ASSESSING VARIABLE ANNUITIES AS A SOLUTION TO AMERICA'S RETIREMENT INCOME CHALLENGE:[1,2]

A closer look at annuity lifetime withdrawal guarantees and target date mutual funds

By[3]

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EXECUTIVE SUMMARY

This paper compares the relative performance of two emerging financial instruments, the Target Retirement Date Fund (TDF), and a Lifetime Guaranteed Withdrawal Annuity (LGWA), as tools for converting savings into retirement income. The salient feature of the annuity is its ability to ensure the retiring investor a guaranteed income stream during his or her lifetime (or spouse) , while permitting more aggressive participation in the growth potential associated with equities. The TDF, in accordance with standard practice, is assumed to allocate 33% of the account balance to an equity pool, while the LGWA allocates as much as 80%; the remainder is invested in a fixed income pool.

The authors observe that the LGWA offers a significantly higher rate of return and a much more substantive withdrawal stream compared to the traditional TDF, yet, is able to mitigate downside risk in a manner that is on par with the more conservative allocations of the TDF. In particular, in terms of the probability of the account running out of money, and the mean and standard deviation of the time at which this happens, the LGWA product provides (even with its greater allocation to equities) an almost identical profile to that of the TDFs. The key to achieving this stems from the lifetime income guarantees provided by the LGWA

The reported comparisons are based on extensive Monte Carlo simulations of the trajectories of the two instruments for several types of stochastic models, using 1926-2006 data and a variety of sensitivity analyses.

1. INTRODUCTION

This paper compares the relative performance of two emerging financial instruments, the Target Retirement Date Fund, and a Lifetime Guaranteed Withdrawal Annuity, as tools for converting retirement savings into retirement income.

Twenty-five years ago, more than 60% of employed Americans enjoyed defined benefit retirement plans, where the employer provides a fixed monthly payment throughout the employee's retirement. Today, more and more companies have eliminated traditional pension plans and have shifted the investment risk to their employees, often foregoing retirement benefits altogether. As a consequence a rapidly declining minority of future retirees continue to enjoy these defined benefit plans.

Many studies[4] have concluded that most households save too little and, in addition, invest too conservatively. In 2005, the national savings rate dipped into negative territory. Analyses of 401(k) asset allocations show that Americans are placing between 55% and 67% of their account into lower-return instruments, such as bond and money market funds, as opposed to equities.   Almost all experts agree that this is overly conservative. Moreover, much of the public's intimidation by the stock market is based on ignorance. The Hoover Institution, for example, reports, based on the above studies, that a majority of Americans do not know that stocks have had the best historical returns of all investments, with a quarter believing that certificates of deposit have. "A majority of Americans do not know why they should diversify, while 45% believe that diversification "guarantees" their investment won't fall when the market does"[5]

[4] These studies have been conducted by private consultants like Hewitt Associates, and think tanks like the Employee Benefit Research Institute and the Hoover Institution..

[5] See, for example, D. Mastio, "Lessons Our 401(k)s Taught Us: How much do investors know about investing for retirement", Policy Review 95 (1999), Hoover Institution.

Many investors face the possibility of depleting their savings or having too little to live on, in their retirement years. This problem is particularly prevalent among retirees and is compounded by the increasing longevity of the population. The tendency to invest too conservatively is based on most investors' averseness to face the risks of equity markets for a large percentage of their investment portfolio, in particular as they face the retirement stage of their life. Indeed, most financial advisors recommend that at the age of 65, investors limit their holdings in equity funds or individual stocks to approximately 33%[6], with further downward adjustments as the investor advances in age.
Target Retirement Date Funds (TDFs) constitute one popular vehicle that implements this strategy by automatically shifting assets over time to more conservative allocations. While these conservative investment strategies protect the investor against the volatility risks associated with equity markets, they provide little opportunity for pre-consumption growth and, hence, have increased potential for the savings to be depleted prematurely. For those investors with somewhat larger savings balances at retirement age, the traditional investment strategies deprive their heirs of the normal growth potential, associated with a "typical" early or mid- career diversification rule.[7]

Variable annuity contracts are emerging as one of the widely used financial solutions that seeks to alleviate some of the shortcomings of TDFs, by providing the investor with one or several guarantees. Historically, these guarantees were of one of the following two types or a combination thereof:

(a) A Guaranteed Minimum Accumulation Benefit (GMAB): the investor is guaranteed at least return of initial investment at some specific future point in time , with some potential for additional returns, based on market performance of the underlying funds.

[6] See, for example, Table 1 in J. Poterba, D. Wise, J. Rauh and S. Venti. "Lifecycle Asset Allocation Strategies and the Distribution of 401(k) Retirement Wealth", (2006), National Bureau of Economic Research (NBER) Working Paper W 11974.
[7] A simple average of current target asset allocations of three major providers of life cycle funds (Fidelity, Vanguard and T. Rowe Price) exhibits a gradual decline of the allocation percentage to equity funds from 91.5% at age 25 to 27.7 % at age 70.

(b) A Guaranteed Minimum Death Benefit (GMDB): here, the investor's beneficiaries are guaranteed to receive a specific minimum sum upon the death of the investor during the contract term.

The expanding market for variable annuities - in the United States alone this market has rapidly grown to the level of approximately $130 billion in annual sales of newly initiated contracts - is evidence that these type of contracts present investors with attractive alternatives to traditional TDF plans.

