



# The inefficiency of refinancing: Why prepayment penalties are good for risky borrowers<sup>☆</sup>

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## ABSTRACT

This paper provides a theoretical analysis of the efficiency of prepayment penalties in a dynamic competitive lending model with risky borrowers and costly default. When considering improvements in the borrower's creditworthiness as one of the reasons for refinancing mortgages, we show that refinancing penalties can be welfare improving and that they can be particularly beneficial to riskier borrowers in the form of lower mortgage rates, reduced defaults, and increased availability of credit. Thus, a high concentration of prepayment penalties among the riskiest borrowers can be an outcome of efficient equilibrium in a mortgage market. We also provide empirical evidence that is consistent with the key predictions of our model.

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## 1. Introduction

Prior to the recent mortgage crisis, prepayment penalties were widely used in mortgages given to less creditworthy, subprime borrowers. As the housing market collapsed, these clauses created a storm of criticism. In response to these

concerns, legislators and regulators have imposed new rules restricting the use of prepayment penalties.<sup>2</sup>

<sup>2</sup> See, among others, the Board of Governors of the Federal Reserve System's final rule amending Regulation Z published in July 2008 and the H.R. 1728: Mortgage Reform and Anti-Predatory Lending Act of 2009.

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The critics of prepayment penalties contend that lenders abused less sophisticated, poorer borrowers by offering them higher mortgage rates than they deserved and locking them into these high cost loans with the use of prepayment penalties. Consequently, the high default rates of these borrowers during the crisis were, in part, blamed on the predatory lending practices facilitated by the usage of prepayment penalties.<sup>3</sup> Importantly, the very high concentration of prepayment penalties among subprime borrowers compared with their low usage among more creditworthy, prime borrowers has been viewed as one of the key pieces of evidence that these clauses were used in a predatory manner.<sup>4</sup>

Motivated by this debate, we provide in this paper a theoretical analysis of the efficiency of prepayment penalties in a dynamic competitive lending model with costly default. When considering improvements in the borrower's creditworthiness (such as positive wealth shocks) as one of the reasons for refinancing mortgages, we show that refinancing penalties can be welfare improving, and that they can be particularly beneficial to riskier borrowers in the form of lower mortgage rates, reduced defaults, and increased availability of credit. Consequently, we argue that a high concentration of prepayment penalties among the riskiest borrowers can be an outcome of an efficient equilibrium in a mortgage market.

To formalize this argument, we develop a simple dynamic competitive lending model with fixed rate mortgages (FRMs), but with no changes in aggregate interest rates. Within the model, we consider borrowers who differ only in their initial wealth (a measure of credit quality). Borrowers obtain FRMs to purchase their homes. Homeownership is assumed to generate positive utility gains for the borrowers. Once a mortgage is originated, the borrower's creditworthiness evolves stochastically over time. When borrowers receive positive credit shocks, they would like to refinance to obtain a lower mortgage rate that is commensurate with their new lower default risk. Borrowers who receive severe enough negative financial shocks will default. Default is assumed to be costly, due to foreclosure costs and other deadweight losses. Over time, the borrowers who choose to refinance are those whose creditworthiness has improved the most. Thus, without restrictions on prepaying a loan, mortgage

pools are increasingly composed of the least creditworthy borrowers; that is, borrowers who have received zero or negative credit shocks since mortgage origination.<sup>5</sup>

Our model allows us to solve for the equilibrium mortgage interest rate. As in all credit models, the mortgage premium increases as observed credit quality falls. Higher mortgage premiums compensate the lender for larger expected losses due to increased defaults by riskier borrowers. However, our model also generates a second reason that lenders charge a higher mortgage premium for lending to risky borrowers. In the face of free refinancing, a rational lender anticipates adverse selection in mortgage pools over time and compensates for it by charging a higher premium on loans that are freely prepayable. The required lending premium to compensate for prepayment risk is highest for the riskiest borrowers, as these borrowers benefit the most from prepayment if they receive a positive credit shock.

In our setting, like in most credit models, imposing a prepayment penalty allows the lender to reduce the mortgage rate. At first, one could think that adding prepayment penalties to mortgages in our competitive equilibrium setting would be welfare neutral. On the one hand, the borrower would benefit from a prepayment penalty by paying an up-front lower mortgage rate. On the other hand, this benefit would be compensated by the borrower's inability to refinance to a lower mortgage rate when his ex post creditworthiness improves.

We show that risky borrowers are strictly better off by choosing mortgages with prepayment penalties. This is because the refinancing generates two types of inefficiencies, both of them due to the fact that lenders must charge ex ante higher mortgage rates for fully prepayable mortgages. First, the higher ex ante mortgage premium makes the ex post less creditworthy borrowers (those who received negative credit shocks) more likely to default, which is socially costly. Second, the required increase in premium to compensate for refinancing leads some particularly high-risk borrowers to be excluded from the credit markets, although these borrowers would otherwise be able to qualify for a loan if refinancing were not allowed (e.g., if prepayment penalties were employed). Both of these effects reduce welfare.

The benefit of a prepayment penalty stems from its role as a commitment device that allows the borrower to credibly remain with the same lender for a longer period of time. Consequently, prepayment penalties facilitate provision of insurance against costly default within the same initial risk category of borrowers. Those of the borrowers who become more creditworthy ex post end up effectively subsidizing those whose creditworthiness has deteriorated, which improves welfare.

Next, we introduce mobility benefits into our setting. We show that the negative effect of prepayment penalties on borrowers' mobility can explain the high concentration of prepayment penalties among the riskiest borrowers.

<sup>3</sup> See, for example, Hillary Clinton's "Speech on Housing and the Mortgage Crisis" delivered on August 7, 2007 or the testimony of Julia Brown, senior policy counsel at the Center for Responsible Lending, before the Congressional Financial Crisis Inquiry Commission in January 2010.

<sup>4</sup> For example, a report from the Center for Responsible Lending by Goldstein and Strohauer Son (2003) states: "While some subprime lenders claim that borrowers actually choose prepayment penalties in order to lower the costs of their loan, borrower choice cannot explain the 80% penetration rate of prepayment penalties in subprime loans in comparison to the 2% penetration rate in the competitive, more transparent, conventional market. The wide disparity between the prime and subprime market penetration rates shows that subprime consumers do not "choose" prepayment penalties in any meaningful sense. Rational subprime borrowers with market power should prefer them no more often, and probably less often, than conventional borrowers so that they can refinance into a conventional loan at a significantly lower rate as soon as credit improves."

<sup>5</sup> This selection in pool quality over time is an important theme to the empirical prepayment literature and ties strongly to the observed path dependence of prepayments (see Richard and Roll, 1989).

For the highest credit quality borrowers, the benefit of a prepayment penalty is minimal. These borrowers already receive the lowest available mortgage rate, so even a small benefit from mobility is likely to tip them in favor of choosing a fully prepayable mortgage. By contrast, low credit quality borrowers receive the largest discounts for accepting a prepayment penalty and, hence, are much more likely to choose a mortgage with a prepayment penalty.

We then argue that an optimal prepayment penalty in our setting with mobility benefits should be contingent, i.e., different for refinancings than for house sales. The penalty for refinancing should be sufficiently high to limit opportunistic refinancing. However, the penalty for house sales should be smaller or zero to allow for beneficial housing turnover.<sup>6</sup>

We provide some evidence concerning the predictions of our model using data on securitized mortgages obtained from LoanPerformance (LP), a subsidiary of First American CoreLogic, Inc. To match the model as closely as possible and to limit the likelihood of refinancing driven by lower aggregate interest rates, we generate a sample of more than 18 thousand FRMs originated in June 2003, when market interest rates were at their lowest point in over two decades preceding the housing crisis that started in 2007.<sup>7</sup> We also focus exclusively on FRMs to avoid empirical complications in those cases in which borrowers choose to refinance to avoid an upward adjustment in mortgage rates when an initial teaser rate expires or when short-term interest rates rise.

The empirical evidence is consistent with the key predictions of our model. First, we examine prepayment behavior in FRMs without prepayment penalties in response to a house price shock (a proxy for an ex post wealth shock). We find that borrowers who receive positive house price shocks are much more likely to prepay their mortgage than borrowers in locations where house prices grew less quickly. Moreover, the prepayment rate of the high risk (subprime) borrowers is much more sensitive to house price changes than the prepayment rate of low risk (prime) borrowers. Similar results hold when we look at non-agency subprime and prime loans without prepayment penalties originated in the years 2003–2005. Finally, using mortgage performance data merged with consumer credit data, we provide additional evidence indicating that changes in the borrower's creditworthiness not related to changes in house prices are also an important factor affecting prepayment behavior.

Next we examine the use of prepayment penalties. Consistent with our model, we show that the riskiest (subprime) borrowers are the most likely to have prepayment

penalties, with about 72% of these loans originated in June 2003 having prepayment penalties (the penalty for early refinancing typically amounts to between 3% and 5% of the prepaid loan balance). By comparison, less than 2% of prime borrowers have prepayment penalties. We also find that similar results hold for other origination periods.

Many critics might argue that the riskiest borrowers are most susceptible to misunderstanding their mortgage and, thus, might unwittingly take on a loan with a prepayment penalty without any benefit. In contrast to that view, but consistent with our model, we find that, among the group of less creditworthy borrowers, mortgages with prepayment penalties carry lower rates compared with loans without penalties, the more so the riskier the loans are.<sup>8</sup>

Our paper is not the first to point out that the prepayment penalties could benefit borrowers. In an important earlier work, [Dunn and Spatt \(1985\)](#) study the role of due-on-sale clauses and prepayment penalties in a two-period setting where the borrowers receive ex post stochastic shocks to the incremental utility received from selling the house (from mobility). They show that such clauses could enhance welfare by improving risk-sharing opportunities of borrowers. We contribute to their analysis in a number of ways. Our dynamic model features costly default, a change in the borrower's creditworthiness not related to moving out, and the possibility of sequentially refinancing a mortgage while staying in the same house. This allows us to study the benefits of prepayment penalties across the measure of borrower creditworthiness and the impact of these clauses on default rates. Our setting also allows us to separate prepayments due to house sales from those due to mortgage refinancings. As pointed out by [Dunn and Spatt \(1985\)](#), prepayment penalties on house sales could diminish beneficial housing turnover, thus reducing the benefit of such clauses. As such concerns do not arise for refinancings, our analysis highlights the efficiency value of prepayment penalties on mortgage refinancings. Finally, we also investigate whether the model predictions are consistent with the data.

Our result on the inefficiency of refinancing is similar in spirit to the observation that lack of consumer commitment can generate inefficiencies in health or life insurance markets (see, e.g., [Palfrey and Spatt, 1985](#); [Cochrane, 1995](#); [Hendel and Lizzeri, 2003](#)).<sup>9</sup> This literature highlights the adverse role of short-term contracts in that they do not offer insurance against "reclassification risk", so bad news about the persistent health status of a consumer can result in increased premiums. In our setting, the anticipated free exit (refinancing of a mortgage) by borrowers with good credit in the future limits the possibility to cross-subsidize, resulting in higher initial mortgage premiums. This insight emphasizing the value of commitment to a long-term mortgage

<sup>6</sup> One potential concern with such contingent prepayment penalties is that a borrower whose creditworthiness improved could sell a home and purchase a new one to effectively refinance a loan and obtain a lower rate without paying a higher penalty. However, as we discuss in Section 5.4, sizable transaction costs associated with house sales would likely limit such opportunistic prepayments for most borrowers.

<sup>7</sup> Conforming mortgage interest rates reached even lower levels following the Federal Reserve Bank's accommodative policy in response to the housing crisis that started in 2007. However, the refinancing market remained virtually inaccessible to most of the risky borrowers in this period.

<sup>8</sup> As we focus on 2003 originations, we recognize that underwriting standards could have changed in the years just preceding the subprime mortgage crisis.

<sup>9</sup> See also [Peterson and Rajan \(1995\)](#), who show that an exclusive relationship resulting from monopolistic lending makes creditors more likely to finance credit-constrained firms because it is easier for these creditors to internalize the benefits of assisting the firms.

contract is similar in spirit to [Dunn and Spatt \(1988\)](#), who point out that long-term mortgages could facilitate intertemporal risk sharing about the creditworthiness of the borrower.

Our paper is also related to the existing work on prepayments. Most of this literature focuses on borrower's incentives to prepay in response to a decline in the aggregate interest rates.<sup>10</sup> Our analysis is related to studies that highlight the improvements in the borrower's creditworthiness as one of the reasons to refinance.<sup>11</sup> More importantly, unlike most of this literature, we focus on the welfare consequences of prepayment penalties.

Finally, our paper is also related to the literature that addresses the implications of various constraints on the design of mortgages and the behavior of borrowers and lenders.<sup>12</sup> It is also related to the recent literature on subprime lending by examining the role of prepayment penalties in this market.<sup>13</sup>

The paper is organized as follows. [Section 2](#) presents the continuous-time setting of the model. [Section 3](#) discusses competitive mortgage lending with FRM contracts with sufficiently high prepayment penalties to discourage refinancing. [Section 4](#) studies the effect of refinancing on mortgage lending. [Section 5](#) provides a computed example, while [Section 6](#) discusses model extensions. [Section 7](#) presents the empirical evidence. [Section 8](#) concludes.

## 2. Setup

A borrower (a household) wants to buy a home at date  $t=0$ . To justify the initial purchase of the home, we assume that the borrower extracts more utility from the house when he owns it than when he rents it. Home ownership delivers to the borrower a public and deterministic utility stream  $\theta$ . We assume that this utility stream remains constant as long as the borrower stays in the

<sup>10</sup> See, for example, [Dunn and McConnell \(1981\)](#), [Schwartz and Torous \(1989\)](#), [Stanton \(1995\)](#), and [Kojien, Van Hemert, and Van Nieuwerburgh \(2009\)](#).

