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**MANAGED COMMODITY FUNDS**

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## MANAGED COMMODITY FUNDS

This paper examines the performance of managed commodity funds during the period 1982 through 1996. By year-end 1996 almost \$30 billion of speculative capital was invested in these funds, a nearly 60-fold increase since 1980. Three types of managed commodity funds are available. First, investors can purchase the shares of public commodity (or futures) funds, which is similar to buying shares in a stock or bond mutual fund except that mutual funds buy and sell securities rather than commodity futures. Public funds are accessible to small (retail) investors because they typically have the lowest minimum-investment requirements. Second, investors can place funds with a commodity pool operator (CPO), who pools all investors' funds together and employs one or more commodity trading advisors (CTAs) to manage the pooled funds. Pools have higher minimum-investment requirements than public funds. Third, investors can directly retain a CTA to manage their funds on an individual basis. This avenue is open only to investors with substantial net worth and to institutional investors, since CTAs typically set high minimum-investment requirements.<sup>1</sup>

The fees charged by commodity funds vary significantly but are in general quite high. CTAs impose both a management fee (about two to three percent of principal) and a profit-based incentive fee (about fifteen to twenty-five percent of net new profits). In addition to passing through to investors the fees charged by CTAs, pools and public funds also have similar management and incentive fees. There may also be one-time front-load and back-load fees of up to 8 percent of principal, and brokerage commissions and trading expenses can amount to as much as 10 percent of principal during a year. Despite these high fees, a general conclusion of the paper is that some types of managed commodity fund investments provide high after-fee returns compared to traditional asset classes, and that when these investments

are included in diversified stock and bond portfolios the performance of those portfolios is significantly enhanced.

This paper provides the most comprehensive analysis of the performance of managed commodity funds to date. It encompasses sixteen years of data on managed commodity funds and includes nearly two hundred thousand monthly return observations. Unlike most prior studies, the data used in this study include comprehensive performance information on non-surviving commodity funds, which permits a more thorough analysis of the effects of survivorship bias on reported fund returns. In addition, the study provides a thorough analysis of the “self-selection” bias in reported returns by being able to make use of unique data on first-reporting dates, and shows how prior procedures for treating the self-selection bias may result in a downward bias in reported returns. The paper also provides a comprehensive analysis of the portfolio allocations that would be given to the alternative commodity fund investments in diversified stock and bond portfolios. A unique aspect of this analysis is the comparison of managed commodity funds with both passive and “momentum” commodity indexes, which shows that those indexes are not a substitute for an investment in managed commodity funds.

The organization of the paper is as follows. Section I describes the data examined and discusses two potential biases that may exist in the data. Section II describes the measures used to evaluate the performance of managed commodity fund investments. Sections III and IV provide performance data for commodity funds, both as “stand-alone” investments and assets in diversified stock and bond portfolios. Section V estimates the optimal allocations that managed commodity funds receive in constrained and unconstrained stock and bond portfolios. Section VI compares the performance of managed commodity funds with both passive and momentum commodity indexes, and section VII summarizes the

conclusions of the paper.

## I. DATA AND POSSIBLE BIASES

The data encompass the monthly returns 1,150 CTAs, 439 commodity pools, and 619 public funds that existed at some time during the period 1980 through 1996. In total, there are 119,481 monthly return observations: 60,054 for CTA's, 24,523 for commodity pools, and 34,904 for public funds.<sup>2</sup> These data include returns for both surviving and non-surviving commodity funds. Reported monthly returns are net of all fees.<sup>3</sup> Information about fees is available only for the current period (April, 1997), and indicates that administrative fees range from 0.1 to twelve percent of principal, with a median of about three percent, and incentive fees range from ten to forty percent of net new profits, with a median fee of twenty percent.

There may be two biases in the MAR data: a "self-selection" bias, and a "survivorship" bias. A "self-selection" bias may exist because the MAR data may include returns prior to the date that a commodity fund first reports to MAR. In particular, the reported performance histories of CTAs commonly include returns prior to their accepting investors' (or "public") funds and registering with the CFTC.<sup>4</sup> The inclusion of pre-reporting returns in the data may result in an upward bias in returns because only successful commodity funds have an incentive to report returns.<sup>5</sup>

The data may also have a "survivorship" bias because not all non-surviving funds may be included in the data. If non-surviving funds have lower returns than surviving funds (which seems likely), omitting non-surviving (or lower-return) funds from the data will result in observed returns that are upwardly-biased. While our data contains returns for both surviving and non-surviving funds, it is not clear that it includes all non-surviving funds, especially for years prior to 1989.<sup>6</sup>

Two procedures are used to eliminate whatever self-selection exists. The first omits the first twenty-eight months of CTA reported returns, but does not omit any performance history for funds and pools. Edwards and Park (1996) find evidence of a self-selection bias in CTA returns but not in the returns of pools and funds, and show that this bias can be eliminated by omitting the first twenty-eight months of a CTA returns. A problem with this procedure, however, is that it also eliminates a large number of small CTAs from the data, which imparts a downward bias to CTA returns because small CTAs generally have higher returns than large CTAs.<sup>7</sup> In particular, the statistics in Appendix 1 show that when no performance data are excluded small CTAs have returns that are significantly higher than do large CTAs. (See column 1) Further, this difference persists even after the first twenty-eight months of performance data are excluded from the data. (See column 3) Thus, omitting the early performance histories of CTAs may unwitting cause a downward bias in CTA returns by eliminating small, high-return, CTAs from the data.

The second procedure uses the “first-reporting” dates contained in the MAR database to construct a “first-reporting” (FR) rule to determine how many months of data to omit. Specifically, because MAR did not collect comprehensive data on commodity funds prior to 1991, only data subsequent to January, 1991, are used to determine the median number of months of pre-reporting performance data included in the MAR data for CTAs, pool, and funds.<sup>8</sup> The respective median number of months are twelve, five, and six months for CTAs, pools and funds, respectively.<sup>9</sup> Thus, to correct for a possible self-selection bias, the first twelve months of returns for all CTAs are omitted, the first five months of returns for all pools are omitted, and the first six months of returns for all public funds returns are omitted. This procedure has the benefit of not arbitrarily eliminating as many small CTAs from the data as does the 28-month exclusion rule.

In subsequent analyses in this paper results for only the first-reporting rule are reported in the text. Except for CTAs, the results are not sensitive to which exclusion rule is used to correct for the potential self-selection bias. Results for CTAs when the first twenty-eight months of returns are omitted are reported in the Appendix 4.

With respect to a possible survivorship bias, data on attrition rates suggest that there may be an upward bias in commodity fund returns during the period 1980-88 because these data may not include all non-surviving funds. Specifically, an analysis of “attrition rates” for commodity funds shows that annual attrition rates are much lower in the years 1980 through 1988 than in 1989 through 1996: about nine percent versus sixteen percent for CTAs, one percent versus fifteen percent for pools, and less than one percent versus twelve percent for public funds.<sup>10</sup> (See Appendix 2) Although the differences in attrition rates pre- and post-1989 could be due to the more turbulent commodity markets that existed during the 1990's, or to the greater supply of capital to commodity markets in the 1990's, the most likely explanation is that prior to 1989 the data do not include all non-survivors.<sup>11</sup> An analysis of the difference in returns of survivors versus non-survivors during 1989-96 (when the data contain most of the non-survivors) indicates that the exclusion of all non-survivors from the data would result in an average annual survivorship bias of 5.17 percentage points for CTAs, 6.74 percentage points for pools, and 3.05 percentage points for public funds (see Appendix 3). Thus, to the extent that some non-survivors are omitted from the data prior to 1989, reported fund returns during those years will be upwardly biased.<sup>12</sup>

## II. PERFORMANCE MEASURES

To evaluate the performance of alternative managed commodity fund investments, both raw returns and risk-adjusted returns are examined for three stylized portfolios of CTAs, pools, and funds: (1) one-fund portfolios, where each month the investor is assumed to

select a single CTA, pool or fund; (2) Equally-Weighted Market Portfolios (EWMP) of all CTAs, pools, or funds in existence in a particular month, where it is assumed that an identical amount is invested each month in each CTA, pool, or fund in the portfolio; and, (3) Value-Weighted Portfolios (VWMP) of all CTAs, pools, or funds in existence in a particular month, where the monthly fund-weights reflect the proportion of total funds under management by all CTAs, pools, or funds in the month that is managed by a particular CTA, pool, or fund. Monthly and annual returns for these stylized portfolios are examined over the seventeen-year period 1980 through 1996.