While the traditional guarantees succeed in limiting the downside risk associated with the value of the annuity at one specific point in time or at the time of death, they do not (directly) address the investor's concern of providing sufficient funds to support ongoing expenditures *during his or her life time and that of the surviving spouse (if applicable).*

Recently, a new type of variable annuity has been introduced to the market. The salient feature of this annuity is that it is able to ensure the retiring investor without annuitization, a guaranteed income stream during his or her lifetime, as well as that of the surviving spouse, while permitting him to participate more aggressively in the growth potential associated with equities. For this reason we shall refer to this contract as a Lifetime Guaranteed Withdrawal Annuity (LGWA). This lifetime withdrawal guarantee is designed to permit the investor to participate more aggressively in equity markets and hence to benefit from its growth potential, while ensuring him or her a minimum income stream, which is in place for as long as the investor is alive. Thereafter, the surviving spouse, if applicable, continues to benefit from a restricted version of the withdrawal guarantee; a more precise description of the contract is given in the next section.

The objective of this paper is to assess the *relative* performance of a hypothetical, variable annuity with a Lifetime Withdrawal Guarantee vis-à-vis that of traditional Target Retirement Date funds. This allows one to evaluate the potential of this new instrument to meet the financial challenges faced by future retirees (at least in comparison to traditional retirement plans). Our assessments are based on extensive Monte Carlo simulations of the alternative product trajectories under a variety of plausible stochastic models for the monthly co-movements of the returns of the underlying equity and fixed income pools over the lifetime of the contract. Each of the returns models uses parameter values reflecting the historical experience in one of three 35 year tranches in the period January 1926- June 2006 over which accurate historical data are available: (1) January 1926-December 1960; (2) August 1948-July 1983 and (3) June 1971- May 2006. The full set of results, along with comprehensive discussions thereof can be found in the unabridged version of this paper.[8] Because of space limitations, we present, here, only a handful of representative graphs and tables, all based on the first type of returns models discussed in Section 3. The tables (graphs) display the results for parameter values reflecting each (the first) of the above 35 year tranches. The precise details of the models used in this study are explained in section 3. The main results are described in section 4.

Summary of the main findings. Our study primarily compares a variant of the LGWA product with a relatively aggressive allocation to equities (80 %), and a simple variant of a TDF, which, in accordance with standard practice, allocates only 33 % to equities in the post-retirement years; the remaining allocation is to fixed income products. We observe, on a consistent basis, that the new variable annuity product offers a significantly higher rate of return and a much more substantive withdrawal stream compared to traditional TDFs, yet, is able to mitigate downside risk in a manner that is on par with the more conservative allocations of the TDFs. The particular simple LGWA variant studied in this paper, provides an increased mean internal rate of return (IRR) which is approximately 50 to 330 basis points higher than that characterizing the variants of TDFs studied here. The median of the total cash flow value (minus the initial investment) is always larger under the LGWA, and this difference can be quite significant. The absolute magnitude of these values depends, as one would expect, on the particulars of the economic environment in which one is testing performance.

In terms of downside risk, measured by the probability of the account running out of money and the mean and standard deviation of the time at which this happens, the

[8] The full paper can be downloaded from www.gsb.columbia.edu/faculty/afedergruen and www.gsb.columbia.edu/faculty/azeevi.

LGWA product provides (even with its greater allocation to equities) an almost identical profile to that of the TDFs. The key to achieving this stems from the lifetime income guarantees provided by the LGWA. Indeed, we observe that even under extremely favorable market conditions this insurance guarantee is activated with a likelihood that ranges from 3% - 8 %. The probability that the insurance guarantee is activated can be as high as 33% under less favorable conditions observed in one of the three tranches of our historical data.

Thus, the hypothetical LGWA allows the investor to benefit from a significantly higher mean IRR and increased withdrawal streams relative to traditional Target Retirement Date funds, yet, by virtue of the lifetime income guarantee provisions, this form of variable annuity is able to mitigate the downside risks to almost identical levels as those found in traditional conservative funds.

Figure 1 displays the deciles of the total $ amount received by the investor, during and at the end of the contract horizon, under the LGWA (L80) and the above benchmark Target Retirement Date fund (TM33), assuming that for both instruments the investor withdraws an annual amount equal to 5% of the highest anniversary value the contract has achieved. As mentioned, the Figure, representative of the general patterns observed for our various stochastic returns models, applies to the first of the three models described in section 3 and uses parameters reflecting the historical data in the first 35 year tranche.It considers both the case where the investor is subject to actuarial lifetime time distributions, as well as where he or she is assumed to outlive the full 35 year contract term. This total cash measure ignores the time value of money; in section 4 we focus on the properties of the (internal) rate of return as well as various downside risk measures.