<sup>11</sup> See, for example, the contingent claim valuation studies building from a stochastic house price process such as [Kau, Keenan, Muller, Epperson \(1992\)](#), [Deng, Quigley, Van Order \(2000\)](#), and the recent work by [Kau, Keenan, Lyubimov, and Slawson \(2010\)](#), among others, where changes in house prices is one of the factors affecting prepayment behavior.

<sup>12</sup> See, for example, [Chari and Jagannathan \(1989\)](#), [LeRoy \(1996\)](#), [Stanton and Wallace \(1998\)](#), and [Ambrose, Bian, and LaCour-Little \(2011\)](#), who study the implications of ex ante asymmetric information between borrower and lender for the menu of offered mortgages; [Cocco \(2004\)](#) and [Campbell and Cocco \(2003, 2007\)](#), who study implications of housing and mortgage choice for household risk management; [Lustig and Van Nieuwerburgh \(2005\)](#), who study the effect of housing collateral on risk sharing and risk premia; [Ortalo-Magné and Rady \(2006\)](#), who study the link between credit constraints and home ownership and house prices; [Sinai and Souleles \(2005\)](#), who study the role of home ownership as a hedge against rental risk; and [Piskorski and Tchisty \(2010, 2011\)](#), who study optimal mortgage design in dynamic settings with costly foreclosure and a risky borrower who requires incentives to repay his debt.

<sup>13</sup> See, among others, [Mayer, Pence, and Sherlund \(2009\)](#), [Mian and Sufi \(2009, forthcoming\)](#), [Keys, Mukherjee, Seru, and Vig \(2010\)](#), [Rajan, Seru, and Vig \(2010\)](#), [Piskorski, Seru, and Vig \(2010\)](#), [Keys, Seru, and Vig \(2010\)](#), [Jiang, Nelson, and Vytlačil \(2011\)](#), [Purnanandam \(forthcoming\)](#), and [Demyanyk and Van Hemert \(2011\)](#).

same house.<sup>14</sup> The price of the home  $P$  is greater than the borrower's initial wealth  $S_0$ .<sup>15</sup> Thus, the borrower must obtain funds from the lender to finance the house purchase. We assume that the borrower and lender's actions have no effect on variables such as house prices or interest rates.<sup>16</sup>

Each of the lenders is risk neutral, has unlimited capital, and values a stochastic cumulative cash flow  $\{f_t\}$  as

$$E \left[ \int_0^{\infty} e^{-rt} df_t \right], \quad (1)$$

where  $r$  is the market interest rate at which the lender discounts cash flows.

A borrower values a stochastic cumulative consumption flow  $\{C_t\}$  as

$$E \left[ \int_0^{\infty} e^{-\gamma t} dC_t \right], \quad (2)$$

where  $dC_t \geq 0$ . We assume that the borrower is more impatient than the lender, i.e.,  $\gamma > r$  for all  $t$ , reflecting in our setting that the intertemporal marginal rate of substitution for a borrowing-constrained household is greater than that of a financial institution. The borrower's consumption at  $t$ ,  $dC_t \geq 0$ , represents the discretionary consumption of goods and services, which, among many other things, could include such items as restaurant dining, vacation trips, and buying a new car. A consumption level of zero ( $dC_t = 0$ ) means that the borrower consumes only goods and services that have priority over debt repayment, which can include items such as food, medicine, transportation, and other goods and services essential for the household.

A borrower must use his income to cover the necessary expenses  $\eta_t$  before spending on discretionary consumption or debt repayment. Let  $\bar{Y}_t \geq 0$  denote the borrower's total cumulative income up to time  $t$ . We focus on the borrower's "excess" cumulative income,  $Y_t \equiv \bar{Y}_t - \eta_t$ , where  $\{\eta_t\}$  is a cumulative level of necessary consumption given by an exogenous stochastic spending process that incorporates shocks such as medical bills, auto repair costs, fluctuations of food and gasoline prices, and so on.<sup>17</sup> Therefore, the borrower's excess income  $Y_t$  is a more relevant determinant of the borrower's ability to pay for a house than his total

<sup>14</sup> For simplicity, we do not consider the possibility that the borrower can make adjustments that either increase or decrease the quality of the house. See, among others, [Spiegel and Strange \(1992\)](#) and [Shiller and Weiss \(2000\)](#), who study the implications of this possibility.

<sup>15</sup> It is reasonable to expect that the home price  $P$  is increasing in its utility  $\theta$  and that the borrower optimizes over the set of available  $(\theta, P)$  pairs. This optimization is not considered in the paper. However, this does not lead to a loss of generality, because our analysis applies to any  $(\theta, P)$  pair.

<sup>16</sup> In a general equilibrium framework, actions of mortgage lenders and home buyers on the aggregate level can affect macroeconomic variables. However, as long as the economic agents on the individual level have no market power, they should regard macroeconomic variables as exogenous in equilibrium.

<sup>17</sup> This specification of preferences has been used by [Piskorski and Tchisty \(2010, 2011\)](#) and is similar in flavor to the one used by [Ait-Sahalia, Parker and Yogo \(2004\)](#) who propose a partial resolution of the equity premium puzzle by distinguishing between the consumption of basic goods and that of luxury goods.

income. From now on we refer to  $Y_t$  simply as the borrower's income.

We assume that a standard Brownian motion  $Z = \{Z_t; 0 \leq t < \infty\}$  drives the borrower's income process. Accordingly, the borrower's excess income up to time  $t$ , denoted by  $Y_t$ , evolves as

$$dY_t = \mu dt + \sigma dZ_t, \quad (3)$$

where  $\mu \geq \theta$  is the drift of the borrower's excess income, and  $\sigma$  is the sensitivity of the borrower's income to its Brownian motion component. We assume that the lender knows  $\mu$  and  $\sigma$  but does not know the realizations of the borrower's excess income shocks  $Z_t$ . Thus, realizations of the borrower's income are not contractible. These assumptions are motivated by the observation that lenders use a variety of methods (e.g., credit score or demographic variables) to determine the type of the borrower before the loan is approved but, henceforth, do not condition the terms of the contract on the realizations of the borrower's income, likely because it is costly or impossible to monitor the borrower's necessary spending shocks and his total income.

The borrower maintains a savings account. The savings account balance  $S$  grows at the interest rate  $r$ . The borrower must maintain a non-negative balance in his account.

In case of a mortgage foreclosure at time  $\tau$ , the borrower receives the value of his outside option,  $A$ , which represents the borrower's continuation utility after the loss of the home plus the value of any savings he might have at the time of default. The value  $A$  incorporates such factors as the consumption value  $\mu/\gamma$  of the borrower's expected future income, financial and intangible moving costs, losses from the damaged credit history, and the option to buy or rent another home in the future and mobility gains. The lender sells the repossessed house at a foreclosure auction and receives a payoff of  $L$ . We assume that  $L < P \leq \theta/r$  and  $A \leq \mu/\gamma$ , which makes the liquidation inefficient, and that  $P - L \geq \gamma(\theta/r - L)$ , which ensures that the default costs are not very small.

The borrower with initial wealth  $S_0$  decides to buy a house whenever the total utility he gets from homeownership is at least as big as the value  $R(S_0)$  he could get by not buying. The value  $R(S_0)$  represents the continuation utility of the borrower with initial wealth  $S_0$  who does not purchase a house.  $R(S_0)$  incorporates such factors as the consumption value ( $\mu/\gamma$ ) of his expected future income, the value of savings  $S_0$ , and the option to buy or rent another home in the future. We assume that  $R(S_0) \geq (\mu/\gamma) + S_0$ , which implies that the outside value of a prospective borrower who does not purchase a house is at least as big as the sum of his initial wealth and the expected value of his excess disposable income.

The borrower could be allowed to sell the house to move to another location. Such moves could take place for employment or family reasons. A moving opportunity arrives according to a standard Poisson process with the intensity  $\delta$ . When moving opportunity number  $i$  arrives, the net gain for the borrower from moving to the new location  $m_i$  is drawn independently from probability density  $f(m)$  and assumed to be positive,  $m_i > 0$ . For simplicity we assume that the Poisson process and the mobility gains are independent from the borrower's income process. If the borrower

decides to move, he has to sell the house and prepay the mortgage. Let  $M \equiv \int_0^\infty mf(m) dm$  denote the average mobility gain.

Before purchasing a house, the borrower and the lender sign a contract that governs their relation after the purchase is made. First, we consider a case in which no prepayments are allowed. Then, we compare it with the case in which the borrower can prepay any time. Finally, we discuss contingent prepayment penalties that differentiate between refinancing and home sales.

### 3. Fixed rate mortgage with no prepayment

Under the terms of the FRM contract, the borrower is required to make payments at the constant rate  $\bar{\phi}$ . If a contract is signed, the lender transfers the funds  $P$  needed to purchase a home to the borrower at time 0. Once the mortgage is originated, if the borrower fails to make the payment, a foreclosure is initiated and the borrower gets the value  $A$  and the lender gets the value  $L$ . In this section, we assume that the prepayment penalty is sufficiently high so that the borrower would not prepay the mortgage under any circumstances.

The borrower's total expected payoff from the mortgage with no refinancing at time zero is given by

$$\bar{U}_0 = E \left[ \int_0^\tau e^{-\gamma s} (\theta dt + dC_t) + e^{-\gamma \tau} A \right], \quad (4)$$

where  $\tau$  is the time when the borrower defaults on the mortgage. The lender's total discounted expected payments from the mortgage as of time zero are given by

$$\bar{V}_0 = E \left[ \int_0^\tau e^{-rs} \bar{\phi} dt + e^{-r\tau} L \right]. \quad (5)$$

The borrower with initial wealth  $S_0$  decides to buy a house whenever the total utility he gets under homeownership is at least as big as the value  $R(S_0)$  he could get by not buying, where  $R(S_0) \geq \mu/\gamma + S_0 \geq A + S_0$ . Given this, the net utility gain for the borrower from homeownership financed by an FRM with coupon  $\bar{\phi}$  and no refinancing allowed is bounded from above by  $\int_0^\tau e^{-\gamma t} (\theta - \bar{\phi}) dt$ . This immediately implies that any coupon payment  $\bar{\phi}$  in the FRM contract with no refinancing satisfies  $\bar{\phi} \leq \theta$ .

Because the borrower's income is stochastic while the mortgage payments are fixed, the borrower needs to save part of his income to be able to make mortgage payments in the future. Thus, for a given FRM, the borrower's continuation payoff  $\bar{U}(S_t, \bar{\phi})$  is a function of the mortgage payments  $\bar{\phi}$  and the balance  $S_t$  on the borrower's savings account. The borrower's savings evolve according to

$$dS_t = rS_t dt + dY_t - \bar{\phi} dt - dC_t. \quad (6)$$

The borrower chooses his consumption and savings strategy  $(C, S)$  to maximize his payoff given the mortgage contract. The borrower saves only if  $\bar{U}'(S_t, \bar{\phi}) \geq 1$  and consumes only if  $\bar{U}'(S_t, \bar{\phi}) \leq 1$ , where prime denotes the derivative with respect to  $S$ . This implies the existence of an upper bound  $S^1(\bar{\phi})$  on the amount the borrower saves. Proposition 1 formalizes this finding.

*Proposition 1.* For a given fixed rate mortgage  $\bar{\phi} \leq \theta$ , the borrower's continuation payoff  $\bar{U}(S_t, \bar{\phi})$  and the maximum

saving level  $S^1(\bar{\phi})$  solve the following problem:

$$\gamma \bar{U}(S, \bar{\phi}) = \theta + (rS + \mu - \bar{\phi}) \bar{U}'(S, \bar{\phi}) + \frac{1}{2} \sigma^2 \bar{U}''(S, \bar{\phi})$$

for  $S \in [0, S^1(\bar{\phi})]$ , (7)

$$\bar{U}(0, \bar{\phi}) = A, \tag{8}$$

$$\bar{U}'(S, \bar{\phi}) = 1 \text{ for } S \geq S^1(\bar{\phi}) \text{ and} \tag{9}$$

$$\bar{U}''(S, \bar{\phi}) = 0 \text{ for } S \geq S^1(\bar{\phi}). \tag{10}$$

The function  $\bar{U}(S_t, \bar{\phi})$  is concave in  $S$  and satisfies  $\bar{U}'(S, \bar{\phi}) > 1$  for  $S \in (0, S^1(\bar{\phi}))$ . The optimal strategy for the borrower is to consume only necessities when  $S_t \in [0, S^1(\bar{\phi})]$ , and to consume  $S_t - S^1(\bar{\phi})$  immediately when  $S_t > S^1(\bar{\phi})$ .

*Proof.* In the Appendix.  $\square$

In Proposition 1,  $\bar{U}(S, \bar{\phi})$  represents the total expected utility of the borrower with initial savings of  $S$ , given the mortgage coupon of  $\bar{\phi}$ . Intuitively,  $\bar{U}(S, \bar{\phi})$  is decreasing in the repayment rate  $\bar{\phi}$ , as the more the borrower is asked to pay, the lower his utility is. Because of the borrower's relative impatience, it is costly to postpone his consumption. However, early consumption lowers the borrower's savings level, increasing the risk of costly default. Consequently, it is optimal for the borrower to maintain some precautionary savings. The maximum (targeted) savings level,  $S^1$ , optimally balances these two forces.

Given the coupon  $\bar{\phi}$  and the borrower's initial wealth  $S_0$ , the value of the mortgage to the lender is equal to

$$V_0(S_0, \bar{\phi}) = E \left[ \int_0^{\tau(S_0, \bar{\phi})} e^{-rt} \bar{\phi} dt + e^{-r\tau(S_0, \bar{\phi})} L \right]$$

$$= \frac{\bar{\phi}}{r} - E \left[ e^{-r\tau(S_0, \bar{\phi})} \left( \frac{\bar{\phi}}{r} - L \right) \right]. \tag{11}$$

In Eq. (11),  $\tau(S_0, \bar{\phi})$  is the default time of the borrower implied by his optimal choice of consumption and savings characterized in Proposition 1, given his initial wealth  $S_0$  and the required mortgage payments of  $\bar{\phi}$ .

