Ride-Hailing Networks with Strategic Drivers: The Impact of Platform Control Capabilities on Performance

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Abstract

Problem Definition: Motivated by ride-hailing platforms such as Uber, Lyft and Didi, we study the problem of matching riders with self-interested drivers over a spatial network. We focus on the performance impact of two operational platform controls—demand-side admission control and supply-side repositioning control—considering the interplay with two practically important challenges: (i) spatial demand imbalances prevail for extended periods of time; and (ii) self-interested drivers strategically decide whether to join the network, and if so, whether to reposition when not serving riders.

Methodology/Results: We develop and analyze the steady-state behavior of a novel game-theoretic fluid model of a two-location, four-route loss network. First, we fully characterize and compare the steady-state system equilibria under three control regimes, from minimal control to centralized control. Second, we provide insights on how and why platform control impacts equilibrium performance, notably with new findings on the role of admission control: the platform may find it optimal to strategically reject demand at the low-demand location even if drivers are in excess supply, to induce repositioning to the high-demand location. We provide necessary and sufficient conditions for this policy. Third, we derive upper bounds on the platform’s and drivers’ benefits due to increased platform control; these are more significant under moderate capacity and significant cross-location demand imbalance.

Managerial Implications: Our results contribute important guidelines on the optimal operations of ride-hailing networks. Our model can also inform the design of driver compensation structures that support more centralized network control.

Keywords: ride-hailing, control, network, matching, strategic drivers, demand imbalance

1 Introduction

We are motivated by the emergence of ride-hailing platforms such as Uber, Lyft, Didi, and Via that face the problem of matching supply (drivers) with demand (riders) over a spatial network. We study the performance impact of operational platform controls, focusing on the interplay with two practically important challenges: (i) Significant demand imbalances prevail across network locations for extended periods of time (see Figure 1), so that the natural supply of drivers at a location either

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falls short of or exceeds the demand for rides originating at this location. These mismatches hurt performance as they lead to lost demand, drivers idling, and/or drivers repositioning (without serving a rider) from a low- to a high-demand location. (ii) Drivers are self-interested and decide strategically whether to join the network, and if so, when and where to reposition, trading off the related travel time and cost against their matching (queueing) delay at their current location. These decentralized supply decisions may not be optimal for the overall network.

**Flow Imbalances: Example Manhattan.** We illustrate the magnitude and duration of the demand imbalances noted above with publicly available data for taxi rides in Manhattan.¹ (We do not have public data for ride-hailing platforms, but they likely experience similar imbalances.) Though the data report censored demand (realized trips), we believe the (uncensored) demand imbalances are likely of the same or even higher order of magnitude as the (censored) flow imbalances.

![Figure 1: Route-level flow imbalances in Manhattan](image)

Figure 1 illustrates the route-level realized flow imbalances for two origin-destination pairs in Manhattan, New York City, over all weekdays for one month. We observe a pronounced imbalance of almost one order of magnitude (about 10x) in the morning rush hour and about half an order of magnitude (about 3x) in the evening rush hour in the reverse direction. Our analysis (not shown here) confirms that (i) these route-level flow imbalances persist after aggregation to the location level, and (ii) these substantial route- and location-level imbalances are also statistically significant. Furthermore, it is important to note that imbalance periods typically persist for a couple of hours, in contrast to the typical 10–15 min trip times between these locations. This suggests that network transients may settle down quickly relative to the imbalance duration; which, in part, motivates

¹New York City TLC Trip Record Data. [https://www1.nyc.gov/site/tlc/about/tlc-trip-record-data.page](https://www1.nyc.gov/site/tlc/about/tlc-trip-record-data.page)
our focus on the steady-state fluid model as opposed to the transient process itself.

**Operational Controls to Manage Demand Imbalances.** Motivated by these observations, we study the value of two operational platform controls to manage these demand imbalances: demand-side admission control and supply-side capacity repositioning. Though financial incentives (prices and wages) and spatial information (on demand and price surges) clearly also play an important role in practice, we hold these levers constant to isolate the effects of operational controls.

Admission control allows the platform to accept or reject requests based on destination; in practice platforms do so both directly and indirectly, through ETA quotation. This non-price control complements pricing, allowing platforms to regulate demand with less drastic price fluctuations. Admission control also affects the car distribution in the network, both directly and indirectly, via drivers’ idling delays at lower-demand locations that in turn shape their repositioning incentives.

Repositioning control allows the platform, rather than drivers, to decide when and whether they relocate from lower- to higher-demand locations. In practice, such centralized repositioning control is characteristic when drivers operate like employees (e.g., when driving for Via in “Blue Mode” for hourly pay) and will also gain in relevance with the proliferation of autonomous vehicles.

To evaluate these controls we study the steady-state behavior of a deterministic fluid model of a ride-hailing network in a game-theoretic framework with riders, drivers and the platform. We provide analytical results for two-location networks (Figure 2) and show through numerical results for three-location ring and four-location star networks that our main findings generalize to multi-location networks. Riders generate demand for each route (with a fixed travel time). Prices are fixed; we assume for simplicity that the price per unit travel time is route-independent, though this is not necessary for our analysis. Drivers decide, based on their (heterogeneous) opportunity cost and their equilibrium expected profit rate from participation, whether to join the network, and if so, whether to wait for a rider at their location or to reposition to the other location. Drivers have homogeneous transportation costs and behave symmetrically if they join the network. The platform receives a fixed commission of the fare paid by riders and seeks to maximize its revenue. We consider three control regimes: (i) Centralized Control of both admission and repositioning; (ii) Minimal Control: no admission control and decentralized repositioning control; and (iii) Optimal Admission Control with centralized admission control and decentralized repositioning control.

**Main Results and Contributions.** First, we propose a novel game-theoretic model that accounts for key features of ride-hailing platforms: the network structure and demand imbalances, the driver incentives, and the interplay of queueing, transportation times, and driver decisions.
Second, we fully characterize the steady-state system equilibria for the three control regimes outlined above, relying on the analysis of equivalent capacity allocation problems.

Third, we provide insights on how and why platform control impacts equilibrium performance. (i) Decentralized repositioning leads to inefficient capacity allocation as a result of excessive driver idling at low-demand locations. (ii) Admission control can significantly reduce these inefficiencies. (iii) Most notably, we identify a novel role for admission control: as a tool to influence drivers’ repositioning decisions. Specifically, the platform may find it optimal to strategically reject demand at the low-demand location, though there is an excess driver supply, to induce repositioning to the high-demand location. We provide intuitive necessary and sufficient optimal conditions for this policy. This finding highlights that operational levers, not only pricing, can shape repositioning. Whereas here admission control influences the relocation of idle strategic capacity, the standard roles of admission control in the queueing literature are (1) to balance myopic rewards with opportunity costs, and (2) to control the relocation of utilized resources.

Fourth, we derive upper bounds on the platform’s and the drivers’ benefits due to increased platform control capability. These bounds show that, at practically relevant levels cross-location demand imbalances, the benefits can be very significant for the platform, of the order of 50%, 100% or even larger improvements, especially when the network operates with moderate capacity. These bounds also point to tension between platform and driver gains, e.g., large platform gains require an increase in driver participation, which limits gains in per-driver profits.

**Related Literature.** This paper is related to the growing literature on ride-hailing platforms. We first survey theoretical studies and then turn to empirical studies. We group the models considered in theoretical studies into three streams: (i) single-location models with strategic driver supply, (ii) multi-location models with centralized supply control, and (iii) multi-location models with strategic driver supply. Most papers belong to (i) and (ii), whereas this paper belongs to (iii).

(i) **Single-location models with strategic driver supply.** This stream either ignores the spatial dimension or captures it in reduced form. Most studies focus on controlling rider prices and driver wages to match demand with supply; some of these papers largely ignore queueing considerations (e.g., Gurvich et al. 2016, Cachon et al. 2017, Hu and Zhou 2020 and Hu et al. 2021), others consider delay-sensitive customers using queueing models (e.g., Banerjee et al. 2016, Taylor 2018, Bai et al. 2019 and Benjaafar et al. 2021a). Castillo et al. (2021) use a stylized model that captures space in reduced form (pickup times decrease in the number of idle cars) to show that surge pricing can help avoid an inefficient “wild goose chase” whereby long pickup times reduce driver earnings.

Garg and Nazerzadeh (2021) study driver surge pricing mechanisms under non-stationary demand.
A few papers study matching under fixed prices, i.e., the design of operational controls (capacity reservation and priority policies) to match strategic drivers with differentiated demand in terms of the value (e.g., Castro et al. 2021) or timing of ride requests (e.g., Zhou et al. 2021).

(ii) Multi-location models with centralized supply control. This stream assumes that platforms fully control the vehicle supply and operation. Most papers model the system as a closed queueing-loss network: nodes correspond to locations, a fixed set of cars circulate among nodes where they queue while waiting for trip matches, and trip requests are lost if not matched upon arrival.

Some papers focus on demand-side controls. Waserhole and Jost (2016) and Banerjee et al. (2021) consider static pricing. Balseiro et al. (2021) and Chen et al. (2020) study state-dependent pricing under stationary and non-stationary demand, respectively. Kanoria and Qian (2020) study the joint problem of state-dependent pricing (or admission control) and matching in the absence of prior knowledge of the demand arrival rates. Assuming fixed pricing, Wang et al. (2019) study admission control based on a pickup-time threshold in a two-sided model with open rider-side queue and a closed driver-side queue that captures space in reduced form (similar to Castillo et al. 2021).

In these studies of demand-side control, cars are only matched with local requests, so only relocate when utilized. In contrast, studies of supply-side controls focus on operational levers to control the flow of empty cars through proactive repositioning and reactive matching.

Papers that focus on repositioning include Iglesias et al. (2017) and Braverman et al. (2019) who study static policies and Benjaafar et al. (2021c) and Hosseini et al. (2021) who consider dynamic policies. Most relevant to our paper is Braverman et al. (2019). They prove an asymptotic limit theorem that justifies the use of a stationary deterministic fluid network model (such as the one in this paper) and then characterize the fluid-based optimal empty-car routing policy that maximizes some function of throughput. In contrast to our paper, they fix the capacity, restrict attention to centralized repositioning (as in our regime C in Section 3) and do not consider admission control.

Papers that focus on matching include Feng et al. (2020), Banerjee et al. (2020), Özkan and Ward (2020), and Hu and Zhou (2021). Feng et al. (2020) compare the performance of two matching systems, on-demand versus street hailing, for a closed circular queueing network. Banerjee et al. (2020) consider state-dependent control in a closed queueing network; Özkan and Ward (2020) consider state-independent control for an open one-sided queueing model (vehicles exit upon matching); Hu and Zhou (2021) consider dynamic control for a discrete-time, two-sided queueing model (supply and demand units queue before abandoning) and match-dependent rewards reflect spatial distance.

Some papers jointly consider repositioning and matching. An early study by Meyer and Wolfe (1961) compares the performance of various policies for special networks (two nodes, or a continuum
of locations with uniformly distributed demand). Ata et al. (2020a) propose and demonstrate the effectiveness of a dynamic policy that hinges on the approximate analysis in the heavy traffic regime.

Some papers study higher-level strategic issues such as capacity sizing (Besbes et al. 2021a, Benjaafar et al. 2021c) and service region design (e.g., He et al. 2017).

(iii) Multi-location models with strategic driver supply. This stream focuses on pricing policies that account for spatial considerations and strategic drivers’ joining and/or location decisions.

Focusing on welfare maximization, Ma et al. (2021) propose an optimal and incentive-aligned spatio-temporal pricing mechanism for the finite-horizon problem with complete information.

Studies of static price and wage policies for revenue maximization include Bimpikis et al. (2019) and Besbes et al. (2021b). Bimpikis et al. (2019) consider a discrete-time stationary network. They ignore driver queueing effects and assume that ample driver supply is available at a fixed cost, and travel times take one time period. They show platform profits and consumer surplus increase when demands are more balanced across the network, consistent with our results that demand imbalances magnify the value of operational platform controls. Besbes et al. (2021b) study short-term location-dependent pricing for a linear city where rational, myopic drivers with exogenous initial locations make one-shot (re)location decisions. Studies of dynamic surge pricing and wage policies under non-stationary demand include Guda and Subramanian (2019) and Afèche et al. (2021).

Unlike these pricing studies, we focus on operational controls. Benjaafar et al. (2021b) adopt our model and extend it by introducing autonomous vehicles (AVs), related operational decisions, and driver wage decisions. They show that if AVs are sufficiently affordable, the platform would deploy them so as to substantially reduce the need for repositioning by human (strategic) drivers.

Empirical studies. Some papers study ride-hailing data, others taxi data. Using Uber data, Chen and Sheldon (2015) show that surge pricing induces drivers to work longer and hence increases efficiency; Hall et al. (2017) find that the driver supply is highly elastic to wage and underlying fare changes, so the per-trip earnings boost of a fare hike is negated by higher driver competition (consistent with our results, as noted above). Yan et al. (2020) review operational matching and dynamic pricing techniques and discuss a dynamic waiting mechanism inspired by Uber. Using NYC taxi data, Buchholz (2021) and Ata et al. (2020b) analyze the dynamic spatial equilibrium with strategic taxi drivers, and study how matching and spatial pricing affect performance. Buchholz (2021) shows that matching technology can improve performance significantly even under optimized pricing, which supports the value of studying the impact of operational controls.

Plan for the Paper. In §2 we present the model and problem formulations. In §3 we study centralized control, in §4 the regimes with decentralized repositioning. In §5 we present theoretical
upper bounds on the performance gains of platform control. In §6 we generalize our results to multi-location networks. In §7 we offer concluding remarks. (All proofs are in the Appendix.)

2 Model and Problem Formulations

We consider a deterministic fluid model of a ride-hailing network in steady state. Braverman et al. (2019) rigorously justify such a fluid model for a stochastic closed queueing network with centralized car control in a “large market regime”, i.e., the number of cars $N$ and the potential demand rates grow linearly with $N$, holding constant travel times. They prove the process-level and steady-state convergence of the scaled queue length process to a fluid limit as $N \to \infty$. Their arguments could be adapted to our setting; we focus directly on a set of (motivated) steady-state flow equations.

2.1 Model Primitives

Figure 2 shows the network schematic and the model primitives that we describe in this section.

[Diagram of a network with nodes and routes labeled with $\Lambda_{lk}$, $t_{lk}$, $\Lambda$, and $\gamma p$]

Figure 2: Model primitives

Network. The network has two locations (nodes), indexed by $l = 1, 2$, and four routes (arcs), indexed by $lk$ for $l, k \in \{1, 2\}$. We denote by $t_{lk}$ the travel time on route $lk$ and by $t$ the travel time vector. We impose no restrictions on travel times; specifically, we allow $t_{12} \neq t_{21}$, to reflect, for example, different uptown/downtown routes. The travel times are constant and, in particular, independent of the number of drivers that serve demand for the platform. This assumes that the number of drivers has no significant effect on road congestion and transportation delays.

Riders. Riders generate demand for trips. We assume that the platform charges a fixed price of $\$p$ per unit travel time for all routes. Given the price $p$, the potential demand rate for route-$lk$ trips is $\Lambda_{lk}$, and $\Lambda$ denotes the potential demand rate vector. The platform keeps a portion $\gamma$
of the total fee as commission and drivers collect the remainder. Riders are impatient, i.e., rider requests are lost if not matched instantly with an available car. We focus on the case of imbalanced cross-location demands, $A_{12} \neq A_{21}$; without loss of generality, we make the following assumption.

**Assumption 1** (Demand imbalance). $0 < A_{12} < A_{21}$.

**Drivers.** Drivers supply capacity to the network. Let $N$ be the pool of (potential) drivers, each equipped with one car (unit of capacity). Drivers are self-interested and seek to maximize their profit rate per unit time. They decide whether to join the network, and, if so, whether to reposition to the other network location, i.e., travel without serving a rider, in anticipation of a faster match.

Each driver has an idiosyncratic opportunity cost rate, denoted by $c_o$, that is assumed to be an independent draw from a common continuous distribution $F$ with connected support $[0, \infty)$. Drivers join the network if their expected profit rate $\pi$ equals or exceeds their outside opportunity cost rate, i.e., the number of participating drivers $n = NP (c_o \leq \pi) = NF(\pi)$. The per-driver profit rate $\pi$ is itself a quantity that emerges in equilibrium and depends on $n$, the drivers’ trip-related earnings and cost, the platform’s controls, and the driver decisions, to be specified in §3 and §4.

Participating drivers incur a common driving cost rate of $c$ independent of the car occupancy. Drivers serving rider demand earn revenue at rate $\bar{\gamma}p$, where $\bar{\gamma} = 1 - \gamma$, and hence profit rate $\bar{\gamma}p - c$. However, their actual profit rate is lower if they spend time waiting for riders (accruing zero profit when idling) and/or repositioning from one location to the other (incurring the driving cost rate $c$). The following assumption ensures that drivers can earn a positive profit by repositioning.

**Assumption 2** (Positive profit from repositioning). $ct_{12} < t_{21}(\bar{\gamma}p - c)$ and $ct_{21} < t_{12}(\bar{\gamma}p - c)$.

Given Assumption 1, only the first condition in Assumption 2 will prove to be relevant.

**Platform.** The platform is operated by a monopolist firm that matches drivers with riders with the objective of maximizing its revenue rate. The platform may have two controls: a) demand-side admission control, and b) supply-side capacity repositioning, as detailed in §2.3.

**Information.** Riders and drivers rely on the platform for matching, that is, potential riders cannot see the available driver capacity, and drivers cannot see the arrivals of rider requests.

The platform knows the model primitives, including the potential demand rates $A_i$ the destination of each trip request, the travel times $t$, the driving cost $c$ and the opportunity cost rate distribution $F$. The driver opportunity cost rates are private information, not known by the platform. Therefore, participating drivers are homogeneous to the platform. The platform knows the state of the system, i.e., each driver’s location, travel direction and status at each point in time.

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2 Assumption 2 also implies $\bar{\gamma}p - c > 0$, hence $F(\bar{\gamma}p - c) > 0$, so at least some drivers choose to join the network.
Drivers do not observe the state of the system, but they have (or can infer) the information required to compute their individual expected profit rates, namely: the travel times $t$; the steady-state delays (possibly zero) until they get matched at each location; the destination (routing) probabilities for matches at each location; and the probabilities that they will choose or be instructed to reposition from one location to the other. These delays and the routing and repositioning probabilities are endogenous quantities consistent with the network equilibrium, as detailed below.

2.2 Matching Supply with Demand

Admission control. Let $\lambda_{lk} \leq A_{lk}$ denote the effective route-$lk$ demand rate, i.e., the rate of trip requests that are served. Let $\lambda$ be the vector of effective demand rates. A trip request may be lost either if there is no available driver capacity at the time and location of the request (recall that riders are impatient), or if the platform chooses to exercise admission control and reject the request (e.g., based on the requested destination) even though driver capacity is available.

Matching at each location. At each location drivers that become available (i.e., do not reposition upon arrival) join a single queue, to be matched with accepted ride requests that originate at this location. The platform matches drivers according to a uniform policy, such as First-In-First-Out (FIFO) or random order. Therefore, in steady state drivers queueing at location $l$ have the same waiting time, denoted by $w_l$, and the same matching probability for a route-$lk$ request, $\frac{\lambda_{lk}}{\lambda_{11} + \lambda_{12}}$. Let $q_l$ denote the steady-state queue length at location $l$. Little’s Law implies that $q_l = w_l (\lambda_{11} + \lambda_{12})$. Let $w$ and $q$ denote, respectively, the vector of steady-state waiting times and queue lengths.

Repositioning of capacity between locations. Let $\nu_{12}$ and $\nu_{21}$ be the aggregate flow rates of drivers repositioning from location 1 and 2, respectively, and $\nu = (\nu_{12}, \nu_{21})$. Up to three flows emanate from location $l$: Drivers that are matched with riders leave at rates $\lambda_{11}$ and $\lambda_{12}$, and drivers that reposition to location $k \neq l$ leave at rate $\nu_{lk}$ (without queueing at location $l$). Therefore, letting $\eta(\lambda, \nu)$ denote the corresponding vector of steady-state repositioning fractions, we have

$$\eta_{1}(\lambda, \nu) = \frac{\nu_{12}}{\lambda_{11} + \lambda_{12} + \nu_{12}} \quad \text{and} \quad \eta_{2}(\lambda, \nu) = \frac{\nu_{21}}{\lambda_{21} + \lambda_{22} + \nu_{21}}.$$ (1)

Repositioning decisions are either centralized or decentralized. Under centralized repositioning the platform controls the repositioning rates $\nu$ (e.g., drivers are employees or autonomous vehicles) and the fractions $\eta$ emerge in response through (1). Under decentralized repositioning each participating driver chooses her repositioning strategy to maximize her steady-state profit rate. A driver’s repositioning strategy is a vector of probabilities, denoted by $\tilde{\eta}$, that specify for each location the fraction of times that the driver will, upon arrival, directly reposition to the other location.
location. The steady-state profit rate of an individual driver circulating through the network, denoted by \( \tilde{\pi}(\tilde{\eta}; \lambda, w) \) and derived explicitly in §4.1, is a function of her repositioning fractions, \( \tilde{\eta} \), the routing probabilities implied by \( \lambda \), and the delays in the matching queues, \( w \). (The rates \( \lambda \) and delays \( w \) emerge as equilibrium quantities that depend on the platform and driver decisions as discussed below.) Since participating drivers are homogeneous, we focus on symmetric strategies where drivers choose the same fractions \( \tilde{\eta} \) to maximize \( \tilde{\pi}(\tilde{\eta}; \lambda, w) \). A set of flow rates \( (\lambda, \nu) \) and delays \( w \) admit a symmetric driver repositioning equilibrium if, and only if, the resulting unique repositioning fractions \( \eta(\lambda, \nu) \) that satisfy (1) agree with every driver’s best response to \( (\lambda, w) \):

\[
\eta(\lambda, \nu) \in \arg\max_{\tilde{\eta}} \tilde{\pi}(\tilde{\eta}; \lambda, w). 
\]

### 2.3 Three Control Regimes: Problem Formulations

We study three regimes, Centralized Control, Minimal Control and Admission Control, that differ in whether (i) repositioning decisions are centralized or decentralized, and (ii) the platform exercises admission control or not. The problems for all regimes have in common the platform’s objective function and drivers’ participation decisions that we formalize in this section, and the system flow constraints described in §2.2. We formulate these problems in terms of the tuple \( (\lambda, \nu, w, n) \).

**Platform revenue.** Let \( \Pi(\lambda) := \gamma_p \lambda \cdot t \) denote the platform’s steady-state revenue rate, where \( \gamma_p \) is its commission rate per busy driver and \( \lambda \cdot t \) is the number of busy drivers (and riders in service). Hence the realized demand (rider welfare) is proportional to the platform revenue.

**Drivers’ participation constraint.** Each driver decides whether to participate by comparing her opportunity cost to her profit rate from joining the system. We compute the per-driver profit rate in two ways: (i) as the profit rate of an individual driver circulating in the network, \( \tilde{\pi}(\tilde{\eta}; \lambda, w) \), as outlined in §2.2 and formalized in §4.1 (Lemma 2), and (ii) as the average of total driver profits:

\[
\pi(\lambda, \nu, n) := \frac{(\gamma_p - c) \sum_{l,k=1,2} \lambda_{lk} t_{lk} - c(\nu_{12} t_{12} + \nu_{21} t_{21})}{n},
\]

where the total profit in the numerator equals total service revenue minus total repositioning cost.

As explained in §2.2, in each regime all participating drivers have symmetric repositioning
fractions $\eta(\lambda, \nu)$ and hence achieve the same profit rate. The approaches (i) and (ii) therefore yield the same profit rate, i.e., $\tilde{\pi}(\eta(\lambda, \nu); \lambda, w) = \pi(\lambda, \nu, n)$ for all $(\lambda, \nu, w, n)$ that satisfy the system flow constraints described in §2.2. A participation equilibrium therefore requires $n = NF(\pi(\lambda, \nu, n))$.

Centralized Control (C). In the centralized benchmark the platform has “maximum” control, over both driver repositioning and demand admission decisions. The platform solves:

\[
\text{(Problem C)} \quad \max_{\lambda, \nu, w, n} \Pi(\lambda) \quad (3a)
\]

\[
\text{s.t.} \quad \lambda_{12} + \nu_{12} = \lambda_{21} + \nu_{21}, \quad (3b)
\]

\[
\sum_{l,k=1,2} \lambda_{lk} t_{lk} + \nu_{12} t_{12} + \nu_{21} t_{21} + \sum_{l=1,2} w_l (\lambda_{l1} + \lambda_{l2}) = n, \quad (3c)
\]

\[
0 \leq \lambda \leq \Lambda, \quad \nu \geq 0, \quad w \geq 0, \quad (3d)
\]

\[
\pi(\lambda, \nu, n) = (\gamma p - c) \sum_{l,k=1,2} \lambda_{lk} t_{lk} - c(\nu_{12} t_{12} + \nu_{21} t_{21}) / n, \quad (3e)
\]

\[
n = NF(\pi(\lambda, \nu, n)), \quad (3f)
\]

where (3b)–(3c) are the system flow constraints and (3e)–(3f) enforce the participation equilibrium.