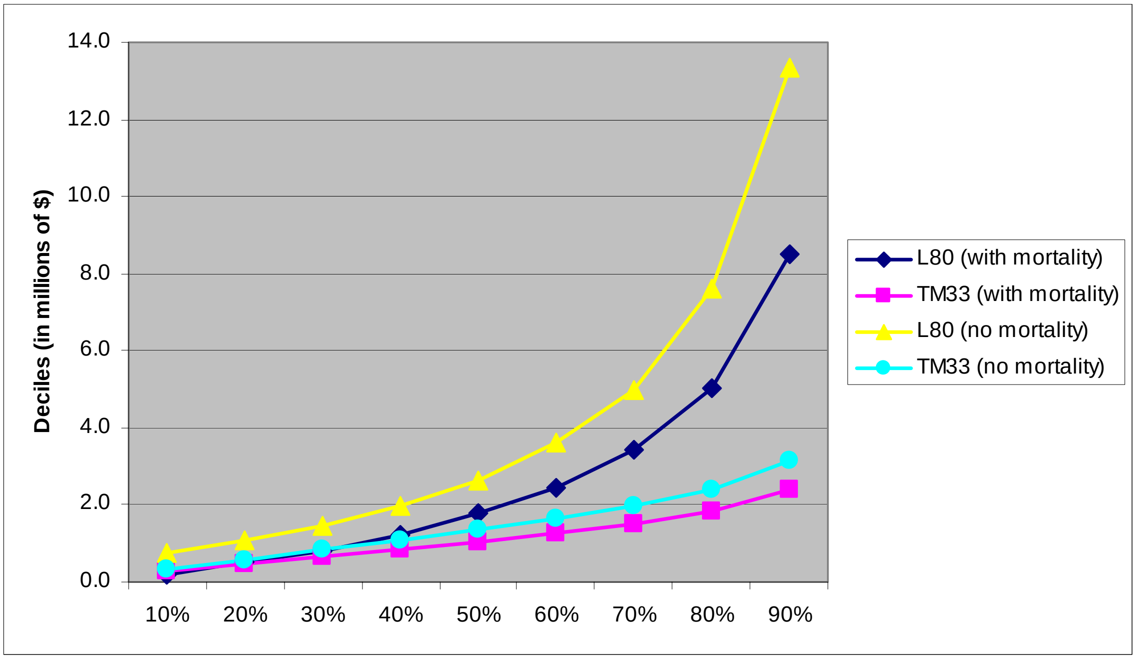

Figure 1: Total NET CASH FLOW for the calibration period of Jan-26 -- Dec-60. The y-axis represents the deciles (in millions of dollars) of the total net cash flow distribution; for each point on the curve, the likelihood of the total net cash flow being at or below the y-coordinate is given by the value of the x-coordinate

2. DESCRIPTION OF THE FINANCIAL INSTRUMENTS

Under a LGWA, the issuing insurance company guarantees that the investor, during his or her entire lifetime, can, in any given year, withdraw, an amount equal to 5%[9] of the *highest* anniversary value the contract has achieved[10]. By adjusting the guaranteed annual

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[9] This corresponds with a *monthly* withdrawal amount of 1/12 of 5%.

[10] This highest value is reassessed, periodically, in accordance with the prevailing contract value. During the first 10 years of the contract, the reassessment, or so-called *step up*-date, coincides with the anniversary date. Thereafter, the investor has the ability to choose the reassessment date, subject to the provision that at least one year must elapse between two consecutive ”step ups”.

withdrawal amount (GAWA) upward, as a percentage of the highest previously observed contract value (at specific potential adjustment or "step up" dates, see below), the GAWA is expected to grow at a rate given by a weighted average of the growth rates in the equity and fixed income pools. Since the GAWA is never reduced, even when the contract value or the underlying equity or fixed income markets decline, the investor never needs to scale down his or her consumption level, at least in nominal terms.

The above annual withdrawal guarantee is in place for as long as the investor is alive. Thereafter, the surviving spouse, if applicable, continues to benefit from a restricted guarantee: the potential monthly withdrawal amount continues to be determined in the same way it is during the lifetime of the primary investor, but the total withdrawal amount is limited to the so-called *Benefit Base.* The latter is initiated at the level of the initial contract value and is reduced, each month, by the prevailing withdrawal amount; the balance is adjusted to the level of the prevailing contract value, at the above mentioned "step up dates," but only if this adjustment results in an increase of the Benefit Base. While the surviving spouse continues to have the opportunity for "step ups" to the Benefit Base, he or she does not receive an increased "lifetime guarantee" of payments. Payments continue only until the Benefit Base is reduced to zero.

When evaluating the traditional Target Retirement Date funds, we assume either that, *for as long as there is sufficient money in the account,*

(i) the investor withdraws the same amount from these funds, as would be the case if the money were invested in the LGWA, or

(ii) the investor withdraws each month, 1/12 of 5 % of the periodically determined highest value the account balance in the Target Retirement Date fund has reached, up until the current month.[11]

Under the first withdrawal rule (i), the investor's cash flows remain identical under the LGWA and the Target Retirement Date fund, for as long as there is money in the *latter*'s account. This allows for a meaningful comparison of terminal account balances, internal

[11] In this context, potential step-ups of the withdrawal amounts are assessed at the anniversary dates of the fund

rates of return and run-out times and probabilities, while holding the investor's consumption pattern constant, for as long as possible. The second rule allows for a comparison with a traditional TDF.

We have chosen to focus on a particular variant of the LGWA which invests 80% of the contract value in equities (, denoted L80 for brevity), and a Target Retirement Date fund with a 33% participation in the equities' pool; the remainder of each account is allocated to fixed income instruments. We evaluate the TDF under withdrawal strategies (i) and (ii) above; the TDF corresponding to the former will be denoted T33, and the latter TM33 (where 'M' is mnemonic for the *modified* withdrawal rule (ii)). We consider these comparisons particularly relevant, since, as discussed above, most financial analysts would, in the absence of income guarantees such as those provided by the LGWA, advise their clients at (or after) retirement age to limit their exposure to equity markets to approximately the 33% level.

The following additional assumptions underlie the performance evaluations:

- For theTDF, an annual management fee of 1.43% of the prevailing account balance is assumed[12]; for the LGWA, the combined annual asset based fee is set at 1.15%, plus a LGWA charge assessed against the Benefit Base, which is set at 0.90 %, per annum.[13,14]
- We assume that 80% of the equity portion of the contract is allocated to the S&P 500 and 20% to Small Caps; the fixed income portion is spread equally between Long Term Corporate Bonds and Long Term Government Bonds.[15]

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[12] We assume that no front end expense load is assessed; these are typically waived if the initial investment is at least $1 million.

[13] The combined asset based management fee consists of three components: (a) a Mortality and Expense Fee of 0.15%; (b) administrative expenses of 0.15%, and (c) Fund expenses of 0.85%. These fees are, in reality, to be charged, daily, in proportion to the prevailing contract value. In our simulations we assess these fees monthly. The special LGWA charge is, in reality, to be charged quarterly: in our simulations the account is charged monthly.

