, 6.1%].

The log-normal hypothesis is derived from a Geometric Brownian motion (GBM) model, by far the most common approach to model the dynamics of equity market movements. In particular, if we let  $S_t$  denote the value of the equity portfolio at time  $t$ , then the log-normal assumption postulates that

$$\log(S_t/S_{t-1}) \sim N(\mu, \sigma^2) \quad \text{for all } t = 1, 2, \dots$$

That is, the price relatives follow a normal distribution with mean  $\mu$  and variance  $\sigma^2$ . Consistent with the common random walk tenet with regard to the dynamics of equity markets, the price relatives are assumed to be independent and identically distributed with the above normal distribution. (This implies that  $S_t$  follows a log-normal distribution, or GBM in continuous time, with the above parameters.) For each tranche of historical data, we estimate the constant return of the fixed income component and the two parameters, drift rate  $\mu$  and volatility  $\sigma$ , that characterize the log-normal distribution. These parameter estimates are then used for the subsequent simulation studies that evaluate the performance of the various investment contracts (12 in total). Starting with the base values for the parameters, we have carried out various sensitivity analyses, where one or several parameters are changed, so as to assess the robustness of our results when deviating from historically observed patterns.

**Approach 2 (the refined model):** Here we model the returns of the fixed income component using a more elaborate stochastic model commonly known as an ARMA/GARCH process.<sup>25</sup> This model is rich enough to capture two key characteristics which are commonly observed in the behavior of interest rates, namely, mean reversion and heteroscedasticity in the volatility. In particular, our model postulates that

$$R_t = r + \rho R_{t-1} + \varepsilon_t \quad \text{for all } t = 1, 2, \dots$$

where  $\varepsilon_t$  follows a normal distribution with mean 0 and conditional variance  $\sigma_t^2$  satisfying:

$$\sigma_t^2 = \kappa + \alpha \sigma_{t-1}^2 + \beta \varepsilon_{t-1}^2 \quad \text{for all } t = 1, 2, \dots$$

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<sup>25</sup>GARCH stands for Generalized AutoRegressive Conditional Heteroscedasticity. This class of models, that were first introduced in a simpler form by Engle (1982), is by far the most common tool for modeling short term behavior of interest rates in financial markets.

These dynamics encode within them the two qualitative features described above. First, note that  $R_t$  has a long term mean which is equal to  $r/(1-\rho)$  and it reverts to this mean at rate  $\rho$ . Second, observe that the volatility at time  $t$  has mean zero but its conditional variance,  $\sigma_t^2$ , evolves over time in a manner which is dictated by the past conditional volatility as well as current realized volatility. In this manner the eventual realized volatility in the interest rate process  $R_t$  is non-homogenous and exhibits features which are more representative of those observed in empirical data from fixed income markets. With regard to the specification of the above model, we impose the following constraints:  $|\rho| < 1$ ,  $\alpha + \beta < 1$ , and  $\alpha, \beta, \kappa \geq 0$ . These ensure that the process is well defined mathematically and exhibits a long term steady-state behavior. For the equity component we continue to use the standard log-normal model described earlier. Again, for each tranche of historical data, we estimate the parameters of the above ARMA/GARCH process  $(r, \rho, \kappa, \alpha, \beta)$  and the two parameters that characterize the log-normal distribution  $(\mu, \sigma^2)$ . As with Approach 1, we have carried out various sensitivity analyses, in which one or several of the base parameter values are varied.

**Approach 3 (historical bootstrapping):** Here we depart from the world of parametric models and propose an alternative that is free of such assumptions. Our approach is based on the concept of bootstrapping<sup>26</sup> which works as follows. For any given tranche of historical data we first partition it into non-overlapping blocks of equal length  $b$ . If  $T$  is the number of months in a tranche, then the number of blocks is  $N = T/b$  (where  $b$  is chosen so that there is no remainder term). We then take  $N$  random draws with replacement from the set  $\{1, 2, \dots, N\}$  (note that each such integer may appear more than once in the randomly drawn set). Each number in the generated index set points to the appropriate block position in the original data. For example,  $n \in \{1, \dots, N\}$  indexes the block in the data tranche that starts at time  $t = (n-1)b + 1$  and ends at  $t = nb$ . We then assemble a so called bootstrap sample by “patching together”  $N$  blocks from the original tranche of data according to the content and order of the index set. The resulting

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<sup>26</sup>Efron, B., and Tibshirani, R. An Introduction to the Bootstrap. Chapman and Hall, 1993.

bootstrap sample has length  $T$  and is comprised of blocks of the original data that have been essentially shuffled out of order and may contain repetitions of any given block (and hence omissions of other blocks).

An important point in the above construction is that we use the same index set to create both the fixed-income bootstrap sample as well as the equity bootstrap sample. In this manner we preserve the historical co-movement (dependence) structure between the two components, while at the same time maintaining the intra-block temporal dependence structure in each time series. The shuffling of blocks introduces sufficient independence that enables the bootstrap theory to be invoked. (Roughly speaking, this theory asserts that by repeating the bootstrap process sufficiently many times one is able to suitably approximate the distribution that governs the original data.) In our study we simulate 10,000 bootstrap samples based on which performance measures are calculated. This provides an adequate level of accuracy for our purposes and is consistent with the 10,000 simulations used in other cases (as explained in detail in section 2). We use two block sizes: 1 month and 12 months. The first choice allows for a larger bootstrap sample at the expense of potentially eliminating all temporal structure; the second choice gives rise to smaller bootstrap samples which preserve the intra-block dependence structure seen in the historical data. (We have tried a variety of other values as well but the results do not seem to be very sensitive to the choice of this tuning parameter.)

In view of the three modeling approaches, the three 35 year tranches within the period January 1926-June 2006, and the various sensitivity analyses carried out within each modeling approach, we report simulation results for a total of twenty stochastic models for the combined equity- and fixed income returns.

#### **4. PERFORMANCE MEASURES**

The cash flow stream for the investor consists of the initial investment, a stream of monthly (though possibly terminating) withdrawals and a final account value at the end

of the contract term. The latter occurs after 35 years or upon the decease of the last surviving spouse and is therefore random itself (at least in the simulation sets in which the investor's and the spouse's lifetime are randomly generated in accordance with the actuarial survival probabilities).

When evaluating a given investment contract under a given combined stochastic model for (a) the monthly co-movements of the returns on the equity-and fixed income pools, as well as (b) the life time distributions of the investor and spouse, we monitor and characterize each of the following performance measures:

- TOTAL=the sum total of all withdrawals, plus the terminal account balance, minus the initial investment:
- IRR= the internal rate of return of the cash flow stream over the course of the contract horizon;
- INCOMERATIO= the ratio of the total income received under the considered Instrument and that received under the TM33 contract.
- PROBGAR= the probability that, at any point in time, the insurance guarantee provided by the LGWA is activated, i.e., the investor is able to withdraw an amount in excess of his or her account balance.
- PROBRUN= probability that the income stream for the investor runs out before the end of the contract term
- RUNOUT= number of months before the end of the contract term, during which the (spouse of) the investor (continuously) receives no income.

The IRR and RUNOUT measures are themselves random. In the Tables presented in the next Section as well as the Appendices, we report their means and standard deviations. For the INCOME RATIO measure, we report only the means. We add two additional characterizations to provide further insight into the variability of these measures: First, we report, in Appendix 2, for the L80, T33 and TM33 instruments, plots that exhibit the likelihood that any given (IRR) return level is beaten<sup>27</sup>. Almost invariably, the

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<sup>27</sup> This is often referred to as the complementary cumulative density function, or *ccdf*.

RUNOUT variable has a high likelihood of being zero, given by  $1 - \text{PROBRUN}$ , i.e., the investor never ceases to draw an income from the variable annuity contract. Conditional upon the investor's income stream "running out" (which occurs with probability  $\text{PROBRUN}$ ) the duration of the run-out time is, invariably, fairly uniformly spread. For the sake of brevity, we therefore omit any graphs displaying the distribution of this measure. The  $\text{PROBGAR}$  probability relates to the LGWA only: it represents the likelihood of the investor or the spouse receiving a monthly income in excess of their account balance. In other words,  $\text{PROBGAR} = 0$  for any of the Target Date funds. Second, we compare the above performance measures across the different investment products, separately for different sets of market scenarios, moving progressively from the most inferior to the most superior scenarios. Anchoring on a given straw-man vehicle<sup>28</sup>, we separate, to this end, the 10,000 scenarios first into 2 sets, i.e., those in which the account runs out under the straw-man vehicle and those in which the account does not run out. Each of these sets is further divided into its four quartiles<sup>29</sup>, i.e., the scenarios with the 25% lowest Internal Rates of Return, those, whose IRR fall between the first quartile and the median, etc.

## 5. RESULTS

In this Section, we discuss the results of our study. As explained in Section 2, we initially focus on the following three investment instruments:

- the L80 product, which, in view of the lifetime withdrawal guarantees, permits bold participation in the equity markets, comparable to generally accepted allocation strategies for young investors, and
- the T33 and TM33 funds, the traditional alternatives for retirees or those approaching retirement age.

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<sup>28</sup> We have selected the Target Date Fund with an 80% participation in equities and the first withdrawal rule (i), as the straw-man vehicle. As explained, this instrument is denoted as T80.

<sup>29</sup> The separation is again based on the IRR values under the T80 instrument.

In the second part of this Section, we discuss all 12 investment vehicles, so as to characterize the impacts of the allocation strategy within each contract type (L, T and TM), as well as to allow for comparisons among contract types, when the allocation strategy is held constant.

### 5.1 THE BASIC CHOICES

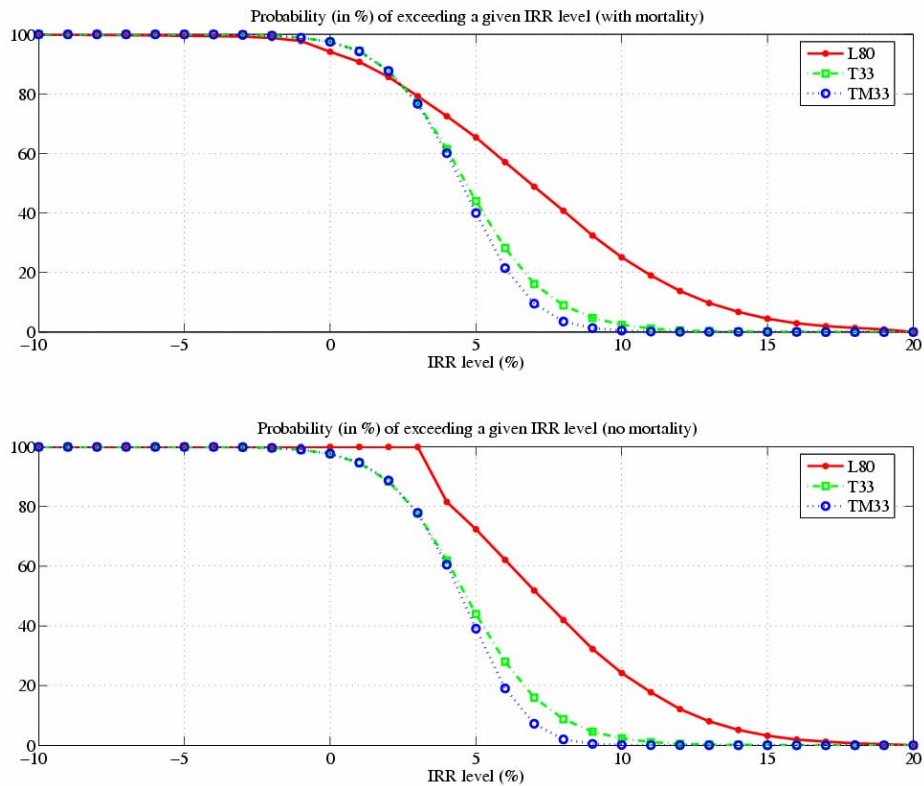
Table 1 exhibits the various performance measures for the L80, T33 and TM33 contracts, under the first of the four stochastic models for the monthly returns of the equity and fixed income pools, i.e., lognormal equity values, combined with constant fixed income returns. (We defer our discussion of the total cash flow measure TOTAL to then next subsection, where it is conducted in the context of a full comparison among all 12 instruments.) As explained in Section 3, the first (middle, last) set of three columns refers to parameter values matching historical data for a stretch of 35 years starting in January 1926 (August 1948, June 71). While the *absolute* values of the various performance measures vary significantly from one set of simulations to the next, the following conclusions hold across the board:

Lifetimes generated using probability tables									
	Jan-26--Dec-60			Aug-48--July-83			June-71--May-06		
	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33
PROBGAR	0.3308	n/a	n/a	0.0285	n/a	n/a	0.0292	n/a	n/a
mean_IRR	7.46%	5.22%	4.91%	8.43%	5.35%	5.23%	9.00%	8.63%	8.53%
std_IRR	4.66%	2.47%	2.11%	2.72%	1.26%	1.11%	2.94%	1.25%	1.16%
mean_RUNOUT	11.0290	62.2196	9.3136	1.1774	49.9757	0.7069	1.2222	7.5424	0.0000
std_RUNOUT	31.4245	69.9894	28.7004	10.4022	57.9143	6.5530	10.7919	25.2829	0.0000
PROBRUN	0.1590	0.6140	0.1460	0.0192	0.5956	0.0195	0.0205	0.1256	0.0000
INCOMERATIO	1.6135	1.1176	1.0000	1.5968	1.2021	1.0000	1.1738	1.1137	1.0000

Investor assumed to live throughout the 35 years									
	Jan-26--Dec-60			Aug-48--July-83			June-71--May-06		
	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33
PROBGAR	0.5445	n/a	n/a	0.0847	n/a	n/a	0.0831	n/a	n/a
mean_IRR	8.12%	5.24%	4.86%	8.49%	5.37%	5.21%	9.07%	8.68%	8.54%
std_IRR	3.52%	2.42%	1.94%	2.26%	1.21%	0.98%	2.46%	1.13%	0.98%
mean_RUNOUT	0.0000	153.3929	30.5296	0.0000	134.3126	3.4250	0.0000	26.0094	0.0000
std_RUNOUT	0.0000	65.3596	51.7199	0.0000	50.4820	14.8924	0.0000	46.9966	0.0000
PROBRUN	0.0000	0.9676	0.3647	0.0000	0.9904	0.0810	0.0000	0.3203	0.0000
INCOMERATIO	1.9804	0.9131	1.0000	1.9019	0.9635	1.0000	1.2174	1.0321	1.0000

Table 1: Lognormal equity values; constant fixed income returns



**Figure 5: Calibration period: Jan-26 -- Dec-60 Lognormal equity values;  
Fixed Income returns constant.**

- (I) The L80 product allows the investor to enjoy, on average, a significantly larger return on his or her investment (as measured by the mean IRR) than the traditional T33 and TM33 funds. Focusing, e.g., on the relative performance with respect to the TM33 contract, the *magnitude* of the *additional* mean IRR varies from 0.5% to 2.6% and 3.2 %, under actuarially distributed life times ( or 0.5%, 3.3% and 3.3% if the investor outlives the 35 year contract term). This difference is always statistically significant; indeed, the estimates of the mean IRR have a margin of error<sup>30</sup> of less than 0.1%, when building 95% confidence intervals.( The relatively low difference of 0.5% pertains to a set of parameters which display a, historically, unusually low value of the equity

<sup>30</sup> The margin of error is defined as the one-sided width of the confidence interval.



premium of 3.6%, i.e., an unusually small difference in the average equity and fixed income returns; under such a set of scenarios, the mean IRR is relatively insensitive to the allocations applied to the equity and fixed income pools.) The *additional mean return* under L80 comes at the expense of a somewhat more elevated *standard deviation* of the IRR. (The increments in the standard deviation of the IRR vary from 1.6% to 1.8% and 2.6% for the three sets of simulations, when comparing the TM33 and the L80 instruments; they are somewhat smaller when comparing T33 with L80.) It should be noted, however, that the *standard deviation of the IRR* is a measure of the general volatility of the IRR-measure which incorporates the potential for *upside* deviations from the mean in an equal and symmetric way to that of *downside* deviations. This is best illustrated by Figure 2, which displays the likelihood of “beating” any given IRR percentage, under the three instruments, when the parameters in the stochastic returns model are chosen to match the actual returns in the first of the three considered intervals of 35 years.. Note that, if the investor is assumed to outlive the full 35 year period, the likelihood of beating *any* IRR level is higher under L80 as compared with the two Target Date funds. Employing terminology in the risk management literature, this implies that the IRR under L80 (*stochastically*) *dominates* the IRR of the other two instruments. Under actuarially distributed lifetimes, there is *near dominance*: the likelihood of beating any given IRR percentage is higher for all but a small range of low IRR values; moreover, in that range the difference between the likelihoods is small. The same patterns are observed, when the parameters in the stochastic returns model are anchored on the other two 35 year time intervals; see Figures A2-1 and A2-2 in Appendix 2.

- (II) Other insightful characterizations of the income risks during the lifetime of the investor and his or her spouse, are provided by the various “run-out” statistics in the last three lines of the tables. These are discussed, in detail, in (III) - (V) below.
- (III) When assuming that the investor’s and the spouse’s life times follow actuarial distributions, the investor enjoys, under the L80 product, the above *additional*

*mean return* on his or her investment, while limiting the *downside risks* to roughly the same values as the TM33 product. These downside risks are measured by PROBRUN (= probability of the income stream coming to a halt) and the mean and standard deviation of RUNOUT, the number of months before the end of the contract term during which no income is received. Interestingly, all three of the run-out statistics, i.e. the likelihood of the income stream coming to a halt (PROBRUN) as well as both the mean and standard deviation of the run-out time (RUNOUT) are virtually identical under L80 and TM33. (Recall that the investor's income stream never runs out during his or her own lifetime, which explains why all of the "run-out" statistics for L80 are zero in the bottom half of Table 1. The fact that they are positive in the upper part of the table reflects the possibility that the original investor is survived by his or her spouse, who subsequently depletes the benefit base.)

- (IV) The above shows that the LGWA provisions mitigate the "run-out" risks almost to the same extent as the conservative allocation rule (33% investment in the equity fund) does for the traditional Target Date Fund TM33, all while allowing for a considerably higher expected IRR. In addition, it should be noted that the TM33 contract manages to limit the "run-out" risks to the same level as the L80 product, but only by reducing the monthly income withdrawals to a much lower level. This can be inferred by considering the INCOMERATIO: on average, if their lifetimes follow actuarial probability distributions, the monthly income withdrawn by the investor and his or her spouse under L80 is approximately 60% larger than under TM33, for the first two sets of simulations and 17% larger under the last set. (The corresponding margins are, understandably, even higher if the investor is assumed to outlive the full 35 years of the contract term, since in this case the LGWA guarantees remain, categorically, operative for the full contract term. Note that under this assumption, the average monthly income is more than 90% higher under the L80 product, for the first two sets of simulations.)

(V) The fact that the income withdrawal potential is considerably lower under the TM33 product can also be inferred by comparing its performance with that of the T33 instrument; recall that, under the latter, the investor is assumed to withdraw, for as long as feasible, a monthly income equal to what he or she would receive under the L80 contract. Under the income stream which the L80 contract provides the investor, there is a dramatically larger likelihood of “running out” when investing in the traditional Target Date Fund and allocating only 33% of the account to the equity fund<sup>31</sup>: in the first set of simulations (reflecting the January 1926-December 1960 period), the likelihood of running out is 61% under T33 versus 16% under the L80 vehicle: these likelihood pairs are 60% versus 2% and 13% versus 2% for the second and third set of simulations, respectively. Similarly, the average duration of the uncovered period, during which no income is received, which is measured by the mean of the RUNOUT measure, is at least 5.5 times as long under T33 as compared to the L80 vehicle. (Under the first two sets of simulations, the additional uncovered period is, on average, at least 4 years longer.)

Note, that, even though the investor is originally permitted to withdraw equal amounts under the T33 contract as he would be able to do under the L80 instrument, in the end, the average monthly income is only 10-20% larger than under a traditional TM33 fund, the consequence of a significantly larger likelihood of the income stream coming to a halt because the account is depleted. (If the investor outlives the full 35 year contract term, the average monthly income is hardly larger than under TM33 and, in two of the three sets of simulations, significantly lower!) However, under TM33 with lower

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<sup>31</sup> For this purpose, we confine ourselves to the results under lifetimes distributed according to the actuarial survival probabilities. Recall again that, in the bottom part of Table 1, where it is assumed that the investor survives the full 35 years of the contract duration, all run-out statistics equal zero under L80., while they are even higher under both the T33 and TM33 vehicles (compared to their counterparts under actuarially determined lifetimes). In the first two sets of simulations, i.e., if the mean equity and fixed income returns reflect the averages over the January 1926-December 1960 or the August 1948-July 1983 periods, it is virtually certain that the T33 fund runs out, i.e., the traditional Target Date fund runs out, if the investor is allowed to withdraw income levels in accordance with the L80 contract.

withdrawals before run-out than T33, TM33 may not be able to keep up with inflation.

- (VI) The PROBGAR measure indicates that under certain historically prevalent parameter combinations, there is a high likelihood that the LGWA guarantees need to be activated, i.e., the investor or the spouse receives income in excess of their prevailing account balance. (Almost invariably, this means that income is received, while the account value has been depleted.) This likelihood is as high as 33% in the first set of simulations (reflecting the volatile period January 26-December 1960), even when assuming that the investor's lifetime and that of the spouse follow actuarial probability distributions. When assuming that the investor survives the full 35 years, the LGWA guarantees are activated, at some point of the 35 year horizon, with a probability of no less than 54%.

Even when the return parameters are anchored on the far less risky horizons August 1948-July 1983 and June 1971- May 2006, the likelihood of the LGWA guarantees being activated is approximately 3% or 8%, depending upon whether the investor's and the spouse's lifetime follow the actuarial probability distributions, or the investor is assumed to survive the full 35 years, respectively.

	Lifetimes generated using probability tables								
	Jan-26--Dec-60			Aug-48--July-83			June-71--May-06		
	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33
<b>PROBGAR</b>	0.3420	n/a	n/a	0.0418	n/a	n/a	0.0344	n/a	n/a
<b>mean_IRR</b>	7.46%	4.69%	4.41%	8.11%	4.47%	4.38%	8.92%	8.00%	8.00%
<b>std_IRR</b>	4.71%	6.94%	6.80%	2.81%	2.67%	2.57%	2.99%	2.15%	2.06%
<b>mean_RUNOUT</b>	11.6207	70.9804	17.7510	1.8141	61.0925	8.6236	1.4356	16.7186	0.6918
<b>std_RUNOUT</b>	32.3627	73.3347	42.3795	12.7735	64.0575	30.5634	11.1595	39.4093	8.2065
<b>PROBRUN</b>	0.1646	0.6577	0.2328	0.0316	0.6524	0.1267	0.0236	0.2277	0.0119
<b>INCOMERATIO</b>	1.6631	1.0951	1.0000	1.6134	1.1401	1.0000	1.2250	1.0946	1.0000

	Investor assumed to live throughout the 35 years								
	Jan-26--Dec-60			Aug-48--July-83			June-71--May-06		
	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33
<b>PROBGAR</b>	0.5555	n/a	n/a	0.1100	n/a	n/a	0.0924	n/a	n/a
<b>mean_IRR</b>	8.11%	4.70%	4.34%	8.21%	4.47%	4.34%	8.99%	8.00%	8.00%
<b>std_IRR</b>	3.54%	6.91%	6.73%	2.32%	2.62%	2.47%	2.49%	1.98%	1.80%
<b>mean_RUNOUT</b>	0.0000	164.3911	50.4816	0.0000	150.9805	26.5660	0.0000	50.0164	2.3600
<b>std_RUNOUT</b>	0.0000	67.7697	67.6079	0.0000	52.9997	51.8570	0.0000	66.1775	16.5113
<b>PROBRUN</b>	0.0000	0.9588	0.4981	0.0000	0.9825	0.3305	0.0000	0.4768	0.0303
<b>INCOMERATIO</b>	2.1238	0.9114	1.0000	1.9861	0.9322	1.0000	1.3275	0.9877	1.0000

**Table 2: Lognormal equity values; fixed income returns from ARMA/GARCH**

	Lifetimes generated using probability tables								
	Jan-26--Dec-60			Aug-48--July-83			June-71--May-06		
	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33
PROBGAR	0.3355	n/a	n/a	0.0375	n/a	n/a	0.0444	n/a	n/a
mean_IRR	7.37%	5.12%	4.80%	8.36%	5.03%	4.92%	8.89%	8.27%	8.24%
std_IRR	4.59%	2.63%	2.24%	2.87%	2.10%	1.92%	3.11%	2.18%	2.09%
mean_RUNOUT	11.1218	64.4930	11.4293	1.6991	56.8581	7.1819	1.8646	11.9501	0.4635
std_RUNOUT	31.3506	70.1679	32.3071	12.4119	61.2522	24.9441	12.7346	32.1450	6.2422
PROBRUN	0.1620	0.6283	0.1717	0.0278	0.6360	0.1200	0.0327	0.1809	0.0086
INCOMERATIO	1.6007	1.1110	1.0000	1.5508	1.1425	1.0000	1.1796	1.0952	1.0000

	Investor assumed to live throughout the 35 years								
	Jan-26--Dec-60			Aug-48--July-83			June-71--May-06		
	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33
PROBGAR	0.5545	n/a	n/a	0.1059	n/a	n/a	0.1114	n/a	n/a
mean_IRR	8.06%	5.14%	4.76%	8.43%	5.03%	4.86%	9.00%	8.31%	8.27%
std_IRR	3.49%	2.58%	2.08%	2.36%	2.04%	1.75%	2.60%	2.00%	1.83%
mean_RUNOUT	0.0000	155.6743	36.2925	0.0000	145.0509	25.2093	0.0000	38.6061	1.5494
std_RUNOUT	0.0000	63.9310	55.8482	0.0000	52.1316	45.9883	0.0000	56.3641	12.5074
PROBRUN	0.0000	0.9671	0.4137	0.0000	0.9769	0.3320	0.0000	0.4276	0.0236
INCOMERATIO	1.9694	0.9130	1.0000	1.9004	0.9330	1.0000	1.2525	1.0059	1.0000

**Table 3: Historical distributions with monthly bootstrapping**

	Lifetimes generated using probability tables								
	Jan-26--Dec-60			Aug-48--July-83			June-71--May-06		
	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33
PROBGAR	0.3650	0.0000	0.0000	0.0625	0.0000	0.0000	0.0418	0.0000	0.0000
mean_IRR	7.77%	5.12%	4.79%	8.26%	4.98%	4.89%	9.12%	8.52%	8.50%
std_IRR	4.91%	2.94%	2.54%	3.05%	2.13%	1.95%	3.06%	2.22%	2.13%
mean_RUNOUT	11.6161	70.5736	13.8572	2.4545	58.8554	7.2860	1.6042	10.5918	0.3054
std_RUNOUT	32.3924	72.8949	36.8176	14.9062	63.4724	25.3350	11.7304	30.2004	4.7602
PROBRUN	0.1665	0.6609	0.1923	0.0401	0.6392	0.1170	0.0273	0.1648	0.0060
INCOMERATIO	1.7147	1.1079	1.0000	1.5809	1.1356	1.0000	1.1836	1.1000	1.0000

	Investor assumed to live throughout the 35 years								
	Jan-26--Dec-60			Aug-48--July-83			June-71--May-06		
	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33
PROBGAR	0.5718	0.0000	0.0000	0.1524	0.0000	0.0000	0.1041	0.0000	0.0000
mean_IRR	8.45%	5.12%	4.72%	8.40%	4.99%	4.85%	9.21%	8.54%	8.52%
std_IRR	3.71%	2.89%	2.37%	2.49%	2.06%	1.79%	2.55%	2.02%	1.85%
mean_RUNOUT	0.0000	167.4388	41.4615	0.0000	147.9807	25.3344	0.0000	36.0043	1.1329
std_RUNOUT	0.0000	63.8842	62.1515	0.0000	54.8772	46.3958	0.0000	54.4670	10.5393
PROBRUN	0.0000	0.9710	0.4280	0.0000	0.9720	0.3301	0.0000	0.4115	0.0173
INCOMERATIO	2.1673	0.9002	1.0000	1.9361	0.9272	1.0000	1.2520	1.0111	1.0000

**Table 4: Historical distributions with bootstrapping in batches of 12 months**

While the absolute values of the various performance measures vary, somewhat, with the *type* of stochastic model used for the joint returns of the equity and fixed income pools, each of the above main conclusions (I)-(VI) continue to hold throughout. Table 2 exhibits

the results when the returns of the fixed income pool are generated by the ARMA/GARCH process described in Section 3(, while the equity fund follows the Lognormal process employed in the first stochastic model.) Tables 3 and 4 exhibit the results when the joint returns are generated from the historical distributions, employing elementary bootstrapping and bootstrapping with 12 month batches respectively. The following additional observations are noteworthy:

- The differences in the standard deviations of the IRR under L80 and TM33 are generally lower than those observed in Table 1 for the first stochastic model. This reflects the fact that the first stochastic model assumes that the fixed income pool generates constant returns and is therefore entirely risk free, per se. Under the three alternative stochastic models, the fixed income pool exhibits a significant degree of volatility; as a consequence, the standard deviations of the IRR of the T33 and TM33 instruments, which invest no less than *two thirds* of the account in the fixed income pool, are significantly higher in Tables 2-4 than the corresponding values in Table 1. (At the same time, the standard deviation of the IRR for L80 is, understandably, far less sensitive to the *type* of stochastic model employed to represent the fixed income pool, as only 20% of the account is invested in it. The net result is that the *difference* in the standard deviation of the IRR of L80 and TM33 is generally lower in Tables 2-4.)

Remarkably, under the ARMA/GARCH model of Table 2, L80 exhibits both a much larger *mean* and a much smaller *standard deviation* of the IRR than T33 or TM33, when the model parameters match the data pertaining to the first 35 year horizon January 1926-December 1960. Under the second set of simulations, with parameters reflecting the August 1948-July 1983 horizon, the standard deviations of the IRR are approximately identical under L80 and T33 or TM33. (Under the first two sets of simulations in Table 2, L80 thus completely dominates the T33 and TM33 instruments when considering traditional mean-variance tradeoffs.)

- The IRR under L80 (nearly), stochastically, dominates those under the T33

and TM33 instruments: the likelihood of beating any given return rate is almost always larger under L80. See Appendix 2 for the relevant plots.

- Since the stochastic models for the returns of the fixed income pool underlying Tables 2-4, exhibit a considerable degree of volatility, while table 1 assumes constant returns for the fixed income pool it is understandable that the PROBGAR values in tables 2-4 are even higher than those discussed, under observation (V).