Monthly returns are measured as the change in the unit-value of the fund (capital gains or losses) during the month plus any cash distributions per unit-value made during the month divided by the unit-value at the end of the preceding month. Monthly returns for an EWMP are the simple arithmetic average of the monthly returns of all CTAs, pools or funds in the portfolio. Monthly returns for a VWMP are the weighted-average of the monthly returns of the CTAs, pools, or funds in the portfolio. As investments, therefore, both EWMPs and VWMPs implicitly assume a one-month investment horizon and that investors re-balance their portfolios at the end of every month to maintain the assumed weights in the portfolio: equal for the EWMP, and dollar-weighted for the VWMP.<sup>13</sup> For randomly-selected, single CTA, pool, and fund portfolios, expected monthly returns are the simple average of the monthly returns of all CTAs, pools, or funds in existence in the month, similar to the returns on an EWMP. (The volatility of monthly returns, however, will be different.<sup>14</sup>

Risk-adjusted returns for commodity fund investments are measured by their Sharpe ratios (SR).<sup>15</sup> Sharpe ratios are calculated as

$$(1) \quad \frac{R_i - R_f}{\sigma_i}$$

where  $R_i$  = the average monthly rate of return on the  $i$ th commodity fund investment during a specified investment period;  $R_f$  = the average monthly risk-free rate of return (or T-Bill return) during the investment period; and  $\sigma_i$  = the standard deviation of monthly rates of return on the  $i$ th commodity fund investment during the investment period.

Returns volatility is also examined. Returns volatility is measured as the standard deviation (SD) of monthly returns for a specific time period. For example, the annual returns volatility for an EWMP (or a VWMP) of CTAs is the standard deviation of the twelve monthly returns during the year on an EWMP (or a VWMP) of CTAs. The returns volatility for a randomly-selected, single CTA, pool, or fund portfolio is more complicated: it is a function of both the time variation in returns and the cross-sectional variation in returns that occurs because a different CTA, pool, or fund is selected every month from the population of CTAs, pools, and funds. For example, the expected annual returns volatility for a one-CTA portfolio is the standard deviation of all individual CTA monthly returns during the year.

In the analyses which follows, average returns, returns volatility, and Sharpe ratios for each of the alternative commodity fund investments are compared to similar performance measures for traditional asset classes. Table 1 provides returns for the benchmark investments large-cap and small-cap common stock indexes (the S&P 500 index and the Russell 2000 index respectively), U.S. Treasury bills, intermediate-term government bonds, long-term government bonds, and long-term corporate bonds; and for two commodity indexes, the CRB and the MLM index.

### III. COMMODITY FUNDS AS STAND-ALONE INVESTMENTS

#### A. Commodity Trading Advisors

Table 2 provides performance statistics for CTAs: randomly-selected, single-CTA



portfolios, EWMPs of CTAs, and VWMPs of CTAs. (Appendix 4 provides comparable performance figures for CTAs after omitting the first twenty-eight months of reported returns.) Four conclusions emerge from this analysis.

First, the returns volatility of single-CTA portfolios is more than twice the returns volatility of either an EWMP or a VWMP of CTAs.<sup>16</sup> This occurs because single-CTA portfolios are not diversified across CTAs. Thus, a much higher returns volatility for single-CTA portfolios makes them an inferior investment to either an EWMP or a VWMP of CTAs, since their mean return is no higher.

Second, average returns on an EWMP of CTAs are generally much higher than on a VWMP of CTAs. For example, in the 1982-96 period an EWMP of CTAs had an annual return of almost 23.2 percent, while a VWMP of CTAs had an annual return of 13.8 percent . This occurs because small CTAs generally have higher returns than large CTAs.<sup>17</sup>

Third, returns on both an EWMP and a VWMP of CTAs have been falling over time. For example, annual returns for an EWMP of CTAs are much higher in 1982-88 (34.3 percent) than in 1989-1996 (13.3 percent).

Fourth, returns volatility, for both EWMP and VWMP returns, is considerably lower in 1993-96 than in earlier years. (It is also largely insensitive to whether twelve or twenty-eight months of returns data are excluded.)

## **B. Pools**

Table 3 provides performance statistics for commodity pools: randomly-selected, single-pool portfolios, an EWMP of pools, and a VWMP of pools. These statistics are similar to those for CTAs in the following respects:

-- the returns volatility for single-pool portfolios is much higher than for either an EWMP or a VWMP of pools;

-- average returns for both an EWMP and a VWMP of pools have been falling over time, and are much lower in the 1989-96 period than in earlier years (for example, an EWMP of pools had an annual return of 28.7 percent in 1982-88 versus an annual return of 9.4 percent in 1989-96); and

-- the returns volatility of both an EWMP and a VWMP of pools is much lower in 1989-96 than in earlier years.

They differ from the performance statistics for CTAs in that the mean return in 1989-1996 for a VWMP of pools is considerably higher (13.9 percent) than the mean return for an EWMP of pools (9.4 percent), the opposite from that for CTAs. This probably occurs because pools can employ whatever size CTAs they believe will deliver the best performance. In addition, there may be economies of scale in pool operations.

#### C. Public Funds

Table 4 provides performance statistics for public funds: randomly-selected, single-fund, portfolios, an EWMP of public funds, and a VWMP of public funds. The pattern of public fund returns is similar to those for pools, with one major exception: public fund returns are much lower than for either pools or CTAs. For example, in 1989-96 the mean annual returns for an EWMP of CTAs, pools, and funds are, respectively, 13.3 percent, 9.4 percent, and 6.2 percent, the latter just barely above the mean Treasury Bill return during this period (5.16 percent). As such, public funds have the lowest returns of all managed commodity fund investments.

#### D. Comparative Risk-Adjusted Returns

In Table 5 managed commodity funds are ranked against each other as well as against traditional asset classes in terms of risk-adjusted returns. An EWMP of CTAs ranks first among all investments in 1982-96, first during 1982-88, and second during 1989-96. CTA

returns during the 1982-88 period, however, are probably upward-biased because of a survivorship bias. But even during 1989-96, when there is probably little survivorship bias, an EWMP of CTAs continues to be a good stand-alone investment, ranking below only common stocks. Given the sensational (and historically abnormal) high returns generated by common stocks during this period, an investment in an EWMP of CTAs appears to provide very attractive returns.<sup>18</sup> An EWMP of pools ranks second during 1982-88, but its ranking falls to a dismal seventh during 1989-1996, making it a poorer stand-alone investment than common stocks and bonds over the entire time period. Public funds also perform poorly, no doubt because of the higher fees associated with those funds and self-imposed restrictions in retaining only “seasoned” CTAs.<sup>19</sup> Finally, as expected, randomly-selected, single-CTA, pool, or fund portfolios never receive a high ranking because higher returns volatility lowers their Sharpe ratios significantly.

Table 5 also shows the performance statistics for VWMPs. A VWMP of pools ranks third for the entire 1982-96 period, behind corporate and government bonds and ahead of common stocks, third in 1982-1989, and first in 1992-96. It is notable that a VWMP of CTAs never ranks highly, reflecting the poorer performance of large CTAs. Thus, in 1989-96 a VWMP of pools provides risk-adjusted returns higher than even the “high-flying” S&P 500 stock index. Further, in 1989-96 (a period relatively free of survivorship bias), the Sharpe ratio for a VWMP of pools is considerably higher than for an EWMP of CTAs (0.955 versus 0.796).<sup>20</sup>

#### E. Summary

The foregoing analyzes suggest five major conclusions. First, a VWMP of pools stands out as an attractive stand-alone investment, with respect to both traditional asset classes and other managed commodity funds, especially during 1989-96. Although a VWMP

of pools earned a somewhat lower average annual return than common stocks during this period (13.9 percent versus 16.0 percent), the lower volatility of pool returns resulted in a higher Sharpe ratio for a VWMP of pools. This performance is especially impressive given the extraordinary high common stock returns during the 1989-96 period. A clear implication is that pool managers add value: they generate higher returns and higher Sharpe ratios than most traditional assets classes, and they outperform other managed commodity funds.

Second, neither single-CTA, pool, nor fund portfolios nor any type of public commodity fund investment make attractive stand-alone investments. Single-CTA, pool or fund portfolios have high returns volatility, and public funds have significantly lower returns.

Third, the strong performance of an EWMP of CTAs during 1982-1988 should probably be given less credibility for two reasons. This period is subject to the greatest survivorship bias, and CTA reported returns are highly sensitive to the data exclusion rule used to control for self-selection bias.