[14] Variable Annuity issuers often reserve the right to increase fees on the LGWA after a specified period of time. We have, however, assumed that the fee rates remain constant.

[15] All returns data were obtained from Ibbotson for the time period 1926-2006.

- The investor has the option to withdraw less than the guaranteed levels; however, in our simulations, we assume that the investor (and spouse) take full advantage of the withdrawal limits.
- As mentioned in Footnote 4, after the first 10 years, the step-up dates, at which the Benefit Base and Withdrawal Guarantee are re-determined, may be selected by the investor with the proviso that at least 12 months must elapse between consecutive step-ups. In our simulations, we assume, that after the first 10 years, the investor always elects the *first* permitted step-up month, which results in an upward adjustment of the benefit base.

We assume throughout that the primary contract holder is male and married; he and his wife are assumed to be 65 and 62 years old, respectively. (Mortality assumptions affecting the lifetime of the contracts will be discussed in the next section.)

3. DESCRIPTION OF THE STOCHASTIC MODELS AND DETAILS OF THE SIMULATION STUDY

In this paper we take three different approaches to modeling the behavior of the equity and fixed income components of both the Target Retirement Date funds and the hypothetical. The first two approaches use simple parametric models to capture the behavior of the fixed income and equity components of the various products; below we explain the details of both these models, noting that the simpler "basic" model is nested within the second one. The third approach is "model free" in so far as it does not assume any specific structure and is only driven by the historical data itself, by means of a bootstrapping methodology.

Approach 1 (the basic model): Here we model the returns of the fixed income component as *constant* and model the behavior of the equity component using a standard log-normal hypothesis. That is, for the fixed income portfolio we assume that

$$R_t = r \quad \text{for all } t = 1, 2, \ldots$$

where R_t denotes the rate of return during month t, and r is a constant.

The log-normal hypothesis is by far the most common approach to model the dynamics of equity market movements. In particular, if we let S_t denote the value of the equity portfolio at time t, then the log-normal assumption postulates that

$$\log(S_t/S_{t-1}) \sim N(\mu, \sigma^2) \quad \text{for all } t = 1, 2, \ldots$$

That is, the price relatives follow a normal distribution with mean μ and variance σ^2. Consistent with the common random walk tenet with regard to the dynamics of equity markets, the price relatives are assumed to be independent and identically distributed with the above normal distribution. For each tranche of historical data, we estimate the constant return of the fixed income component and the two parameters, the drift rate μ and volatility σ, that characterize the log-normal distribution. These parameter estimates are then used for the subsequent simulation studies that evaluate the performance of the various investment contracts.

Approach 2 (the refined model): Here we model the returns of the fixed income component using a more elaborate stochastic model commonly known as an ARMA/GARCH process.[16] This model is rich enough to capture two key characteristics which are commonly observed in the behavior of interest rates, namely, mean reversion and heteroscedasticity in the volatility. In particular, our model postulates that

$$R_t = r + \rho R_{t-1} + \varepsilon_t \quad \text{for all } t = 1, 2, \ldots$$

where ε_t follows a normal distribution with mean 0 and conditional variance σ_t^2 satisfying:

$$\sigma_t^2 = \kappa + \alpha\sigma_{t-1}^2 + \beta\varepsilon_{t-1}^2 \quad \text{for all } t = 1, 2, \ldots$$

These dynamics encode within them the two qualitative features described above. First, R_t has a long term mean which is equal to $r/(1-\rho)$ and it reverts to this mean at rate ρ. Second, the volatility at time t has mean zero but its conditional variance, σ_t^2, evolves over time in a manner which is dictated by the past conditional volatility as well as

[16]GARCH stands for Generalized AutoRegressive Conditional Heteroscedasticity. This class of models, that were first introduced in a simpler form by Engle (1982), is by far the most common tool for modeling short term behavior of interest rates in financial markets.

current realized volatility. In this manner the eventual realized volatility in the interest rate process R_t is non-homogenous and exhibits features which are more representative of those observed in empirical data from fixed income markets. With regard to the specification of the above model, we impose the following constraints: $|\rho|<1$, $\alpha+\beta<1$, and $\alpha,\beta,\kappa\geq 0$. These ensure that the process is well defined mathematically and exhibits a long term steady-state behavior. For the equity component we continue to use the standard log-normal model described earlier. Again, for each tranche of historical data, we estimate the parameters of the above ARMA/GARCH process ($r,\rho,\kappa,\alpha,\beta$) and the two parameters that characterize the log-normal distribution.

Approach 3 (historical bootstrapping): Here we depart from the world of parametric models and propose an alternative that is free of such assumptions. Our approach is based on the concept of bootstrapping[17] which roughly works as follows. For any given tranche of historical data we first partition it into non-overlapping blocks of equal length. We then assemble a bootstrap sample by randomly drawing from these blocks (with replacement); this resulting bootstrap sample has the same length as the original tranche, and is comprised of blocks of the original data that have been essentially shuffled out of order and may contain repetitions of any given block (and hence omissions of other blocks). By executing this procedure in sync for both the fixed income and equity data, we preserve the historical co-movement (dependence) structure between the two components, while at the same time maintaining the intra-block temporal dependence structure in each time series. The shuffling of blocks introduces sufficient independence that enables the bootstrap theory to be invoked. In our study we simulate 10,000 bootstrap samples, based on which all performance measures are calculated. This provides an adequate level of accuracy for our purposes and is consistent with the 10,000 simulations used in other cases. (We have tried a variety of block sizes in our study and the results do not seem to be very sensitive to the choice of this tuning parameter.)