Minimal Control (M). In this regime drivers control repositioning and the platform does not exercise demand admission control. Problem M augments Problem C with the driver repositioning equilibrium constraints (1)–(2), and the following constraints that capture the absence of admission control. First, the platform matches trip requests to drivers in pro-rata (or FIFO) manner. That is, requests originating at the same location have equal, destination-independent, service probabilities:

\[
\frac{\lambda_{l1}}{A_{l1}} = \frac{\lambda_{l2}}{A_{l2}}, \quad l = 1, 2. \quad (4)
\]

Second, the platform never turns away requests when there are drivers available to serve them. That is, drivers cannot reposition out of a location that has unmet demand, i.e.,

\[
(A_{l1} + A_{l2} - \lambda_{l1} - \lambda_{l2}) \nu_{lk} = 0, \quad l = 1, 2, k \neq l, \quad (5)
\]

and demand requests can only be lost at a location where no drivers are waiting, i.e.,

\[
(A_{l1} + A_{l2} - \lambda_{l1} - \lambda_{l2}) w_{l} = 0, \quad l = 1, 2. \quad (6)
\]

In the Minimal Control regime, the platform therefore solves:

\[
\text{(Problem M)} \quad \max_{\lambda, \nu, w, n} \{ \Pi(\lambda) : (1) - (2), (3b) - (3f), (4) - (6) \}. \quad (7)
\]

Admission Control (A). This regime differs from the centralized benchmark only in that repo-
sitioning is decentralized, i.e., subject to the driver repositioning equilibrium constraints (1)–(2):

\[
(\text{Problem A}) \quad \max_{\lambda, \nu, w, n} \{ \Pi(\lambda) : (1) - (2), (3b) - (3f) \}. \tag{8}
\]

2.4 Reformulation to Capacity Allocation Problems

It is intuitive and analytically convenient to reformulate the above problems in terms of the driver capacities allocated to serving riders, repositioning (without riders), and queueing for riders.

For route \(lk\), let \(S_{lk}\) denote the offered load of trips, and \(s_{lk}\) denote the (effective) capacity serving riders. Let \(S\) and \(s\) denote the corresponding vectors, \(S = \sum_{lk} S_{lk}\) the total offered load, and \(\bar{s} = \sum_{lk} s_{lk}\) the total service capacity. From Little’s Law,

\[
S_{lk} = \Lambda_{lk} t_{lk} \quad \text{and} \quad s_{lk} := \lambda_{lk} t_{lk}, \quad l, k \in \{1, 2\}. \tag{9}
\]

Let \(r_{lk}\) be the capacity repositioning from location \(l\) to \(k\), \(r = (r_{12}, r_{21})\), and \(\bar{r} = r_{12} + r_{21}\), where

\[
r_{lk} = \nu_{lk} t_{lk}, \quad l \neq k, \tag{10}
\]

and \(q_l\) be the capacity queueing at location \(l\). Let \(q = (q_1, q_2)\) and \(\bar{q} = q_1 + q_2\), where

\[
q_l = (\lambda_{l1} + \lambda_{l2}) w_l, \quad l = 1, 2. \tag{11}
\]

Using (9)–(11) we transform the problems in (\(\lambda, \nu, w, n\)) presented in §2.3 into equivalent problems in \((s, r, q, n)\). With some abuse of notation, we write the platform revenue function as \(\Pi(s) = \gamma p \bar{s}\) instead of \(\Pi(\lambda)\), the per-driver profit functions as \(\tilde{\pi}(\tilde{\eta}; s, q)\) instead of \(\tilde{\pi}(\tilde{\eta}; \lambda, w)\) and \(\pi(s, r, n)\) instead of \(\pi(\lambda, \nu, n)\), and the repositioning fractions in (1) as \(\eta(s, r)\) instead of \(\eta(\lambda, \nu)\).

Using (9)–(11) and the definitions of \(\bar{s}\) and \(\bar{r}\), the constraints (3b)–(3f) are equivalent to:

\[
\frac{s_{12} + r_{12}}{t_{12}} = \frac{s_{21} + r_{21}}{t_{21}}, \tag{12a}
\]

\[
\bar{s} + \bar{r} + \bar{q} = n, \tag{12b}
\]

\[
0 \leq s \leq S, \quad r \geq 0, \quad q \geq 0, \tag{12c}
\]

\[
\pi(s, r, n) = \frac{(\gamma p - c) \bar{s} - c \bar{r}}{n}, \tag{12d}
\]

\[
n = NF(\pi(s, r, n)). \tag{12e}
\]

2.5 Two-Step Solution Approach

For regime \(X \in \{C, M, A\}\) and capacity \(n\), let \(C_X(n)\) denote the set of capacity allocations \((s, r, q)\) that satisfy all the constraints, except the driver participation constraints (12d)–(12e). That is, \(C_C(n) = \{(s, r, q) : (12a) - (12c)\}\), \(C_M(n) = \{(s, r, q) : (1) - (2), (4) - (6), (9) - (11), (12a) - (12c)\}\),
and $\mathcal{C}_A(n) = \{(s, r, q) : (1) - (2), (9) - (11), (12a) - (12c)\}$. Therefore, in regime $X$ the platform’s optimization problem and the optimal revenue rate, denoted by $\Pi_X$, are given by

$$\Pi_X := \max_{s, r, q, n} \{\Pi(s) : (s, r, q) \in \mathcal{C}_X(n), (12d) - (12e)\}, \quad X \in \{C, M, A\}. \quad (13)$$

We solve the platform’s capacity allocation problems (13) in two steps as follows.

**Step 1:** Solve for the optimal allocation $(s, r, q)$ of a fixed capacity of participating drivers, $n$:

$$\Pi_X(n) := \max_{s, r, q} \{\Pi(s) : (s, r, q) \in \mathcal{C}_X(n)\}, \quad n \in [0, N]. \quad (14)$$

Let $\mathcal{C}_X^*(n) := \arg\max_{s, r, q} \{\Pi(s) : (s, r, q) \in \mathcal{C}_X(n)\}$. Let $(s_X(n), r_X(n), q_X(n)) \in \mathcal{C}_X^*(n)$ be a solution of (14) that maximizes the per-driver profit in $(12d)$, i.e., $\pi(s_X(n), r_X(n), n) = \max_{(s, r, q) \in \mathcal{C}_X(n)} \pi(s, r, n)$. Let $\pi_X(n) := \pi(s_X(n), r_X(n), n)$ be the resulting per-driver profit as a function of $n$.

**Step 2:** Solve for the unique equilibrium participating capacity $n_X^*$: With per-driver profit function $\pi_X(n)$ non-increasing, there exists a unique solution of the participation equilibrium equation $(12e)$:

$$n_X^* = NF(\pi_X(n_X^*)). \quad (15)$$

Hence $\Pi_X = \Pi_X(n_X^*)$ and the equilibrium per-driver profit $\pi_X^* := \pi_X(n_X^*)$.

**Lemma 1** (Sufficient conditions for optimality of two-step solution). For regime $X$ the two-step solution (14)–(15) identifies the optimal solution of (13), i.e., $\Pi_X^* = \Pi_X(n_X^*)$, if the per-driver profit $\pi_X(n)$ is (i) non-increasing in $n$, and (ii) attains the maximum per-driver profit at every $n$:

$$\pi_X(n) = \max_{(s, r, q) \in \mathcal{C}_X(n)} \pi(s, r, n), \quad n \in [0, N]. \quad (16)$$

Condition (16) implies the platform’s and drivers’ incentives are aligned. We show that (16) holds in regimes C and M, but may not hold in regime A, which calls for modifying this two-step approach.

### 3 Centralized Control (C)

Under centralized control (C) the platform controls demand admission and driver repositioning.

Problem C, given in (3a)–(3f), is equivalent to the capacity allocation problem (13) for regime $X = C$. We solve this problem in two steps, as outlined in §2.5. In Step 1, Proposition 1 characterizes the optimal allocation of fixed capacity $n$, that is, $\Pi_C(n) := \max_{s, r, q} \{\Pi(s) : (s, r, q) \in \mathcal{C}_C(n)\}$. In Step 2, Corollary 1 shows that the resulting per-driver profit function (12d) satisfies the optimality conditions of Lemma 1 and yields a unique equilibrium capacity, $n_C^*$, satisfying (12e).

**Proposition 1** (Regime C: Optimal allocation of fixed driver capacity). Define the constants

$$n_1^C := \mathcal{S} - (A_{21} - A_{12}) t_{21} \quad \text{and} \quad n_2^C := \mathcal{S} + (A_{21} - A_{12}) t_{12}. \quad (17)$$
In regime C, problem (14) yields the following optimal allocation \((s, r, q)\) of the driver capacity \(n\):

1. **Scarce capacity** \((n \leq n^C_1)\). All drivers serve riders: \(s = n; \ r = 0; \ q = 0\).

2. **Moderate capacity** \((n^C_1 < n \leq n^C_2)\). Drivers serve riders or reposition from the low- to the high-demand location: \(s + r_{12} = n\) where \(r_{12} = t_{12}/(t_{12} + t_{21})(n - n^C_1), \ r_{21} = 0; \ q = 0\).

3. **Ample capacity** \((n > n^C_2)\). Drivers serve all riders, reposition from the low- to the high-demand location, or wait in queue: \(s = S; \ r_{12} = n^C_2 - S, r_{21} = 0; \ q = n - n^C_2\).

**Remark 3.** The equilibrium participating capacity increases in the driver pool size \(N\). The intervals in \(n\) of Proposition 1 hence map to intervals in \(N\) with respective capacity allocations in equilibrium.

Figure 4 (a) illustrates Proposition 1. Importantly, centralized control makes efficient use of capacity: drivers idle in queue only if capacity is ample to serve all demand, i.e., \(n > n^C_2\).

The threshold \(n^C_1\) is the maximum offered load that can be served without repositioning, i.e., all local demand and the balanced cross-location demand; destination-based admission control at the high-demand location allows the platform to serve all local requests while rejecting excess demand to the low-demand location. The threshold \(n^C_2\) is the minimum capacity needed to serve the total offered load \(S\), including the excess cross-location demand that requires empty-car repositioning. For ample capacity \((n > n^C_2)\) the solution is not unique, but it is the unique one that maximizes the per-driver profit (condition (16) in Lemma 1) by precluding repositioning from location 2 to 1.

Substituting \(s\) and \(r\) from Proposition 1 into (12d) yields the following per-driver profit function:

\[
\pi_C(n) = \frac{(\bar{\gamma}p - c)S - cn}{n} = \begin{cases} 
\frac{\bar{\gamma}p - c}{n}, & \text{zone 1 } (n \leq n^C_1), \\
\frac{1}{n}(\bar{\gamma}p(n^C_1 + (n - n^C_1)\frac{t_{21}}{t_{12} + t_{21}})) - c, & \text{zone 2 } (n^C_1 < n \leq n^C_2), \\
\frac{1}{n}(\bar{\gamma}pS - cn^C_2), & \text{zone 3 } (n > n^C_2).
\end{cases}
\]

(18)

This profit rate reflects the drivers' utilization profile: in zone 1 they serve riders all the time (net margin rate \(\bar{\gamma}p - c\)); in zone 2 they still drive around all the time but serve riders only a fraction of the time; and in zone 3 they also queue a fraction of the time.

**Corollary 1** (Regime C: Driver participation equilibrium). Under optimal centralized admission control and repositioning, (i) the per-driver profit in (18) satisfies the conditions in Lemma 1 for optimality of the two-step approach, i.e., \(\pi_C(n)\) is decreasing and maximizes the per-driver profit at every \(n\); (ii) there exists a unique equilibrium capacity of participating drivers, \(n^*_C = NF(\pi_C(n^*_C))\).
4 Regimes with Decentralized Repositioning

In this section we characterize the properties of a driver repositioning equilibrium in §4.1, and then the equilibria for two regimes, Minimal Control (M) in §4.2; and Admission Control in §4.3. We summarize the key differences between the equilibria of regimes C, M, and A in §4.4.

4.1 Driver Repositioning Equilibrium

Recall from §2.2 that under decentralized repositioning, drivers symmetrically choose their repositioning fractions \( \eta \) to maximize their profit rate \( \pi(\eta; \lambda, w) \). By (1) and (2), the flow rates \( \lambda(w) \) and delays \( w \) admit a driver repositioning equilibrium if, and only if, the corresponding unique repositioning fractions \( \eta(\lambda, w) \), in (1), are every driver’s best response to \( (\lambda, w) \), i.e., satisfy (2). Using (9)–(10) to map \( (\lambda, w) \) to \( (s, r) \), we henceforth express the functions \( \eta(\lambda, w) \) as \( \eta(s, r) \).

Remark 1. Under Assumption 1, it is not optimal to reposition from the high-demand location \( (2) \) to the low-demand location \( (1) \). We therefore focus on driver repositioning equilibria with \( \nu_{21} = r_{21} = 0 \), so that \( \eta_2(s, r) = \tilde{\eta}_2 = 0 \). Using (9) and (11) to map \( (\lambda, w) \) to \( (s, q) \), we henceforth express the profit-rate \( \pi(\tilde{\eta}; \lambda, w) \) for simplicity as the univariate function \( \tilde{\pi}(\tilde{\eta} \tilde{1}; s, q) \).

Remark 2. Without loss of optimality, we restrict attention to cases with non-zero served cross-traffic demand; in such cases, \( \lambda_{21} > 0 \) in light of Remark 1 and flow balance, so that \( s_{21} > 0 \).

Lemma 2 (Per-driver profit rate). Consider a driver who circulates with repositioning fractions \( \tilde{\eta} \in [0, 1] \) and \( \tilde{\eta}_2 = 0 \) through a network with offered loads \( s \) and queue lengths \( q \). If \( s_1 := s_{11} + s_{12} = 0 \) and \( \tilde{\eta}_1 < 1 \), the driver’s expected steady-state profit rate \( \pi(\tilde{\eta}_1; s, q) = 0 \). Otherwise,

\[
\tilde{\pi}(\tilde{\eta}_1; s, q) = \frac{(\tilde{\gamma}p - c)T^s(\tilde{\eta}_1; s) - cT^s(\tilde{\eta}_1)}{T^s(\tilde{\eta}_1; s) + T^r(\tilde{\eta}_1) + T^q(\tilde{\eta}_1; s, q)},
\]

where \( T^s(\tilde{\eta}_1; s) \), \( T^r(\tilde{\eta}_1) \), and \( T^q(\tilde{\eta}_1; s, q) \) are explicit functions (given in the proof) that denote the expected times that the driver spends in steady state serving riders, repositioning, and queueing, respectively, during a cycle between consecutive arrivals to the same location.

The equilibrium definition (1)–(2) is equivalent to the following definition in terms of \( (s, r, q) \).

Definition 1 (Repositioning equilibrium). A capacity allocation \( (s, r, q) \) admits a symmetric driver repositioning equilibrium with \( \eta_2(s, r) = 0 \) if and only if \( r_{21} = 0 \) and \( \eta(s, r) \) is a driver’s best response:

\[
\eta_1(s, r) = \frac{r_{12}}{s_{11} + s_{12} + r_{12}} \quad \text{and} \quad \eta_2(s, r) = \frac{r_{21}}{s_{21} + s_{22} + r_{21}} = 0,
\]

\[
\eta_1(s, r) \in \arg\max_{\tilde{\eta}_1} \tilde{\pi}(\tilde{\eta}_1; s, q).
\]
Proposition 2 establishes that condition (21) in Definition 1 implies a mapping from service and repositioning capacities to an explicitly defined set of driver-incentive compatible queue lengths.

**Proposition 2** (Driver-incentive compatible queue lengths). A service and repositioning capacity allocation \((s, r)\) admits a symmetric driver repositioning equilibrium with \(\eta_2(s, r) = 0\), if and only if \(r_{12} = \frac{s_1 s_2}{t_{21}} - s_{12}, r_{21} = 0\) and the queue lengths \(q\) are driver-incentive compatible, that is,

\[
q \in D(s) := \begin{cases} 
\{ q : q_1 \leq q_1^*(s) + k(s)q_2 \} & \text{if } \bar{s}_1 > 0 = r_{12}(s), \text{so } \eta_1(s, r) = 0; \\
\{ q : q_1 = q_1^*(s) + k(s)q_2 \} = \{ q : \bar{\pi}(0; s, q) \geq \bar{\pi}(1; s, q) \} & \text{if } \bar{s}_1, r_{12}(s) > 0, \text{so } \eta_1(s, r) \in (0, 1); \\
\{ q : q_1 = 0 \}, \text{and } \bar{\pi}(0; s, q) = 0 < \bar{\pi}(1; s, q), & \text{if } \bar{s}_1 = 0 < r_{12}(s), \text{so } \eta_1(s, r) = 1,
\end{cases}
\]

where \(q_1^*(s) > 0\) and \(k(s) > 0\) are explicit functions that are specified in (A.6).

Inducing drivers not to reposition from location 1, the first case in (22), requires a sufficiently short location-1 queue \((q_1 \leq q_1^*(s) + k(s)q_2)\), specifically, such that the pure strategy of never repositioning weakly dominates that of always repositioning \((\bar{\pi}(0; s, q) \geq \bar{\pi}(1; s, q))\).

Inducing drivers to reposition from location 1 requires one of two conditions: (i) If any location-1 demand is served \((s_1 > 0)\), the second case in (22), drivers reposition a fraction of the time (identified by (20)) if the queues in the two locations make them indifferent between queueing at and repositioning from location 1. (ii) If no location-1 demand is served \((s_1 = 0)\), the third case in (22), drivers prefer to always reposition from location 1, so \(q_1 = 0\).

Proposition 2 foreshadows the critical role of demand admission control in shaping drivers’ repositioning incentives through the set \(D(s)\), as discussed in detail in §4.3. In light of Proposition 2, (22) alone ensures for regimes M and A that a capacity allocation \((s, r, q)\) admits a symmetric driver repositioning equilibrium; (20) merely determines the corresponding repositioning fractions.

### 4.2 Minimal Control (M)

Under minimal control the platform exercises no admission control and drivers make repositioning and participation decisions. Problem M in (7) is equivalent to the capacity allocation problem (13) for regime \(X = M\). We solve (13) in two steps as outlined in §2.5. In Step 1, for fixed participating capacity, Proposition 3 presents the solution of (14), that is, \(\Pi_M(n) := \max_{s, r, q} \{ II(s) : (s, r, q) \in C_M(n) \}\). In Step 2, Corollary 2 shows that the resulting per-driver profit (12d) satisfies the optimality conditions of Lemma 1 and yields a unique equilibrium capacity that satisfies (12e).

We first simplify the set \(C_M(n) = \{(s, r, q) : (1) - (2), (4) - (6), (9) - (11), (12a) - (12c)\}\), defined
in §2.5. Using Proposition 2 to substitute (22) for (1)–(2), and (9)–(11) to translate (4)–(6) into

\[
\frac{s_{l1}}{S_{l1}} = \frac{s_{l2}}{S_{l2}}, \quad l = 1, 2, \tag{23}
\]

\[(S_{l1} + S_{l2} - s_{l1} - s_{l2})r_{lk} = 0, \quad l = 1, 2, k \neq l, \tag{24}\]

\[(S_{l1} + S_{l2} - s_{l1} - s_{l2})q_l = 0, \quad l = 1, 2, \tag{25}\]

we have

\[C_M(n) = \{(s, r, q) : (12a) - (12c), (22), (23) - (25)\}. \tag{26}\]

**Proposition 3** (Regime M: Unique feasible allocation of fixed driver capacity). Define

\[n_1^M := n_1^C - \left(1 - \frac{A_{12}}{A_{21}}\right) S_{22}, \quad n_2^M := n_1^M + q_1^* (S), \quad \text{and} \quad n_3^M := n_2^C + q_1^* (S), \tag{27}\]

where \(n_1^C\) and \(n_2^C\) are defined in (17) and \(n_1^M < n_1^C < S < n_2^C < n_3^M\).

In regime M, problem (14) has the following unique feasible allocation of the driver capacity, \(C_M(n)\):

1. **Scarc capacity** (\(n \leq n_1^M\)). All drivers serve riders: \(\bar{s} = n; \ r = 0; \ q = 0\).
2. **Moderate capacity**—no repositioning but queuing (\(n_1^M < n \leq n_2^M\)). Drivers serve all riders at the low- and a fraction \(\frac{A_{12}}{A_{21}}\) of riders at the high-demand location, or queue at the low-demand location: \(\bar{s} = n_1^M\) where \(s_{1k} = S_{1k}, \ s_{2k} = S_{2k} \frac{A_{12}}{A_{21}}\) for \(k = 1, 2; \ r = 0; \ q_1 = n - n_1^M < q_1^* (S), \ q_2 = 0\).
3. **Moderate capacity**—repositioning and queuing (\(n_2^M < n \leq n_3^M\)). Drivers serve all riders at the low- and more than a fraction \(\frac{A_{12}}{A_{21}}\) of riders at the high-demand location, reposition from the low- to the high-demand location, or queue at the low-demand location: \(\bar{s} > n_1^M\) where \(s_{1k} = S_{1k}\) for \(k = 1, 2; \ r_{12} > 0, \ r_{21} = 0; \ q_1 = q_1^* (S), \ q_2 = 0\).
4. **Ample capacity** (\(n > n_3^M\)). Drivers serve all riders, reposition from the low- to the high-demand location, or queue at both locations: \(\bar{s} = S; \ r_{12} = n_2^C - S, \ r_{21} = 0; \ q > 0\) and \(q_1 = q_1^* (S) + k(S)q_2\).

These capacity zones map to driver pool intervals (N) with respective *equilibrium* capacity allocations (cf. Remark 3 for regime C). Figure 4 (b) illustrates Proposition 3. Compared to Centralized Control (Proposition 1), Minimal Control reduces the driver utilization at moderate capacity (zones 2 and 3), i.e., both local and cross-location demand is lost at the high-demand location while drivers idle in queue at the low-demand location: (1) Because the platform cannot use admission control to prioritize local rides at the high-demand location, the maximum load it can serve without repositioning is lower than with admission control (regime C), i.e., \(n_1^M < n_1^C\). (2) Because repositioning is decentralized, drivers will reposition from the low-demand location only if the queueing delay there is sufficiently long, i.e., in zone 3 where \(q_1 = n_2^M - n_1^M\). Therefore, \(n_3^M\), the minimum capacity required to serve the total offered load, exceeds the corresponding capacity under centralized control,
Corollary 2 (Regime M: Driver participation equilibrium). Under no admission control and decentralized repositioning, (i) the per-driver profit $\pi_M(n)$ satisfies the conditions in Lemma 1 for optimality of the two-step approach, i.e., $\pi_M(n)$ is decreasing and maximizes the per-driver profit at every $n$; (ii) there exists a unique equilibrium capacity of participating drivers, $n^*_M = NF(\pi_M(n^*_M))$.

4.3 Admission Control (A)

4.3.1 Network Equilibrium

In regime $A$ the platform controls demand admission and drivers make repositioning and participation decisions. Problem $A$ in (8) is equivalent to the capacity allocation problem (13) for regime $X = A$. To solve (13) we modify the two steps outlined in §2.5 to accommodate the following, potentially optimal, admission control feature: strategic demand rejection at the low-demand location, to encourage drivers to reposition to the high-demand location. When strategic demand rejection is optimal at some fixed capacity, $n$, this may reduce the resulting per-driver profit, $\pi_A(n)$, which violates the optimality condition in Lemma 1. We accommodate this issue as follows.

In Step 1, Proposition 4 characterizes for fixed capacity, $n$, the optimal capacity allocation $(s, r, q)$ that solves (14) for regime $X = A$, that is, $\Pi_A(n) := \max_{s, r, q} \{H(s) : (s, r, q) \in C_A(n)\}$, where $C_A(n) = \{(s, r, q) : (12a) - (12c), (22)\}$ by the definition in §2.5 and Proposition 2.

In Step 2, Lemma 3 establishes (i) that any optimal equilibrium capacity $n^*_A \in [n_A, \hat{n}_A]$, where $n_A$ and $\hat{n}_A$ are, respectively, the unique equilibrium capacity levels under optimal admission control and the restricted optimal policy without strategic demand rejection, and (ii) a sufficient condition for optimality of strategic demand rejection and uniqueness of the optimal equilibrium capacity.