We assume that the lending market is competitive. Therefore, the lender chooses the smallest coupon  $\bar{\phi}$  so that he breaks even. This leads to Definition 1.

**Definition 1.** The competitive mortgage repayment rate on the FRM with no refinancing for the borrower with initial level of wealth (savings)  $S_0$  satisfies

$$\bar{\phi}^*(S_0) = \min_{\bar{\phi} \geq 0} \{V_0(S_0, \bar{\phi}) = P\}, \tag{12}$$

and the implied mortgage premium is given by  $\rho^*(S_0) = \frac{\bar{\phi}^*(S_0)}{P} - r$ . The borrower takes the loan whenever  $\bar{U}(S_0, \bar{\phi}^*(S_0)) \geq R(S_0)$ .

Let  $\underline{S}$  be the minimum level of wealth such that the borrower takes the loan and the lender breaks even. Let  $\bar{S}$  be the initial level of wealth corresponding to the lowest coupon payment  $\bar{\phi}^*$ .

**Lemma 1.** The competitive mortgage repayment rate on the FRM with no refinancing,  $\bar{\phi}^*$ , and the borrower's maximum savings level,  $S^1$ , are strictly decreasing in the borrower's

initial savings on  $S_0 \in [\underline{S}, \bar{S}]$ . Hence

$$\bar{S} = \min_{S \geq \underline{S}} \{S^1(\bar{\phi}^*(S))\}. \tag{13}$$

*Proof.* In the Appendix.  $\square$

Lemma 1 is intuitive. The mortgage premium compensates the lender for the loss due to default. The higher the initial wealth (savings) of the borrower, the lower the probability of default, and so the lower the risk premium charged on the loan. Also, intuitively, the borrower's maximum savings level,  $S^1$ , decreases with his initial creditworthiness (savings). Higher initial savings imply a lower mortgage rate, which translates into lower risk of default. Consequently, it is optimal for the borrower to lower the maximum precautionary savings level when his mortgage rate is lower.

#### 4. Fixed rate mortgage with prepayment

In this section, we allow for refinancing of mortgage loans. It is convenient to first consider the case in which the borrower is not allowed to refinance the mortgage but can freely prepay it to take advantage of mobility benefits. That is, the only difference from the case considered in Section 3 is that the borrower prepays the mortgage when a moving opportunity arrives, receiving positive expected gain  $M$ .

**Proposition 2.** Consider a borrower who prepays his mortgage only due to mobility gains. Given the coupon  $\tilde{\phi}$  and the borrower's savings level  $S_0$ , the borrower's continuation payoff  $\tilde{U}(S_t, \tilde{\phi})$  and the maximum saving level  $S_{ref}^1(\tilde{\phi})$  solve the following problem:

$$\gamma \tilde{U}(S, \tilde{\phi}) = \theta + (rS + \mu - \tilde{\phi}) \tilde{U}'(S, \tilde{\phi}) + \frac{1}{2} \sigma^2 \tilde{U}''(S, \tilde{\phi}) + \delta M$$

for  $S \in [0, S_{ref}^1(\tilde{\phi})]$ , (14)

$$\tilde{U}(0, \tilde{\phi}) = A, \tag{15}$$

$$\tilde{U}'(S, \tilde{\phi}) = 1 \text{ for } S \geq S_{ref}^1(\tilde{\phi}) \text{ and} \tag{16}$$

$$\tilde{U}''(S, \tilde{\phi}) = 0 \text{ for } S \geq S_{ref}^1(\tilde{\phi}). \tag{17}$$

The function  $\tilde{U}(S_t, \tilde{\phi})$  is concave in  $S$  and satisfies  $\tilde{U}'(S, \tilde{\phi}) > 1$  for  $S \in (0, S_{ref}^1(\tilde{\phi}))$ . The optimal strategy for the borrower is to consume only necessities when  $S_t \in [0, S_{ref}^1(\tilde{\phi})]$  and to consume  $S_t - S_{ref}^1(\tilde{\phi})$  immediately when  $S_t > S_{ref}^1(\tilde{\phi})$ . The borrower defaults when his savings reaches zero for the first time. The corresponding value of the mortgage to the lender equals to

$$\tilde{V}(S_0, \tilde{\phi}) = E \left[ \int_0^{\min(\tau, \tau^m)} e^{-rt} \tilde{\phi} dt + e^{-r\tau} 1_{\tau < \tau^m} L + e^{-r\tau^m} 1_{\tau^m < \tau} P \right], \tag{18}$$

where  $\tau$  is the default time of the borrower implied by his optimal choice of consumption and savings characterized above and  $\tau^m$  is the arrival time of first moving opportunity.

The competitive market coupon rate is given by

$$\tilde{\phi}^*(S_0) = \{\inf \phi \geq 0 : \tilde{V}(S_0, \phi) = P\}. \quad (19)$$

The borrower takes the loan whenever  $\tilde{U}(S_0, \tilde{\phi}^*) \geq R(S_0)$ .

*Proof.* In the Appendix.

Proposition 2 is very similar to Proposition 1, as in both cases the borrower does not refinance the mortgage due to improved creditworthiness. The only difference from the case considered in Section 3 is that the borrower prepays the mortgage when a moving opportunity arrives, receiving positive expected gain  $M$ . Again, intuitively,  $\tilde{U}$  is decreasing in the repayment rate  $\tilde{\phi}$ .

Let  $\tilde{S}$  be the minimum level of wealth such that the borrower who prepays the mortgage only due to mobility shocks takes the loan and the lender breaks even. Let  $\bar{S}^{ref}$  be the initial level of wealth corresponding to the lowest coupon payment for a borrower who prepays the mortgage only due to mobility shocks. Lemma 2 follows.

*Lemma 2.* The competitive mortgage repayment rate on the FRM when the borrower prepays only due to mobility shocks,  $\tilde{\phi}^*$ , and the borrower's maximum savings level,  $S^1$ , are strictly decreasing in the borrower's initial savings on  $S \in [\tilde{S}, \bar{S}^{ref}]$ . Hence:

$$\bar{S}^{ref} = \min_{S \geq \tilde{S}} \{S_{ref}^1(\tilde{\phi}^*(S))\}. \quad (20)$$

*Proof.* The proof follows similar steps as the proof of Lemma 1.  $\square$

The intuition behind Lemma 2 is very similar to that of Lemma 1.

So far we allow the borrower to prepay the mortgage only due to mobility shocks. Now we consider the case in which the borrower can freely prepay the loan due to mobility shocks as well as improvements in his creditworthiness. Each time his creditworthiness sufficiently improves, the borrower can refinance the loan, i.e., replace the existing loan with a new loan of the same amount, but with a lower coupon payment. The borrower sticks to the existing loan when his creditworthiness deteriorates, as then refinancing would imply a higher interest rate premium on the mortgage, and, thus would make him worse off. In addition, the borrower prepays the mortgage when he sells the house to take advantage of positive mobility gains.

To model this formally, we assume that the borrower refinances the loan when his creditworthiness improves by a certain amount.<sup>18</sup> This is represented by an increasing sequence of savings cutoffs  $\{S^i\}_{i=1}^K$ , where  $S^1 = 0$  and  $S^K = \bar{S}_{ref}$ . The borrower refinances each time his savings level reaches the next cutoff point. The number of relevant cutoff points for the borrower with initial wealth level  $S_0$  is given by

$$N_{S_0} = \#\{S > S_0 : S \in \{S^i\}_{i=1}^K\}. \quad (21)$$

<sup>18</sup> This assumption can be justified by the discrete nature of the credit scoring technology or some potential refinancing costs the borrower has to bear. See Section 6.2 for a discussion.

Define a sequence  $\{S_{S_0}^n\}_{n=0}^{N_{S_0}}$  as  $S_{S_0}^0 = S_0$  and, if  $N_{S_0} > 0$ ,

$$S_{S_0}^n = S^{K+n-N_{S_0}} \quad \text{for } n = 1, \dots, N_{S_0}. \quad (22)$$

Then the borrower with initial wealth  $S_0$  refinances for the  $n$ th time when his wealth reaches  $S_{S_0}^n$  for the first time.

We first consider the case in which the borrower's initial savings level  $S_0$  is greater than  $\bar{S}^{ref}$ . In this case, the optimal behavior of this borrower and the associated competitive mortgage rate are determined by Proposition 2. The borrower's initial rate is low enough that he never refinances in the future due to possible improvements in his creditworthiness. However, the borrower prepays the mortgage to take advantage of mobility benefits.

When  $S_0 < \bar{S}^{ref}$  and  $N_{S_0} > 0$ , the borrower prepays the mortgage not only when he moves to a new location, but also when his wealth reaches the next wealth level  $\{S_{S_0}^n\}_{n=1}^{N_{S_0}}$ . When the borrower prepays, the lender gets paid  $P$  and the mortgage contract is terminated.

*Proposition 3.* If  $S_0 < \bar{S}^{ref}$ , then under the competitive lending market it is optimal for the borrower to refinance the loan each time his wealth reaches the next wealth level  $\{S_{S_0}^n\}_{n=1}^{N_{S_0}}$ . The borrower's continuation payoff after the  $n$ th refinancing,  $U^n(S, \phi^n)$  for  $n = 0, \dots, N_{S_0}$ , is given by a concave, twice continuously differentiable function in  $S$ . The function  $U^n(S, \phi^n)$  solves for  $n = 0, \dots, N-1$ :

$$\begin{aligned} \gamma U^n(S, \phi^n) &= \theta + (rS + \mu - \phi^n)(U^n)'(S, \phi^n) \\ &\quad + \frac{1}{2}\sigma^2(U^n)''(S, \phi^n) + \delta M \quad \text{for } S \in [0, S_{S_0}^{n+1}], \end{aligned} \quad (23)$$

$$U^n(0, \phi^n) = A, \quad (24)$$

$$U^n(S_{S_0}^n, \phi^n) = U^{n+1}(S_{S_0}^n, \phi^{n+1}), \quad (25)$$

and  $U^{N_{S_0}}(S, \phi^{N_{S_0}}) = \tilde{U}(S, \phi^{N_{S_0}})$  for all  $S \geq 0$ , where  $\tilde{U}$  is given by Proposition 2. The optimal strategy for the borrower is to consume only necessities when  $S_t \in [0, \bar{S}^{ref}]$ , and to consume  $S_t - \bar{S}^{ref}$  immediately when  $S_t > \bar{S}^{ref}$ . The corresponding market value of the mortgage with free refinancing and coupon  $\phi^n$  solves for  $S \in [0, S_{S_0}^{n+1}]$ :

$$\begin{aligned} rV^n(S, \phi^n) &= \phi^n + (rS + \mu - \phi^n)(V^n)'(S, \phi^n) \\ &\quad + \frac{1}{2}\sigma^2(V^n)''(S, \phi^n) + \delta(P - V^n(S, \phi^n)), \end{aligned} \quad (26)$$

$$V^n(0, \phi^n) = L \quad \text{and} \quad (27)$$

$$V^n(S_{S_0}^n, \phi^n) = P. \quad (28)$$

The competitive market coupon payments  $\{\phi^n\}_{n=0}^{N_{S_0}}$  are given by

$$\phi^n = \min_{\phi \geq 0} \{V^n(S_{S_0}^n, \phi) = P\} \quad \text{for } n = 0, \dots, N_{S_0} - 1 \quad \text{and} \quad (29)$$

$$\phi^{N_{S_0}} = \tilde{\phi}^*(\bar{S}^{ref}), \quad (30)$$

where  $\tilde{\phi}^*$  is determined in Proposition 2. The borrower takes the loan whenever  $U^0(S_0, \phi^0) \geq R(S_0)$ .

*Proof.* In the Appendix.  $\square$

In Proposition 3,  $U^n(S, \phi^n)$  represents the total expected utility of the borrower after the  $n$ th refinancing who has savings of  $S$  and faces the mortgage coupon of  $\phi^n$ . The left-hand side of Eq. (23) says that the borrower's expected

utility grows on average at the rate  $\gamma$ . The right-hand side of Eq. (23) shows that for  $S \in [0, S_0^{n+1}]$  the borrower's expected utility growth comes from utility flow  $\theta$  from owning the home, expected mobility benefits  $M$  arriving with intensity  $\delta$ , and the effect of changes in the borrower's savings level  $S$  on the borrower's expected utility  $U^n(S, \phi^n)$ . Eqs. (24) and (25) are the boundary conditions for the borrower's expected utility in the event of default and refinancing, respectively. Similarly,  $V^n(S, \phi^n)$  represents the value of the mortgage  $\phi^n$  to the lender when the borrower's savings level is equal to  $S \in [0, S_0^{n+1}]$ . Eq. (26) shows that, to receive  $V^n$ , the lender must earn total expected return of  $rV^n$ . The lender earns this return from coupon  $\phi^n$ , expected appreciation of the mortgage value  $V^n(S, \phi^n)$  due to changes in the borrower's savings, and the loan repayment due to a mobility shock that arrives with intensity  $\delta$ . Eqs. (27) and (28) are the boundaries for the lender's payoff in the event of default and refinancing, respectively. Finally, the conditions (29) and (30) determine the competitive mortgage rate at the  $n$ th refinancing.

## 5. Benefits of prepayment penalties

In this section, we use results of Propositions 1, 2, and 3 to compare the borrower's expected utility under the mortgages with and without prepayment penalties assuming the competitive lending market. We first consider the case without mobility benefits ( $\delta = 0$ ).

Due to their complexity, Propositions 1, 2 and 3 do not allow for analytical solutions. We proceed with numerical computations to compare the fixed rate mortgage contract with prepayment and the fixed rate mortgage contract of the same amount with no prepayment. Property 1 is satisfied for a very large range of parameters that we tried in our computations. We have not been able to discover a counterexample that would not satisfy this property.