Fourth, returns on all types of managed commodity funds fell substantially in 1989-1996, compared to 1982-88, for reasons that remain unclear. A possible “data” explanation is that returns in 1982-88 may have been artificially inflated because of an upward survivorship bias. The elimination of this bias in 1989-96 makes it appear that returns fell in 1989-96. Another possibility is that market conditions in 1989-96 may not have been favorable to commodity traders. In particular, most commodity traders are to a greater or lesser degree “trend followers,” and in 1989-96 commodity prices exhibited less trending behavior (or more choppiness) than in earlier years, making it difficult for traders to capitalize on price trends. Finally, during 1989-1996 there was undoubtedly greater competition. With more capital and traders competing for trading profits, commodity markets may have become more efficient, resulting in lower returns.

Fifth, despite the decline in returns in 1989-96, Sharpe ratios for a VWMP of pools rose significantly from 1982-88 to 1989-96 (from 0.694 to 0.955) because lower returns were more than offset by a lower volatility of returns. However, this was not true for an EWMP of pools or for either an EWMP or a VWMP of CTAs. Thus, large pools appear to have been more successful in managing risk than were either small pools or individual CTAs.

#### IV. COMMODITY FUNDS AS PORTFOLIO ASSETS

An alternative way to view managed commodity funds is as a separate asset class in a diversified portfolio, and then determine whether portfolio performance is enhanced by the inclusion of commodity funds in the portfolio. Institutional investors, such as pension funds, have begun to experiment with including managed commodity funds in their portfolios in an effort to enhance performance.<sup>21</sup> A reason to think that portfolio performance would be enhanced by the inclusion of commodity funds is the low correlation between the returns on commodity funds and the returns on most other financial assets.

Table 6 shows the simple correlation coefficients between managed commodity fund returns and the returns on other asset classes. In general, these correlations are very low (generally below 0.10) and are often not significantly different from zero. Some correlations are even negative. For example, returns on a VWMP of pools are negatively correlated with S&P 500 common stock returns in all time periods, although they are never significantly different from zero. The highest correlation observed for the 1982-96 period is 0.15, between a VWMP of funds and long-term government bonds. Thus, including managed commodity funds in a diversified asset portfolio should provide diversification benefits.

Adding a new asset class to a portfolio enhances portfolio performance (or increases the portfolio's Sharpe ratio) if the asset in question satisfies the following condition:

$$(2) \quad \left[ \frac{\text{Sharpe Ratio of Candidate Asset}}{\sigma_c} \right] \geq \left[ \frac{\text{Correlation Coefficient}}{\rho_{pc}} \right] \times \left[ \frac{\text{Sharpe Ratio of Portfolio}}{\sigma_p} \right]$$

where the correlation coefficient reflects the correlation between the returns on the new asset and the returns on the existing portfolio.<sup>22</sup> If this correlation is zero, the above equation reduces to the simple condition:

$$(3) \quad \left[ \frac{\text{Sharpe Ratio of Candidate Asset}}{\sigma_c} \right] \geq 0$$

Thus, an asset will satisfy this condition if its return is greater than the risk-free rate of return.<sup>23</sup>

Table 7 provides “break-even” returns for the alternative commodity fund investments. Specifically, the minimum (or “break-even”) rate of return that a commodity fund must earn in order to enhance portfolio performance can be determined by rewriting equation (2) and solving for  $R_c$ , the required rate of return:

$$\left[ \frac{R_c - R_f}{\sigma_c} \right] \geq \rho_{pc} \left[ \frac{R_p - R_f}{\sigma_p} \right]$$

$$(4) \quad R_c \geq \rho_{pc} \left[ \frac{\sigma_c}{\sigma_p} \right] (R_p - R_f) + R_f$$

where  $R_c$  = the average monthly rate of return on commodity fund investment c;  $R_f$  = the average monthly riskless rate of return;  $\sigma_c$  = the standard deviation of monthly rates of return on commodity fund investment c;  $R_p$  = the average monthly rate of return on portfolio p;  $\sigma_p$  =

the standard deviation of the monthly rates of return on portfolio p; and  $\tilde{\rho}_{cp}$  = the simple correlation between monthly returns on the commodity fund investment c and monthly returns on portfolio p. For given  $\sigma_c$ ,  $\tilde{\rho}_{cp}$ ,  $\sigma_p$ ,  $R_p$ , and  $R_f$ , therefore, the required rate of return on a commodity fund investment is  $R_c$ .

Break-even returns for two hypothetical portfolios are shown in Table 7: one that is 100 percent invested in the S&P 500 common stock index, and one that consists of 60 percent S&P 500 stocks and 40 percent long-term corporate bonds.<sup>24</sup> Also shown are actual returns on the alternative commodity fund investments. If the actual return on a commodity fund investment is greater than the break-even return for that investment, including the investment in a diversified portfolio will increase the portfolio's Sharpe ratio. Over the entire 1982-96 period, as well as for the sub-period 1982-88, all commodity fund investments satisfy this criterion for both benchmark portfolios. The only exception occurs in 1989-96, when an EWMP of public funds fails to satisfy this criterion for either benchmark portfolio. (A VWMP of public funds barely satisfies it). Thus, a break-even analysis indicates that including commodity fund investments in diversified stock and bond portfolios will enhance the performance of those portfolios.

## V. OPTIMAL PORTFOLIO ALLOCATIONS

The previous findings raise the issue of what allocations should be given to managed commodity fund investments in order to optimize portfolio performance. Elton, Gruber, and Rentzler (1987) show that optimal allocations can be obtained by solving the following constrained optimization if the objective is to maximize a portfolio's Sharpe-ratio:<sup>25</sup>

$$(5) \quad \text{Maximize } \gamma_p = \frac{R_p - R_f}{\sigma_p}$$

subject to

$$R_p = \sum_{i=1}^N R_i X_i, \sum_{i=1}^N X_i = 1, X_i \geq 0 \text{ for all } i$$

where  $\tilde{a}_p$  = Sharpe ratio of portfolio p;  $R_p$  = the expected rate of return on portfolio p;  $\sigma_p$  = the standard deviation of the monthly rates of return on portfolio p;  $R_f$  = the risk-free rate of return;  $X_i$  = the proportion of asset i in portfolio p; and  $R_i$  = the expected rate of return on asset i.

Because the objective function represented by equation (5) is non-linear, the optimization solution must be obtained by using a numerical algorithm. In addition, optimal allocations are estimated for both unconstrained and constrained portfolios.<sup>26</sup> In the constrained-estimation procedure, the minimum and maximum portfolio allocations for stocks and bonds are set equal to the minimum and maximum U.S. capital-market-value-weights over the 1970-84 period.<sup>27</sup>

Table 8 shows the optimum allocations for CTAs, pools, and funds for 1982-96 and for two sub-periods, 1982-88 and 1989-96. The allocations are generated by assuming that a particular commodity fund investment (or combination of such investments) is included in a diversified portfolio consisting of S&P 500 stocks, small-cap stocks, intermediate-term government bonds, long-term government bonds, and long-term corporate bonds. Optimal allocations for portfolios that do not include any commodity funds are shown in the column labeled w/o. In general, unconstrained portfolios tend to have from 15 to 47 percent invested in S&P 500 stocks and the remainder of the portfolio in long-term corporate bonds and intermediate-term government bonds, depending on the time period analyzed. Neither small-



cap stocks nor long-term government bonds enter the unconstrained optimal portfolios. In 1982-1996 an unconstrained portfolio consisting of 74 percent bonds and 26 percent stock has an average annual return of 12.9 percent, a standard deviation of monthly returns of 7.1 percent, and a Sharpe ratio of 0.922. This portfolio is used as a benchmark against which to evaluate the benefits of incorporating managed commodity funds into the portfolio.

Columns one through six in Table 8 show for both constrained and unconstrained portfolios the optimal allocations when each of the alternative managed commodity funds is included in the portfolio. For unconstrained portfolios in 1982-96, an EWMP of CTAs and a VWMP of pools receive the highest allocations (twenty-nine and twenty-eight percent respectively). For constrained portfolios during this period these two commodity fund investments as well as an EWMP of pools all receive the highest permissible portfolio allocation: twenty-seven percent. Further, the allocations for an EWMP of CTAs and a VWMP of pools rise sharply in the unconstrained portfolios in 1989-96 (to thirty-nine and forty-eight percent respectively). Taking only 1989-96, a period for which the quality of the data is the highest, inclusion of a VWMP of pools increases the Sharpe ratio of the benchmark unconstrained portfolio by a surprising 45.4 percent (from 0.979 to 1.423), and increases the Sharpe ratio of the benchmark constrained portfolio by 30.3 percent (from 0.954 to 1.256). (See column 5 in Table 8) These increases in Sharpe ratios occur largely because of a reduction in portfolio returns volatility, although there is a small increase in average portfolio returns as well.

