A Theory of Bank Liquidity Requirements*

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Abstract

We develop a theory of bank liquidity (cash reserve) requirements. The model encompasses three motives for requiring bank cash holdings as part of a prudential regulatory framework: (1) cash is observable and verifiable, (2) because the riskiness of cash is invariant to bankers' decisions about whether to invest resources in risk management, greater cash holdings improve incentives to manage risk in the non-cash asset portfolio of risky assets held by the bank, and (3) maintaining cash in advance saves on liquidation costs. In an autarkic banking equilibrium, cash is held voluntarily by banks as a commitment device to manage risk properly; increasing cash holdings in response to adverse news stems depositors' incentives to withdraw funds early. In a model with multiple banks and information externalities, deposit insurance may be optimal, and cash reserve requirements are needed to incentivize prudent behavior by banks. In a model with multiple banks subject to liquidity shocks, the coalition of banks will commit to lend each other funds in response to bank-specific needs to accumulate cash; in that equilibrium, cash requirements will be imposed by the group to prevent free riding on efficient interbank liquidity assistance.

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1 Introduction

In response to the global financial crisis of 2007-2009, the Basel Committee has proposed a new global set of liquidity requirements, the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NFSR), to complement its revised framework of international capital requirements. The primary and obvious motivation for the new interest in managing banks' liquidity is concern about liquidity risk: "The objective of the LCR is to promote the short-term resilience of the liquidity risk profile of banks. It does this by ensuring that banks have an adequate stock of unencumbered high-quality liquid assets that can be converted easily and immediately in private markets into cash to meet their liquidity needs for a 30 calendar day liquidity stress scenario" (Basel Committee on Banking Supervision, 2013). After the malfunctioning of interbank markets and the heavy reliance of banks on central bank lending during the crisis, policy makers understandably would like to reduce the likelihood of systemic crises, fire sales, and the dependence of banks on the lender-of-last-resort.

In this paper, we develop a model of liquidity requirements in which the proper focus of regulation is on the asset side of the bank's balance sheet, and in which the liquidity requirement takes the form of a narrow cash reserve requirement. Rather than the Basel III approach - which seeks to limit rollover risk - we show that the role of liquidity requirements should be conceived in a more nuanced way, not just as an insurance policy to deal with liquidity risk that can arise in a financial crisis, but also as a prudential regulatory tool to make crises less likely. In our framework, cash requirements limit default risk and encourage good risk management. We show that the primary benefits derived from cash requirements (like capital requirements) relate to the special role of cash in incentivizing improvements in bank risk profiles by encouraging proper risk management. Cash holdings reduce the probability of a liquidity crisis by making banking systems more resilient from a default risk perspective.

Our framework also provides a resolution to "Goodhart's Paradox" of liquidity regulation (Goodhart 2008, Weidmann 2014). Goodhart argues that because cash requirements force banks to hold cash, they may limit the usefulness of having cash to resolve liquidity problems. In our model, the incentive benefits of cash holdings and cash reserve requirements enable markets to function better, and their usefulness does not depend on a bank's ability to payout the cash to resolve liquidity risk.

Unlike the Basel III approach to liquidity regulation - in which capital standards regulate default risk and liquidity standards are conceived as protecting against additional exogenous liquidity shocks - our approach recognizes that illiquidity in markets is almost always a direct consequence of severe increases in default risk. For example, during the recent U.S. banking crisis of 2007-2009, the ratios of the market value of equity to assets fell gradually over a period of more than two years, implying a gradual increase in counterparty risk. At the time of the failure of Lehman Brothers in September 2008, the counterparty risks of banks such as Citibank had risen to a critical level (Calomiris and Herring, 2013). The collapse of short-term funding in repos, commercial paper, and interbank lending in the fall of 2008 was not an exogenous shock, but rather reflected that rise in counterparty risk; Lehman's failure was the proverbial "match in the tinder box," just as the 2007 asset-backed commercial paper collapse had reflected the arrival of adverse news about counterparty risk (Heider, Hoerova and Holthausen, 2009; Gorton and Metrick, 2012; Covitz, Liang, and Suarez, 2013).

The recent crisis was not new in this respect. The history of banking crises is a history of endogenous collapses of market liquidity that are caused by asymmetric information about bank default risk. Because bank liabilities have short duration and mainly take the form of money market instruments, they respond to even small increases in default risk with severe credit rationing (Calomiris and Gorton, 1991). Indeed, it is noteworthy that the history of prudential regulation prior to the 1980s focused on cash ratio requirements, not capital ratio requirements, as the primary regulatory tool. In that sense, our model points toward the need to restore cash requirements to their rightful place in the prudential regulatory toolkit.

Another empirical observation that motivates our approach is the importance of risk management for explaining cross-sectional differences in the extent to which banks suffer losses during financial crises. Ellul and Yerramilli (2013) show that the centrality of the chief risk officer (CRO) within the bank (as measured by the ratio of CRO compensation relative to CEO compensation) predicts the extent of bank risk taking ex ante and losses ex post. Fahlenbrach, Prilmeier and Stulz (2012) similarly show that there is a bank fixed effect in risk management: banks that had the relatively greatest losses in 2008 tended to be those that also suffered the most severe losses in 1998.

We construct a simple framework of cash requirements in the context of a model in which both default and liquidity risk arise. We show that, under the assumptions of a parsimonious model that allows bankers to choose their balance sheet structure and their risk profile, there is an optimal amount of observable cash assets that a bank holds.

There are two essential differences between capital on the liability side and cash on the asset side that drive our results: (1) The value of cash held at the central bank is observable (verifiable) at all times; the value of capital, in contrast, depends on the value of risky assets. (2) Cash is a riskless asset and so, when banks hold cash, they commit to removing default risk from a portion of their portfolio. Because cash is both observable and riskless, and is available to repay senior claim-holders (depositors) in the event of a bank liquidation, the commitment to hold cash has important implications for bankers' incentives toward risk in the future. That commitment affects the way outsiders – who lack information about bank assets and bankers' behavior – view the risk management of the bank, which has immediate consequences for the bank's access to funding.

In our model, there is a conflict of interest between the banker/owner and outside investor with respect to risk management; the banker suffers a private cost from managing risk, and does not always gain enough as the owner to offset that cost. We demonstrate the relative importance of cash for mitigating both default and liquidity risks by solving our model under three alternative policy regimes: (1) autarky, in which each bank chooses both its asset and liability structure in combination with a decision to manage the risk of high-return assets (e.g., loans) in an environment without liquidity risk, (2) a deposit insurance system created to address information externalities in risk management, and (3) collective action, where a group of banks forms a coalition to coinsure against liquidity risk.

Under autarky, regulation of liquidity plays no role because there are no externalities to motivate regulation. The bank voluntarily chooses the socially and individually optimal combination of cash holdings and loans, trading off the benefit from committing to "incentive-compatible" risk management from higher cash holdings against the cost of holding low-return cash assets.

An important insight from the autarky model is that the ability of outsiders to withdraw their claims plays an important role in encouraging banks to maintain adequate amounts of cash, especially when "bad" states of the world necessitate a topping up of cash balances. Without the discipline of a withdrawal threat, the bank has no incentive to increase cash holdings in bad states of the world. Instead, it would promise to do so but renege on that promise after outsiders had deposited their funds with the bank. That is, the bank must be exposed to potential liquidity risk in order for cash to be able to improve risk management.

We next consider a model of the economic efficiency gains from deposit insurance in the presence of an information externality among banks (depositors cannot distinguish between sound and weak banks). We show that deposit insurance is optimal when depositors are sufficiently risk averse, and the subsidy risky banks receive from safe banks is not too high. We also show that a cash reserve requirement is an important tool in the presence of deposit insurance (even though deposit insurance eliminates all liquidity risk in the banking system). Cash requirements are necessary in the presence of deposit insurance as a means of controlling default risk.

We then examine a role for cash reserve requirements in a model of a banking system subject to liquidity shocks. We show that the coalition of banks (which we refer to as the clearinghouse because bank clearinghouses historically were the form of collective action that managed liquidity risk) is able to achieve a better outcome than banks can achieve under autarky because liquidity risk is diversifiable across banks. As before, giving depositors a withdrawal option is necessary to implement the socially optimal amount of cash holdings, which result in optimal risk management. To prevent free riding on collective protection, the bank coalition will therefore impose a cash reserve requirement on its members (as discussed, for example, in Gorton, 1985).

Our framework has several implications for the design of liquidity regulation. Rather than viewing the regulation as ex post insurance, we view it as providing ex ante incentives to reduce credit risk, which in turn enables banks to better access markets for low-risk, short-term debt. Our framework stresses the need to regulate cash holdings, not rollover risk, and crucially, it requires that cash holdings be continuously observable, which requires that they be held outside the bank (for example, at the central bank). If instead cash was held inside banks, holding more liquidity might actually lead to greater risk-taking (see Myers and Rajan, 1998).[1]

We also emphasize the need to jointly consider liquidity and capital regulation. The banks in our model make joint decisions about their asset and liability structure. Because cash reserve requirements ensure efficient and credible risk management, they also make the book value of equity reported by the bank a more reliable indicator of the true value of equity. Thus, an important implication of our model is that the use of cash reserve requirements makes equity ratio requirements more effective.

Finally, the Basel LCR approach views greater reliance on insured deposits as resulting in less need for cash assets because there is less withdrawal risk. In sharp contrast, in our model, insured deposits undermine market discipline; required holdings of cash are therefore needed to compensate for the absence of market discipline.