Lifetimes generated using probability tables															
	increase sigma by 50%			increase sigma by 100%			increase mu, sigma by 50%			decrease mu, sigma by 50%					
	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33
PROBGAR	0.3308	n/a	n/a	0.6621	n/a	n/a	0.8350	n/a	n/a	0.4121	n/a	n/a	0.3833	n/a	n/a
mean_IRR	7.46%	5.22%	4.91%	6.80%	4.88%	4.33%	6.37%	4.18%	3.42%	10.70%	7.37%	6.71%	3.67%	3.09%	3.02%
std_IRR	4.66%	2.47%	2.11%	6.11%	4.05%	3.40%	7.27%	5.79%	4.83%	7.09%	4.01%	3.24%	2.22%	1.11%	1.04%
mean_RUNOUT	11.03	62.22	9.31	22.16	78.04	28.67	29.40	92.07	51.08	12.30	88.36	10.19	16.87	25.41	16.56
std_RUNOUT	31.42	69.99	28.70	42.98	79.47	52.53	48.15	87.12	70.74	33.29	82.87	33.14	36.95	41.96	32.71
PROBRUN	0.1590	0.6140	0.1460	0.2959	0.6528	0.3370	0.3727	0.6858	0.4826	0.1707	0.7027	0.1321	0.2509	0.3939	0.3040
INCOMERATIO	1.6135	1.1176	1.0000	1.6565	1.0556	1.0000	1.7371	1.0238	1.0000	2.3863	1.0779	1.0000	1.1186	1.0413	1.0000

Investor assumed to live throughout the 35 years															
	increase sigma by 50%			increase sigma by 100%			increase mu, sigma by 50%			decrease mu, sigma by 50%					
	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33
PROBGAR	0.5445	n/a	n/a	0.8650	n/a	n/a	0.9622	n/a	n/a	0.6060	n/a	n/a	0.7422	n/a	n/a
mean_IRR	8.12%	5.24%	4.86%	8.18%	4.89%	4.24%	8.28%	4.18%	3.32%	11.40%	7.41%	6.65%	4.65%	3.08%	3.00%
std_IRR	3.52%	2.42%	1.94%	4.56%	4.01%	3.21%	5.48%	5.74%	4.63%	5.77%	3.95%	2.99%	1.20%	1.07%	0.95%
mean_RUNOUT	0.00	153.39	30.53	0.00	175.68	73.74	0.00	192.20	115.01	0.00	183.68	28.50	0.00	107.93	61.01
std_RUNOUT	0.00	65.36	51.72	0.00	75.23	78.31	0.00	84.35	91.93	0.00	80.82	56.71	0.00	38.98	47.90
PROBRUN	0.0000	0.9676	0.3647	0.0000	0.9490	0.6113	0.0000	0.9307	0.7562	0.0000	0.9360	0.2836	0.0000	0.9984	0.7938
INCOMERATIO	1.9804	0.9131	1.0000	2.1528	0.9054	1.0000	2.3867	0.9201	1.0000	3.2824	0.8067	1.0000	1.3420	0.9700	1.0000

**Table 5: Sensitivity analyses: Lognormal equity values; constant fixed income returns**

Lifetimes generated using probability tables															
	increase sigma by 50%			increase sigma by 100%			increase mu, sigma by 50%			decrease mu, sigma by 50%					
	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33
PROBGAR	0.3420	n/a	n/a	0.6703	n/a	n/a	0.8370	n/a	n/a	0.4267	n/a	n/a	0.4002	n/a	n/a
mean_IRR	7.46%	4.69%	4.41%	6.76%	4.37%	3.84%	6.47%	3.74%	2.99%	10.67%	6.84%	6.19%	3.63%	2.53%	2.50%
std_IRR	4.71%	6.94%	6.80%	6.17%	7.66%	7.33%	7.33%	8.77%	8.14%	7.14%	7.69%	7.26%	2.25%	6.50%	6.50%
mean_RUNOUT	11.62	70.98	17.75	22.11	84.36	35.66	28.89	97.30	56.86	12.22	93.85	16.02	16.82	38.84	29.87
std_RUNOUT	32.36	73.33	42.38	42.94	82.40	59.62	47.83	88.52	73.50	33.24	85.63	42.39	36.77	55.09	48.97
PROBRUN	0.1646	0.6577	0.2328	0.2951	0.6795	0.3921	0.3704	0.7126	0.5301	0.1726	0.7210	0.1912	0.2542	0.4769	0.4122
INCOMERATIO	1.6631	1.0951	1.0000	1.7381	1.0490	1.0000	1.8319	1.0262	1.0000	2.5171	1.0643	1.0000	1.1622	1.0220	1.0000

Investor assumed to live throughout the 35 years															
	increase sigma by 50%			increase sigma by 100%			increase mu, sigma by 50%			decrease mu, sigma by 50%					
	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33
PROBGAR	0.5555	n/a	n/a	0.8664	n/a	n/a	0.9643	n/a	n/a	0.6154	n/a	n/a	0.7512	n/a	n/a
mean_IRR	8.11%	4.70%	4.34%	8.17%	4.37%	3.75%	8.37%	3.73%	2.88%	11.38%	6.87%	6.13%	4.66%	2.51%	2.46%
std_IRR	3.54%	6.91%	6.73%	4.61%	7.65%	7.23%	5.57%	8.75%	8.01%	5.85%	7.66%	7.15%	1.22%	6.49%	6.47%
mean_RUNOUT	0.00	164.39	50.48	0.00	182.63	88.80	0.00	199.37	125.97	0.00	190.87	42.21	0.00	121.99	88.93
std_RUNOUT	0.00	67.77	67.61	0.00	77.28	83.90	0.00	83.27	92.85	0.00	80.70	68.84	0.00	57.21	62.64
PROBRUN	0.0000	0.9588	0.4981	0.0000	0.9457	0.6793	0.0000	0.9389	0.7909	0.0000	0.9399	0.3803	0.0000	0.9469	0.8518
INCOMERATIO	2.1238	0.9114	1.0000	2.3292	0.9136	1.0000	2.5672	0.9289	1.0000	3.5663	0.8178	1.0000	1.4761	0.9778	1.0000

**Table 6: Sensitivity analyses: Lognormal equity values: fixed income returns from ARMA/GARCH process**

To further test whether the observations under (I)-(VI) are robust, we have carried out a variety of sensitivity analyses in which one of the parameters in the stochastic model for the monthly joint equity and fixed income returns is varied. In particular, we have investigated, for the stochastic models with lognormal equity values what impact a more volatile equity market would have or one in which  $\sigma / \mu$ , the ratio of the mean and standard deviation of the monthly equity return is preserved but both parameters are increased or decreased (in the same proportion). Tables 5 and 6 show the results for the first two stochastic models, with fixed income returns constant or generated by the above ARMA/GARCH model respectively, and with the base set of parameters anchored on the first 35 year horizon (January 1926-December 1960).

All of our observations under (I)-(VI) continue to apply, with the following additional remarks:

- Table 5: while in the base case, the run-out statistics are slightly worse for the L80 product compared with the TM33 instrument, the relative comparison favors the L80 instrument under increased volatility of the equity markets.<sup>32</sup> The same applies to the INCOMERATIO measure, i.e., the more volatile the equity markets are, the *larger* the *relative* difference between the average monthly income enjoyed under L80 versus TM33. Since the above observations about the additional mean and standard deviation of the IRR continue to apply, the overall conclusion is that the *relative* benefits of the L80 product (compared to T33 and TM33) become progressively larger as the volatility of the equity markets increase. This is, at first, somewhat surprising since the L80 instrument is primarily invested in equities while the T33 and TM33 instruments have limited exposure to the equity markets. Nevertheless, the results indicate that under increased volatility of the equity markets, a traditional Target Date Fund would need to shift the allocation to equities even below the 33% level to achieve the same reduction of downside risks as

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<sup>32</sup> Recall from the discussion above, and the contents of Table 5, that the TM33 instrument is, in the base case, able to reduce the downside risks to (slightly lower) levels than the L80 instrument but only by simultaneously sacrificing much of the expected IRR, and by reducing the investor's average income by some 38%., in the first two sets of simulations.



the L80 instrument, at the expense of even larger sacrifices in the mean IRR and the average monthly withdrawals from the fund. When the mean and standard deviation of the monthly equity returns grow (decline) in proportion, the differences of both the mean and standard deviation of the IRR of the L80 product -compared with either T33 or TM33- grow (decline); the run-out statistics shift somewhat in favor of TM33 (L80).

- The same conclusions apply to Table 6: in particular, increased volatility of the equity markets further enhances the *relative* performance of the L80 instrument. Under this stochastic model, the L80 instrument unambiguously dominates the T33 and TM33 products, in terms of a higher mean IRR, a lower standard deviation of the IRR, much higher average monthly income and better run-out statistics, irrespective of the large variations in the assumed mean and standard deviation of the monthly equity returns. Note also that under this stochastic model for the fixed income markets, the INCOMERATIO is systematically even larger than it is, when assuming constant interest rates.

Lifetimes generated according to lifetables												
	Q1			Q2			Q3			Q4		
	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33
PROBGAR	1.0000	n/a	n/a	1.0000	n/a	n/a	1.0000	n/a	n/a	1.0000	n/a	n/a
mean_IRR	1.07%	0.34%	0.36%	2.33%	1.84%	1.87%	4.54%	3.12%	3.06%	9.09%	6.26%	4.97%
std_IRR	1.77%	1.42%	1.41%	1.56%	0.82%	0.83%	1.35%	0.73%	0.72%	2.54%	1.91%	1.11%
mean_RUNOUT	53.45	63.80	78.75	39.10	52.38	49.83	30.87	85.20	35.36	20.29	153.15	24.14
std_RUNOUT	60.60	68.01	63.49	52.05	63.56	51.86	42.78	59.74	41.77	33.45	55.88	35.73
PROBRUN	0.5610	0.6248	0.8179	0.5182	0.5709	0.6782	0.5046	0.8848	0.6124	0.3752	0.9927	0.5064
INCOMERATIO	1.1486	0.9722	1.0000	1.1269	0.9682	1.0000	1.2483	0.9766	1.0000	1.8058	0.9438	1.0000

**Table 7: Performance measures by quartile; Historical Distributions with monthly bootstrapping; scenarios with run-outs under T80**

Lifetimes generated according to lifetables												
	Q1			Q2			Q3			Q4		
	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33
PROBGAR	0.3393	n/a	n/a	0.1363	n/a	n/a	0.0702	n/a	n/a	0.0379	n/a	n/a
mean_IRR	2.85%	3.26%	3.25%	6.84%	4.94%	4.82%	9.63%	6.28%	5.96%	13.88%	8.51%	7.73%
std_IRR	2.37%	1.10%	1.09%	0.93%	0.74%	0.59%	0.87%	0.88%	0.60%	2.44%	1.74%	1.15%
mean_RUNOUT	13.08	11.37	5.38	3.32	42.67	0.87	0.91	72.13	0.21	0.31	98.47	0.18
std_RUNOUT	33.94	29.02	19.12	16.19	53.21	6.29	7.52	64.35	2.61	4.14	75.30	3.83
PROBRUN	0.1871	0.2127	0.1138	0.0692	0.5605	0.0323	0.0226	0.7473	0.0087	0.0082	0.8170	0.0041
INCOMERATIO	1.0786	1.0459	1.0000	1.3522	1.1557	1.0000	1.7362	1.2028	1.0000	2.5203	1.2215	1.0000

**Table 8: Performance measures by quartile; Historical Distributions with monthly bootstrapping; scenarios *without run-outs* under T80**

As mentioned in Section 4, we have compared the various performance measures, not just in the aggregate across all 10,000 scenarios generated within a given stochastic model, but also within each of four quartiles of the IRR performance (of the T80 straw man product), separately for the scenarios in which the investor’s income stream runs out and those where it does not. We have done this systematically for all simulations based on historical distributions which employ the basic bootstrapping technique, and for all 12 investment vehicles considered. Here, we focus on the simulations that are anchored on the January 1926 till December 1960 data, assuming the lifetimes of the investor and the spouse are generated according to the actuarial probability distributions. As before, we continue to focus on the L80, T33 and TM33 instruments. (See Appendix 3 for a complete set of results comparing all 12 instruments, under the three 35 year historical horizons, and under the two alternative assumptions regarding the investor and the spouse’s lifetimes.)

Under the chosen stochastic model for the market returns, there is a 22% chance of the investor running out of income when investing in our straw-man T80 product. Table 7 reports on the set of scenarios in which, under the straw man vehicle, the investor runs out of income and Table 8 on the remaining 7808 scenarios. One notices in Table 7 that the L80 instrument outperforms TM33, in *each* of the four quartiles considered there, both in terms of the mean IRR and in terms of *each* of the run-out statistics. Thus the LGWA provisions limit the downside risks as effectively as the conservative TM33

vehicle, not just in the aggregate but specifically in the set of troublesome scenarios that are associated with run-outs (under the straw man instrument), and this in each of the quartiles of this set of scenarios.

## *5.2 LIFETIME INCOME GUARANTEE ANNUITIES VERSUS TARGET DATE FUNDS: ALTERNATIVE ALLOCATION STRATEGIES*

Thus far, we have focused on a comparison of the L80 instrument and the traditional Target Date funds T33 and TM33. As explained, we consider these the basic choices, since the LGWA guarantees are designed specifically to allow the investor to participate boldly (with 80% of the account balance) in the equity markets, while limiting his or her downside risks; similarly, in the absence of such income guarantees, financial advisors will, understandably, advise investors to limit their exposure to the equity markets to 33% or less, when approaching retirement age. Indeed the statistical analysis, discussed in the previous subsection, substantiates the rationale for this practice.

In this subsection, we compare, nevertheless, all *four* of the above considered allocation strategies (33%, 40%, 60% and 80% participation in the equity markets, with continuous rebalancing of the account), for the instrument with the LGWA provisions (L), as well as the Target Date funds T and TM. This gives rise to a total of 12 instruments, which have been compared systematically, for all 20 stochastic models describing the monthly returns in the equity and fixed income pool, both under the assumption that the investor's and the spouse's lifetime follow the actuarial probability distributions, and assuming the investor outlives the full 35 years of the contract horizon. Consider, for example, Table 9, below, which compares all previously discussed performance measures for the model with Lognormal equity values and constant fixed income returns, anchored on the January 1926- December 1960 data.

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.1931	0.2072	0.2642	0.3308	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.37%	4.87%	6.25%	7.46%	5.22%	5.69%	6.78%	7.43%	4.91%	5.37%	6.57%	7.46%
std_IRR	1.98%	2.37%	3.52%	4.66%	2.47%	2.93%	4.25%	5.60%	2.11%	2.56%	4.02%	5.64%
mean_RUNOUT	8.92	8.88	9.66	11.03	62.22	53.15	27.95	21.96	9.31	10.88	17.72	26.84
std_RUNOUT	26.84	27.31	29.21	31.42	69.99	65.07	51.53	52.80	28.70	32.32	44.48	56.71
PROBRUN	0.1493	0.1429	0.1452	0.1590	0.6140	0.5607	0.3465	0.2141	0.1460	0.1565	0.2049	0.2694

D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.5171	0.4887	0.4911	0.5445	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.88%	5.37%	6.81%	8.12%	5.24%	5.71%	6.78%	7.31%	4.86%	5.32%	6.50%	7.37%
std_IRR	1.22%	1.56%	2.55%	3.52%	2.42%	2.86%	4.11%	5.30%	1.94%	2.37%	3.77%	5.36%
mean_RUNOUT	0.00	0.00	0.00	0.00	153.39	138.44	84.44	57.85	30.53	33.20	46.10	63.38
std_RUNOUT	0.00	0.00	0.00	0.00	65.36	66.69	74.83	85.79	51.72	56.60	72.06	86.56
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9676	0.9420	0.7410	0.4053	0.3647	0.3581	0.3943	0.4619

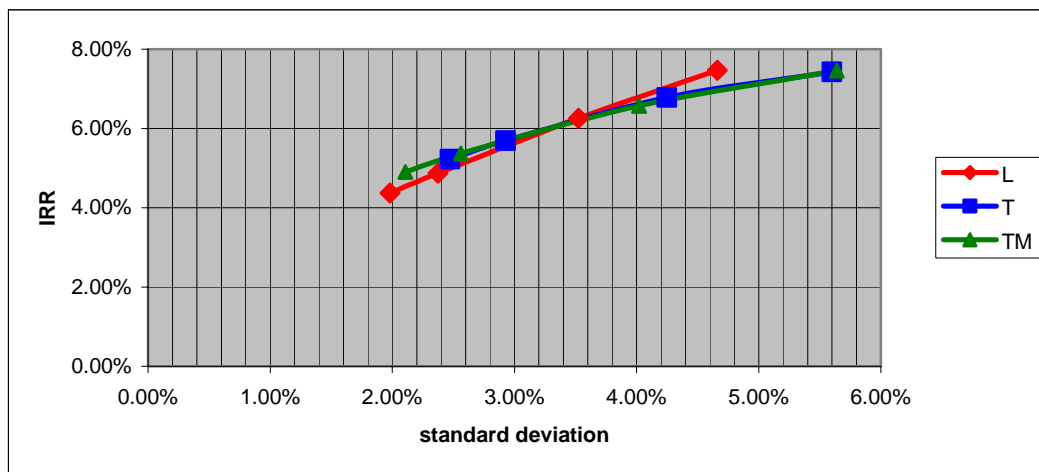
**Table 9: Calibration period: Jan-26 -- Dec-60 Lognormal Equity Values; FI returns constant**

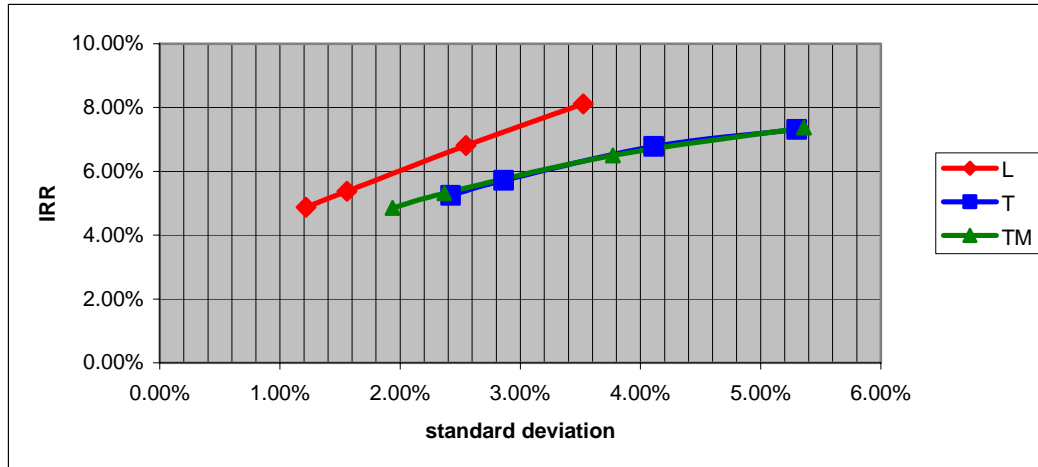
As is typically the case, both the mean and the standard deviation of the IRR increase, for all three contract types, as the portfolio allocation is shifted towards increased participation in the equity markets. However, when focusing on the above discussed *downside risks*, these do *not* necessarily increase with the allocation percentage in the equity fund.

For example, for the T instrument, under which the investor, for as long as feasible, withdraws equal amounts as he or she would under the corresponding L product, the mean RUNOUT period and the likelihood of the account running out (PROBRUN) actually *decrease*, with increased participation in the equity fund. (The standard deviation of the RUNOUT measure exhibits no clear pattern). This is explained by the fact that, on average, larger allocations to the equity fund allow for larger growth rates of the account balance, and hence for extensions of the amount of time during which the withdrawal stream associated with the L80 account can be sustained. This factor *dominates* the fact that, under increased allocations to the equity fund, the returns become more volatile, and, hence more prone to run-outs. For the L- and T- instruments, we observe an increase in the run-out statistics as the allocation percentage to the equity fund is increased.

Interestingly, the increase in the run-out statistics is rather modest for the L-products: for example, the probability of the investor's income stream coming to an end, sometime prior to the decease of his or her spouse, is, in this set of scenarios, approximately 15% under the very conservative alternative L33 and 16% under L80. (Recall that under the LGWA guarantees, the investor's income stream never terminates, which explains why PROBRUN=0 in the bottom half of the table, for each of the L-instruments. The run-out probability is unusually high under this set of scenarios, whose parameters reflect the highly volatile era from January 1926- December 1960; for the same stochastic model anchored on the other two 35 year horizons, the run-out probability grows from 0% to 2% as one shifts from L33, via L40 and L60 to L80.)

While, for the L-instruments, the run-out statistics increase only slightly as the allocation to the equity fund is increased, the PROBGAR measure increases quickly with the latter: under L33, the likelihood of the LGWA guarantees being activated at any point during the contract term, i.e., the investor receiving income in excess of his account balance, is 19%, when using L33, but it grows to 33% when choosing L80. This demonstrates that, in some historically based stochastic returns models, the income guarantees are used with a likelihood as high as 1/3.





**Figure 6: Calibration period: Jan-26 – Dec-60; Lognormal Equity Values; FI returns constant. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit assumes the investor outlives the 35 year horizon.**

Figure 6 displays the means and standard deviations of the IRR for the 12 instruments considered. Three curves (one for each of the instrument types L, T, and TM) connect the 4 points associated with the 4 considered allocation strategies between the equity fund and the fixed income fund. Each curve may be viewed as the *efficient frontier* in traditional *mean-variance tradeoff analyses*. Note that, under actuarially based lifetimes for the investor and the spouse, the efficient frontiers cross each other: for most equity allocation rules, the L-instrument has a slightly lower mean and a slightly lower standard deviation of the IRR than the corresponding T- or TM- instrument. (In this stochastic model, the L80 instrument dominates the T80 and TM80 product in terms of both measures.) In Appendix 1, we exhibit the counterparts of Table 9 and Figure 6 for each of the remaining 19 stochastic returns models considered. The following observations apply to all of the models considered:

- comparing the L- instrument with the T-products under *identical* equity allocation percentages, both the mean and the standard deviation of the IRR are, almost invariably lower for the former, compared with the latter. However, the differences, in the mean IRR values are relatively small. Thus, almost invariably, the Lifetime Guaranteed Withdrawal Annuity (LGWA)

allows the investor to reduce the risk as measured by the standard deviation of the IRR, at the expense of sacrificing *little* in terms of the *expected* Internal Rate of Return. In some stochastic models and for some of the equity allocation percentages, the L-instrument actually *dominates* the corresponding T- choices, in terms of both the mean and the standard deviation of the IRR.

- In general, the *relative* performance of an L-contract, compared with the T- or TM-instruments with the same equity allocation percentage, becomes stronger when considering larger volatilities in the equity and fixed income markets. For example, in all of the 5 stochastic models with an ARMA/GARCH process governing the fixed income returns, is the efficient frontier for the L-contract entirely *dominating* the efficient frontiers for the T- and TM-instruments: in other words, regardless of the chosen equity allocation, the L-contract has both a superior mean and standard deviation of the IRR.
- Almost invariably, the L- instrument performs considerably better in terms of the run-out statistics when compared to the corresponding T- and TM-product; this, in addition to exhibiting a lower standard deviation of the IRR.
- Within the L-line of contracts, the run-out statistics vary insignificantly with the chosen equity allocation percentage; this implies that considerably larger expected returns can be achieved by adopting L80, as compared to any of the other L-instruments, all while sacrificing very little in terms of increased downside risks.
- In contrast, the PROBGAR measure typically varies more significantly with the chosen equity allocation percentage, as in Table 9: the larger the participation in the equity markets, the more likely the LGWA guarantees are activated at some point in the contract term.

We conclude this Section with a discussion of the TOTAL measure, which aggregates all income withdrawals and the terminal account value at the end of the contract term ( , net of the initial investment). Tables10-12 exhibit the deciles of this measure for all 12

instruments considered, under the basic stochastic returns model, with Lognormal equity values and fixed income returns. (Each of the tables corresponds with one of the three 35 year tranches used to calibrate the model parameters.) Appendix 4 contains tables with the mean and standard deviations of the TOTAL measure, for all 12 instruments, as well as *all* of the stochastic returns models considered in this study.<sup>33</sup> The following general conclusions are apparent from Tables 10-12:

- Similar to what can be seen in Figure 1 for a specific 35 year calibration interval, L80, almost invariably, dominates the T33 and TM33 instruments, when considering the median or higher deciles. For the lower deciles, the differences tend to be relatively small.
- Almost without exception, for the L instrument, all decile values increase when increasing the percentage of the account allocated to equities; this implies that, almost invariably, the L80 contract (stochastically) *dominates* the L60 product, which in turn dominates L40, and the latter dominates L33. The same observations do not necessarily apply to the target fund instruments; in the absence of income guarantees, increased exposure to the equity markets may have an adverse effect on the TOTAL measure.
- When comparing decile values, or even the means of the TOTAL measure<sup>34</sup> between the L instrument and the T and TM instruments, under a *given* allocation strategy between the equity and fixed income pools- e.g., L80 and TM80- the latter often outperform the L product, somewhat. This is a consequence of the somewhat higher management and insurance fees required to provide the LGWA guarantees, and the resulting major reductions in the downside risks.

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<sup>33</sup> The same appendix tables provide the same information regarding the total management fees paid under the various instruments.

<sup>34</sup> See Appendix 4.



D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
10%	0.2246	0.2241	0.2070	0.1898	0.2638	0.2469	0.1532	0.0301	0.2762	0.2548	0.1547	0.0277
20%	0.3899	0.4154	0.4709	0.4958	0.4323	0.4595	0.4913	0.4302	0.4832	0.5031	0.4998	0.4183
30%	0.5286	0.5879	0.7260	0.8149	0.5668	0.6366	0.8114	0.8948	0.6668	0.7227	0.8457	0.8652
40%	0.6628	0.7542	1.0182	1.2265	0.6876	0.8023	1.1552	1.4449	0.8432	0.9530	1.2301	1.3898
50%	0.8232	0.9692	1.3909	1.7757	0.7987	0.9599	1.5378	2.1455	1.0407	1.2079	1.6831	2.0730
60%	0.9967	1.1934	1.8313	2.4584	0.8990	1.1151	1.9770	3.1008	1.2532	1.4877	2.2293	2.9571
70%	1.2031	1.4728	2.4031	3.4407	1.0021	1.2788	2.5218	4.3389	1.5142	1.8330	2.9040	4.1128
80%	1.4884	1.8651	3.2697	5.0276	1.1073	1.4474	3.2214	6.4907	1.8445	2.2896	3.9496	6.0383
90%	1.9618	2.5436	4.9362	8.5135	1.2542	1.6878	4.4252	11.0695	2.4134	3.1077	5.9281	10.1603

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
10%	0.7458	0.7458	0.7458	0.7458	0.2929	0.2775	0.1821	0.0436	0.3247	0.3026	0.1893	0.0435
20%	0.7648	0.8153	0.9571	1.0702	0.4718	0.5100	0.5683	0.5071	0.5840	0.6114	0.6092	0.4955
30%	0.8651	0.9565	1.2352	1.4706	0.6140	0.7037	0.9676	1.1270	0.8267	0.9132	1.0783	1.0639
40%	0.9739	1.1127	1.5527	1.9636	0.7421	0.8919	1.4270	1.9788	1.0812	1.2520	1.6473	1.8558
50%	1.1088	1.3138	1.9814	2.6478	0.8499	1.0493	1.9156	3.2003	1.3509	1.6130	2.3456	2.9004
60%	1.2843	1.5710	2.5647	3.6221	0.9483	1.2041	2.4760	4.7381	1.6423	1.9916	3.1408	4.2500
70%	1.5139	1.9025	3.2985	4.9570	1.0407	1.3523	3.0964	6.9509	1.9595	2.4335	4.1520	6.1304
80%	1.8729	2.4157	4.5894	7.6200	1.1376	1.5097	3.9064	10.8599	2.4104	3.0894	5.7723	9.4342
90%	2.4675	3.3040	7.0895	13.3325	1.2789	1.7460	5.1400	19.5001	3.1263	4.1520	8.8003	16.6549

**Table 10: Deciles for the TOTAL (net cash flow) measure, in millions of \$.; e.g., the total net cash flow is, with 90% probability, at or below the 90-th percentile.**

**Calibration period: Jan-26 -- Dec-60 Lognormal Equity Values; FI returns constant**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
10%	0.4745	0.5339	0.7232	0.9002	0.5568	0.6313	0.8837	1.1546	0.6144	0.6877	0.9119	1.1329
20%	0.6212	0.7223	1.0533	1.4243	0.6833	0.8057	1.2508	1.8062	0.8033	0.9228	1.3171	1.7484
30%	0.7430	0.8827	1.3614	1.9256	0.7708	0.9287	1.5535	2.3959	0.9538	1.1166	1.6621	2.3166
40%	0.8533	1.0294	1.6472	2.4121	0.8316	1.0237	1.8441	3.0263	1.0900	1.2945	2.0135	2.9101
50%	0.9663	1.1849	1.9644	2.9891	0.8843	1.1101	2.1389	3.7619	1.2358	1.4854	2.3917	3.5757
60%	1.0935	1.3528	2.3320	3.6772	0.9305	1.1835	2.4604	4.6689	1.3906	1.6958	2.8266	4.4180
70%	1.2458	1.5579	2.7849	4.5820	0.9745	1.2584	2.8232	5.8520	1.5719	1.9401	3.3658	5.4779
80%	1.4299	1.8158	3.3786	5.8156	1.0251	1.3369	3.2548	7.5063	1.7948	2.2441	4.0772	6.9723
90%	1.7102	2.2091	4.3660	8.0236	1.0912	1.4506	3.8395	10.4993	2.1301	2.7131	5.2633	9.5952

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
10%	0.7497	0.8298	1.1672	1.6116	0.6588	0.7878	1.3589	2.3393	0.8908	1.0491	1.5863	2.1611
20%	0.8556	1.0155	1.6734	2.4790	0.7709	0.9530	1.8910	3.6169	1.1552	1.4025	2.2436	3.2592
30%	0.9807	1.2226	2.1276	3.3002	0.8346	1.0478	2.2544	4.6859	1.3633	1.6719	2.7686	4.2416
40%	1.1121	1.4181	2.5694	4.1541	0.8833	1.1236	2.5791	5.8964	1.5329	1.9097	3.2957	5.2776
50%	1.2630	1.6245	3.0130	5.0591	0.9226	1.1844	2.8843	7.1195	1.7094	2.1365	3.8060	6.3185
60%	1.4240	1.8389	3.4954	6.0934	0.9583	1.2438	3.1808	8.5437	1.8900	2.3786	4.3825	7.6120
70%	1.5951	2.0761	4.0739	7.3862	0.9994	1.3064	3.4845	10.3298	2.0820	2.6538	5.0651	9.1439
80%	1.8051	2.3665	4.8111	9.1280	1.0425	1.3738	3.8466	12.7556	2.3210	2.9858	5.9468	11.2660
90%	2.1026	2.7918	6.0039	12.1271	1.1059	1.4762	4.3844	16.8750	2.6615	3.4865	7.3630	14.8436