Proposition 4 (Regime $A$: Optimal allocation of fixed driver capacity). Define the constants

$$n^A_1 := n^C_1 \quad \text{and} \quad n^A_3 := n^C_2 + q^*_1(S),$$

where $n^C_1$ and $n^C_2$ are defined in (17) and $n^C_1 < \overline{S} < n^C_2$. In regime $A$, problem (14) yields the following optimal capacity allocation, where the threshold $n^A_2 \in (n^A_1, n^A_3)$ is implicitly defined:

1. **Scarce capacity** ($n \leq n^A_1$). All drivers serve riders: $\overline{s} = n$; $r = 0$; $q = 0$.

2. **Moderate capacity—no repositioning but queueing** ($n^A_1 < n \leq n^A_2$). Drivers serve all riders except a fraction $1 - \frac{q^*_1}{\Lambda_{22}}$ from the high- to the low-demand location, and queue at the low-demand location: $\overline{s} = n^A_1$; $r = 0$; $q_1 = n - n^A_1 < q^*_1(S)$, $q_2 = 0$.

3. **Moderate capacity—repositioning, with or without queueing** ($n^A_2 < n \leq n^A_3$). Compared to zone
2, drivers serve more riders at the high- but possibly fewer riders at the low-demand location, they reposition from the low- to the high-demand location, and may queue at the low-demand location: $\bar{s} > n_A^1; \ r_{12} > 0, \ r_{21} = 0; \ q_1 = q_1^*(s) \geq 0, \ q_2 = 0$.

(4) **Ample capacity** ($n > n_A^3$): Drivers serve all riders, reposition from the low- to the high-demand location, or queue at both locations: $\bar{s} = S; \ r_{12} = n_C^2 - S, \ r_{21} = 0; \ q_1 = q_1^*(S) + k(S)q_2, \ q_2 > 0$.

These capacity zones map to driver pool intervals ($N$) with respective *equilibrium* capacity allocations (cf. Remark 3 for regime C). Figure 4 (c) illustrates Proposition 4. Compared to Minimal Control (Proposition 3), Regime A improves driver utilization at moderate capacity (zones 2 and 3) in three ways: (1) With admission control, the platform can serve all local demand at the high-demand location without repositioning, like under Centralized Control, hence $n_A^1 = n_C^1 > n_M^1$. (2) More local demand admitted at the high-demand location makes this location a more profitable destination for drivers, in turn reducing the wasteful queueing at the low-demand location. (3) Most importantly, in zone 3 the platform may *reject rider requests at the low-demand location* even if it has available drivers, in order to make it less attractive for drivers to queue there and induce them instead to reposition to, and serve *more* riders at, the high-demand location.\(^3\) Note that under this policy, the demand served at the low-demand location decreases in the capacity, i.e., in zone 3 compared to zone 2. We call this policy *strategic demand rejection* as it serves to regulate the incentives of strategic drivers. We elaborate on the rationale and optimality conditions in §4.3.2.

Whereas strategic demand rejection may benefit the platform, it may reduce driver profits, as drivers spend less time queueing and more time repositioning, which is costly. In this case, the platform faces the following trade-off: Increase revenue through strategic demand rejection at the expense of lower driver participation due to reduced per-driver profits; or increase revenue with no (or less) strategic demand rejection, to boost per-driver profits and driver participation. In light of this trade-off, Lemma 3 provides bounds on the (optimal) equilibrium participating capacity $n_A^*$, and a sufficient condition for optimality of strategic demand rejection and uniqueness of $n_A^*$.

**Lemma 3** (Regime A: Driver participation equilibrium). Under optimal admission control and decentralized repositioning, the driver participation equilibrium has the following properties:

(i) Under the unrestricted optimal capacity allocation of Proposition 4, the per-driver profit $\pi_A(n)$ is decreasing and yields a unique equilibrium capacity of participating drivers, $n_A$, which solves

$$NF(\pi_A(n_A^+)) \leq n_A \leq NF(\pi_A(n_A)), \text{ where } n_A^+ = \lim_{\epsilon \downarrow 0} n_A + \epsilon.$$  

\(^3\)This property of the optimal control policy seems to be in contrast to the literature in ride-hailing networks.
(ii) For \( n \in (n^A_1, n^A_3] \), let the restricted optimal revenue without strategic demand rejection be
\[
\hat{\Pi}_A(n) := \max_{s,r,q} \{ \Pi(s) : (s,r,q) \in C_A(n), s_{11} = S_{11}, s_{12} = S_{12}, n \in (n^A_1, n^A_3] \},
\]
and \( \hat{\pi}_A(n) \) be the resulting per-driver profit. For \( n \in [0, n^A_1] \cup (n^A_3, N] \) let \( \hat{\pi}_A(n) = \pi_A(n) \). The per-driver profit \( \hat{\pi}_A(n) \) is decreasing, yields a unique equilibrium capacity, \( \hat{n}_A \), which solves
\[
\hat{n}_A = NF(\hat{\pi}_A(\hat{n}_A)),
\]
and maximizes the per-driver profit, i.e., satisfies optimality condition (16) in Lemma 1:
\[
\pi_A(n) \leq \hat{\pi}_A(n) = \max_{s,r,q} \{ \pi(s,r,n) : (s,r,q) \in C_A(n) \} \quad \text{for} \ n \in [0, N].
\]

(iii) We have \( n_A \leq \hat{n}_A \), the optimal equilibrium participating capacity \( n^*_A \in [n_A, \hat{n}_A] \), the per-driver profit \( \pi^*_A \) solves \( n^*_A = NF(\pi^*_A) \) with \( \pi_A(n_A) \leq \pi^*_A \leq \pi_A(\hat{n}_A) \), and \( \Pi^*_A \geq \max \{ \Pi_A(n_A), \hat{\Pi}_A(\hat{n}_A) \} \).

(iv) If \( \Pi_A(n) = \hat{\Pi}_A(n) \) for \( n \in (n^A_1, n^A_3] \), strategic demand rejection is suboptimal and \( n^*_A = n_A = \hat{n}_A \).

(v) If \( \Pi_A(n_A) > \hat{\Pi}_A(\hat{n}_A) \), strategic demand rejection is optimal and the optimal equilibrium capacity \( n^*_A < \hat{n}_A \). If in addition \( n_A < \hat{n}_A \), the optimal policy harms drivers: \( \pi_A(n^*_A) < \pi_A(\hat{n}_A) \).

The equilibrium condition (29) in Part (i) of Lemma 3 reflects that the per-driver profit \( \pi_A(n) \) is discontinuous if strategic demand rejection is optimal at some capacity levels. Whereas Part (i) characterizes the equilibrium capacity \( n_A \) under the unrestricted solution of Proposition 4, Part (ii) characterizes the “conditional” equilibrium capacity \( \hat{n}_A \) if strategic demand rejection is precluded. Part (iii) shows that the optimal equilibrium capacity \( n^*_A \) is “sandwiched” between these two capacity levels. Parts (ii) and (v) capture the trade-off inherent in strategic demand rejection: by Part (ii) it may reduce the per-driver profit \( \pi_A(n) \leq \hat{\pi}_A(n) \) by (32)), yet by Part (v) it may increase the platform revenue \( \Pi_A(n_A) > \hat{\Pi}_A(\hat{n}_A) \), possibly at lower equilibrium capacity \( n_A < \hat{n}_A \).

### 4.3.2 Optimal Strategic Demand Rejection to Induce Driver Repositioning

The strategic demand rejection under moderate capacity (zone 3) introduced above may at first seem counterintuitive, as the platform rejects some or all rider requests at the low-demand location (1), even though there is an excess supply of drivers. However, by sacrificing revenue at the low-demand location, the platform may incentivize drivers to reposition to, and generate more revenue at, the high-demand location. Specifically, rejecting rider requests at the low-demand location creates an artificial demand shortage that drivers offset by choosing to reposition more frequently to the high-demand location, rather than joining the queue at the low-demand location; the result
is a shorter queue there (the waiting time may increase or decrease). In terms of Proposition 2, rejecting demand at location 1 alters the driver-incentive compatible capacity allocation by reducing the queue-length threshold $q^*_1(s)$, which frees up driver capacity to reposition and serve riders at the high-demand location. By controlling congestion, the platform has an operational lever to incentivize drivers to reposition, as opposed to, for example, increasing their wage.

Next, we identify optimality conditions for strategic demand rejection in two steps: (i) at fixed participating capacity, which depends on the trade-off between revenue at the two locations; (ii) at the equilibrium capacity, which depends on the size of the driver pool and the trade-off between revenue-per-driver and the number of participating drivers.

**Optimality of Strategic Demand Rejection at Fixed Capacity Levels.** Proposition 5 identifies a necessary and sufficient condition in terms of the model primitives, for the optimality of strategic demand rejection at some fixed capacity levels in the moderate-capacity zone (3). To simplify notation and highlight the structural imbalances, define

$$
\rho_1 \ := \ \frac{S_{11}}{S_{11} + S_{12}}, \quad \rho_2 \ := \ \frac{S_{22}}{S_{21} + S_{22}}, \quad \tau \ := \ \frac{t_{21}}{t_{12}}, \quad \kappa \ := \ \frac{c}{\gamma p} < 1,
$$

where $\rho_1$ and $\rho_2$ are the shares of the local-demand offered load at location 1 and 2, respectively, $\tau$ is the ratio between cross-location travel times, and $\kappa$ is the ratio of driving cost to drivers’ service revenue (“relative driving cost”). Assumption 2 implies that $\kappa < \tau/(1 + \tau)$.

**Proposition 5** (Regime A: Optimality of strategic demand rejection at fixed capacity level). Under optimal admission control and decentralized repositioning, it is optimal at moderate capacity, i.e., for some $n \in (n^*_A, n^*_A)$, to strategically reject rider requests at the low-demand location so as to induce repositioning to the high-demand location, if and only if the following condition holds:

$$
\frac{A_{12}}{A_{21}} \frac{1 - \rho_1 \kappa}{1 - \rho_1} < \frac{\tau - (\tau + 1 - \rho_2)\kappa}{1 - \rho_2} \left( \frac{\kappa + \tau}{\tau} \frac{1 - \rho_2}{\rho_2} \right).
$$

Condition (34) is necessary and sufficient for strategic demand rejection to be optimal at some fixed capacity, i.e., $\Pi_A(n) > \tilde{\Pi}_A(n)$ for some $n$. However, optimality of strategic demand rejection at the equilibrium capacity, i.e., $\Pi_A(n^*_A) > \tilde{\Pi}_A(\hat{n}_A)$, the condition in Lemma 3 (v), requires additional conditions that we present in Proposition 6. First, consider the intuition for (34), assuming $\tau = 1$ for simplicity. Strategic demand rejection can only be optimal for some $n$, i.e., (34) holds, if:

*The share of the local-demand offered load at the high-demand location, $\rho_2$, is not too high.*

Under this condition, drivers have a weak incentive to reposition to the high-demand location as they are likely to get matched there to a rider going to the low-demand location. Therefore,
encouraging drivers to reposition requires rejecting demand at the low-demand location. Without
local demand at the high-demand location ($\rho_2 = 0$), condition (34) holds regardless of other factors.

The share of the local-demand offered load at the low-demand location, $\rho_1$, is not too high.\(^5\) Under this condition, drivers have a strong incentive to queue at the low-demand location as they are likely to be assigned a rider going to the high-demand location. Therefore, encouraging drivers to reposition to the high-demand location requires rejecting demand at the low-demand location.

The cross-location demand imbalance, $A_{12}/A_{21}$, is sufficiently large.\(^6\) More cross-location demand at the high-demand location increases the value of rejecting demand at the low-demand location in order to induce drivers to reposition to the high-demand location.

The relative driving cost, $\kappa$, is sufficiently large: When repositioning is expensive, drivers have no incentive to drive empty; therefore, the platform needs to strengthen their incentive to reposition to the high-demand location over queueing at the low-demand location by rejecting demand there.

**Optimality of Strategic Demand Rejection at Equilibrium Capacity.** As discussed above, for fixed driver participation level, strategic demand rejection may reduce drivers’ profits as they pay for repositioning but not for queueing. Therefore, even when (34) holds (so that strategic demand rejection is optimal at some fixed capacity levels) the platform may be able to increase revenue without rejecting location-1 demand. Proposition 6 identifies intuitive sufficient conditions for strategic demand rejection to be optimal in equilibrium, i.e., for Lemma 3 (v) to hold.

**Proposition 6** (Regime A: Sufficient optimality conditions for strategic demand rejection). There exists an interval $(\bar{N}, \bar{N})$ with $0 < \bar{N} < \bar{N} < \infty$ and a threshold function $\bar{p}_2(N) : (\bar{N}, \bar{N}) \rightarrow (0, \infty)$ such that strategic demand rejection is optimal in equilibrium if the driver pool size $N \in (\bar{N}, \bar{N})$ and the share of the local-demand offered load at the high-demand location $\rho_2 \in [0, \bar{p}_2(N))$.

Figure 3 illustrates Proposition 6 with a numerical example. Fixing $\rho_1 = 0.75$, $A_{21}/A_{12} = 4$, $\tau = 1$, $\kappa = 0.3$, the left panel shows the region in the $(N, \rho_2)$-parameter space, i.e., the combination of driver pool size and local-demand offered load share at the high-demand location, that yield optimal strategic demand rejection in equilibrium. The horizontal line $\rho_2 = 0.53$ indicates the maximum value of $\rho_2$ implied by condition (34) in Proposition 5 for optimality of strategic demand rejection at some capacity level. The right panel presents this optimality region and threshold line in a way that highlights their connection to the local-demand share at the high-demand location ($A_{22}/(A_{21} + A_{22})$).

---

\(^5\)The left-hand side (LHS) of (34) increases in $\rho_1$ from $A_{12}/A_{21}$ for $\rho_1 = 0$ to $\infty$ as $\rho_1 \rightarrow 1$, so that condition (34) holds if both local-demand shares, $\rho_1$ and $\rho_2$, are below some threshold.

\(^6\)The LHS of (34) is positive and decreases to zero as $A_{12}$ increases from $A_{12}$ to $\infty$ ($A_{12}/A_{21} < 1$ by Assumption 1). Therefore, (34) holds for sufficiently large $A_{21}$, provided the RHS is positive, i.e., $\rho_2$ is below some threshold.
Figure 3: Proposition 6: Optimal strategic demand rejection in equilibrium ($\rho_1 = 0.75, A_{21}/A_{12} = 4, \tau = 1, \gamma = 0.25, \kappa = 0.3, F \sim U(0, p - c = 2.55)$)

assuming that local trips originating at the high-demand location last about 20% of cross-location trips (i.e., $t_{22}/t_{21} = 0.2$). This panel shows that strategic demand rejection is optimal even when the local-demand share at the high-demand location is relatively high, up to 60% of total demand.

4.4 Graphical Summary of Capacity Allocation under Regimes C, M, and A

Figure 4 visualizes for the regimes C, M, and A the optimal capacity allocations specified in Propositions 1, 3 and 4, respectively. To make these graphs comparable we show these allocations as a function of the same equilibrium capacity ($n^*$ on the horizontal axes), though we note that the equilibrium capacities typically differ across regimes (e.g., for moderate driver pool $N$, more control increases the equilibrium capacity: $n^*_M < n^*_A < n^*_C$, Proposition 7). (i) For scarce capacity (Zone 1), all drivers are busy serving riders in all regimes. (ii) For ample capacity (Zones 3 or 4), all riders are served, and all regimes agree again. But, importantly, (iii) in regime C the platform can serve all the demand with less capacity and no queueing; in contrast, regimes M and A require the buildup of queues (Zone 2) to induce driver repositioning. (iv) Admission control (A) allows the platform to increase driver utilization vs. M, by prioritizing demand at the high-demand location based on destination, and also by rejecting demand at the low-demand location to boost repositioning.

Furthermore, for each regime, the model primitives have the following effects (ceteris paribus): The equilibrium driver participation increases in the driver pool ($N$), the total offered load ($S$) (for fixed route ratios), and in the revenue per unit time ($\tilde{\gamma}p$), whereas it decreases in the driving cost.
5 The Impact of Platform Controls on System Performance

In this section we study how platform controls affect equilibrium performance. In §5.1 we rank the key metrics; in §5.2 we provide upper bounds on the platform’s and the drivers’ gains from control.

5.1 Ranking of Platform Revenue, Per-Driver Profit, and Driver Capacity

Proposition 7 shows that more control always benefits the platform but may hurt drivers.

**Proposition 7** (Ranking of equilibrium profits and capacity). Define the driver pool thresholds

\[ N_1^M := n_1^M / F(\gamma_p - c), \quad N_1^A := n_1^A / F(\gamma_p - c), \quad N_3 := n_3^A / F(\pi_A(n_3^A)) = n_3^M / F(\pi_M(n_3^M)), \]

where \( n_1^M, n_3^M \) are defined in (27), \( n_1^A, n_3^A \) are defined in (28), and \( N_1^M < N_1^A < N_3 \).

1. More platform control increases the equilibrium platform revenue: \( \Pi^*_M \leq \Pi^*_A \leq \Pi^*_C \), where
   \[ \Pi^*_M < \Pi^*_A \iff N \in (N_1^M, N_3) \quad \text{and} \quad S_{22} > 0, \]
   \[ \Pi^*_A < \Pi^*_C \iff N \in (N_1^A, N_3). \]

2. Centralized control maximizes driver participation and per-driver profit rate: \( \max\{n_M^*, n_A^*\} \leq n_C^* \) and \( \max\{\pi_M^*, \pi_A^*\} \leq \pi_C^* \), where the inequalities are strict iff \( N \in (N_1^A, N_3) \).

3. With decentralized repositioning, admission control affects driver capacity and profits as follows:
   (a) No change \( n_A^* = n_M^*, \pi_A^* = \pi_M^* \), if the driver pool is scarce \( N \leq N_1^M \) or ample \( N \geq N_3 \).
(b) Increase \((n^*_A > n^*_M, \pi^*_A > \pi^*_M)\), if the driver pool is moderate \((N \in (N^1_M, N_3))\) and strategic demand rejection is suboptimal.

(c) Decrease \((n^*_A < n^*_M, \pi^*_A < \pi^*_M)\), if the driver pool is moderate \((N \in (N, N') \subset (N^1_A, N_3))\), strategic demand rejection is optimal, and the share of the local-demand offered load at the high-demand location is below some threshold \((\rho_2 \in [0, \bar{\rho}_2(N)), \ 0 < \bar{\rho}_2(N) < \infty)\).

Three points emerge from Proposition 7. First, platform control improves performance only if the driver pool is moderate \((N \in (N^1_M, N_3))\). Otherwise, all regimes yield full driver utilization if the pool is scarce \((N \leq N^1_M)\) or serve all demand if the pool ample \((N \geq N_3)\).

Second, for moderate driver pool, more platform control \((M \rightarrow A \rightarrow C)\) generally improves the platform revenue (Part 1) and the driver participation and per-driver profit (Parts 2 and 3a).

Third, drivers may be hurt under decentralized repositioning, in that admission control reduces their participation and profits (Part 3c) when strategic demand rejection is optimal and stronger conditions hold than those in Proposition 6. Specifically, the conditions on the driver pool \((N' < N)\) and the local demand at the high-demand location \((\bar{\rho}_2(N) < \bar{\rho}_2(N))\) imply that both the availability and the value of additional capacity are so low that the platform prefers to boost revenue through strategic demand rejection, even at the expense of restricting driver participation.

5.2 Upper Bounds on the Gains in Platform Revenue and Per-Driver Profit

Next we provide upper bounds on the gains from control for the platform and drivers.

**Proposition 8** (Upper bounds on platform revenue gains). Fix \(N \geq n^3_M = n^3_A\).

1. **Platform revenue gain due to admission control (regime A over M):** If (34) is not satisfied,

   \[
   \max_{F(\cdot)} \frac{\Pi^*_A - \Pi^*_M}{\Pi^*_M} \leq \frac{S}{n^1_M} - 1 = \left( \frac{A_{21}}{A_{12}} - 1 \right) \frac{1}{1 + \frac{1}{1 - \rho_2} \frac{1}{1 - \rho_1}}. \tag{35}
   \]

2. **Platform revenue gain due to centralized repositioning control (regime C over A):**

   \[
   \max_{F(\cdot)} \frac{\Pi^*_C - \Pi^*_A}{\Pi^*_A} \leq \frac{S}{n^1_A} - 1 = \left( \frac{A_{21}}{A_{12}} - 1 \right) \frac{1}{1 + \frac{1}{1 - \rho_2} \frac{1}{1 - \rho_1}} + \frac{\rho_2}{1 - \rho_2} \frac{A_{21}}{A_{12}}. \tag{36}
   \]

These bounds can be approached arbitrarily closely for specific choices of the opportunity cost distribution \(F(\cdot)\), see Supplemental Material S3. The condition \(N \geq n^3_M = n^3_A\) requires a sufficiently large driver pool to serve all riders under decentralized repositioning. The upper bound on the gain from admission control (the RHS in (35)) is attained if, under minimal control only the demand that does not require repositioning is served, and admission control increases driver participation enough to serve all demand. The upper bound for repositioning control in (36) has a similar interpretation.
The key insight from (35) and (36) is that the potential revenue gains increase in the cross-location demand imbalance ratio. Table 1 highlights that these gains are very substantial at imbalance ratios such as 2 and 5 that are practically very common, e.g., see Figure 1 for Manhattan.

<table>
<thead>
<tr>
<th>cross demand imbalance $\frac{\Lambda_{21}}{\Lambda_{12}}$</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>from admission control (35)</td>
<td>0%</td>
<td>43%</td>
<td>150%</td>
<td>319%</td>
</tr>
<tr>
<td>from central. repositioning (36)</td>
<td>0%</td>
<td>25%</td>
<td>100%</td>
<td>225%</td>
</tr>
</tbody>
</table>

(a) Balanced cross-local demand at low-demand location ($\rho_1 = 0.5$)

<table>
<thead>
<tr>
<th>cross demand imbalance $\frac{\Lambda_{21}}{\Lambda_{12}}$</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>from admission control (35)</td>
<td>0%</td>
<td>53%</td>
<td>189%</td>
<td>407%</td>
</tr>
<tr>
<td>from central. repositioning (36)</td>
<td>0%</td>
<td>30%</td>
<td>120%</td>
<td>270%</td>
</tr>
</tbody>
</table>

(b) Imbalanced cross-local demand at low-demand location ($\rho_1 = 0.25$)

Table 1: Upper bounds in (35) and (36) on platform revenue gain ($t_{lk} = 1, \forall lk, A_{12} = A_{22} = 1$)

**Proposition 9** (Upper bound on per-driver profit gains). Fix $N \geq n_3^M = n_3^A$ and assume that (34) is not satisfied. The per-driver profit gain from admission control (under regime A or C) satisfies:

$$\max_{F(\cdot)} \frac{\pi_A^* - \pi_M^*}{\pi_M^*} = \max_{F(\cdot)} \frac{\pi_C^* - \pi_M^*}{\pi_M^*} \leq \frac{1 - \rho_2}{\tau - (1 - \rho_2 + \tau)\kappa}. \tag{37}$$

Whereas the bounds on the platform gains in Proposition 8 can only be attained when more control yields repositioning, attaining the bound on the per-driver profit gain in (37) requires the absence of repositioning. This contrast points to the following key tension between the drivers’ and the platform’s gains from control: If a small change in the per-driver profit increases the number of participating drivers significantly, then the platform may extract significant gains while drivers are only marginally better off; conversely, if a large change in the per-driver profit only yields a small increase in their number, then drivers extract significant gains while the platform does not.

6 Robustness of Results for Multi-Location Networks

In this section, we present numerical results for three-location ring and four-location star networks. These suggest our analytical results for two-location networks are robust and reveal how they generalize to multi-location networks. We illustrate the key points with selected examples and relegate the mathematical formulations and further numerical results to Supplemental Material S2.

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7 Supplemental Material S3 illustrates this tension and how it depends on the opportunity cost distribution $F(\cdot)$. 

26
To make our discussion precise, define the net flow of location (node) \(i\) as the difference between its total potential demand inflows and outflows, i.e., \(\text{NetFlow}_i = \sum_k A_{ki} - A_{ik}\). We call location \(i\) an inflow node if \(\text{NetFlow}_i > 0\), an outflow node if \(\text{NetFlow}_i < 0\), or a balanced node if \(\text{NetFlow}_i = 0\).

**Three-location ring networks.** Figure 5 shows the four possible types of three-location ring networks; these types differ in the net flow configuration of their nodes.

![Figure 5: Four types of three-location ring networks.](image)

(The numbers indicate the potential demand rates of arcs and the net flows of nodes.)