Section 2 provides a brief review of the literature. Section 3 describes the model. Section 4 presents the benchmark case in which effort is observable and there is no moral hazard problem. We then consider the problem with moral hazard and autarky in Section 5. Sections

[1]The failure of MF Global illustrated how financial firms can "window dress" their accounts on quarterly reporting dates to appear to hold a large amount of cash assets, but those assets can disappear quickly at the discretion of management when management chooses to invest them in risky assets. See, for example, "MF Global Masked Debt Risks," Reuters (November 3, 2011).

6 and 7 solve for equilibrium under deposit insurance and under collective action, respectively. Section 8 concludes. All proofs are in the Appendix.

2 Literature Review

A helpful place to begin any discussion of cash reserve requirements is with the Black-Scholes-Merton framework (e.g., Merton 1977). In that framework, there are no transaction or information costs, all information that can be known is known equally to all parties, and all securities are equally (perfectly) liquid – by which we mean that they can be sold for their true value, which is common knowledge, without incurring any physical liquidation cost. In that framework, the only special property of cash is its lack of risk. From the standpoint of prudential regulation – which seeks to avoid the default risk of banks – greater cash holdings and greater reliance on equity finance each reduce the default risk of the bank.

When one relaxes the assumptions of the Black-Scholes-Merton framework, however, the effects of cash and equity on default risk are not the same. For example, in Diamond and Dybvig (1983) physical costs of liquidation make liquidity risk (the possible need to finance early consumption) costly, which could motivate the holding of inventories of liquid assets. In Calomiris and Kahn (1991), depositors receive noisy and independent signals about the risky portfolio outcome of a bank. By holding reserves, banks insulate themselves against the liquidity risk of a small number of "misinformed" early withdrawals in states of the world where the outcome is actually good. Without those reserves, banks offering demandable debt contracts (which are optimal in the Calomiris-Kahn model) would unnecessarily subject themselves to physical liquidation costs when they fail to meet depositors' requests for early withdrawal.

In many models of banking, equity is an undesirable means of controlling risk because of the high costs of raising equity. Typically, the optimal contract between the banker and his or her funding sources is a debt contract, and sometimes a demandable debt contract.

Debt contracts economize on the cost of ex post verification (Townsend 1979, Diamond 1984, Gale and Hellwig 1985, Calomiris and Kahn 1991), reduce the negative signaling of bank type (Myers and Majluf 1984), allow the trading of the outside claim (Gorton and Pennacchi, 1990) and they can limit the hold-up problem between bankers and their borrowers (Diamond and Rajan, 2001). In our model equity is assumed to be supplied by the banker, and not by outsiders because of its prohibitively high cost. The outsiders claim is senior to the banker's claim and can therefore be thought of as debt. Importantly, it will be optimal to make the debt demandable.

Given the high costs of external finance via equity, cash holdings can provide a more cost-effective means of reducing default risk even in the absence of moral-hazard. Moral-hazard makes default risk endogenous and creates a conflict of interest between bank insiders and outside investors. The conflict of interest creates a wedge between physical cash-flows and those that can be credibly pledged to outside investors (see Holmström and Tirole 1997, 1998).

Biais, Heider and Hoerova (2012) show that in the context of derivative trading with moral-hazard in risk-management, optimal hedging contracts may benefit from the use of variation margins. As we show in the model below, cash holdings can be an especially useful means of reducing risk through the effects of cash holdings on the incentives of bankers to expend effort on risk management, especially in economic downturns. Because cash holdings limit the extent to which debtholders lose from high-risk strategies and the extent to which bankers can pursue high-risk strategies, more cash helps to better align managers' incentives with the interest of debtholders.

Our paper builds on the insights of these various models. Like Diamond and Dybvig (1983) we assume that exogenous demand for early liquidation can be physically costly, which gives rise to liquidity risk in our model. Like Holmström and Tirole (1997, 1998), we assume that risk management entails private costs to the bank manager. In such a context, cash is doubly important, not just because of its lack of risk, but also because of its observability. Outsiders investors can be confident that managers will exert effort for proper risk management irrespective of the unobservable state, if and only if cash margins are sufficiently high. As in Biais, Heider and Hoerova (2012), setting aside cash on a separate account has the benefit of curbing the moral-hazard in risk-management but carries the (opportunity) cost of having less invested in a high-return asset. In equilibrium, a withdrawable senior claim (e.g, deposits) for outsiders and a sufficient amount of cash reserves provide an optimal contractual solution to the banking problem that combines all those elements: That solution minimizes the overall costs associated with early liquidation, shirking in risk management and foregone opportunities for profits (from cash holdings).

The role of cash in our banking environment resembles at first glance the role of collateral in environments with adverse selection (Besanko and Thakor 1987) or moral hazard (Boot and Thakor 1994). There are, however, important differences. First, the use of cash can easily be made contingent on the state of the economic environment. While this in principle is also possible for traditional collateral, say a banker's house, it is unlikely to be feasible in practice. The value of the house will also depend on the economic environment and, moreover, the value will probably not be enough relative to the size of the incentive problem of a typical bank. This relates to the second difference. While the amount of collateral is usually taken as given, the amount of cash is endogenous. It depends on the portfolio choice of the banker. Moreover, and again related, the value of collateral is usually *assumed* to be lower in the hands of lenders than in the hands of borrowers. In contrast, the cost of cash is an opportunity cost. It arises from not having invested in high-risk/high-return assets such as loans.

It is also interesting to note the long tradition in banking – despite the absence of formal modeling – that has focused on cash requirements as a prudential device. Indeed, with few exceptions, historical prudential regulation prior to the 1980s has concentrated on requirements for cash rather than requirements for capital. For example, the New York Clearing

House maintained a 25% cash reserve requirement against deposits for its members.[2] In the famous 1873 Coe Report, authored by George Coe, the President of the New York Clearing House, cash was seen as the essential tool for managing systemic risk (Wicker 2000). In contrast, neither regulators nor bank coalitions set minimum equity capital-to-asset ratios for banks, with few exceptions, until the 1980s.[3]

The observability advantage of cash has been particularly obvious in the recent history of financial crises in our current highly regulated and protected banking environment, where government both insures deposits and regulates banks' risk-based capital requirements. In this environment, depositors or other banks have little incentive to monitor banks or penalize high risk. Supervisors, who are charged with protecting taxpayers from the costs of bailing out banks, typically have not identified bank losses in a timely way. That fact reflects a combination of problems, including the absence of private incentives for supervisors to invest in monitoring, and the presence of political pressures on supervisors to avoid recognizing losses (Calomiris and Herring 2013). For example, in December 2008, as an insolvent Citibank was bailed out by the U.S. government, its ratio of book capital to risk-weighted assets still exceeded 11%.

In private interbank coalitions, it is easy to see why cash would be especially valued over equity capital as a tool. Not only does cash mitigate the costs of raising equity ex ante and mitigate liquidity risks ex post, a greater reliance on observable cash (rather than hard-to-observe equity) to manage free-riding problems has additional advantages in the context of a private coalition of competing banks. For bankers to verify each other's capital in a precise way they would have to be able to demonstrate the value of each bank's capital to the group. Given the private information inherent in bank lending, doing so would require a significant expenditure of cost. Furthermore, a full analysis of the value of a bank's loan

[2]An additional motivation for New York City banks to maintain high required reserves was that city's position at the peak of the "reserve pyramid" that connected banks throughout the United States even though it generally did not employ its reserves to protect banks in other regions or non-clearing house members during financial crises.

[3]Some of those exceptions were the state experiments with deposit insurance, for example, in the early 20th century (see Calomiris, 1990).

portfolio would require the sharing of information about loan customers, and such sharing would destroy quasi rents banks earn from investing in private information (Rajan 1992); thus, a policy of collective monitoring of each bank's capital would entail huge information costs to banks, and erode the incentive for banks to invest in client-specific information. Our models of collective action developed below do not consider these quasi-rent-erosion costs of enforcing capital requirements in a banking coalition, but obviously, if they were included, they would magnify the gains from the use of cash to manage default risk.

3 The Model

There are six dates, $t = 0, 1, 2, 3, 4,$ and 5, and two types of agents, bankers and depositors. Bankers are risk-neutral. We consider both the case of risk-neutral depositors, as well as the case of risk-averse depositors.

At time 0, a banker is endowed with knowledge about a prospective loan making opportunity, and with own equity $E_0 > 0$. A banker accepts deposits D at time 0, which are in perfectly elastic supply to a banker up to a maximum of $\bar{D}$.[4] Instead of depositing funds in a bank, depositors can store their funds, which yields a per-unit return of one for sure.

At time 0, a banker can invest a bank's resources, $D + E_0$, in cash (the amount invested in cash is denoted C_0) or in loans (the amount invested in loans is denoted L_0, which is also the number of loans since the loan size is normalized to 1). Hence, $C_0 + L_0 = D + E_0$. The payoff from the loan will occur at time 5 if the loan is allowed to mature. That payoff per unit invested will either be $Y > 1$ or 0. The probability of the high payoff depends on a banker's costly risk-management effort at time 4. If he exerts effort, the high payoff Y occurs for sure and if he does not exert effort, Y occurs only with probability $p < 1$. The outcome for each bank that does not exert effort is i.i.d. and by the law of large numbers it is certain that p proportion of the banks not exerting effort will receive the high payoff. When

[4] For simplicity, we assume a maximum supply of deposits. This is a reduced form to capture an increasing cost of attracting more deposits due to, e.g., geographical distance. Alternatively, we could assume a limited amount of loan-making opportunities.

a banker chooses not to exert effort, he derives a private benefit B per loan (equivalently, when not exerting effort, a banker saves on costly risk management). The effort choice of a banker is unobservable to depositors.