**Table 11: Deciles for the TOTAL (net cash flow) measure, in millions of \$.; e.g., the total net cash flow is, with 90% probability, at or below the 90-th percentile. Calibration period: Aug-48 -- July-83 Lognormal Equity Values; FI returns constant**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
10%	1.3764	1.3545	1.1924	0.9556	1.5654	1.5545	1.4493	1.2263	1.6022	1.5776	1.4425	1.1898
20%	1.8566	1.8604	1.7670	1.5850	2.0656	2.1228	2.1291	2.0097	2.1764	2.1816	2.1224	1.9492
30%	2.2453	2.2785	2.2798	2.2058	2.4159	2.5427	2.7360	2.7493	2.6338	2.6807	2.7312	2.6473
40%	2.6098	2.6763	2.8008	2.8201	2.6981	2.9324	3.3314	3.5543	3.0761	3.1649	3.3314	3.3856
50%	2.9588	3.0710	3.3679	3.5287	2.9459	3.2730	3.9743	4.4947	3.5066	3.6395	3.9930	4.2389
60%	3.3534	3.5186	3.9887	4.3576	3.2148	3.6228	4.6741	5.5603	3.9958	4.1907	4.7524	5.2291
70%	3.7857	4.0183	4.7455	5.4626	3.5179	3.9827	5.4929	6.9721	4.5416	4.7982	5.6540	6.4921
80%	4.3442	4.6690	5.8016	6.9664	3.9033	4.4099	6.6048	9.0316	5.2175	5.6017	6.9139	8.3456
90%	5.2056	5.7400	7.6876	9.9488	4.5169	5.0816	8.3715	13.0098	6.2486	6.9057	9.1816	11.9516

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
10%	3.1282	3.0014	2.4485	1.8016	2.8453	3.2507	3.3037	2.6646	3.8996	3.7748	3.2239	2.4453
20%	3.6219	3.6037	3.3820	2.9216	3.2025	3.7441	4.4270	4.2572	4.4767	4.4725	4.2929	3.8266
30%	4.0135	4.0829	4.1193	3.9121	3.5364	4.0858	5.2449	5.5748	4.9344	5.0391	5.1698	4.9996
40%	4.3658	4.5220	4.8469	4.9113	3.8631	4.3994	6.0737	6.9931	5.3659	5.5774	6.0491	6.2164
50%	4.7204	4.9781	5.6370	6.0645	4.1779	4.7079	6.8591	8.5662	5.7868	6.1084	6.9763	7.6003
60%	5.1034	5.4622	6.5002	7.3659	4.5099	5.0347	7.7248	10.3713	6.2478	6.6985	8.0138	9.1559
70%	5.5472	6.0487	7.5883	9.1062	4.8657	5.4140	8.6935	12.8029	6.7760	7.3869	9.3145	11.2710
80%	6.0936	6.7688	8.9764	11.4212	5.2897	5.8616	9.9005	16.0625	7.4295	8.2483	10.9905	14.0702
90%	6.9191	7.8772	11.3106	15.5910	5.8481	6.5079	11.6007	21.8426	8.4229	9.5908	13.8006	19.1108

**Table 12: Deciles for the TOTAL (net cash flow) measure, in millions of \$.; e.g., the total net cash flow is, with 90% probability, at or below the 90-th percentile. Calibration period: June-71 – May-06 Lognormal Equity Values; FI returns constant**

## Appendix 1: Performance Measures Under Twenty Stochastic Returns Models

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.4044	0.4405	0.5549	0.6621	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.24%	4.69%	5.85%	6.80%	4.88%	5.09%	4.97%	3.83%	4.33%	4.53%	4.62%	3.93%
std_IRR	2.76%	3.27%	4.73%	6.11%	4.05%	4.85%	7.32%	10.17%	3.40%	4.21%	6.91%	10.08%
mean_RUNOUT	16.55	17.20	19.48	22.16	78.04	74.81	68.34	74.05	28.67	35.10	57.44	81.80
std_RUNOUT	36.91	37.94	40.52	42.98	79.47	77.61	79.26	88.56	52.53	59.36	77.60	90.46
PROBRUN	0.2422	0.2447	0.2659	0.2959	0.6528	0.6429	0.5903	0.5605	0.3370	0.3722	0.4921	0.6108

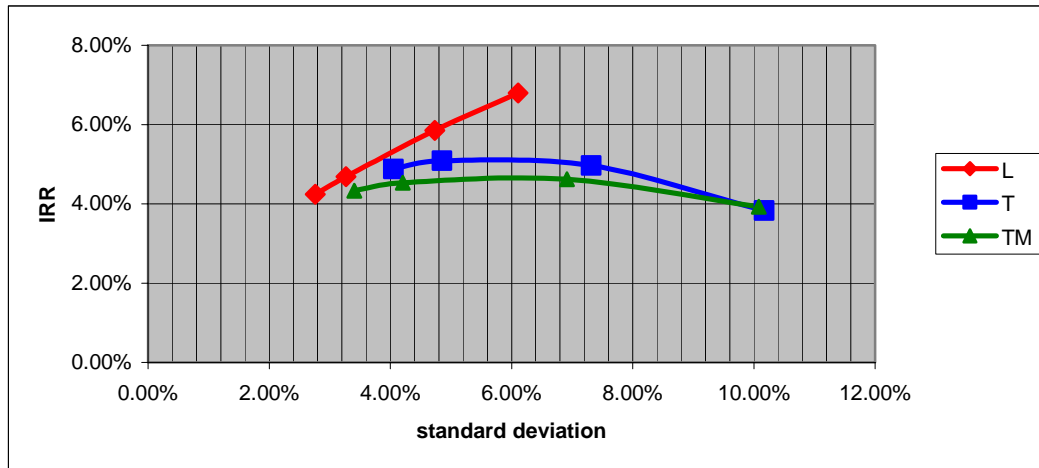
D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.7126	0.7242	0.7943	0.8650	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	5.22%	5.71%	7.05%	8.18%	4.89%	5.09%	4.92%	3.67%	4.24%	4.43%	4.49%	3.78%
std_IRR	1.66%	2.09%	3.34%	4.56%	4.01%	4.80%	7.19%	9.75%	3.21%	3.99%	6.67%	9.84%
mean_RUNOUT	0.00	0.00	0.00	0.00	175.68	169.81	153.79	151.41	73.74	84.45	121.02	158.75
std_RUNOUT	0.00	0.00	0.00	0.00	75.23	76.80	91.04	111.72	78.31	85.51	101.01	107.56
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9490	0.9329	0.8787	0.7662	0.6113	0.6352	0.7326	0.8213

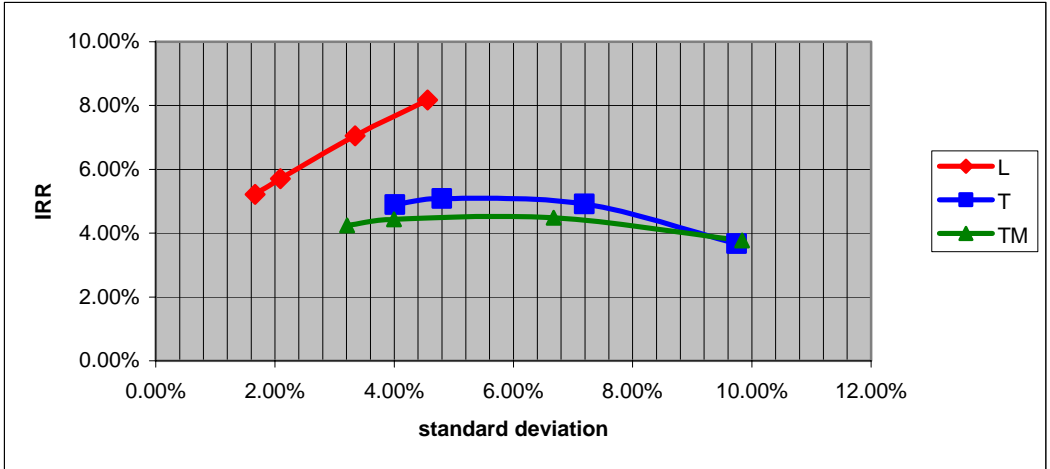
**Table A1-1: Calibration period: Jan-26 -- Dec-60 Lognormal Equity Values; FI returns constant. Volatility of Equity returns increased by 50%**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.5548	0.6096	0.7377	0.8350	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.17%	4.58%	5.61%	6.37%	4.18%	3.98%	2.13%	-1.60%	3.42%	3.23%	1.68%	-1.36%
std_IRR	3.41%	4.01%	5.71%	7.27%	5.79%	6.98%	10.89%	15.97%	4.83%	6.05%	10.31%	15.56%
mean_RUNOUT	21.30	22.41	25.75	29.40	92.07	93.66	102.73	120.66	51.08	62.19	96.85	129.33
std_RUNOUT	41.83	42.94	45.69	48.15	87.12	87.42	93.20	100.75	70.74	78.16	93.47	100.38
PROBRUN	0.2942	0.3041	0.3371	0.3727	0.6858	0.6949	0.7213	0.7669	0.4826	0.5415	0.6896	0.8030

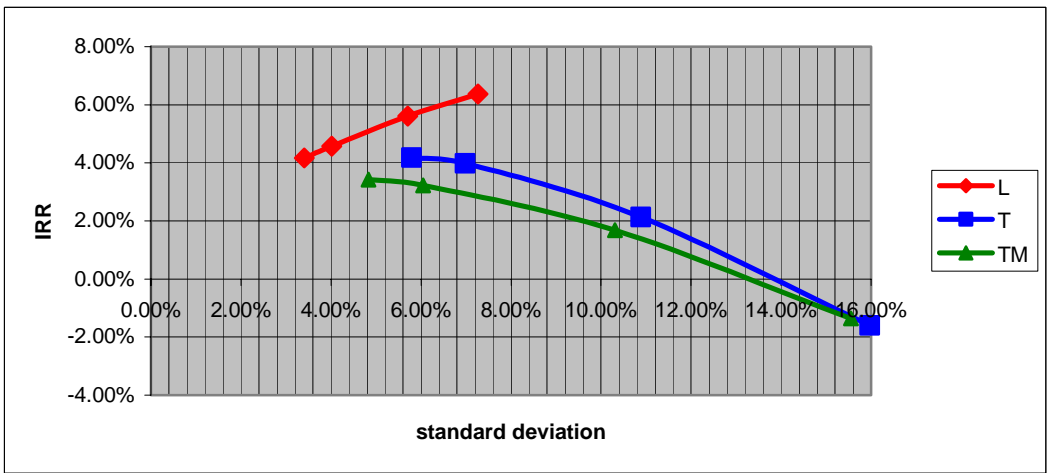
D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.8239	0.8475	0.9167	0.9622	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	5.52%	6.02%	7.30%	8.28%	4.18%	3.96%	2.05%	-1.62%	3.32%	3.12%	1.55%	-1.50%
std_IRR	2.11%	2.61%	4.08%	5.48%	5.74%	6.92%	10.72%	15.25%	4.63%	5.84%	10.09%	15.35%
mean_RUNOUT	0.00	0.00	0.00	0.00	192.20	193.35	201.86	221.11	115.01	132.29	182.80	226.91
std_RUNOUT	0.00	0.00	0.00	0.00	84.35	85.48	94.44	103.68	91.93	97.28	102.90	97.34
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9307	0.9246	0.9229	0.9182	0.7562	0.7935	0.8838	0.9472

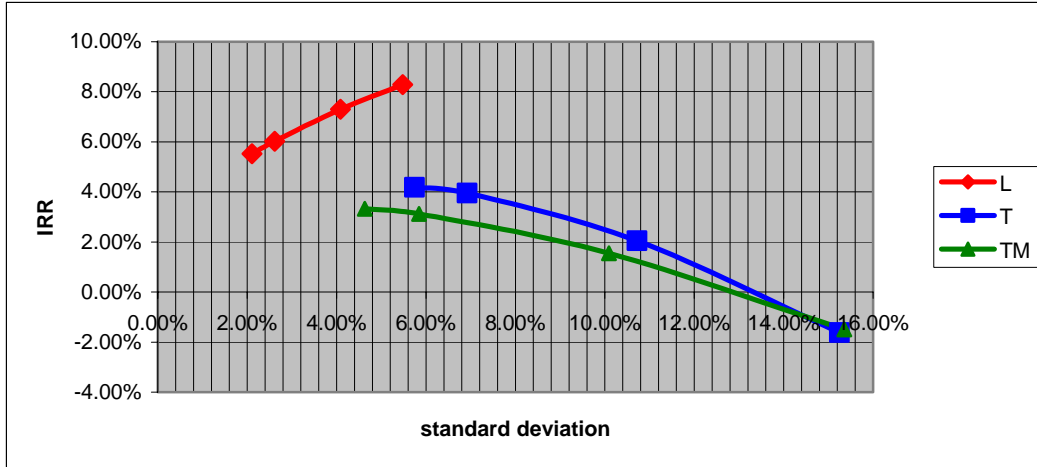
**Table A1-2: Calibration period: Jan-26 -- Dec-60 Lognormal Equity Values; FI returns constant. Volatility of Equity returns increased by 100%**





**Figure A1-1: Calibration period: Jan-26 – Dec-60; Lognormal Equity Values with volatility increased by 50%; FI returns constant. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**





**Figure A1-2: Calibration period: Jan-26 – Dec-60; Lognormal Equity Values with volatility increased by 100%; FI returns constant. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.1773	0.2028	0.2979	0.4121	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	6.21%	6.99%	9.05%	10.70%	7.37%	8.09%	9.51%	9.86%	6.71%	7.42%	9.04%	9.91%
std_IRR	3.00%	3.60%	5.36%	7.09%	4.01%	4.76%	6.99%	9.40%	3.24%	3.98%	6.49%	9.46%
mean_RUNOUT	7.27	7.78	9.73	12.30	88.36	78.37	47.39	36.70	10.19	13.20	25.67	42.14
std_RUNOUT	25.29	26.53	29.86	33.29	82.87	78.92	67.59	70.85	33.14	38.99	57.59	74.25
PROBRUN	0.1121	0.1152	0.1395	0.1707	0.7027	0.6634	0.4809	0.3009	0.1321	0.1555	0.2416	0.3478

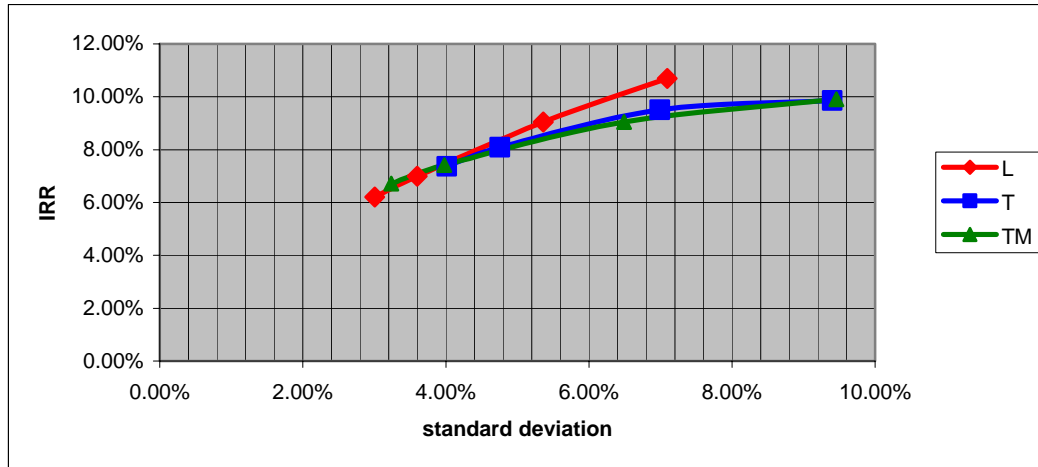
D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.3821	0.3972	0.4870	0.6060	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	6.58%	7.39%	9.58%	11.40%	7.41%	8.14%	9.55%	9.71%	6.65%	7.35%	8.95%	9.79%
std_IRR	2.19%	2.70%	4.23%	5.77%	3.95%	4.67%	6.85%	9.06%	2.99%	3.70%	6.18%	9.14%
mean_RUNOUT	0.00	0.00	0.00	0.00	183.68	169.69	117.99	80.65	28.50	34.37	58.06	88.28
std_RUNOUT	0.00	0.00	0.00	0.00	80.82	81.22	86.64	105.53	56.71	64.43	87.19	105.13
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9360	0.9137	0.7995	0.4619	0.2836	0.3060	0.4082	0.5336

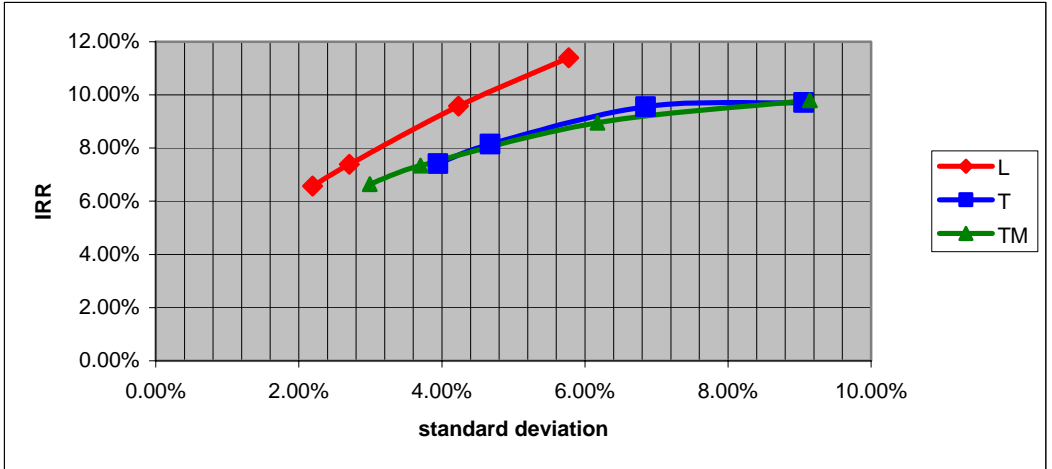
**Table A1-3: Calibration period: Jan-26 -- Dec-60 Lognormal Equity Values; FI returns constant. Mean and Volatility of Equity returns increased by 50%**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.3145	0.3283	0.3552	0.3833	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	2.55%	2.71%	3.20%	3.67%	3.09%	3.24%	3.63%	3.91%	3.02%	3.18%	3.59%	3.92%
std_IRR	0.98%	1.16%	1.69%	2.22%	1.11%	1.32%	1.95%	2.60%	1.04%	1.25%	1.92%	2.62%
mean_RUNOUT	19.52	18.49	17.04	16.87	25.41	21.68	14.64	15.36	16.56	16.74	19.50	24.07
std_RUNOUT	36.61	36.30	36.30	36.95	41.96	39.21	35.96	40.27	32.71	34.07	39.79	46.53
PROBRUN	0.3190	0.3000	0.2665	0.2509	0.3939	0.3504	0.2201	0.1866	0.3040	0.2922	0.2885	0.3127

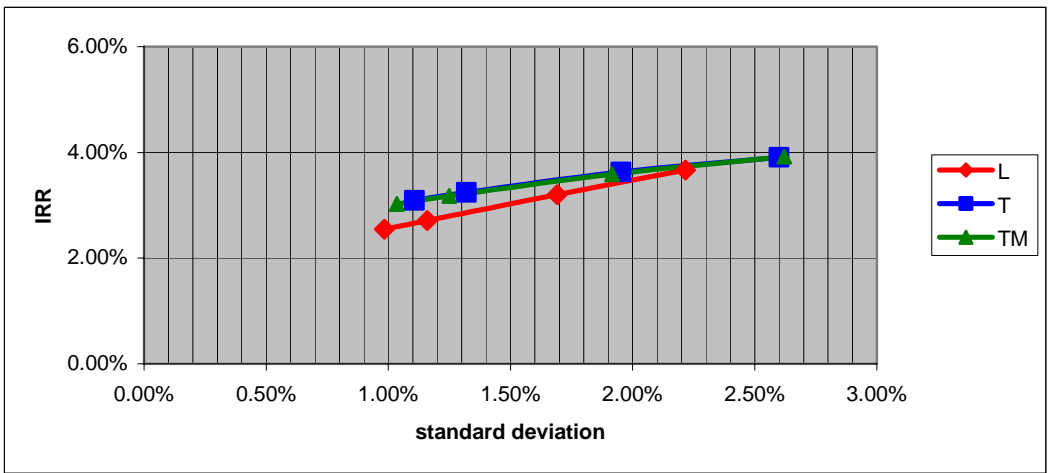
D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.9404	0.8913	0.7846	0.7422	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	3.69%	3.79%	4.19%	4.65%	3.08%	3.23%	3.59%	3.83%	3.00%	3.15%	3.54%	3.86%
std_IRR	0.23%	0.35%	0.77%	1.20%	1.07%	1.27%	1.85%	2.42%	0.95%	1.15%	1.78%	2.45%
mean_RUNOUT	0.00	0.00	0.00	0.00	107.93	98.15	71.73	62.70	61.01	58.86	60.10	66.94
std_RUNOUT	0.00	0.00	0.00	0.00	38.98	44.32	61.15	70.73	47.90	51.91	61.81	70.57
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9984	0.9774	0.7615	0.5780	0.7938	0.7364	0.6472	0.6247

**Table A1-4: Calibration period: Jan-26 -- Dec-60 Lognormal Equity Values; FI returns constant. Mean and Volatility of Equity returns decreased by 50%**

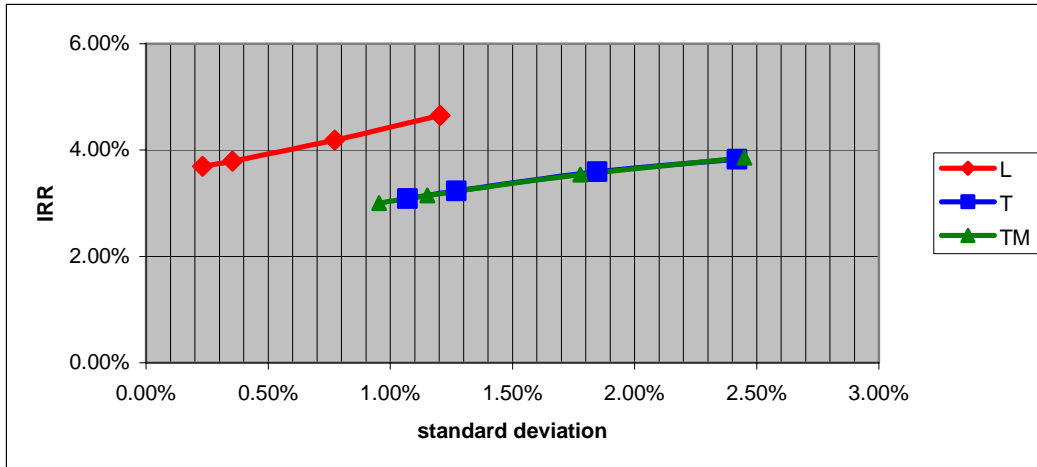




**Figure A1-3: Calibration period: Jan-26 – Dec-60; Lognormal Equity Values with mean and volatility increased by 50%; FI returns constant. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**







**Figure A1-4: Calibration period: Jan-26 – Dec-60; Lognormal Equity Values with mean and volatility decreased by 50%; FI returns constant. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.2933	0.2820	0.2893	0.3420	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.16%	4.68%	6.15%	7.46%	4.69%	5.24%	6.53%	7.34%	4.41%	4.94%	6.33%	7.37%
std_IRR	2.33%	2.62%	3.62%	4.71%	6.94%	7.03%	6.89%	6.61%	6.80%	6.87%	6.75%	6.64%
mean_RUNOUT	12.80	11.76	10.93	11.62	70.98	61.33	32.38	23.13	17.75	17.40	20.47	28.09
std_RUNOUT	32.46	31.56	30.98	32.36	73.33	68.76	55.03	54.18	42.38	42.43	47.99	57.85
PROBRUN	0.2001	0.1803	0.1602	0.1646	0.6577	0.6126	0.3869	0.2221	0.2328	0.2204	0.2313	0.2781

D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.6297	0.5787	0.5250	0.5555	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.90%	5.34%	6.75%	8.11%	4.70%	5.24%	6.51%	7.20%	4.34%	4.86%	6.23%	7.26%
std_IRR	1.42%	1.69%	2.59%	3.54%	6.91%	6.99%	6.79%	6.33%	6.73%	6.79%	6.58%	6.38%
mean_RUNOUT	0.00	0.00	0.00	0.00	164.39	149.65	93.57	59.71	50.48	48.12	52.01	65.37
std_RUNOUT	0.00	0.00	0.00	0.00	67.77	68.65	76.44	87.17	67.61	68.43	76.23	87.87
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9588	0.9404	0.7793	0.4127	0.4981	0.4557	0.4283	0.4687

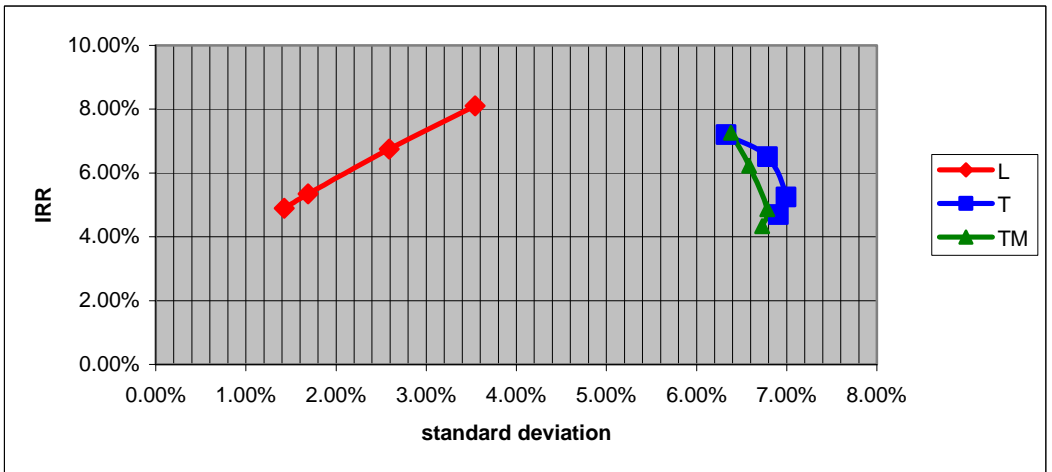
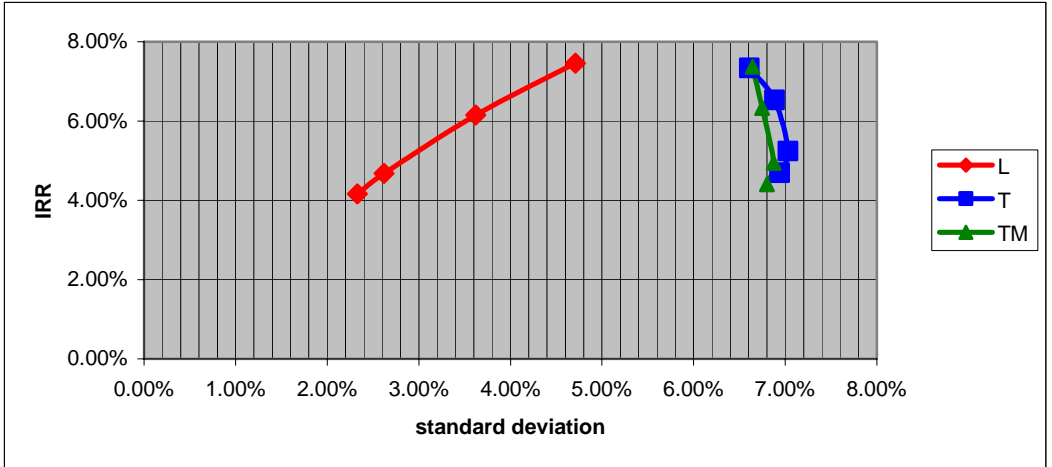
**Table A1-5: Calibration period: Jan-26 -- Dec-60 Lognormal Equity Values; FI returns ARMA/GARCH.**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.4683	0.4882	0.5679	0.6703	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.09%	4.55%	5.78%	6.76%	4.37%	4.66%	4.77%	3.84%	3.84%	4.12%	4.42%	3.93%
std_IRR	2.94%	3.39%	4.78%	6.17%	7.66%	8.03%	9.15%	10.24%	7.33%	7.64%	8.83%	10.21%
mean_RUNOUT	18.15	18.11	19.50	22.11	84.36	80.10	70.46	73.83	35.66	39.98	58.48	81.01
std_RUNOUT	38.51	38.69	40.41	42.94	82.40	80.42	80.18	88.20	59.62	63.81	77.78	89.46
PROBRUN	0.2628	0.2601	0.2684	0.2951	0.6795	0.6667	0.6044	0.5636	0.3921	0.4155	0.5087	0.6123

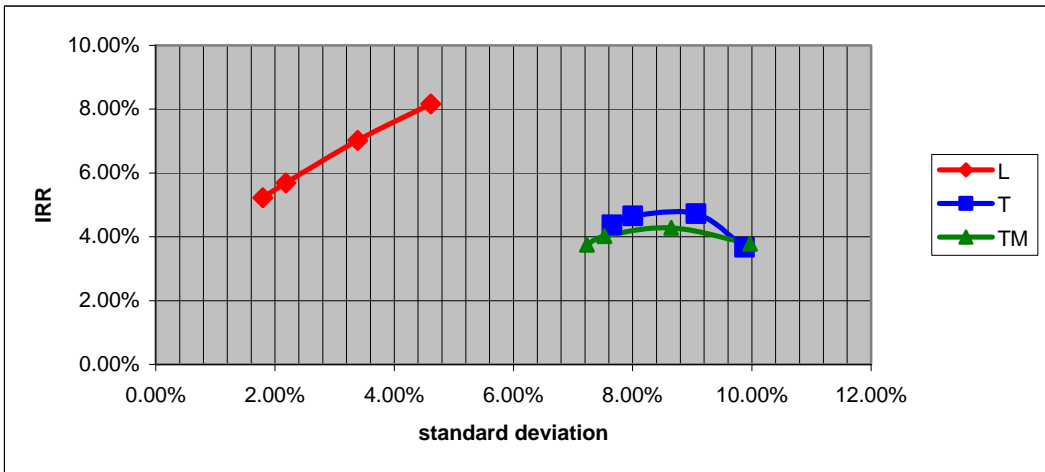
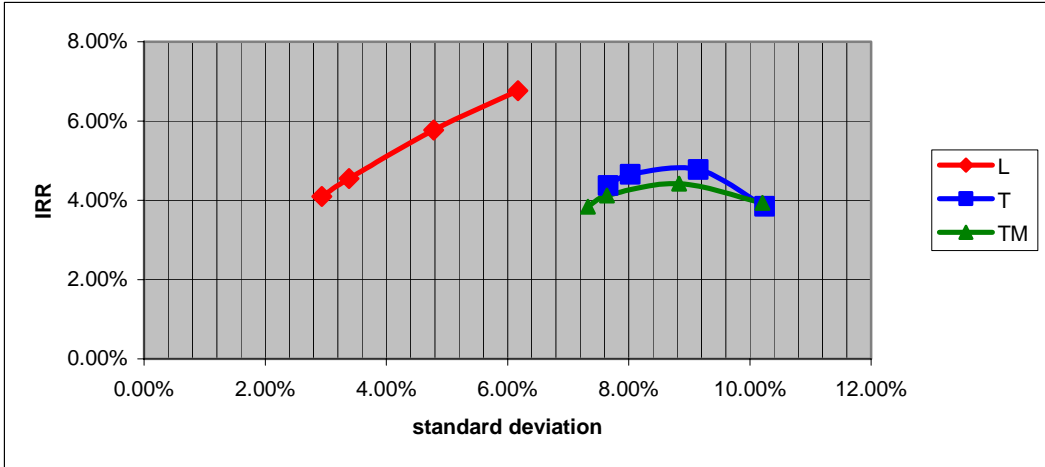
  