We illustrate the key points with representative results\(^8\) for network type I. We focus on the optimal capacity allocation for fixed capacity \(n\), as the equilibrium capacity depends on the driver opportunity cost distribution \(F\) and pool size \(N\). For network I the offered load \(\sum l_k S_{lk} = 12\). Balanced demand accounts for 9 units, and total excess demand for 3 units that go to inflow node 1 from outflow nodes 2 (2 units) and 3 (1 unit). Table 2 shows the capacity allocation for Admission Control (A), the most interesting regime (see Supplemental Material S2.4 for Minimal Control), for \(n \geq 10\), where driver utilization drops below 100%.

1. Compared to Centralized Control, decentralized repositioning reduces the capacity utilization at moderate capacity as it requires wasteful queueing to motivate drivers to reposition; furthermore, optimal admission control mitigates these losses vs. FIFO admission. Under Centralized Control the minimum capacity to serve the offered load \(\sum l_k S_{lk} = 12\) is \(n^C_2 = 15\) (80% service utilization, 20% for repositioning) but much larger in Regimes A and M, specifically \(n^A_3 = n^M_3 = 24\) (50% utilization, 12.5% for repositioning and 37.5% for queueing). Comparing regimes A and M, a capacity of \(n = 12\) serves 9 demand units in regime A (Table 2) but fewer than 7 units in regime M (Table 3 in S2), corresponding to service utilizations of 75% vs. 56%, respectively.

2. Under decentralized repositioning, drivers only reposition from inflow nodes (here, node 1) to

\(^8\)Types II and III yield similar results, see Supplemental Material S2. We omit type IV as it is balanced.
outflow nodes (here, nodes 2 and 3) and the buildup of driver queues obeys the following pattern. As the capacity $n$ increases, driver queues appear at nodes in decreasing order of their net flows:

(i) Queues first form at **inflow nodes** to induce repositioning to outflow nodes: for $n \in [10, 20]$, a queue forms only at the inflow node 1, where for $n > 15$ it is long enough to induce repositioning.

(ii) Queues then form at **lower-imbalance outflow nodes**, to incentivize further repositioning to outflow nodes with higher imbalance: For $n \in [21, 24]$, the queue ($q_3 > 0$) at the low-imbalance outflow node 3 reduces this node’s attractiveness as a repositioning destination, so drivers at the inflow node 1 are encouraged to reposition to the other (high-imbalance) outflow node 2.

(iii) Queues finally also form at the **higher-imbalance outflow node** (2) at ample capacity ($n > 24$).

3. **Strategic demand rejection may be optimal at inflow nodes**: For $n \in [18, 19]$, some demand is rejected at inflow node 1 to boost repositioning to outflow nodes; this shrinks the node-1 queue.

![Figure 6: A star network and its optimal capacity allocation under Admission Control (A)](image-url)
**Star networks.** These observations also generalize to star networks where the hub is the only inflow node. Figure 6 shows an example: the hub node (1) has a net inflow of 6 and the spoke nodes (2, 3 and 4) have net outflows of 1, 2 and 3, respectively. As the capacity $n$ increases, we observe similar patterns as in ring network I: queues appear at nodes in decreasing order of their net flow ($1 \rightarrow 2 \rightarrow 3 \rightarrow 4$) while repositioning between the inflow hub node and the outflow spoke nodes helps serve all demand in the end. We also find strategic demand rejection at the inflow hub node under some intermediate values of $n$ ($s_{11} < S_{11}$ as red colored in the table).

7 Discussion and Concluding Remarks

We study the performance impact of operational platform controls for ride-hailing networks with strategic drivers under significant demand imbalances. Our equilibrium analysis of a stationary fluid model yields the following key results: (i) Decentralized repositioning leads to inefficient capacity allocation as a result of excessive driver idling at low-demand locations. (ii) Admission control significantly reduces these inefficiencies. (iii) Most notably, we identify a novel role for admission control: as a tool to influence strategic drivers’ repositioning decisions via demand rejection at low-demand locations. The practical implication is that admission control must also consider this effect on the distribution of empty cars, not only its immediate effect on busy cars. (iv) We provide upper bounds on the platform’s and drivers’ benefits due to increased control. These bounds show that these benefits can be very significant and point to tension between platform and driver gains.

An important direction of future research is to study the interplay of financial and operational controls. The following questions regarding variability and information are also important.

**Steady-state fluid model.** Ride-hailing services face two types of demand variability: (i) non-stationary average demand rates (e.g., per hour) that reflect significant time-of-day patterns, and (ii) stochastic fluctuations around these time-varying rates. Our model simplifies this setting in two ways, (i) by focusing on a “stationary time slice” during which the demand rates are (approximately) constant, and (ii) by ignoring the stochastic fluctuations. Though we make these simplifications for the sake of analytical tractability, we think the resulting steady-state fluid model provides a reasonable approximation, given the following two key features of demand in operational ride-hailing networks. First, the demand (and supply) rates are large, certainly in major metropolitan areas during rush hour, relative to the effects of stochastic fluctuations. This provides informal support for approximating the stochastic discrete model by a deterministic fluid model. Second, whereas intraday variation in demand rates can be very significant, the duration of different intraday
demand regimes is long (e.g., a couple of hours) when compared with the typical transportation
time (e.g. 10-15 minutes). This suggests that within each demand regime (e.g., morning rush hour
vs midday vs. evening rush hour), there is enough time for transients to settle down and the system
to reach steady state, or at least that using the steady state may be a reasonable approximation.

Though our steady-state fluid model ignores demand variability, it is useful, because it provides
a reasonable approximation and yields an analytically tractable formulation that generates impor-
tant and robust structural results. Specifically, our model allows us to characterize key aspects
of strategic drivers’ equilibrium behavior and the equilibrium capacity allocation. Whereas this
behavior would be intractable under time-varying and stochastic demand, we think in such settings
our key insights would continue to hold, notably, (i) decentralized repositioning leads to inefficient
capacity allocation due to excessive driver idling, (ii) admission control can significantly mitigate
these inefficiencies, and (iii) how and why admission control may involve strategic demand rejection.
Specifically, the key driver of these results is the prevalence of substantial demand imbalances, and
empirical data show that such demand imbalances are prevalent in urban traffic. Therefore, we
think our key insights would be sufficiently robust under time-varying and/or stochastic demand.

Variability and information design. In our stationary fluid model, the equilibrium system
state is constant over time. Therefore, information is irrelevant as a control lever: drivers must
simply be informed about (or correctly anticipate) the constant equilibrium values of the key
variables that affect their profits, i.e., the queueing delays and the demand mix at each location.

An interesting direction for future research is to study the role and value of information design
when the equilibrium system state fluctuates due to stochastic and/or time-varying demand.
Under stochastic stationary demand the platform’s information design problem is to decide which
queue-length information (if any) to share with drivers. The design where drivers do not observe
the idle-car queues is close to our model; namely, in equilibrium drivers make their repositioning
decisions upon arrival to each location, these decisions depend on the steady-state average queue
lengths at both locations, and repositioning in equilibrium typically involves mixed strategies and
the queue lengths equal some indifference thresholds. The case where drivers observe the real-
time queue lengths at both locations gives rise to a dynamic network game with competing long-
lived and forward-looking strategic agents. The analysis of this game is challenging for several
reasons: (i) drivers can make state-dependent decisions; (ii) the relevant system state depends on
all drivers’ strategies and is multi-dimensional (queue lengths at each location, plus number of cars
traveling on each route and/or vector of their arrival times at destination); (iii) forecasting their
expected queue position upon their next tip completion is difficult for drivers (because it depends
on the multi-dimensional state and its evolution is subject to demand uncertainty); and (iv) fully forward-looking drivers need to optimize beyond their next trip completion. In sum, it seems imperative to simplify this problem, e.g., by restricting drivers’ strategy space and/or simplifying their information processing so that they act somewhat myopically, e.g., by maximizing their payoff until the next trip completion. Under *non-stationary* demand the problem is even more intricate. For example, in addition to the aforementioned challenges, drivers now also need to forecast the effects of changing demand rates on the queue length they can expect at the other location.

**References**


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Appendix. Proofs

Proof of Lemma 1. For regime $X$, let $(s_X(n^*_X), r_X(n^*_X), q_X(n^*_X))$ denote an optimal solution of (14) at the equilibrium participating capacity $n^*_X$ obtained from (15).

Clearly, $(s_X(n^*_X), r_X(n^*_X), q_X(n^*_X), n^*_X)$ is a feasible solution of (13), with objective value $\Pi_X(n^*_X)$. Note that better solutions to (13), if any, can only be achieved at $n > n^*_X$ since $\Pi_X(n)$ increases in $n$ (as will be shown later in each regime $X$). However, any such solution $(s, r, q, n) \in \mathcal{C}_X$ at $n > n^*_X$ does not satisfy constraint (12e) because

$$\pi(s, r, n) \leq \pi_X(n) \leq \pi_X(n^*_X) = F^{-1}\left(\frac{n^*_X}{N}\right) < F^{-1}\left(\frac{n}{N}\right),$$

where the first inequality follows from (16), the second inequality is due to $\pi_X(n)$ decreasing in $n$ (as will be shown later in each regime $X$), and the last inequality follows from the assumption that $F()$ is continuously increasing on $[0, \infty)$. Therefore, $(s(n^*_X), r(n^*_X), q(n^*_X), n^*_X)$ is also the optimal solution to (13) and we have $\Pi^*_X = \Pi_X(n^*_X)$. \hfill \Box

Proof of Proposition 1. We start with the following observations about the optimal solution.

(i) Allocating all capacity $n$ towards serving riders (i.e., $r = 0, q = 0$) is feasible (hence optimal) if and only if $n \leq n^C_1$. To see this, let

$$S^C_1 = (S_{11}, S_{12}, S_{21}, S_{22}).$$

Then $n^C_1 = S^C_1 \cdot 1 = S - (A_{21} - A_{12})t_{21}$ is the maximum service capacity without repositioning ($r = 0$). Therefore with $r = 0$ and $q = 0$, $\pi \leq n^C_1 \Leftrightarrow n \leq n^C_1$ by (12b); if $n > n^C_1$, then $r = 0, q = 0$ is not feasible.

(ii) Allowing for repositioning capacity $r_{12} \geq 0$, the maximum service capacity achievable (assuming $n$ is sufficiently large) is

$$\begin{cases} n^C_1 + r_{12} \frac{t_{21}}{t_{12}}, & \text{with } r_{21} = 0 \quad \text{if } r_{12} \in [0, n^C_1 - S] \\ S, & \text{with } r_{21} = (S_{12} + r_{12}) \frac{t_{21}}{t_{12}} - S_{21} > 0 \quad \text{if } r_{12} > n^C_1 - S \end{cases}$$

To see this, from (12a) we have $s_{21} = (s_{12} + r_{12}) \frac{t_{21}}{t_{12}} - r_{21}$, hence for $r_{12} \in [0, n^C_1 - S]$,

$$\max_{0 \leq s \leq S, r_{21} \geq 0} \pi = \max_{0 \leq s \leq S, r_{21} \geq 0} \left\{ s_{11} + s_{12} + (s_{12} + r_{12}) \frac{t_{21}}{t_{12}} - r_{21} + s_{22} = n^C_1 + r_{12} \frac{t_{21}}{t_{12}} \right\}$$

where the maximum is achieved at $s_{11} = S_{11}, s_{12} = S_{12}, s_{22} = S_{22}, r_{21} = 0$. Note that the service capacity reaches its upper bound $S$ when $r_{12}$ reaches $n^C_1 - S$, i.e., $n^C_1 + (n^C_1 - S) \frac{t_{21}}{t_{12}} = S$. For $r_{12} > n^C_1 - S$, the maximum service capacity stays at $S$ with $s = S$, but $r_{21} = (S_{12} + r_{12}) \frac{t_{21}}{t_{12}} - S_{21} > 0$ by (12a).

With these observations, we can derive the optimal structure given by the Proposition. Zone (1) follows directly from observation (i). In zone (2) and (3) where $n > n^C_1$, by observation (ii), the optimization problem with least repositioning travel cost (i.e., avoiding unnecessary repositioning capacity) can be simplified as

$$\max_{r_{12}} \left\{ n^C_1 + r_{12} \frac{t_{21}}{t_{12}} \quad \text{if } r_{12} \leq n, \quad r_{12} \in [0, n^C_1 - S], \quad r_{21} = 0 \right\}.$$  

When $n^C_1 < n \leq n^C_2$ (zone (2)), the inequality constraint is binding and the optimal solution is

$$r_{12} = (n - n^C_1) \frac{t_{12}}{t_{12} + t_{21}}, \quad r_{21} = 0, \quad s = S^C_1 + \left(0, 0, (n - n^C_1) \frac{t_{21}}{t_{12} + t_{21}}\right), \quad q = 0.$$

For $n > n^C_2$ (zone (3)), the inequality constraint is not binding. With all the demand served ($s = S$), the extra
capacity waits in queues and the optimal solution is
\[ r_{12} = n_2^C - S, \quad r_{21} = 0, \quad s = S, \quad q \in \{ (q_1, q_2) : q_1 + q_2 = n - n_2^C \}. \]

**Proof of Corollary 1.** (i) The optimality condition (16) in Lemma 1 requires that the per-driver profit is maximized subject to (12a)–(12c) at any \( n > 0 \) under the platform’s optimal capacity allocation prescribed by Proposition 1. Note that
\[
\pi(s, r, n) = \frac{(\gamma p - c)\bar{s} - c\bar{r}}{n}.
\]
By Proposition 1, \( \bar{s} = n, r = 0 \) for \( n \) in the scarce capacity zone \( (0, n_1^C] \), clearly \( \pi(s, r, n) \) is maximized; for \( n \) in the ample capacity zone \( (n_2^C, \infty) \), \( \bar{s} = S \) and \( r = (n_2^C - S, 0) \) involves the minimum repositioning capacity \( \bar{r} = n_2^C - S \), hence \( \pi(s, r, n) \) is also maximized.

For fixed \( n \) in the moderate capacity zone \( (n_1^C, n_2^C] \), further increasing \( \pi(s, r, n) \) requires
\[
(\gamma p - c)\Delta s - c\Delta r > 0 \quad \Rightarrow \quad \Delta \bar{r} < \frac{\gamma p - c}{c}\Delta \bar{s}.
\]
Since \( \Delta \bar{s} \) is maximized under the platform’s optimal capacity allocation, it can only be decreased or remain unchanged, i.e., \( \Delta \bar{s} \leq 0 \). Hence
\[
\Delta \bar{r} < \frac{\gamma p - c}{c}\Delta \bar{s} \leq \frac{t_{12}}{t_{21}} \Delta \bar{s} \leq 0,
\]
where the second inequality follows from Assumption 2. We next show that (A.2) cannot hold due to the platform’s optimal capacity allocation and the flow balance constraint:

- By Proposition 1, the platform’s optimal capacity allocation in the moderate capacity zone \( (n_1^C, n_2^C] \) has \( s_{11} = S_{11}, s_{12} = S_{12}, s_{22} = S_{22} \) and \( r_{12} > 0, r_{21} = 0 \), hence \( s_{11}, s_{12}, s_{22} \) cannot be increased while \( r_{21} \) cannot be reduced. It then follows from \( \Delta r_{12} + \Delta r_{21} = \Delta \bar{r} < 0 \) by (A.2) that \( r_{12} \) must be reduced. To conclude, any change of these capacity variables must satisfy
\[
\Delta s_{11}, \Delta s_{12}, \Delta s_{22} \leq 0, \quad \Delta r_{12} < 0, \quad \Delta r_{21} \geq 0.
\]

- By flow balance constraint (12a), i.e., \( (s_{12} + r_{12})/t_{12} = (s_{21} + r_{21})/t_{21} \), its change satisfies
\[
\frac{\Delta s_{12} + \Delta r_{12}}{t_{12}} = \frac{\Delta s_{21} + \Delta r_{21}}{t_{21}},
\]
which implies
\[
\Delta s_{21} = \frac{t_{21}}{t_{12}} (\Delta s_{12} + \Delta r_{12}) - \Delta r_{21} \leq \frac{t_{21}}{t_{12}} (\Delta r_{12} + \Delta r_{21}) - \left( 1 + \frac{t_{21}}{t_{12}} \right) \Delta r_{21} \leq \frac{t_{21}}{t_{12}} \Delta \bar{r},
\]
where the two inequalities follow from \( \Delta s_{12} \leq 0 \) and \( \Delta r_{21} \geq 0 \) in (A.3), respectively.

By (A.3) and (A.4), we have
\[
\Delta \bar{r} = \sum \Delta s_{ij} \leq \Delta s_{21} \leq \frac{t_{21}}{t_{12}} \Delta \bar{s},
\]
which is a clear contradiction with \( \Delta \bar{s} > \frac{t_{21}}{t_{12}} \Delta \bar{r} \) from (A.2). Therefore \( \pi(s, r, n) \) is indeed maximized subject to (12a)–(12c) and the optimality condition holds.

(ii) The existence and uniqueness of the participation equilibrium follows directly from the fact that \( \pi_C(n) \) is continuously decreasing in \( n \) with zero limit value as \( n \to \infty \).
Proof of Lemma 2. If \( \bar{s}_1 = s_1 + s_{12} = 0 \) and \( \bar{\gamma}_1 < 1 \), then any driver who chooses not to not reposition at location 1 will stay there forever. In steady state, all drivers queue at location 1, \( W_1 = \infty \), \( T^v(\bar{\gamma}_1; s, q) = \infty \), and hence \( \bar{\pi}(\bar{\gamma}_1; s, q) = 0 \). Otherwise, we are not in this degenerate case and by Remark 2, \( s_{21} > 0 \), so that the two locations are commuting and drivers’ expected steady-state profit rate follows from the Renewal Reward Theory. Without loss of generality, we calculate the time functions over cycles starting and ending at the low-demand location (1). Let \( p_{1k} = \lambda_{1k}/(\lambda_{11} + \lambda_{12}) \) denote the probability of serving a \( 1k \)-ride at location \( l \). The expected service, repositioning and queueing time functions are as follows:

- The expected service time in a cycle is given by
  \[
  T^s(\bar{\gamma}_1; s) = (1 - \bar{\gamma}_1) \left[ p_{11} t_{11} + p_{12} \left( t_{12} + \frac{1 - p_{21}}{p_{21}} t_{22} + t_{21} \right) \right] + \bar{\gamma}_1 \left( \frac{1 - p_{21}}{p_{21}} t_{22} + t_{21} \right),
  \]
  where \( \frac{1 - p_{21}}{p_{21}} t_{22} \) gives the expected time serving local demand at location 2. This follows from the fact that the number of local rides at location 2 a driver serves (“failures”) before picking a ride back to location 1 (“success”) follows a geometric distribution of “success” probability \( p_{21} \).

- The expected repositioning time in a cycle is simply \( T^r(\bar{\gamma}_1) = \bar{\gamma}_1 t_{12} \).

- The expected queueing delay in a cycle is given by
  \[
  T^q(\bar{\gamma}_1; s, q) = (1 - \bar{\gamma}_1) \left[ W_1 + p_{12} \frac{1}{p_{21}} W_2 \right] + \bar{\gamma}_1 \frac{1}{p_{21}} W_2,
  \]
  where \( \frac{1}{p_{21}} W_2 \) gives the expected queueing time at location 2. This follows from the fact that the number of queueing delays (“trials”) at location 2 a driver encounters before picking a ride back to location 1 (“success”) follows a geometric distribution of “success” probability \( p_{21} \). \( W_1 = Q_1/(\lambda_{11} + \lambda_{12}) \) is the queueing time at location \( l \) due to Little’s Law.

Substituting the above time functions in (19) we have:

\[
\bar{\pi}(\bar{\gamma}_1; s, q) = \frac{(\gamma p - c) \left[ (1 - \bar{\gamma}_1) s_{21} \frac{1 + \lambda_{21}}{\lambda_{21}} (s_{11} + s_{12}) + \bar{\gamma}_1 s_{11} \frac{1 + \lambda_{11}}{\lambda_{11}} + s_{12} (s_{21} + s_{22}) \right] - c \bar{\gamma}_1 s_{21} \frac{1 + \lambda_{21}}{\lambda_{21}} (s_{11} + s_{12})}{(1 - \bar{\gamma}_1) s_{12} \frac{1 + \lambda_{12}}{\lambda_{12}} (s_{11} + s_{12} + q_1) + (\bar{\gamma}_1 s_{11} \frac{1 + \lambda_{11}}{\lambda_{11}} + s_{12} (s_{21} + s_{22} + q_2) + \bar{\gamma}_1 s_{21} \frac{1 + \lambda_{21}}{\lambda_{21}} (s_{11} + s_{12})}. \tag{A.5}
\]

Proof of Proposition 2. First, note that with \( \eta_2(s, r) = 0 \), \( r_{21} = 0 \) by (20), and \( r_{12}(s) \) follows from the flow balance constraint (12a). The driver-incentive compatible capacity allocation must satisfy (21). Differentiating \( \bar{\pi}(\bar{\gamma}_1; s, q) \) given in (A.5) wrt \( \bar{\gamma}_1 \), we get

\[
\frac{\partial \bar{\pi}}{\partial \bar{\gamma}_1} = \frac{s_{21} \frac{1 + \lambda_{21}}{\lambda_{21}} (s_{11} + s_{12}) \left[ (s_{21} + s_{22}) \gamma p - (s_{21} + s_{22} + s_{21} \frac{1 + \lambda_{21}}{\lambda_{21}}) c \right]}{[1 - \bar{\gamma}_1] s_{21} + s_{12} \frac{1 + \lambda_{21}}{\lambda_{21}} + s_{12} (s_{21} + s_{22} + q_1) + (\bar{\gamma}_1 s_{11} \frac{1 + \lambda_{11}}{\lambda_{11}} + s_{12} (s_{21} + s_{22} + q_2) + \bar{\gamma}_1 s_{21} \frac{1 + \lambda_{21}}{\lambda_{21}} (s_{11} + s_{12})]^2} \times [q_1 - q_1^*(s) + k(s) q_2],
\]

where

\[
q_1^*(s) = \frac{(s_{11} + s_{12}) s_{21} \frac{1 + \lambda_{21}}{\lambda_{21}} + (s_{21} + s_{22}) s_{12} (s_{21} + s_{22} - (s_{21} + s_{22} + s_{21} \frac{1 + \lambda_{21}}{\lambda_{21}}) / \gamma p}, k(s) = \frac{(s_{11} + s_{12}) - s_{11} \frac{1 + \lambda_{11}}{\lambda_{11}} + s_{12}}{(s_{21} + s_{22}) - (s_{21} + s_{22} + s_{21} \frac{1 + \lambda_{21}}{\lambda_{21}}) / \gamma p}. \tag{A.6}
\]

The sign of \( \partial \pi / \partial \bar{\gamma}_1 \) only depends on the sign of \( q_1 - q_1^*(s) + k(s) q_2 \). Note that \( \eta_1(s, r) = r_{12}/(s_{11} \frac{1 + \lambda_{11}}{\lambda_{11}} + s_{12} + r_{12}) \) from (20), so by (21):
We make the following observations about the feasible solution.

(i) When \( \pi_1 > 0 = r_{12}, \eta_1(s, r) = 0 \), which requires \( \partial \pi / \partial \eta_1 \leq 0 \), hence \( q_1 \leq q_1^*(s) + k(s)q_2 \);

(ii) When \( \pi_1, r_{12} > 0, \eta_1(s, r) \in (0,1) \), which requires \( \partial \pi / \partial \eta_1 = 0 \) (equivalently, \( \pi(0; s, q) = \pi(1; s, q) \)), hence \( q_1 = q_1^*(s) + k(s)q_2 \) with \( q_1^*(s), k(s) > 0 \);

(iii) When \( \pi_1 = 0 < r_{12}, \eta_1(s, r) = 1 \) and \( q_1^*(s) = k(s) = 0 \), all drivers reposition at location 1 without waiting in a queue \( (q_1 = 0) \) and \( \pi(0; s, q) = 0 < \pi(1; s, q) \).