At time 1, a fraction π of depositors experience an exogenous liquidity shock requiring them to transfer μD cash "abroad" from their deposit account. We set $\pi = 0$ for most of our modeling, but subsequently relax that assumption in Section 7.

At time 2, there are two possible aggregate states s, good or bad, $s = \{g, b\}$. The probability of the good state occurring is denoted by q (the bad state occurs with probability $1 - q$). The aggregate state is observed for free by a banker. If depositors want to observe the aggregate state, they need to spend m (e.g., they invest in a perfectly revealing signal about the aggregate state). In the bad aggregate state, the private benefit of a banker is larger, making risk management more costly, than in the good aggregate state, $B_g < B_b$. This assumption can be viewed as capturing, in a reduced-form, a set-up with costly loan monitoring (whereby the frequency of monitoring should be higher in bad times to achieve the same probability of repayment) or a set-up with searching for prospective loan applicants and/or screening out bad borrowers (which is more difficult in bad economic times). We let B denote the expected private benefit of a banker, i.e. $B \equiv qB_g + (1 - q) B_b$. We assume that

$$Y > 1 > qY + (1 - q)(pY + B_b). \tag{1}$$

The first inequality implies that when a banker exerts risk-management effort, loan making has a positive NPV. The second inequality implies that unless a banker does risk-management effort in *both* aggregate states, loan making is socially wasteful: even after accounting for the (maximum) private benefit of a banker, it is more profitable to store funds.[5]

At $t = 3$, after learning the aggregate state, a banker can increase his cash holdings by $\Delta C(s)$ by liquidating an amount $\Delta L(s)$ of loans. But converting loans into cash at

[5]The expression on the right-hand side of (1) is the return from exerting effort in the good state and not exerting effort in the bad state. Since $B_b > B_g$ and, by (1), $Y > pY + B_b$, this is the maximum expected return that can be generated if a banker does not exert effort in at least one aggregate state.

$t = 3$ is costly, $\Delta C(s) = (1-l)\,\Delta L(s)$, where l is the cost of liquidation per unit of loans when loans are liquidated prior to maturity. A banker's cash holdings at time 3 and state s are $C_3(s) = C_0 + \Delta C(s)$, while his loan holdings are reduced to $L_3(s) = L_0 - \Delta L(s) = L_0 - \Delta C(s)\,\frac{1}{1-l}$. Note that the liquidation of loans reduces the value of equity, $E_3 = C_3 + L_3 - D = E_0 - \Delta C(s)\,\frac{l}{1-l}$.

The purpose of having liquid assets prior to time 4 is to incentivize the banker to exert risk-management effort. The banker's incentive to exert effort, and thus to increase the probability of high payoff, is lower in the bad aggregate state. By increasing liquid assets at time 3, the banker will be able to respond to the observed bad aggregate state by increasing liquid assets, and thereby make risk-management effort at time 4 credible.

At the end of $t = 3$, depositors choose whether or not to "run" their bank, i.e., whether or not to shut down the bank. We assume that one depositor's announcement is enough to shut down the bank. This can be understood, for example, as an environment in which one depositor has a low cost m of observing the aggregate state, while the other depositors face prohibitive costs (so that only one of the depositors is informed). The informed depositor can be seen as causing the bank to close by withdrawing his or her funds and triggering imitative withdrawals by other depositors (who, in equilibrium, would not otherwise run the bank). For simplicity, we adopt a liquidation rule that when a bank is shut down, a banker surrenders all claims to the bank's assets. Depositors liquidate the loan portfolio at a cost v per unit of loans. We assume that depositors face a higher cost of liquidation cost than the banker, i.e., $v > l$. Hence, if the bank is shut down, depositors receive the liquidation value of loans plus the cash in the bank, $(1-v)\,L_3(s) + C_3(s)$.

At time 4, a banker chooses whether or not to exert risk-management effort. Effort is not contractible since it is not observable by anyone but the banker himself.

At time 5, the loan outcome Y or 0 is realized. If the high payoff is realized, then depositors receive R and the banker receives the residual, $YL_3(s) + C_3(s) - R$. If the low payoff is realized, depositors receive $R_f \leq C_3(s)$. Note that holding cash allows the banker to pay depositors a positive amount even if the loan returns nothing.[6]

In sum, the timeline is as follows:

Time 0: E_0 and D are invested in L_0 and C_0 and the contract promises (R, R_f).

Time 1: Exogenous deposit withdrawals of μD occur with probability π.

Time 2: The state is revealed and a banker updates his private benefit B_s based on the aggregate state. Depositors choose whether or not to spend m to observe the state.

Time 3: A banker can increase cash-holdings by liquidating some loans, where the new amount of cash is given by $C_3(s)$. Depositors choose whether or not to shut down the bank.

Time 4: A banker decides whether to exert unobservable risk-management effort.

Time 5: A banker pays R if outcome is Y and R_f if outcome is 0.

4 First-best Equilibrium

We now consider the case in which a banker's risk-management effort is publicly observable so that there is no moral hazard problem. While implausible, it offers a benchmark against which we will identify the inefficiencies generated by moral hazard. We solve for the optimum under the assumption of depositor risk-neutrality, and then show that the equilibrium is the same under risk aversion.

In the first-best, efficiency requires that a banker does risk-management effort since loan making is only productive when a banker exerts effort (condition (1)). Moreover, cash is not used. This is because cash is dominated in terms of rate-of-return compared to loan making when accompanied by risk-management effort, and there are no incentive benefits of cash since incentive problems are absent in the first-best. Hence, $E_0 + D = L_0$, $C_0 = C_3(s) = 0$ and loans always return Y per unit invested. Note also that depositors do not need to spend m to observe the aggregate state (to gauge how costly risk-management is for the banker):

[6]Note that with our two-point distribution, there is no real difference in the cash-flows between debt and equity for the outside claim held by depositors. We can explicitly derive the cash-flows of the outside claim to be like in a debt contract using costly state verification. Cash would then serve an additional function in the model, namely economizing on verification costs. For the sake of simplicity, and without loss of generality, we avoid this complication in our model.

They can observe their banker's effort directly and write contracts that guarantee the banker will exert effort.

Depositors are willing to deposit with a banker as long as the contract offers them at least the same payoff as storage. That is, the participation constraint of depositors is given by:

$$R \geq D. \tag{2}$$

Since a banker has a unique ability to make loans, he will not leave depositors any rents, i.e., $R = D$. Every deposit brought in earns $Y - 1 > 0$ implying that it is optimal to operate the bank at the maximum scale. Hence, if a banker chooses to take in deposits, his payoff at $t = 5$ is:

$$YE_0 + (Y - 1)\bar{D}. \tag{3}$$

A banker prefers to take in deposits and operate the bank rather than invest solely his own equity as long as he earns more by operating the bank with deposits compared to investing his equity only:

$$YE_0 + (Y - 1)\bar{D} \geq YE_0, \tag{4}$$

which is satisfied since $Y > 1$. Moreover, it is optimal not to allow depositors to withdraw early.

Proposition 1 summarizes the first-best outcome.

Proposition 1 *When risk-management effort is observable, a banker exerts effort, cash is never used, and depositors do not spend the cost of observing the aggregate state.*

Note that the optimal contract entails a deterministic payoff D for depositors and therefore, this is also the optimal contract under depositor risk-aversion.

5 Equilibrium under Moral Hazard

In this section, we solve for the autarkic banking equilibrium when risk-management effort is unobservable. We show that the banker may choose to hold cash as a commitment device for proper risk management. We solve for the optimum under the assumption of depositor risk-neutrality, and then show that the equilibrium is the same under risk aversion.

Unlike in the first-best, the banker must now be induced to exert risk management effort since his effort is no longer contractible by depositors. There are two incentive-compatibility constraints, one for each aggregate state, which ensure that the banker prefers to exert risk-management effort. The constraints are conditional on the aggregate state since the banker observes the realization of the aggregate state before he makes his effort decision.

The incentive-compatibility constraint in state s is given by:

$$YL_3(s) + C_3(s) - R \geq p\left[YL_3(s) + C_3(s) - R\right] + (1-p)\left[C_3(s) - R_{f,s}\right] + B_s L_3(s)$$

The expression on the left-hand side is the banker's payoff when he exerts risk-management effort. The expression on the right-hand side is his (out-of-equilibrium) expected payoff if he does not exert effort. With probability $1-p$, the loans return zero and the payments to depositors cannot exceed the banker's cash holdings, $R_{f,s} \leq C_3(s)$. Simplifying the incentive constraint we get:

$$\mathcal{P}_s L_3(s) \geq R - R_{f,s} \tag{5}$$

where

$$\mathcal{P}_s \equiv Y - \frac{B_s}{1-p}. \tag{6}$$

Following Holmström and Tirole (1997, 1998), we refer to $\mathcal{P}$ as the "pledgeable income" of the banker, i.e., the share of the return per unit of loans that can be pledged to depositors without jeopardizing the incentives of the banker to properly manage the loan portfolio. Note that $\mathcal{P}_s > 0$ (with $\mathcal{P}_g > \mathcal{P}_b$) under our assumption that effort is productive in both aggregate states (see (1)).