D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.7662	0.7610	0.8008	0.8664	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	5.23%	5.69%	7.02%	8.17%	4.37%	4.65%	4.72%	3.68%	3.75%	4.02%	4.29%	3.78%
std_IRR	1.80%	2.18%	3.39%	4.61%	7.65%	8.00%	9.06%	9.88%	7.23%	7.52%	8.64%	9.97%
mean_RUNOUT	0.00	0.00	0.00	0.00	182.63	175.99	157.21	151.21	88.80	95.37	124.34	158.44
std_RUNOUT	0.00	0.00	0.00	0.00	77.28	78.27	90.47	111.31	83.90	88.68	101.10	107.35
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9457	0.9359	0.8853	0.7665	0.6793	0.6806	0.7415	0.8193

**Table A1-6: Calibration period: Jan-26 -- Dec-60 Lognormal Equity Values; FI returns ARMA/GARCH. Volatility of Equity returns increased by 50%**



**Figure A1-5: Calibration period: Jan-26 – Dec-60; Lognormal Equity Values; ARMA/GARCH FI returns. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**



**Figure A1-6: Calibration period: Jan-26 – Dec-60; Lognormal Equity Values with volatility increased by 50%; ARMA/GARCH FI returns. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.5945	0.6325	0.7464	0.8370	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.10%	4.53%	5.62%	6.47%	3.74%	3.64%	2.08%	-1.30%	2.99%	2.88%	1.62%	-1.11%
std_IRR	3.55%	4.11%	5.78%	7.33%	8.77%	9.52%	12.17%	15.72%	8.14%	8.84%	11.68%	15.48%
mean_RUNOUT	22.52	23.17	25.98	28.89	97.30	97.92	104.54	120.66	56.86	66.29	97.70	129.04
std_RUNOUT	42.81	43.51	45.81	47.83	88.52	88.21	92.67	100.06	73.50	79.26	92.76	99.54
PROBRUN	0.3081	0.3130	0.3414	0.3704	0.7126	0.7190	0.7339	0.7702	0.5301	0.5705	0.6999	0.8066

D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.8536	0.8670	0.9249	0.9643	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	5.55%	6.03%	7.33%	8.37%	3.73%	3.61%	1.99%	-1.38%	2.88%	2.76%	1.47%	-1.25%
std_IRR	2.22%	2.69%	4.15%	5.57%	8.75%	9.49%	12.07%	15.17%	8.01%	8.69%	11.49%	15.28%
mean_RUNOUT	0.00	0.00	0.00	0.00	199.37	199.22	204.95	221.04	125.97	139.92	185.20	226.78
std_RUNOUT	0.00	0.00	0.00	0.00	83.27	84.09	92.08	102.95	92.85	96.99	101.22	96.54
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9389	0.9372	0.9300	0.9201	0.7909	0.8169	0.8963	0.9504

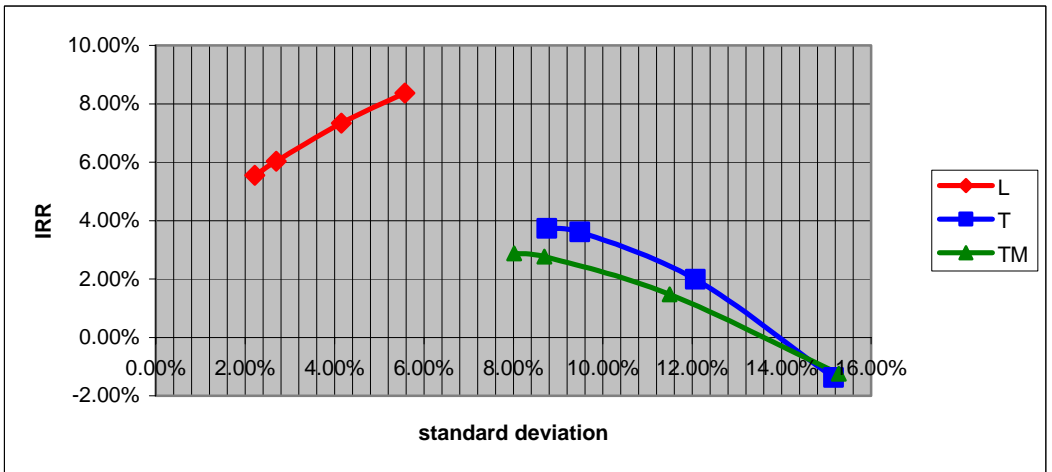
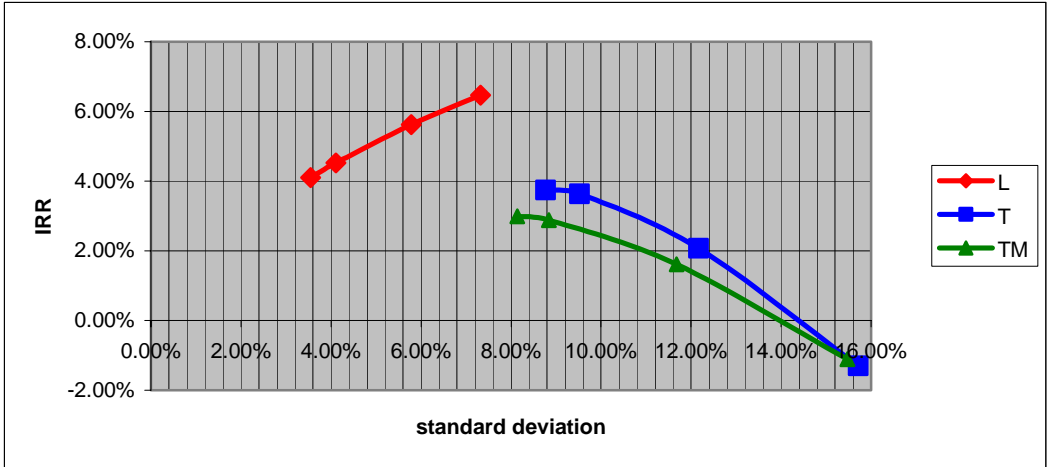
**Table A1-7: Calibration period: Jan-26 -- Dec-60 Lognormal Equity Values; FI returns ARMA/GARCH. Volatility of Equity returns increased by 100%**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.2481	0.2568	0.3245	0.4267	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	5.93%	6.75%	8.91%	10.67%	6.84%	7.64%	9.28%	9.82%	6.19%	6.97%	8.80%	9.88%
std_IRR	3.21%	3.74%	5.42%	7.14%	7.69%	8.01%	8.91%	9.46%	7.26%	7.54%	8.51%	9.50%
mean_RUNOUT	9.25	9.04	9.98	12.22	93.85	83.80	51.78	37.71	16.02	17.58	27.47	43.10
std_RUNOUT	28.28	28.13	29.94	33.24	85.63	81.68	69.36	70.62	42.39	45.23	58.72	73.59
PROBRUN	0.1429	0.1375	0.1475	0.1726	0.7210	0.6820	0.5114	0.3145	0.1912	0.2007	0.2635	0.3627

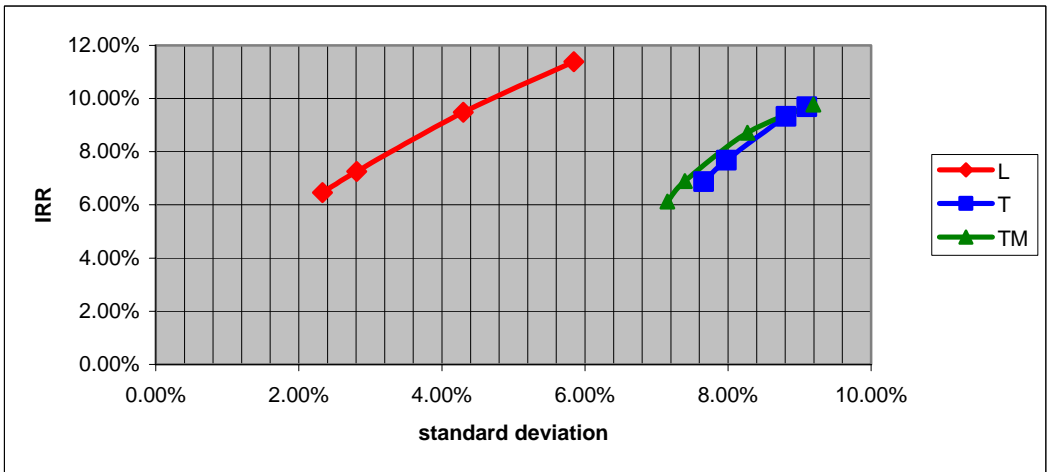
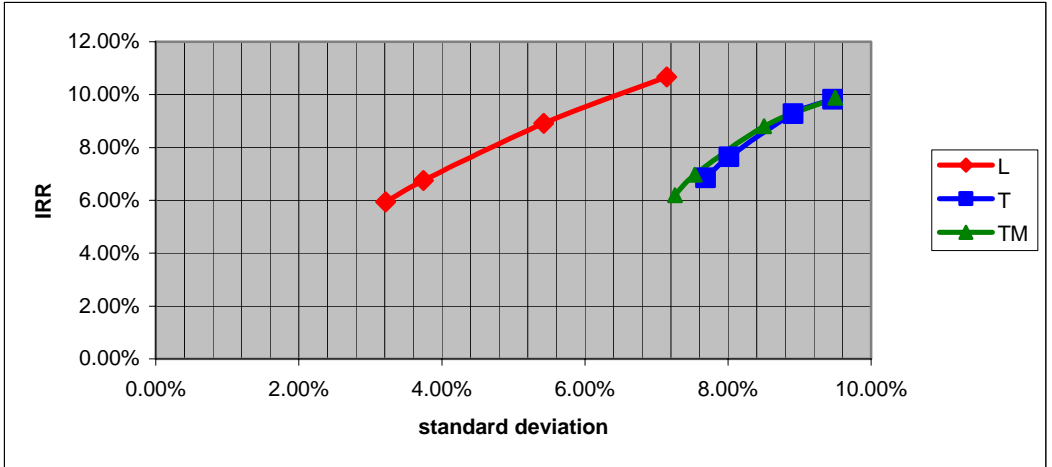
  

D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.4826	0.4724	0.5190	0.6154	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	6.46%	7.26%	9.49%	11.38%	6.87%	7.68%	9.33%	9.69%	6.13%	6.90%	8.72%	9.78%
std_IRR	2.33%	2.81%	4.30%	5.85%	7.66%	7.98%	8.81%	9.10%	7.15%	7.40%	8.27%	9.19%
mean_RUNOUT	0.00	0.00	0.00	0.00	190.87	176.78	124.23	82.44	42.21	44.71	62.55	90.39
std_RUNOUT	0.00	0.00	0.00	0.00	80.70	81.40	86.81	105.24	68.84	72.62	88.84	104.68
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9399	0.9219	0.8169	0.4746	0.3803	0.3768	0.4371	0.5500

**Table A1-8: Calibration period: Jan-26 -- Dec-60 Lognormal equity values; FI returns ARMA/GARCH. Mean and Volatility of Equity returns increased by 50%**



**Figure A1-7: Calibration period: Jan-26 – Dec-60; Lognormal Equity Values with volatility increased by 100%; ARMA/GARCH FI returns. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**



**Figure A1-8: Calibration period: Jan-26 – Dec-60; Lognormal Equity Values with mean and volatility increased by 50%; ARMA/GARCH FI returns. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.4664	0.4454	0.4075	0.4002	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	2.42%	2.59%	3.11%	3.63%	2.53%	2.76%	3.34%	3.80%	2.50%	2.72%	3.31%	3.82%
std_IRR	1.51%	1.55%	1.82%	2.25%	6.50%	6.45%	6.45%	3.76%	6.50%	6.44%	6.44%	3.78%
mean_RUNOUT	23.60	21.97	18.50	16.82	38.84	33.85	21.00	17.01	29.87	27.48	24.13	25.13
std_RUNOUT	41.53	40.28	37.78	36.77	55.09	51.57	43.35	42.85	48.97	47.36	45.60	48.19
PROBRUN	0.3494	0.3303	0.2826	0.2542	0.4769	0.4388	0.2899	0.2000	0.4122	0.3835	0.3307	0.3225

D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.9399	0.9122	0.8148	0.7512	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	3.86%	3.92%	4.22%	4.66%	2.51%	2.74%	3.29%	3.71%	2.46%	2.68%	3.25%	3.74%
std_IRR	0.70%	0.70%	0.86%	1.22%	6.49%	6.43%	6.41%	3.64%	6.47%	6.41%	6.40%	3.66%
mean_RUNOUT	0.00	0.00	0.00	0.00	121.99	112.56	82.56	66.54	88.93	82.34	71.49	70.79
std_RUNOUT	0.00	0.00	0.00	0.00	57.21	57.85	65.85	72.65	62.64	63.85	67.27	72.35
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9469	0.9322	0.7910	0.5942	0.8518	0.8074	0.6917	0.6415

**Table A1-9: Calibration period: Jan-26 -- Dec-60 Lognormal equity values; FI returns ARMA/GARCH. Mean and Volatility of Equity returns decreased by 50%**

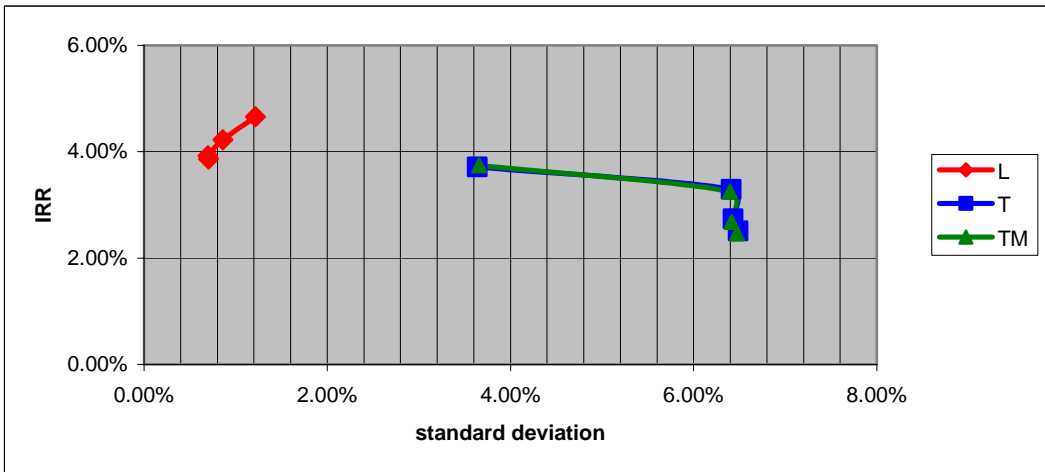
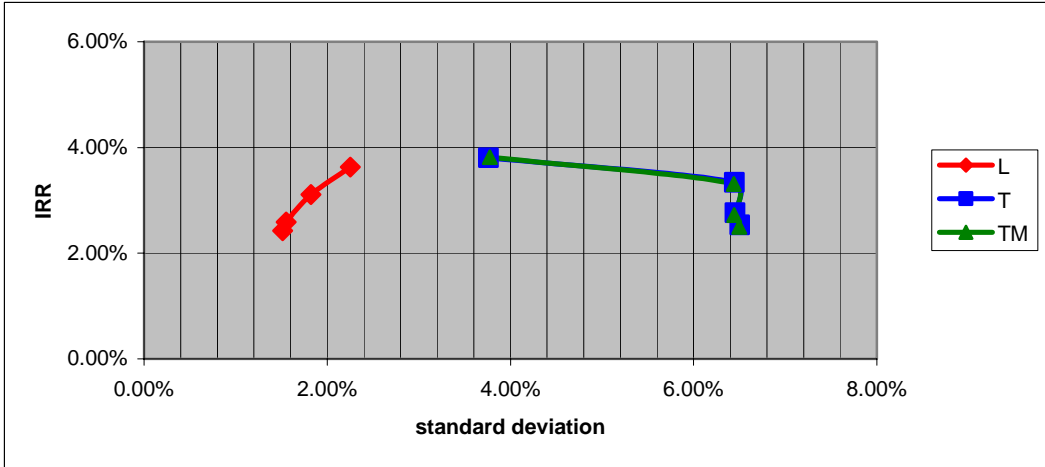
D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.2246	0.2261	0.2730	0.3355	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.30%	4.79%	6.16%	7.37%	5.12%	5.58%	6.67%	7.33%	4.80%	5.26%	6.46%	7.36%
std_IRR	2.07%	2.42%	3.51%	4.59%	2.63%	3.04%	4.26%	5.55%	2.24%	2.66%	4.03%	5.59%
mean_RUNOUT	10.15	9.74	9.97	11.12	64.49	55.35	29.22	22.49	11.43	12.52	18.34	27.10
std_RUNOUT	28.48	28.32	29.48	31.35	70.17	65.38	52.00	53.31	32.31	34.95	45.31	56.89
PROBRUN	0.1681	0.1570	0.1506	0.1620	0.6283	0.5774	0.3629	0.2192	0.1717	0.1746	0.2098	0.2723

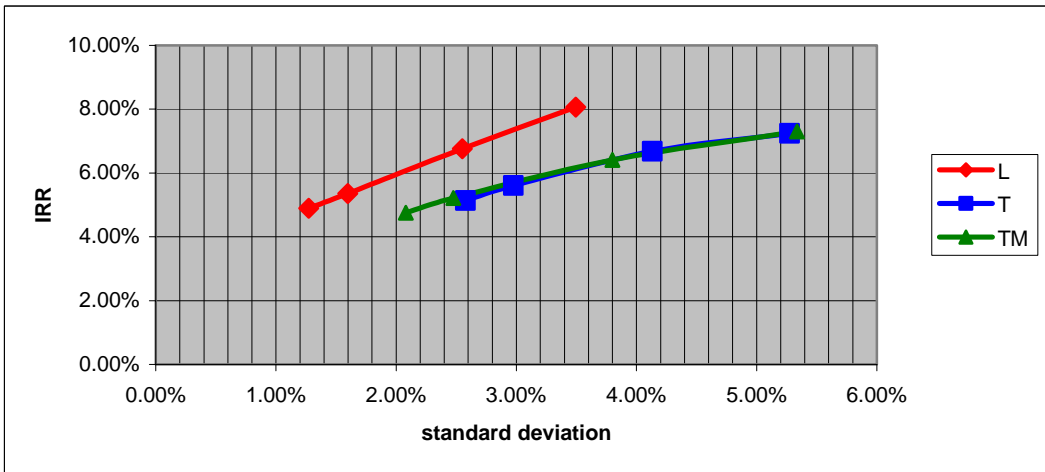
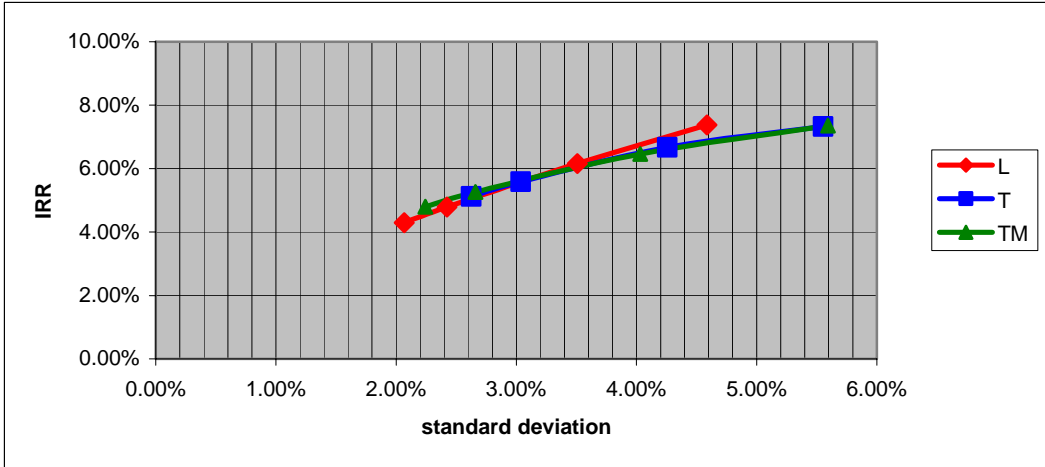
D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.5567	0.5206	0.5084	0.5545	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.89%	5.36%	6.76%	8.06%	5.14%	5.61%	6.68%	7.24%	4.76%	5.22%	6.41%	7.30%
std_IRR	1.27%	1.60%	2.55%	3.49%	2.58%	2.98%	4.13%	5.28%	2.08%	2.48%	3.80%	5.34%
mean_RUNOUT	0.00	0.00	0.00	0.00	155.67	140.74	86.13	58.63	36.29	37.48	47.99	64.29
std_RUNOUT	0.00	0.00	0.00	0.00	63.93	65.39	74.79	85.72	55.85	59.53	72.76	86.47
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9671	0.9457	0.7499	0.4126	0.4137	0.3947	0.4114	0.4717

**Table A1-10: Sampling period: Jan-26 -- Dec-60 Historical distributions with monthly Bootstrapping**





**Figure A1-9: Calibration period: Jan-26 – Dec-60; Lognormal Equity Values with mean and volatility decreased by 50%; ARMA/GARCH FI returns. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**



**Figure A1-10: Sampling period: Jan-26 – Dec-60; Historical distributions with monthly Bootstrapping. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.0000	0.0001	0.0055	0.0292	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	7.93%	8.12%	8.61%	9.00%	8.63%	8.82%	9.29%	9.62%	8.53%	8.73%	9.23%	9.63%
std_IRR	1.17%	1.41%	2.17%	2.94%	1.25%	1.49%	2.18%	2.91%	1.16%	1.40%	2.14%	2.94%
mean_RUNOUT	0.00	0.01	0.26	1.22	7.54	4.28	0.20	0.80	0.00	0.00	0.18	1.37
std_RUNOUT	0.00	1.11	4.70	10.79	25.28	18.17	4.28	9.93	0.00	0.00	4.12	12.26
PROBRUN	0.0000	0.0002	0.0052	0.0205	0.1256	0.0815	0.0044	0.0103	0.0000	0.0000	0.0034	0.0200

D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.0000	0.0015	0.0218	0.0831	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	7.94%	8.13%	8.63%	9.07%	8.68%	8.88%	9.33%	9.61%	8.54%	8.73%	9.24%	9.63%
std_IRR	0.99%	1.20%	1.84%	2.46%	1.13%	1.33%	1.92%	2.56%	0.98%	1.19%	1.84%	2.60%
mean_RUNOUT	0.00	0.00	0.00	0.00	26.01	16.55	1.41	3.46	0.00	0.00	0.65	4.13
std_RUNOUT	0.00	0.00	0.00	0.00	47.00	36.21	10.89	21.15	0.00	0.18	8.47	22.55
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.3203	0.2412	0.0291	0.0387	0.0000	0.0001	0.0083	0.0482

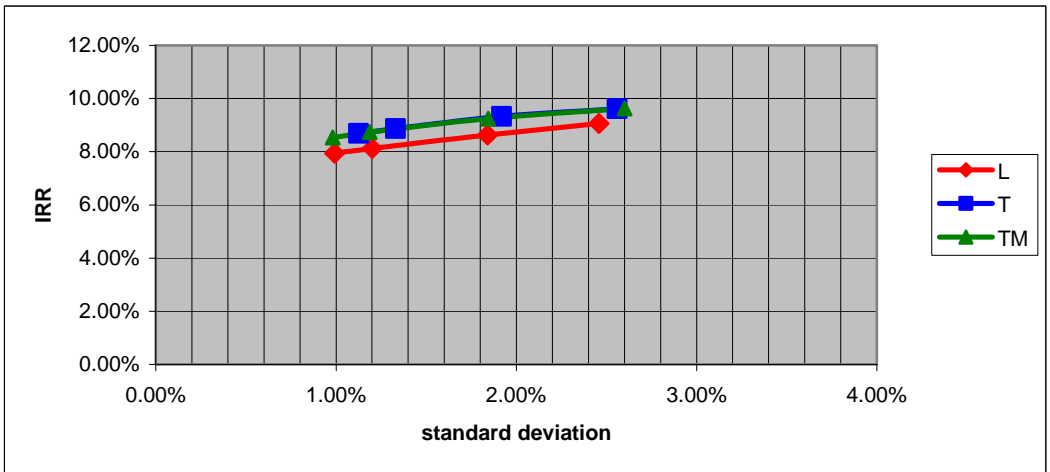
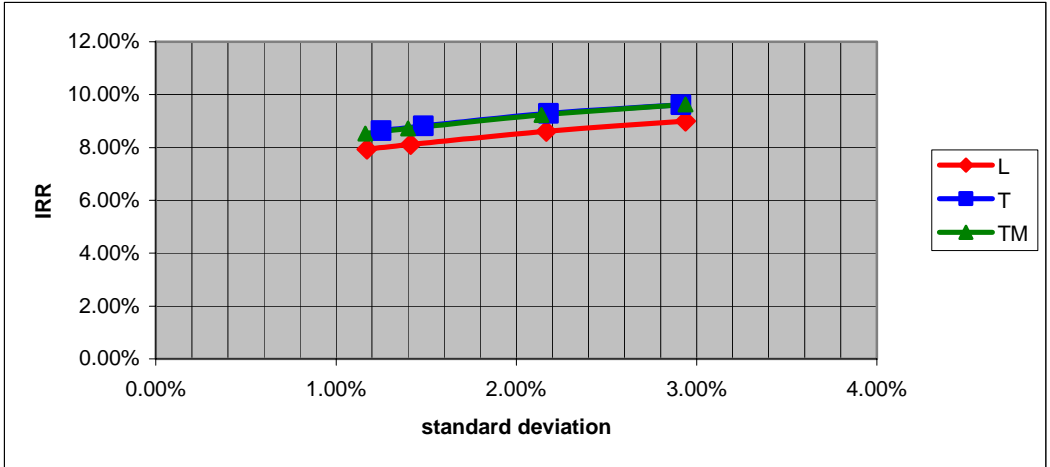
**Table A1-11: Calibration period: June-71 – May-06; Lognormal equity values; FI constant**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.0172	0.0147	0.0170	0.0344	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	7.38%	7.65%	8.35%	8.92%	8.00%	8.29%	9.02%	9.53%	8.00%	8.27%	8.98%	9.54%
std_IRR	2.07%	2.09%	2.41%	2.99%	2.15%	2.15%	2.42%	2.97%	2.06%	2.07%	2.38%	2.99%
mean_RUNOUT	0.84	0.64	0.73	1.44	16.72	11.11	0.99	0.95	0.69	0.53	0.58	1.62
std_RUNOUT	8.25	7.24	7.76	11.16	39.41	31.21	8.80	10.70	8.21	7.07	7.40	13.12
PROBRUN	0.0158	0.0132	0.0136	0.0236	0.2277	0.1702	0.0237	0.0114	0.0119	0.0092	0.0100	0.0228

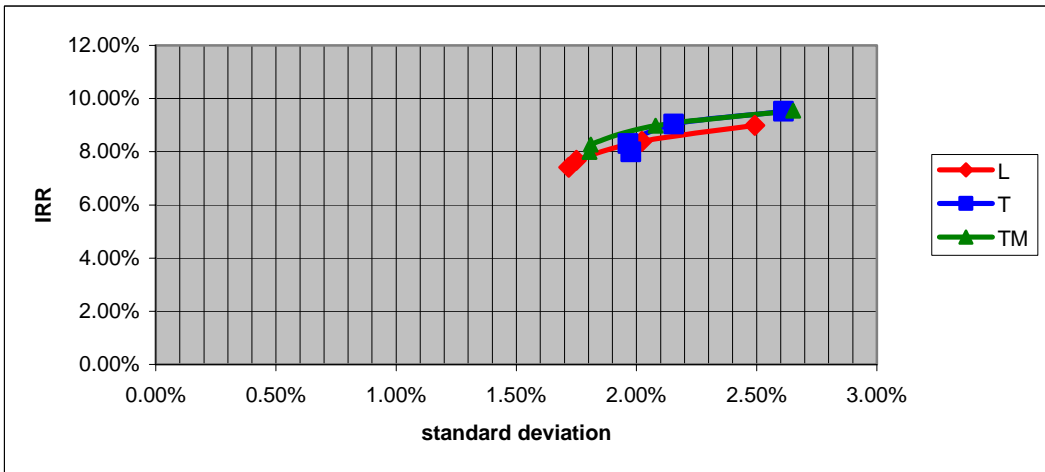
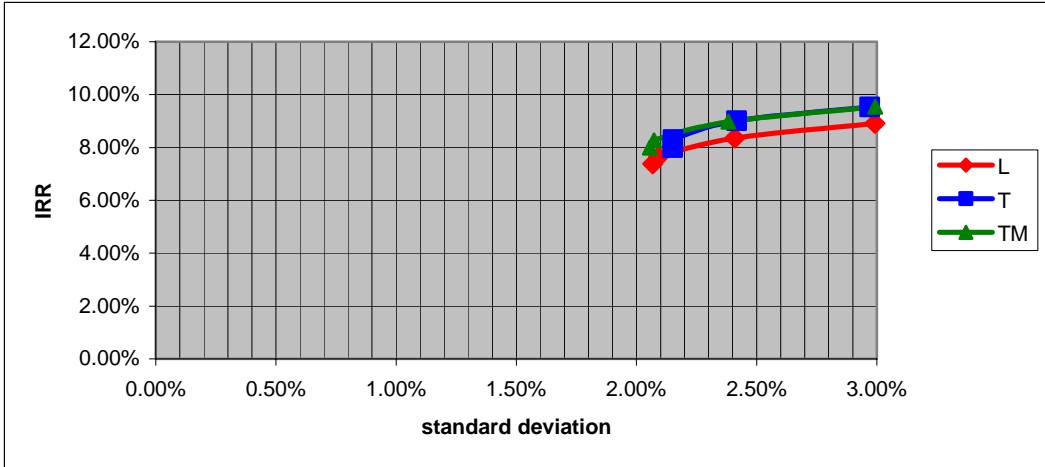
  

D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.0636	0.0516	0.0518	0.0924	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	7.42%	7.68%	8.39%	8.99%	8.00%	8.30%	9.04%	9.52%	8.00%	8.27%	8.98%	9.55%
std_IRR	1.72%	1.75%	2.02%	2.49%	1.98%	1.96%	2.15%	2.61%	1.80%	1.81%	2.08%	2.65%
mean_RUNOUT	0.00	0.00	0.00	0.00	50.02	36.70	5.53	3.96	2.36	1.86	1.98	4.72
std_RUNOUT	0.00	0.00	0.00	0.00	66.18	56.28	21.67	22.73	16.51	14.33	14.61	24.20
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.4768	0.4035	0.0983	0.0414	0.0303	0.0249	0.0260	0.0544