*Property 1. Assume there are no mobility benefits, i.e.,  $\delta = 0$ . Let  $\underline{S}^{ref}$  be the minimum wealth (savings) such that the borrower takes the FRM loan with refinancing and the lender breaks even. Then, for  $S_0 \in [\underline{S}^{ref}, \bar{S}^{ref}]$ , the expected utility for the borrower under competitive lending is greater when refinancing is not allowed (e.g., in a regime with prepayment penalties):*

$$\bar{U}(S_0, \bar{\phi}^*(S_0)) > U^0(S_0, \phi^0(S_0)), \quad (31)$$

and more so for riskier borrowers:

$$\frac{d(\bar{U}(S_0, \bar{\phi}^*(S_0)) - U^0(S_0, \phi^0(S_0)))}{dS_0} < 0. \quad (32)$$

The rate discount from accepting a prepayment penalty is larger for less creditworthy borrowers:

$$\frac{d(\bar{\phi}^*(S_0) - \phi^0(S_0))}{dS_0} < 0. \quad (33)$$

Moreover, allowing for refinancing leads to an exclusion from the lending market of riskier borrowers:  $\underline{S}^{ref} > \underline{S}$ .

Property 1 states that borrowers are worse off under the FRM contract when refinancing is allowed and there are no mobility benefits. Moreover, the worse a borrower's

creditworthiness (his initial wealth), the larger the benefit from no refinancing (sufficiently high prepayment penalty).

To explain this point consider a group of ex ante identical borrowers obtaining FRM loans to purchase identical homes. Suppose that their initial wealth level (initial creditworthiness) is such that they would qualify for a loan under both regimes (with and without refinancing allowed). As the borrowers are ex ante identical, they are charged the same premiums on their loans. Under the FRM contract without a prepayment penalty, borrowers who become more creditworthy over time would refinance to obtain cheaper premiums on their loans, while borrowers whose creditworthiness deteriorates would stay with the original lender. Like in most credit models, the rational lender would anticipate and compensate for refinancing by charging a higher premium on the loans without prepayment penalty.

At first, one could think that allowing for refinancing is welfare neutral for those borrowers who would qualify for credit under both regimes. On the one hand, ex post less creditworthy borrowers (those who received negative shocks to their financial position) would be worse off than under the contract with a prepayment penalty, as they would have to pay a higher premium. On the other hand, those borrowers whose creditworthiness would sufficiently improve ex post (those who received positive shocks to their financial position) would refinance to lower premiums and thus be better off than under the contract with a prepayment penalty.

Property 1 states that the expected gains if the borrower's creditworthiness improves are not sufficient to offset the expected losses when the borrower's creditworthiness deteriorates.<sup>19</sup> Why is this so? Charging a higher premium makes borrowers more likely to default, and the likelihood of default is more sensitive to premiums for those who are less creditworthy. Consequently, the decrease in the likelihood of default due to lower premiums for those who refinance is not sufficient to offset an increase in the likelihood of default due to higher premiums paid by those who cannot refinance due to deterioration in their creditworthiness. As a result, the possibility of refinancing increases the expected number of defaults in a given pool of borrowers. Because defaults are costly, refinancing leads to welfare losses. Moreover, allowing mortgage refinancing causes some risky borrowers to be unable to obtain credit, unlike under the contracts with no refinancing. As homeownership is assumed to generate positive utility gains for borrowers, this exclusion from credit leads to lower utility for those who would otherwise qualify for credit without refinancing.

### 5.1. A numerical example

In this section we illustrate the features of competitive FRM contract lending, with and without refinancing, in a

<sup>19</sup> Property 1 is similar in spirit to the finding of Manso, Strulovici, and Tchistyi (2010), who show that performance-sensitive debt (PSD), the class of debt obligations whose interest payments depend on the borrower's performance, is inefficient compared with fixed rate debt of the same market value.

parametrized example. We choose the model parameters as follows. We set  $r = 3\%$  (interest rate),  $\gamma = 5\%$  (borrower's discount rate),  $\mu = 1$  (mean borrower's income),  $\sigma = 1$  (volatility of borrower's income),  $\theta = 1$  (utility flow from homeownership),  $A = 20$  (the borrower's reservation value following default),  $R(S_0) = \mu/\gamma + S_0$  (the borrower's reservation value in case of no house purchase),  $P = 18$  (house price),  $L = 14$  (liquidation value of the house). The refinancing wealth cutoffs are set as  $\{S^1\}_{i=1}^{K-1} = \{0.02i\}_{i=1}^{K-1}$ , with  $S^K = \bar{S}^{ref} = 0.33$ .

The left-hand side of Fig. 1 shows the FRM mortgage premiums under a competitive lending market, without refinancing and with refinancing allowed, as a function of the borrower's creditworthiness (initial wealth level). As before, no refinancing is justified by the imposition of a sufficiently high prepayment penalty.

The borrower's creditworthiness increases in his savings level  $S$  (the likelihood of default decreases with the wealth level). The vertical lines show the wealth cutoffs below which the borrower does not qualify for a loan. We observe that mortgage premiums are always decreasing in the wealth level, reflecting a lower likelihood of loss due to default, whether or not refinancing is allowed. The mortgage premiums with refinancing are larger compared with without refinancing, and more so for riskier borrowers (with lower wealth levels).

We also observe that allowing for refinancing leads to a significant exclusion of riskier types (those with lower wealth levels) from the lending market. In the regime with refinancing, the riskiest borrowers that qualify for credit (those with initial wealth of  $\bar{S}^{ref}$ ) have a large net positive utility gain from homeownership. However, lenders cannot break even on less creditworthy borrowers (those with  $S_0 < \bar{S}^{ref}$ ), because the required higher mortgage premium would be more than offset by costs associated with a higher likelihood of default.

The right-hand side of Fig. 1 shows the borrower's net utility gain from homeownership, financed by the FRM mortgage without and with refinancing allowed, as a function of the borrower's initial wealth level. We observe that the borrower's net utility gain is lower when refinancing is

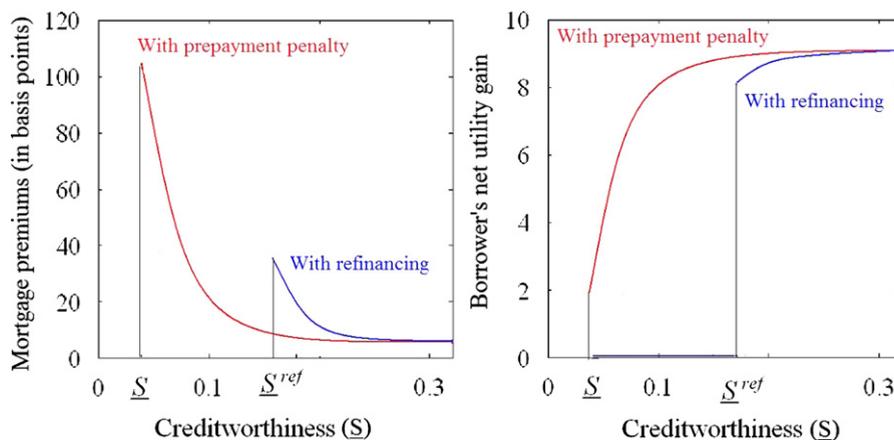
allowed, particularly for riskier borrowers. However, a large efficiency loss from refinancing in this example comes from the exclusion of riskier types who cannot enjoy the benefits of homeownership. Also, one could see more high cost (high premium) loans and more defaults in the aggregate if prepayment penalties are used, just because more risky borrowers would be able to qualify for a loan.

## 5.2. Prepayment penalties and mobility

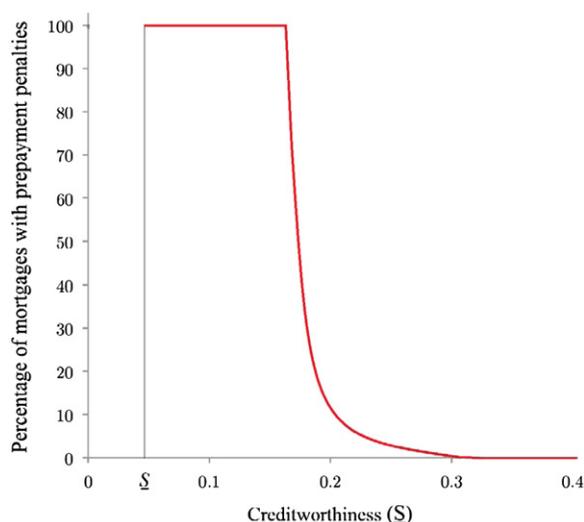
Property 1 is predicated on the assumption that borrowers have no reason to prepay their mortgage except to receive a lower mortgage premium. This analysis ignores other likely reasons that borrowers prepay mortgages, such as move to another location. Such moves could take place for employment or family reasons. As shown by Dunn and Spatt (1985), in a setting with mobility, the prepayment penalties could diminish beneficial housing turnover, thus reducing benefits of such clauses.

It is straightforward to incorporate mobility benefits into the results of Property 1. Consider the case of the borrower with a given initial savings level  $S_0$  with positive expected benefits from moving out ( $\delta > 0$  and  $M > 0$ ). If this borrower chooses a loan with no refinancing (with sufficiently high prepayment penalty), his utility and behavior are determined by Proposition 1, while his coupon rate is determined according to Definition 1. If this borrower chooses a loan without the prepayment penalty, his utility, behavior, and coupon rate are determined according to Proposition 3.

For the highest credit quality borrowers (those with high savings levels), the benefit of a prepayment penalty in terms of a lower rate premium is minimal. These borrowers already receive the lowest available mortgage rate, so even a small benefit from moving is likely enough to tip them in favor of choosing a fully prepayable mortgage. Thus, we would expect that the highest quality borrowers almost surely want to avoid prepayment penalties. By contrast, low credit quality borrowers receive the largest discounts for accepting a prepayment penalty. For these borrowers, the



**Fig. 1.** Mortgage premiums and the borrower's net utility gain from homeownership. This figure plots the mortgage premiums in basis points (left panel) and the borrower's net utility gain from homeownership (right panel) as a function of initial savings of the borrower ( $S$ ) for fixed rate mortgages with and without prepayment penalty. The model parameters are given in Section 5.1.



**Fig. 2.** Model with mobility benefits: percentage of mortgages with prepayment penalties. This figure plots the fraction of loans with prepayment penalty as a function of borrowers' initial savings level ( $\underline{S}$ ) in a setting with mobility benefits. The mobility benefits arrive at Poisson rate 0.2 and the expected gains  $M$  from mobility are distributed uniformly on  $[0, 1/2]$  across the borrowers. The other model parameters are given in Section 5.1.

likelihood of mobility or its benefits must be high for them to choose a fully prepayable mortgage.

*Property 2.* For a given level of mobility benefits  $\delta$  and  $M$ , there exists a saving threshold  $S^p < \bar{S}^{ref}$ , such that borrowers with high savings levels  $S_0 \in (S^p, \bar{S}^{ref}]$  are better off without the prepayment penalties, while borrowers with low savings levels  $S_0 \in [\underline{S}, S^p]$  benefit from prepayment penalties.

A direct consequence of Property 2 is that prepayment penalties should be concentrated among the riskiest borrowers. To illustrate this point, we add mobility gains to the parametrized example from Section 5.1. In particular, we assume that the mobility benefits arrive at Poisson rate  $\delta = 0.2$ , while the expected gains  $M$  from mobility are distributed uniformly on  $[0, \frac{1}{2}]$  across the borrowers. Fig. 2 displays the percentage of borrowers obtaining mortgages with prepayment penalties as a fraction of total mortgages originated. As borrowers become less creditworthy (as their initial wealth decreases), the larger the fraction of mortgages with prepayment penalties becomes.

Interestingly, 100% of mortgages taken by the least creditworthy borrowers have prepayment penalties. This is because borrowers cannot qualify for the mortgage without the prepayment penalty if their wealth is below  $\underline{S}^{ref}$ , but they can take the mortgage with the prepayment penalty if their wealth is above  $\underline{S}$ . However, almost 100% of the most creditworthy borrowers choose the mortgage without the prepayment penalty, because the benefits associated with prepayment penalties are almost nonexistent for them.

### 5.3. Contingent prepayment penalties

Our previous discussion suggests that prepayment penalties are beneficial for borrowers, when they are used

to prohibit refinancing. However, prepayment penalties also have a negative effect on the borrowers' welfare, because they reduce mobility benefits. Thus, an optimal prepayment penalty should be contingent, i.e., different for refinancings than for house sales. The penalty for refinancing should be sufficiently high to discourage refinancing. Meanwhile, the penalty for house sales should be smaller or zero.

Depending on the parameters, the optimal prepayment penalty can be zero or positive for house sales. A positive prepayment penalty would result in some extra payments from those borrowers who obtain sufficiently high benefits from moving. Although in our setting the borrower is risk neutral, his expected utility is a concave function of the saving level  $S_t$ , because default is a costly and inefficient outcome. As a result, for a reason very similar to the risk-sharing argument of Dunn and Spatt (1985), a sufficiently small prepayment penalty conditional on the house sale could have risk-sharing benefits that outweigh some expected losses in the beneficial housing turnover.

One potential concern with a contingent prepayment penalty is that a borrower whose creditworthiness improved could sell a home and purchase a new one to effectively refinance a loan and obtain a lower rate without paying a higher penalty. However, in practice, sizable transaction costs associated with house sales would likely limit such opportunistic prepayments for most borrowers.<sup>20</sup>

Examples exist of mortgage contracts with contingent prepayment penalties that discriminate between refinancing and house sales. Some residential mortgages in the US have soft prepayment penalties that allow prepayment without penalty under the terms of selling the home. In Canada it is common for a borrower to pay the prepayment penalty regardless of the reason he pays off his mortgage early. However, a borrower who takes out a mortgage on a new house from the same lender as his original property typically has the prepayment penalty refunded. Another example is assumable mortgages, which were common in the US in the late 1970s and are still commonly used for commercial mortgages. With an assumable mortgage, the owner can always sell the property subject to the existing mortgage, thus avoiding the prepayment penalty.