When all commodity fund investments are permitted to enter the portfolio, the results are strikingly dichotomous. In both 1982-1996 and the sub-period 1982-88, an EWMP of CTAs is the only commodity fund investment to enter the portfolio, receiving a portfolio allocation of between twenty-six to twenty-nine percent in the constrained and unconstrained

portfolios (see column 9 in Table 8). The results for 1989-1996 are quite different: only a VWMP of pools enters the portfolio, receiving an allocation of forty-eight and twenty-seven percent in the unconstrained and constrained portfolios respectively.

This result occurs because the returns on an EWMP of CTAs fall sharply from 1982-88 to 1989-96, from an average annual return of 34.3 percent to an annual return of 13.3 percent (see Table 5). Returns on a VWMP of pools also fall over this period, but by considerably less, and this decline is coupled with an even greater decline in returns volatility, resulting in an increase in the Sharpe ratio for a VWMP of pools from 1982-88 to 1989-96.

Further, the high returns in 1982-88 on an EWMP of CTAs may be subject to an upward bias because of a survivorship bias in the CTA data during those years. For this reason the most credibility should be given to the findings for the 1989-96 period. In that period a VWMP of pools is the best commodity fund investment. When this investment is given the optimal allocation of forty-eight percent in an unconstrained portfolio, or twenty-seven percent in a constrained portfolio, portfolio Sharpe ratios increase by 45.4 percent and 30.3 percent respectively. (See column 9 in Table 8)

The next best alternative in 1989-96 is an EWMP of CTAs. An EWMP of CTAs receives a portfolio allocation of thirty-nine percent in an unconstrained portfolio and twenty-seven percent in a constrained portfolio. Including those allocations in the benchmark unconstrained and constrained portfolios increases portfolio Sharpe ratios by 27.6 and 22.7 percent respectively.<sup>28</sup>

## VI. MANAGED COMMODITY FUNDS VERSUS PASSIVE COMMODITY INDEXES

Since the stylized managed commodity fund investments examined in this paper are essentially different kinds of indexes of managed commodity fund returns, a fundamental question is whether a passive commodity index exists that would provide a good substitute for

managed commodity funds. This section compares the returns on two passive commodity indexes -- the CRB index and the MLM index -- to the returns on managed commodity fund investments. (Average returns and the volatility of returns on the CRB and MLM indexes are reported in Table 1.)

The CRB index is a passive, “buy-and-hold” futures (commodity) index. Returns on the index reflect returns from holding equal long futures positions in twenty-one different commodities plus the Treasury bill rate.<sup>29</sup> Significant positive returns on this index typically occur when commodities are in short supply and commodity prices are rising. The MLM index is a completely different type of commodity index: it is a dynamically-constructed, passive, index that permits both long and short positions. Specifically, it employs a simple moving-average technical trading rule to identify impending upward and downward price trends and takes either a long or short position in each of the twenty-five commodities in the index. Thus, returns on the MLM index reflect the returns on these short and long positions plus the Treasury bill rate.<sup>30</sup> Because returns on the MLM index typically are significantly positive when there are sharp price trends (either up or down) in commodity markets, this index can be viewed as a “price momentum” index.

Of the two commodity indexes, the MLM index more closely simulates what managed commodity funds do. Most managed commodity funds employ trend-following or “market-momentum” technical trading methodologies to identify price trends, and then take either long or short positions to capitalize on rising or falling commodity prices. (Fung and Hsieh, 1997) The correlation coefficients reported in Table 9 confirm this impression. There is a significant positive correlation between MLM index returns and the returns on all of the stylized managed commodity fund investments, whereas there is no correlation between CRB index returns and returns on those investments.

To determine whether either the CRB or the MLM index is a substitute for managed commodity funds in a diversified portfolio, optimal portfolio allocations are re-estimated allowing for the inclusion of these indexes in the portfolio. Table 10 reports those results. In unconstrained portfolios the MLM index receives a 26 percent allocation in 1982-96 and a 39 percent allocation in 1989-96. However, in both periods at least one of the managed commodity fund investments continues to receive a substantial portfolio allocation. (See Table 10, left panel, col. 6) To a large extent the inclusion of the MLM index substantially reduces the allocations to traditional assets classes rather than to managed commodity funds. In constrained portfolios, the MLM index does not enter the portfolio at all, while managed commodity funds continue to receive substantial allocations. (See Table 10, right panel, col. 6) This occurs because the imposed portfolio constraints do not permit a reduction in the allocations to traditional asset classes. Thus, neither the CRB nor the MLM commodity index is substitute for a managed commodity fund investment.

## VII. CONCLUSION

This study examines the performance of managed commodity fund investments during the years 1982 through 1996, both as stand-alone investments and as assets in diversified stock and bond portfolios. The performance of nine stylized commodity fund investments are examined: randomly-selected, single-CTAs, pool and fund portfolios; equally-weighted market portfolios (EWMPs) of CTAs, pools and funds; and, value-weighted (VWMP) of CTAs, pools, and funds. In addition, two subperiods are examined: 1982-88 and 1989-96.

The key finding is that several types of managed commodity funds make both good stand-alone investments and good portfolio assets. Based on an analysis of Sharpe ratios, an EWMP of CTAs and a VWMP of pools receive the highest ranking among the alternative commodity fund investments. In all time periods one of those outperforms even large-cap

common stocks, despite the sensational returns on common stock in the 1980's and 1990's. In addition, with the exception of public funds, including commodity fund investments in a diversified stock and bond portfolio significantly increases portfolio Sharpe ratios. For example, if either a VWMP of pools or an EWMP of CTAs is included in diversified stock and bond portfolios in 1989-96, portfolio Sharpe ratios increase by 22.7 to 45.4 percent. In addition, in diversified asset portfolios managed commodity fund investments receive portfolio allocations of between 27 and 48 percent, depending on whether the portfolio is constrained or unconstrained.

It is also shown that commodity indexes are not a substitute for a managed commodity fund investment. Managed commodity fund returns are compared to the returns on two commodity indexes, the CRB and the MLM indexes, and it is shown that the inclusion of these commodity indexes in a diversified portfolio does not supplant managed commodity funds in the portfolio.

These findings raise a number of issues that warrant further study. First, since it is unrealistic to believe that investors could assemble a portfolio consisting of all CTAs or pools, and re-balance these portfolios every month to achieve the designated portfolio weighting, there remains the issue of how to construct a more feasible managed commodity fund investment for investors. In particular, how many CTAs or pools should investors hold and how should those CTAs and pools be chosen?<sup>31</sup> Second, the high returns earned by CTAs and pools raise the issue of the sources of those returns. How could such high speculative returns be earned in efficient commodity markets? Are commodity markets not efficient? These returns also do not appear to be due to the existence of significant systematic risk (as commonly measured by either “betas” or the correlation between CTA and pool returns and returns on other financial assets). Finally, are CTA and pool returns high because commodity

**fund managers have superior trading skill? An important issue for future research is to determine whether in fact CTAs do possess such skill.**

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TABLE 1  
Returns and Standard Deviations for Alternative Asset Classes