It follows that \( q \in \mathcal{D}(s) \) as defined in (22). \( \square \)

**Proof of Proposition 3.** We want to show there is a unique feasible capacity utilization satisfying (12a)–(12c), (23)–(25) and (22), i.e., \((s, r, q, n) \in \mathcal{M}\) given by (26). Note that \( r_{21} = 0 \) by (22). By (12a) and (23) we can express service capacities in terms of \( s_{12} \) and \( r_{12} \):

\[
s_{11} = s_{12} \frac{S_{11}}{S_{12}}, \quad s_{21} = (s_{12} + r_{12}) \frac{t_{21}}{t_{12}}, \quad s_{22} = (s_{12} + r_{12}) \frac{t_{21} S_{22}}{t_{12} S_{21}}. \quad (A.7)
\]

We will focus on \((s_{12}, r_{12}, q, n) \), and recover the remaining quantities using (A.7). The other constraints, (12b), (22), (24), (25) and (12c), are rewritten below.

\[
\frac{n_1^M}{S_{12}} s_{12} + \left[ \frac{r_{21}}{t_{12}} \left( 1 + \frac{S_{22}}{S_{21}} \right) + 1 \right] r_{12} + q_1 + q_2 = n, \quad (A.8)
\]

\[
\begin{align*}
q_1 & \leq q_1^*(s) + k(s)q_2 & \text{if } r_{12} = 0, \\
& = q_1^*(s) + k(s)q_2 & \text{if } r_{12} > 0,
\end{align*} \quad (A.9)
\]

\[
(S_{12} - s_{12}) r_{12} = 0, \quad (A.10)
\]

\[
(S_{12} - s_{12}) q_1 = 0, \quad \left( S_{21} - (s_{12} + r_{12}) \frac{t_{21}}{t_{12}} \right) q_2 = 0, \quad (A.11)
\]

\[
0 \leq s_{12} \leq S_{12}, \quad 0 \leq \frac{t_{21}}{t_{12}} (s_{12} + r_{12}) \leq S_{21}, \quad r_{12} \geq 0, \quad q \geq 0. \quad (A.12)
\]

We make the following observations about the feasible solution.

(i) **The allocation where all capacity serves rider demand, i.e.,** \( r_{12} = 0, q = 0, s_{12} = \frac{n_1^M}{n_1^M} S_{12}, \) **is feasible if and only if** \( n \leq n_1^M. \)** (a) “\( \Rightarrow \)” : this is immediate; this also implies that if \( n > n_1^M \), then \( r_{12} > 0 \) or \( q > 0 \).

(b) “\( \Leftarrow \)” : given \( n \leq n_1^M \), suppose first that \( r_{12} > 0 \), then (A.10) implies that \( s_{12} = S_{12}, \) thus \( \pi > n_1^M \) and \( n > n_1^M, \) a contradiction; second, that \( q > 0, \) then (A.11) implies that \( s_{21} = S_{12}, \) thus \( \pi \geq n_1^M \) and \( n > n_1^M, \) a contradiction; or third, that \( q_2 > 0, \) then (A.11) implies that \( s_{21} = S_{21}, r_{12} > 0, \) thus \( n > n_1^M \) is still a contradiction. Hence \( n \leq n_1^M \Rightarrow r_{12} = 0, q = 0, s_{12} = \frac{n_1^M}{n_1^M} S_{12}, \) which satisfies (A.8)–(A.12) and is thus feasible.

(ii) **When \( n > n_1^M, \) any feasible solution must serve all demand at location 1, i.e.,** \( s_{12} = S_{12}, \) **and moreover,** \( q_1^*(s) \equiv q_1^*(S) \). By (i), \( n > n_1^M \) implies that \( r_{12} > 0 \) and/or \( q \neq 0 \), and either of these assertions implies that \( s_{12} = S_{12} \) by (A.10) and (A.11). To show that \( q_1^*(s) \equiv q_1^*(S) \), we substitute \( s_{11}, s_{21}, s_{22} \) in \( q_1^*(s) \) in (A.6) by (A.7),

\[
q_1^*(s) = \left( \frac{S_{12}}{S_{12}} + 1 \right) \frac{t_{21}}{t_{12}} + \left( 1 + \frac{S_{22}}{S_{21}} \right) \frac{t_{21} S_{22}}{t_{12} S_{21}} s_{12},
\]

which is equal to a constant multiplying \( s_{12} \). For \( s_{12} = S_{12} \), we have that \( q_1^*(s) \equiv q_1^*(S) \).

With these two observations, we derive the feasible solution given by the Proposition.
Proof of Proposition 4. 

Lemma 1 is satisfied under the constraints in (26), hence the per-driver profit is naturally maximized and the optimality condition (16) in

Proof of Corollary 2. (i) By Proposition 3, at any \( n > 0 \) there is a \textit{unique} feasible driver capacity allocation under the constraints in (26), hence the per-driver profit is naturally maximized and the optimality condition (16) in Lemma 1 is satisfied.

(ii) Substituting \( \bar{s} \) and \( \bar{r} \) from Proposition 3 into (12d) yields

\[
\pi_M(n) = \frac{\gamma p - c}{n} \bar{s} - c\bar{r} = \begin{cases} 
\gamma p - c & \text{zone (1) (n \leq n_1^M),} \\
\frac{n^M}{12} (\gamma p - c) & \text{zone (2) (n_1^M < n \leq n_2^M),} \\
\frac{S_{12} + S_{22}}{S_{12} + S_{22} + S_{21}} \gamma p - c & \text{zone (3) (n_2^M < n \leq n_3^M),} \\
\left( \frac{1}{n} (\gamma p \bar{s} - c n_2^M) \right) & \text{zone (4) (n > n_3^M).}
\end{cases}
\]

(A.13)

It is easy to see that \( \pi_M(n) \) is continuously decreasing in \( n \) and that \( \lim_{n \to \infty} \pi_M(n) = 0 \). Therefore, the participation equilibrium condition (12e), \( n = NF(\pi_M(n)) \), has a unique solution \( n_3^M \).

Proof of Proposition 4. We have the following observations about the optimal solution.

(i) \textit{Allocating all capacity n towards serving riders (i.e., r_{12} = 0, q = 0) is feasible (hence optimal) if and only if n \leq n_1^A := n_1^C.} This is the same as observation (i) in the Proof of Proposition 1. Also notice that constraint (22) is satisfied.

(ii) \textit{If for some capacity level n_1 the service capacity \( \bar{s} > n_1^A \), then for all capacity levels \( n_2 \geq n_1 \) the optimal solution involves repositioning.} First, note that by the definition of \( n_1^A, \bar{s} > n_1^A \) implies that \( r_{12} > 0 \) (which holds at \( n_1 \)), and hence we only need to show that the optimal service capacity at \( n_2 \) is higher than \( n_1^A \). It suffices to find one feasible solution at \( n_2 \) that has the same service capacity as at \( n_1 \), which is higher than \( n_1^A \). To achieve
this, let the service capacity vector \( s \) and the repositioning capacity \( r_{12} > 0 \) at \( n_2 \) be the same as those at \( n_1 \), respectively, and put the extra capacity \( n_2 - n_1 \) into \( q \) satisfying \( q_1 = q_1^*(s) + k(s)q_2 \). In this way all constraints are still satisfied while the service capacity \( \sigma > n_1^A \) remains unchanged.

(iii) \( r_{12} > 0 \) for all \( n > n_1^A + q_1^*(S_1^A) \), where \( S_1^A = S_1^C \) defined in (A.1) such that \( S_1^A \cdot 1 = n_1^A \). It suffices to show that at capacity levels in the right neighborhood of \( n_1^A + q_1^*(S_1^A) \), the optimal service capacity is higher than \( n_1^A \), which then, by observation (ii), will prove the result. To show this, for an arbitrarily small \( \epsilon > 0 \), let \( n_\ast \) be the minimum feasible total capacity to provide service vector \( S_1^A + (0,0,\epsilon,0) \), and hence service capacity \( n_1^A + \epsilon > n_1^A \). Following constraints (12a)–(12c) and (22), we have

\[
n_\ast = n_1^A + \epsilon + \frac{t_{12}}{t_{12}} \epsilon + q_1^*(S_1^A + (0,0,\epsilon,0)),
\]

and \( n_0 = n_1^A + q_1^*(S_1^A) \) for \( \epsilon = 0 \). It is easy to see that \( n_\ast \) increases in \( \epsilon \), since by definition (A.6),

\[
\frac{\partial q_1^*(s)}{\partial s_{21}} = \frac{((s_{11} + s_{12})\gamma p - s_{11}\epsilon)s_{22} + \frac{t_{12}}{t_{12}}}{[(s_{21} + s_{22})\gamma p - (s_{21} + s_{22} + s_{21} + s_{22})\epsilon]}> 0, \quad \forall s_{22} > 0, s_{11} + s_{12} > 0,
\]

i.e., \( q_1^*(s) \) increases wrt \( s_{21} \) when \( s_{22}, s_{11} + s_{12} > 0 \). Therefore, the optimal service capacity must be higher than \( n_1^A \) at capacity levels in the right neighborhood of \( n_1^A + q_1^*(S_1^A) \).

(iv) \( r_{12} = 0 \) for \( n \in [n_1^\ast, n_1^A + \delta] \) for a small \( \delta > 0 \). We first prove this for the optimal solution at \( n = n_1^A + \delta \) by establishing that for any feasible \( q_1 \) it must be that \( q_1 < q_1^*(s) \), from which we deduce \( r_{12} = 0 \) from (22). Then, observation (ii) yields the same result for \( n \in [n_1^\ast, n_1^A + \delta] \). Pick

\[
\delta = \min \left\{ q_1^*(S_1^A), \frac{(S_1^A)^2}{S_{12}} \right\},
\]

so that \( n_1^A + \delta \leq n_1^A + q_1^*(S_1^A), \delta < S_{12} \) and

\[
\delta < \frac{(S_1^A)^2}{S_{12} \left( 1 + \frac{t_{12}}{t_{12}} \right) + S_{21} + S_{22}} \left( 1 + \frac{t_{12}}{t_{12}} \right) \Rightarrow S_{12}(S_{12} - \delta) \leq \frac{S_{12}(S_{12} - \delta)}{S_{21} + S_{22}} \left( 1 + \frac{t_{12}}{t_{12}} \right) > \delta.
\]

First note that the optimal solution at \( n_1^A + \delta \) must have \( \bar{\sigma} \geq n_1^A \), since a feasible solution \( s = S_1^A, r_{12} = 0, q_1 = 0, q_2 = \delta \) yields \( \bar{\sigma} = n_1^A \). Therefore

\[
r_{12}, q_1 \leq \delta, \quad s_{21} \geq \frac{t_{12}}{t_{12}}, \quad s_{12} \geq S_{12} - \delta > 0,
\]

where the first inequality follows from \( \bar{\sigma} \geq n_1^A \) and capacity constraint (12b), the second is by \( \bar{\sigma} \geq n_1^A \), and the third follows from the second and (12a) in that \( s_{12} = s_{21} + s_{22} - r_{12} \geq S_{12} - \delta \).

Then,

\[
q_1^*(s) = \frac{(s_{11} + s_{12})s_{21} \frac{t_{12}}{t_{12}} + (s_{21} + s_{22})s_{12}}{(s_{21} + s_{22}) - (s_{21} + s_{22} + s_{21} + s_{22}) \sigma} \geq \frac{s_{12} s_{21} \frac{t_{12}}{t_{12}} + s_{21} s_{12}}{S_{21} + S_{22}} \geq \frac{S_{12}(S_{12} - \delta) \frac{t_{12}}{t_{12}} (1 + \frac{t_{12}}{t_{12}})}{S_{21} + S_{22}} > \delta,
\]

where the the second inequality follows from the second and third inequalities in (A.16), and the last inequality is by (A.15). This, together with the first inequality in (A.16), yields \( q_1 \leq \delta < q_1^*(s) \), which implies \( r_{12} = 0 \) by constraint (22). Hence we have shown \( r_{12} = 0 \) at \( n = n_1^A + \delta \). By observation (ii), the optimal solution at \( n \in [n_1^\ast, n_1^A + \delta] \) has service capacity \( \bar{\sigma} = n_1^A \) and no repositioning.

(v) An optimal solution can serve all demand \( (s = S) \) if and only if \( n \geq n_2^C + q_1^*(S) \). “⇐”: given \( n \geq n_2^C + q_1^*(S) \), it is easy to verify that the capacity allocation \( s = S, r = (n_2^C - \bar{\sigma}, 0) \) and \( q_1 = q_1^*(S) + k(S)q_2 \)
with \( q_1 + q_2 = n - n_2^C \) is feasible and serves all demand (hence optimal). “\( \Rightarrow \)” an optimal (hence feasible) solution that serves all demand must have \( s = S, r = (n_2^C - S, 0) \) and \( q_1 = q_1^*(S) + k(S)q_2 \). By (12b) this yields \( n = n_2^C + q_1^*(S) + k(S)q_2 + q_2 \geq n_2^C + q_1^*(S) = n_3^A \).

With these five observations, we can derive the optimal solution given by the Proposition. Zone (1) follows directly from observation (i) and zone (4) follows from observation (v). In \( (n_1^A, n_2^A) \), not all drivers are serving riders and not all riders are served. There exists a threshold \( n_3^A \) such that \( n_1^A < n_3^A < n_2^A \) which separates zone (2) and (3) apart: in zone (2), \( (n_1^A, n_2^A) \), optimal solution has service capacity \( s = n_1^A \), no repositioning \( (r = 0) \), and extra capacity queues at location 1 with \( q = (n - n_1^A, 0) \); whereas in zone (3), \( (n_2^A, n_3^A) \), optimal solution involves repositioning \( (r_{12} > 0) \), serves \( s = n_3^A \), and extra capacity queues at location 1 with \( q = (q_1^*(s), 0) \). Note that \( n_2^A > n_3^A \) by observation (iv). \( n_2^A < n_3^A \) follows from \( n_2^A \leq n_1^A + q_1^*(S_1^A) \) by observation (iii) and \( n_1^A + q_1^*(S_1^A) < n_2^C + q_1^*(S) = n_3^A \) by property (A.14). Furthermore, the fact that optimal solution involves repositioning at any capacity level in zone (3) follows from observation (ii).

The following proofs of Lemma 3 and Propositions 6 and 7 refer to two technical lemmas, Lemmas S-1 and S-2. The statements and proofs of these lemmas as well as the proof of Proposition 5 are relegated to the Supplemental Material S1.

**Proof of Lemma 3.**  
(i) Under the unrestricted optimal policy where strategic demand rejection might prevail, we show in Lemma S-2 (see Supplemental Material S1) that \( \pi_A(n) \) is decreasing in \( n \) and \( \lim_{n \rightarrow \infty} \pi_A(n) = 0 \). Hence there is a unique equilibrium participating capacity \( n_A \) that satisfies (29).

(ii) Under the restricted optimal policy where strategic demand rejection is disallowed, the platform’s optimal capacity allocation for \( n \in (\hat{n}_2^A, n_3^A) \) follows pattern (1) in Lemma S-1 (see Supplemental Material S1): only \( s_{21} \) is increasing and \( \hat{\pi}_A(n) \) is continuous. Then case (i) in the proof of Lemma S-2 shows that \( \hat{\pi}_A(n) \) decreases in \( n \) (note that Lemma S-2 adopts the general notation \( \pi_A(n) \) which means \( \hat{\pi}_A(n) \) in this case), and hence there is a unique solution \( \hat{n}_A \) that solves (31). The optimality condition (16) in Lemma 1 can be proven similarly as under regime C (in the proof of Corollary 1), which shows that any feasible deviation from the platform’s optimal capacity allocation cannot increase the per-driver profit. First, the logic for the scarce and ample capacity zones, \( (0, n_1^A) \) and \( (n_2^A, \infty) \), is identical to that in the proof of Corollary 1. Second, in the moderate capacity zone (without repositioning), \( (n_1^A, \hat{n}_2^A] \), there is zero repositioning capacity, i.e., \( \bar{r} = 0 \), which cannot be reduced, hence the key inequality (A.2) in the proof of Corollary 1 is immediately violated, and it follows that the per-driver profit is maximized. Third, in the moderate capacity zone (with repositioning), \( (\hat{n}_2^A, n_3^A] \), the argument is identical to that in zone 3 of regime C. We have thus proven that for any \( n \), the per-driver profit is maximized without strategic demand rejection (which may not be the case with strategic demand rejection), i.e., (32) holds.

(iii) \( n_A \leq \hat{n}_A \) follows immediately from (32). To see the second part, first note that, by definition, \( n_A \) and \( \hat{n}_A \) (and their associated optimal capacity allocations) are both feasible solutions to problem (13) with \( X = A \). Next,
we show that no better equilibrium can be established at \( n < n_A \) or \( n > \hat{n}_A \). On the one hand, if there exists an equilibrium at \( n < n_A \) with associated optimal service vector \( s \), then it must be suboptimal since

\[
P(n) \leq P_A(n) \leq P_A(n_A),
\]

where the first inequality is by definition that \( P_A(n) \) is the maximum platform revenue at \( n \) subject to \((s, r, q, n) \in C_A\), and the second inequality follows from the monotonicity of \( P_A(\cdot) \) implied by Proposition 4. On the other hand, any solution \((s, r, q, n) \in C_A\) at \( n > \hat{n}_A \) does not satisfy the driver participation constraint because

\[
\pi(s, r, n) \leq \hat{\pi}_A(n) \leq \hat{\pi}_A(\hat{n}_A) = F^{-1}\left(\frac{\hat{n}_A}{N}\right) < F^{-1}\left(\frac{n}{N}\right),
\]

where the first inequality follows from the optimality condition (16) in Lemma 1 that is proved in part (ii), the second inequality is due to \( \hat{\pi}_A(n) \) decreasing in \( n \) shown in part (i) above, and the last inequality follows from the assumption that \( F(\cdot) \) is continuously increasing on \([0, \infty)\).

Therefore, the equilibrium participating capacity \( n^*_A \) must lie in \([n_A, \hat{n}_A]\). Equilibrium per-driver profit \( \pi^*_A \) follows from the participation equilibrium constraint (12e). The inequalities \( \pi_A(n_A) \leq \pi^*_A \leq \hat{\pi}_A(\hat{n}_A) \) follows from the monotonicity of \( F(\cdot) \).

The inequality \( \Pi_A^S \geq \max\{\Pi_A(n_A), \hat{\Pi}_A(\hat{n}_A)\} \) follows from the fact that \( n_A \) and \( \hat{n}_A \) (and their associated optimal capacity allocations) are both feasible solutions to problem (13) with \( X = A \), which achieve objective values \( \Pi_A(n_A) \) and \( \hat{\Pi}_A(\hat{n}_A) \), respectively.

(iv) With \( \Pi_A(n) \equiv \hat{\Pi}_A(n) \), there is no need for strategic demand rejection, \( \pi_A(n) \equiv \hat{\pi}_A(n) \), and the result follows directly.

(v) Since \( \hat{n}_A \) is the unique participating equilibrium disallowing strategic demand rejection (any equilibrium participating capacity less than \( \hat{n}_A \) will involve strategic demand rejection to some degree), it follows that if \( \hat{\Pi}_A(\hat{n}_A) < \Pi_A(n_A) \), disallowing strategic demand rejection yields lower platform revenue than allowing so and hence \( n^*_A \neq \hat{n}_A \). In other words, \( \Pi_A(n_A) > \hat{\Pi}_A(\hat{n}_A) \) is sufficient for strategic demand rejection to be optimal and \( n^*_A < \hat{n}_A \).

To see the second part, first note that under \( \Pi_A(n_A) > \hat{\Pi}_A(\hat{n}_A) \) and \( n_A \leq \hat{n}_A \) given in part (ii), \( n_A \) cannot be a discontinuity point of \( \pi_A(\cdot) \) where \( \Pi_A = \hat{\Pi}_A \). Then with \( n_A < \hat{n}_A \), it follows from the monotonicity of \( F(\cdot) \) that \( \pi_A(n_A) = F^{-1}(n_A/N) < F^{-1}(\hat{n}_A/N) = \hat{\pi}_A(\hat{n}_A) \).

\[\square\]

**Proof of Proposition 6.** We first prove the case \( \rho_2 = 0 \) and obtain the range \((N, N)\), and then show the existence of the threshold level \( \hat{\rho}_2(N) > 0 \). Note that since \( S_{21} > 0 \) by Assumption 1, the positivity of \( \rho_2 \) is equivalent to the positivity of \( S_{22} \). There is essentially a one-to-one mapping between \( \rho_2 \) and \( S_{22} \) given \( S_{21} \), and we will focus on \( S_{22} \) in what follows.

In the following three steps, under \( S_{22} = 0 \), we first prove several properties of the platform revenue and per-driver profit rate functions when strategic demand rejection is allowed or disallowed, then use the properties to show that strategic demand rejection is optimal when the equilibrium driver participation is in the intermediate region, and finally obtain the corresponding range of driver pool size.
(1) When $S_{22} = 0$, we have the following four properties (i)--(iv) regarding the platform revenue and per-driver profit under optimal capacity allocation allowing or disallowing strategic demand rejection:

(i) Condition (34) in Proposition 5 holds.

(ii) $n_2^4 < \hat{n}_2^4 < n_3^4$.

(iii) $\Pi_A(n)$ strictly increases on $(n_2^4, n_3^4)$; $\hat{\Pi}_A(n)$ stays constant on $(n_2^4, \hat{n}_2^4)$ and strictly increases on $[\hat{n}_2^4, n_3^4]$; and $\Pi_A(n) > \hat{\Pi}_A(n)$ on $(n_2^4, n_3^4)$.

(iv) $\pi_A(n)$ remains constant on $(n_2^4, n_3^4)$; $\hat{\pi}_A(n)$ strictly decreases on $[n_2^4, \hat{n}_2^4]$ and stays constant on $(\hat{n}_2^4, n_3^4)$; $\pi_A(n) < \hat{\pi}_A(n)$ on $(n_2^4, \hat{n}_2^4)$ and $\pi_A(n) = \hat{\pi}_A(n)$ on $[\hat{n}_2^4, n_3^4]$.

Property (i) is immediate by setting $S_{22} = 0$ in (34). To obtain properties (ii) and (iii), we first show that under $S_{22} = 0$ (hence $s_{22} = 0$), we can significantly simplify the 3 patterns of optimal capacity allocation as a function of participating capacity $n \in (n_2^4, n_3^4)$ established in Lemma S-1 and its proof in the Supplemental Material S1.

Setting $s_{22} = 0$ in the derivatives (S.4)–(S.6) and using $n = g(s)$ by (S.9), we get

$$\frac{\partial n}{\partial s_{11}} = \frac{\partial g(s)}{\partial s_{11}} = \left(1 + \frac{t_{12}}{t_{21}}\right) \frac{\tilde{g}p - c}{\tilde{g}p - \left(1 + \frac{2c}{t_{21}}\right)c},$$

$$\frac{\partial n}{\partial s_{12}} = \frac{\partial g(s)}{\partial s_{12}} = \left(1 + \frac{t_{12}}{t_{21}}\right) \frac{\tilde{g}p}{\tilde{g}p - \left(1 + \frac{2c}{t_{21}}\right)c},$$

$$\frac{\partial n}{\partial s_{21}} = \frac{\partial g(s)}{\partial s_{21}} = 1 + \frac{t_{12}}{t_{21}}.$$

Clearly $\frac{\partial n}{\partial s_{21}} < \frac{\partial n}{\partial s_{11}} < \frac{\partial n}{\partial s_{12}}$ and all are constants that only depend on model primitives. Noticing that $\pi_7(n_2^4) = \pi_2(n_3^4) = S$ at the right end $n_3^4$, and the patterns specified in Lemma S-1 (i.e., with $n$ increasing towards $n_3^4$), in pattern (1) only $s_{21}$ increases towards $S_{21}$; in pattern (2), first $s_{21}$ increases to $S_{21}$ and then $s_{12}$ increases to $S_{12}$; and in pattern (3), first $s_{21}$ increases to $S_{21}$, then $s_{11}$ increases to $S_{11}$ and last $s_{12}$ increases to $S_{12}$, a direct implication of the derivatives’ ranking is that the left ends of each pattern (at which the service capacity is $n_2^4$ and repositioning is about to start) are ordered as $\pi_3^{-1}(n_3^4) \leq \pi_2^{-1}(n_3^4) < \pi_1^{-1}(n_2^4)$, and, furthermore, pattern (3) yields the largest service capacity for $n \in (\pi_3^{-1}(n_2^4), n_3^4)$ and pattern (1) which disallows strategic demand rejection yields the smallest service capacity for $n \in (\pi_3^{-1}(n_2^4), n_3^4)$. Consequently, the platform’s optimal capacity allocation follows pattern (3) when strategic demand rejection is allowed and follows pattern (1) when disallowed. Hence,

$$n_2^4 := \pi_3^{-1}(n_2^4) < \pi_1^{-1}(n_2^4) =: \hat{n}_2^4 < n_3^4,$$

which proves property (ii), and property (iii) follows immediately.

To see (iv), first note that $\pi_A(n)$ and $\hat{\pi}_A(n)$ correspond with patterns (3) and (1) established above, respectively.

We can then similarly simplify the per-driver profit rate as a function of participating capacity $n \in (n_2^4, n_3^4)$ under each pattern established in the proof of Lemma S-2 in Supplemental Material S1. In specific, setting $S_{22} = 0$, we get $\pi'(n) = 0$ within each pattern given by (S.20)–(S.24). Note that under pattern (1), $\pi'(n) < 0$ for $n < \pi_1^{-1}(n_2^4) = \hat{n}_2^4$. Property (iv) hence follows.