## 6. Extensions

In this section, we discuss some possible extensions of our model. We argue that, even if we take into account additional factors, refinancing penalties can still benefit borrowers.

<sup>20</sup> For example, most residential real estate brokerage firms in the United States have charged single-family home sellers a commission on the order of 6% of the house value (Delcour and Miller, 2002). This does not include additional costs of selling the home (opportunity costs of time, closing costs, moving costs, and new mortgage origination fees). Hence, just to cover the brokerage commission fee on the house sale, the borrower who on average expects to stay in a new home for about six years would need to obtain interest savings of more than 100 basis points per year. This is a sizable reduction. For example the difference between the average rate in the bottom (riskiest) and the top (safest) quarter of FRMs originated in 2003 is around 200 basis points according to the LP database.

### 6.1. Stochastic house prices and interest rates

So far we have considered a time-homogeneous setting in which agents are infinitely lived and the borrower's average income and the liquidation and reservation values do not change over time. In a stochastic house price environment, an increase in the house value would increase the borrower's creditworthiness (due to an increase in the value of collateral and his total wealth) and, thus, provide an additional incentive to refinance if this option is allowed. Similarly, a decline in the house value would lower the borrower's creditworthiness and could lead to a strategic default: the borrower may default even when he is still able to make mortgage payments.

Some borrowers could also choose to cash out part of their home equity following an increase in home value, refinancing not just to get a lower mortgage rate. However, as such refinancings are also induced by positive credit shocks, a freely prepayable mortgage would still have the undesirable feature we identified. It would allow those with ex post good credit shocks to freely exit from the credit arrangement, thus limiting the ability to share credit risk among borrowers.<sup>21</sup>

Overall, our argument supporting refinancing penalties would likely be stronger in a stochastic house price environment, due to an additional source of variation in ex post credit quality of the borrowers. In fact, in our empirical work we use house price shocks as a proxy for ex post wealth shocks.

For tractability, we also assume that the market interest rate is constant. In a setting with stochastic aggregate interest rates, the borrowers would have incentive to refinance if the rates sufficiently decline. However, the rationale for benefits of prepayment penalties we focus on would also be present in such a richer environment.

### 6.2. Endogenous refinancing cutoffs

For simplicity, we assume that a borrower wants to refinance his loan when his creditworthiness improves by a certain exogenously specified amount. Such a grid of refinancing cutoffs could be endogenized by explicitly modeling refinancing costs.<sup>22</sup> Because our results hold for any such grid, this does not change the predictions of our model. However, it also generates a new prediction. Because the benefits of refinancing are higher for riskier borrowers, these borrowers would refinance their mortgages more often in response to positive wealth shocks. We test this prediction using house price shocks as a proxy for ex post wealth shocks.

In a model with explicit refinancing costs, the utility of the borrower in the regime with refinancing would be lower relative to our setting, as the borrower would also

need to incur these additional costs. Hence, considering these costs explicitly in our setting would make the benefits of prepayment penalties larger for risky borrowers as these penalties would also prevent some dead-weight refinancing costs.

### 6.3. General risk aversion

For the sake of tractability, we assume risk neutrality of the borrower with respect to discretionary consumption. A more general form of risk aversion on the borrower's side would likely strengthen our results, as such borrowers would value insurance more, increasing the benefits of refinancing penalties.

### 6.4. Endogenous downpayment

We assume a constant (zero) downpayment by the borrower. In the health and life insurance context, front-loading contributions can facilitate provision of long-term insurance (Hendel and Lizzeri, 2003). In the lending context, the beneficial role of frontloading through downpayment or points would be much weaker. Those who can spend significantly on points or higher downpayments are likely already low-risk borrowers. Thus, as these borrowers already receive low mortgage rates, making a sizable downpayment might not benefit them significantly. This is an important difference from the health or life insurance context, in which the agent's ability to frontload an insurance premium does not necessarily indicate lower likelihood of death or serious illness.

### 6.5. Other mortgage contracts

For clarity of exposition, and partly motivated by our empirical work, we focus our attention on a simple FRM contract. These loans are more evenly spread across the credit distribution, allowing us to investigate the relation between the prepayment penalty status and the borrower's creditworthiness within the same class of loans. Also, FRMs have a simpler pricing schedule, allowing for an easier comparison of mortgage cost.

A FRM could be potentially improved by a more flexible loan in which a lender provides the borrower with some additional access to credit.<sup>23</sup> However, such more complex contracts would have the same rationale for including refinancing penalties.

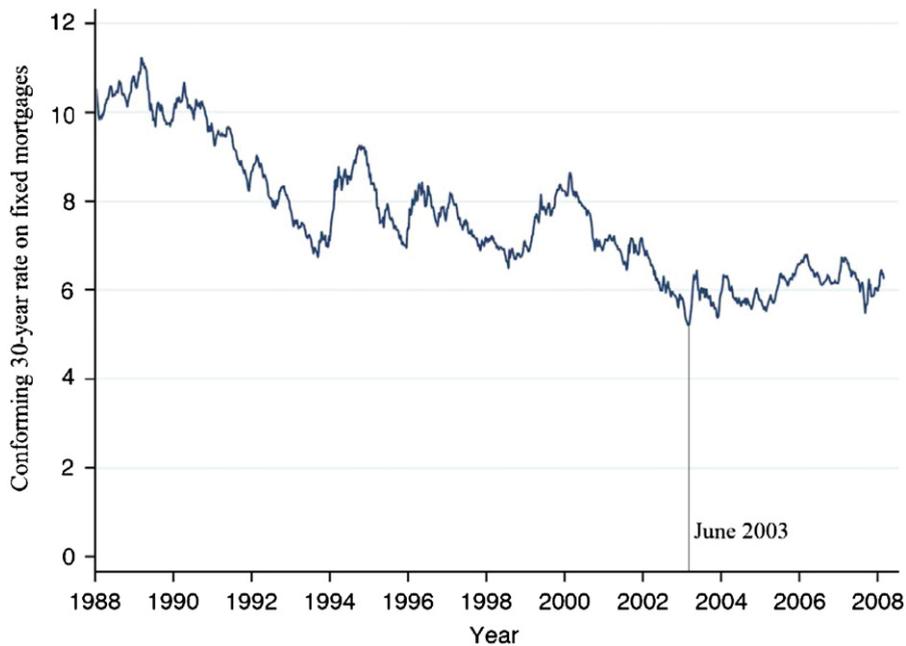
Given our simple setting, we also did not consider the so-called hybrid mortgages with initial teaser rates, such as the 2/28 adjustable-rate mortgages (ARMs).<sup>24</sup>

<sup>21</sup> A moderate refinancing penalty could be optimal if there are sizable utility gains for the borrowers from cashing-out their home equity. The penalty amount could be chosen to be sufficiently low to allow the borrowers to withdraw some of their home equity when needed at a reasonable cost, but at the same time to be sufficiently high to discourage some opportunistic prepayments.

<sup>22</sup> See Dunn and Spatt (2005), who solve simultaneously for the value of the mortgage and the borrower's refinancing strategy in the presence of refinancing costs.

<sup>23</sup> For example, Piskorski and Tchisty (2010) show that the optimal mortgage contract in an environment with risky and unobservable borrower's income and costly foreclosure would have the lender providing the borrower with some additional access to credit (e.g., through a home equity line of credit or a second mortgage).

<sup>24</sup> The 2/28 ARM is a mortgage that carries introductory rate for the first two years before resetting to a possibly higher adjustable rate. Piskorski and Tchisty (2011) provide a theoretical analysis of such products and show that mortgages with teaser rates and flexible payment schedule most benefit the least creditworthy borrowers in locations where house prices and income are expected to grow.



**Fig. 3.** Conforming 30-year rates on fixed rate mortgages. This figure plots the monthly average quoted interest rate for a conforming 30-year fixed rate mortgage (in percentage terms), as collected in a survey of 125 mortgage lenders nationwide conducted by Freddie Mac (Federal Home Loan Mortgage Corporation). About 25 lenders of varying types are surveyed in each of five regions of the United States and the average rates are weighted according to transaction frequency (as measured by the number of conforming single-family mortgage originations reported in annual Housing Mortgage Disclosure Act data) to obtain a national average.

The teaser structure of these loans would provide an additional rationale for the prepayment penalty. For the lender to break even on such a loan, the borrower has to either pay a higher rate after reset or be subject to a prepayment penalty if he refinances early to compensate for the initial rate reduction. Consistent with this observation, the majority of these loans carry prepayment penalties, and they are almost exclusively concentrated among the riskiest borrowers.<sup>25</sup>

#### 6.6. Non-competitive lending

Our results are derived in a competitive mortgage lending market. We conjecture that the benefits of refinancing penalties would also be present in other models of competition. The relative bargaining powers of the borrower and lender would then determine how they would share the additional surplus generated by the use of refinancing penalties.

### 7. Empirical evidence

The model Section 6 makes a number of predictions that we now examine using data on FRMs. We use house price changes as a proxy for ex post credit shocks.

<sup>25</sup> Among the 2/28 ARM loans originated between 2003 and 2006, more than 74% carry prepayment penalties (LP database). Mayer, Pence, and Sherlund (2009) report that among the least creditworthy (subprime) borrowers, over 75% of mortgages originated over the 2003–2007 period had teaser rates while, among more creditworthy (Alt-A) borrowers, these loans constituted only about 10%.

One real-world complication in empirically investigating the predictions of our model is that borrowers might prepay to obtain a lower mortgage rate due to lower market interest rates, rather than because of a change in credit status. Our model does not explicitly consider how the addition of interest rate shocks could impact prepayment, though as discussed in Section 6.1, we are confident that all of our main predictions would still hold. To ensure that our empirical work is not biased by the impact of negative interest rate shocks on prepayment, we focus on mortgages originated in June 2003. During that month, mortgage rates were at their lowest level of the period between 1988 and 2008 (see Fig. 3), minimizing the potential value of the prepayment option due to market interest rate changes. By focusing on borrowers who obtained mortgages when the market rate was the lowest in years preceding the housing crisis, any observed refinancings must be due to factors unrelated to market interest rate declines.

#### 7.1. Empirical predictions

We develop predictions that are consistent with our model.

*Borrowers who receive positive credit shocks are more likely to prepay their mortgages.* We examine mortgages without prepayment penalties to see how likely these mortgages are to be prepaid in response to house price changes. Our model predicts that borrowers who receive the largest house price increases should be the most likely to prepay early.

*The sensitivity of prepayment risk to a positive credit shock is larger for lower credit quality households.* Our

**Table 1**  
Variables description.

Variable	Description
Pays Off in X Qs	One if the loan pays off in full in the first X (eight or 16) quarters from origination and zero otherwise
Average Quarterly HPI Growth	The quarterly, real housing price index at the respective horizon calculated using Office of Federal Housing Enterprise Oversight MSA-level data
Prepayment Penalty	One if the loan contains a prepayment penalty and zero otherwise
Coupon Rate	Monthly interest rate charged to the borrower
Origination Amount	Origination amount in thousands of dollars
FICO	Credit score of the borrower using the FICO credit profile
Vantage	Credit score of the borrower using the Vantage credit profile
Percentage Change in Vantage	Captures the difference (in percentage terms) between the last observed Vantage credit score of the borrower before the month of prepayment and initial Vantage at loan origination
Lien	Either <i>Senior</i> for loans in the first position or <i>Subordinate</i> for loans that are not in the senior position
Senior Has Juniors	One when the first lien loan is known to have an associated junior lien and zero otherwise
Loan Is Subprime	One when the loan is classified as subprime and zero otherwise
CLTV	Combined loan-to-value: for junior liens, the loan-to-value (LTV) ratio calculated as the sum of the value of all liens against the home over the value of the home; for senior (first) lien loans, due to the fact that we do not observe the value of additional liens, we use as the CLTV the ratio of the first loan amount to the value of the home
Loan Purpose	Either <i>For Purchase</i> , <i>Cash Out Refinance</i> , or <i>No Cash Out Refinance</i>
Documentation	Variable that is <i>Full Documentation</i> , <i>Low Documentation</i> , or <i>No Documentation</i>
Term	Categorical variable for the length of the term, either <i>10 years</i> , <i>15 years</i> , or <i>30 years</i>

model implies that the worst credit risk borrowers benefit the most from positive credit shocks such as an increase in the value of their house and, thus, they should be most likely to prepay their mortgages.

The next two predictions are based on our results showing that the benefits from a prepayment penalty are larger for less creditworthy borrowers.

*Prepayment penalties should be most prevalent among the riskiest borrowers.* We examine whether the distribution of prepayment penalties as a function of borrowers' credit-worthiness in the data corresponds to the one implied by our model.

*Conditional on qualifying for credit, the riskiest borrowers choosing loans with prepayment penalties obtain the largest reductions in mortgage rates relative to the rates they would have obtained with fully prepayable mortgages.*

## 7.2. Data summary

Our primary data come from LoanPerformance, which provides loan-level data on a large number of securitized mortgages. Mayer, Pence, and Sherlund (2009) show that the LP data appear relatively representative of the universe of high-cost risky loans, with the exception that refinancings appear to be somewhat overrepresented in LP.

LP collects its data at two different times. LP collects data on contract terms at the time of origination. In addition, LP collects data from servicers throughout the life of the mortgage on whether or not the loan has paid off or become delinquent. We create a combined data set that includes both the characteristics of a loan at origination as well as its monthly payment history.

Within the LP database, we consider loans with the following characteristics: loans originated in June 2003; fixed interest rate; term lengths of 15, 20 or 30 years; known prepayment penalty status; located in a Metropolitan Statistical Area (MSA) with housing price index (HPI) data; and collateralized by an owner-occupied one to four unit home. We collect HPI data from the Office of Federal

Housing Enterprise Oversight. The data are reported at the MSA level and are mapped onto the LP data using a ZIP code to MSA correspondence. The HPI is normalized to reflect real dollars. Table 1 defines the variables we use in our analysis.