Note that setting the (out-of-equilibrium) payoff to depositors $R_{f,s}$ as high as possible relaxes the incentive constraint. Hence, it is optimal to set $R_{f,s} = C_3(s)$. Moreover, the participation constraint of depositors is given by (2) and, as before, the banker does not want to leave any surplus to depositors, so the participation constraint binds, $R = D$. Using $L_3(s) = L_0 - \Delta C(s)\frac{1}{1-l}$, $C_3(s) = C_0 + \Delta C(s)$, and $L_0 = D + E_0 - C_0$, we can re-write (5) as

$$\mathcal{P}_s(D + E_0) + (1 - \mathcal{P}_s) C_0 + \left(1 - \frac{\mathcal{P}_s}{1-l}\right)\Delta C(s) \geq D. \tag{7}$$

Note that $1 - \mathcal{P}_s > 1 - \frac{\mathcal{P}_s}{1-l}$. Moreover, if $\mathcal{P}_s \geq 1$ in both aggregate states, then cash holdings cannot help with incentives as they tighten the incentive constraints. Hence, cash will not be held in this case, implying that the incentive constraints simplify to $\mathcal{P}_s(D + E_0) \geq D$. This is always satisfied for $\mathcal{P}_s \geq 1$. We can thus state the following lemma.

Lemma 1 *When risk-management effort is not observable, but $\mathcal{P}_s \geq 1$ in both aggregate states, the first-best is always reached.*

Moreover, note that even when $\mathcal{P}_s < 1$, as long as $\frac{\mathcal{P}_s}{1-\mathcal{P}_s} \geq \frac{\bar{D}}{E_0}$, the first-best can be reached with $C_0 = \Delta C(s) = 0$. In what follows, we focus on the case in which the first-best is not attainable, i.e., $\mathcal{P}_s < 1$ and $\frac{\mathcal{P}_s}{1-\mathcal{P}_s} < \frac{\bar{D}}{E_0}$ in at least one aggregate state. Since $\mathcal{P}_g > \mathcal{P}_b$, we assume that the first-best is not attainable in the bad aggregate state.

5.1 Aggregate State Observable at No Cost

Here we demonstrate that the optimal second-best contract under the assumptions above has the following key attributes: the bank offers the depositor a senior claim D on the bank (when the bank is insolvent, the depositor receives the whole value of the bank, which is less than D), and gives the depositor an option to close down (run) the bank after the non-contractible state is revealed.[7] By offering this contract to the depositor, the banker

[7] Outside funding is given a senior claim in equilibrium. Thus equity is limited to inside equity in our model, and banks operate at a corner solution, utilizing all inside equity in the bank. It would be possible

ensures that he will generate sufficient amounts of cash in the bad state so as to incentivize himself to undertake the privately costly risk-management effort. In equilibrium, there are no depositor runs because the banker always creates sufficient reserves in the bad state, and there is no bank insolvency because the bank's investments always generate Y when the banker invests in risk management.

The social cost of this arrangement is the opportunity cost of generating cash, which is the amount of foregone lending opportunities and the physical liquidation cost that results from transforming loans into cash reserves. We first show that the contract described here is feasible. We then consider, in Section 5.2, whether there is an alternative contract based on banker communication via cash reserve accumulation (a form of "messaging") that would be less wasteful because it would economize on cash in the bad state. A contract that would credibly lower promised payments to depositors in the bad state would reduce the amount of cash reserves that need to be accumulated. We show that such a contract is not feasible, which implies that the second-best contract pays a fixed deposit amount D and needs to be enforced by the threat of a depositor run.

We start by characterizing the case in which depositors can observe the aggregate state at zero cost, $m = 0$. We consider the case in which $m > 0$, and the depositor's choice of whether or not to expend m and observe the state, in Section 5.3.

When $\mathcal{P}_s < 1$, we can re-write the incentive constraint (7) as

$$\frac{\mathcal{P}_s}{1-\mathcal{P}_s}E_0 + \frac{1}{1-\mathcal{P}_s}\left(1 - \frac{\mathcal{P}_s}{1-l}\right)\Delta C\left(s\right) \geq D - C_0 \tag{8}$$

An increase in $t = 3$ cash holdings, $\Delta C\left(s\right) > 0$, can only relax the incentive constraint when

to construct a model where some outside funding could take the form of equity. For example, this could be achieved by allowing some suppliers of capital ("directors") to monitor bank risk management at a cost by participating in corporate governance (e.g., as members of the bank's loan review committee monitoring bank risk-management). We conjecture that it would be possible in such a model to derive interior solutions for total outside equity and total cash, where total outside equity and cash would be chosen endogenously, by equating the marginal costs of each.

the term in brackets is positive, which is the case when

$$\mathcal{P}_s < 1 - l. \tag{9}$$

Liquidating loans only helps with incentives when the liquidation return is higher than the pledgeable return. Moreover, (8) implies that while both cash at $t = 0$ and extra cash at $t = 3$ relax the incentive constraint, they do not have the same role in relaxing incentives. C_0 has a per unit benefit of one but is not contingent on the state. $\Delta C(s)$ is contingent on the state (i.e., it is flexible), but its per unit benefit is less than one:

$$\frac{1}{1-\mathcal{P}_s}\left(1-\frac{\mathcal{P}_s}{1-l}\right) < 1.$$

In equilibrium, the banker always exerts effort, and his payoff from taking in deposits and operating the bank is:

$$q\left[YL_3(g) + C_3(g) - D\right] + (1-q)\left[YL_3(b) + C_3(b) - D\right]$$

which, using $L_3(s) = L_0 - \Delta C(s)\frac{1}{1-l}$, $C_3(s) = C_0 + \Delta C(s)$, and $L_0 = D + E_0 - C_0$, re-writes as:

$$YE_0 + (Y-1)(D-C_0) - \left(\frac{Y}{1-l} - 1\right)\left[q\Delta C(g) + (1-q)\Delta C(b)\right]. \tag{10}$$

The banker maximizes (10) subject to the incentive constraints (7) (one for each aggregate state), the feasibility constraints $D \leq \bar{D}$ and $0 \leq \Delta C(s) \leq (1-l)(D + E_0 - C_0)$, the banker's participation constraint (requiring that he does better by taking in deposits rather than by investing only his own equity):

$$YE_0 + (Y-1)(D-C_0) - \left(\frac{Y}{1-l} - 1\right)\left[q\Delta C(g) + (1-q)\Delta C(b)\right] \geq YE_0 \tag{11}$$

and the no run condition for depositors requiring that $(1-v)L_3(s) + C_3(s) \leq D$ or, after substituting for $L_3(s)$ and $C_3(s)$ and simplifying:

$$\frac{1-v}{v}E_0 + \Delta C(s)\frac{v-l}{v(1-l)} \leq D - C_0. \tag{12}$$

For ease of exposition, we will now consider the case in which there is no incentive problem in the good aggregate state, $\mathcal{P}_g > 1$, and in which the liquidation costs for depositors are sufficiently high (see condition (13) below). Then, $\Delta C(g) = 0$. At this point, it is useful to introduce some notation. Define the net return on loans as $r \equiv Y - 1$, the outside financing capacity of equity in the bad state as $f_b \equiv \frac{\mathcal{P}_b}{1-\mathcal{P}_b}$, the ratio of return-to-cost of liquidation for the banker as $\lambda \equiv \frac{1-l}{l}$, and the ratio of return-to-cost of liquidation for depositors as $\nu \equiv \frac{1-v}{v}$, with:[8]

$$\nu < \lambda - (1-q)\left(\frac{1+r}{r} + \lambda\right). \tag{13}$$

With this notation, we can now spell out our assumption about the maximum available amount of deposits, $\bar{D}$. We posit that

$$\frac{\bar{D}}{E_0} < \lambda < \frac{\lambda + f_b}{1 + f_b}. \tag{14}$$

This assumption avoids an automatic liquidation of the bank in the bad aggregate state and makes it impossible for bankers with severe moral hazard (low f_b) to attract any deposits.

The following proposition summarizes the case when depositors can observe the aggregate state at no cost.

Proposition 2 *When risk-management effort is unobservable and the first-best is unattainable, the banker is incentivized to exert effort by holding cash. If*

$$f_b \geq \lambda - (1-q)\left(\frac{1+r}{r} + \lambda\right) \tag{15}$$

[8]This condition rules out a set of parameters for which deposit-taking is not feasible at intermediate levels of financing capacity f_b.

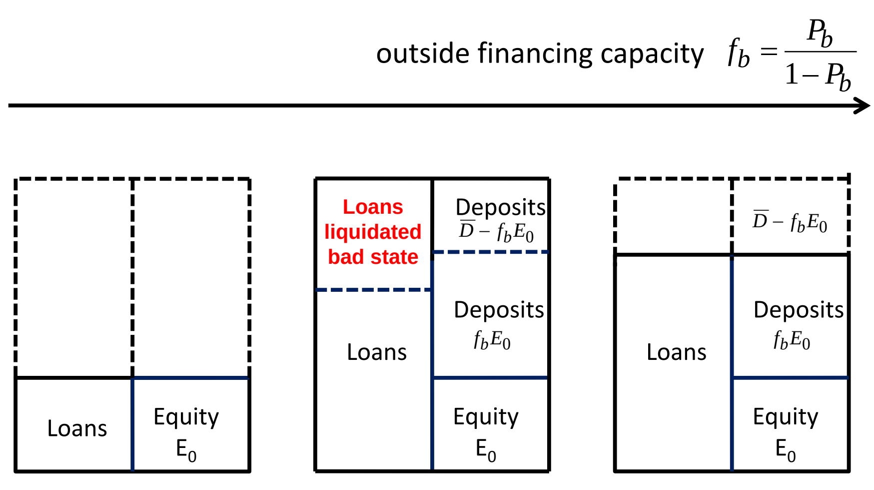
holds, then adjusting cash at $t = 3$ is too costly and the banker builds incentive-compatible cash holdings at $t = 0$. If condition (15) does not hold and $f_b \geq \nu$, then the banker increases his cash holdings at $t = 3$ if the bad aggregate state is realized. For $f_b < \nu$, the banker will not raise deposits and will invest solely his own equity.