Further details concerning the simulation study. Since the new annuity is designed for retirees, or those approaching retirement age, we have modeled the investor's and the

[17] Efron, B., and Tibshirani, R. An Introduction to the Bootstrap. Chapman and Hall, 1993.

spouse's mortality in accordance with actuarial survival rates. All simulations are carried out by generating a pair of remaining life times for the investor and spouse, in accordance with these survival probabilities.[18] However, in parallel, each simulated scenario has been evaluated under the assumption that the investor outlives the full 35 years, i.e., the full contract term. The first version provides a statistically realistic representation of the return and risk profile which a typical investor faces. The second set of simulations exhibits, however, how the investor would fare under the different investment vehicles if he faced what, in terms of the onus to finance retirement income, may be viewed as a *worst case* scenario.

Each set of simulations, for a given combination of an investment vehicle, stochastic model of equity and fixed income returns, and a given assumption about the investor's (and the spouse's) remaining life time (actuarially based, or in excess of the full 35 year contract limit) is carried out 10,000 times to generate high precision estimators, see Section 4[19]. Since there are 480 sets of simulations, a grand total of 4,800,000 35-year trajectories (of 420 months each) have been evaluated.

4. RESULTS

When comparing the performance of the LGWA with the traditional TDF, we focus on the following measures:

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[18] For this purpose, we have employed the most recent (2002) actuarial tables published by the Social Security Office, see http://www.ssa.gov/OACT/STATS/table4c6.html.

[19] For example, with a sample of 10000 instances, the standard error of the estimate of the internal rate of return IRR, see Section 4, is invariably less than 0.05%, resulting in a margin of error of 0.1%. This implies, for example, that if the estimate of the mean IRR is 6%, say, the 95% confidence interval for the mean is contained within the interval [5.9%, 6.1%].

- IRR= the internal rate of return of the complete cash flow stream, including the initial investment, monthly income withdrawals and the terminal account value;
- INCOMERATIO= the mean ratio of the total income received under the considered instrument and that received under the TM33 contract.
- PROBGAR= the probability that, at any point in time, the insurance guarantee provided by the LGWA is activated, i.e., the investor is able to withdraw an amount in excess of his or her contract value.
- PROBRUN= probability that the income stream for the investor runs out before the end of the contract term
- RUNOUT= number of months before the end of the contract term, during which the (spouse of) the investor (continuously) receives no income.

The IRR and RUNOUT are random variables, which we characterize via their means and standard deviations, and in the IRR case, via the complete cumulative distribution plot.

Table 1 exhibits the various performance measures, under the first of the four stochastic returns models, i.e., lognormal equity values, combined with constant fixed income returns. The first (middle, last) set of three columns refers to parameter values matching historical data for a stretch of 35 years starting in January 1926 (August 1948, June 71). While the *absolute* values of the various performance measures vary significantly with the parameters and type of stochastic returns model employed, the following conclusions hold across the board: (We mostly confine ourselves to the results for an investor whose life time, as well as that of his spouse, follow the actuarial probability distributions; for any of the above measures, the *relative* performance of L80 is (further) enhanced under the assumption that the investor outlives the full 35 year contract horizon.)

	Lifetimes generated using probability tables								
	Jan-26--Dec-60			Aug-48--July-83			June-71--May-06		
	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33
PROBGAR	0.3308	n/a	n/a	0.0285	n/a	n/a	0.0292	n/a	n/a
mean_IRR	7.46%	5.22%	4.91%	8.43%	5.35%	5.23%	9.00%	8.63%	8.53%
std_IRR	4.66%	2.47%	2.11%	2.72%	1.26%	1.11%	2.94%	1.25%	1.16%
mean_RUNOUT	11.0290	62.2196	9.3136	1.1774	49.9757	0.7069	1.2222	7.5424	0.0000
std_RUNOUT	31.4245	69.9894	28.7004	10.4022	57.9143	6.5530	10.7919	25.2829	0.0000
PROBRUN	0.1590	0.6140	0.1460	0.0192	0.5956	0.0195	0.0205	0.1256	0.0000
INCOMERATIO	1.6135	1.1176	1.0000	1.5968	1.2021	1.0000	1.1738	1.1137	1.0000

	Investor assumed to live throughout the 35 years								
	Jan-26--Dec-60			Aug-48--July-83			June-71--May-06		
	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33
PROBGAR	0.5445	n/a	n/a	0.0847	n/a	n/a	0.0831	n/a	n/a
mean_IRR	8.12%	5.24%	4.86%	8.49%	5.37%	5.21%	9.07%	8.68%	8.54%
std_IRR	3.52%	2.42%	1.94%	2.26%	1.21%	0.98%	2.46%	1.13%	0.98%
mean_RUNOUT	0.0000	153.3929	30.5296	0.0000	134.3126	3.4250	0.0000	26.0094	0.0000
std_RUNOUT	0.0000	65.3596	51.7199	0.0000	50.4820	14.8924	0.0000	46.9966	0.0000
PROBRUN	0.0000	0.9676	0.3647	0.0000	0.9904	0.0810	0.0000	0.3203	0.0000
INCOMERATIO	1.9804	0.9131	1.0000	1.9019	0.9635	1.0000	1.2174	1.0321	1.0000