**Table A1-12: Calibration period: June-71 – May-06; Lognormal equity values; FI ARMA/GARCH**



**Figure A1-11: Calibration period: June-71 – May-06; Lognormal equity values; FI constant. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**



**Figure A1-12: Calibration period: June-71 – May-06; Lognormal equity values; FI ARMA/GARCH. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.0121	0.0123	0.0199	0.0444	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	7.62%	7.84%	8.42%	8.89%	8.27%	8.51%	9.09%	9.50%	8.24%	8.46%	9.05%	9.51%
std_IRR	2.11%	2.18%	2.56%	3.11%	2.18%	2.23%	2.57%	3.11%	2.09%	2.16%	2.54%	3.13%
mean_RUNOUT	0.63	0.63	0.93	1.86	11.95	7.47	0.91	1.28	0.46	0.43	0.79	2.14
std_RUNOUT	6.63	6.73	8.75	12.73	32.15	24.64	9.22	12.53	6.24	6.08	8.85	15.28
PROBRUN	0.0130	0.0122	0.0168	0.0327	0.1809	0.1306	0.0176	0.0154	0.0086	0.0075	0.0128	0.0300

D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.0505	0.0487	0.0648	0.1114	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	7.67%	7.89%	8.49%	9.00%	8.31%	8.55%	9.13%	9.51%	8.27%	8.49%	9.08%	9.53%
std_IRR	1.78%	1.85%	2.16%	2.60%	2.00%	2.03%	2.30%	2.76%	1.83%	1.89%	2.24%	2.81%
mean_RUNOUT	0.00	0.00	0.00	0.00	38.61	26.94	4.45	5.38	1.55	1.47	2.51	6.35
std_RUNOUT	0.00	0.00	0.00	0.00	56.36	46.54	20.80	26.61	12.51	12.34	16.90	28.34
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.4276	0.3513	0.0745	0.0544	0.0236	0.0214	0.0330	0.0693

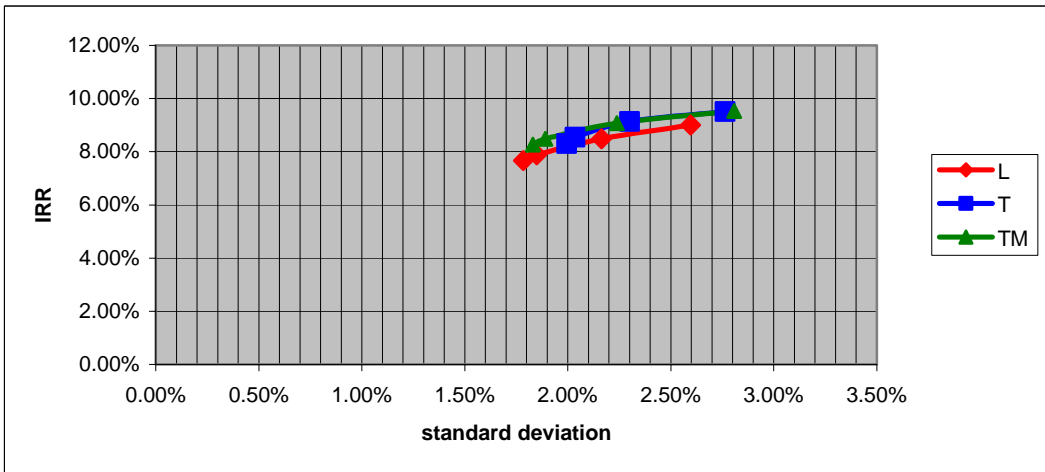
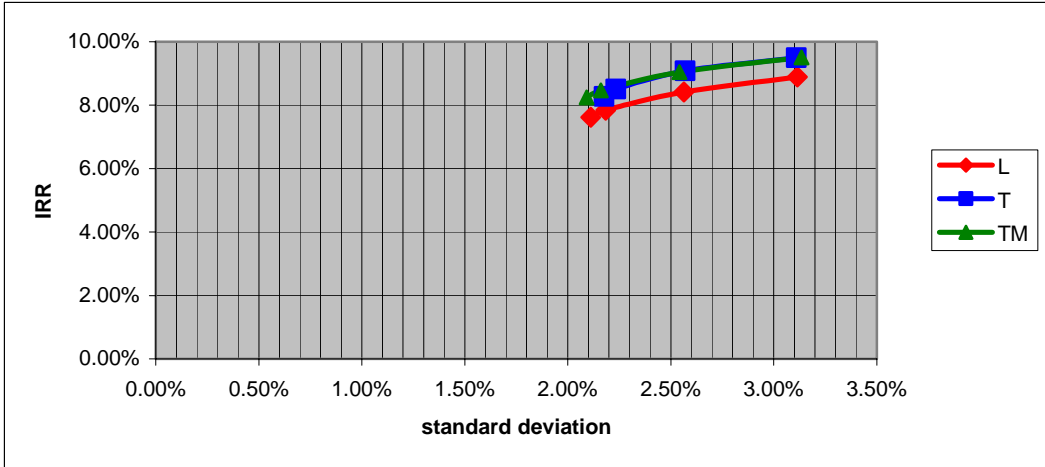
**Table A1-13: Sampling period: June-71 – May-06; Historical Distributions with monthly bootstrapping**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.0266	0.0224	0.0227	0.0285	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.61%	5.18%	6.84%	8.43%	5.35%	5.92%	7.54%	9.04%	5.23%	5.80%	7.46%	9.05%
std_IRR	1.12%	1.35%	2.03%	2.72%	1.26%	1.48%	2.09%	2.68%	1.11%	1.33%	2.01%	2.71%
mean_RUNOUT	1.73	1.40	1.13	1.18	49.98	37.10	3.78	0.65	0.71	0.65	0.79	1.20
std_RUNOUT	10.70	9.95	9.80	10.40	57.91	50.10	14.99	8.46	6.55	6.64	8.29	11.10
PROBRUN	0.0404	0.0298	0.0203	0.0192	0.5956	0.5085	0.0978	0.0094	0.0195	0.0158	0.0147	0.0183

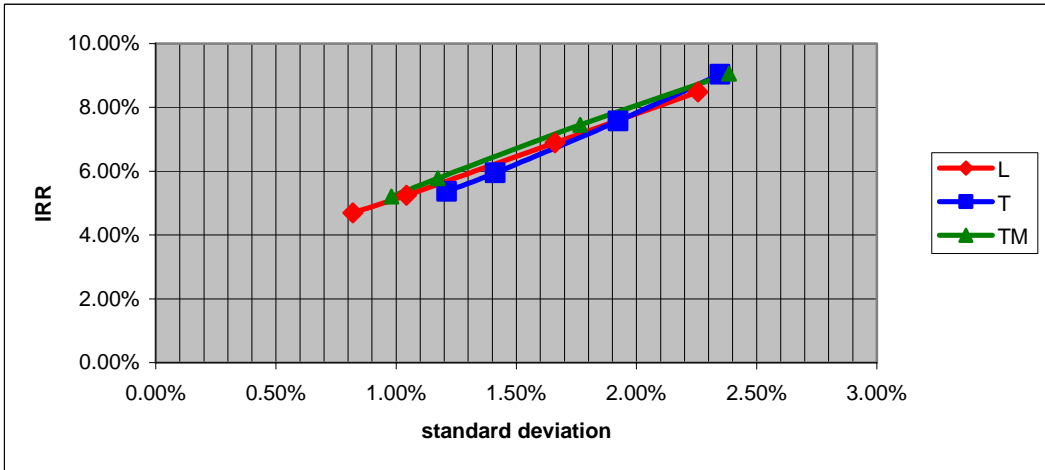
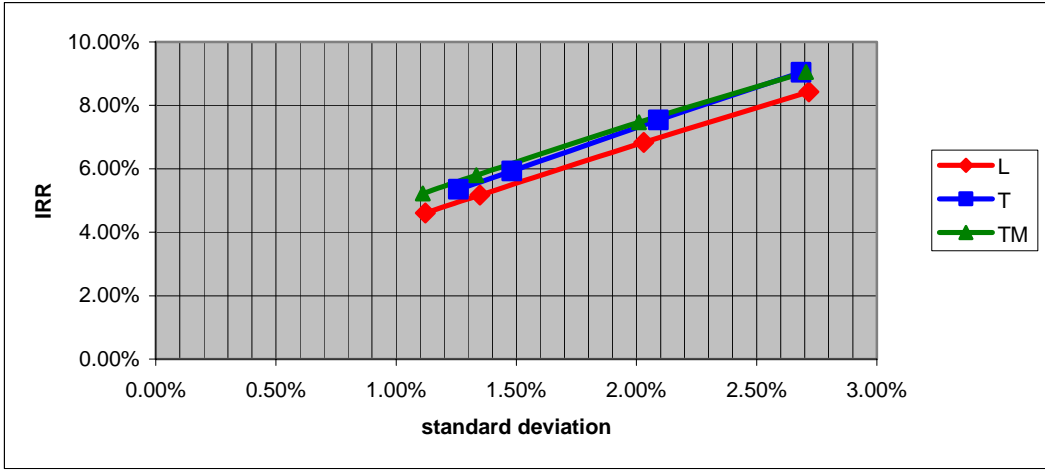
  

D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.2133	0.1480	0.0920	0.0847	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.70%	5.24%	6.89%	8.49%	5.37%	5.94%	7.57%	9.03%	5.21%	5.78%	7.45%	9.05%
std_IRR	0.82%	1.04%	1.66%	2.26%	1.21%	1.41%	1.92%	2.35%	0.98%	1.17%	1.77%	2.38%
mean_RUNOUT	0.00	0.00	0.00	0.00	134.31	111.24	20.40	3.25	3.43	2.96	2.98	3.89
std_RUNOUT	0.00	0.00	0.00	0.00	50.48	51.20	32.86	19.90	14.89	14.71	16.97	21.27
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9904	0.9615	0.4287	0.0370	0.0810	0.0603	0.0461	0.0482

**Table A1-14: Calibration period: Aug-48 -- July-83; Lognormal equity values; FI constant**



**Figure A1-13: Sampling period: June-71 -- May-06; Historical distribution with monthly bootstrapping. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**



**Figure A1-14: Calibration period: Aug-48 -- July-83; Lognormal equity values; FI constant. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**



D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.1524	0.1114	0.0567	0.0418	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	3.95%	4.55%	6.35%	8.11%	4.47%	5.13%	6.99%	8.71%	4.38%	5.03%	6.93%	8.72%
std_IRR	1.62%	1.75%	2.23%	2.81%	2.67%	2.59%	2.55%	2.83%	2.57%	2.48%	2.48%	2.85%
mean_RUNOUT	7.89	5.51	2.57	1.81	61.09	47.78	8.43	1.34	8.62	6.14	2.89	2.03
std_RUNOUT	24.73	20.93	14.77	12.77	64.06	57.32	26.53	13.35	30.56	26.36	18.77	15.45
PROBRUN	0.1416	0.1006	0.0459	0.0316	0.6524	0.5782	0.1607	0.0149	0.1267	0.0869	0.0406	0.0275

D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.5180	0.3775	0.1675	0.1100	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.42%	4.86%	6.49%	8.21%	4.47%	5.13%	7.02%	8.72%	4.34%	5.00%	6.92%	8.74%
std_IRR	0.96%	1.16%	1.75%	2.32%	2.62%	2.53%	2.41%	2.53%	2.47%	2.36%	2.28%	2.56%
mean_RUNOUT	0.00	0.00	0.00	0.00	150.98	129.44	35.38	4.93	26.57	18.43	8.26	5.70
std_RUNOUT	0.00	0.00	0.00	0.00	53.00	54.06	45.69	26.27	51.86	45.54	32.58	27.43
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9825	0.9695	0.5875	0.0495	0.3305	0.2230	0.0975	0.0625

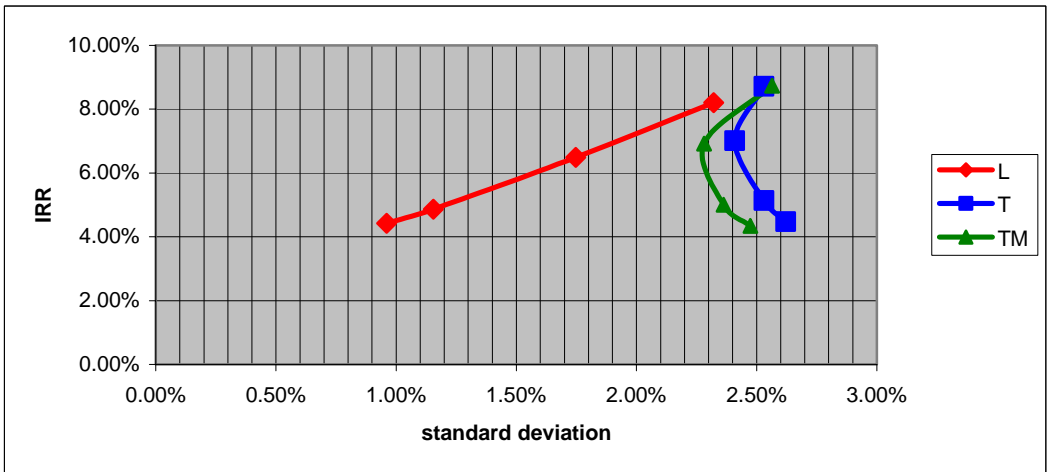
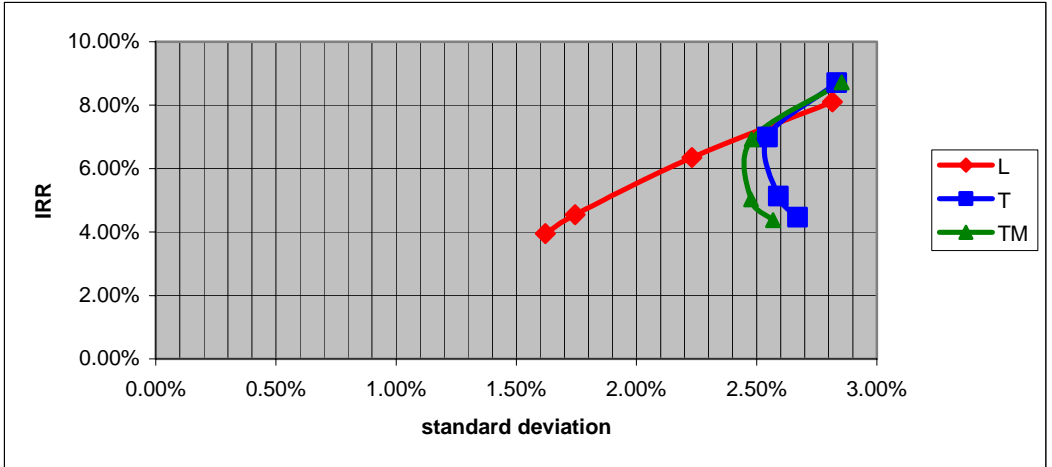
**Table A1-15: Calibration period: Aug-48 -- July-83; Lognormal Equity Values; FI ARMA/GARCH**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.1672	0.1168	0.0495	0.0375	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.36%	4.94%	6.68%	8.36%	5.03%	5.64%	7.37%	8.97%	4.92%	5.53%	7.30%	8.98%
std_IRR	1.82%	1.94%	2.36%	2.87%	2.10%	2.16%	2.44%	2.84%	1.92%	2.00%	2.36%	2.87%
mean_RUNOUT	7.60	5.33	2.44	1.70	56.86	43.76	6.69	0.82	7.18	4.87	2.18	1.75
std_RUNOUT	24.79	21.08	14.59	12.41	61.25	54.18	21.22	9.19	24.94	20.66	14.08	13.06
PROBRUN	0.1358	0.0956	0.0417	0.0278	0.6360	0.5586	0.1458	0.0124	0.1200	0.0812	0.0343	0.0265

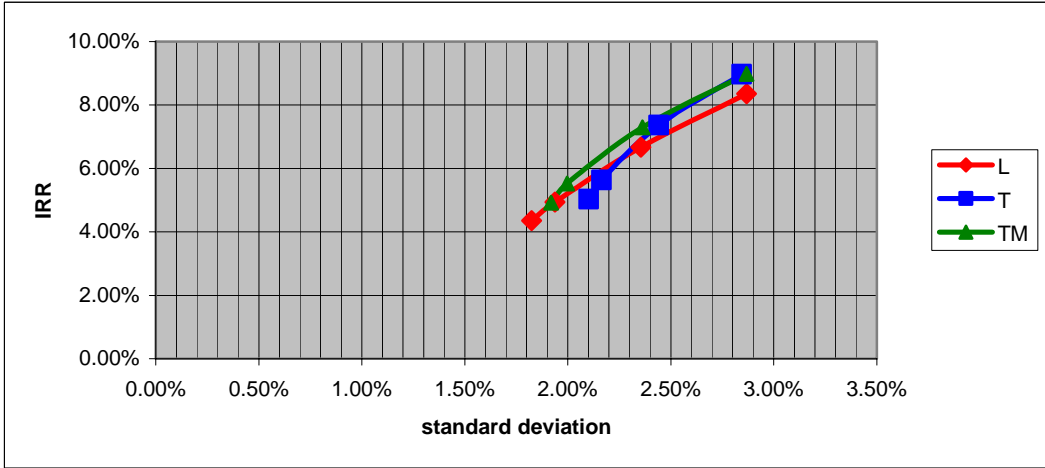
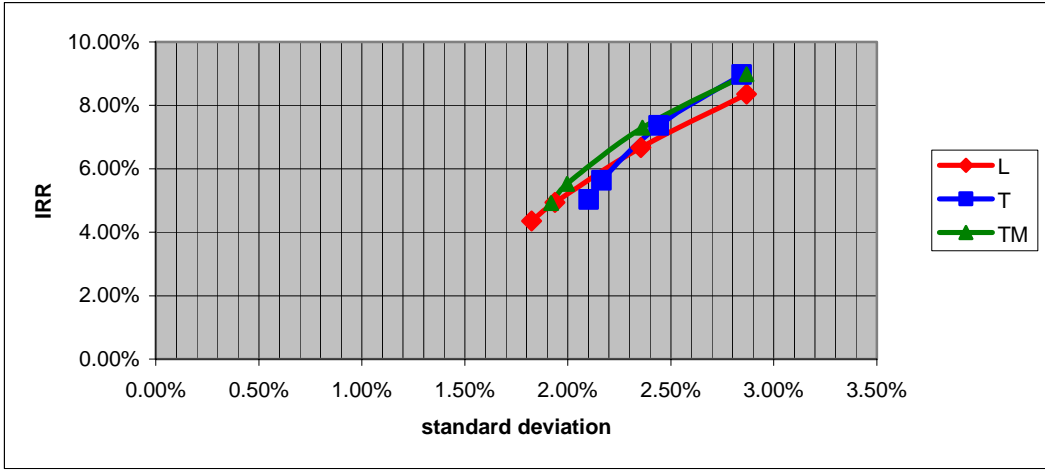
  

D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.4929	0.3685	0.1657	0.1059	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.78%	5.22%	6.78%	8.43%	5.03%	5.64%	7.38%	8.95%	4.86%	5.48%	7.28%	8.97%
std_IRR	1.14%	1.34%	1.87%	2.36%	2.04%	2.08%	2.27%	2.50%	1.75%	1.81%	2.11%	2.54%
mean_RUNOUT	0.00	0.00	0.00	0.00	145.05	123.27	30.61	4.58	25.21	17.03	7.31	5.41
std_RUNOUT	0.00	0.00	0.00	0.00	52.13	52.77	41.57	23.92	45.99	39.25	27.68	25.53
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9769	0.9589	0.5319	0.0491	0.3320	0.2293	0.0970	0.0620

**Table A1-16: Sampling period: Aug-48 -- July-83; Historical distributions with monthly bootstrapping**



**Figure A1-15: Calibration period: Aug-48 -- July-83; Lognormal Equity Values; FI ARMA/GARCH. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**



**Figure A1-16: Sampling period: Aug-48 -- July-83; Historical distributions with monthly bootstrapping. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.2413	0.2460	0.2990	0.3650	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
mean_IRR	4.37%	4.90%	6.41%	7.77%	5.12%	5.60%	6.72%	7.33%	4.79%	5.27%	6.50%	7.37%
std_IRR	2.24%	2.61%	3.76%	4.91%	2.94%	3.41%	4.90%	6.61%	2.54%	3.02%	4.67%	6.65%
mean_RUNOUT	10.52	10.13	10.44	11.62	70.57	61.76	35.28	28.20	13.86	15.37	22.84	33.50
std_RUNOUT	29.72	29.54	30.53	32.39	72.89	68.80	58.30	60.86	36.82	40.14	52.10	64.48
PROBRUN	0.1682	0.1586	0.1555	0.1665	0.6609	0.6156	0.4059	0.2574	0.1923	0.1953	0.2413	0.3095

D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.5614	0.5280	0.5233	0.5718	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
mean_IRR	4.98%	5.48%	7.01%	8.45%	5.12%	5.61%	6.70%	7.17%	4.72%	5.19%	6.39%	7.24%
std_IRR	1.37%	1.70%	2.71%	3.71%	2.89%	3.35%	4.79%	6.34%	2.37%	2.83%	4.44%	6.40%
mean_RUNOUT	0.00	0.00	0.00	0.00	167.44	153.62	99.62	68.62	41.46	43.29	56.24	74.83
std_RUNOUT	0.00	0.00	0.00	0.00	63.88	65.58	79.15	95.04	62.15	66.44	81.23	95.34
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9710	0.9540	0.8098	0.4426	0.4280	0.4118	0.4384	0.5006

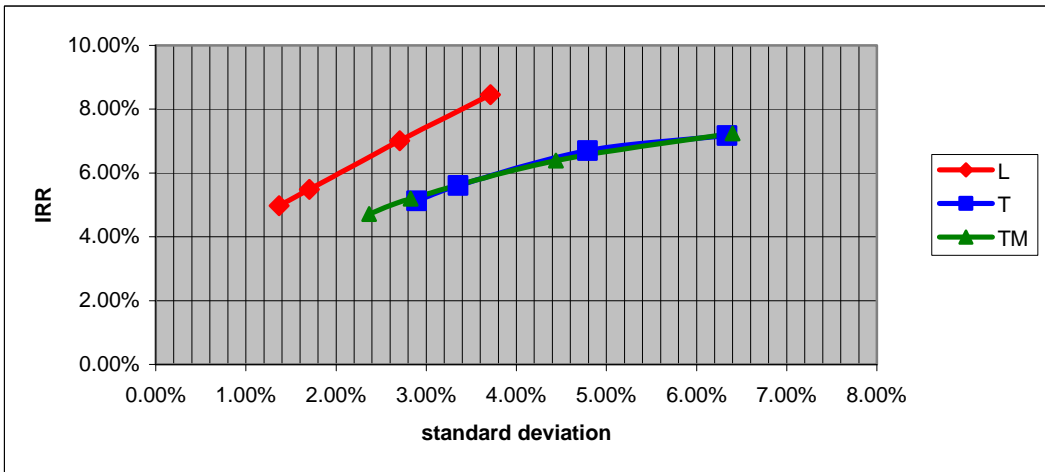
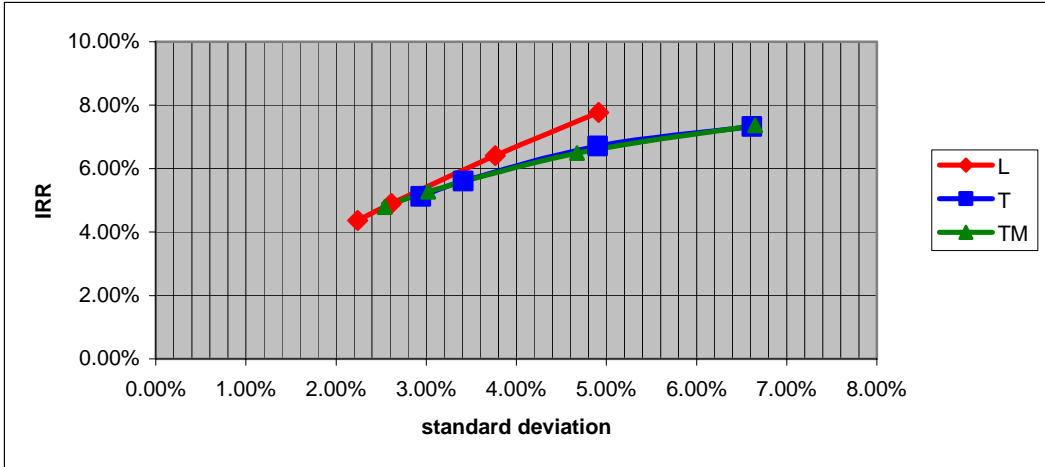
**Table A1-17: Sampling period: Jan-26 -- Dec-60 Historical Distributions; bootstrapping with 12 month batches**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.0087	0.0090	0.0178	0.0418	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
mean_IRR	7.88%	8.10%	8.66%	9.12%	8.52%	8.75%	9.32%	9.73%	8.50%	8.72%	9.29%	9.74%
std_IRR	2.16%	2.22%	2.55%	3.06%	2.22%	2.26%	2.56%	3.06%	2.13%	2.19%	2.54%	3.09%
mean_RUNOUT	0.50	0.53	0.78	1.60	10.59	6.52	0.80	1.26	0.31	0.35	0.79	1.97
std_RUNOUT	6.28	6.52	8.12	11.73	30.20	22.99	8.84	12.36	4.76	5.26	9.07	14.97
PROBRUN	0.0088	0.0092	0.0141	0.0273	0.1648	0.1161	0.0146	0.0150	0.0060	0.0066	0.0109	0.0272

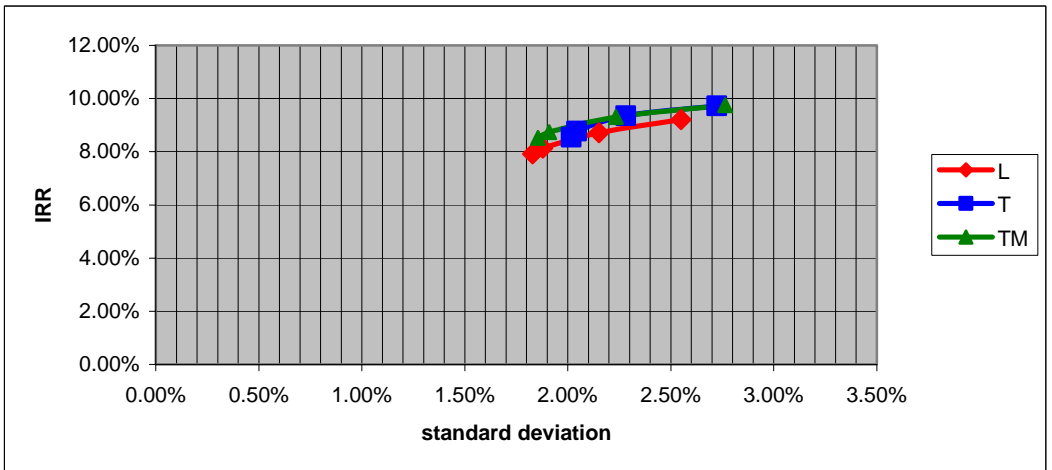
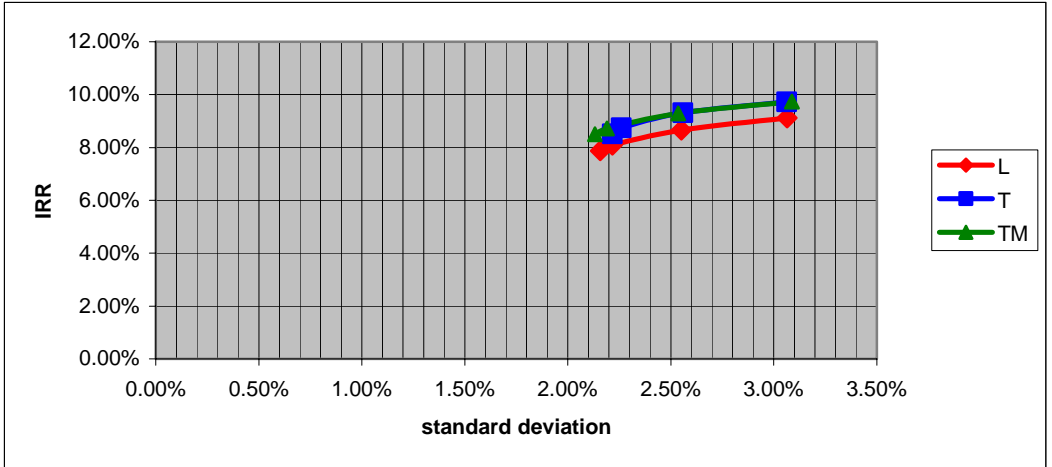
  

D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.0391	0.0392	0.0567	0.1041	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
mean_IRR	7.92%	8.14%	8.72%	9.21%	8.54%	8.78%	9.35%	9.73%	8.52%	8.74%	9.30%	9.75%
std_IRR	1.83%	1.88%	2.15%	2.55%	2.02%	2.04%	2.28%	2.72%	1.85%	1.91%	2.23%	2.76%
mean_RUNOUT	0.00	0.00	0.00	0.00	36.00	24.78	4.18	5.01	1.13	1.23	2.45	5.98
std_RUNOUT	0.00	0.00	0.00	0.00	54.47	44.62	20.61	26.00	10.54	11.23	16.98	27.73
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.4115	0.3274	0.0669	0.0491	0.0173	0.0181	0.0303	0.0655

**Table A1-18: Sampling period: June-71 -- May-06; Historical Distributions; bootstrapping with 12 month batches.**



**Figure A1-17: Sampling period: Jan-26 -- Dec-60 Historical Distributions; bootstrapping with 12 month batches. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**