Figure 1 illustrates the cases described in Proposition 2. When the moral hazard problem in the bad state is small, which means a large outside financing capacity f_b, the banker accumulates cash at $t = 0$. The optimal cash holdings are given by the binding incentive-compatibility constraint in the bad state:

$$C_0 = D - f_b E_0$$

where $f_b E_0 \leq D \leq \bar{D}$ (any D in this range generates the same net surplus for the banker).

Figure 1: Bank balance sheets as a function of the severity of the moral hazard problem



When the moral hazard problem is severe (low outside financing capacity f_b), then the banker cannot raise any deposits and invests only his equity.

For an intermediate moral hazard problem (intermediate financing capacity f_b), the banker does not hold any cash ex ante and instead increase his cash holdings at $t = 3$ only if the bad state is realized. The optimal amount of cash in that case is given by:

$$C_3^* = \Delta C(b) = \frac{\lambda}{\lambda - f_b}\left(\bar{D} - f_b E_0\right). \tag{16}$$

Finally, note that the optimal contract entails a deterministic payoff D for depositors and therefore, this is also the optimal contract under depositor risk aversion.

5.2 Cash-contingent contract without depositor discipline

We now consider an alternative contract which specifies payments to the depositor that vary depending on the amount of cash reserves accumulated by the banker. As before, we solve for the optimum under the assumption of depositor risk-neutrality first, and then discuss how the optimal contract is affected by risk aversion.

The accumulation of cash in the bade state can be seen as a "message" from the banker to the depositor that the bad state has occurred and that, consequently, the depositor receives a different deposit payment than in the good state. As we will show, there is no credible messaging contract that is consistent with the reservation return required by the depositor and the truthtelling incentive constraints faced by the banker. Thus, there is no feasible contract superior (i.e., that involves less costly reserves) to the contract characterized in the previous subsection, i.e., the contract that pays a fixed deposit amount and needs to be enforced by the threat of a depositor run.

Consider the following deviation from the optimal contract with a fixed payment D:

$$\begin{aligned} D_g &= D + \varepsilon \\ D_b &= D - \frac{\varepsilon q}{1-q} \\ C_g &= 0 \\ C_b &< C_3^*. \end{aligned}$$

Note that the depositor participation constraint can be written as $qD_g + (1-q)D_b = D$. That is, the deviation we consider satisfies the participation constraint of depositors if and only if the promised payments are always made in each of the two states, i.e., the banker exerts effort in the bad state (he always does in the good state by assumption). This will occur if and only if the banker truthfully reveals the bad state through his selection of the amount of reserves. The truth-telling constraints that must be satisfied consider whether the banker is better off truthfully signaling the state (and specifying the contractual payment accordingly) through his choice of reserves.

In the good state, the banker must get more value by not accumulating reserves:

$$YL_3(g) - D_g \geq YL_3(b) + C_b - D_b \tag{17}$$

where $L_3(g) = L_0 = D + E_0$ and $L_3(b) = L_0 - C_b\frac{1}{1-l} = D + E_0 - C_b\frac{1}{1-l}$. (Recall that L_3 only depends on the announcement of the state but not the true state).

In the bad state, the truthtelling constraint is:

$$YL_3(b) + C_b - D_b \geq p\,(YL_3(g) - D_g) + BL_3(g). \tag{18}$$

Note that if the banker announces the good state even though it is the bad one, and thus does not accumulate reserves, he will shirk on risk-management effort (because the promised payment is too high relative to the accumulated reserves to elicit risk-management effort).

Hence, loans fail with probability p but he receives the private benefit B on each loan.

It turns out, that it is not optimal to save on cash reserves by offering a contract with state-contingent deposit repayment but no ability to run.

Proposition 3 *No deviation* (ε, C_b) *from the contract with fixed deposit repayment satisfies the truth-telling constraints (17) and (18).*

5.3 Aggregate State Observable at a Cost

In this subsection, we consider the case in which depositors can observe the aggregate state at a cost $m > 0$. Hence, they have to decide whether or not it is in their interest to expend this cost.

If depositors spend m and observe the aggregate state, then the solution to the banker's problem is the same as the one characterized in the previous subsection, except that the promised payout to depositors has to be increased by m (so that depositors still break-even on the contract, even though they have to pay m to observe the aggregate state). Hence, the banker's surplus is reduced by m.

If depositors decide not to spend m, then the solution to the banker's problem is as follows. Since depositors cannot distinguish the good aggregate state from the bad one, the banker has to hold the same amount of cash in both states. The banker's cash holdings have to be high enough to ensure that he exerts risk-management effort in the bad aggregate state (then, he would also exert effort in the good state). If the banker did not hold enough cash to satisfy his incentive constraint in the bad state, the equilibrium with banking would not be viable. It follows that the banker will find it optimal to satisfy his incentive constraint with $t = 0$ cash only. This is because there is no flexibility benefit of increasing cash at $t = 3$ (since depositors cannot tell the states apart) and increasing cash holdings at $t = 3$ is always more costly than building sufficient cash reserves at $t = 0$.

This logic leads to the following proposition.

Proposition 4 *If the cost of observing the aggregate state m is low, depositors become informed about the aggregate state and the banker increases his cash holdings if the bad aggregate state realizes, as long as the liquidation is not too costly. If the cost m is high, depositors do not become informed and the banker only uses cash at $t = 0$ as a commitment for proper risk management.*

6 Deposit Insurance

In the autarkic banking equilibria we have discussed thus far, banks choose the optimal level of reserves voluntarily. We now consider an environment with heterogeneous banks in which deposit insurance emerges as a part of the optimal contract. The deposit insurance equilibrium entails social benefits as well as social costs. The benefit is insuring depositors against the consequence of contracting with a weak bank. Cash reserve requirements are an integral part of this deposit insurance equilibrium as they limit the social cost of insurance by limiting the subsidy that weak banks extract from sound banks.

In the autarky set-up considered thus far, there is no purpose to deposit insurance. In equilibrium (conditional on the incentive-compatible supply of risk-management effort by bankers), all banks receive payout $YL_3(s)$ at $t = 5$ and all depositors are paid the amount D promised to them at $t = 0$ from those proceeds.

In order to motivate deposit insurance, we introduce heterogeneity in the banking system which potentially creates an externality among banks.[9] Specifically, we assume that some

[9] An alternative modeling approach would rely instead on political economy rather than efficiency to motivate deposit insurance. For example, a subset of bankers might lobby for a deposit insurance system that subsidizes them at the expense of other banks. Politically influential "risky bankers" receive a cross-subsidy from low-risk bankers through the provision of deposit insurance. This approach to motivating deposit insurance is defensible on historical grounds. Federal deposit insurance began in the United States as the result of the Banking Act of 1933 (Calomiris and White 1994; Calomiris and Haber 2014, Chapter 6). It was understood by advocates and opponents to have resulted from special interest advocacy by small, risky rural banks at the expense of other banks. President Roosevelt, Senator Carter Glass, the Federal Reserve, the Treasury and the American Bankers Association all opposed it, but acquiesced as part of a political compromise with Representative Henry Steagall and others who championed the interests of small rural bankers. The spread of deposit insurance internationally in the latter half of the twentieth century is also understandable as a political process (Demirguc-Kunt, Kane and Laeven 2008). Furthermore, there is a large

fraction (α) of bankers have unobservably higher private cost of managing risk in the bad state of the world (B_b^h rather than B_b). We refer to banks whose private cost of risk management is equal to B_b as "sound" banks and to those whose private cost of risk management equal to B_b^h as "weak" banks. We assume that B_b^h is sufficiently high so that weak banks choose not to invest in risk management in the bad state of the world, i.e.,

$$B_b^h > (1-p)\,Y. \tag{19}$$

Because weak banks have a higher default probability in the bad state than sound banks, and because the bank type is privately observed, this can create an externality between banks. We show that one way to deal with this externality is for bankers to insure each other's deposits so that depositors receive a riskless payment even if their bank's loan proceeds are zero (Proposition 5). An alternative way is a contract in which all banks pay a risk premium on deposits that compensates depositors for the risk associated with having deposited with one of the weak bankers. We show that such a contract dominates deposit insurance under risk neutrality. However, for a sufficiently large depositor risk-aversion, the deposit insurance equilibrium dominates the equilibrium without deposit insurance (Proposition 6).

6.1 The Deposit Insurance Equilibrium

In the deposit insurance equilibrium, banks agree to provide deposit insurance to one another so that the banks that forego risk management and earn a zero return at $t = 5$ will have their promised deposit payments covered by surviving banks. A condition for receiving deposit insurance is a bank's willingness to hold the amount of required reserves established by the deposit insurance system.

Referring to Proposition 2, we assume, without loss of generality, that liquidation costs

literature establishing that risk subsidization has been a predictable consequence of most deposit insurance arrangements around the world (Demirguc-Kunt and Detragiache 2002, Demirguc-Kunt and Huizinga 2004, Cull, Senbet and Sorge 2005).

are low enough so that reserves are accumulated only at $t = 3$. As before, the state is not contractible. Furthermore, unlike the uninsured depositors in Section 5, who are willing to invest resources to observe the state, insured depositors no longer have an incentive to monitor the state to ensure that banks top up their cash reserves in the bad state. However, sound banks who wish to participate in the deposit insurance system with a cash reserve requirement can now be relied upon to truthfully reveal the bad state, which entails that all banks participating in the deposit insurance system accumulate reserves of C_3^* at $t = 3$ (given in (16)). Their announcements of the bad state are reliable because they benefit individually from identifying the bad state if and only if it occurs, given their liability for insuring the depositors of failed weak banks. The revelation of the bad state can be implemented by specifying a rule saying that if any bank that is a member of the deposit insurance system claims that the bad state has occurred, it will be deemed to have occurred.