Table 1: Lognormal equity values; constant fixed income returns

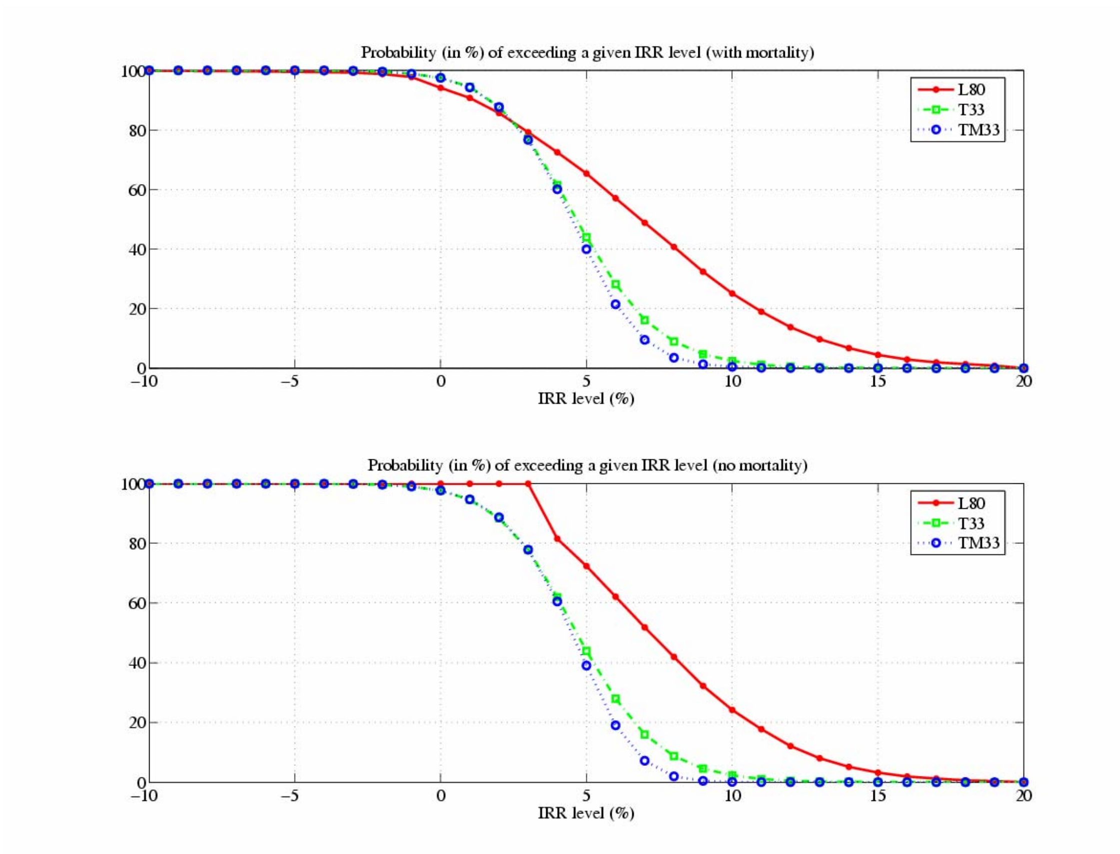


Figure 2: Calibration period: Jan-26 -- Dec-60 Lognormal equity values; Fixed Income returns constant.

Properties of the rate of return. We first note that the mean IRR of the L80 product is significantly larger than that of the traditional T33 and TM33 funds. This difference is always statistically significant, as the estimates of the mean IRR have a margin of error[20] of less than 0.1%, when building 95% confidence intervals. (The relatively low difference of 0.5% pertains to a set of parameters which display an unusually small equity premium, i.e., difference in the average equity and fixed income returns of 3.6%; under such a set of scenarios, the mean IRR is relatively insensitive to the allocations applied to the equity and fixed income pools.) The *additional mean* IRR under L80 comes at the expense of a somewhat larger *standard deviation*. However, the *standard deviation* is a measure of general volatility, which incorporates the potential for *upside* deviations from the mean in an equal and symmetric way to *downside* deviations. A more meaningful comparison is provided by Figure 2, which displays the likelihood of "beating" any given IRR percentage, under the three instruments, when the parameters in the stochastic returns model are anchored on the first of the three 35 year intervals. If the investor outlives the full 35 year period, the likelihood of beating *any* IRR level is higher under L80. In risk management terms, the IRR under L80 *(stochastically) dominates* that of the other two instruments. Under actuarially distributed lifetimes, there is *near dominance*: the likelihood of beating any given IRR percentage is higher for all but a small range of low IRR values; moreover, in that range the difference between the likelihoods is small.

Downside risk properties. The second point to note is that the investor enjoys, under the L80 product, the above *additional mean* IRR, while limiting the *downside risks* to roughly the same values as the TM33 product. These downside risks are measured by PROBRUN (= probability of the income stream coming to a halt) and the mean and standard deviation of RUNOUT, the number of months before the end of the contract term during which no income is received. This illustrates that the LGWA provisions mitigate the "run-out" risks almost to the same extent as the conservative allocation rule (33% investment in the equity fund) does for the traditional Target Retirement Date fund

[20] The margin of error is defined as the one-sided width of the confidence interval.

TM33, while allowing for a considerably higher expected IRR. In addition, the TM33 contract limits the "run-out" risks to the same level as the L80 product, but only by reducing the monthly income withdrawals to a much lower level: the INCOMERATIO measure shows that the monthly income withdrawn by the investor and his or her spouse under L80 is, on average, approximately 60% larger than under TM33, for the first two sets of simulations and 17% larger under the last set.

It is possible to rephrase the previous point in the following manner. If within a traditional Target Retirement Date fund which allocates only 33% of the account to the equity pool, one allows the investor to withdraw, for as long as possible, equal amounts as under L80, there is a dramatically *larger* likelihood of "running out". In the first set of simulations, the likelihood of running out is 61% under T33 versus 16% under L80. These likelihood pairs are 60% versus 2% and 13% versus 2% for the second and third set of simulations, respectively. Similarly, the average duration of the uncovered period, during which no income is received, i.e., the RUNOUT mean, is at least 5.5 times as long under T33 as compared to L80. Even though the investor is originally permitted to withdraw equal amounts under T33 as under L80, in the end, the average monthly income is only 10-20% larger than under a traditional TM33 fund, the consequence of a significantly larger likelihood of the income stream coming to a halt because the account is depleted. (If the investor outlives the full 35 year contract term, the average monthly income is hardly larger than under TM33 and, in two of the three sets of simulations, significantly lower!) However, under TM33 with lower withdrawals before run-out than T33, TM33 may not be able to keep up with inflation.