We consider two subsets of the LP database: prime and subprime. Prime loans are classified by the type of pool they belong to, using definitions that are reported by the issuer of the mortgage-backed security (MBS). Prime MBS are backed by high-quality mortgages (that is, mortgages for borrowers with relatively low loan-to-value ratios and with very good credit scores), whose initial balance typically exceeds the maximum limits for participation by government-sponsored enterprises. Subprime loans consist of mortgages belonging to pools classified as subprime and having an origination FICO score less than 620.<sup>26</sup>

We focus on FRMs due to complications in understanding prepayment behavior for ARMs that often have teaser rates or other features that complicate the empirical analysis by giving borrowers reasons to refinance their mortgages other than changes in their creditworthiness. After all our restrictions, we are left with a sample of 9,046 subprime FRMs (of which 2,517 carry no prepayment penalty) and 9,628 prime FRMs that carry no prepayment penalties to investigate the refinancing propensity of the borrowers.<sup>27</sup>

Table 2 reports summary statistics for the securitized loans in our sample that carry no prepayment penalties. In general, loans in prime pools are safe along all dimensions and have a mean origination FICO score of 738. Subprime loans are much riskier with an average origination FICO score equal to 574. These statistics suggest

<sup>26</sup> Most lenders define a borrower as subprime if the borrower's FICO credit score is below 620 (see, e.g., the Office of the Comptroller of the Currency Mortgage Metrics Reports; Keys, Mukherjee, Seru, and Vig, 2010). We impose this additional restriction as some of the subprime pools may also contain few better quality loans (e.g., for credit enhancement).

<sup>27</sup> Due to the paucity of prime mortgages with prepayment penalties, we do not examine rate differences for prime mortgages with and without prepayment penalties.

variation across multiple risk factors that complicate our analysis. Among the categories of loans, subprime loans are prepaid in the first 16 quarters at much higher rates (70%) compared with prime loans (31%).

Houses also experience varied rates of price appreciation depending on their location. The mean quarterly real appreciation rate was 1.3% (about 5% annualized), reflecting the strong growth of house prices over our sample period. Thus, we would expect relatively few defaults, as borrowers who get into financial trouble can respond by paying off

their mortgage by selling their house, often at a profit. However, there is wide dispersion in house price growth rates. The highest-appreciation markets experienced quarterly appreciation rates as high as 3.3% (almost 14% per year for more than 4 years). Slightly less than 10% of markets saw negative real appreciation rates over this time period.

Table 3 reports summary statistics for all securitized subprime loans in our sample, whether or not these mortgages contain a prepayment penalty, and also separately for senior and junior liens. As Table 3 shows, more than 72% of subprime loans have a prepayment penalty.

**Table 2**

Summary of fixed rate mortgage (FRM) loans with no prepayment penalties.

The table covers owner occupied FRM loans without a prepayment penalty originated during June 2003 from the LoanPerformance (LP) database.

Variables	Subprime		Prime	
	Mean	Standard Deviation	Mean	Standard Deviation
Pays Off in 16 Qs	0.702	0.457	0.314	0.464
Average Quarterly HPI Growth	0.011	0.009	0.018	0.009
Coupon Rate	9.075	1.905	5.478	0.361
Origination Amount	110.6	88.8	485.5	164.9
CLTV	80	15	63	15
FICO	574.4	32.045	738.9	40.932
No Cash Refinance	0.124	0.329	0.636	0.481
Cash Out Refinance	0.648	0.478	0.193	0.394
Low Documentation	0.18	0.384	0.289	0.453
No Documentation	0.004	0.063	0.067	0.25
15 Year Term	0.132	0.338	0.395	0.489
20 Year Term	0.162	0.369	0.008	0.09
Senior has Juniors	0.009	0.093	0.009	0.286
Subordinate Lien	0.205	0.404	0	0
Number of Loans	2,517		9,631	

### 7.3. Empirical results

The first two predictions of our model state that the borrowers who receive positive credit shocks are more likely to prepay their mortgages. We begin our analysis by exploring whether the changes in borrowers' credit-worthiness, using changes in house prices as a proxy, are an important factor affecting refinancing behavior.

Table 4 presents regressions that explore the prediction that higher house prices spur increased prepayments, so that over time, pools of outstanding loans are disproportionately composed of mortgages in locations with below-average house price appreciation. We use a logit specification with a dependent variable that equals one if a loan in a given category pays off in the next four years and zero otherwise. The independent variable of interest is the annualized rate of house price appreciation. The specifications include a variety of control variables that are commonly associated with loan payoff and risk, including the coupon rate on the mortgage, the borrower's FICO score, loan-to-value ratio, and whether the loan was a refinancing or cash-out refinancing at origination. We run separate regressions for prime and subprime

**Table 3**

Summary statistics for fixed rate mortgage (FRM) subprime loans both with and without prepayment penalties.

The table covers owner occupied FRMs originated during June 2003 from the LoanPerformance (LP) database.

Variables	All liens		Senior liens		Junior liens	
	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
Pays Off in 16 Qs	0.719	0.448	0.697	0.459	0.903	0.296
Average Quarterly HPI Growth in 16 Qs	0.016	0.012	0.016	0.012	0.016	0.012
Prepayment Penalty	0.722	0.448	0.748	0.434	0.53	0.499
Coupon Rate	8.363	1.635	7.956	1.204	11.304	1.281
Origination Amount	133.8	92.44	147.09	90.74	38.42	20.47
CLTV	78	16	75	15	96	10
FICO	576.6	31.218	574.61	31.613	590.7	23.850
No Cash Refinance	0.14	0.346	0.152	0.359	0.047	0.212
Cash Out Refinance	0.693	0.461	0.749	0.434	0.292	0.455
Low Documentation	0.183	0.387	0.2	0.4	0.061	0.239
No Documentation	0.005	0.072	0.005	0.073	0.004	0.06
15 Year Term	0.109	0.312	0.092	0.289	0.233	0.423
20 Year Term	0.12	0.325	0.054	0.226	0.597	0.491
Senior Has Juniors	0.013	0.115	0.015	0.123	0	0
Subordinate Lien	0.122	0.327	0	0	1	0
Number of Loans	9,046		7,945		1,101	

**Table 4**

Effect of house price growth for prepayments.

The table covers owner occupied FRMs without a prepayment penalty originated during June 2003 from the LoanPerformance database. Payoff is defined as voluntary prepayment. House price growth is calculated using Office of Federal Housing Enterprise Oversight MSA-level data. Other controls include coupon rate, FICO score, origination amount, CLTV (combined loan-to-value), loan purpose, documentation level, term length, and junior or senior status. Coefficients reported are the marginal effects from a logistic regression multiplied by the standard deviation of the right-hand side variable. *t*-statistics are in parentheses.

	Subprime	Prime
Dependent variable	Pays Off in 16 Qs	Pays Off in 16 Qs
Dependent variable mean	0.702	0.314
Average Quarterly HPI Growth	0.083 (7.19)	0.006 (1.18)
Other Controls	Yes	Yes
Number of Loans	2,517	9,628

loans. For ease of interpretation, coefficients in all tables reflect marginal effects of a 1 standard deviation change in the variable about its mean for continuous variables, or a 1 unit change in the case of dummy variables.

Table 4 shows that, consistent with the first empirical prediction of our model, borrowers in locations where house prices appreciated sharply are more likely to prepay their mortgages compared with borrowers in locations where house prices grew less quickly. The coefficient on annualized house price appreciation is positive and statistically different from zero for subprime borrowers and is positive for prime borrowers.

Our second empirical prediction is that the sensitivity of prepayment risk to a positive credit shock is larger for lower credit quality household. Consistent with this prediction, we find that the effect of house price appreciation on payoff is much larger (both on an absolute and relative basis) for the riskier subprime loans. As Table 4 shows for prime mortgages, a 1 standard deviation change in house price appreciation (1.0%) leads to a modest 0.6% increase in the payoff rate, an increase of about 2% of the mean payoff rate of 31%. For the riskier subprime loans, the marginal effect rises to 8.3%, a much larger relative increase of 11.8% from the mean payoff rate of 70%.

While Table 4 focuses on mortgages originated in June 2003, similar results hold when we look at non-agency subprime and prime loans without prepayment penalties originated in the years 2003–2005.<sup>28</sup> Estimating the specification in Table 4 for subprime FRMs without prepayment penalties originated in 2003, we find that a 1% increase in quarterly growth of house prices is associated with a 5.74% increase in the payoff rate during the two years after origination (an increase of 13.8% relative to a mean payoff rate). Among non-agency prime FRMs originated in 2003, this sensitivity is much smaller. A 1% increase in quarterly growth of house prices is associated

with a 1.67% increase in the payoff rate during two years after origination (an increase of 5.6% relative to a mean payoff rate). Similarly, for subprime FRMs without prepayment penalties originated in 2004, we find that a 1% increase in quarterly growth of house prices is associated with a 6.13% absolute increase in the payoff rate during the two years after origination, while among prime loans this effect is much smaller and equal to 1.19%. Finally, for FRMs originated in 2005, a 1% increase in quarterly growth rate of house prices is associated with a 4.4% increase in the payoff rate of subprime loans in the first two years after origination but only a 0.5% increase in the payoff rate for prime loans. Overall, these results support the conventional view in the mortgage industry that house price appreciation is an economically important factor in explaining payoff rates for the risky mortgages.

To provide further evidence that changes in borrower's creditworthiness are an important factor affecting prepayments, we focus on a sample of subprime loans originated in 2005. The data in this sample come from BlackBox Logic's securitized mortgage data set and consist of 12,103 subprime FRMs originated in 2005 with no indication of a prepayment penalty.<sup>29</sup> These loans have been merged with high confidence to Equifax credit data. Equifax is a credit reporting agency that provides monthly data on borrowers' current credit scores, payments and balances on mortgage and installment debt, and balances and credit utilization for revolving debt (such as credit cards). Equifax reports Vantage as the credit score. Intended to be comparable to FICO, the Vantage score was designed by the three credit reporting bureaus (Equifax, Experian, and TransUnion) to measure overall borrower credit health. Vantage scores range from 501 to 990. An improvement in Vantage score is associated with improvement in the borrower's creditworthiness (e.g., paying down credit card debt).

The merged data allow us to investigate the relation between the change in the borrower's creditworthiness as measured by the change in their Vantage credit scores and their prepayment behavior. Table 5 presents regressions

<sup>28</sup> This sample consists of subprime and prime loans from the LP database originated in the years 2003–2005 that carry no prepayment penalties and satisfy the other restrictions defined in Section 7.2. We exclude the 2006 origination vintage as these loans faced limited refinancing opportunities due to the collapse of the housing market in 2007.

<sup>29</sup> We do not have access to merged data with sufficient coverage for loans originated prior to 2005.

that explore the prediction that higher house prices and improvements in borrower creditworthiness as measured by change in credit score are associated with increased prepayments. The dependent variable in this regression is a dummy variable that is one if the mortgage exits the sample via prepayment within 8 quarters of origination and zero otherwise. Quarterly HPI growth is calculated using changes in the Office of Federal Housing Enterprise Oversight's MSA-level price indices from origination until the loan either exits the samples or eight quarters pass. The change in

**Table 5**

Effect of house price growth and the borrowers' credit score changes for prepayments.

The table contains a sample of subprime FRMs with no indication of prepayment penalty originated during 2005 from the BlackBox-Equifax database. Payoff is defined as voluntary prepayment. House price growth is calculated using Office of Federal Housing Enterprise Oversight MSA-level data. The change in Vantage is the variable capturing the difference (in percentage terms) between the last observed Vantage score before the month of prepayment and initial Vantage at loan origination. Other controls include interest rate, original vantage, original balance, CLTV (combined loan-to-value), loan purpose, documentation level, term length, and junior or senior status. Coefficients reported are marginal effects from a probit regression. *t*-statistics are in parentheses.

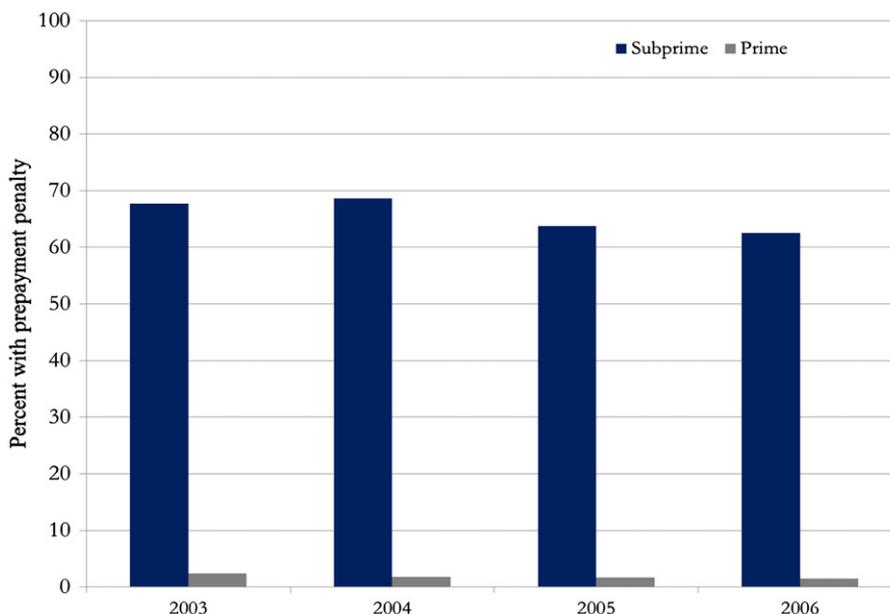
Dependent variable	Pays Off in 8 Qs
Dependent variable mean	0.215
Average Quarterly HPI Growth	0.102 (33.25)
Percentage Change in Vantage	0.00357 (9.59)
Other Controls	Yes
Number of Loans	12,103

Vantage is the variable capturing the difference (in percentage terms) between the last observed Vantage score before the month of prepayment (to avoid the credit impact of prepayment) and initial Vantage at loan origination.