We first consider the case of a pooling equilibrium in which both types of banks (B_b, B_b^h) choose to participate in the deposit insurance scheme. We derive conditions under which pooling occurs. We assume that any deviation from equilibrium behavior is interpreted as coming from a weak bank.

Let us consider the behavior of sound banks (B_b) first. Sound banks will have to bail out the deposits of weak banks when the bad state and the bad outcome occur. By assumption (19), weak banks will not choose to invest in risk management in bad states of the world because B_b^h is sufficiently high. The aggregate expected cost of insuring weak banks' deposits in the bad state is:

$$d = \alpha(1 - p)(D - C_3^*),$$

which is the product of the proportion of expected weak bankers at $t = 0$, the probability of the bad outcome, and the amount of deposit liabilities of weak banks in excess of their cash reserves. A sound banker is willing to participate in deposit insurance as long as his payoff from doing so is higher than his payoff from not participating in deposit insurance:

$$[qYL_3(g) + (1-q)(YL_3(b) + C_3^*) - D] - (1-q)\frac{d}{(1-\alpha) + \alpha p} \geq YE_0. \tag{20}$$

The left-hand side is a sound banker's profit from running a bank minus the expected cost of participating in deposit insurance. The deposit insurance cost d must be paid in the bad state (which occurs with probability $1-q$) and $\frac{1}{(1-\alpha)+\alpha p}$ is each sound bank's share of the deposit insurance cost. Note that αp proportion of the weak banks will survive and share the bailout costs with the sound banks. If a sound banker chooses not to participate in deposit insurance (right-hand side of (20)), depositors believe that he is a weak banker and therefore do not deposit in his bank (recall that, by assumption (1), it is not worthwhile for depositors to fund a bank that they know does not engage in risk management in both states). Hence, the banker invests solely his own equity and obtains YE_0.

Weak banks (B_b^h) will also be willing to participate in the deposit insurance equilibrium as long as doing so is profitable for them. The condition can be written, for a particular weak bank, as:

$$q[YL_3(g) - D] + (1-q)p\left[YL_3(b) + C_3^* - D - \frac{d}{(1-\alpha) + \alpha p}\right] + (1-q)B_b^h L_3(b) \geq E_0. \tag{21}$$

The left-hand side is a weak banker's profit from running a bank and pooling with sound bankers. In the good state (which occurs with probability q), weak banks do risk management and repay their depositors. In the bad state (which occurs with probability $1-q$), weak banks do not manage risk and therefore earn private benefit B_b^h per unit of their loans $L_3(b)$. A p proportion of weak banks generates the return Y on loans, repays their depositors and shares the bailout costs d. The rest of the weak banks fail to generate a positive return on the loans and are only left with the reserves C_3^*. Those reserves are used to pay off their depositors, and the shortfall $D - C_3^*$ is then covered by the deposit insurance. The right-hand side of (21) is the payoff a weak banker earns when not participating in the deposit insurance. In this case, he is identified as a weak banker and therefore cannot attract any deposits (assumption (1)). The payoff of a weak bank when not joining the deposit insurance is E_0. He does not make loans because they have negative NPV as he does not exert risk-management effort.

We can now state the condition under which both sound and weak banks are willing to participate in deposit insurance.

Proposition 5 *Both sound banks and weak banks are willing to participate in a deposit insurance system in which pooling occurs if the good state is sufficiently likely (q is sufficiently high). Sound banks will truthfully declare the bad state when it occurs, thereby requiring that all banks accumulate reserves C_3^*.*

For completeness, we mention properties of a separating equilibrium in which sound banks are willing to participate in deposit insurance while weak banks are not willing to participate. In this case, the willingness to participate in deposit insurance signals sound bank quality. Obviously, sound banks are willing to participate in deposit insurance because in so doing they achieve separation and therefore avoid any cost of subsidizing weak banks. This equilibrium is the same as in the model without weak banks.

6.2 Optimality of deposit insurance

Here we show that the deposit insurance pooling equilibrium is not dominated by a pooling equilibrium without deposit insurance as long as depositors are sufficiently risk averse. When depositors are sufficiently risk-averse, both sound banks and weak banks will prefer the deposit insurance pooling equilibrium to pooling without deposit insurance. Both types of banks earn more rents from pooling with deposit insurance.

If there is pooling without deposit insurance, then risk-neutral depositors are willing to accept a higher payment $(D + \varepsilon)$ when their bank is solvent in order to compensate them for the loss when their bank is insolvent. The higher payment is given by the depositor participation constraint:

$$[(1-\alpha)+\alpha q+\alpha p(1-q)](D+\varepsilon)+(1-q)(1-p)\alpha C_3^*=D. \tag{22}$$

In words, without deposit insurance, depositors earn the promised payment $(D+\varepsilon)$ if they are lucky enough to deposit with a sound bank, or if they deposit with a weak bank in the good state, or if they deposit with the weak bank in the bad state but are lucky enough for the good outcome to occur. When none of those is true, they receive only C_3^*.

In pooling without deposit insurance, sound banks no longer have to pay their share of the deposit obligations of failed weak banks, but they do have to pay their own depositors more as the result of the inability of depositors to distinguish sound from weak banks. In the pooling equilibrium without deposit insurance, each sound bank will earn rent equal to:

$$qYL_3(g)+(1-q)(YL_3(b)+C_3^*)-(D+\varepsilon)-YE_0.$$

Proposition 6 *Suppose q is sufficiently high so that both sound and weak banks are willing to participate in a deposit insurance system with pooling. If depositors are sufficiently risk averse, then pooling with deposit insurance is optimal.*

7 Mutual Liquidity Risk Insurance

We now relax the assumption made in the prior sections of the paper to permit depositors also to make exogenous withdrawals at $t=1$ in response to a need for liquidity that is unrelated to the condition of banks. Specifically, we assume that at $t=1$, each bank is subject to an exogenous liquidity shock, which is i.i.d. across banks. As before, we assume, without loss of generality, that liquidation costs are low enough so that for the purpose of ensuring credible risk-management effort, banks only accumulate cash reserves at $t=3$ (Proposition 2). In the presence of exogenous liquidity risk, which occurs at $t=1$, banks must, however, also accumulate cash reserves at $t = 0$.

We show that adding this feature to the model results in bank coinsurance of the liquidity risk related to exogenous withdrawals of deposits. As part of the coinsurance arrangements among banks, bankers will require one another to maintain cash reserves to avoid free-riding on the collective coinsurance of exogenous liquidity risk. This is true whether or not the deposit insurance exists, and therefore provides an independent motivation for cash reserve requirements.

For tractability, we assume that the size of the liquidity shock is identical across banks. With probability π, each bank experiences a withdrawal of a fraction μ of its deposits (D) at $t = 1$, and with probability $(1 - \pi)$, each bank experiences zero exogenous withdrawals of deposits at $t = 1$.

We interpret the exogenous liquidity shock as a withdrawal necessitated by a bank depositor's need to purchase an item of value μD from "abroad." The international feature of the transfer has important consequences for the cash needs of banks. In our model, unlike other models of liquidity demand shocks in banking (such as Diamond and Dybvig 1983) we do not assume that shocks to depositor's desire to purchase goods and services necessarily result in disintermediation. In our model, "domestic" transfers among individual depositors would not require the physical interbank exchange of cash. Depositors' domestic purchases can be accomplished by interbank accounting transfers of deposits among the banks within the domestic banking system because, in equilibrium, banks have an incentive to agree to offset any individual deposit flows with countervailing interbank deposit flows at $t = 1$. This can be accomplished without any transfer of cash. When a depositor makes a domestic purchase of μD, she uses a check drawn on her bank (Bank A) to execute that purchase. The seller of the domestic good deposits that check in his bank (Bank B), and an offsetting interbank accounting transaction can occur (a deposit claim of Bank B on Bank A for μD), which balances the books between the banks to take account of and offset the individual deposits being debited and credited at the two banks. This is feasible because the domestic transaction is observable to the domestic banks (i.e., the transaction is routed through the clearing house owned by all the banks collectively).

Because the source of domestic interbank transfers is observably exogenous to any banker's decisions, and because there is no moral hazard arising as the result of any change in any bank's balance sheet (the bank losing deposit dollars experiences no net change in its total deposits, which consist of the sum of individual plus interbank deposits; and similarly the bank receiving the deposits experiences no net change in deposits), there is no need for domestic banks to limit their willingness to offset individual domestic deposit flows with interbank deposit flows. Depositors' domestic purchases, therefore, will not necessitate any cash transfers among domestic banks at $t = 1$, or any cash reserve holdings at $t = 0$.