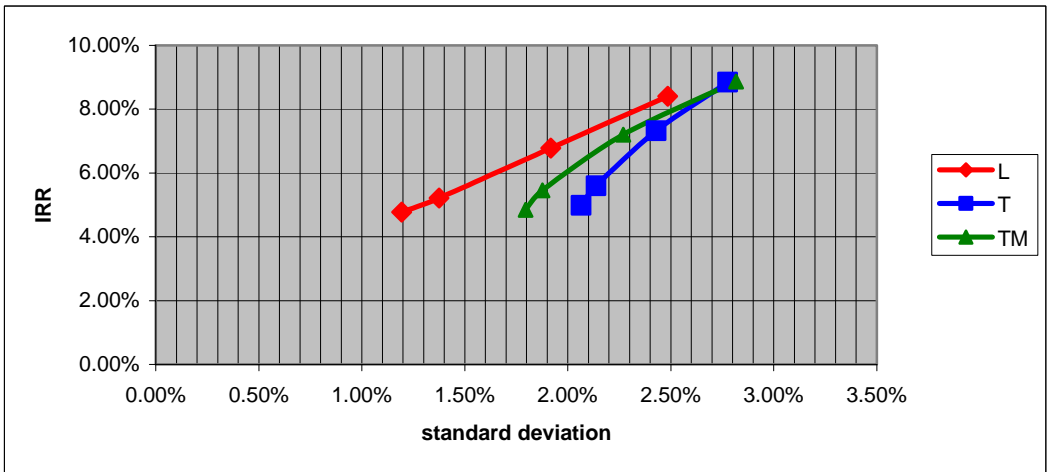
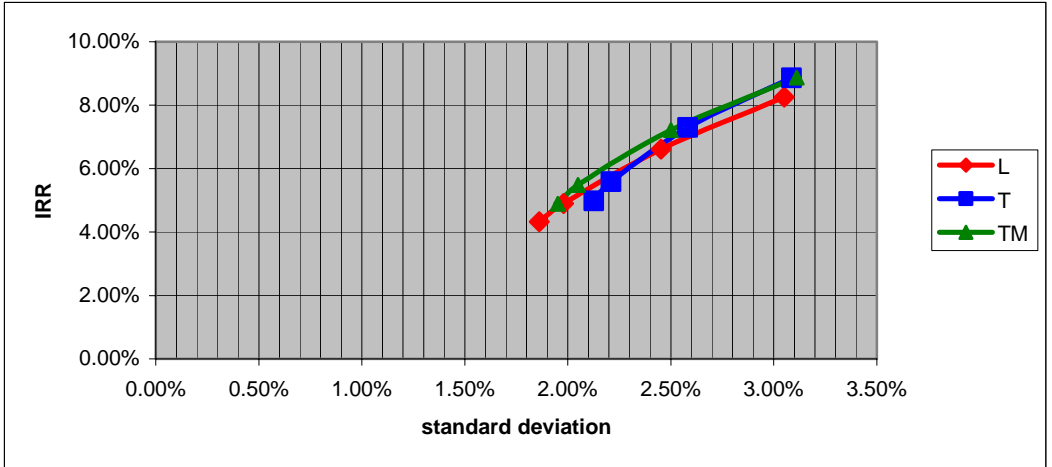
**Figure A1-18: Sampling period: June-71 -- May-06; Historical Distributions; bootstrapping with 12 month batches. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.1556	0.1161	0.0693	0.0625	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
mean_IRR	4.32%	4.90%	6.61%	8.26%	4.98%	5.59%	7.29%	8.86%	4.89%	5.48%	7.22%	8.87%
std_IRR	1.86%	1.98%	2.45%	3.05%	2.13%	2.21%	2.58%	3.09%	1.95%	2.05%	2.50%	3.11%
mean_RUNOUT	7.71	5.62	3.01	2.45	58.86	46.16	9.19	2.11	7.29	5.33	3.30	3.20
std_RUNOUT	24.88	21.42	16.23	14.91	63.47	56.85	26.28	16.18	25.34	22.12	18.39	19.07
PROBRUN	0.1348	0.0983	0.0519	0.0401	0.6392	0.5596	0.1799	0.0250	0.1170	0.0860	0.0487	0.0428

D=35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
PROBGAR	0.4932	0.3733	0.1996	0.1524	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
mean_IRR	4.77%	5.22%	6.78%	8.40%	4.99%	5.60%	7.32%	8.85%	4.85%	5.45%	7.21%	8.87%
std_IRR	1.19%	1.37%	1.92%	2.49%	2.06%	2.14%	2.43%	2.78%	1.79%	1.88%	2.27%	2.82%
mean_RUNOUT	0.00	0.00	0.00	0.00	147.98	127.19	38.40	8.13	25.33	18.22	10.49	9.49
std_RUNOUT	0.00	0.00	0.00	0.00	54.88	55.48	47.77	32.95	46.40	41.25	34.42	34.83
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9720	0.9547	0.5852	0.0798	0.3301	0.2382	0.1233	0.0989

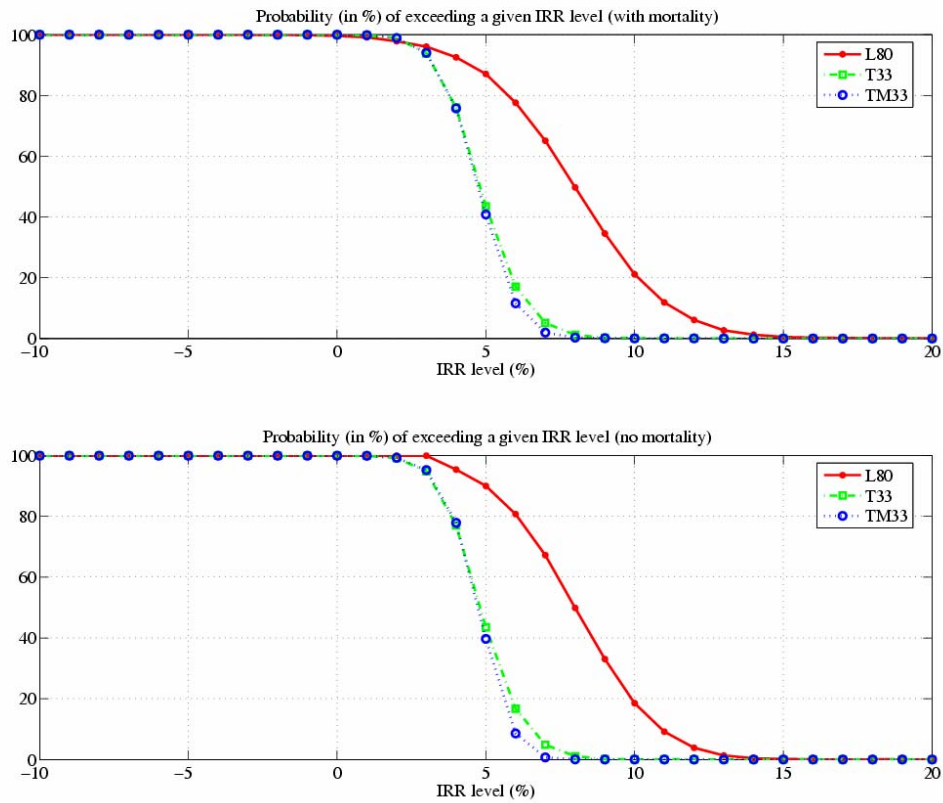
**Table A1-19: Sampling period: Aug-48 -- July-83; Historical Distributions; bootstrapping with 12 month batches.**



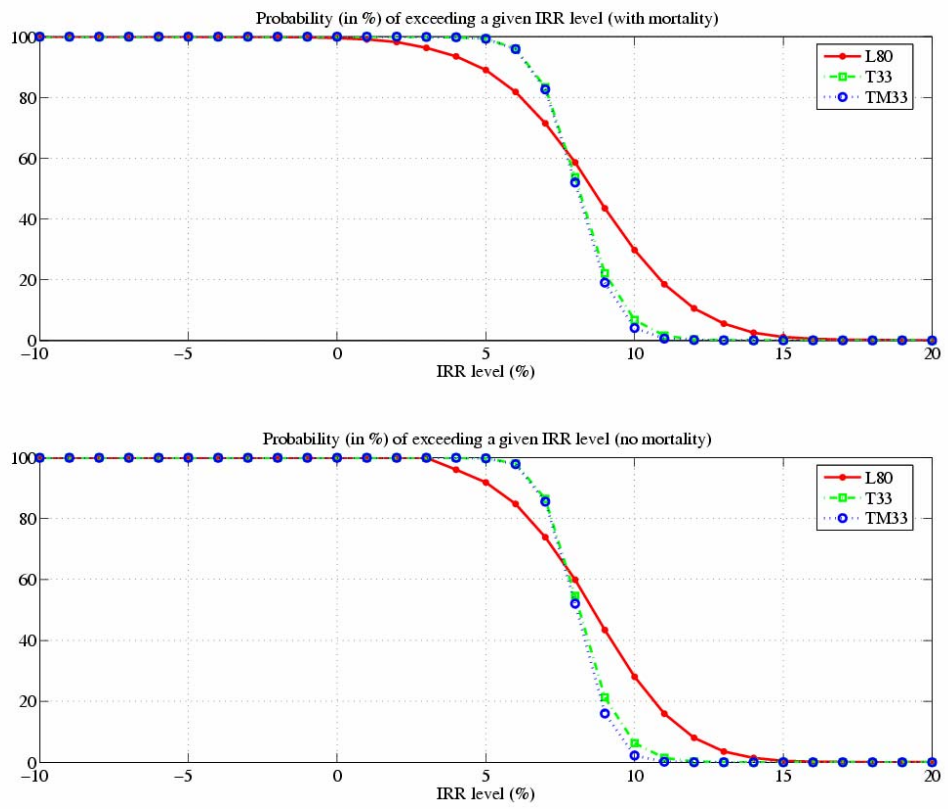
**Figure A1-19: Sampling period: Aug-48 -- July-83; Historical Distributions; bootstrapping with 12 month batches. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit to a 35 year horizon.**



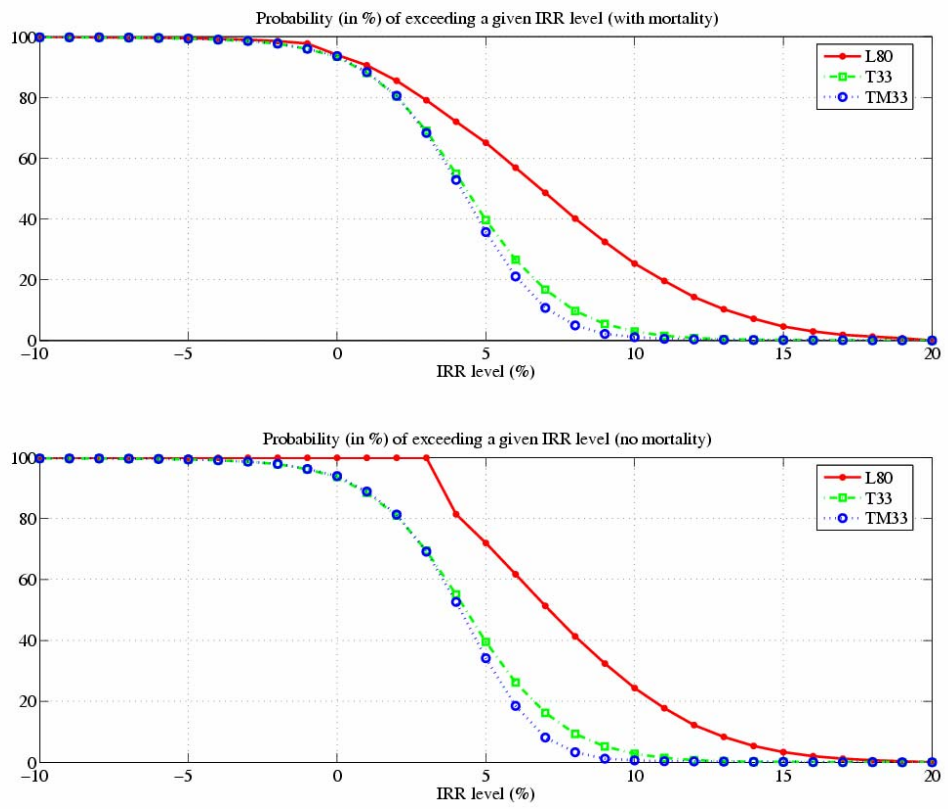
## Appendix 2: Distributions of the Internal Rate of Return



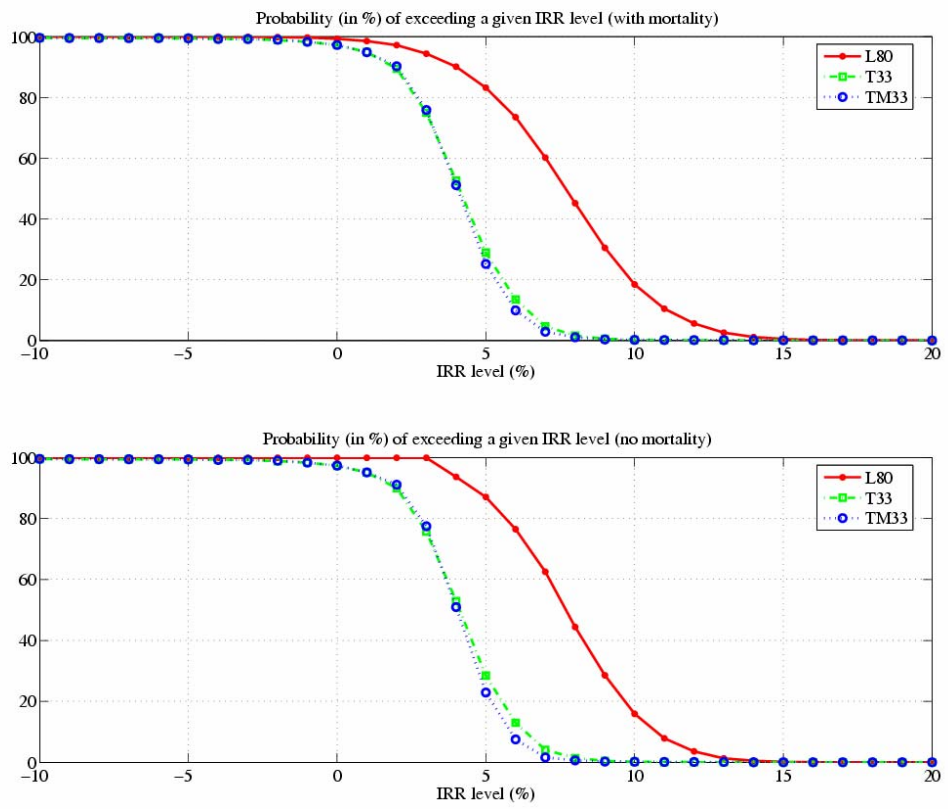
**Figure A2-1: Calibration period: Aug-48 -- July-83; Lognormal equity values; FI constant**



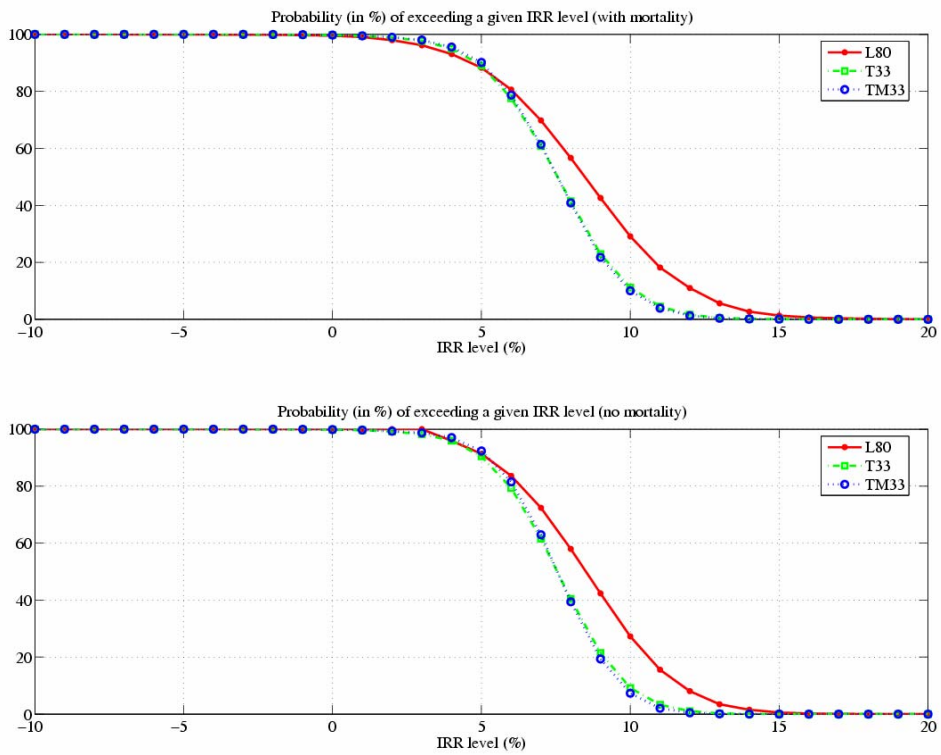
**Figure A2-2: Calibration period: June-71 -- May-06; Lognormal equity values; FI constant**



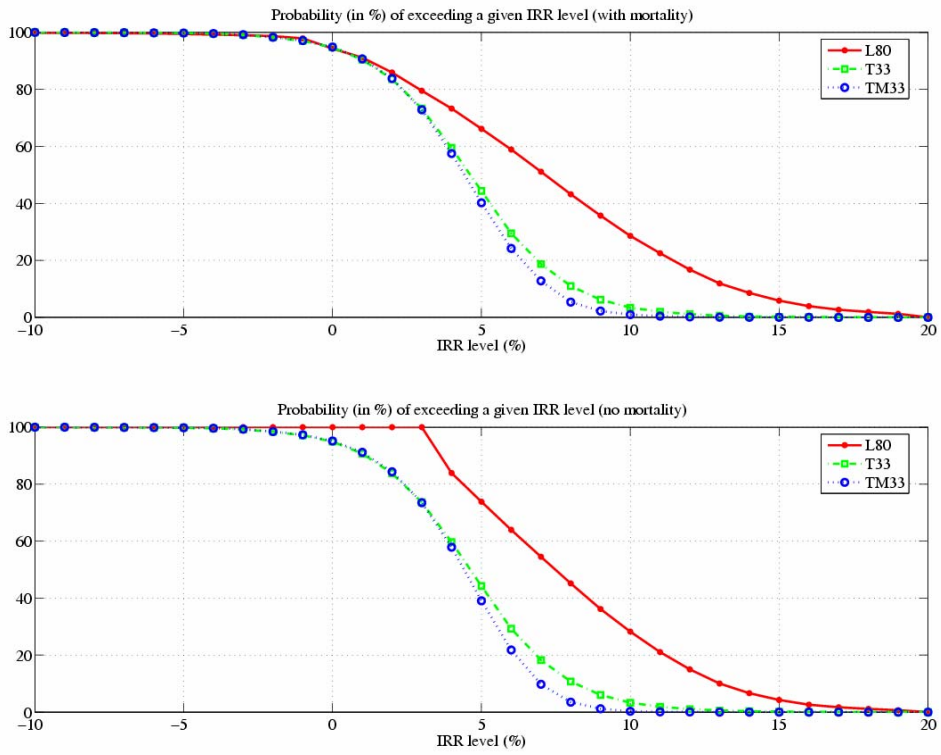
**Figure A2-3: Calibration period: Jan-26 -- Dec-60 Lognormal Equity Values; FI returns ARMA/GARCH**



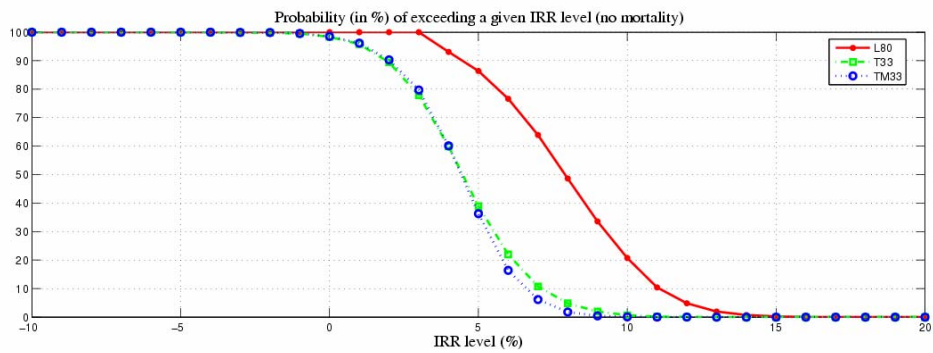
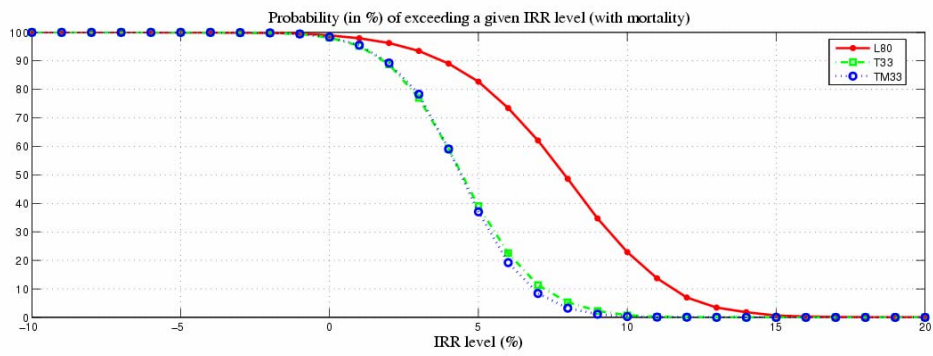
**Figure A2-4: Calibration period: Aug-48 -- July-83; Lognormal equity values; FI returns ARMA/GARCH**



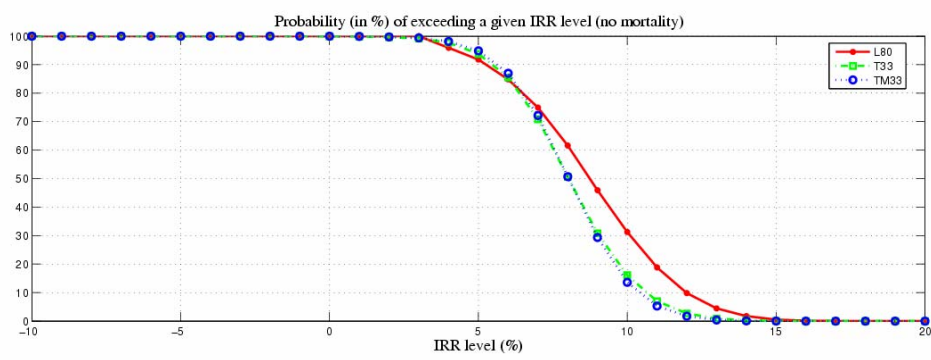
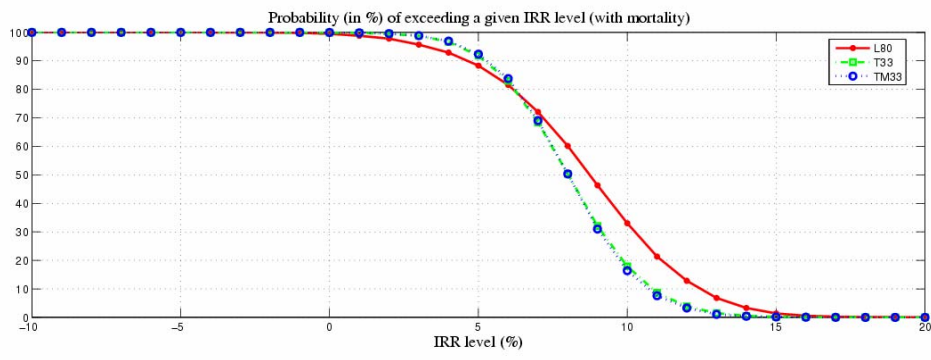
**Figure A2-5: Calibration period: June-71 -- May-06; Lognormal equity values; FI returns ARMA/GARCH**



**Figure A2-6: Sampling period: Jan-26 -- Dec-60 Historical distributions with monthly bootstrapping**



**Figure A2-7: Sampling period: Aug-48 -- July-83; Historical distributions with monthly bootstrapping**



**Figure A2-8: Sampling period: June-71 -- May-06; Historical distributions with monthly bootstrapping**



### Appendix 3: Performance Measures by Quartile; Run-out Scenarios vs. Non-Run-Out Scenarios

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
<b>Q1</b>												
PROBGAR	0.8175	0.8704	0.9781	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	1.07%	0.99%	0.90%	1.00%	0.29%	-0.25%	-2.50%	-5.62%	0.31%	-0.23%	-2.48%	-5.63%
std_IRR	1.36%	1.49%	1.89%	2.05%	1.50%	1.73%	2.54%	3.69%	1.48%	1.70%	2.53%	3.69%
mean_RUNOUT	45.49	46.69	48.54	48.86	68.82	79.36	118.09	151.36	76.53	87.32	123.09	157.39
std_RUNOUT	54.66	55.42	56.87	57.08	71.69	74.34	76.49	75.01	65.09	68.40	74.10	74.08
PROBRUN	0.5566	0.5620	0.5620	0.5639	0.6296	0.6880	0.9033	1.0000	0.7719	0.8047	0.9252	1.0000
<b>Q2</b>												
PROBGAR	0.7482	0.7974	0.9781	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	1.86%	1.89%	2.00%	2.29%	1.96%	1.87%	1.29%	0.15%	2.00%	1.91%	1.31%	0.14%
std_IRR	0.86%	0.91%	1.18%	1.43%	0.80%	0.82%	0.89%	1.00%	0.82%	0.86%	0.93%	1.00%
mean_RUNOUT	40.30	40.79	43.96	45.27	49.07	53.08	69.93	104.29	48.04	53.15	76.90	111.48
std_RUNOUT	49.71	50.24	53.07	54.24	61.95	62.84	65.72	61.91	50.73	53.31	60.87	61.78
PROBRUN	0.5347	0.5438	0.5456	0.5474	0.5438	0.5858	0.7281	1.0000	0.6551	0.7080	0.8449	1.0000
<b>Q3</b>												
PROBGAR	0.7847	0.8358	0.9818	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	2.81%	3.02%	3.73%	4.50%	3.08%	3.21%	3.40%	3.25%	3.01%	3.15%	3.36%	3.28%
std_IRR	0.66%	0.70%	0.92%	1.21%	0.68%	0.68%	0.74%	0.92%	0.69%	0.70%	0.77%	0.95%
mean_RUNOUT	30.74	30.31	31.01	31.93	89.51	85.30	77.98	84.84	36.10	39.99	62.26	95.35
std_RUNOUT	41.89	42.13	43.68	44.52	60.74	60.19	59.43	56.99	42.94	45.36	53.74	56.12
PROBRUN	0.4872	0.4763	0.4690	0.4781	0.9015	0.8996	0.8923	1.0000	0.6186	0.6606	0.8303	1.0000
<b>Q4</b>												
PROBGAR	0.6661	0.7318	0.9891	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.56%	5.14%	7.00%	8.93%	6.17%	6.67%	7.70%	8.25%	4.91%	5.44%	6.95%	8.42%
std_IRR	1.04%	1.22%	1.82%	2.49%	1.89%	2.08%	2.45%	2.61%	1.08%	1.26%	1.89%	2.67%
mean_RUNOUT	19.19	19.43	20.53	21.08	149.44	137.86	103.13	69.85	23.17	27.92	51.44	84.12
std_RUNOUT	29.99	30.52	31.92	32.82	51.66	50.39	47.22	50.40	33.20	36.47	46.18	48.97
PROBRUN	0.4215	0.4142	0.4161	0.4234	0.9982	0.9982	0.9964	1.0000	0.4982	0.5547	0.8066	0.9964

**Table A3-1: Sampling period: Jan-26 -- Dec-60 Historical Distributions with monthly bootstrapping; model with mortality; statistics conditional on T80 running out**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
<b>Q1</b>												
PROBGAR	0.1901	0.1655	0.1870	0.3468	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	2.55%	2.69%	2.89%	2.80%	3.22%	3.39%	3.69%	3.63%	3.21%	3.38%	3.68%	3.60%
std_IRR	1.14%	1.28%	1.79%	2.43%	1.14%	1.28%	1.78%	2.40%	1.13%	1.27%	1.79%	2.42%
mean_RUNOUT	10.22	8.94	8.72	11.85	12.00	8.26	2.20	0.00	5.82	4.86	5.07	9.82
std_RUNOUT	26.62	24.97	25.97	31.64	30.12	24.64	13.11	0.00	19.66	18.55	21.17	30.57
PROBRUN	0.1921	0.1675	0.1537	0.1808	0.2228	0.1711	0.0569	0.0000	0.1260	0.1025	0.0886	0.1644
<b>Q2</b>												
PROBGAR	0.0712	0.0523	0.0594	0.1347	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.15%	4.62%	5.85%	6.80%	4.93%	5.41%	6.63%	7.57%	4.81%	5.29%	6.57%	7.58%
std_IRR	0.61%	0.60%	0.66%	0.92%	0.72%	0.70%	0.69%	0.80%	0.59%	0.58%	0.63%	0.82%
mean_RUNOUT	2.67	1.62	1.12	2.41	42.06	30.87	6.45	0.00	0.75	0.50	0.43	1.59
std_RUNOUT	11.52	8.90	7.52	13.07	52.13	45.04	19.57	0.00	6.12	4.70	4.37	9.11
PROBRUN	0.0743	0.0492	0.0343	0.0502	0.5630	0.4657	0.1573	0.0000	0.0246	0.0164	0.0159	0.0543
<b>Q3</b>												
PROBGAR	0.0328	0.0225	0.0343	0.0748	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	5.30%	5.99%	7.90%	9.58%	6.26%	6.96%	8.76%	10.22%	5.94%	6.63%	8.56%	10.27%
std_IRR	0.61%	0.60%	0.66%	0.86%	0.88%	0.87%	0.78%	0.79%	0.59%	0.58%	0.64%	0.81%
mean_RUNOUT	0.79	0.56	0.54	0.95	72.90	57.41	12.27	0.00	0.20	0.14	0.22	1.05
std_RUNOUT	5.97	4.99	5.48	8.03	65.98	59.70	28.25	0.00	2.58	2.36	3.24	7.29
PROBRUN	0.0266	0.0210	0.0174	0.0246	0.7382	0.6547	0.2444	0.0000	0.0108	0.0061	0.0082	0.0353
<b>Q4</b>												
PROBGAR	0.0097	0.0097	0.0154	0.0394	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	7.13%	8.16%	11.12%	13.90%	8.57%	9.62%	12.30%	14.42%	7.74%	8.77%	11.73%	14.51%
std_IRR	1.18%	1.34%	1.87%	2.46%	1.76%	1.90%	2.17%	2.42%	1.17%	1.33%	1.86%	2.44%
mean_RUNOUT	0.25	0.23	0.25	0.46	103.25	87.18	25.13	0.00	0.16	0.14	0.17	0.50
std_RUNOUT	3.78	3.65	3.95	5.81	76.46	71.80	39.99	0.00	2.83	2.49	2.61	5.05
PROBRUN	0.0067	0.0061	0.0067	0.0092	0.8320	0.7761	0.4124	0.0000	0.0041	0.0036	0.0056	0.0190

**Table A3-2: Sampling period: Jan-26 -- Dec-60 Historical Distributions with monthly bootstrapping; model with mortality; statistics conditional on T80 *not* running out**