(2) If $n_2^4 \in (n_2^4, \hat{n}_2^4)$, there must be $n_2^4 < n_A < \hat{n}_2^4 < \hat{n}_2^4$ following from $\pi_A(n) < \hat{\pi}_A(n)$ in (1.iv) and the monotonicity assumption on $F(\cdot)$. Hence by $\Pi_A(n) > \Pi_A(n_2^4)$, $\forall n \in (n_2^4, \hat{n}_2^4)$ and $\hat{\Pi}_A(n) \equiv \Pi_A(n_2^4)$, $\forall n \in (n_2^4, \hat{n}_2^4)$ due to (1.iii), there must be $\hat{\Pi}_A(n) = \Pi_A(n_2^4) < \Pi_A(n_A)$. By Lemma 3 (iv), strategic demand rejection is optimal.
If \( n_A^* \in [n_3^A, n_3^A] \), it follows from \( \pi_A(n) = \hat{\pi}_A(n) \) in (1.iv) that \( n_A^* = n_A = \hat{n}_A \) and thus \( \hat{H}_A(n_A^*) < H_A(n_A^*) \). By Lemma 3 (iv), strategic demand rejection is optimal.

(3) For a given driver opportunity cost distribution \( F \), let \((N, \overline{N})\) be the range of driver pool size such that \( N \in (N, \overline{N}) \) if \( \hat{n}_A^* \in (n_3^A, n_3^A) \) under \( S_{22} = 0 \). The range exists and is unique because of the monotonicity assumption on \( F(\cdot) \) and that \( \pi_A(\cdot) \) is decreasing.

We have shown that strategic demand rejection is optimal if \( S_{22} = 0 \) and \( N \in (N, \overline{N}) \). Now fix any \( N \in (N, \overline{N}) \), we show that by continuity, the above results still hold for sufficiently small \( S_{22} \). Specifically, we have the following properties:

(i) The platform’s optimal revenues allowing or disallowing strategic demand rejection, \( \Pi_A(n) \) and \( \hat{\Pi}_A(n) \), are both continuous in \( n \) and \( S_{22} \). This follows from Berge’s maximum theorem, because the maximand \( \Pi(s) \) is continuous and the feasible sets given by the constraints in \( \Pi_A(n) \) and \( (30) \) are both continuous correspondences of \( (s, r, q) \).

(ii) The resulting per-driver profits allowing or disallowing strategic demand rejection, \( \pi_A(n) \) and \( \hat{\pi}_A(n) \), are both continuous in \( n \in (n_3^A, n_3^A) \) and \( S_{22} \). Given the definition of \( \pi(s, r, n) \) in (12d) and the definition of \( \pi_X(n) \) in Step 1 of the two-step approach described in §2.5, since the optimal solution \( s(n) \) and \( r(n) \) are both continuous for \( n \in (n_3^A, n_3^A) \) under either pattern (3) (optimal capacity allocation allowing strategic demand rejection) or pattern (1) (optimal capacity allocation disallowing strategic demand rejection) specified in Lemma S-1, \( \pi_A(n) \) and \( \hat{\pi}_A(n) \) are also both continuous for \( n \in (n_3^A, n_3^A) \). The continuity in \( S_{22} \) follows because the optimal solution \( s(n) \) and \( r(n) \) are both continuous in parameter \( S_{22} \) under either pattern (3) and pattern (1) for \( n \in (n_3^A, n_3^A) \).

(iii) The drivers’ opportunity cost distribution \( F(\cdot) \) is continuous and strictly increasing.

(iv) Due to (ii) and (iii), the equilibrium participating capacities allowing or disallowing strategic demand rejection, \( n_A \) and \( \hat{n}_A \), are both continuous in \( S_{22} \).

Since we have shown in the first part that \( \hat{H}_A(\hat{n}_A) < H_A(n_A) \) always holds for any \( N \in (N, \overline{N}) \) at \( S_{22} = 0 \), it follows from (i) and (iv) that \( \hat{H}_A(\hat{n}_A) < H_A(n_A) \) still holds for sufficiently small \( S_{22} > 0 \). We also know that condition (34) in Proposition 5 does not hold for sufficiently large \( S_{22} \), hence by continuity there exists \( \hat{S}_{22}(N) > 0 \) such that \( \hat{H}_A(\hat{n}_A) < H_A(n_A) \), \( \forall S_{22} \in [0, \hat{S}_{22}(N)) \), i.e, strategic demand rejection is optimal in a neighborhood of \( S_{22} = 0 \). This translates to a threshold level \( \hat{\rho}_2(N) \) on \( \rho_2 \).

Proof of Proposition 7. We start by ranking the platform revenue and per-driver profit under the three control regimes for fixed participating capacity. Lemma 4 below establishes that both are higher when the platform has more control capabilities.

**Lemma 4 (Ranking of equilibrium profits for fixed capacity).** For fixed participating capacity \( n \), platform controls have the following impact on profits:

1. More platform control increases the platform revenue: \( \Pi_M(n) \leq \Pi_A(n) \leq \Pi_C(n) \), where
   \[
   \Pi_M(n) < \Pi_A(n) \quad \text{iff} \quad n \in (n_1^M, n_3^M) = (n_1^A, n_3^A) \quad \text{and} \quad S_{22} > 0,
   \]
   \[
   \Pi_A(n) < \Pi_C(n) \quad \text{iff} \quad n \in (n_1^A, n_3^A).
   \]
(2) Centralized control maximizes the per-driver profit rate: \( \pi_M(n) \leq \pi_C(n) \) with strict inequality iff \( n \in (n_1^M, n_2^M) \), \( \pi_A(n) \leq \tilde{\pi}_A(n) \leq \pi_C(n) \) with strict second inequality iff \( n \in (n_1^A, n_2^A) \).

(3) Under decentralized repositioning, optimal admission control affects per-driver profit as follows:

(a) No change \((\pi_A(n) = \pi_M(n))\) under small \( (n \leq n_1^M) \) or large \( (n \geq n_3^M = n_3^A) \) capacity.

(b) Increase \((\pi_A(n) > \pi_M(n))\) under intermediate capacity \((n_1^M < n < n_3^M = n_3^A)\) if (34) is not satisfied.

(c) Decrease \((\pi_A(n) < \pi_M(n))\) under intermediate capacity \((n_2^A < n < n_3^M)\) if \( S_{22} = 0 \).

Proof. (1) The formulation of \( \Pi_X(n) \) given by (14) has the same objective function but shrinking constraint sets \( C_X(n) \) (i.e., fewer constraints) for \( X \) ranging \( M \to A \to C \), hence the ranking holds. The conditions for strict inequalities follow from Propositions 1, 3 and 4.

(2) \( \pi_M(n) \leq \pi_C(n) \) follows directly from the specification of \( \pi_C(n) \) and \( \pi_M(n) \) in (A.13) and (18), respectively. \( \pi_A(n) \leq \tilde{\pi}_A(n) \) is given by (32). To see \( \tilde{\pi}_A(n) \leq \pi_C(n) \): for \( n \leq \hat{n}_2^A \) and \( n \geq n_4^A \) (zone (1), (2) and (4) under regime \( A \) disallowing strategic demand rejection), \( \tilde{\pi}_A(n) = \pi_A(n) \leq \pi_C(n) \) follows directly from the specification of \( \pi_C(n) \) and \( \pi_A(n) \) in (18) and (S.19), respectively; for \( \hat{n}_2^A < n < n_4^A \) (zone (3)), \( \tilde{\pi}_A(n) \leq \pi_C(n) \) follows from Lemma S-1. The strict inequality conditions also follow from the above specification.

(3) Part (a) follows directly from the specification of \( \pi_M(n) \) and \( \pi_A(n) \) in (A.13) and (S.19), respectively, and the fact that \( n_1^M \leq n_1^A \). To see part (b), first note that for (34) to be not satisfied, there must be \( \rho_2 > 0 \) and equivalently \( S_{22} > 0 \), which implies \( n_1^M < n_1^C = n_1^A \) by (27). Thus \( \pi_M(n) < \pi_A(n) \) for \( n_3^M < n \leq n_1^A \) by the specification of \( \pi_M(n) \) and \( \pi_A(n) \) in (A.13) and (S.19), respectively. Then given (34) is not satisfied (no strategic demand rejection in regime \( A \)), it follows from Lemma S-1 that \( \pi_M(n) < \tilde{\pi}_A(n) \) for \( n_3^A < n < n_4^A = n_3^M \). Together we have proven part (b). For part (c), given \( S_{22} = 0 \), one can see that the optimal capacity allocation in the absence of strategic demand rejection is equivalent to that under regime \( M \), i.e., \( \hat{\Pi}_A(n) = \Pi_M(n), \hat{\pi}_A(n) = \pi_M(n) \) and \( \hat{n}_2^A = n_2^M \), see the proof of Proposition 6. As a result, we may simply compare \( \pi_A(n) \) with \( \hat{\pi}_A(n) \) under \( S_{22} = 0 \). It then follows immediately from properties (ii) and (iv) in Part (1) of the proof of Proposition 6 that \( \pi_A(n) < \tilde{\pi}_A(n) = \pi_M(n) \) for \( n_2^A < n < \hat{n}_2^A = n_2^M \), and \( \pi_A(n) = \pi_M(n) \) for \( n \leq n_2^A \) or \( n \geq n_2^M \).

Remark: Part (1) of Lemma 4 also implies that riders benefit from increasing platform control capability: An important performance metric for the riders is the network-wise service level, defined as the fraction of the total rider demand that is served, i.e., \( \pi/N \). Since the platform revenue rate is proportional to the total service capacity \( \Pi(s) = \gamma pS \), the network-wise service level is proportional to the platform revenue rate, and therefore increases with platform controls.

Back to the proof of Proposition 7. Note the following properties of \( \Pi_X(\cdot), \pi_X(\cdot) \) and \( n_X^\gamma \):

(i) \( \Pi_X(n) \) is increasing in \( n \) for \( X \in \{M, A, C\} \). This follows immediately for regime \( C \) from Proposition 1. For regimes \( M \) and \( A \), one can verify that increasing capacity can be allocated into IC queues as in (22) without any reduction in the capacity that serves rider demand.

(ii) \( \pi_M(n), \pi_A(n), \pi_C(n) \) are continuously decreasing in \( n \), and \( \pi_A(n) \) is continuously decreasing for \( n > n_2^A \). This follows from (18), (A.13), (S.19) and the proof of Lemma S-2.

(iii) For \( \pi_1(n) \leq \pi_2(n) \) both continuously decreasing for \( n \in (A, B) \), let \( n_i^* \in (A, B) \) be the solution to \( n_i^* = NF(\pi_i(n_i^*)) \), \( i = 1, 2 \).

Then \( n_1^* \leq n_2^* \) and \( \pi_1(n_1^*) \leq \pi_2(n_2^*) \), where the inequalities are strict iff \( \pi_1(n_1^*) < \pi_2(n_2^*) \) or \( \pi_1(n_2^*) < \pi_2(n_2^*) \). To
With these three properties and Lemma 4, we are ready to prove the three parts in Proposition 7.

(1) The first inequality, \( \Pi_M \leq \Pi_A \), can be shown by discussing whether (34) is satisfied. If (34) is not satisfied, strategic demand rejection is suboptimal, then \( \Pi_M = \Pi_M(n'_M) \leq \Pi_A(n'_M) \leq \Pi_A(n'_A) = \Pi_A \), where the first inequality follows from Lemma 4 (1), which is strict if \( S_{22} \neq 0 \) and \( n'_M \in (n'_1M, n'_3) \) (equivalently \( N \in (N'_1M, N_3) \)), and the second inequality follows from the monotonicity of \( \Pi_A(\cdot) \) in Property (i) and the ranking \( n'_M \leq n'_A \) given by (a) and (b) in Part (3) of the proposition. If (34) is satisfied, consider the value of \( n'_M \) (equivalently the value of \( N \)): in zone (1) and (4) \( n'_M \leq n'_M < n'_1M \) and \( n'_M \geq n'_M = n'_A \), respectively, where \( \Pi_M(\cdot) = \Pi_A(\cdot) \). Therefore \( \Pi_M = \Pi_M(n'_M) = \Pi_A(n'_M) \), where the inequality is strict if \( S_{22} \neq 0 \) (so that \( n'_M < n'_1M \)); in zone (3) \( n'_M < n'_M < n'_M = n'_A \), since \( \hat{\pi}_A(n) \geq \pi_A(n) \geq \pi_M(n'_M) = \pi_M(n'_M) \) for \( n < n'_M = n'_A \), where the first inequality follows from (32) and the second inequality is strict if \( S_{22} \neq 0 \) (see proof of Proposition 6), there must be \( n'_M \leq n_A \leq n'_A \leq \hat{n}_A \) and hence \( \Pi_M \leq \Pi_A(n'_M) \leq \Pi_A(n'_A) \leq \Pi_A \), with first two inequalities being strict if \( S_{22} \neq 0 \). This completes the proof of \( \Pi_M \leq \Pi_A \).

The second inequality, \( \Pi_A \leq \Pi_C \), holds because \( \Pi_A \leq \Pi_A(n'_A) \leq \Pi_C(n'_A) \leq \Pi_C(n'_C) = \Pi_C \), where the first inequality follows from the fact that \( \Pi_A(n) \), as the unrestricted optimal platform revenue, is the highest at any \( n \), including \( n'_A \), the second inequality follows from Lemma 4 (1), which is strict if \( n'_A \in (n'_1A, n'_3) \) (equivalently \( N \in (N'_1A, N_3) \)), and the third inequality follows from the monotonicity of \( \Pi_C(\cdot) \) in Property (i) and the ranking \( n'_A \leq n'_A \) given in Part (2).

(2) This follows from properties (ii), (iii) and Lemma 4 (2) directly. To see the iff condition for strict inequalities, note that \( N \in (N'_1A, N_3) \) \( \iff \) \( n'_A \in (n'_1A, n'_3) \) \( \iff \) \( \pi_A(n'_A) < \pi_C(n'_A) \), and also \( N \in (N'_1A, N_3) \) \( \iff \) \( n'_M \in (n'_1M, n'_3) \) \( \iff \) \( \pi_M(n'_M) < \pi_C(n'_M) \). It then follows from Property (iii).

(3) Cases (a) and (b) follow directly from properties (ii), (iii) and cases (a) and (b) in Lemma 4 (3). For case (c), first consider \( \rho_2 = 0 \) (equivalently \( S_{22} = 0 \)), it follows from case (c) (and its proof) in Lemma 4 (3) that \( \pi_A(n) \leq \pi_M(n) = \hat{\pi}_A(n) \) for \( n \in (n'_2M, n'_2M) \), with strictly inequality when \( n \in (n'_2M, n'_3) \). This, together with properties (ii) and (iii), implies \( n'_M > n'_A \) and \( \pi'_M > \pi'_A \) for \( n'_M \in (n'_2M, n'_3) \), or equivalently \( N \in (N_2M, \hat{N}_3) \), where \( N, \hat{N} \) are values of \( N \) corresponding to \( n'_A = n'_A \) and \( n'_A = \hat{n}'_A = n'_3 \), respectively. Then, fixing \( N \in (N_2M, \hat{N}_3) \), we can show by continuity that the above results still hold for sufficiently small \( \rho_2 \in [0, \hat{\rho}_2(N)] \) like the last part of the proof of Proposition 6.

\[ \square \]
Proof of Proposition 8. (1) According to Proposition 7 (and Lemma 4 in its proof), if strategic demand rejection is suboptimal ((34) not satisfied), the platform revenue rate gain from admission control is positive only when $n_M^* \in (n_1^M, n_3^M)$. In this range we have

\[ \Pi_M^* \geq \Pi_M(n_1^M) = \gamma p n_1^M, \]  
(\ref{eq:prop8_1})

where the equality holds for $n_M^* \in (n_1^M, n_3^M)$. By Proposition 7 and Lemma 4 in its proof, $n_M^* \in (n_1^M, n_3^M)$ and (34) not satisfied also imply that $\pi_M^* < \pi_A^*$, thus $n_A^* \in (n_1^A, n_3^A)$ since otherwise $n_M^* = n_A^*$ and $\pi_M^* = \pi_A^*$. This yields

\[ \Pi_A^* \leq \Pi_A(n_3^M) = \Pi_A(n_3^A) = \gamma p S, \]  
\(\text{(A.19)}\)

where the equality is approached by $n_A^* \to n_3^M = n_3^A$.

Given that $N \geq n_3^M = n_3^A$, $\pi_X = NF(\pi_X(n_3^X))$ can take on values in $[0, n_3^M]$ depending on the choice of $F(\cdot)$, for $X \in \{M, A, C\}$. Consequently, the bounds in (\ref{eq:prop8_1}) and (\ref{eq:prop8_2}) can be approached and therefore

\[ \max_{F(\cdot)} \frac{\Pi_A - \Pi_A^*}{\Pi_M^*} \leq \frac{\gamma p S - \gamma p n_1^M}{\gamma p n_1^M} = \left( \frac{A_{21}}{A_{12}} - 1 \right) \frac{1}{1 + \frac{1}{1 - \rho_1} + \frac{\rho_2}{1 - \rho_2} \frac{A_{21}}{A_{12}}}. \]

To approach this upper bound, we need $n_A^* \in (n_1^M, n_2^M)$ and $n_A^* \to n_3^M = n_3^A$ so that $\Pi_A^* = \gamma p n_1^M$ and $\Pi_A \to \gamma p S$.

(Refer to Figure 7 (a) and (c) for an illustration.) This holds for opportunity cost distributions $F(\cdot)$ satisfying

\[ F^{-1}(n_2^M/N) = \pi_M(n_2^M) \quad \text{and} \quad F^{-1}(n_3^M/N) = \pi_M(n_3^M), \]

i.e., the value of $F$ at $\pi_M(n_2^M)$ is fixed at $n_2^M/N \Rightarrow n_2^M = n_2^A$ and $F$ grows sufficiently fast to $n_3^M/N = n_3^A/N$ at $\pi_M(n_2^M)^+ = \pi_A(n_3^A)^+$ ($\Rightarrow n_A^* \to n_3^A = n_3^A$); in words, there is a sufficiently large mass of potential drivers with opportunity cost around $\pi_M(n_2^M)^+$.

(2) According to Proposition 7 (and Lemma 4 in its proof), the platform revenue rate gain from centralized repositioning is positive only when $n_A^* \in (n_1^A, n_3^A)$. This yields

\[ \Pi_A^* \geq \Pi_A(n_1^A) = \gamma p n_1^A, \]  
\(\text{(A.20)}\)

where the equality holds for $n_A^* \in (n_1^A, n_2^A)$. By Proposition 7 and Lemma 4 in its proof, $n_A^* \in (n_1^A, n_3^A)$ also implies that $\pi_A^* < \pi_C^*$. Thus, $n_C^* \in (n_1^A, n_3^A)$, since otherwise $n_A^* = n_C^*$ and $\pi_A^* = \pi_C^*$. This yields

\[ \Pi_C^* \leq \Pi_C(n_3^A) = \gamma p S, \]  
\(\text{(A.21)}\)

where the equality holds for $n_C^* \in [n_1^C, n_3^A]$.

Given that $N \geq n_3^M = n_3^A$, $\pi_X = NF(\pi_X(n_3^X))$ can take on values in $[0, n_3^A]$ depending on the choice of $F(\cdot)$, for $X \in \{M, A, C\}$. Using the bounds in (\ref{eq:prop8_20}) and (\ref{eq:prop8_21}) we have

\[ \max_{F(\cdot)} \frac{\Pi_C - \Pi_C^*}{\Pi_A} \leq \frac{\gamma p S - \gamma p n_1^A}{\gamma p n_1^A} = \left( \frac{A_{21}}{A_{12}} - 1 \right) \frac{1}{1 + \frac{1}{1 - \rho_1} + \frac{\rho_2}{1 - \rho_2} \frac{A_{21}}{A_{12}}}. \]

To achieve this upper bound, we need $n_A^* \in (n_1^A, n_2^A)$ and $n_C^* \in [n_1^C, n_2^A]$ so that $\Pi_A^* = \gamma p n_1^A$ and $\Pi_C^* \to \gamma p S$.

Noticing Lemma 4 (2) and Property (ii) in the proof of Proposition 7 about $\pi_A(\cdot), \pi_C(\cdot)$, this holds for opportunity cost distributions $F(\cdot)$ satisfying

\[ F(\pi_A(n_2^A)) \leq n_2^A/N \quad \text{and} \quad F(\pi_C(n_2^A)) \geq n_2^C/N \]
\(\text{(A.22)}\)

when $\pi_A(n_2^A) < \pi_C(n_2^A)$. When $\pi_A(n_2^A) \geq \pi_C(n_2^A)$, (A.22) cannot be satisfied by any $F$ and hence the upper bound is not tight.

\[ \square \]
Proof of Proposition 9. Since $\pi_M(n) = \pi_A(n) = \pi_C(n)$ for $n \leq n^M_1$ and $n \geq n^M_3$, the per-driver profit rate gain from admission control only (regime $A$ over $M$) and from admission control plus centralized repositioning (regime $C$ over $M$) can be positive only for $n^*_M \in (n^M_1, n^M_3)$, and $n^*_A, n^*_C \in (n^M_1, n^M_3)$ simultaneously. It follows from the (decreasing) monotonicity of $\pi_X(\cdot)$, $X \in \{M, A, C\}$ that

$$\pi^*_M \geq \pi_M(n^M_3), \quad \pi^*_A \leq \pi_A(n^M_1) = \tilde{\gamma}p - c, \quad \pi^*_C \leq \pi_C(n^M_1) = \tilde{\gamma}p - c. \quad (A.23)$$

Therefore,

$$\max_{F(\cdot)} \frac{\pi^*_A - \pi^*_M}{\pi^*_M} \leq \frac{\tilde{\gamma}p - c}{\pi_M(n^M_3)} - 1 = \frac{1 - \rho_2}{\tau - (1 - \rho_2 + \tau)\kappa}. \quad (A.24)$$

To achieve this upper bound, we need $n^*_M \in [n^M_2, n^M_3)$ and $n^*_A, n^*_C \in (n^M_1, n^A_1]$ so that the equalities in (A.23) are satisfied. If $n^M_2 \leq n^A_1 = n^C_1$, these conditions hold for $F(\cdot)$ satisfying

$$F(\pi_M(n^M_2)) \geq n^M_2 / N \quad \text{and} \quad F(\pi_A(n^A_1)) \leq n^A_1 / N. \quad (A.24)$$

If $n^M_2 > n^A_1 = n^C_1$, then (A.24) cannot be satisfied by any $F$ and the upper bound is not tight. □
Supplemental Materials

S1 Supplemental Lemmas and Proofs for Control Regime A

Under regime A, the lower capacity threshold $n_A^4$ of zone (3)—moderate capacity with repositioning—and the optimal capacity allocation within this zone, do not have explicit expressions (see Proposition 4). Lemmas S-1 and S-2 fill in the remaining details.

Given a level of participating capacity $n$, Proposition 4 shows that the optimal capacity allocation in zone (3) has $r_{12} > 0, r_{21} = 0$ and $q = (q_1^*(s), 0)$. Therefore, the constraints of Problem A at a given capacity $n$, (12a)–(12c) and (22), simplify to

$$\pi + \left( \frac{t_{12}}{t_{31}} s_{21} - s_{12} \right) + q_1^*(s) = n$$

(S.1)

and $0 \leq s \leq S, \frac{t_{12}}{t_{31}} s_{21} > s_{12}$. Note that $s$ determines $r_{12}$ by the second term and $q$ by $q_1^*(s)$.

Lemma S-1 shows that there are three possible optimal capacity allocation patterns at any level of participating capacity in zone (3). These patterns differ in terms of whether demand is rejected at the low-demand location, and if so, for which route(s).

Lemma S-1. Under control regime A, the optimal capacity allocation as a function of the participating capacity $n \in (n_2^A, n_3^A]$ (moderate capacity zone with repositioning) follows one of three patterns, denoted by $s_i(n), i = 1, 2, 3$. Let $\pi(n)$ denote the total service capacity under pattern $i$, and $\pi^{-1}(\cdot)$ denote its inverse. The optimal pattern $i^*$ is the one that attains the maximum service capacity, i.e., $i^* = \arg\max_{i \in \{1, 2, 3\}} \pi(n)$.