| Year    | S&P500 (Large-cap) |                    |               | Russell 2000 (Small-cap) |                    |               | Intermediate-Term Government Bonds |                    |               | Long-Term Government Bonds |                    |               |
|---------|--------------------|--------------------|---------------|--------------------------|--------------------|---------------|------------------------------------|--------------------|---------------|----------------------------|--------------------|---------------|
|         | Monthly Return     | Standard Deviation | Annual Return | Monthly Return           | Standard Deviation | Annual Return | Monthly Return                     | Standard Deviation | Annual Return | Monthly Return             | Standard Deviation | Annual Return |
| 1980    | 2.50               | 5.29               | 29.95         | 3.06                     | 7.88               | 36.72         | 0.41                               | 4.60               | 4.96          | -0.17                      | 6.16               | -2.02         |
| 1981    | -0.36              | 3.72               | -4.26         | 0.30                     | 5.29               | 3.55          | 0.80                               | 3.12               | 9.59          | 0.33                       | 6.40               | 3.96          |
| 1982    | 1.76               | 5.52               | 21.16         | 2.04                     | 6.11               | 24.44         | 2.17                               | 2.03               | 26.03         | 2.91                       | 3.03               | 34.87         |
| 1983    | 1.74               | 2.86               | 20.92         | 2.27                     | 4.98               | 27.19         | 0.61                               | 1.54               | 7.30          | 0.10                       | 3.25               | 1.20          |
| 1984    | 0.58               | 4.05               | 6.96          | -0.53                    | 4.70               | -6.38         | 1.11                               | 1.83               | 13.32         | 1.26                       | 3.35               | 15.08         |
| 1985    | 2.41               | 3.51               | 28.86         | 2.39                     | 4.92               | 28.62         | 1.57                               | 1.63               | 18.79         | 2.33                       | 3.52               | 27.95         |
| 1986    | 1.55               | 5.18               | 18.54         | 0.57                     | 4.75               | 6.79          | 1.19                               | 1.65               | 14.28         | 1.96                       | 5.00               | 23.47         |
| 1987    | 0.81               | 8.80               | 9.72          | -0.13                    | 10.79              | -1.60         | 0.25                               | 1.39               | 2.96          | -0.19                      | 2.97               | -2.26         |
| 1988    | 1.34               | 2.91               | 16.09         | 1.94                     | 4.07               | 23.28         | 0.50                               | 1.36               | 6.04          | 0.81                       | 2.91               | 9.72          |
| 1989    | 2.36               | 3.57               | 28.32         | 1.31                     | 3.09               | 15.67         | 1.06                               | 1.56               | 12.67         | 1.42                       | 2.35               | 17.04         |
| 1990    | -0.14              | 5.31               | -1.67         | -1.57                    | 6.88               | -18.83        | 0.78                               | 1.26               | 9.36          | 0.53                       | 2.69               | 6.36          |
| 1991    | 2.34               | 4.56               | 28.07         | 3.33                     | 5.21               | 39.94         | 1.21                               | 0.88               | 14.50         | 1.50                       | 1.79               | 17.94         |
| 1992    | 0.64               | 2.13               | 7.66          | 1.50                     | 4.27               | 18.00         | 0.59                               | 1.58               | 7.09          | 0.67                       | 2.04               | 8.00          |
| 1993    | 0.81               | 1.77               | 9.73          | 1.49                     | 2.79               | 17.87         | 0.90                               | 1.16               | 10.76         | 1.43                       | 2.07               | 17.10         |
| 1994    | 0.15               | 3.04               | 1.81          | -0.11                    | 3.13               | -1.30         | -0.43                              | 1.36               | -5.16         | -0.64                      | 2.51               | -7.70         |
| 1995    | 2.70               | 1.50               | 32.34         | 2.14                     | 2.81               | 25.72         | 1.31                               | 0.98               | 15.67         | 2.34                       | 2.17               | 28.07         |
| 1996    | 1.79               | 3.15               | 21.48         | 1.37                     | 4.31               | 16.40         | 0.18                               | 1.52               | 2.14          | -0.04                      | 2.70               | -0.53         |
| 1982-96 | 1.39               | 4.17               | 16.68         | 1.20                     | 5.02               | 14.40         | 0.87                               | 1.52               | 10.44         | 1.09                       | 2.98               | 13.08         |
| 1982-88 | 1.46               | 4.92               | 17.52         | 1.22                     | 6.02               | 14.64         | 1.06                               | 1.71               | 12.72         | 1.31                       | 3.54               | 15.72         |
| 1989-96 | 1.33               | 3.40               | 15.96         | 1.18                     | 4.34               | 14.16         | 0.70                               | 1.33               | 8.40          | 0.90                       | 2.40               | 10.80         |
| 1989-92 | 1.30               | 4.09               | 15.60         | 1.11                     | 5.20               | 13.37         | 0.91                               | 1.33               | 10.92         | 1.03                       | 2.22               | 12.36         |
| 1993-96 | 1.36               | 2.59               | 16.32         | 1.22                     | 3.32               | 14.64         | 0.49                               | 1.31               | 5.88          | 0.77                       | 2.59               | 9.24          |

Monthly Return: Arithmetic average of monthly returns during the year.

Standard Deviation: Standard deviation of monthly returns during the year.

Annual Return: Monthly return multiplied by 12.

TABLE 1 (cont.)  
Returns and Standard Deviations for Alternative Asset Classes

| Year    | Long-Term Corporate Bonds |                    |               | Treasury Bills |                    |               | CRB Index*     |                    |               | MLM Index      |                    |               |
|---------|---------------------------|--------------------|---------------|----------------|--------------------|---------------|----------------|--------------------|---------------|----------------|--------------------|---------------|
|         | Monthly Return            | Standard Deviation | Annual Return | Monthly Return | Standard Deviation | Annual Return | Monthly Return | Standard Deviation | Annual Return | Monthly Return | Standard Deviation | Annual Return |
| 1980    | -0.08                     | 5.81               | -1.00         | 0.89           | 0.26               | 10.70         | 1.70           | 5.08               | 20.34         | 0.67           | 3.66               | 7.98          |
| 1981    | 0.04                      | 5.75               | 0.53          | 1.15           | 0.13               | 13.80         | -0.29          | 3.20               | -3.44         | 3.22           | 2.68               | 38.67         |
| 1982    | 3.06                      | 3.68               | 36.72         | 0.84           | 0.21               | 10.06         | 0.16           | 2.58               | 1.92          | 1.97           | 2.31               | 23.58         |
| 1983    | 0.55                      | 2.94               | 6.55          | 0.71           | 0.05               | 8.46          | 2.19           | 3.50               | 26.33         | 0.04           | 2.53               | 0.43          |
| 1984    | 1.35                      | 3.22               | 16.20         | 0.79           | 0.09               | 9.42          | -0.23          | 3.21               | -2.75         | 1.87           | 1.76               | 22.41         |
| 1985    | 2.26                      | 2.99               | 27.07         | 0.62           | 0.05               | 7.45          | 0.13           | 2.28               | 1.52          | 0.95           | 2.10               | 11.42         |
| 1986    | 1.55                      | 2.36               | 18.54         | 0.50           | 0.05               | 5.99          | -0.27          | 2.53               | -3.25         | 1.02           | 3.08               | 12.25         |
| 1987    | 0.01                      | 2.78               | 0.16          | 0.44           | 0.06               | 5.28          | 1.40           | 2.77               | 16.80         | 1.11           | 1.31               | 13.26         |
| 1988    | 0.88                      | 2.38               | 10.51         | 0.51           | 0.10               | 6.17          | 1.26           | 4.13               | 15.10         | 0.85           | 2.79               | 10.14         |
| 1989    | 1.28                      | 1.79               | 15.30         | 0.67           | 0.06               | 8.06          | -0.05          | 2.51               | -0.65         | 1.43           | 1.37               | 17.15         |
| 1990    | 0.57                      | 2.04               | 6.80          | 0.63           | 0.05               | 7.54          | 0.38           | 2.12               | 4.58          | 1.27           | 1.70               | 15.20         |
| 1991    | 1.53                      | 1.24               | 18.36         | 0.45           | 0.05               | 5.45          | -0.12          | 2.09               | -1.42         | 0.10           | 1.74               | 1.15          |
| 1992    | 0.76                      | 1.53               | 9.12          | 0.29           | 0.04               | 3.44          | 0.11           | 1.52               | 1.36          | -0.05          | 1.21               | -0.58         |
| 1993    | 1.05                      | 1.42               | 12.55         | 0.24           | 0.01               | 2.86          | 1.18           | 2.33               | 14.19         | 0.77           | 1.46               | 9.22          |
| 1994    | -0.47                     | 2.04               | -5.69         | 0.32           | 0.07               | 3.83          | 0.70           | 1.56               | 8.44          | 0.91           | 1.44               | 10.90         |
| 1995    | 1.99                      | 1.81               | 23.82         | 0.45           | 0.04               | 5.45          | 0.69           | 1.07               | 8.25          | 0.89           | 1.23               | 10.71         |
| 1996    | 0.14                      | 2.16               | 1.65          | 0.42           | 0.02               | 5.08          | 0.28           | 2.55               | 3.38          | 0.88           | 1.31               | 10.51         |
| 1982-96 | 1.10                      | 2.46               | 13.20         | 0.53           | 0.19               | 6.36          | 0.52           | 2.55               | 6.25          | 0.93           | 1.92               | 11.18         |
| 1982-88 | 1.38                      | 2.99               | 16.56         | 0.63           | 0.17               | 7.56          | 0.66           | 3.08               | 7.95          | 1.11           | 2.33               | 13.36         |
| 1989-96 | 0.85                      | 1.87               | 10.20         | 0.43           | 0.15               | 5.16          | 0.40           | 2.00               | 4.77          | 0.77           | 1.47               | 9.28          |
| 1989-92 | 1.03                      | 1.67               | 12.36         | 0.51           | 0.16               | 6.12          | 0.08           | 2.03               | 0.97          | 0.69           | 1.62               | 8.23          |
| 1993-96 | 0.67                      | 2.05               | 8.04          | 0.36           | 0.10               | 4.32          | 0.71           | 1.93               | 8.56          | 0.86           | 1.32               | 10.34         |

Monthly Return: Arithmetic average of monthly returns during the year.  
Standard Deviation: Standard deviation of monthly returns during the year.  
Annual Return: Monthly return multiplied by 12.  
\* The CRB Future Price Index returns plus the returns on Treasury bills.