In contrast, transfers of funds abroad occur outside of the domestic banking system. We assume (on the basis of unspecified factors that limit international contracting) that it is not physically possible for interbank deposits to offset international flows among individual depositors. Thus, international transactions necessitate cash disbursements abroad by banks experiencing an outflow of individual deposits.[10]

Domestic banks can benefit from coinsuring one another against liquidity shocks involving international transactions. Those shocks are i.i.d. across banks, implying that, by the law of large numbers, the amount of reserve outflows is $\pi\mu D$. Domestic banks will agree to the following mutually beneficial arrangement: each bank holds $\pi\mu D$ in cash reserves at $t = 0$. All banks that experience international withdrawals of deposits can request a zero-interest loan of cash from all the other banks in the amount $(1-\pi)\mu D$. The proceeds of this interbank loan to any requesting bank is channeled through the clearing house at $t = 1$, which combines the interbank loan proceeds of the lending banks of $(1-\pi)\mu D$ with the borrowing bank's own holdings of $\pi\mu D$, and sends μD abroad to fund the transfers of cash to foreign banks that are receiving international deposit inflows from the domestic bank members of the clearing

[10]This could take place, for example, via a transfer of gold balances from Bank A to a foreign bank, which could be accomplished through the export of gold, or the transfer of Bank A's gold holdings already located in an account based abroad.

house. The loan is repaid by the disbursing bank at $t = 5$. Because the loan is riskless in equilibrium, and because there is nothing to be gained by establishing rules that would limit banks' demands for liquidity, interbank loans for funding liquidity disbursements at $t = 1$ are given at a zero interest rate.

Two aspects of our assumptions relating to this arrangement, and the role of the clearing house, should be emphasized. First, we assume that the clearing house can enforce the reserve requirements credibly because it can monitor member banks' holdings of cash and require members to maintain a cash balance of $\pi\mu D$ at $t = 0$. Second, because the clearing house aggregates the interbank loans of cash at $t = 1$ and sends them abroad as agent for the disbursing banks, there is no moral hazard associated with deception by the bank that is sending the gold abroad. If the clearing house was not acting as agent for the transfer, then one could worry that domestic banks might have an incentive to overstate their needs for gold transfers. By doing so they could obtain gold at $t = 1$ (at the expense of other banks' foregone opportunity costs from lending) which would reduce that bank's cost of maintaining reserve holdings at $t = 3$ to avoid the threat of a deposit run if the bad state occurs.

Because banks coinsure the liquidity risk of withdrawals abroad, each bank's total debt (consisting of customer plus interbank deposits) falls by $\pi\mu D$ at $t = 1$. Because $\bar{D}$ is the binding constraint on the scale of banks, the existence of the liquidity shock implies that L_0 will be slightly lower (by the amount $\pi\mu\bar{D}$) than in the equilibrium where $\pi = 0$.[11]

Note that in our framework we separate the timing of the exogenous liquidity shock from the timing of other events. Exogenous transfers abroad occur prior to the revelation of the state (which occurs at $t = 2$), prior to bankers' decisions about "topping up" cash balances and the endogenous withdrawals by depositors (at $t = 3$), and prior to any risk-management action by bankers (at $t = 4$). Cash reserves used to satisfy foreign needs flow out at $t = 1$. Our results do not depend crucially on this timing convention. By assuming that exogenous withdrawals precede endogenous withdrawals, we are able to model their

[11]Note that because L_0 is lower than before, the amount of loans that must be liquidated in the bad state at $t = 3$ is also lower than before.

consequences for reserve balances as independent. If, however, the sequencing of the two sources of deposit withdrawal (exogenous and endogenous withdrawals) were reversed, the amounts of reserves held for the two purposes would not always be independent. For example, reserves accumulated by banks in response to the revelation of the bad state could be available (once risk-management decisions have been made) to fund international transfers of cash. This scope economy, however, complicates the analysis while not affecting the key results of the model.

Consistent with Proposition 2, in a model where $\pi = 0$ (that is, a model without exogenous liquidity risk), under some parameterizations of our model (e.g., when l is high), banks will choose to maintain positive reserves at $t = 0$ in order to avoid the costs of "topping up" at $t = 3$. In that case the clearing house will require that banks hold $C_0^* + \pi\mu D$, where C_0^* is the amount of reserves banks choose to hold in the absence of the exogenous liquidity shock.[12]

8 Conclusion

We consider the role that cash reserves play in optimal banking arrangements. Cash reserves reduce the vulnerability of banks to liquidity risks that arise from granting depositors the option to withdraw their funds. Liquidity risk is of two types, exogenous (where withdrawal behavior is unrelated to depositors' beliefs about bank conditions) and endogenous (where withdrawals reflect deterioration in bank conditions). The role of cash in mitigating exogenous liquidity risk is straightforward: holding cash sufficient to cover exogenous unpredictable needs of depositors makes it possible to avoid destabilizing bank failures or high

[12]Thus far, the aggregate amount of liquidity that needs to be withdrawn from the banking system has been fixed at μD. We could allow for the possibility of an adverse systemic shock: In addition to μD, with probability ϕ, an amount x must be shipped abroad by each bank. Banks can deal with this risk in one of two ways. They could either maintain additional cash reserves at $t = 0$ in the amount x or liquidate loans at $t = 1$ to generate sufficient cash when the systemic shock is realized. Similar to the logic underlying Proposition 2, which of these solutions the banks choose will depend on the costs of liquidation l and the probability ϕ. In states where the systemic shock occurs, banks will experience a reduction in deposits at $t = 1$, and therefore, the amount of liquidation needed at $t = 3$ in the bad state will be less than in the model without systemic liquidity risk.

liquidation costs of loan portfolios that would occur in the absence of cash.

In the case of endogenous liquidity risk, the role of cash is to reduce insolvency risk by incentivizing efficient risk management by bankers. The fact that cash is impervious to moral hazard makes its possession by banks (e.g., in the form of deposits at the central bank) uniquely useful in promoting good risk management by banks with respect to their risky assets. Banks that hold sufficient cash are able to gain market confidence in their risk management, and thereby attract and retain deposits. In bad states of the world (i.e., recessions) banks that might otherwise be tempted to allow risky assets to become riskier will raise their cash holdings to preserve market confidence in their low risk. The use of cash in our model expands efficient bank lending.

We begin with autarkic models of single banks and their optimal contracts with depositors. In such environments, although cash reserves play a crucial role in bank contracting, there is no need for cash reserve requirements imposed either by a coalition of banks or by the government. Individual banks face the incentive to offer optimal contracts that create the incentive for them to maintain cash reserves.

Once we allow for heterogeneity among multiple banks, however, cash reserve requirements – imposed either by private coalitions of banks or by the government – become necessary as a means of preventing free riding on collective protection. In the case of endogenous liquidity risk, in an environment where banks are of heterogeneous quality, and where depositors are sufficiently risk averse, deposit insurance arises as part of the optimal banking contract because it insulates depositors from the consequences of bank heterogeneity. In that environment, a cash reserve requirement is needed to limit the extent of the subsidy weak banks receive from sound banks.

In the case of exogenous liquidity risk, in an environment where banks can benefit from coinsuring each other's diversifiable liquidity risk, a cash reserve requirement is necessary to prevent free riding on other banks and thereby make collective protection credible.

Our model points to important shortcomings in existing regulatory frameworks for liquidity requirements, such as Basel III. That framework's liquidity standards view reductions in short-term debt as interchangeable with increases in cash reserves. In contrast, in our model, the combination of a large amount of short-term debt and a high cash reserve requirement is efficient and superior to a regulatory rule that would encourage lower levels of both.

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9 Appendix

Proof of Proposition 2 The incentive constraint in the bad aggregate state must be binding since cash holdings are costly and hence the banker will choose to hold just enough cash to satisfy the incentive constraint. The binding incentive constraint in the bad state (8) can be written as:

$$f_b E_0 + \frac{\lambda - f_b}{\lambda} \Delta C\,(b) = D - C_0. \tag{23}$$

Note that the no-run condition (12) can be written as:

$$\nu E_0 + \frac{\lambda - \nu}{\lambda} \Delta C\,(b) \le D - C_0. \tag{24}$$

From (10), we get that the net surplus of the banker (over the return from investing only his own equity, YE_0) is given by:

$$r(D - C_0) - (1-q)\frac{1 + r + r\lambda}{\lambda} \Delta C(b). \tag{25}$$

Substituting the binding incentive constraint (23) into (25) yields:

$$r f_b E_0 + \left[\frac{\lambda - f_b}{\lambda} r - (1-q) \frac{1 + r + r\lambda}{\lambda} \right] \Delta C(b). \tag{26}$$

It follows that if the expression in the square brackets above is non-positive,

$$\frac{\lambda - f_b}{\lambda} r - (1-q) \frac{1 + r + r\lambda}{\lambda} \le 0$$

or, equivalently,

$$f_b \ge \lambda - (1-q) \left(\frac{1+r}{r} + \lambda \right), \tag{27}$$

then the banker will never increase his cash holdings at $t = 3$ since his surplus is decreasing in ΔC.

Suppose condition (27) holds. From the binding incentive constraint (23), we get that:

$$f_b E_0 = D - C_0 \tag{28}$$

It follows that no run condition (24) is satisfied since $f_b E_0 = D - C_0 > \nu E_0$ holds by (27) and (13). By (26), the banker's surplus in this case is given by $r f_b E_0$. Note that cash holdings do not affect the banker's surplus since he only earns the net return r on loans. Hence, when condition (27) holds, the banker will raise deposits and invest $L_0 = E_0 + f_b E_0$ into loans. He can raise deposits beyond $f_b E_0$ (the financing capacity of his equity in the bad state) as long as he invests the extra deposits into cash at $t = 0$ so that the incentive constraint (28) continues to hold.