The results in Table 5 again confirm that positive growth in house prices is associated with increased prepayments. A 1% increase in the quarterly growth of the house prices is associated with a 10% increase in the payoff rate during the two years after origination. Similarly, the changes in the borrower's creditworthiness as measured by the changes in credit score are also associated with increased prepayments. As Table 5 indicates, a 10% relative improvement in the Vantage score is associated with a 3.5% increase in the payoff rate during the two years after origination (16.2% relative increase in the mean payoff rate). As the standard deviation of the percentage change in the Vantage score is equal to 10.8% in our data, this estimate indicates that changes in the borrower's creditworthiness unrelated to changes in house prices are also an important factor affecting prepayment behavior.

Our third empirical prediction says that the riskiest borrowers are the most likely to obtain loans with prepayment penalties. Across the two types of loans, the evidence strongly supports this prediction. First, among FRMs originated in June 2003, less than 2% of mortgages in prime pools (1.5%) have prepayment penalties, and 72.2% of subprime mortgages have prepayment penalties. The summary statistics of mortgages with and without prepayment penalties also show that mortgages with prepayment penalties are much riskier on observable dimensions, as measured by variables such as FICO or combined loan-to-value (CLTV).

These results also hold in other origination periods. To illustrate this, Fig. 4 shows the fraction of loans with prepayment penalties among non-agency subprime and



**Fig. 4.** Fraction of fixed rate mortgages (FRMs) with prepayment penalties by origination year. This figure presents the percentage of loans that carry prepayment penalties among non-agency securitized subprime and prime FRMs originated from 2003 to 2006 from the LoanPerformance database. The subprime and prime categories are defined as in Section 7.2.

prime FRMs originated from 2003 to 2006. In each of these years, the majority of subprime borrowers chose mortgages with prepayment penalties. At the same time, less than 3% of prime borrowers end up with loans carrying prepayment penalties. Finally, a monotonic relation exists between the FICO credit score and the presence of a prepayment penalty in the data. The majority of loans with low FICO scores have a prepayment penalty, while only a small fraction of loans with high FICO scores have such clauses.

The fourth prediction of our model is that borrowers choosing prepayment penalties obtain rates that are lower than they would have obtained with fully prepayable mortgages (conditional on qualifying for credit), with larger reductions going to riskier borrowers. Empirical investigation of this prediction is challenging as we do not observe whether a borrower would qualify for credit for a given house without a prepayment penalty and if so, what the corresponding mortgage rate would be. Nevertheless, to shed some light on this question, we compare the rates on loans with and without prepayment penalties, controlling for a number of observable risk characteristics. Our model implies that borrowers choosing prepayment penalties are likely to be less creditworthy (potentially on dimensions not captured by our set of risk controls). Thus, this potential selection on unobservables would make it harder to show that the borrowers with prepayment penalties obtain lower rates.

Table 6 presents the result of regressing the subprime coupon rates on the prepayment penalty status. Consistent with our model, we find that among risky borrowers, mortgages with prepayment penalties carry lower rates compared with loans with free refinancing, with the largest difference for the riskiest loans. Column 1 reports results for all borrowers. The subprime borrowers receive mortgages with a rate that is almost 20 basis points lower (0.195%) when a loan has a prepayment penalty. To further investigate this, we compare rates for borrowers for senior and junior liens. Column 2 reports the safer senior liens (the highest priority mortgages) while Column 3 presents results for riskier second liens. These junior mortgages fall behind the first liens in priority and have a very high average CLTV of about 100%. Thus, our

model would predict larger discounts for prepayment penalties for riskier second lien borrowers than for first lien borrowers. Consistent with our theory, the mortgage rates of the riskiest junior liens are 62 basis points lower (0.625%) when their loan has a prepayment penalty. For the first lien borrowers this difference is 14 basis points (0.139%).

These differences in coupon rates between loans with and without prepayment penalties are sizable given that the period of applicability of a typical prepayment penalty is limited. For example, among subprime loans originated in 2003, the average penalty lasted for about three years (Chomsisengphet and Pennington-Cross, 2006).

While the percentage of subprime borrowers with fully prepayable loans is relatively low, the question naturally arises as to why low credit quality borrowers would not always have prepayment penalties if the higher mortgage rates often lead to greater defaults. Our earlier discussion suggests that restricted mobility could impose additional costs that might more than offset the benefit of the lower mortgage rates that usually accompany a prepayment penalty. Thus, some borrowers with a high likelihood of moving might accept higher rates and a greater risk of default in return for not avoiding being locked into their homes. In addition, a number of states have legal restrictions on the usage of prepayment penalties by lenders (our empirical specifications for coupon rate include location specific (MSA) fixed effects).

## 8. Concluding remarks

Critics of subprime mortgages often point to a high concentration of prepayment penalties among less creditworthy borrowers. They argue that prepayment penalties unfairly lock less creditworthy borrowers into mortgages with high interest payments.

This paper shows that, in a competitive lending model, refinancing penalties can be welfare improving and that they can be particularly beneficial to riskier borrowers in the form of lower mortgage rates, reduced defaults, and increased availability of credit. Thus, a high concentration of prepayment penalties among the least creditworthy

**Table 6**

Mortgage coupon and prepayment penalty for subprime loans.

The table covers owner occupied fixed rate mortgage loans originated during June 2003 from the LoanPerformance database. Coupon rate is the monthly interest rate charged to the borrower (in percentage terms). Prepayment penalty is a dummy variable that takes value of one if the loan contains a prepayment penalty and is zero otherwise. Other controls include FICO score, origination amount, CLTV (combined loan-to-value), loan purpose, documentation level, term length, junior or senior status, and MSA fixed effects. Coefficients reported are from an ordinary least squares regression. *t*-statistics are in parentheses.

	All liens	Senior liens	Junior liens
Dependent variable	Coupon Rate	Coupon Rate	Coupon Rate
Dependent variable mean	8.363	7.956	11.304
Prepayment Penalty	−0.195 (−6.90)	−0.139 (−4.55)	−0.624 (−7.93)
Other Controls	Yes	Yes	Yes
Number of Loans	9,046	7,945	1,101

borrowers can be an outcome of efficient and fair lending. Also our model implies that one could see more high cost (high premium) loans and more defaults in the aggregate if prepayment penalties are used, just because more risky borrowers would be able to qualify for credit. Overall, our model highlights the importance of considering dynamic features of credit contracts to understand the impact of specific lending terms.<sup>30</sup> We also provide empirical evidence that is consistent with the key predictions of our model.

Our analysis suggests that the refinancing penalties could be a part of a refashioned mortgage product that helps riskier borrowers obtain credit. This might require appropriate disclosure and counseling to ensure that borrowers understand the terms of their mortgages and the implications of a prepayment penalty.

In deriving our results, we have not considered that risky lending could have some negative externalities. In such a setting, a restriction on the usage of prepayment penalties can lower the amount of risky lending and, hence, lead to some welfare benefits. However, if the objective of the regulators is to reduce risky lending, our results suggest that a better policy would be to impose direct restrictions on lending based on borrowers' creditworthiness while still allowing the usage of prepayment penalties for qualified borrowers.

**Appendix A**

*A.1. Proof of Proposition 1*

First we show that the solution to Eqs. (7)–(10) is concave in  $S$ . Consider a function

$$F(S, \bar{\phi}) = \bar{U}(S, \bar{\phi}) - S, \tag{34}$$

where  $\bar{U}(S, \bar{\phi})$  is a solution to Eqs. (7)–(10). From Eqs. (7)–(10) we have that

$$\gamma F(S, \bar{\phi}) = \mu + \theta - \bar{\phi} + (rS + \mu - \bar{\phi})F'(S, \bar{\phi}) - (\gamma - r)W + \frac{1}{2}\sigma^2 F''(S, \bar{\phi}) \quad \text{for } S \in [0, S^1(\bar{\phi})], \tag{35}$$

$$F(0, \bar{\phi}) = A, \tag{36}$$

$$F'(S, \bar{\phi}) = 0 \quad \text{for } S \geq S^1(\bar{\phi}) \quad \text{and} \tag{37}$$

$$F''(S, \bar{\phi}) = 0 \quad \text{for } S \geq S^1(\bar{\phi}). \tag{38}$$

As the solution  $F$  is smooth, we can differentiate it with respect to  $S$  to obtain

$$(\gamma - r)F'(S, \bar{\phi}) = (rS + \mu - \bar{\phi})F''(S, \bar{\phi}) - (\gamma - r) + \frac{1}{2}\sigma^2 F'''(S, \bar{\phi}) \tag{39}$$

for  $S \in [0, S^1(\bar{\phi})]$ .

Evaluating Eq. (39) at  $S = S^1(\bar{\phi})$  yields

$$F_{-}'''(S^1(\bar{\phi}), \bar{\phi}) = \frac{2(\gamma - r)}{\sigma^2} > 0. \tag{40}$$

As  $F''(S^1(\bar{\phi}), \bar{\phi}) = 0$  and  $F_{-}'''(S^1(\bar{\phi}), \bar{\phi}) > 0$ , the implication is that there exists  $\varepsilon > 0$  such that  $F'' < 0$  over the interval  $(S^1(\bar{\phi}) - \varepsilon, S^1(\bar{\phi}))$ . Also as  $F'(S^1(\bar{\phi}), \bar{\phi}) = 0$  and  $F''(S^1(\bar{\phi}), \bar{\phi})$

$< 0$  over the interval  $(S^1(\bar{\phi}) - \varepsilon, S^1(\bar{\phi}))$ , the implication is that  $F' > 0$  over the interval  $(S^1(\bar{\phi}) - \varepsilon, S^1(\bar{\phi}))$ .

From Eq. (7) we have that

$$F''(S, \bar{\phi}) = \frac{2[\gamma F(S, \bar{\phi}) - [\mu + \theta - \bar{\phi} - (\gamma - r)S] - (rS + \mu - \bar{\phi})F'(S, \bar{\phi})]}{\sigma^2}. \tag{41}$$

As  $\bar{\phi} \leq \theta$  and as by assumption  $\theta \leq \mu$  we have that  $\mu - \bar{\phi} \geq 0$ . Hence, from Eq. (41) whenever (i)  $\gamma F(S, \bar{\phi}) < [\mu + \theta - \bar{\phi} - (\gamma - r)S]$  and (ii)  $F'(S, \bar{\phi}) > 0$  holds, it follows that  $F'' < 0$ . Now because  $\gamma F(S^1(\bar{\phi}), \bar{\phi}) = \mu + \theta - \bar{\phi} - (\gamma - r)S^1(\bar{\phi})$  and  $F' > 0$  over the interval  $(S^1(\bar{\phi}) - \varepsilon, S^1(\bar{\phi}))$  it follows that these conditions are satisfied over the interval  $(S^1(\bar{\phi}) - \varepsilon, S^1(\bar{\phi}))$ . Moreover, (i) holds for any  $S \in [0, S^1(\bar{\phi})]$  as long as  $F$  remains strictly increasing, i.e., as long as  $F' > 0$ .

Now suppose by contradiction that  $F' \leq 0$  for some  $S \leq S^1(\bar{\phi}) - \varepsilon$  and let

$$\tilde{S} = \sup\{S \leq S^1(\bar{\phi}) - \varepsilon \mid F' \leq 0\}. \tag{42}$$

Then it follows that  $F'(\tilde{S}, \bar{\phi}) = 0$  and that, for all  $S \in (\tilde{S}, S^1(\bar{\phi}))$ , we have that  $F' > 0$  and so (i) and (ii) hold. But this implies that  $F'' < 0$  for  $S \in (\tilde{S}, S^1(\bar{\phi}))$ . From the Fundamental Theorem of Calculus it follows that

$$S'(b) + \int_{\tilde{S}}^{S^1(\bar{\phi})} F''(S, \bar{\phi}) dS, \tag{43}$$

which given that  $S'(S^1(\bar{\phi}), \bar{\phi}) = 0$  implies that

$$F'(\tilde{S}, S^1(\bar{\phi})) = - \int_{\tilde{S}}^{S^1(\bar{\phi})} F''(S, \bar{\phi}) dS. \tag{44}$$

As  $F'' < 0$  for  $S \in (\tilde{S}, S^1(\bar{\phi}))$ , Eq. (44) implies that  $F'(\tilde{S}, S^1(\bar{\phi})) > 0$ , which is a contradiction. Hence, we have that  $F' > 0$  for  $S \in (0, S^1(\bar{\phi}))$ , (i) and (ii) hold and  $F'' < 0$  for  $S \in (0, S^1(\bar{\phi}))$ . But this implies that  $\bar{U}'(S, \bar{\phi}) > 1$  and  $\bar{U}''(S, \bar{\phi}) < 0$  for  $S \in (\tilde{S}, S^1(\bar{\phi}))$ .