(1) No demand rejection at the low-demand location: only $s_{21}$ is increasing in this zone.

$$s_1(n) = (S_{11}, S_{12}, s_{21}, S_{22}) \text{ subject to (S.1), } n \in (\pi_1^{-1}(n_1^A), n_2^A].$$

(2) Rejecting cross-traffic demand at the low-demand location: for small $n$, $s_{21}$ is increasing while $s_{12} = 0$; for large $n$, $s_{21} = S_{21}$ and $s_{12}$ is increasing.

$$s_2(n) = (S_{11}, s_{12}, s_{21}, S_{22}) \text{ subject to } (S_{21} - s_{21}) s_{12} = 0 \text{ and (S.1), } n \in (\pi_2^{-1}(n_1^A), n_2^A].$$

(3) Rejecting local and cross-traffic demand at the low-demand location: for small $n$, $s_{21}$ is increasing while $s_{11} = s_{12} = 0$; for medium $n$, $s_{21} = S_{21}$, $s_{11}$ is increasing and $s_{12} = 0$; for large $n$, $s_{21} = S_{21}, s_{11} = S_{11}$ and $s_{12}$ is increasing.

$$s_3(n) = (s_{11}, s_{12}, s_{21}, S_{22}) \text{ subject to } (S_{21} - s_{21}) s_{11} = (S_{11} - s_{11}) s_{12} = 0 \text{ and (S.1), } n \in (\pi_3^{-1}(n_1^A), n_2^A].$$

Proof. By Proposition 4, the optimal capacity allocation of given participating capacity $n \in (n_2^A, n_3^A]$ has $r_{12} > 0, r_{21} = 0$ and $q = (q_1^*(s), 0)$. Therefore, for fixed $n$, Problem A reduces to $\max_{s,r,q} \{ \Pi(s) : (12a) - (12c), (22) \}$, and it can be reformulated as maximizing the total service capacity over feasible service capacity vector $s$:

$$\max_{s} \pi$$

(S.2a)

s.t. $g(s) := \pi + \left( \frac{t_{12}}{t_{31}} s_{21} - s_{12} \right) + q_1^*(s) \leq n$

(S.2b)

$$0 \leq s_{ij} \leq S_{ij}, \forall i, j.$$ 

(S.2c)
Note that \( g(s) \) is the total capacity expressed as a function of \( s \). Relaxing the equality constraint (S.2b) to the inequality constraint (S.2b) does not matter since positive \( q_2 \) and \( q_1 = q_1(s) + k(s)q_2 \) are feasible by (22) (but not optimal by Proposition 4). Constraint \( r_{ij} = \frac{1}{t_{21}} s_{21} - s_{12} > 0 \) is omitted since a violation results in \( s_{21} \leq \frac{1}{t_{21}} \Rightarrow \pi \leq n_1^4 \), clearly suboptimal in zone (3).

Let \( \alpha, \beta_{ij}, \bar{\beta}_{ij} \) be the dual variables associated with the capacity constraint (S.2b), the upper and lower bound constraints (S.2c), respectively. The KKT conditions are

\[
\begin{align*}
\text{(stationarity)} & \quad \alpha \frac{\partial g(s)}{\partial s_{ij}} + \beta_{ij} - \bar{\beta}_{ij} = 1, \forall i, j, \\
\text{(complementary slackness)} & \quad \alpha(n - g(s)) = \bar{\beta}_{ij}(S_{ij} - s_{ij}) = \beta_{ij}s_{ij} = 0, \forall i, j, \\
\text{(dual feasibility)} & \quad \alpha, \beta_{ij}, \bar{\beta}_{ij} \geq 0, \forall i, j, \\
\text{(primal feasibility)} & \quad g(s) \leq n, \\
\text{(primal feasibility)} & \quad 0 \leq s_{ij} \leq S_{ij}, \forall i, j. 
\end{align*}
\]

The complementary slackness constraints (S.3b) and dual feasibility constraints (S.3c) establish the relationship between primal and dual variables: \( \bar{\beta}_{ij} = 0 (\beta_{ij} = 0) \) when \( s_{ij} \) is at its upper (lower) bound; \( s_{ij} \) must be at its upper (lower) bound when \( \beta_{ij} > 0 (\bar{\beta}_{ij} > 0) \); \( \alpha = 0 \) when \( g(s) < n \) and \( g(s) = n \) when \( \alpha > 0 \). Moreover, \( \beta_{ij}, \bar{\beta}_{ij} = 0 \). We omit explicit references to the primal and dual feasibility constraints (S.3c)–(S.3e) in the following proof.

To prove the lemma, we will use the above KKT conditions to establish that any optimal solution to problem (S.2a)–(S.2c) for \( n \in (n_1^2, n_2^2) \) must satisfy four necessary conditions (a)–(d) stated below. Before that, we calculate some first and second partial derivatives of \( g(s) \) that will also be used to prove the four conditions:

\[
\begin{align*}
\frac{\partial g(s)}{\partial s_{11}} & = \frac{s_{21}(1 + \frac{t_{12}}{t_{21}})}{(s_{21} + s_{22})} + s_{22} - (\frac{s_{21} + s_{22} + s_{21}}{t_{21}}) (\gamma p - c) > 1, \\
\frac{\partial g(s)}{\partial s_{12}} & = \frac{s_{21}(1 + \frac{t_{12}}{t_{21}})}{(s_{21} + s_{22})} + s_{22} - (\frac{s_{21} + s_{22} + s_{21}}{t_{21}}) (\gamma p - c) > 1, \\
\frac{\partial g(s)}{\partial s_{21}} & = 1 + \frac{t_{12}}{t_{21}} + \frac{(s_{11}(\gamma p - c) + s_{12}(\gamma p))s_{21}}{(s_{21} + s_{22})(\gamma p - (s_{21} + s_{22} + s_{21}) \frac{t_{12}}{t_{21}})} > 1, \\
\frac{\partial g(s)}{\partial s_{22}} & = 1 - \frac{(s_{11}(\gamma p - c) + s_{12}(\gamma p))s_{21}}{(s_{21} + s_{22})(\gamma p - (s_{21} + s_{22} + s_{21}) \frac{t_{12}}{t_{21}})} < 1. 
\end{align*}
\]

It follows that

\[
\frac{\partial^2 g(s)}{\partial s_{11}^2} = \frac{\partial^2 g(s)}{\partial s_{12}^2} = \frac{\partial^2 g(s)}{\partial s_{11} \partial s_{21}} = \frac{\partial^2 g(s)}{\partial s_{12} \partial s_{21}} = 0, \quad \frac{\partial^2 g(s)}{\partial s_{11} \partial s_{12}} > 0, \quad \frac{\partial^2 g(s)}{\partial s_{21}^2} \leq 0,
\]

where the last inequality is strict when \( s_{11} + s_{22} \).

Now we are ready to state and prove the four necessary conditions.

(a) All capacity is used within this zone and \( s_{21} \) has a lower bound:

\[
\begin{align*}
g(s) & = n, \\
S_{12} \frac{t_{21}}{t_{12}} & \leq s_{21}. 
\end{align*}
\]

To prove (S.9), note that when \( s \neq S \), pick any \( s_{ij} < S_{ij} \), then \( \bar{\beta}_{ij} = 0 \) by (S.3b). By (S.3a) this implies that \( \alpha \neq 0 \) and hence \( g(s) = n \) by (S.3b). When \( s = S \), \( g(s) = n = n_1^4 \). For (S.10), \( s_{21} \geq S_{12} \frac{t_{21}}{t_{12}} \) follows directly from \( \pi > n_1^4 \) in zone (3). By (S.3b), \( s_{21} > 0 \) also implies \( \bar{\beta}_{ij} = 0 \).
(b) Rejecting local demand at the high-demand location ($s_{22}$) is suboptimal:

$$s_{22} = S_{22}.$$  \hspace{1cm} (S.11)

Using $\beta_{21} = 0$ from part (a) and $\frac{\partial g(s)}{\partial s_{22}} > 1$, stationarity constraints (S.3a) imply that $\alpha < 1$. Putting this and $\frac{\partial g(s)}{\partial s_{22}} < 1$ back to (S.3a), we obtain $\beta_{22} > 0$. Therefore it follows from (S.3b) that $s_{22} = S_{22}$ and $\beta_{22} = 0$.

(c) Rejecting cross-traffic demand ($s_{12}$) is more profitable than rejecting local demand ($s_{11}$) at the low-demand location:

$$(S_{11} - s_{11})s_{12} = 0.$$  \hspace{1cm} (S.12)

We prove this by contradiction using (S.3a) and (S.3b). Suppose on the contrary $(S_{11} - s_{11})s_{12} > 0$ for some $s_{11} < S_{11}$ and $s_{12} > 0$, then (S.3b) require $\beta_{11} = \beta_{12} = 0$ and hence (S.3a) yield

$$\alpha \frac{\partial g(s)}{\partial s_{11}} - \beta_{11} = \alpha \frac{\partial g(s)}{\partial s_{12}} + \beta_{12} = 1.$$  

This cannot happen due to $\frac{\partial g(s)}{\partial s_{11}} < \frac{\partial g(s)}{\partial s_{12}}$ and (S.3c). Therefore we must have $(S_{11} - s_{11})s_{12} = 0$.

(d) Neither demand stream at the low-demand location is partially served unless $s_{21}$ is fully served:

$$s_{12}(S_{12} - s_{12})(S_{21} - s_{21}) = 0,$$  \hspace{1cm} (S.13)

$$s_{11}(S_{11} - s_{11})(S_{21} - s_{21}) = 0.$$  \hspace{1cm} (S.14)

We prove the two equations in similar ways by showing that any violation will lead to suboptimality. For (S.13), suppose on the contrary $s_{12}(S_{12} - s_{12})(S_{21} - s_{21}) > 0$ for some $0 < s_{12} < S_{12}$ and $s_{21} < S_{21}$, then (S.3b) and $\beta_{21} = 0$ from part (a) require $\beta_{12} = \beta_{21} = 0$. It hence follows from (S.3a) that

$$\alpha \frac{\partial g(s)}{\partial s_{11}} = \alpha \frac{\partial g(s)}{\partial s_{21}} = 1,$$

thus $\alpha > 0$ and $1 < \frac{\partial g(s)}{\partial s_{11}} < \frac{\partial g(s)}{\partial s_{12}} = \frac{\partial g(s)}{\partial s_{21}}$. By the second derivatives in (S.8), increasing $s_{12}$ and decreasing $s_{21}$ will always maintain the inequality

$$1 < \frac{\partial g(s)}{\partial s_{11}} < \frac{\partial g(s)}{\partial s_{12}} < \frac{\partial g(s)}{\partial s_{21}}.$$  \hspace{1cm} (S.15)

Therefore we can keep increasing $s_{12}$ ($\Delta s_{12} > 0$) and decreasing $s_{21}$ ($\Delta s_{21} < 0$) simultaneously such that the following equality holds at any subsequent $s_{12}$ and $s_{21}$:

$$\Delta s_{12} \frac{\partial g(s)}{\partial s_{12}} + \Delta s_{21} \frac{\partial g(s)}{\partial s_{21}} = 0.$$  \hspace{1cm} (S.16)

In this way we can maintain

$$\Delta g(s) = \sum_{i,j} \frac{\partial g(s)}{\partial s_{ij}} \Delta s_{ij} = \Delta s_{12} \frac{\partial g(s)}{\partial s_{12}} + \Delta s_{21} \frac{\partial g(s)}{\partial s_{21}} = 0,$$ \hspace{1cm} i.e., keep $g(s)$ constant, while improving the objective function (service capacity) by

$$\Delta \beta = \Delta s_{12} + \Delta s_{21} = \Delta s_{12} \left(1 - \frac{\partial g(s)}{\partial s_{12}} \frac{\partial g(s)}{\partial s_{21}} \right) > 0,$$

which follows from (S.15) and (S.16), until $s_{12}(S_{12} - s_{12})(S_{21} - s_{21}) = 0$ is satisfied.

Similarly, for (S.14), suppose on the contrary $s_{11}(S_{11} - s_{11})(S_{21} - s_{21}) > 0$ for some $0 < s_{11} < S_{11}$ and $s_{21} < S_{21}$,
We have thus shown all the three possible patterns. (2) \( s_{21} = \) constant service capacity in zone (2), and up to (1) \( n \) as how service capacity varies with an increasing

**Lemma S-2.** Per-driver profit rate under control regime \( A \), \( \pi_A(n) \), is decreasing in \( n \) for \( n \in (n_2^A, n_3^A) \).
Proof. Substituting \( \bar{s} \) and \( \bar{r} \) from Proposition 4 into (12d) yields

\[
\pi_A(n) = \frac{(\bar{\gamma}p - c)\bar{s} - cr}{n} = \begin{cases} 
\bar{\gamma}p - c & \text{zone (1) } (n \leq n_1^3), \\
\frac{n^4}{n} (\bar{\gamma}p - c) & \text{zone (2) } (n_1^3 < n \leq n_2^3), \\
\frac{1}{n} [(\bar{\gamma}p - c)\bar{s} - cr] & \text{zone (3) } (n_2^3 < n \leq n_3^3), \\
\frac{1}{n} (\bar{\gamma}p\bar{s} - cn_2^3) & \text{zone (4) } (n > n_3^3). 
\end{cases} \tag{S.19}
\]

Lemma S-1 shows that for participating capacity \( n \in (n_1^3, n_3^3) \), the optimal capacity allocation may alternate among three patterns characterized by \( s_i(n) \), with service capacity \( \bar{\pi}(n) \) for \( i = 1, 2, 3 \). To prove this lemma, we show that the per-driver profit rate is decreasing for \( n \) varying within each of the 3 patterns or at feasible transitions between patterns. First, note that \( n = g(s) \) in zone (3) (see (S.9)) from the proof of Lemma S-1. Hence \( \partial n / \partial s_{ij} = \partial g(s) / \partial s_{ij} \).

Then:

(i) **Within pattern (1):** only \( s_{21} \) is increasing,

\[
\pi'(n) = \frac{\left( \bar{\gamma}p - c - \frac{t_{12}}{t_{21}} \right) \frac{\partial g(s)}{\partial s_{21}}}{n^2} n - \left( \bar{\gamma}p - c - cr_{12} \right)
= - \frac{S_{22}\bar{\gamma}pt_{12}}{n^2 t_{21}} \left( \frac{\partial g(s)}{\partial s_{21}} \right)^{-1} \left( 1 + \frac{S_{11}(\bar{\gamma}p - c) + S_{12}\bar{\gamma}p}{(s_{21} + S_{22})\bar{\gamma}p - (s_{21} + S_{22} + s_{21} t_{12} t_{21})} \right)^2 < 0. \tag{S.20}
\]

(ii) **Within pattern (2):** for small \( n, s_{21} \) is increasing while \( s_{12} = 0 \),

\[
\pi'(n) = \frac{\left( \bar{\gamma}p - c - \frac{t_{12}}{t_{21}} \right) \frac{\partial g(s)}{\partial s_{21}}}{n^2} n - \left( \bar{\gamma}p - c - cr_{12} \right)
= - \frac{S_{22}\bar{\gamma}pt_{12}}{n^2 t_{21}} \left( \frac{\partial g(s)}{\partial s_{21}} \right)^{-1} \left( 1 + \frac{S_{11}(\bar{\gamma}p - c)}{(s_{21} + S_{22})\bar{\gamma}p - (s_{21} + S_{22} + s_{21} t_{12} t_{21})} \right)^2 < 0. \tag{S.21}
\]

For large \( n, s_{21} = S_{21} \) and \( s_{12} \) is increasing,

\[
\pi'(n) = \frac{\bar{\gamma}p \left( \frac{\partial g(s)}{\partial s_{12}} \right)^{-1}}{n^2} n - \left( \bar{\gamma}p - c - cr_{12} \right) = 0. \tag{S.22}
\]

Note that \( \pi(n) \) is continuous at the turning (non-differentiable) point where \( s = (S_{11}, 0, S_{21}, S_{22}) \).

(iii) **Within pattern (3):** for small \( n, s_{21} \) is increasing while \( s_{11} = s_{12} = 0 \),

\[
\pi'(n) = \frac{\left( \bar{\gamma}p - c - \frac{t_{12}}{t_{21}} \right) \frac{\partial g(s)}{\partial s_{11}}}{n^2} n - \left( \bar{\gamma}p - c - cr_{12} \right)
= - \frac{S_{22}\bar{\gamma}pt_{12}}{n^2 t_{21}} \left( \frac{\partial g(s)}{\partial s_{11}} \right)^{-1} < 0. \tag{S.23}
\]

For medium \( n, s_{21} = S_{21}, s_{11} \) is increasing and \( s_{12} = 0 \),

\[
\pi'(n) = \frac{\bar{\gamma}p - c \left( \frac{\partial g(s)}{\partial s_{11}} \right)^{-1}}{n^2} n - \left( \bar{\gamma}p - c - cr_{12} \right) = 0. \tag{S.24}
\]

For large \( n, s_{21} = S_{21}, s_{11} = S_{11} \) and \( s_{12} \) is increasing, we have the same (S.22). Note that \( \pi(n) \) is continuous at the two turning (non-differentiable) points where \( s = (0, 0, S_{21}, S_{22}) \) and \( s = (S_{11}, 0, S_{21}, S_{22}) \).

As \( n \) increases, an optimal transition from pattern \( i \) to \( j \) at \( n \) must satisfy

\[
\bar{s}(n) = \bar{s}_j(n) \quad \text{and} \quad \bar{s}_i(n^-) < \bar{s}_j(n^+). \tag{S.25}
\]

Namely, patterns \( i \) and \( j \) have the same service capacity at transition \( n \), and the service capacity increases faster after the transition. We then discuss all three possible transitions.
(i) Between patterns (1) and (2), it is only optimal to transit from (1) to (2). Recall pattern (2) given in Lemma S-1, \( n \) can be small or large at the transition. If \( n \) is small such that \( s_{21} \) is increasing while \( s_{12} = 0 \) at the transition, we have \( s_1 = (S_{11}, S_{12}, s_{21}^{(1)}, S_{22}) \), \( s_2 = (S_{11}, 0, s_{21}^{(2)}, S_{22}) \).\(^{10}\) By \( \varpi = \varpi^n \) in (S.25), there must be \( s_{21}^{(1)} < s_{21}^{(2)} \) and hence

\[
\varpi'(n) = \left( \frac{\partial g(s)}{\partial s_{21}^{(1)}} \right)^{-1} < \left( \frac{\partial g(s)}{\partial s_{21}^{(2)}} \right)^{-1} = \varpi_n',
\]

where the inequality follows from \( \partial^2 g(s)/\partial s_{21}^2 < 0 \) given in (S.8). Therefore, (S.25) implies that the transition must be from pattern (1) to (2): \( (S_{11}, S_{12}, s_{21}^{(1)}, S_{22}) \to (S_{11}, 0, s_{21}^{(2)}, S_{22}) \). Obviously \( r_{12} \) jumps up and \( \pi(n) \) jumps down at the transition.

If \( n \) is large such that \( s_{21} = S_{21} \) and \( s_{12} \) is increasing (or just starts increasing from 0) at the transition, we have \( s_1 = (S_{11}, S_{12}, s_{21}, S_{22}) \), \( s_2 = (S_{11}, s_{12}, S_{21}, S_{22}) \). There must be \( \varpi'(n^+) < \varpi_n'^+ \) since otherwise

\[
\varpi(n^+) = \varpi(n) + \int_n^{n^+} \varpi'(x)dx > \varpi(n) + \int_n^{n^+} \varpi_n'(n)dx \\
\geq \varpi(n) + \int_n^{n^+} \varpi_n'(n^+)dx = \varpi_n(n^+) + \int_n^{n^+} \varpi_n'(n)dx = \varpi_n(n^+),
\]

where the first inequality follows from \( \varpi'(x) = (\partial g(s)/\partial s_{21})^{-1} \) with \( g(s) = x \) and \( \partial^2 g(s)/\partial s_{21}^2 < 0 \) given in (S.8), the second inequality is by the opposite assumption that \( \varpi_{n^+} = \varpi_n(n^+) \), and the next equality follows from \( \varpi(n) = \varpi(n) \) (at transition), \( \varpi_n(x) = (\partial g(s)/\partial s_{21})^{-1} \) with \( g(s) = x \), and \( \partial^2 g(s)/\partial s_{21}^2 = 0 \) given in (S.8). Therefore, (S.25) implies that the transition must be from pattern (1) to (2): \( (S_{11}, S_{12}, s_{21}, S_{22}) \to (S_{11}, s_{12}, S_{21}, S_{22}) \). Similarly, \( r_{12} \) jumps up and \( \pi(n) \) jumps down at the transition.

(ii) Between patterns (1) and (3), it is only optimal to transit from (1) to (3). Recall pattern (3) given in Lemma S-1, \( n \) can be small, medium or large at the transition. The case where \( n \) is small (such that \( s_{21} \) is increasing) or large (such that \( s_{12} \) is increasing) can be shown identically as above. For the case where \( n \) is medium, such that \( s_{11} \) is increasing (or just starts increasing from 0) while \( s_{12} = 0, s_{21} = S_{21} \) at the transition, we have \( s_1 = (S_{11}, S_{12}, s_{21}, S_{22}) \), \( s_2 = (s_{11}, 0, S_{21}, S_{22}) \). There must be \( \varpi'(n^+) < \varpi_n'(n^+) \) since otherwise

\[
\varpi(n^+) = \varpi(n) + \int_n^{n^+} \varpi'(x)dx > \varpi(n) + \int_n^{n^+} \varpi_n'(n)dx \\
\geq \varpi(n) + \int_n^{n^+} \varpi_n'(n^+)dx = \varpi_n(n^+) + \int_n^{n^+} \varpi_n'(n)dx = \varpi_n(n^+),
\]

where \( \int_n^{n^+} \varpi_n'(x)dx \) is an integration from \( n \) to \( \varpi_n^{-1}((S_{11}, 0, S_{21}, S_{22})) \) and then to \( n^+ \). The reasoning for the (in)equalities is analogous to that in above part (i). Therefore, (S.25) implies that the transition must be from pattern (1) to (3): \( (S_{11}, S_{12}, s_{21}, S_{22}) \to (s_{11}, 0, S_{21}, S_{22}) \). Apparently \( r_{12} \) jumps up and \( \pi(n) \) jumps down at the transition.

(iii) Between patterns (2) and (3), it is only optimal to transit from (2) to (3). Recall patterns (2) and (3) given in Lemma S-1, the capacity allocations are the same in the case where \( n \) is large (such that \( s_{11} = S_{11}, s_{12} = S_{12} \) while \( s_{12} \) is increasing), thus transitions must happen when \( n \) is not large, where \( s_{12} \equiv 0 \) is in common.

Furthermore, when \( n \) is not large, there is only one case under pattern (2) where \( s_{21} \) is increasing, and there

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\(^{10}\) We use superscript numbers to distinguish patterns under consideration.
are two cases under pattern (3): small \( n \) where \( s_{11} = 0 \) and \( s_{21} \) is increasing, and medium \( n \) where \( s_{21} = S_{21} \) and \( s_{11} \) is increasing. This shares a similar structure as the discussion between patterns (1) and (2) in part (i) above, so we omit the details here. Similarly, note that the transitions, if any, are always from pattern (2) to (3).

**Proof of Proposition 5.** From Lemma S-1, it is optimal to reject rider requests at the low-demand location for some \( n \) in zone (3) if and only if pattern (2) or (3) provides the largest service capacity at some \( n \in (n_2^A, n_3^A) \), i.e., \( \exists n \in (n_2^A, n_3^A) \) such that \( \hat{s}_1(n) < \max_{i \in \{2,3\}} \hat{s}_i(n) \). We need to compare the three patterns in terms of their service capacity \( \hat{s}_i(n) \), \( i = 1, 2, 3 \). We have the following three observations.

(i) At the right end of zone (3), \( \hat{s}_1(n_3^A) = \hat{s}_2(n_3^A) = \hat{s}_3(n_3^A) = \hat{S} \).

(ii) For \( n \) close to \( n_3^A \) (\( n \to n_3^A^- \)), it follows from Lemma S-1 that pattern (1) has \( s_1(n) = (S_{11}, S_{12}, s_{21}, S_{22}) \) with \( s_{21} \) varying, while pattern (2) and (3) both have \( s_2(n) = s_3(n) = (S_{11}, s_{12}, S_{21}, S_{22}) \) with \( s_{12} \) varying. Therefore we have
\[
\hat{s}_1'(n_3^A) = \left. \frac{\partial \hat{s}_1(n)}{\partial s_{12}} \right|_{s_{12}=S} = \left( \left. \frac{\partial g(s)}{\partial s_{12}} \right|_{s_{12}=S} \right) > 0,
\]
\[
\hat{s}_2'(n_3^A) = \left. \frac{\partial \hat{s}_2(n)}{\partial s_{21}} \right|_{s_{21}=s_{21}^S} = \left. \frac{\partial^2 g(s)}{\partial s_{21}^2} \right|_{s_{21}=s_{21}^S} < 0,
\]
i.e., \( \hat{s}_1(n) \) is strictly convex and increasing in \( n \) near \( n_3^A \). And
\[
\hat{s}_2'(n_3^A) = \hat{s}_3'(n_3^A) = \left. \frac{\partial \hat{s}_2(n)}{\partial s_{12}} \right|_{s_{12}=s_{12}^S} = \left. \frac{\partial^2 g(s)}{\partial s_{12}^2} \right|_{s_{12}=s_{12}^S} = 0,
\]
i.e., \( \hat{s}_2(n) \) and \( \hat{s}_3(n) \) both increase linearly in \( n \) near \( n_3^A \).