On the role of the lifetime withdrawal guarantee. A third point concerns the PROBGAR measure. Our results indicate that under certain historically prevalent parameter combinations, there is a high likelihood that the LGWA guarantees need to be activated, i.e., the investor or the spouse receives income in excess of their prevailing contract value. (Almost invariably, this means that income is received, while the contract value has been depleted.) This likelihood is as high as 33% in the first set of simulations.

	Lifetimes generated using probability tables								
	Jan-26--Dec-60			Aug-48--July-83			June-71--May-06		
	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33
PROBGAR	0.3420	n/a	n/a	0.0418	n/a	n/a	0.0344	n/a	n/a
mean_IRR	7.46%	4.69%	4.41%	8.11%	4.47%	4.38%	8.92%	8.00%	8.00%
std_IRR	4.71%	6.94%	6.80%	2.81%	2.67%	2.57%	2.99%	2.15%	2.06%
mean_RUNOUT	11.6207	70.9804	17.7510	1.8141	61.0925	8.6236	1.4356	16.7186	0.6918
std_RUNOUT	32.3627	73.3347	42.3795	12.7735	64.0575	30.5634	11.1595	39.4093	8.2065
PROBRUN	0.1646	0.6577	0.2328	0.0316	0.6524	0.1267	0.0236	0.2277	0.0119
INCOMERATIO	1.6631	1.0951	1.0000	1.6134	1.1401	1.0000	1.2250	1.0946	1.0000

	Investor assumed to live throughout the 35 years								
	Jan-26--Dec-60			Aug-48--July-83			June-71--May-06		
	L80	T33	TM33	L80	T33	TM33	L80	T33	TM33
PROBGAR	0.5555	n/a	n/a	0.1100	n/a	n/a	0.0924	n/a	n/a
mean_IRR	8.11%	4.70%	4.34%	8.21%	4.47%	4.34%	8.99%	8.00%	8.00%
std_IRR	3.54%	6.91%	6.73%	2.32%	2.62%	2.47%	2.49%	1.98%	1.80%
mean_RUNOUT	0.0000	164.3911	50.4816	0.0000	150.9805	26.5660	0.0000	50.0164	2.3600
std_RUNOUT	0.0000	67.7697	67.6079	0.0000	52.9997	51.8570	0.0000	66.1775	16.5113
PROBRUN	0.0000	0.9588	0.4981	0.0000	0.9825	0.3305	0.0000	0.4768	0.0303
INCOMERATIO	2.1238	0.9114	1.0000	1.9861	0.9322	1.0000	1.3275	0.9877	1.0000

Table 2: Lognormal equity values; fixed income returns from ARMA/GARCH

As mentioned, the *relative* performance of the L80 product is comparably, or at least equally favorable under any of the alternative stochastic returns models. As an example, Table 2 exhibits the results when the returns of the fixed income pool are generated by the ARMA/GARCH process described above, (while the equity fund follows the Lognormal process employed in the first stochastic model.) Here, in the first set of simulations, L80 exhibits both a much *larger mean* and a much *smaller standard deviation* of the IRR than T33 or TM33. Under the second set of simulations, the standard deviations of the IRR are approximately identical under L80 and T33 or TM33. (Under the first two sets of simulations, L80 thus completely dominates T33 and TM33 when considering traditional mean-variance tradeoffs.) In fact, the IRR under L80, stochastically, dominates those under the T33 and TM33 instruments: the likelihood of beating any given return rate is always larger under L80. Due to the increased volatility of the fixed income returns, the PROBGAR values are even higher than those discussed, previously.

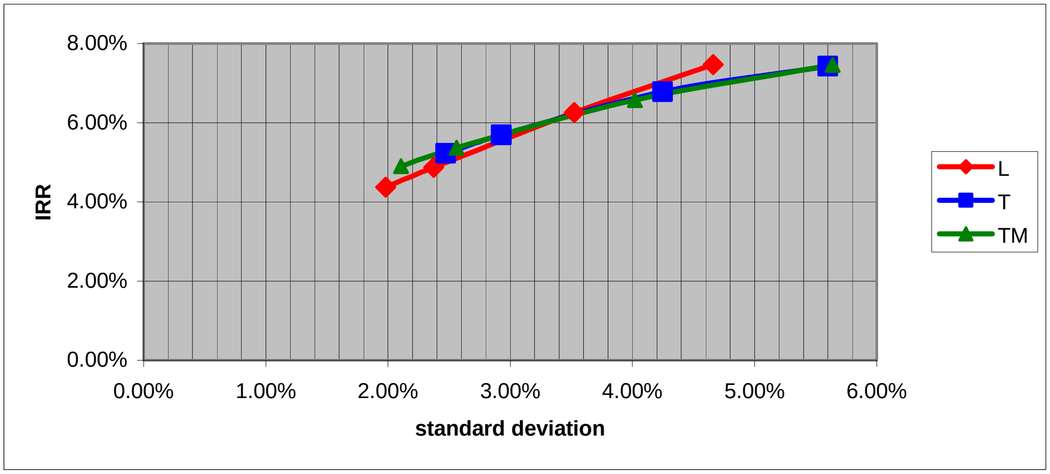
Robustness of the results. The robustness of the above conclusions has been confirmed via a variety of sensitivity analyses in which one of the parameters in the stochastic