Conditions (9)–(10) imply that

$$\gamma \bar{U}(S^1(\bar{\phi}), \bar{\phi}) = \theta + \mu - \bar{\phi} + rS^1(\bar{\phi}). \tag{45}$$

The borrower's savings (wealth) evolves according to

$$dS_t = (rS_t + \mu - \bar{\phi}) dt + \sigma dZ_t - dC_t. \tag{46}$$

For an arbitrary feasible strategy  $(C, S)$ , consider

$$G_t \equiv \int_0^t e^{-\gamma s} (dC_s + \theta ds) + e^{-\gamma t} \bar{U}(S_t, \bar{\phi}), \tag{47}$$

where function  $\bar{U}$  satisfies all the conditions outlined in Proposition 1. We show that  $G_t$  is a supermartingale. Differentiating Eq. (47) with respect to  $t$  and using Ito's lemma gives

$$\begin{aligned} e^{\gamma t} dG_t = & [-\gamma \bar{U}(S_t, \bar{\phi}) + \theta + (rS_t + \mu - \bar{\phi})\bar{U}'(S_t, \bar{\phi}) \\ & + 0.5\sigma^2 \bar{U}''(S_t, \bar{\phi})] dt \\ & + (1 - \bar{U}'(S_t, \bar{\phi})) dC_t + \bar{U}'(S_t, \bar{\phi}) \sigma dZ_t. \end{aligned} \tag{48}$$

When  $S_t \leq S^1(\bar{\phi})$ , the first term in the right-hand side of Eq. (48) is zero, because of Eq. (7). When  $S_t > S^1(\bar{\phi})$ , the first term is negative, because of Eqs. (9), (10) and (45) and the fact  $\gamma > r$ . Because  $\bar{U}'(S_t, \bar{\phi}) \geq 1$  and  $dC_t \geq 0$ ,  $G_t$  is a supermartingale.

$$E \left[ \int_0^\tau e^{-\gamma s} (dC_s + \theta ds) + e^{-\gamma \tau} A \right] = E[G_\tau] \leq G_0 = \bar{U}(S_0, \bar{\phi}). \tag{49}$$

<sup>30</sup> See note 11, which discusses some of the recent literature that considers dynamic features of mortgage contracts.

Thus, the agent's payoff associated with strategy (C,S) is less than or equal to  $\bar{U}(S_0, \bar{\phi})$ .

If the agent has zero consumption whenever  $S_t \in [0, S^1(\bar{\phi})]$  and consumes  $S_t - S^1(\bar{\phi})$  immediately whenever  $S_t > S^1(\bar{\phi})$ , then  $G_t$  is a martingale and Eq. (49) holds with equality. Hence, this is the optimal strategy, which results in  $\bar{U}(S_0, \bar{\phi})$  payoff to the agent.  $\square$

## A.2. Proof of Lemma 1

We first show that the maximum savings level of the borrower,  $S^1(\bar{\phi}^*)$ , is strictly increasing in the competitive mortgage repayment rate with no refinancing  $\bar{\phi}^*$ . Analogously to Lemma 6 in DeMarzo and Sannikov (2006), using Eqs. (45) and (9), we have that

$$\frac{\partial \bar{U}(S^1(\bar{\phi}^*), \bar{\phi}^*)}{\partial \bar{\phi}^*} = -\mathbb{E} \left[ \int_0^{\tau(S^1(\bar{\phi}^*), \bar{\phi}^*)} e^{-\gamma t} \bar{U}'(S_t, \bar{\phi}^*) dt \mid S_0 = S^1(\bar{\phi}^*) \right]. \quad (50)$$

Because  $\partial \bar{U}'(S_t, \bar{\phi}^*) \geq 1$ , we have that

$$\begin{aligned} -\frac{\partial \bar{U}(S^1(\bar{\phi}^*), \bar{\phi}^*)}{\partial \bar{\phi}^*} &\geq \mathbb{E} \left[ \int_0^{\tau(S^1(\bar{\phi}^*), \bar{\phi}^*)} e^{-\gamma t} dt \right] \\ &= \frac{1}{\gamma} (1 - \mathbb{E}[e^{-\gamma \tau(S^1(\bar{\phi}^*), \bar{\phi}^*)}]). \end{aligned} \quad (51)$$

However, because  $S_0 \leq S^1(\bar{\phi}^*)$  and  $r < \gamma$ , we can write that

$$\begin{aligned} P = V(S_0, \bar{\phi}^*) &\leq V(S^1(\bar{\phi}^*), \bar{\phi}^*) = \frac{\bar{\phi}^*}{r} - \left( \frac{\bar{\phi}^*}{r} - L \right) \mathbb{E}[e^{-r\tau(S^1(\bar{\phi}^*), \bar{\phi}^*)}] \\ &< \frac{\bar{\phi}^*}{r} - \left( \frac{\bar{\phi}^*}{r} - L \right) \mathbb{E}[e^{-\gamma \tau(S^1(\bar{\phi}^*), \bar{\phi}^*)}], \end{aligned} \quad (52)$$

which gives

$$\mathbb{E}[e^{-\gamma \tau(S^1(\bar{\phi}^*), \bar{\phi}^*)}] \leq \frac{\frac{\bar{\phi}^*}{r} - P}{\frac{\bar{\phi}^*}{r} - L}. \quad (53)$$

Hence,

$$-\frac{\partial \bar{U}(S^1(\bar{\phi}^*), \bar{\phi}^*)}{\partial \bar{\phi}^*} \geq \frac{1}{\gamma} \left( 1 - \frac{\frac{\bar{\phi}^*}{r} - P}{\frac{\bar{\phi}^*}{r} - L} \right) = 1 + \frac{P - \left( \gamma \frac{\bar{\phi}^*}{r} + (1-\gamma)L \right)}{\gamma \left( \frac{\bar{\phi}^*}{r} - L \right)}. \quad (54)$$

Noting that  $\bar{\phi}^* \leq \theta$  and  $\theta \leq r[P - (1-\gamma)L]/\gamma$  [as by assumption  $P - L \geq \gamma(\theta/r - L)$ ], we have that  $P \geq \gamma \bar{\phi}^*/r + (1-\gamma)L$ . But this implies that  $\partial \bar{U}(S^1(\bar{\phi}^*), \bar{\phi}^*)/\partial \bar{\phi}^* < -1$  and so  $\partial S^1(\bar{\phi}^*)/\partial \bar{\phi}^* > 0$ .

Now take any  $S_0 \leq S^1(\bar{\phi}^*(S_0))$ . Consider  $S'_0 < S_0$ . Suppose that by contradiction  $\bar{\phi}^*(S'_0) < \bar{\phi}^*(S_0)$ . But this given the above implies that  $S^1(\bar{\phi}^*(S'_0)) < S^1(\bar{\phi}^*(S_0))$ , which implies that  $\tau(S_0, \bar{\phi}^*(S_0)) > \tau(S'_0, \bar{\phi}^*(S'_0))$  [as both  $S'_0 < S_0$  and  $S^1(\bar{\phi}^*$

$(S'_0)) < S^1(\bar{\phi}^*(S'_0))$ ]. But this implies that

$$\begin{aligned} P &= \underbrace{\frac{\bar{\phi}^*(S_0)}{r} - \mathbb{E} \left[ e^{-r\tau(S_0, \bar{\phi}^*(S_0))} \left( \frac{\bar{\phi}^*(S_0)}{r} - L \right) \mathcal{F}_0 \right]}_{V_0(S_0, \bar{\phi}^*(S_0))} \\ &> \underbrace{\frac{\bar{\phi}^*(S'_0)}{r} - \mathbb{E} \left[ e^{-r\tau(S'_0, \bar{\phi}^*(S'_0))} \left( \frac{\bar{\phi}^*(S'_0)}{r} - L \right) \mathcal{F}_0 \right]}_{V_0(S'_0, \bar{\phi}^*(S'_0))}, \end{aligned} \quad (55)$$

which contradicts  $V_0(S'_0, \bar{\phi}^*(S'_0)) = P$ . Therefore, we have that the competitive mortgage rate on FRM with no refinancing,  $\bar{\phi}^*$ , as well as the borrower's maximum savings level,  $S^1$ , are strictly decreasing in the borrower's initial savings on  $S_0 \in [\underline{S}, \bar{S}]$ . Moreover

$$\bar{S} = \min_{S \geq \underline{S}} \{S^1(\bar{\phi}^*(S))\}, \quad (56)$$

as we have that  $d\bar{\phi}^*(S)/dS < 0$  for  $S \in [\underline{S}, \bar{S}]$  and  $\partial S^1(\bar{\phi}^*)/\partial \bar{\phi}^* > 0$ .

## A.3. Proof of Proposition 2

The proof is similar to the proof of Proposition 1. Function  $\tilde{U}(S, \tilde{\phi})$  is concave in  $S$  because it solves the same equation as  $\bar{U}(S, \bar{\phi})$  does, if we replace  $\theta$  with  $\theta + \delta M$ .

Let  $Q(t)$  denote the Poisson process that governs arrival of the mobility opportunities and  $\tau^m$  be the time of the first mobility opportunity, i.e.,  $\tau^m = \inf\{t : Q(t) = 1\}$ . Let

$$\tilde{U}(S_t, \tilde{\phi}, Q(t)) = \begin{cases} \tilde{U}(S_t, \tilde{\phi}) & \text{for } t < \tau^m, \\ \tilde{U}(S_t, \tilde{\phi}) + M & \text{for } t \geq \tau^m, \end{cases} \quad (57)$$

where function  $\tilde{U}$  satisfies all the conditions outlined in Proposition 2 and  $M$  is the average mobility gain.

For an arbitrary feasible strategy (C,S), consider

$$\tilde{G}_t \equiv \int_0^t e^{-\gamma s} (dC_s + \theta ds) + e^{-\gamma t} \tilde{U}(S_t, \tilde{\phi}, Q(t)). \quad (58)$$

The borrower's savings (wealth) evolves according to

$$dS_t = (rS_t + \mu - \tilde{\phi}) dt + \sigma dZ_t - dC_t \quad (59)$$

Differentiating  $\tilde{G}_t$  with respect to  $t$  and using Ito's lemma gives

$$\begin{aligned} e^{\gamma t} d\tilde{G}_t &= [-\gamma \tilde{U}(S_t, \tilde{\phi}) + \theta + (rS_t + \mu - \tilde{\phi}) \tilde{U}'(S_t, \bar{\phi}) \\ &\quad + 0.5\sigma^2 \tilde{U}''(S_t, \tilde{\phi}) + \delta M] dt \\ &\quad + (1 - \tilde{U}'(S_t, \tilde{\phi})) dC_t + \tilde{U}'(S_t, \tilde{\phi}) \sigma dZ_t, \end{aligned} \quad (60)$$

for  $0 \leq t < \min(\tau, \tau^m)$ . When  $S_t \leq S^1_{ref}(\tilde{\phi})$  and  $0 \leq t < \min(\tau, \tau^m)$ , the first term in the right-hand side of Eq. (60) is zero, because of Eq. (14). When  $S_t > S^1_{ref}(\tilde{\phi})$  and  $0 \leq t < \min(\tau, \tau^m)$ , the first term is negative, because of Eqs. (14), (16)–(17) and the fact  $\gamma > r$ . Because  $\tilde{U}'(S_t, \tilde{\phi}) \geq 1$  and  $dC_t \geq 0$ , the second term is non-positive.

The agent's payoff associated with strategy (C,S) is given by

$$E \left[ \int_0^{\min(\tau, \tau^m)} e^{-\gamma s} (dC_s + \theta ds) + e^{-\gamma \tau} 1_{\tau < \tau^m} A + e^{-\gamma \tau^m} 1_{\tau^m < \tau} (\tilde{U}(S_t, \tilde{\phi}) + m) \right] = E[\tilde{G}_{\min(\tau, \tau^m)}] \leq \tilde{G}_0 = \tilde{U}(S_0, \tilde{\phi}). \tag{61}$$

Thus, the agent's payoff associated with strategy (C,S) is less than or equal to  $\tilde{U}(S_0, \tilde{\phi})$ .

If the agent has zero consumption whenever  $S_t \in [0, S_{ref}^1(\tilde{\phi})]$  and consumes  $S_t - S_{ref}^1(\tilde{\phi})$  immediately whenever  $S_t > S_{ref}^1(\tilde{\phi})$ , then  $\tilde{G}_t$  is a martingale and Eq. (61) holds with equality. Hence, this is the optimal strategy, which results in  $\tilde{U}(S_0, \tilde{\phi})$  payoff to the agent.

A.4. Proof of Proposition 3

The proof of Eqs. (23)–(25) is essentially the same as the proof of Proposition 2, except that the boundary condition (16)–(17) is replaced with (25).

To prove (26)–(28), let  $Q(t)$  denote the Poisson process that governs arrival of the mobility opportunities, and  $\tau^m$  be the time of the first mobility opportunity, i.e.,  $\tau^m = \inf\{t : Q(t) = 1\}$  and  $\tau^n = \inf\{t > 0 : S_t = S_0^{n+1}\}$ . Let

$$\hat{V}^n(S_t, \phi^n, Q(t)) = \begin{cases} V^n(S_t, \phi^n) & \text{for } t < \min(\tau^n, \tau^m), \\ P & \text{for } t \geq \min(\tau^n, \tau^m), \end{cases} \tag{62}$$

where function  $V^n$  satisfies Eqs. (26)–(28). Consider process

$$X_t = \int_0^t e^{-rs} \phi^n ds + e^{-rt} \hat{V}^n(S_t, \phi^n, Q(t)). \tag{63}$$

The borrower's savings evolve according to  $dS_t = rS_t dt + \mu dt + \sigma dZ_t - \phi^n dt$ , for  $t \leq \min(\tau, \tau^n, \tau^m)$ .

Differentiating  $X_t$  with respect to  $t$  and using Ito's lemma gives

$$dX_t = e^{-rt} \left( -rV^n(S_t, \phi^n) + \phi^n + (rS_t + \mu - \phi^n)(V^n)'(S_t, \phi^n) + \frac{1}{2} \sigma^2 (V^n)''(S_t, \phi^n) + \delta(P - V^n(S_t, \phi^n)) \right) dt + e^{-rt} (V^n)'(S_t, \phi^n) \sigma dZ_t. \tag{64}$$

Because  $V^n$  solves (26), the drift is equal to zero for  $t \leq \min(\tau, \tau^n, \tau^m)$ . Thus,  $X_0 = E[X_{\min(\tau, \tau^n, \tau^m)}]$ , or

$$V^n(S_0, \phi^n) = E \left[ \int_0^{\min(\tau, \tau^n, \tau^m)} e^{-rt} \phi^n dt + e^{-r \min(\tau, \tau^n, \tau^m)} (L + 1_{\min(\tau^n, \tau^m) < \tau} (P - L)) \right], \tag{65}$$

which means that  $V^n(S, \phi^n)$  is equal to the market value of the mortgage when the savings level is equal to  $S$ .

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