D = 35 years												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
<b>Q1</b>												
PROBGAR	0.9981	0.9981	1.0000	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	3.69%	3.73%	3.86%	4.00%	0.94%	0.57%	-1.00%	-3.33%	1.00%	0.63%	-0.97%	-3.34%
std_IRR	0.24%	0.31%	0.51%	0.72%	1.41%	1.64%	2.55%	3.73%	1.43%	1.67%	2.56%	3.73%
mean_RUNOUT	0.00	0.00	0.00	0.00	160.89	169.74	201.17	234.09	148.39	159.67	197.77	234.99
std_RUNOUT	0.00	0.00	0.00	0.00	47.81	48.27	44.55	35.63	44.97	46.52	44.47	35.35
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9893	0.9893	0.9952	1.0000	0.9884	0.9884	0.9942	1.0000
<b>Q2</b>												
PROBGAR	0.9981	0.9981	1.0000	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	3.83%	3.92%	4.24%	4.58%	2.66%	2.73%	2.70%	2.21%	2.71%	2.78%	2.73%	2.20%
std_IRR	0.38%	0.47%	0.78%	1.09%	0.67%	0.66%	0.69%	0.83%	0.69%	0.70%	0.73%	0.84%
mean_RUNOUT	0.00	0.00	0.00	0.00	125.90	123.10	124.19	144.49	89.78	91.98	112.51	150.01
std_RUNOUT	0.00	0.00	0.00	0.00	59.10	59.83	60.02	52.18	42.48	45.62	53.78	51.00
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9593	0.9573	0.9593	1.0000	0.9476	0.9447	0.9573	1.0000
<b>Q3</b>												
PROBGAR	0.9787	0.9845	1.0000	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.22%	4.48%	5.35%	6.21%	3.95%	4.21%	4.76%	5.00%	3.75%	4.03%	4.67%	5.05%
std_IRR	0.50%	0.59%	0.87%	1.18%	0.74%	0.75%	0.79%	0.92%	0.64%	0.65%	0.74%	0.97%
mean_RUNOUT	0.00	0.00	0.00	0.00	156.82	146.83	120.64	103.95	57.90	60.44	83.14	119.42
std_RUNOUT	0.00	0.00	0.00	0.00	45.60	47.41	52.84	58.97	41.99	45.02	52.55	52.36
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9971	0.9961	0.9845	1.0000	0.8643	0.8663	0.9390	1.0000
<b>Q4</b>												
PROBGAR	0.9273	0.9476	0.9981	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	5.51%	6.16%	8.20%	10.25%	6.96%	7.60%	8.93%	9.64%	5.48%	6.12%	8.01%	9.85%
std_IRR	0.98%	1.16%	1.76%	2.44%	1.89%	2.09%	2.46%	2.54%	1.07%	1.24%	1.86%	2.60%
mean_RUNOUT	0.00	0.00	0.00	0.00	213.58	198.62	147.74	85.79	39.27	44.03	69.95	104.13
std_RUNOUT	0.00	0.00	0.00	0.00	32.57	34.15	40.79	56.91	40.04	42.84	48.94	50.11
PROBRUN	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	0.7110	0.7507	0.9534	0.9990

**Table A3-3: Sampling period: Jan-26 -- Dec-60 Historical Distributions with monthly bootstrapping; model without mortality; statistics conditional on T80 running out**

D = 35 years												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
<b>Q1</b>												
PROBGAR	0.6739	0.5289	0.3839	0.4786	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	3.90%	4.10%	4.79%	5.37%	4.18%	4.55%	5.49%	6.14%	4.20%	4.57%	5.52%	6.10%
std_IRR	0.36%	0.47%	0.75%	1.04%	0.72%	0.73%	0.81%	1.03%	0.65%	0.66%	0.79%	1.05%
mean_RUNOUT	0.00	0.00	0.00	0.00	76.18	55.73	11.27	0.00	9.45	4.14	0.68	4.46
std_RUNOUT	0.00	0.00	0.00	0.00	52.75	49.30	23.28	0.00	20.39	13.39	3.95	12.97
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.8434	0.7304	0.2757	0.0000	0.2628	0.1484	0.0545	0.1688
<b>Q2</b>												
PROBGAR	0.2371	0.1519	0.1315	0.2193	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.69%	5.21%	6.72%	7.98%	5.40%	5.98%	7.50%	8.69%	5.28%	5.86%	7.42%	8.71%
std_IRR	0.52%	0.54%	0.56%	0.77%	0.79%	0.77%	0.65%	0.60%	0.54%	0.51%	0.51%	0.65%
mean_RUNOUT	0.00	0.00	0.00	0.00	132.93	108.20	24.84	0.00	1.40	0.54	0.34	2.67
std_RUNOUT	0.00	0.00	0.00	0.00	49.73	51.60	37.78	0.00	7.30	3.97	2.81	10.67
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9714	0.9421	0.4312	0.0000	0.0552	0.0272	0.0252	0.1015
<b>Q3</b>												
PROBGAR	0.0919	0.0688	0.0776	0.1511	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	5.56%	6.28%	8.32%	10.16%	6.54%	7.29%	9.26%	10.76%	6.17%	6.91%	8.97%	10.83%
std_IRR	0.53%	0.52%	0.56%	0.76%	0.94%	0.95%	0.81%	0.64%	0.51%	0.50%	0.54%	0.69%
mean_RUNOUT	0.00	0.00	0.00	0.00	172.66	149.56	42.90	0.00	0.49	0.26	0.15	1.89
std_RUNOUT	0.00	0.00	0.00	0.00	37.94	40.80	43.62	0.00	4.18	2.87	1.78	8.25
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9993	0.9980	0.6780	0.0000	0.0218	0.0123	0.0143	0.0776
<b>Q4</b>												
PROBGAR	0.0470	0.0361	0.0606	0.1172	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	7.05%	8.06%	11.01%	13.80%	8.67%	9.75%	12.44%	14.24%	7.64%	8.66%	11.61%	14.40%
std_IRR	0.92%	1.04%	1.50%	2.04%	1.77%	1.90%	2.06%	1.95%	0.91%	1.03%	1.48%	2.01%
mean_RUNOUT	0.00	0.00	0.00	0.00	216.73	196.59	90.47	0.00	0.24	0.12	0.10	1.31
std_RUNOUT	0.00	0.00	0.00	0.00	32.10	33.85	45.88	0.00	2.93	2.06	1.46	6.84
PROBRUN	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000	0.9550	0.0000	0.0109	0.0061	0.0075	0.0552

**Table A3-4: Sampling period: Jan-26 -- Dec-60 Historical Distributions with monthly bootstrapping; model with mortality; statistics conditional on T80 *not* running out**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
<b>Q1</b>												
PROBGAR	0.3590	0.4615	0.8205	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	3.27%	2.78%	1.29%	0.98%	3.72%	3.24%	1.06%	-2.26%	3.56%	3.06%	0.89%	-2.26%
std_IRR	1.77%	1.67%	1.23%	1.35%	2.34%	2.24%	1.99%	1.96%	2.34%	2.22%	1.83%	1.96%
mean_RUNOUT	21.72	28.64	53.77	62.69	18.23	19.59	52.72	127.31	28.64	35.13	73.38	134.18
std_RUNOUT	39.42	44.78	57.02	62.97	46.11	49.06	70.17	72.19	49.06	53.07	68.06	69.07
PROBRUN	0.2821	0.3590	0.6154	0.6410	0.1538	0.1538	0.4359	1.0000	0.3590	0.3846	0.6410	1.0000
<b>Q2</b>												
PROBGAR	0.3158	0.3421	0.7895	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	3.87%	3.54%	2.68%	2.42%	4.52%	4.19%	2.92%	1.11%	4.49%	4.13%	2.83%	1.09%
std_IRR	1.40%	1.25%	0.81%	0.88%	1.54%	1.43%	1.04%	0.64%	1.53%	1.40%	1.01%	0.63%
mean_RUNOUT	11.87	18.82	27.74	35.13	4.61	5.39	20.82	77.32	7.87	11.63	39.76	88.29
std_RUNOUT	24.29	33.06	46.01	48.60	18.95	20.03	43.45	50.80	19.50	22.54	50.73	54.27
PROBRUN	0.2632	0.2895	0.3684	0.4211	0.0789	0.1053	0.3158	1.0000	0.2105	0.2895	0.5000	1.0000
<b>Q3</b>												
PROBGAR	0.3077	0.3590	0.6923	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.55%	4.37%	3.84%	3.80%	4.97%	4.83%	4.18%	3.04%	5.02%	4.86%	4.15%	3.03%
std_IRR	1.15%	1.04%	0.72%	0.83%	1.57%	1.43%	0.96%	0.55%	1.51%	1.37%	0.89%	0.56%
mean_RUNOUT	6.00	9.67	19.23	25.59	20.10	20.23	24.03	63.44	10.18	10.77	26.51	71.44
std_RUNOUT	15.62	22.54	34.59	38.12	39.87	39.82	41.07	42.86	28.67	28.44	38.07	45.42
PROBRUN	0.1795	0.1795	0.3077	0.4359	0.2821	0.2821	0.3846	1.0000	0.1538	0.1795	0.5128	1.0000
<b>Q4</b>												
PROBGAR	0.2632	0.3421	0.7895	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	6.29%	6.30%	6.46%	7.07%	6.97%	6.97%	6.84%	6.55%	6.87%	6.86%	6.78%	6.63%
std_IRR	1.69%	1.68%	1.73%	1.91%	1.91%	1.89%	1.91%	2.02%	1.73%	1.74%	1.85%	2.07%
mean_RUNOUT	7.26	9.39	19.32	25.08	45.55	45.84	42.11	64.03	7.84	8.37	26.21	79.97
std_RUNOUT	21.74	24.23	32.83	33.91	54.14	52.13	47.36	37.56	22.02	23.70	35.54	34.77
PROBRUN	0.1579	0.2368	0.4737	0.5789	0.5263	0.5526	0.5789	1.0000	0.1579	0.1579	0.6053	1.0000

**Table A3-5: Sampling period: June-71 -- May-06; Historical Distributions with monthly bootstrapping; model with mortality; statistics conditional on T80 running out**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
<b>Q1</b>												
PROBGAR	0.0244	0.0236	0.0280	0.0963	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	5.55%	5.54%	5.41%	5.09%	6.19%	6.20%	6.13%	5.86%	6.21%	6.21%	6.12%	5.84%
std_IRR	1.61%	1.53%	1.48%	1.73%	1.59%	1.51%	1.43%	1.66%	1.59%	1.51%	1.43%	1.68%
mean_RUNOUT	1.57	1.38	1.69	4.59	2.59	1.62	0.24	0.00	0.97	0.68	0.55	2.48
std_RUNOUT	10.54	9.65	10.80	19.37	14.89	11.53	4.40	0.00	8.69	7.17	5.95	13.80
PROBRUN	0.0309	0.0272	0.0337	0.0841	0.0459	0.0329	0.0053	0.0000	0.0191	0.0142	0.0134	0.0479
<b>Q2</b>												
PROBGAR	0.0028	0.0016	0.0028	0.0142	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	7.14%	7.31%	7.76%	8.08%	7.74%	7.94%	8.41%	8.73%	7.77%	7.94%	8.39%	8.74%
std_IRR	1.21%	1.06%	0.68%	0.60%	1.23%	1.07%	0.67%	0.57%	1.19%	1.04%	0.67%	0.58%
mean_RUNOUT	0.13	0.06	0.15	0.51	6.10	3.68	0.36	0.00	0.00	0.00	0.03	0.24
std_RUNOUT	2.15	1.29	2.61	5.61	22.78	16.96	5.06	0.00	0.06	0.00	0.72	3.64
PROBRUN	0.0049	0.0032	0.0045	0.0118	0.1032	0.0703	0.0081	0.0000	0.0004	0.0000	0.0024	0.0069
<b>Q3</b>												
PROBGAR	0.0016	0.0008	0.0012	0.0061	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	8.20%	8.49%	9.28%	9.98%	8.83%	9.13%	9.93%	10.58%	8.81%	9.10%	9.89%	10.60%
std_IRR	1.17%	1.02%	0.65%	0.55%	1.22%	1.05%	0.65%	0.53%	1.15%	1.01%	0.64%	0.54%
mean_RUNOUT	0.10	0.05	0.05	0.13	12.28	6.88	0.41	0.00	0.04	0.03	0.03	0.09
std_RUNOUT	2.35	1.38	1.50	2.60	31.14	22.86	5.19	0.00	1.76	1.61	1.16	1.84
PROBRUN	0.0028	0.0016	0.0016	0.0033	0.2080	0.1361	0.0122	0.0000	0.0012	0.0004	0.0008	0.0037
<b>Q4</b>												
PROBGAR	0.0285	0.0202	0.0202	0.0349	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	9.79%	10.24%	11.53%	12.73%	10.54%	10.99%	12.21%	13.28%	10.39%	10.84%	12.12%	13.33%
std_IRR	1.51%	1.44%	1.39%	1.57%	1.63%	1.54%	1.42%	1.54%	1.50%	1.43%	1.38%	1.56%
mean_RUNOUT	0.03	0.03	0.01	0.02	26.20	16.77	0.51	0.00	0.01	0.00	0.00	0.02
std_RUNOUT	1.33	1.21	0.37	0.52	45.14	35.38	4.95	0.00	0.48	0.00	0.00	0.88
PROBRUN	0.0004	0.0008	0.0008	0.0012	0.3616	0.2743	0.0191	0.0000	0.0004	0.0000	0.0000	0.0008

**Table A3-6: Sampling period: June-71 -- May-06; Historical Distributions with monthly bootstrapping; model with mortality; statistics conditional on T80 not running out**

D = 35 years												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
<b>Q1</b>												
PROBGAR	0.5956	0.6691	0.9853	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.43%	4.25%	3.84%	3.85%	4.29%	3.95%	2.45%	0.23%	4.26%	3.93%	2.45%	0.22%
std_IRR	1.01%	0.86%	0.46%	0.49%	2.04%	1.96%	1.76%	2.02%	1.98%	1.92%	1.75%	2.01%
mean_RUNOUT	0.00	0.00	0.00	0.00	40.35	48.88	93.83	173.22	40.60	48.39	93.25	174.64
std_RUNOUT	0.00	0.00	0.00	0.00	61.49	64.28	71.62	41.33	59.58	62.27	71.13	41.18
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.3824	0.4632	0.7574	1.0000	0.4044	0.4632	0.7647	1.0000
<b>Q2</b>												
PROBGAR	0.4559	0.5000	0.8088	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.78%	4.63%	4.17%	4.11%	5.12%	4.97%	4.29%	3.02%	5.12%	4.97%	4.27%	2.99%
std_IRR	0.98%	0.88%	0.65%	0.73%	1.47%	1.35%	0.93%	0.38%	1.43%	1.31%	0.90%	0.38%
mean_RUNOUT	0.00	0.00	0.00	0.00	20.76	21.49	31.04	87.66	14.89	16.18	30.59	93.58
std_RUNOUT	0.00	0.00	0.00	0.00	45.58	44.96	48.28	42.42	34.83	34.15	43.82	43.75
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.2574	0.2868	0.4044	1.0000	0.2353	0.2868	0.4412	1.0000
<b>Q3</b>												
PROBGAR	0.3824	0.4632	0.8235	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	5.40%	5.32%	5.18%	5.36%	5.72%	5.64%	5.25%	4.57%	5.86%	5.76%	5.30%	4.55%
std_IRR	0.94%	0.85%	0.70%	0.86%	1.14%	1.03%	0.73%	0.59%	1.09%	1.00%	0.73%	0.62%
mean_RUNOUT	0.00	0.00	0.00	0.00	32.89	33.15	40.03	75.74	11.35	12.35	31.97	92.35
std_RUNOUT	0.00	0.00	0.00	0.00	53.54	52.07	51.76	47.59	31.09	32.17	43.97	43.70
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.3824	0.4118	0.5000	1.0000	0.1765	0.1985	0.5294	1.0000
<b>Q4</b>												
PROBGAR	0.3088	0.4044	0.8603	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	6.84%	6.95%	7.41%	8.18%	7.61%	7.71%	7.84%	7.78%	7.31%	7.41%	7.68%	7.89%
std_IRR	1.28%	1.28%	1.39%	1.62%	1.60%	1.56%	1.55%	1.69%	1.41%	1.37%	1.42%	1.75%
mean_RUNOUT	0.00	0.00	0.00	0.00	79.92	73.44	58.30	58.76	8.88	9.51	24.42	82.32
std_RUNOUT	0.00	0.00	0.00	0.00	70.67	65.74	50.75	39.97	25.65	25.78	33.50	34.50
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.7132	0.7059	0.7647	1.0000	0.1471	0.1838	0.5294	1.0000

**Table A3-7: Sampling period: June-71 -- May-06; Historical Distributions with monthly bootstrapping; model without mortality; statistics conditional on T80 not running out**

D = 35 years												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
<b>Q1</b>												
PROBGAR	0.0904	0.0731	0.0613	0.1843	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	6.08%	6.13%	6.19%	6.09%	6.65%	6.74%	6.90%	6.84%	6.70%	6.77%	6.88%	6.81%
std_IRR	1.20%	1.13%	1.00%	1.10%	1.30%	1.19%	0.97%	1.05%	1.27%	1.16%	0.96%	1.06%
mean_RUNOUT	0.00	0.00	0.00	0.00	11.38	7.41	1.05	0.00	1.99	1.18	0.21	0.98
std_RUNOUT	0.00	0.00	0.00	0.00	31.73	24.68	7.63	0.00	13.14	9.92	3.51	5.43
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.1602	0.1179	0.0296	0.0000	0.0359	0.0237	0.0089	0.0452
<b>Q2</b>												
PROBGAR	0.0135	0.0102	0.0072	0.0368	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	7.33%	7.52%	8.01%	8.38%	7.91%	8.13%	8.66%	9.03%	7.96%	8.15%	8.65%	9.03%
std_IRR	1.08%	0.95%	0.60%	0.52%	1.16%	1.00%	0.59%	0.47%	1.06%	0.92%	0.58%	0.49%
mean_RUNOUT	0.00	0.00	0.00	0.00	20.74	12.54	0.88	0.00	0.09	0.01	0.00	0.21
std_RUNOUT	0.00	0.00	0.00	0.00	41.37	31.70	7.22	0.00	1.77	0.40	0.00	2.66
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.2912	0.2091	0.0250	0.0000	0.0047	0.0008	0.0000	0.0097
<b>Q3</b>												
PROBGAR	0.0068	0.0038	0.0051	0.0140	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	8.24%	8.52%	9.31%	10.00%	8.87%	9.17%	9.96%	10.59%	8.85%	9.14%	9.92%	10.62%
std_IRR	1.03%	0.90%	0.58%	0.50%	1.15%	0.99%	0.58%	0.46%	1.02%	0.89%	0.57%	0.48%
mean_RUNOUT	0.00	0.00	0.00	0.00	39.56	24.85	0.95	0.00	0.10	0.06	0.03	0.20
std_RUNOUT	0.00	0.00	0.00	0.00	52.07	41.17	7.17	0.00	2.71	2.16	1.34	2.76
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.4882	0.3799	0.0296	0.0000	0.0025	0.0008	0.0004	0.0072
<b>Q4</b>												
PROBGAR	0.0025	0.0017	0.0004	0.0059	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	9.56%	9.99%	11.21%	12.37%	10.40%	10.84%	11.96%	12.88%	10.16%	10.58%	11.81%	12.96%
std_IRR	1.25%	1.19%	1.14%	1.30%	1.54%	1.44%	1.24%	1.24%	1.24%	1.18%	1.13%	1.28%
mean_RUNOUT	0.00	0.00	0.00	0.00	81.66	59.01	3.13	0.00	0.02	0.00	0.00	0.00
std_RUNOUT	0.00	0.00	0.00	0.00	64.79	57.66	12.77	0.00	0.61	0.00	0.00	0.15
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.7698	0.6720	0.0914	0.0000	0.0013	0.0000	0.0000	0.0008

**Table A3-8: Sampling period: June-71 -- May-06; Historical Distributions with monthly bootstrapping; model without mortality; statistics conditional on T80 not running out**



D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
<b>Q1</b>												
PROBGAR	0.9677	1.0000	1.0000	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	1.19%	1.20%	1.19%	1.22%	-0.59%	-0.62%	-0.83%	-1.29%	-0.56%	-0.59%	-0.82%	-1.30%
std_IRR	1.06%	1.05%	1.05%	1.08%	1.55%	1.48%	1.36%	1.45%	1.57%	1.49%	1.36%	1.44%
mean_RUNOUT	37.74	38.65	39.29	40.10	87.32	90.45	94.32	103.77	88.00	89.19	95.87	106.45
std_RUNOUT	50.43	51.47	51.01	51.07	59.15	55.88	53.51	48.46	58.20	57.14	53.20	50.42
PROBRUN	0.5161	0.5161	0.5161	0.5484	0.9032	0.9355	0.9355	1.0000	0.9677	0.9677	1.0000	1.0000
<b>Q2</b>												
PROBGAR	0.9355	0.9032	1.0000	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	2.06%	2.09%	2.22%	2.46%	0.69%	0.82%	1.14%	1.35%	0.83%	0.97%	1.25%	1.32%
std_IRR	1.04%	0.98%	0.90%	0.88%	1.42%	1.27%	0.76%	0.58%	1.38%	1.23%	0.76%	0.57%
mean_RUNOUT	29.81	30.52	30.42	29.29	76.55	76.45	73.74	69.32	81.74	79.23	78.42	82.32
std_RUNOUT	34.99	34.43	38.33	37.29	55.23	51.99	49.02	49.59	48.61	47.09	43.29	50.60
PROBRUN	0.5484	0.5484	0.5484	0.5484	0.8065	0.8387	0.8710	1.0000	0.8710	0.8710	0.9355	1.0000
<b>Q3</b>												
PROBGAR	0.9355	0.9355	1.0000	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	2.52%	2.64%	3.09%	3.68%	1.77%	2.00%	2.63%	3.17%	1.83%	2.07%	2.68%	3.13%
std_IRR	0.84%	0.78%	0.64%	0.66%	1.25%	1.11%	0.71%	0.51%	1.22%	1.08%	0.70%	0.50%
mean_RUNOUT	26.68	26.26	22.68	21.55	85.90	80.94	64.35	44.39	66.84	62.42	56.58	64.00
std_RUNOUT	39.48	38.82	34.46	34.70	61.06	58.14	48.52	38.95	59.18	56.44	49.78	43.63
PROBRUN	0.5484	0.4839	0.4839	0.4194	0.9355	0.9355	0.9355	1.0000	0.8387	0.8710	0.8387	1.0000
<b>Q4</b>												
PROBGAR	1.0000	1.0000	1.0000	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	3.77%	4.07%	5.12%	6.24%	3.76%	4.12%	5.06%	5.89%	3.28%	3.69%	4.85%	5.95%
std_IRR	0.94%	0.97%	1.16%	1.47%	1.61%	1.60%	1.56%	1.60%	1.15%	1.16%	1.34%	1.66%
mean_RUNOUT	23.03	22.68	19.94	18.87	140.06	130.52	97.42	48.16	65.58	62.19	60.23	64.65
std_RUNOUT	42.03	40.81	35.58	34.34	38.51	36.80	33.37	39.02	47.23	43.86	37.31	36.90
PROBRUN	0.2903	0.3226	0.3226	0.3226	1.0000	1.0000	1.0000	1.0000	0.9032	0.9355	1.0000	1.0000

**Table A3-9: Sampling period: Aug-48 -- July-83; Historical Distributions with monthly bootstrapping; model with mortality; statistics conditional on T80 running out**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
<b>Q1</b>												
PROBGAR	0.3664	0.2919	0.1275	0.0915	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	2.53%	2.84%	3.85%	4.86%	2.99%	3.39%	4.54%	5.61%	2.99%	3.39%	4.54%	5.59%
std_IRR	1.27%	1.25%	1.33%	1.60%	1.43%	1.36%	1.32%	1.53%	1.39%	1.33%	1.32%	1.55%
mean_RUNOUT	18.30	14.85	7.59	5.17	34.73	26.40	6.01	0.00	18.17	12.99	4.87	2.94
std_RUNOUT	37.90	34.32	25.65	21.70	50.76	44.30	21.33	0.00	37.71	32.02	19.91	15.87
PROBRUN	0.2850	0.2417	0.1223	0.0818	0.4607	0.3883	0.1219	0.0000	0.2866	0.2150	0.0834	0.0514
<b>Q2</b>												
PROBGAR	0.1806	0.0996	0.0158	0.0081	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	3.85%	4.38%	6.01%	7.55%	4.49%	5.05%	6.68%	8.20%	4.45%	5.02%	6.66%	8.20%
std_IRR	1.02%	0.92%	0.62%	0.56%	1.16%	1.02%	0.64%	0.53%	1.04%	0.91%	0.59%	0.54%
mean_RUNOUT	8.19	4.31	0.70	0.23	51.02	37.58	3.96	0.00	5.73	2.56	0.22	0.09
std_RUNOUT	24.15	17.67	6.08	3.41	56.71	49.13	15.39	0.00	20.70	13.78	3.80	2.13
PROBRUN	0.1628	0.0936	0.0186	0.0061	0.6205	0.5281	0.0968	0.0000	0.1166	0.0535	0.0061	0.0041
<b>Q3</b>												
PROBGAR	0.0665	0.0296	0.0057	0.0016	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.83%	5.50%	7.47%	9.34%	5.58%	6.25%	8.16%	9.94%	5.46%	6.14%	8.09%	9.96%
std_IRR	1.05%	0.92%	0.60%	0.53%	1.24%	1.08%	0.66%	0.50%	1.03%	0.90%	0.59%	0.52%
mean_RUNOUT	2.41	0.83	0.16	0.10	66.35	50.53	5.39	0.00	1.21	0.40	0.09	0.07
std_RUNOUT	12.02	6.92	3.33	2.71	61.96	55.29	17.40	0.00	8.57	4.94	2.34	2.26
PROBRUN	0.0669	0.0247	0.0041	0.0016	0.7245	0.6394	0.1378	0.0000	0.0324	0.0130	0.0020	0.0016
<b>Q4</b>												
PROBGAR	0.0154	0.0036	0.0012	0.0004	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	6.32%	7.15%	9.57%	11.92%	7.24%	8.06%	10.35%	12.48%	6.94%	7.76%	10.17%	12.52%
std_IRR	1.32%	1.27%	1.27%	1.47%	1.56%	1.48%	1.35%	1.45%	1.30%	1.26%	1.26%	1.46%
mean_RUNOUT	0.41	0.12	0.01	0.00	73.30	57.97	7.58	0.00	0.17	0.07	0.00	0.00
std_RUNOUT	4.48	2.56	0.40	0.00	66.54	59.93	20.37	0.00	3.12	2.01	0.00	0.00
PROBRUN	0.0113	0.0036	0.0004	0.0000	0.7246	0.6602	0.1871	0.0000	0.0053	0.0016	0.0000	0.0000

**Table A3-10: Sampling period: Aug-48 -- July-83; Historical Distributions with monthly bootstrapping; model with mortality; statistics conditional on T80 not running out**

D = 35 years												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
<b>Q1</b>												
PROBGAR	1.0000	1.0000	1.0000	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	3.65%	3.66%	3.71%	3.79%	0.37%	0.45%	0.61%	0.60%	0.42%	0.51%	0.65%	0.58%
std_IRR	0.20%	0.21%	0.29%	0.42%	1.59%	1.50%	1.33%	1.48%	1.58%	1.50%	1.34%	1.47%
mean_RUNOUT	0.00	0.00	0.00	0.00	168.20	166.91	165.23	165.49	162.66	161.34	161.78	167.05
std_RUNOUT	0.00	0.00	0.00	0.00	48.18	45.68	37.01	34.58	47.33	45.57	36.66	34.32
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9837	0.9919	1.0000	1.0000	0.9756	0.9837	1.0000	1.0000
<b>Q2</b>												
PROBGAR	1.0000	1.0000	1.0000	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	3.73%	3.75%	3.86%	4.02%	1.62%	1.84%	2.45%	2.97%	1.64%	1.86%	2.46%	2.95%
std_IRR	0.30%	0.31%	0.42%	0.59%	1.17%	1.03%	0.62%	0.33%	1.16%	1.02%	0.62%	0.33%
mean_RUNOUT	0.00	0.00	0.00	0.00	140.85	133.98	111.63	84.42	130.39	124.17	106.60	90.11
std_RUNOUT	0.00	0.00	0.00	0.00	49.42	47.69	41.55	41.62	44.43	42.01	35.95	43.40
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9919	0.9837	0.9919	1.0000	0.9837	0.9919	0.9919	1.0000
<b>Q3</b>												
PROBGAR	1.0000	1.0000	1.0000	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.10%	4.19%	4.56%	5.04%	2.51%	2.79%	3.57%	4.33%	2.49%	2.78%	3.59%	4.29%
std_IRR	0.50%	0.51%	0.56%	0.68%	1.18%	1.06%	0.72%	0.48%	1.13%	1.01%	0.68%	0.53%
mean_RUNOUT	0.00	0.00	0.00	0.00	157.08	148.28	116.82	68.70	118.02	110.80	94.62	85.13
std_RUNOUT	0.00	0.00	0.00	0.00	40.72	39.57	36.62	41.68	40.38	39.01	33.90	37.98
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9836	0.9836	0.9836	1.0000	0.9754	0.9754	0.9754	1.0000
<b>Q4</b>												
PROBGAR	0.9837	0.9837	1.0000	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.67%	4.95%	6.00%	7.18%	4.37%	4.79%	5.86%	6.78%	3.81%	4.27%	5.59%	6.86%
std_IRR	0.81%	0.86%	1.03%	1.28%	1.50%	1.44%	1.35%	1.31%	1.16%	1.12%	1.14%	1.37%
mean_RUNOUT	0.00	0.00	0.00	0.00	183.67	171.59	127.61	53.98	89.10	82.92	75.79	77.33
std_RUNOUT	0.00	0.00	0.00	0.00	35.36	34.83	32.58	38.68	40.57	38.60	33.54	33.45
PROBRUN	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	0.9675	0.9512	0.9756	1.0000

**Table A3-11: Sampling period: Aug-48 -- July-83; Historical Distributions with monthly bootstrapping; model without mortality; statistics conditional on T80 running out**

D = 35 years												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
<b>Q1</b>												
PROBGAR	0.8710	0.7856	0.3913	0.1904	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	3.89%	3.99%	4.67%	5.75%	3.32%	3.78%	5.15%	6.47%	3.33%	3.79%	5.17%	6.44%
std_IRR	0.45%	0.51%	0.74%	0.97%	1.31%	1.21%	0.98%	0.96%	1.23%	1.14%	0.94%	0.97%
mean_RUNOUT	0.00	0.00	0.00	0.00	111.09	90.55	21.34	0.00	55.65	37.55	7.15	0.78
std_RUNOUT	0.00	0.00	0.00	0.00	56.55	55.51	34.53	0.00	53.82	46.97	20.25	4.83
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9281	0.8815	0.3926	0.0000	0.6927	0.5389	0.1711	0.0387
<b>Q2</b>												
PROBGAR	0.5888	0.3695	0.0661	0.0282	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.31%	4.67%	6.22%	7.83%	4.59%	5.18%	6.88%	8.48%	4.54%	5.14%	6.87%	8.48%
std_IRR	0.66%	0.71%	0.56%	0.47%	1.16%	1.03%	0.62%	0.43%	0.99%	0.85%	0.52%	0.45%
mean_RUNOUT	0.00	0.00	0.00	0.00	132.07	108.33	16.63	0.00	17.58	7.06	0.59	0.16
std_RUNOUT	0.00	0.00	0.00	0.00	47.81	49.01	30.17	0.00	32.84	20.73	4.89	1.98
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9819	0.9592	0.3556	0.0000	0.3266	0.1557	0.0198	0.0084
<b>Q3</b>												
PROBGAR	0.3015	0.1451	0.0240	0.0143	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	4.97%	5.56%	7.48%	9.35%	5.63%	6.30%	8.22%	9.94%	5.47%	6.15%	8.10%	9.97%
std_IRR	0.84%	0.80%	0.53%	0.48%	1.27%	1.11%	0.65%	0.44%	0.94%	0.80%	0.51%	0.46%
mean_RUNOUT	0.00	0.00	0.00	0.00	153.30	129.75	22.48	0.00	5.65	1.78	0.27	0.12
std_RUNOUT	0.00	0.00	0.00	0.00	41.21	42.87	31.77	0.00	18.16	9.74	3.13	1.89
PROBRUN	0.0000	0.0000	0.0000	0.0000	0.9950	0.9891	0.5008	0.0000	0.1341	0.0534	0.0101	0.0059
<b>Q4</b>												
PROBGAR	0.1061	0.0438	0.0088	0.0059	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
mean_IRR	6.11%	6.89%	9.23%	11.51%	7.15%	7.95%	10.15%	12.03%	6.69%	7.49%	9.83%	12.11%
std_IRR	1.06%	1.02%	0.99%	1.14%	1.54%	1.46%	1.23%	1.10%	1.06%	1.01%	0.98%	1.13%
mean_RUNOUT	0.00	0.00	0.00	0.00	180.18	157.88	41.40	0.00	1.30	0.46	0.07	0.02
std_RUNOUT	0.00	0.00	0.00	0.00	33.48	35.15	35.63	0.00	8.03	4.70	1.36	0.47
PROBRUN	0.0000	0.0000	0.0000	0.0000	1.0000	0.9996	0.7832	0.0000	0.0412	0.0147	0.0034	0.0013