TABLE 2  
CTAs: Returns and Standard Deviations  
Exclude First 12 Months

| Year    | Number<br>of<br>CTAs | One-CTA<br>Portfolio* | Equal-Weighted Market<br>Portfolio of CTAs |                       |                  | Value-Weighted Market<br>Portfolio of CTAs |                   |                       |                  |
|---------|----------------------|-----------------------|--|-----------------------|------------------|--|-------------------|-----------------------|------------------|
|         |                      | Standard<br>Deviation | Monthly<br>Return                          | Standard<br>Deviation | Annual<br>Return | Number<br>of<br>CTAs                       | Monthly<br>Return | Standard<br>Deviation | Annual<br>Return |
| 1980    | 35**                 | 14.50                 | NA   | NA                    | NA               | 32**                                       | NA                | NA                    | NA               |
| 1981    | 46                   | 13.88                 | 2.51                                       | 7.18                  | 30.12            | 43   | 1.52              | 6.48                  | 18.20            |
| 1982    | 60                   | 14.63                 | 2.70                                       | 5.65                  | 32.35            | 55   | 1.27              | 7.16                  | 15.23            |
| 1983    | 65                   | 13.30                 | 2.10                                       | 6.45                  | 25.22            | 59   | 0.11              | 6.36                  | 1.28             |
| 1984    | 74                   | 13.71                 | 2.10                                       | 7.06                  | 25.16            | 68   | 1.89              | 8.59                  | 22.68            |
| 1985    | 115                  | 16.65                 | 3.19                                       | 5.59                  | 38.29            | 112  | 2.06              | 6.17                  | 24.76            |
| 1986    | 145                  | 16.67                 | 2.39                                       | 6.74                  | 28.73            | 140  | 0.13              | 5.95                  | 1.50             |
| 1987    | 179                  | 27.44                 | 4.94                                       | 6.52                  | 59.23            | 171  | 3.77              | 5.38                  | 45.18            |
| 1988    | 212                  | 16.67                 | 2.61                                       | 8.13                  | 31.32            | 204  | 1.28              | 6.22                  | 15.34            |
| 1989    | 237                  | 11.44                 | 1.40                                       | 4.51                  | 16.74            | 229  | 0.38              | 5.08                  | 4.56             |
| 1990    | 291                  | 9.69                  | 2.38                                       | 3.02                  | 28.55            | 283  | 1.97              | 3.44                  | 23.64            |
| 1991    | 336                  | 9.21                  | 1.09                                       | 3.66                  | 13.08            | 329  | 1.17              | 4.73                  | 14.04            |
| 1992    | 410                  | 7.09                  | 0.61                                       | 2.88                  | 7.26             | 403  | 0.27              | 3.97                  | 3.24             |
| 1993    | 472                  | 8.29                  | 1.08                                       | 2.19                  | 13.00            | 460  | 1.09              | 2.45                  | 13.08            |
| 1994    | 483                  | 8.54                  | 0.36                                       | 1.94                  | 4.30             | 471  | -0.24             | 2.27                  | -2.83            |
| 1995    | 462                  | 7.90                  | 1.08                                       | 2.08                  | 13.00            | 453  | 1.01              | 2.67                  | 12.16            |
| 1996    | 424                  | 7.72                  | 0.90                                       | 2.88                  | 10.86            | 421  | 1.07              | 3.26                  | 12.88            |
| 1982-96 |                      | 11.55                 | 1.93                                       | 4.97                  | 23.16            |  | 1.15              | 5.12                  | 13.80            |
| 1982-88 |                      | 18.97                 | 2.86                                       | 6.46                  | 34.32            |  | 1.50              | 6.48                  | 18.00            |
| 1989-96 |                      | 8.58                  | 1.11                                       | 2.95                  | 13.32            |  | 0.84              | 3.54                  | 10.08            |
| 1989-92 |                      | 9.23                  | 1.37                                       | 3.52                  | 16.44            |  | 0.95              | 4.27                  | 11.40            |
| 1993-96 |                      | 8.13                  | 0.86                                       | 2.25                  | 10.32            |  | 0.74              | 2.66                  | 8.88             |

Monthly Return: Arithmetic average of monthly returns during the year.

Standard Deviation: Standard deviation of monthly returns during the year.

Annual Return: Monthly Return multiplied by 12.

\* The return on a one-CTA randomly-selected portfolio is identical to the return on an equally-weighted portfolio of CTAs.

TABLE 3  
Pools: Returns and Standard Deviations  
Exclude First 5 Months

| Year    | Number<br>of<br>Pools | One-Pool<br>Portfolio* | Equal-Weighted Market<br>Portfolio of Private Pools |                       |                  | Value-Weighted Market<br>Portfolio of Private Pools | Number<br>of<br>Pools | Monthly<br>Return | Standard<br>Deviation | Annual<br>Return |
|---------|-----------------------|------------------------|---|-----------------------|------------------|---|-----------------------|-------------------|-----------------------|------------------|
|         |                       | Standard<br>Deviation  | Monthly<br>Return                                   | Standard<br>Deviation | Annual<br>Return | Standard<br>Deviation                               |                       |                   |                       |                  |
| 1980    | 10                    | 11.30                  | 5.20  | 8.86                  | 62.45            | 5   | 3.37                  | 5.86              | 40.40                 |                  |
| 1981    | 11                    | 12.53                  | 2.11  | 7.73                  | 25.34            | 6   | 2.63                  | 6.65              | 31.58                 |                  |
| 1982    | 15                    | 10.11                  | 1.64  | 6.89                  | 19.64            | 11  | 1.84                  | 5.79              | 22.14                 |                  |
| 1983    | 24                    | 13.02                  | 2.56  | 8.14                  | 30.67            | 20  | 0.60                  | 4.27              | 7.18                  |                  |
| 1984    | 31                    | 14.82                  | 1.92  | 10.16                 | 23.05            | 27  | 1.52                  | 5.72              | 18.22                 |                  |
| 1985    | 42                    | 14.58                  | 2.04  | 5.86                  | 24.52            | 37  | 1.92                  | 6.19              | 23.04                 |                  |
| 1986    | 65                    | 16.18                  | 2.54  | 5.10                  | 30.49            | 57  | 0.58                  | 5.41              | 6.98                  |                  |
| 1987    | 94                    | 13.84                  | 4.59  | 5.27                  | 55.02            | 83  | 4.04                  | 4.59              | 48.50                 |                  |
| 1988    | 122                   | 20.38                  | 1.47  | 6.29                  | 17.64            | 105   | 1.04                  | 3.63              | 12.51                 |                  |
| 1989    | 157                   | 10.24                  | 0.67  | 4.27                  | 8.05             | 128   | 1.28                  | 3.25              | 15.36                 |                  |
| 1990    | 175                   | 7.91                   | 1.95  | 2.42                  | 23.45            | 159   | 2.84                  | 3.00              | 34.07                 |                  |
| 1991    | 196                   | 8.64                   | 0.41  | 4.30                  | 4.89             | 191   | 0.74                  | 3.07              | 8.90                  |                  |
| 1992    | 203                   | 7.19                   | 0.01  | 2.72                  | 0.12             | 201   | 0.59                  | 2.23              | 7.02                  |                  |
| 1993    | 222                   | 6.91                   | 1.01  | 2.62                  | 12.14            | 219   | 1.24                  | 1.84              | 14.92                 |                  |
| 1994    | 202                   | 6.81                   | -0.04   | 2.24                  | -0.50            | 199   | 0.48                  | 2.19              | 5.71                  |                  |
| 1995    | 191                   | 6.77                   | 1.01  | 2.45                  | 12.06            | 191   | 0.90                  | 2.08              | 10.76                 |                  |
| 1996    | 169                   | 7.63                   | 1.18  | 3.99                  | 14.20            | 168   | 1.18                  | 2.87              | 14.14                 |                  |
| 1982-96 |                       | 10.04                  | 1.53  | 5.26                  | 18.36            |   | 1.39                  | 3.96              | 16.68                 |                  |
| 1982-88 |                       | 15.06                  | 2.39  | 6.83                  | 28.68            |   | 1.65                  | 5.09              | 19.80                 |                  |
| 1989-96 |                       | 7.74                   | 0.78  | 3.18                  | 9.36             |   | 1.16                  | 2.61              | 13.92                 |                  |
| 1989-92 |                       | 8.47                   | 0.76  | 3.50                  | 9.12             |   | 1.36                  | 2.96              | 16.32                 |                  |
| 1993-96 |                       | 7.04                   | 0.79  | 2.86                  | 9.48             |   | 0.95                  | 2.23              | 11.40                 |                  |

Monthly Return: Arithmetic average of monthly returns during the year.