returns model is varied, one at a time. The general observation is that increased volatility in the equity markets further enhances the *relative* performance of the L80 instrument.
In Federgruen and Zeevi (2006)[21], we have compared the various performance measures, not just in the aggregate across all 10,000 scenarios generated within a given stochastic model, but also within each of the four quartiles of the IRR (of the T80 instrument, the straw man product); the comparisons have been carried out separately for the scenarios in which the investor's income stream runs out and those where it does not. We have done this systematically for all simulations based on historical distributions employing the basic bootstrapping technique. Focusing on the set of scenarios in which, under the straw man vehicle, the investor runs out of income, we have observed that the L80 instrument outperforms TM33, in *each* of the four quartiles considered there, both in terms of the mean IRR and in terms of *each* of the run-out statistics. Thus the LGWA provisions limit the downside risks as effectively as the conservative TM33 vehicle, not just in the aggregate but specifically in the set of troublesome scenarios that are associated with run-outs (under the straw man instrument,) and this in each of the quartiles of this set of scenarios.

Different allocation schemes. Thus far we have focused on a comparison of the L80 instrument and the traditional Target Retirement Date funds T33 and TM33. As explained, we consider these the basic choices, since the LGWA guarantees are designed specifically to allow the investor to participate boldly in the equity markets, while limiting downside risks; similarly, in the absence of such income guarantees, financial advisors will, understandably, advise retirement age investors to limit their exposure to the equity markets to 33% or less. The statistical analysis, discussed in the previous subsection, substantiates the rationale for this practice.
In Federgruen and Zeevi (2006), we compare, nevertheless, the following four allocation strategies (33%, 40%, 60% and 80% participation in the equity markets, with continuous rebalancing of the account), for the instrument with the LGWA provisions (L), as well as

[21] Federgruen and Zeevi (2006) " Assessing Financial Product Solutions to America's Retirement Income Challenge", the unabridged version of this paper can be obtained from the authors.

the Target Retirement Date funds T and TM. This gives rise to a total of 12 instruments, which have been compared systematically, for all 20 stochastic returns models.



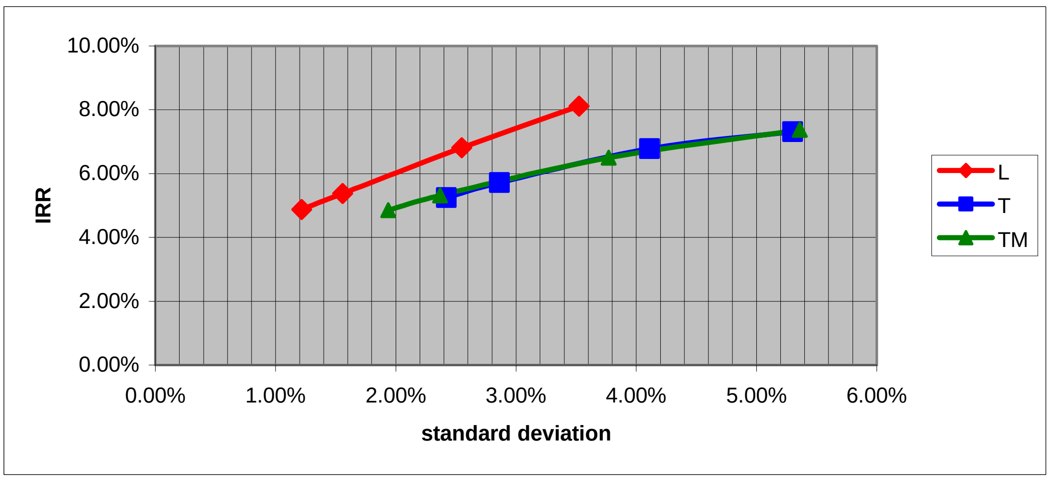


Figure 3: Calibration period: Jan-26 – Dec-60; Lognormal Equity Values; FI returns constant. The plot displays the efficient frontier for the 3 instruments based on 4 different allocation strategies; the top exhibit corresponds to actuarial lifetimes, while the bottom exhibit assumes the investor outlives the 35 year horizon.

As an example, Figure 3 displays the means and standard deviations of the IRR for the 12 instruments considered, using three *efficient frontier* curves - one for each of the instrument types L, T, and TM. Under actuarially based lifetimes, the efficient frontiers cross each other: for most equity allocation rules, the L-instrument has a slightly lower mean and a considerably lower standard deviation of the IRR than the corresponding T- or TM- instrument. When comparing the L- instrument with the T-products under *identical* equity allocation percentages, the following observations apply categorically: Both the mean and the standard deviation of the IRR are, almost invariably lower for the former, compared with the latter. However, the differences in the means are relatively small. Thus, almost invariably, the Lifetime Guaranteed Withdrawal Annuity (LGWA) reduces the risk, as measured by the standard deviation of the IRR, considerably, at the expense of sacrificing *little* in terms of the *expected* IRR. Sometimes, the L-instrument actually *dominates* the corresponding T- choices, in terms of both the mean and the standard deviation. .

In general, the *relative* performance of an L-contract becomes stronger when considering larger volatilities in the equity and fixed income markets. For example, in all of the 5 stochastic models with an ARMA/GARCH process governing the fixed income returns, is the efficient frontier for the L-contract entirely *dominating* the other efficient frontiers, i.e., regardless of the chosen equity allocation, the L-contract has both a superior mean and standard deviation of the IRR. Almost invariably, the L- instrument performs considerably better in terms of the run-out statistics; this, in addition to exhibiting a lower standard deviation of the IRR.