Suppose condition (27) does not hold. As the banker's surplus (26) is increasing in $\Delta C\,(b)$ in this case, the banker would want to set $\Delta C\,(b)$ as high as possible. From the binding incentive constraint (23), we get:

$$\Delta C\left(b\right)=\frac{\lambda}{\lambda-f_{b}}\left(D-C_{0}-f_{b}E_{0}\right) \tag{29}$$

To make $\Delta C\left(b\right)$ as high as possible, the banker will set $D=\bar{D}$ and $C_0=0$. Hence, $\Delta C\left(b\right)=\frac{\lambda}{\lambda-f_{b}}\left(\bar{D}-f_{b}E_{0}\right)$. By (14), the banker will not liquidate all the loans as he will exhaust the available deposits $\bar{D}$ before full liquidation. That is, $\Delta C\left(b\right)<\left(1-l\right)\left(D+E_{0}\right)$ holds. Lastly, we check that that the no run condition (24) is satisfied. Substituting for $\Delta C\left(b\right)$, D, and C_0 in (24), we have that:

$$\frac{\nu-f_{b}}{\lambda-f_{b}}\left[\bar{D}-\lambda E_{0}\right]\geq 0 \tag{30}$$

must hold. Since we assume that $\bar{D}\leq\lambda E_0$, the equilibrium with deposit-taking is only viable for $f_b\geq\nu$, i.e., when the banker has a sufficiently high financing capacity of the inside equity compared to the liquidation return of depositors. For $f_b<\nu$, the banker will not raise deposits and will run the bank using his own equity only.

Proof of Proposition 3

Substituting the values of (D_g,C_g) and (D_b,C_b) into the truth-telling constraints and rearranging yields:

$$C_{b}\geq\varepsilon\left(\frac{1}{1-q}\right)\left(\frac{1}{\frac{Y}{1-l}-1}\right)$$

and

$$C_{b}\leq\varepsilon\left(\frac{q}{1-q}-p\right)\left(\frac{1}{\frac{Y}{1-l}-1}\right)+\left(1-p\right)\left(\frac{\mathcal{P}_{b}\left(D+E_{0}\right)-D}{\frac{Y}{1-l}-1}\right).$$

We know that $C_3^*>0$, which from the binding incentive constraint implies that $\mathcal{P}_b\left(D+E_0\right)-D<0$. To see why, note that the incentive constraint (8) can be written as

$$\left(1-\frac{\mathcal{P}_{b}}{1-l}\right)C_{3}^{*}=D-\mathcal{P}_{b}(D+E_{0}),$$

where we know that $\mathcal{P}_b<1-l$.

Moreover, since $\frac{1}{1-q}>\frac{q}{1-q}-p$ always holds, there is no combination of C_b and ε that satisfies both truthtelling constraints.

Proof of Proposition 5

Recall that we are considering the set of parameters such that liquidation costs are low enough so bankers do not use ex ante cash, $C_0=0$:

$$\frac{1-v}{v}\leq f_{b}<\lambda-\left(1-q\right)\left[\frac{1+r}{r}+\lambda\right]$$

Therefore, we have:

$$C_3^* = C_0 + \Delta C\left(b\right) = \Delta C\left(b\right) = \frac{\lambda}{\lambda - f_b}\left(\bar{D} - f_b E_0\right)$$

where $f_b = \frac{P_b}{1-P_b}$ and $\lambda = \frac{1-l}{l}$. Furthermore,

$$\begin{aligned} D &= \bar{D} \\ L_3(g) &= E_0 + \bar{D} \\ L_3(b) &= L_0 - \Delta C\left(b\right)\frac{1}{1-l} = E_0 + \bar{D} - C_3^*\frac{1}{1-l} = E_0 + \bar{D} - \frac{1}{\lambda - f_b}\frac{1}{l}\left(\bar{D} - f_b E_0\right) \end{aligned}$$

For a weak banker, we need to show that

$$q[YL_3(g) - D] + (1-q)p\left[YL_3(b) + C_3^* - D - \frac{d}{(1-\alpha)+\alpha p}\right] + (1-q)B_b^h L_3(b) \geq E_0$$

Substituting for D, $L_3(g)$, $L_3(b)$ and d, and collecting terms, this is equivalent to showing that:

$$\begin{aligned} &\left[qY + (1-q)\left(pY + B_b^h\right) - 1\right]E_0 + \left[q\left(Y-1\right) + (1-q)\left(p\left(Y-1\right) + B_b^h\right) - \alpha\frac{p(1-p)(1-q)}{(1-\alpha)+\alpha p}\right]\bar{D} \\ &\qquad - (1-q)\left[\frac{pY + B_b^h}{1-l} - \frac{p}{(1-\alpha)+\alpha p}\right]C_3^* \geq 0 \end{aligned}$$

Substituting for C_3^* and collecting terms, this is equivalent to showing that:

$$\begin{aligned} &\left[qY + (1-q)\left(pY + B_b^h\right) - 1 + (1-q)\frac{\lambda f_b}{\lambda - f_b}\left(\frac{pY + B_b^h}{1-l} - \frac{p}{(1-\alpha)+\alpha p}\right)\right]E_0 + \\ &\qquad \left[q\left(Y-1\right) - (1-q)\left(\frac{pY + B_b^h}{1-l}\frac{f_b + l\left(\lambda - f_b\right)}{\left(\lambda - f_b\right)} - \frac{p}{(1-\alpha)+\alpha p}\frac{f_b}{\lambda - f_b}\right)\right]\bar{D} \geq 0 \end{aligned}$$

Putting together terms multiplied by $(1-q)\frac{f_b}{\lambda - f_b}$ yields:

$$\begin{aligned} &\left(qY + (1-q)\left(pY + B_b^h\right) - 1\right)E_0 + q\left(Y-1\right)\bar{D} \\ &\qquad + (1-q)\frac{f_b}{\lambda - f_b}\left(\frac{pY + B_b^h}{1-l} - \frac{p}{(1-\alpha)+\alpha p}\right)\left(\lambda E_0 - \bar{D}\right) - (1-q)\left(pY + B_b^h\right)\frac{\bar{D}}{\lambda} \end{aligned}$$

Note that the first term is positive,

$$qY + (1-q)\left(pY + B_b^h\right) - 1 > 0$$

since $pY + B_b^h > pY + (1-p)\,Y > 1$ (assumption (19)). The second term is also positive as $Y > 1$. The third term is positive since $\frac{pY + B_b^h}{1-l} > 1$ while $\frac{p}{(1-\alpha)+\alpha p} < 1$, and $\lambda E_0 > \bar{D}$ by assumption (14). So, all terms above are positive except the last one. Using $E_0 > \frac{\bar{D}}{\lambda}$, we have

$$\left(qY+(1-q)\left(pY+B_b^h\right)-1\right)E_0+q\left(Y-1\right)\bar{D}$$
$$+(1-q)\frac{f_b}{\lambda-f_b}\left(\frac{pY+B_b^h}{1-l}-\frac{p}{(1-\alpha)+\alpha p}\right)\left(\lambda E_0-\bar{D}\right)-(1-q)\left(pY+B_b^h\right)\frac{\bar{D}}{\lambda}>$$
$$(qY-1)\frac{\bar{D}}{\lambda}+q\left(Y-1\right)\bar{D}+(1-q)\frac{f_b}{\lambda-f_b}\left(\frac{pY+B_b^h}{1-l}-\frac{p}{(1-\alpha)+\alpha p}\right)\left(\lambda E_0-\bar{D}\right)=$$
$$\left(q\left(Y-1\right)+\frac{1}{\lambda}\left(qY-1\right)\right)\bar{D}+(1-q)\frac{f_b}{\lambda-f_b}\left(\frac{pY+B_b^h}{1-l}-\frac{p}{(1-\alpha)+\alpha p}\right)\left(\lambda E_0-\bar{D}\right)$$

As argued above, the second term is positive. A sufficient condition for the first term to be positive is $qY>1$ which holds for q sufficiently high (since $Y>1$).

For a sound bank, we need to show that

$$[qYL_3(g)+(1-q)(YL_3(b)+C_3^*)-D]-(1-q)\frac{d}{(1-\alpha)+\alpha p}\geq YE_0$$

Substituting for D, $L_3(g)$, $L_3(b)$ and d, and collecting terms, this is equivalent to showing that:

$$\left(Y-1\right)\bar{D}+(1-q)C_3^*-(1-q)\frac{Y}{1-l}C_3^*-(1-q)\frac{\alpha(1-p)}{(1-\alpha)+\alpha p}\left(\bar{D}-C_3^*\right)\geq 0. \qquad (31)$$

The first three terms on the left-hand side of (31) give the net profit of a sounds banker if he does not participate in the deposit insurance: the return on the loan-making activity $(Y-1)\,\bar{D}$, the return on cash holdings in the bad state $(1-q)C_3^*$, net of the opportunity cost of liquidating loans in the bad state, $(1-q)\frac{Y}{1-l}C_3^*$. The last term is the cost of the deposit insurance to be paid in the bad state.

Clearly, for $q\to 1$ (or $\alpha\to 0$), (31) is satisfied, as deposit insurance costs vanish. Moreover, the left-hand side of (31) is continuous (and increasing) in q. By continuity, (31) is satisfied for q sufficiently close to 1.

Therefore, for a q sufficiently close to 1, both weak and sound bankers are willing to participate in the deposit insurance.

Proof of Proposition 6 By the depositor participation constraint (22), we have that

$$\varepsilon=\frac{(1-q)\,d}{1-\alpha(1-p)(1-q)}$$

under depositor risk-neutrality. By (20), a sound banker's cost of participating in deposit insurance is given by $\frac{(1-q)d}{(1-\alpha)+\alpha p}$. Thus, under risk neutrality, the cost of participating is greater than ε. In order for deposit insurance to be preferred, risk aversion must be large enough such that $\varepsilon>\frac{(1-q)d}{(1-\alpha)+\alpha p}$.