Standard Deviation: Standard deviation of monthly returns during the year.

Annual Return: Monthly return multiplied by 12.

\* The return on a one-pool randomly-selected portfolio is identical to the return on an equally-weighted portfolio of pools.

The standard deviation of one-pool portfolio returns is the standard deviation of all possible one-pool portfolio returns.

TABLE 4  
Public Funds: Returns and Standard Deviations  
Exclude First 6 Months

| Year    | Number<br>of<br>Funds | One-Fund<br>Portfolio* | Equal-Weighted Market<br>Portfolio of Public Funds |                       |                  | Number<br>of<br>Funds | Value-Weighted Market<br>Portfolio of Public Funds |                       |                  |
|---------|-----------------------|------------------------|--|-----------------------|------------------|-----------------------|--|-----------------------|------------------|
|         |                       | Standard<br>Deviation  | Monthly<br>Return                                  | Standard<br>Deviation | Annual<br>Return |                       | Monthly<br>Return                                  | Standard<br>Deviation | Annual<br>Return |
| 1980    | 13                    | 11.27                  | 2.59   | 6.24                  | 31.07            | 5                     | 3.19   | 7.98                  | 38.31            |
| 1981    | 22                    | 8.77                   | 0.91   | 5.87                  | 10.94            | 19                    | 1.87   | 6.05                  | 22.49            |
| 1982    | 33                    | 8.79                   | 0.93   | 5.47                  | 11.12            | 30                    | 1.09   | 5.40                  | 13.05            |
| 1983    | 48                    | 10.90                  | -0.13  | 7.29                  | -1.58            | 45                    | -0.60  | 6.11                  | -7.21            |
| 1984    | 61                    | 10.40                  | 1.55   | 7.30                  | 18.55            | 59                    | 1.21   | 7.68                  | 14.56            |
| 1985    | 74                    | 8.11                   | 1.85   | 5.13                  | 22.16            | 74                    | 1.67   | 5.80                  | 20.04            |
| 1986    | 93                    | 10.41                  | -0.40  | 6.06                  | -4.79            | 92                    | -1.05  | 6.97                  | -12.60           |
| 1987    | 113                   | 18.21                  | 3.52   | 5.64                  | 42.24            | 112                   | 3.05   | 5.68                  | 36.54            |
| 1988    | 138                   | 10.21                  | 0.90   | 6.95                  | 10.82            | 135                   | 0.48   | 5.70                  | 5.79             |
| 1989    | 180                   | 22.11                  | 0.39   | 5.48                  | 4.74             | 171                   | 0.55   | 4.89                  | 6.62             |
| 1990    | 209                   | 6.05                   | 1.58   | 3.03                  | 18.96            | 198                   | 1.22   | 2.78                  | 14.64            |
| 1991    | 233                   | 7.47                   | 0.53   | 4.64                  | 6.36             | 230                   | 0.80   | 5.10                  | 9.58             |
| 1992    | 257                   | 6.63                   | -0.10  | 3.46                  | -1.23            | 256                   | -0.05  | 3.53                  | -0.64            |
| 1993    | 309                   | 5.33                   | 0.84   | 2.30                  | 10.05            | 306                   | 1.26   | 2.14                  | 15.16            |
| 1994    | 315                   | 5.20                   | -0.60  | 1.90                  | -7.19            | 312                   | -0.47  | 1.70                  | -5.58            |
| 1995    | 324                   | 5.50                   | 0.83   | 2.07                  | 9.98             | 322                   | 0.71   | 2.34                  | 8.57             |
| 1996    | 296                   | 6.03                   | 0.69   | 3.08                  | 8.30             | 293                   | 0.98   | 2.61                  | 11.78            |
| 1982-96 |                       | 9.27                   | 0.82   | 4.90                  | 9.84             |                       | 0.72   | 4.82                  | 8.64             |
| 1982-88 |                       | 12.15                  | 1.17   | 6.21                  | 14.04            |                       | 0.84   | 6.15                  | 10.08            |
| 1989-96 |                       | 8.41                   | 0.52   | 3.38                  | 6.24             |                       | 0.63   | 3.27                  | 7.56             |
| 1989-92 |                       | 11.44                  | 0.60   | 4.17                  | 7.20             |                       | 0.63   | 4.08                  | 7.56             |
| 1993-96 |                       | 5.56                   | 0.44   | 2.38                  | 5.28             |                       | 0.62   | 2.25                  | 7.44             |

Monthly Return: Arithmetic average of monthly returns during the year.

Standard Deviation: Standard deviation of monthly returns during the year.

Annual Return: Monthly return multiplied by 12.

\* The return on a one-fund randomly-selected portfolio is identical to the return on an equally-weighted portfolio of funds.

The standard deviation of one-fund portfolio returns is the standard deviation on all possible one-fund portfolio returns.

TABLE 5  
Average Annual Sharpe Ratios, Rank by Sharpe Ratio, and Average Annual Returns, 1982-1996  
12-month rule for CTAs, 5-month rule for pools, and 6-month rule for funds

|                                    | 1982:1-1996:12 |                   |                        | 1982:1-1988:12 |                   |                        | 1989:1-1996:12 |                   |                        |
|------------------------------------|----------------|-------------------|------------------------|----------------|-------------------|------------------------|----------------|-------------------|------------------------|
|                                    | Sharpe Ratio   | Sharpe Ratio Rank | Average Annual Returns | Sharpe Ratio   | Sharpe Ratio Rank | Average Annual Returns | Sharpe Ratio   | Sharpe Ratio Rank | Average Annual Returns |
| Equally-Weighted Market Portfolios |                |                   |                        |                |                   |                        |                |                   |                        |
| RS CTAs                            | 0.421          | 8                 | 23.2%                  | 0.407          | 7                 | 34.3%                  | 0.274          | 8                 | 13.3%                  |
| RS Private Pools                   | 0.346          | 9                 | 18.4%                  | 0.406          | 8                 | 28.7%                  | 0.152          | 9                 | 9.4%                   |
| RS Public Funds                    | 0.112          | 11                | 9.8%                   | 0.155          | 11                | 14.0%                  | 0.035          | 11                | 6.2%                   |
| EW CTAs                            | 0.977          | 1                 | 23.2%                  | 1.196          | 1                 | 34.3%                  | 0.796          | 2                 | 13.3%                  |
| EW Private Pools                   | 0.662          | 5                 | 18.4%                  | 0.894          | 2                 | 28.7%                  | 0.371          | 7                 | 9.4%                   |
| EW Public Funds                    | 0.211          | 10                | 9.9%                   | 0.303          | 10                | 14.1%                  | 0.088          | 10                | 6.2%                   |
| S&P 500 (large cap)                | 0.717          | 4                 | 16.7%                  | 0.581          | 6                 | 17.5%                  | 0.912          | 1                 | 16.0%                  |
| Long-term Corporate Bonds          | 0.806          | 2                 | 13.2%                  | 0.867          | 4                 | 16.5%                  | 0.777          | 3                 | 10.2%                  |
| Intermediate-term Government F     | 0.775          | 3                 | 10.4%                  | 0.869          | 3                 | 12.7%                  | 0.689          | 4                 | 8.4%                   |
| Long-term Government Bonds         | 0.657          | 6                 | 13.1%                  | 0.667          | 5                 | 15.7%                  | 0.670          | 5                 | 10.8%                  |
| Russell 2000 (small cap)           | 0.451          | 7                 | 14.4%                  | 0.339          | 9                 | 14.6%                  | 0.597          | 6                 | 14.2%                  |
| Value-Weighted Market Portfolios   |                |                   |                        |                |                   |                        |                |                   |                        |
| RS CTAs                            | 0.421          | 8                 | 23.2%                  | 0.407          | 7                 | 34.3%                  | 0.274          | 8                 | 13.4%                  |
| RS Private Pools                   | 0.346          | 9                 | 18.4%                  | 0.406          | 8                 | 28.7%                  | 0.152          | 10                | 9.4%                   |
| RS Public Funds                    | 0.112          | 11                | 9.9%                   | 0.155          | 10                | 14.1%                  | 0.035          | 11                | 6.2%                   |
| VW CTAs                            | 0.422          | 7                 | 13.8%                  | 0.465          | 6                 | 18.0%                  | 0.399          | 7                 | 10.1%                  |
| VW Private Pools                   | 0.752          | 3                 | 16.7%                  | 0.694          | 3                 | 19.8%                  | 0.955          | 1                 | 13.9%                  |
| VW Public Funds                    | 0.142          | 10                | 8.6%                   | 0.116          | 11                | 10.1%                  | 0.202          | 9                 | 7.6%                   |
| S&P 500 (large cap)                | 0.717          | 4                 | 16.7%                  | 0.581          | 5                 | 17.5%                  | 0.912          | 2                 | 16.0%                  |
| Long-term Corporate Bonds          | 0.806          | 1                 | 13.2%                  | 0.867          | 2                 | 16.5%                  | 0.777          | 3                 | 10.2%                  |
| Intermediate-term Government F     | 0.775          | 2                 | 10.4%                  | 0.869          | 1                 | 12.7%                  | 0.689          | 4                 | 8.4%                   |
| Long-term Government Bonds         | 0.657          | 5                 | 13.1%                  | 0.667          | 4                 | 15.7%                  | 0.670          | 5                 | 10.8%                  |
| Russell 2000 (small cap)           | 0.451          | 6                 | 14.4%                  | 0.339          | 9                 | 14.6%                  | 0.597          | 6                 | 14.2%                  |