**Table A3-12: Sampling period: Aug-48 -- July-83; Historical Distributions with monthly bootstrapping; model without mortality; statistics conditional on T80 not running out**

## Appendix 4: Fees and Total Cash Flows Under Twenty Stochastic Returns Models

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	0.9908	1.2297	2.1873	3.6811	0.7768	0.9686	1.9883	4.7169	1.2313	1.5067	2.6121	4.3486
std_TOTAL	0.7599	1.0684	2.7161	6.6479	0.3783	0.5524	1.8350	8.9653	0.9254	1.2945	3.2745	8.0045
mean_TOTAL_FEES	0.4256	0.4667	0.6164	0.8280	0.2097	0.2332	0.3611	0.6910	0.3310	0.3616	0.4747	0.6369
std_TOTAL_FEES	0.1882	0.2411	0.4928	1.0145	0.0442	0.0581	0.1871	0.9271	0.1525	0.1948	0.3971	0.8168

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	1.3983	1.7415	3.2418	5.8469	0.8152	1.0321	2.3661	7.8577	1.5801	1.9901	3.7523	6.8286
std_TOTAL	0.8220	1.2593	3.8591	11.2612	0.3778	0.5597	2.0629	16.5256	1.1877	1.7204	4.9188	14.0422
mean_TOTAL_FEES	0.5233	0.5913	0.8485	1.2411	0.2202	0.2489	0.4312	1.1418	0.4246	0.4772	0.6803	0.9945
std_TOTAL_FEES	0.2421	0.3217	0.7347	1.7230	0.0433	0.0585	0.2122	1.6631	0.1961	0.2609	0.6008	1.4149

**Table A4-1: Calibration period: Jan-26 -- Dec-60 Lognormal equity values; FI returns constant**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	0.9997	1.2362	2.1797	3.6455	0.7207	0.8858	1.7300	4.2712	1.1595	1.4041	2.3748	3.8900
std_TOTAL	1.0891	1.5784	4.7463	15.6101	0.5943	0.8545	2.7097	23.0271	1.3485	1.9463	5.8465	19.5847
mean_TOTAL_FEES	0.4093	0.4452	0.5772	0.7631	0.1933	0.2117	0.3122	0.6115	0.3091	0.3337	0.4257	0.5580
std_TOTAL_FEES	0.2450	0.3246	0.7618	1.9636	0.0628	0.0831	0.2475	1.9056	0.1975	0.2607	0.6120	1.6134

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	1.5378	1.8945	3.4327	6.1255	0.7614	0.9521	2.0222	7.1298	1.4537	1.8055	3.3148	6.0625
std_TOTAL	1.2404	1.9823	7.8857	33.5779	0.6404	0.9436	3.2577	51.0690	1.7729	2.6848	9.9587	42.5884
mean_TOTAL_FEES	0.4954	0.5524	0.7709	1.1030	0.2041	0.2273	0.3652	0.9879	0.3877	0.4291	0.5907	0.8436
std_TOTAL_FEES	0.3312	0.4542	1.1830	3.3798	0.0733	0.0998	0.3081	3.4318	0.2715	0.3713	0.9722	2.8666

**Table A4-2: Calibration period: Jan-26 -- Dec-60 Lognormal equity values; FI returns constant. Volatility of Equity returns increased by 50%**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	1.0467	1.3033	2.3418	3.9197	0.6819	0.8292	1.5788	3.9849	1.0983	1.3247	2.2360	3.6086
std_TOTAL	1.4986	2.2669	7.5166	23.7343	0.8302	1.1887	4.0251	31.1172	1.8464	2.7569	8.9706	27.8818
mean_TOTAL_FEES	0.3990	0.4326	0.5571	0.7270	0.1849	0.2002	0.2842	0.5622	0.2922	0.3133	0.3945	0.5092
std_TOTAL_FEES	0.3128	0.4293	1.1216	3.0350	0.0895	0.1165	0.3327	2.5742	0.2491	0.3390	0.8684	2.2557

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	1.7103	2.1164	3.8804	7.0497	0.7373	0.9154	1.8876	6.9847	1.3654	1.6903	3.1406	5.9473
std_TOTAL	1.8673	3.1688	15.3768	70.9382	0.9469	1.4130	5.2921	103.0179	2.5229	4.0914	19.2410	88.7950
mean_TOTAL_FEES	0.4784	0.5316	0.7492	1.1450	0.1993	0.2200	0.3364	1.0057	0.3618	0.3981	0.5528	0.8436
std_TOTAL_FEES	0.4461	0.6632	2.6379	12.2928	0.1154	0.1554	0.4534	11.2905	0.3631	0.5361	2.0955	9.3200

**Table A4-3: Calibration period: Jan-26 -- Dec-60 Lognormal equity values; FI returns constant. Volatility of Equity returns increased by 100%**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	2.0411	2.7752	6.6235	15.6079	1.1323	1.5158	4.2128	21.2180	2.4740	3.3388	7.9170	18.7372
std_TOTAL	2.0814	3.4302	15.4219	72.0343	0.6423	1.0093	5.0703	99.0443	2.5043	4.1304	18.6579	87.4789
mean_TOTAL_FEES	0.5997	0.7077	1.2165	2.2649	0.2145	0.2509	0.5050	2.0204	0.4659	0.5498	0.9494	1.7817
std_TOTAL_FEES	0.3968	0.5875	2.0757	8.6520	0.0641	0.0870	0.3629	8.1998	0.3210	0.4763	1.6948	7.1187

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	2.9600	4.1865	11.5310	32.2648	1.2029	1.6433	5.1964	48.5235	3.5409	5.0237	14.0129	39.7289
std_TOTAL	2.8280	5.0712	30.0556	201.3542	0.6842	1.0989	6.0532	298.2839	3.6059	6.3711	37.0877	244.7395
mean_TOTAL_FEES	0.8269	1.0254	2.0626	4.5458	0.2285	0.2731	0.6251	4.5410	0.6690	0.8297	1.6783	3.7277
std_TOTAL_FEES	0.5673	0.8927	3.7961	18.8348	0.0783	0.1082	0.4480	18.7697	0.4626	0.7331	3.1518	15.6477

**Table A4-4: Calibration period: Jan-26 -- Dec-60 Lognormal equity values ;FI returns constant. Mean and Volatility of Equity returns increased by 50%**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	0.3982	0.4421	0.5884	0.7553	0.4547	0.5047	0.6878	0.9289	0.5113	0.5618	0.7215	0.8964
std_TOTAL	0.2164	0.2666	0.4539	0.7035	0.1950	0.2549	0.5084	0.9270	0.2688	0.3356	0.5712	0.8777
mean_TOTAL_FEES	0.2983	0.3087	0.3389	0.3688	0.2074	0.2161	0.2487	0.2905	0.2331	0.2404	0.2608	0.2806
std_TOTAL_FEES	0.0732	0.0865	0.1288	0.1778	0.0373	0.0456	0.0870	0.1564	0.0625	0.0728	0.1056	0.1440

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	0.7825	0.8166	0.9779	1.1997	0.4800	0.5388	0.7793	1.1469	0.5738	0.6396	0.8501	1.0844
std_TOTAL	0.0802	0.1357	0.3648	0.6836	0.1986	0.2654	0.5811	1.1815	0.3057	0.3894	0.6876	1.0838
mean_TOTAL_FEES	0.3232	0.3387	0.3849	0.4307	0.2192	0.2310	0.2819	0.3588	0.2618	0.2738	0.3074	0.3396
std_TOTAL_FEES	0.0781	0.0976	0.1587	0.2289	0.0321	0.0431	0.1017	0.2124	0.0694	0.0849	0.1327	0.1886

**Table A4-5: Calibration period: Jan-26 -- Dec-60 Lognormal equity values; FI returns constant. Mean and Volatility of Equity returns decreased by 50%**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	0.9355	1.1667	2.1219	3.6566	0.7474	0.9351	1.9329	4.6869	1.1288	1.4034	2.5222	4.3120
std_TOTAL	0.9006	1.1843	2.8032	6.7600	0.7150	0.8554	1.9957	9.2208	1.1001	1.4409	3.3892	8.1722
mean_TOTAL_FEES	0.4077	0.4495	0.6023	0.8206	0.2077	0.2302	0.3540	0.6844	0.3131	0.3450	0.4620	0.6300
std_TOTAL_FEES	0.2085	0.2566	0.5010	1.0378	0.1034	0.1109	0.2092	0.9574	0.1686	0.2070	0.4035	0.8366

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	1.3891	1.6982	3.1460	5.7765	0.7963	1.0101	2.3004	7.7148	1.4226	1.8237	3.5788	6.7105
std_TOTAL	1.1745	1.5182	3.8735	10.5739	0.8263	1.0021	2.4754	15.4873	1.6644	2.0966	4.9588	13.1949
mean_TOTAL_FEES	0.4933	0.5620	0.8267	1.2380	0.2218	0.2493	0.4232	1.1353	0.3939	0.4485	0.6592	0.9879
std_TOTAL_FEES	0.2777	0.3507	0.7447	1.6858	0.1401	0.1522	0.2756	1.6258	0.2268	0.2860	0.6079	1.3777

**Table A4-6: Calibration period: Jan-26 -- Dec-60 Lognormal equity values; FI returns ARMA/GARCH.**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	0.9594	1.1947	2.1670	3.7538	0.6948	0.8562	1.7019	4.3991	1.0728	1.3213	2.3400	4.0001
std_TOTAL	1.1582	1.6335	4.8110	15.3300	0.7966	1.0203	2.8562	22.1838	1.4239	1.9950	5.8822	18.9672
mean_TOTAL_FEES	0.3975	0.4350	0.5741	0.7755	0.1934	0.2111	0.3091	0.6230	0.2968	0.3232	0.4223	0.5676
std_TOTAL_FEES	0.2586	0.3345	0.7578	1.8692	0.1070	0.1206	0.2629	1.7625	0.2068	0.2663	0.6022	1.5010

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	1.5191	1.8543	3.3444	5.9803	0.7422	0.9275	1.9766	6.7915	1.3153	1.6608	3.1598	5.8103
std_TOTAL	1.3449	2.0257	6.7308	22.1035	0.9473	1.2305	3.4908	33.3686	1.8612	2.7072	8.5413	27.9671
mean_TOTAL_FEES	0.4731	0.5317	0.7603	1.1201	0.2065	0.2283	0.3608	0.9971	0.3653	0.4086	0.5798	0.8516
std_TOTAL_FEES	0.3531	0.4742	1.1896	3.1721	0.1477	0.1687	0.3494	3.1164	0.2880	0.3850	0.9638	2.5860

**Table A4-7: Calibration period: Jan-26 -- Dec-60 Lognormal equity values; FI returns ARMA/GARCH. Volatility of Equity returns increased by 50%**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	1.0204	1.2752	2.3490	4.0755	0.6594	0.8056	1.5670	4.1955	1.0280	1.2588	2.2136	3.7521
std_TOTAL	1.5561	2.3371	8.3750	28.1611	0.9721	1.3041	4.1763	45.0376	1.8969	2.8297	10.3533	36.5356
mean_TOTAL_FEES	0.3906	0.4256	0.5571	0.7443	0.1839	0.1988	0.2820	0.5801	0.2831	0.3058	0.3929	0.5226
std_TOTAL_FEES	0.3235	0.4383	1.1625	3.1359	0.1194	0.1408	0.3441	3.1632	0.2547	0.3431	0.9146	2.5685

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	1.6880	2.0710	3.7531	6.5538	0.7091	0.8801	1.8220	6.1166	1.2375	1.5458	2.8819	5.1344
std_TOTAL	1.8755	3.0089	11.4015	37.4743	1.1464	1.5842	5.2352	59.2047	2.4754	3.8089	14.1477	47.8213
mean_TOTAL_FEES	0.4609	0.5142	0.7247	1.0403	0.1986	0.2181	0.3320	0.9079	0.3438	0.3805	0.5284	0.7607
std_TOTAL_FEES	0.4503	0.6380	1.8466	5.1712	0.1695	0.2037	0.4790	5.3238	0.3618	0.5073	1.4777	4.2263

**Table A4-8: Calibration period: Jan-26 -- Dec-60 Lognormal equity values; FI returns ARMA/GARCH. Volatility of Equity returns increased by 100%**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	1.9030	2.6194	6.5247	16.1722	1.0968	1.4678	4.0608	22.1142	2.2781	3.1239	7.7618	19.3475
std_TOTAL	2.1793	3.5207	16.6031	89.8827	1.0078	1.3652	5.2052	129.4395	2.6297	4.2407	20.1100	109.0315
mean_TOTAL_FEES	0.5738	0.6819	1.1975	2.2765	0.2136	0.2487	0.4919	2.0394	0.4422	0.5263	0.9316	1.7890
std_TOTAL_FEES	0.4147	0.5977	2.0016	7.9496	0.1191	0.1420	0.3958	7.9081	0.3343	0.4822	1.6367	6.6305

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	2.7868	3.9424	11.0172	31.2644	1.1858	1.6195	5.0787	46.6727	3.2058	4.6142	13.2773	38.2531
std_TOTAL	3.1104	5.2713	26.8870	145.5420	1.3153	1.7887	7.3805	216.5973	4.0591	6.6365	33.4318	180.2156
mean_TOTAL_FEES	0.7746	0.9701	1.9991	4.4794	0.2310	0.2746	0.6151	4.4473	0.6213	0.7792	1.6180	3.6525
std_TOTAL_FEES	0.6086	0.9289	3.5507	15.0883	0.1804	0.2200	0.5932	15.2379	0.4978	0.7594	2.9423	12.6431

**Table A4-9: Calibration period: Jan-26 -- Dec-60 Lognormal equity values; FI returns ARMA/GARCH. Mean and Volatility of Equity returns increased by 50%**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	0.3942	0.4319	0.5718	0.7504	0.4202	0.4695	0.6546	0.9156	0.4551	0.5073	0.6816	0.8838
std_TOTAL	0.3515	0.3660	0.4828	0.7125	0.4230	0.4327	0.5712	0.9407	0.4285	0.4540	0.6083	0.8896
mean_TOTAL_FEES	0.2899	0.3001	0.3324	0.3667	0.2043	0.2122	0.2437	0.2883	0.2212	0.2293	0.2536	0.2784
std_TOTAL_FEES	0.0993	0.1058	0.1345	0.1787	0.0871	0.0851	0.0989	0.1573	0.0821	0.0872	0.1098	0.1447

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	0.8440	0.8647	0.9870	1.1987	0.4520	0.5083	0.7398	1.1196	0.5049	0.5698	0.7928	1.0595
std_TOTAL	0.3642	0.3431	0.4172	0.6985	0.5004	0.5083	0.6748	1.2007	0.4903	0.5275	0.7337	1.1003
mean_TOTAL_FEES	0.3127	0.3269	0.3744	0.4267	0.2197	0.2299	0.2759	0.3541	0.2453	0.2575	0.2957	0.3353
std_TOTAL_FEES	0.1149	0.1242	0.1667	0.2314	0.1160	0.1118	0.1248	0.2147	0.0979	0.1055	0.1392	0.1905

**Table A4-10: Calibration period: Jan-26 -- Dec-60 Lognormal equity values; FI returns ARMA/GARCH. Mean and Volatility of Equity returns decreased by 50%**



D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	0.9709	1.2034	2.1360	3.5904	0.7677	0.9551	1.9417	4.6122	1.2010	1.4712	2.5531	4.2473
std_TOTAL	0.7900	1.0919	2.6997	6.5009	0.4288	0.5975	1.8520	8.9237	0.9671	1.3299	3.2784	7.9201
mean_TOTAL_FEES	0.4201	0.4608	0.6084	0.8145	0.2088	0.2320	0.3578	0.6793	0.3257	0.3563	0.4682	0.6265
std_TOTAL_FEES	0.1930	0.2442	0.4848	0.9702	0.0501	0.0635	0.1918	0.8958	0.1566	0.1977	0.3924	0.7883

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	1.3963	1.7256	3.1761	5.6752	0.8094	1.0230	2.3283	7.5940	1.5357	1.9378	3.6543	6.6101
std_TOTAL	0.8512	1.2760	3.7230	10.3007	0.4388	0.6179	2.1076	15.1738	1.2402	1.7564	4.7742	12.9274
mean_TOTAL_FEES	0.5137	0.5808	0.8336	1.2129	0.2202	0.2485	0.4283	1.1150	0.4152	0.4677	0.6678	0.9723
std_TOTAL_FEES	0.2492	0.3255	0.7112	1.5703	0.0523	0.0673	0.2201	1.5285	0.2031	0.2653	0.5841	1.2997

**Table A4-11: Sampling period: Jan-26 -- Dec-60 Historical distributions with monthly bootstrapping**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	3.1674	3.3710	4.0318	4.7872	3.0169	3.3204	4.5305	6.2140	3.7711	4.0161	4.8179	5.7405
std_TOTAL	1.5359	1.8144	2.9484	4.6859	1.1653	1.3763	2.8359	6.3290	1.8718	2.1980	3.5353	5.6089
mean_TOTAL_FEES	0.7967	0.8232	0.9077	1.0027	0.5018	0.5369	0.6760	0.8595	0.6255	0.6478	0.7179	0.7948
std_TOTAL_FEES	0.3320	0.3691	0.5173	0.7333	0.1832	0.1977	0.3371	0.6861	0.2735	0.3038	0.4248	0.6022

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	4.9100	5.2676	6.4480	7.8496	4.2782	4.8105	7.2388	11.0514	6.0203	6.4689	7.9596	9.7174
std_TOTAL	1.5528	2.0316	3.9523	6.8938	1.1752	1.2944	3.3097	9.6771	1.8491	2.4211	4.7415	8.3548
mean_TOTAL_FEES	1.2370	1.2888	1.4541	1.6424	0.7130	0.7799	1.0834	1.5327	1.0002	1.0454	1.1889	1.3490
std_TOTAL_FEES	0.2693	0.3469	0.6405	1.0523	0.1900	0.1754	0.3311	1.0067	0.2183	0.2814	0.5226	0.8660

**Table A4-12: Calibration period: June-71 -- May-06; Lognormal equity values; FI constant**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	2.7691	3.0046	3.7737	4.6592	2.7567	3.0424	4.2425	6.0362	3.3382	3.6139	4.5234	5.5856
std_TOTAL	1.9232	2.1045	2.9735	4.4979	2.1008	2.1925	3.1158	6.0333	2.3001	2.5174	3.5536	5.3718
mean_TOTAL_FEES	0.7281	0.7627	0.8689	0.9834	0.4741	0.5069	0.6447	0.8410	0.5730	0.6008	0.6865	0.7787
std_TOTAL_FEES	0.3743	0.4005	0.5181	0.7094	0.2913	0.2939	0.3716	0.6599	0.3054	0.3271	0.4241	0.5810

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	4.1584	4.5763	5.9807	7.6666	3.8969	4.3701	6.6687	10.7968	5.1703	5.6815	7.4050	9.4923
std_TOTAL	2.5172	2.7735	4.1459	6.8194	3.1887	3.2278	4.2301	9.5740	3.0341	3.3360	4.9965	8.2741
mean_TOTAL_FEES	1.0902	1.1597	1.3758	1.6144	0.6704	0.7286	1.0142	1.5056	0.8885	0.9456	1.1252	1.3252
std_TOTAL_FEES	0.4592	0.4929	0.6887	1.0637	0.4439	0.4309	0.4698	1.0158	0.3695	0.3977	0.5612	0.8740

**Table A4-13: Table A44: Calibration period: June-71 -- May-06; Lognormal equity values; FI ARMA/GARCH**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	2.9944	3.2147	3.9249	4.7297	2.9737	3.2700	4.4682	6.1501	3.6070	3.8667	4.7127	5.6789
std_TOTAL	2.1409	2.3691	3.3521	4.9277	2.1599	2.3240	3.5178	6.6511	2.5638	2.8367	4.0091	5.8960
mean_TOTAL_FEES	0.7608	0.7923	0.8894	0.9946	0.4960	0.5301	0.6676	0.8522	0.5996	0.6250	0.7034	0.7875
std_TOTAL_FEES	0.4056	0.4380	0.5712	0.7729	0.2858	0.2965	0.4072	0.7219	0.3314	0.3581	0.4677	0.6337

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	4.5727	4.9660	6.2606	7.7745	4.2492	4.7620	7.1391	10.9683	5.6877	6.1691	7.7621	9.6291
std_TOTAL	2.8148	3.1484	4.6833	7.2974	3.0703	3.2158	4.7394	10.2654	3.3848	3.7860	5.6532	8.8629
mean_TOTAL_FEES	1.1566	1.2191	1.4129	1.6253	0.7075	0.7709	1.0657	1.5184	0.9444	0.9960	1.1575	1.3343
std_TOTAL_FEES	0.5015	0.5485	0.7676	1.1276	0.4115	0.4101	0.5151	1.0781	0.4045	0.4437	0.6263	0.9268

**Table A4-14: Sampling period: June-71 -- May-06; Historical distributions with monthly bootstrapping**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	1.0413	1.2977	2.3234	3.9329	0.8516	1.0725	2.2747	5.0805	1.3175	1.6160	2.8183	4.7217
std_TOTAL	0.4946	0.6806	1.5840	3.4562	0.2110	0.3165	1.1545	4.6436	0.6038	0.8209	1.8939	4.1366
mean_TOTAL_FEES	0.4488	0.4952	0.6594	0.8889	0.2297	0.2593	0.4190	0.7552	0.3539	0.3891	0.5181	0.7022
std_TOTAL_FEES	0.1442	0.1784	0.3230	0.5823	0.0342	0.0435	0.1430	0.5410	0.1187	0.1454	0.2628	0.4764

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	1.3663	1.7462	3.3974	6.2687	0.9004	1.1561	2.8877	8.7793	1.7582	2.2284	4.2457	7.7689
std_TOTAL	0.5359	0.7834	2.0526	4.9769	0.1791	0.2745	1.1656	6.9616	0.6971	0.9743	2.4848	6.0299
mean_TOTAL_FEES	0.5711	0.6543	0.9574	1.4104	0.2432	0.2799	0.5331	1.3040	0.4726	0.5370	0.7808	1.1553
std_TOTAL_FEES	0.1573	0.1993	0.3990	0.8064	0.0247	0.0294	0.1206	0.7642	0.1195	0.1526	0.3183	0.6590

**Table A4-15: Calibration period: Aug-48 -- July-83; Lognormal equity values; FI constant**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	0.8199	1.0529	2.0476	3.7000	0.7038	0.9023	2.0001	4.7852	1.0285	1.3114	2.4921	4.4483
std_TOTAL	0.5620	0.7227	1.5835	3.5647	0.4680	0.5465	1.2365	4.7539	0.6971	0.8824	1.8910	4.2497
mean_TOTAL_FEES	0.3929	0.4394	0.6076	0.8462	0.2124	0.2388	0.3846	0.7178	0.3075	0.3439	0.4766	0.6681
std_TOTAL_FEES	0.1518	0.1822	0.3191	0.5880	0.0762	0.0825	0.1558	0.5444	0.1247	0.1484	0.2588	0.4804

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	1.1574	1.4516	2.9807	5.9167	0.7491	0.9760	2.4865	8.2941	1.3092	1.7439	3.7089	7.3383
std_TOTAL	0.5697	0.8092	2.0841	5.2205	0.5285	0.6168	1.3927	7.2854	0.8673	1.1194	2.5668	6.3268
mean_TOTAL_FEES	0.4766	0.5574	0.8667	1.3420	0.2263	0.2586	0.4789	1.2406	0.3909	0.4564	0.7073	1.0997
std_TOTAL_FEES	0.1891	0.2297	0.4208	0.8499	0.0957	0.1022	0.1705	0.8074	0.1540	0.1839	0.3373	0.6956

**Table A4-16: Calibration period: Aug-48 -- July-83; Lognormal equity values; FI ARMA/GARCH**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	0.9673	1.2136	2.2428	3.8842	0.8293	1.0460	2.2269	5.0241	1.2098	1.5087	2.7291	4.6699
std_TOTAL	0.6908	0.8656	1.7303	3.5273	0.4869	0.5913	1.4085	4.7117	0.8402	1.0425	2.0641	4.2153
mean_TOTAL_FEES	0.4219	0.4694	0.6416	0.8791	0.2265	0.2554	0.4117	0.7465	0.3293	0.3670	0.5036	0.6943
std_TOTAL_FEES	0.1750	0.2061	0.3436	0.5936	0.0665	0.0773	0.1727	0.5492	0.1418	0.1667	0.2787	0.4854

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	1.3470	1.6681	3.2637	6.1839	0.8794	1.1299	2.8062	8.6745	1.5532	2.0175	4.0652	7.6726
std_TOTAL	0.7346	0.9981	2.2794	5.1025	0.5136	0.6276	1.6047	7.1539	1.0648	1.3425	2.8056	6.2021
mean_TOTAL_FEES	0.5196	0.6024	0.9201	1.3925	0.2405	0.2763	0.5202	1.2885	0.4240	0.4920	0.7511	1.1412
std_TOTAL_FEES	0.2235	0.2647	0.4537	0.8406	0.0718	0.0834	0.1897	0.7961	0.1799	0.2112	0.3641	0.6869

**Table A4-17: Sampling period: Aug-48 -- July-83; Historical distributions with monthly bootstrapping**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	1.0084	1.2676	2.3512	4.1709	0.7608	0.9571	2.0449	5.2782	1.2302	1.5278	2.7682	4.8528
std_TOTAL	0.8668	1.2078	3.0977	7.9277	0.4782	0.6621	2.0799	10.7097	1.0609	1.4687	3.7385	9.5161
mean_TOTAL_FEES	0.4250	0.4697	0.6371	0.8875	0.2034	0.2269	0.3603	0.7392	0.3276	0.3610	0.4878	0.6795
std_TOTAL_FEES	0.2075	0.2653	0.5467	1.1479	0.0549	0.0695	0.2073	1.0505	0.1684	0.2145	0.4406	0.9249

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	1.4614	1.8362	3.5791	6.8990	0.7984	1.0190	2.4241	9.1878	1.5850	2.0354	4.0730	7.9606
std_TOTAL	0.9653	1.4683	4.6036	14.1104	0.4957	0.6918	2.3531	20.5187	1.3861	1.9964	5.8490	17.5467
mean_TOTAL_FEES	0.5268	0.6022	0.9004	1.3927	0.2136	0.2418	0.4282	1.2806	0.4225	0.4812	0.7166	1.1103
std_TOTAL_FEES	0.2748	0.3644	0.8531	2.1242	0.0592	0.0755	0.2383	2.0612	0.2244	0.2972	0.6991	1.7502

**Table A4-18: Sampling period: Jan-26 -- Dec-60 Historical distributions; bootstrapping with 12 month batches**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	3.2237	3.4478	4.1536	4.9288	3.2340	3.5424	4.7585	6.4108	3.8703	4.1336	4.9738	5.9106
std_TOTAL	2.3848	2.5983	3.4923	4.8681	2.5188	2.6813	3.8175	6.5945	2.8556	3.1120	4.1835	5.8398
mean_TOTAL_FEES	0.7945	0.8261	0.9210	1.0207	0.5243	0.5593	0.6967	0.8752	0.6268	0.6520	0.7280	0.8076
std_TOTAL_FEES	0.4457	0.4758	0.5971	0.7759	0.3285	0.3394	0.4429	0.7251	0.3647	0.3896	0.4893	0.6360

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	5.0162	5.4288	6.7517	8.2557	4.7092	5.2674	7.7910	11.6622	6.2207	6.7222	8.3468	10.2101
std_TOTAL	3.1505	3.4706	4.9199	7.3103	3.5851	3.7442	5.2723	10.3133	3.7849	4.1721	5.9384	8.8823
mean_TOTAL_FEES	1.2316	1.2957	1.4894	1.6955	0.7627	0.8311	1.1391	1.5867	1.0058	1.0585	1.2191	1.3910
std_TOTAL_FEES	0.5465	0.5896	0.7900	1.1112	0.4667	0.4672	0.5702	1.0626	0.4428	0.4790	0.6458	0.9140

**Table A4-19: Sampling period: June-71 -- May-06; Historical distributions; bootstrapping with 12 month batches**

D and DS generated according to lifetables												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	0.9647	1.2105	2.2270	3.8578	0.8162	1.0271	2.1755	5.0078	1.2083	1.5037	2.7063	4.6404
std_TOTAL	0.7217	0.9034	1.8265	3.8215	0.5217	0.6291	1.4657	5.1473	0.8770	1.0896	2.1866	4.5803
mean_TOTAL_FEES	0.4209	0.4679	0.6364	0.8714	0.2236	0.2512	0.4017	0.7403	0.3287	0.3657	0.4984	0.6866
std_TOTAL_FEES	0.1794	0.2120	0.3580	0.6343	0.0731	0.0836	0.1778	0.5901	0.1459	0.1719	0.2909	0.5187

D = 35												
	L33	L40	L60	L80	T33	T40	T60	T80	TM33	TM40	TM60	TM80
mean_TOTAL	1.3552	1.6809	3.2672	6.1767	0.8721	1.1169	2.7399	8.6641	1.5633	2.0244	4.0427	7.6357
std_TOTAL	0.7956	1.0701	2.4547	5.6406	0.5744	0.6970	1.7412	7.9624	1.1301	1.4289	3.0400	6.8857
mean_TOTAL_FEES	0.5207	0.6033	0.9146	1.3824	0.2387	0.2730	0.5062	1.2814	0.4253	0.4924	0.7449	1.1304
std_TOTAL_FEES	0.2306	0.2750	0.4832	0.9211	0.0833	0.0949	0.2046	0.8775	0.1865	0.2205	0.3898	0.7543

**Table A4-20: Sampling period: Aug-48 -- July-83; Historical distributions; bootstrapping with 12 month batches**