(iii) The proof of Lemma S-2 establishes that any optimal pattern transitions as \( n \) increases in zone (3) must be from pattern (1) to (2), from pattern (1) to (3), or from pattern (2) to (3)—not vice versa.

Using the above observations, the sufficient and necessary condition that pattern (2) or (3) provides the largest service capacity at some \( n \in (n_2^A, n_3^A) \) is
\[
\hat{s}_2'(n_3^A) = \hat{s}_3'(n_3^A) < \hat{s}_1'(n_3^A).
\]
(S.26)

To see this, if (S.26) holds, observation (i) and (ii) immediately imply that \( \hat{s}_2(n_3^A^-) = \hat{s}_1(n_3^A^-) > \hat{s}_3(n_3^A^-) \), hence patterns (2) and (3) provide the (same) largest service capacity close to \( n_3^A \). On the other hand, if (S.26) does not hold, observation (i) and (ii) imply that \( \hat{s}_2(n_3^A^-) = \hat{s}_3(n_3^A^-) < \hat{s}_1(n_3^A^-) \), i.e., pattern (1) is optimal near \( n_3^A \). It then follows from observation (iii) that there is no transition from pattern (2) or (3) to pattern (1) as \( n \) increases in zone (3), hence pattern (1) is optimal throughout zone (3). Therefore (S.26) is sufficient and necessary for pattern (2) or (3) to be optimal somewhere in zone (3).

We now translate (S.26) to condition (34) in the proposition. Putting the first derivatives in observation (ii) into
The steady-state system constraints include: (i) flow balance at each location (outflows equal inflows), and (ii) fractions resulting from length vectors, respectively. Denoting \( R \) (with zero diagonal elements) the repositioning rate matrix, and where the number of participating drivers satisfies the participation equilibrium \( \eta(\lambda, \nu) \in H := \{ \eta \in \mathbb{R}^{L \times L} : \eta 1 = 1 \} \) as the matrix of steady-state repositioning fractions resulting from \( \lambda \) and \( \nu \), we have

\[
\eta_{ij}(\lambda, \nu) = \begin{cases} \frac{\sum_{k \in V} \lambda_{ik} \nu_{jk}}{\sum_{k \in V} (\lambda_{ik} + \nu_{jk})}, & j = i, i, j \in V. \\ \frac{\nu_{ij}}{\sum_{k \in V} (\lambda_{ik} + \nu_{jk})}, & j \neq i. \end{cases}
\]  

(S.27)

The steady-state system constraints include: (i) flow balance at each location (outflows equal inflows), \( \sum_{i \in V} (\lambda_{ij} + \nu_{ij}) = \sum_{k \in V} (\lambda_{ik} + \nu_{ik}), \forall j \in V \), and (ii) total capacity \( \sum_{i,j \in V} (\lambda_{ij} + \nu_{ij})t_{ij} + \sum_{i \in V} [w_i \sum_{j \in V} \lambda_{ij}] = n \).

S2 Extension to General Networks

In this section we extend the model to general networks and provide additional numerical results for the three-location networks in Figure 5.

S2.1 Model Primitives

Consider a general \( L \)-location network with location (node) set \( V = \{1, \ldots, L\} \) and route (arc) set \( V \times V \). Denote by \( t \in \mathbb{R}^{L \times L} \) the (constant) travel time matrix and by \( A \in \mathbb{R}^{L \times L} \) the potential demand rate matrix (given rider price \( p \) per unit of travel time). Drivers’ profit and participation have the same structure as described in §2.1. As direct extensions from the two-location network, denote by \( \lambda \in \mathbb{R}^{L \times L} \) (\( \lambda \leq A \)) the effective demand rate matrix, \( \nu \in \mathbb{R}^{L \times L} \) (with zero diagonal elements) the repositioning rate matrix, and \( w, q \in \mathbb{R}_+^L \) the steady-state waiting time and queue length vectors, respectively. Denoting \( \eta(\lambda, \nu) \) \( H \) \( \eta 1 = 1 \) as the matrix of steady-state repositioning fractions resulting from \( \lambda \) and \( \nu \), we have

\[
\eta_{ij}(\lambda, \nu) = \begin{cases} \frac{\sum_{k \in V} \lambda_{ik} \nu_{jk}}{\sum_{k \in V} (\lambda_{ik} + \nu_{jk})}, & j = i, i, j \in V. \\ \frac{\nu_{ij}}{\sum_{k \in V} (\lambda_{ik} + \nu_{jk})}, & j \neq i. \end{cases}
\]  

(S.27)

The steady-state system constraints include: (i) flow balance at each location (outflows equal inflows), \( \sum_{i \in V} (\lambda_{ij} + \nu_{ij}) = \sum_{k \in V} (\lambda_{ik} + \nu_{ik}), \forall j \in V \), and (ii) total capacity \( \sum_{i,j \in V} (\lambda_{ij} + \nu_{ij})t_{ij} + \sum_{i \in V} [w_i \sum_{j \in V} \lambda_{ij}] = n \).

Similar to the two-location network model, per-driver profit rate can be computed in two ways: (i) as the per-driver proportion of the cumulative driver profits:

\[
\pi(\lambda, \nu, n) = \frac{(\gamma p - c) \sum_{i,j \in V} \lambda_{ij}t_{ij} - c \sum_{i,j \in V} \nu_{ij}t_{ij}}{n},
\]

where the number of participating drivers satisfies the participation equilibrium \( n = NF(\pi(\lambda, \nu, n)) \), and (ii) from the perspective of an individual driver circulating through the network, \( \bar{\pi}(\bar{\eta}; \lambda, w) \), as a function of her own repositioning strategy (fractions) \( \bar{\eta} \) given the routing probabilities implied by \( \lambda \) and the queueing delays \( w \). In equilibrium, the repositioning fractions \( \eta(\lambda, \nu) \) induced by the aggregate flow rates \( (\lambda, \nu) \) through (S.27) maximizes the individual drivers’ profit rate, i.e.,

\[
\eta(\lambda, \nu) \in \arg\max_{\eta \in H} \bar{\pi}(\bar{\eta}; \lambda, w).
\]  

(S.28)

Since every driver chooses \( \eta(\lambda, \nu) \), each earns the same profit rate, so that \( \bar{\pi}(\eta(\lambda, \nu); \lambda, w) = \pi(\lambda, \nu, n) \) for all \( (\lambda, \nu, w, n) \) tuples that satisfy the system flow constraints described above.

Reformulating the Repositioning Equilibrium Constraint (S.28). Let \( E^{ij} \) be the matrix operator that replaces a matrix’s \( s \)th row by the basis vector \( e_j \), e.g., \( E^{ij}\eta \) denotes the altered repositioning strategy \( \eta \) where the driver chooses a pure strategy at location \( i \) that repositions only to \( j \). At any location \( i \), let \( \mathcal{R}_i(\eta) := \{ j \in V : \)
the subset of locations that are assigned positive probabilities by repositioning strategy \( \bar{\eta} \), so 
\(|R_i(\bar{\eta})| = 1\) is a pure strategy and 
\(|R_i(\bar{\eta})| > 1\) is a mixed strategy. Since all locations in \( R_i(\bar{\eta}) \), if adopted as the only repositioning destination under pure strategies, must yield equal profit rate that is higher than locations outside \( R_i(\bar{\eta}) \), we have

\[
\bar{\pi}(\mathcal{E}^{ij}_{\bar{\eta}}; \lambda, w) = \bar{\pi}(\mathcal{E}^{il}_{\bar{\eta}}; \lambda, w) > \bar{\pi}(\mathcal{E}^{il}_{\bar{\eta}}; \lambda, w), \quad \forall j, k \in R_i(\bar{\eta}), \ l \notin R_i(\bar{\eta}).
\]

Therefore, the repositioning equilibrium constraint (S.28) is equivalent to the following constraints (S.29)–(S.31), where \( \eta \) represents the repositioning fractions \( \eta(\lambda, \nu) \) determined in (S.27).

\[
\begin{align*}
\bar{\pi}(\mathcal{E}^{ij}_{\bar{\eta}}; \lambda, w) & = \xi - \zeta_{ij}, \ \forall i, j \in V, \quad (S.29) \\
\zeta_{ij}\eta_{ij} & = 0, \ \forall i, j \in V, \quad (S.30) \\
\xi & \in \mathbb{R}^L, \ \zeta \in \mathbb{R}^{L \times L}_+, \ \eta \in \mathbb{R}^{L \times L}_+. \quad (S.31)
\end{align*}
\]

### S2.2 Steady-State Per-Driver Profit Rate

Given the steady-state system characterized by \((\lambda, w)\), we can formulate an individual driver’s location visiting process as a Semi-Markov Process (SMP), where the state is the latest location (node) the driver has visited and a transition occurs when the driver arrives at a location, before making a repositioning decision on whether to join the (potential) queue or reposition to another location. Then the driver’s cumulative profit process is a Markov Renewal-Reward Process \( \{\bar{\pi}(t)\} \) described by a sequence \( \{(Y_k, X_k, W_k)\}_{k \in \mathbb{N}} \) as \( \bar{\pi}(\sum_{i=1}^{k} X_i) = \sum_{i=1}^{k} W_i \), where state \( Y_k \) is the location after the \( k \)th transition, \( X_k \) is the sojourn time between the \((k-1)\)th and \( k \)th transition (which includes potential queuing delay and travel time in service or in repositioning), and reward \( W_k \) is the driver profit collected between the \((k-1)\)th and \( k \)th transition (which includes potential service revenue and driving cost).

The transition probability matrix of the embedded DTMC \( \{Y_k\} \) is a function of the driver’s repositioning strategy \( \bar{\eta} \) given the routing probabilities implied by \( \lambda, P(\bar{\eta}; \lambda) \), with elements

\[
P_{ij}(\bar{\eta}; \lambda) = \begin{cases} 
\bar{\eta}_{ii} \sum_{k \in V} \lambda_{ik} & \text{if } j = i \\
\bar{\eta}_{ij} + \bar{\eta}_{ji} \sum_{k \in V} \lambda_{ik} & \text{if } j \neq i,
\end{cases} \quad i, j \in V, \ \sum_{k \in V} \lambda_{ik} > 0. \quad (S.32)
\]

If \( \sum_{k \in V} \lambda_{ik} = 0 \) for some \( i \), then \( P_{ii}(\bar{\eta}; \lambda) = 1(\bar{\eta}_{ii} > 0) > 0 \) and \( P_{ij}(\bar{\eta}; \lambda) = \bar{\eta}_{ij}1(\bar{\eta}_{ii} = 0) \) for \( j \neq i \). In the case where \( \sum_{k \in V} \lambda_{ik} = 0 \) and \( \bar{\eta}_{ii} > 0 \), state \( i \) is absorbing. We assume \( \sum_{k \in V} \lambda_{ik} > 0 \) for \( i \in V \) so that the Markov chain is irreducible. Let \( p(\bar{\eta}; \lambda) \in \Delta^{L-1} \) be the associated stationary distribution.

The expected reward (driver profit) after a transition into location \( i \) is then given by

\[
R_i(\bar{\eta}; \lambda) := E(W_{k+1} | Y_k = i) = (\gamma p - c)\bar{\eta}_{ii} \sum_{j \in V} \lambda_{ij} t_{ij} - c \sum_{j \in V \setminus \{i\}} \bar{\eta}_{ij} t_{ij}, \quad i \in V, \quad (S.33)
\]

and the expected sojourn time after a transition into location \( i \) is

\[
T_i(\bar{\eta}; \lambda, w) := E(X_{k+1} | Y_k = i) = \bar{\eta}_{ii} \left[ w_i + \sum_{j \in V} \lambda_{ij} t_{ij} \right] + \sum_{j \in V \setminus \{i\}} \bar{\eta}_{ij} t_{ij}, \quad i \in V. \quad (S.34)
\]

Let \( R(\bar{\eta}; \lambda) \in \mathbb{R}^L \) and \( T(\bar{\eta}; \lambda, w) \in \mathbb{R}^{L}_+ \) be the corresponding vectors, respectively.\(^{11}\) The following proposition gives

\[P(\bar{\eta}; \lambda) = \text{diag}(D(\bar{\eta})) \left[ \text{diag}(\lambda 1)^{-1} \lambda \right] + \bar{\eta} - \text{diag}(D(\bar{\eta})),\]
It follows from renewal reward theory that
\[ \Pi = \lim_{t \to \infty} \frac{\pi(t)}{t} = \frac{e^T_1 [I - P^o(\bar{\eta}; \lambda)]^{-1} R(\bar{\eta}; \lambda)}{e^T_1 [I - P^o(\eta; \lambda)]^{-1} T(\eta; \lambda, w)}, \] (S.35)
where \( \Pi \) and \( \tau_1 \) are the expected cycle profit and cycle length, respectively, with cycles defined as consecutive arrivals at location 1 (before joining the queue or repositioning), and \( P^o = [0, P_2, \ldots, P_n] \) is the \( P(\bar{\eta}; \lambda) \) matrix with first column replaced by 0.

**Proposition 10.** An individual driver’s expected steady-state profit rate is a function of her repositioning strategy \( \bar{\eta} \) for system state \((\lambda, w)\), given by
\[
\bar{\pi}(\bar{\eta}; \lambda, w) = \frac{\Pi}{\tau_1} = \frac{e^T_1 [I - P^o(\bar{\eta}; \lambda)]^{-1} R(\bar{\eta}; \lambda)}{e^T_1 [I - P^o(\eta; \lambda)]^{-1} T(\eta; \lambda, w)},
\]
where \( \Pi \) and \( \tau_1 \) are the expected cycle profit and cycle length, respectively, with cycles defined as consecutive arrivals at location 1 (before joining the queue or repositioning), and \( P^o = [0, P_2, \ldots, P_n] \) is the \( P(\bar{\eta}; \lambda) \) matrix with first column replaced by 0.

**Proof.** Let \( \Pi_i, i \in V \) be the expected profit collected by a driver starting from location \( i \) (before joining the queue or repositioning) and ending at coming back to location 1, and \( \Pi \) the corresponding vector. We have
\[
\Pi_1 = \bar{\eta}_1 \sum_{j \in V} \sum_{l \in V} \lambda_{lj} [(\bar{\gamma} p - c) t_{lj} + \Pi_j 1(j \neq 1)] + \sum_{j \in V \setminus \{1\}} \bar{\eta}_{lj} (-c t_{lj} + \Pi_j),
\]
\[
\Pi_j = \bar{\eta}_{jj} \sum_{i \in V} \sum_{l \in V} \lambda_{il} [(\bar{\gamma} p - c) t_{il} + \Pi_i 1(l \neq 1)] + \sum_{i \in V \setminus \{j\}} \bar{\eta}_{ij} (-c t_{ij} + \Pi_i 1(l \neq 1)), \quad j \in V \setminus \{1\}.
\]
In vector form and using (S.32) and (S.33), we can derive
\[
\Pi = P^o(\bar{\eta}; \lambda) \Pi + \Pi(\bar{\eta}; \lambda) \Rightarrow \Pi = [I - P^o(\bar{\eta}; \lambda)]^{-1} R(\bar{\eta}; \lambda).
\]

Let \( \tau_i, i \in V \) be the expected duration starting from location \( i \) (before joining the queue or repositioning) and ending at coming back to location 1, and \( \tau \) the corresponding vector. We have
\[
\tau_1 = \bar{\eta}_1 \left[ w_1 + \sum_{j \in V} \sum_{l \in V} \lambda_{lj} (t_{lj} + \tau_j 1(j \neq 1)) \right] + \sum_{j \in V \setminus \{1\}} \bar{\eta}_{lj} (t_{lj} + \tau_j),
\]
\[
\tau_j = \bar{\eta}_{jj} \left[ w_j + \sum_{i \in V} \sum_{l \in V} \lambda_{il} (t_{il} + \tau_i 1(l \neq 1)) \right] + \sum_{i \in V \setminus \{j\}} \bar{\eta}_{ij} (t_{ij} + \tau_i 1(l \neq 1)), \quad j \in V \setminus \{1\}.
\]
In vector form and using (S.32) and (S.34), we can derive
\[
\tau = P^o(\bar{\eta}; \lambda) \tau + T(\bar{\eta}; \lambda, w) \Rightarrow \tau = [I - P^o(\bar{\eta}; \lambda)]^{-1} T(\bar{\eta}; \lambda, w).
\]
It follows from renewal reward theory that
\[
\bar{\pi}(\bar{\eta}; \lambda, w) = \frac{\Pi}{\tau_1} = \frac{e^T_1 [I - P^o(\bar{\eta}; \lambda)]^{-1} R(\bar{\eta}; \lambda)}{e^T_1 [I - P^o(\eta; \lambda)]^{-1} T(\eta; \lambda, w)}.
\]

**S2.3 Three Control Regimes: Problem Formulations**

We now formulate the platform’s revenue maximization problem under the three control regimes.
\[
R(\bar{\eta}; \lambda) = (\bar{\gamma} p - c) \text{diag}(D(\bar{\eta})) \left[ \text{diag}(\lambda) \right]^{-1} ((\lambda \circ t) 1) - c \left[ (\bar{\eta} - \text{diag}(D(\bar{\eta}))) \circ t \right] 1,
\]
\[
T(\bar{\eta}; \lambda, w) = \text{diag}(D(\bar{\eta})) \left[ w + \text{diag}(\lambda) \right]^{-1} ((\lambda \circ t) 1) + \left[ (\bar{\eta} - \text{diag}(D(\bar{\eta}))) \circ t \right] 1,
\]
where \( \text{diag}(a) \) generates a diagonal matrix from vector \( a \), and \( D(A) \) generates a vector from the diagonal elements of matrix \( A \).
Centralized Control (C). This benchmark regime can be formulated as the following optimization problem.

\[
\begin{align*}
\text{(Problem C)} \quad \max_{\lambda, \nu, w, n} \quad & \Pi(\lambda) := \gamma p \sum_{i,j \in V} \lambda_{ij} t_{ij} \\
\text{s.t.} \quad & \sum_{i \in V} (\lambda_{ij} + \nu_{ij}) = \sum_{k \in V} (\lambda_{jk} + \nu_{jk}), \quad \forall j \in V, \\
& \sum_{i,j \in V} (\lambda_{ij} + \nu_{ij}) t_{ij} + \sum_{i \in V} [w_i \sum_{j \in V} \lambda_{ij}] = n, \\
& 0 \leq \lambda \leq A, \; \nu \geq 0, \; w \geq 0, \\
& \pi(\lambda, \nu, n) = \left(\bar{\gamma} p - c\right) \sum_{i,j \in V} \lambda_{ij} t_{ij} - c \sum_{i,j \in V} \nu_{ij} t_{ij}, \\
& n = NF(\pi(\lambda, \nu, n)).
\end{align*}
\]

For fixed capacity \(n\) the problem for regime C is a simple LP given by

\[
\Pi_C(n) = \max_{\lambda, \nu, w} \{ \Pi(\lambda) : (S.36b)–(S.36d) \}. \tag{S.37}
\]

Admission Control (A). Representing the repositioning equilibrium constraint (S.28) using (S.29)–(S.31) together with (S.27) and (S.35), the platform’s problem can be formulated as an MPEC (Mathematical Program with Equilibrium Constraints):

\[
\begin{align*}
\text{(Problem A)} \quad \max_{\lambda, \nu, w, n, \xi, \zeta} & \quad \{ \Pi(\lambda) : (S.27), (S.29)–(S.31), (S.35), (S.36b)–(S.36f) \}. \tag{S.38}
\end{align*}
\]

For fixed capacity \(n\) the problem for regime A is a nonlinear problem given by

\[
\Pi_A(n) = \max_{\lambda, \nu, w, n, \xi, \zeta} \{ \Pi(\lambda) : (S.27), (S.29)–(S.31), (S.35), (S.36b)–(S.36d) \}. \tag{S.39}
\]

Minimal Control (M). Under pro-rata (FIFO) matching, we need the following additional admission constraints:

The effective demand rates are proportional to the potential demand at each location:

\[
\lambda_{ij} = k_i \Lambda_{ij}, \quad 0 \leq k_i \leq 1, \quad \forall i, j \in V, \tag{S.40}
\]

where \(k_i\) is the service rate at location \(i\). Drivers cannot be repositioning out of location \(i\) if the potential rider demand at that location has not been fully served, i.e.,

\[
(1 - k_i) \nu_{ij} = 0, \quad \forall i, j \in V, \tag{S.41}
\]

and demand requests originating at location \(i\) can only be lost if this location has no supply buffer, so no drivers are waiting, i.e.,

\[
(1 - k_i) w_i = 0, \quad \forall i \in V. \tag{S.42}
\]

Under this regime, the platform’s problem can be formulated as:

\[
\begin{align*}
\text{(Problem M)} \quad & \max_{\lambda, \nu, w, n, \xi, \zeta} \{ \Pi(\lambda) : (S.27), (S.29)–(S.31), (S.35), (S.36b)–(S.36f), (S.40)–(S.42) \}. \tag{S.43}
\end{align*}
\]

For fixed capacity \(n\) the problem for regime M is the nonlinear problem

\[
\Pi_M(n) = \max_{\lambda, \nu, w, n, \xi, \zeta} \{ \Pi(\lambda) : (S.27), (S.29)–(S.31), (S.35), (S.36b)–(S.36d), (S.40)–(S.42) \}. \tag{S.44}
\]
S2.4 Additional Numerical Results for Networks in Figure 5

In §6 we introduced several types of three-location ring network in Figure 5 and discussed main findings from network I under Admission Control. Here we present detailed capacity allocation for network I under Minimal Control (Table 3), as well as for networks II and III under Minimal Control and Admission Control (Tables 4 to 7). Note the common setting of unit travel times, rider price \( p = 4 \), commission rate \( \gamma = 25\% \) and driving cost \( c = 1 \).

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Table 3: Optimal capacity allocation for network I under Minimal Control (M)

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Table 4: Optimal capacity allocation for network II under Minimal Control (M)
### Table 5: Optimal capacity allocation for network II under Admission Control (A)

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### S3 Driver Supply and Actual Gains in Platform Revenue and Per-Driver Profit

In this section we illustrate the impact of the driver supply characteristics, specifically, the outside opportunity cost distribution $F$, on the actual platform revenue and per-driver profit gains, compared to the upper bounds in Propositions 8 and 9, and on the tension between the drivers’ and the platform’s gains. For simplicity we focus on the gains from admission control, i.e., regime A over M. (Similar effects determine the actual gains from repositioning.)

Figure 7 illustrates these gains for two opportunity cost distributions. Panel (a) presents a case where admission control yields large benefits for the platform as a result of a large increase in driver participation, and consequently only small benefits for individual drivers. Specifically, the top chart in panel (a) shows for the three control regimes the per-driver profits that are non-increasing functions of the capacity, and the increasing marginal opportunity cost function $F^{-1}(n/N)$. Achieving the upper bound on platform revenue gains from admission control requires two conditions, namely, $n^*_M = n^*_2$ or equivalently, $F^{-1}(n^*_2/N) = \pi_M(n^*_2)$, and $n^*_A = n^*_1$. The first condition holds in the example, the second condition requires infinitely elastic supply around the profit level $\pi_M(n^*_2)$, i.e., that $F$ grows sufficiently fast around this point such that $n^*_2 - n^*_2$ additional drivers join if the per-driver profit is slightly larger, so that $F^{-1}(n^*_2/N) = \pi_M(n^*_2)$. The example depicted in Figure 7 (a) shows how the upper bound can be approached if the supply increases substantially for a moderate change in per-driver profit rate.

Panel (b), in contrast, presents a case where admission control (under regime A or C) yields the maximum achievable per-driver profit gains as a result of a small increase in driver participation, and consequently only modest platform revenue gains. As shown in the top chart of panel (b), in this case the marginal opportunity cost function yields the same equilibrium capacity under minimal control as in panel (a), i.e., $F^{-1}(n^*_2/N) = \pi_M(n^*_2)$; however, the driver supply is so inelastic that the number of drivers willing to participate at the maximum profit rate $(\gamma p - c)$ is smaller than the minimum number required to serve all riders without repositioning, that is, $n^*_A < n^*_1$ where $F^{-1}(n^*_1/N) = c$. The platform’s commission is too high to entice more drivers to participate.
Table 6: Optimal capacity allocation for network III under Minimal Control (M)

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Table 7: Optimal capacity allocation for network III under Admission Control (A)
Figure 7: Impact of admission control on the equilibrium capacity, per-driver profit, and platform revenue ($S = (3, 1, 4, 6), \overline{S} = 14, N = 21, t = 1, \gamma = 0.25, p = 3, c = 0.45$)