RS - Randomly-Selected, single-CTA, pool, or fund portfolios.

EW - Equally-Weighted Market Portfolio; VW - Value-Weighted Market Portfolio.

Annual Sharpe ratios are computed from monthly observations: multiply the monthly Sharpe ratio by the square root of 12.

Average annual returns are the average monthly returns multiplied by 12.

Table 6  
Correlation Coefficients for 1982-1996 and Sub-Periods 1982-1988 and 1989-1996  
12-month rule for CTAs, 5-month rule for pools, and 6-month rule for funds

|                                 | EW     | EW     | EW     | VW     | VW     | VW    | Long          | Inter.      | Long        | Small       |              |      |
|---------------------------------|--------|--------|--------|--------|--------|-------|---------------|-------------|-------------|-------------|--------------|------|
| 1982:1-1996:12                  | CTAs   | Pools  | Funds  | CTAs   | Pools  | Funds | Commo: Stocks | Corp. Bonds | Gov't Bonds | Gov't Bonds | Cap. T-bills |      |
| EW CTAs                         | 1.00   |        |        |        |        |       |               |             |             |             |              |      |
| EW Private Pools                | 0.93** | 1.00   |        |        |        |       |               |             |             |             |              |      |
| EW Public Funds                 | 0.90** | 0.89** | 1.00   |        |        |       |               |             |             |             |              |      |
| VW CTAs                         | 0.91** | 0.91*  | 0.95** | 1.00   |        |       |               |             |             |             |              |      |
| VW Private Pools                | 0.84** | 0.86** | 0.82** | 0.90** | 1.00   |       |               |             |             |             |              |      |
| VW Public Funds                 | 0.89** | 0.86** | 0.96** | 0.96** | 0.87** | 1.00  |               |             |             |             |              |      |
| Common Stock Returns (S&P500)   | -0.01  | -0.03  | 0.08   | 0.02   | -0.05  | 0.08  | 1.00          |             |             |             |              |      |
| Long-term Corporate Bonds       | 0.07   | 0.07   | 0.06   | 0.06   | 0.09   | 0.09  | 0.40**        | 1.00        |             |             |              |      |
| Intermediate-term Government Bc | 0.04   | 0.03   | 0.05   | 0.03   | 0.04   | 0.08  | 0.34**        | 0.92**      | 1.00        |             |              |      |
| Long-term Government Bonds      | 0.11   | 0.09   | 0.11   | 0.10   | 0.14*  | 0.15* | 0.39**        | 0.94**      | 0.92**      | 1.00        |              |      |
| Treasury Bills                  | 0.09   | 0.09   | 0.05   | 0.04   | 0.06   | 0.02  | -0.02         | 0.18**      | 0.23**      | 0.14*       | 1.00         |      |
| Russell 2000 (Small Cap. Index) | -0.08  | -0.10  | -0.02  | -0.06  | -0.13* | -0.03 | 0.85**        | 0.22**      | 0.15*       | 0.19**      | -0.10        | 1.00 |

  

| Sub-period                      | EW     | EW     | EW     | VW     | VW     | VW     | Long          | Inter.      | Long        | Small       |              |         |
|---------------------------------|--------|--------|--------|--------|--------|--------|---------------|-------------|-------------|-------------|--------------|---------|
| 1982:1-1988:12\1989:1-1996:12   | CTAs   | Pools  | Funds  | CTAs   | Pools  | Funds  | Commo: Stocks | Corp. Bonds | Gov't Bonds | Gov't Bonds | Cap. T-bills |         |
| EW CTAs                         | 1.00   | 0.96** | 0.92** | 0.96** | 0.84** | 0.91** | -0.02         | 0.15        | 0.14        | 0.16        | 0.14         | -0.20** |
| EW Private Pools                | 0.92** | 1.00   | 0.92** | 0.94** | 0.90** | 0.89** | 0.01          | 0.19*       | 0.16        | 0.21**      | 0.10         | -0.16   |
| EW Public Funds                 | 0.91** | 0.89** | 1.00   | 0.96** | 0.77** | 0.97** | 0.12          | 0.21**      | 0.21**      | 0.23**      | 0.08         | -0.11   |
| VW CTAs                         | 0.91** | 0.90** | 0.94** | 1.00   | 0.83** | 0.95** | 0.01          | 0.18*       | 0.17*       | 0.20**      | 0.05         | -0.16   |
| VW Private Pools                | 0.85** | 0.85** | 0.83** | 0.92** | 1.00   | 0.76** | -0.11         | 0.11        | 0.10        | 0.15        | 0.15         | -0.26** |
| VW Public Funds                 | 0.90** | 0.86** | 0.96** | 0.97** | 0.91** | 1.00   | 0.14          | 0.25**      | 0.25**      | 0.27**      | 0.04         | -0.10   |
| Common Stock Returns (S&P500)   | -0.01  | -0.06  | 0.06   | 0.03   | -0.02  | 0.06   | 1.00          | 0.52**      | 0.43**      | 0.49**      | 0.08         | 0.77**  |
| Long-term Corporate Bonds       | 0.03   | 0.01   | 0.00   | 0.01   | 0.07   | 0.03   | 0.34**        | 1.00        | 0.92**      | 0.98**      | 0.12         | 0.24**  |
| Intermediate-term Government Bc | -0.03  | -0.05  | -0.03  | -0.05  | 0.01   | 0.00   | 0.29**        | 0.93**      | 1.00        | 0.92**      | 0.17*        | 0.12    |
| Long-term Government Bonds      | 0.08   | 0.04   | 0.06   | 0.05   | 0.13   | 0.10   | 0.33**        | 0.92**      | 0.92**      | 1.00        | 0.10         | 0.19*   |
| Treasury Bills                  | -0.06  | -0.03  | -0.02  | -0.02  | -0.02  | 0.00   | -0.12         | 0.16        | 0.22**      | 0.13        | 1.00         | -0.08   |
| Russell 2000 (Small Cap. Index) | -0.05  | -0.08  | 0.02   | -0.01  | -0.08  | 0.00   | 0.89**        | 0.22**      | 0.16        | 0.19*       | -0.14        | 1.00    |

Correlations are computed using monthly returns.

EW - Equally-Weighted Market Portfolio; VW - Value-Weighted Market Portfolio.

\* significant at the 10% level.

\*\* significant at the 5% level.

Test statistic  $t(n-2) = r / ((1-r^2)/(n-2))^{0.5}$ .

For 1982-1996, the critical values at the 5% and 10% level are 1.9759 and 1.6551, respectively.

For 1982-1988, the critical values at the 5% and 10% level are 1.9886 and 1.6632, respectively.

For 1989-1996, the critical values at the 5% and 10% level are 1.985 and 1.6609, respectively.

Table 7  
Break-Even Analysis  
12-month rule for CTAs, 5-month rule for pools, and 6-month rule for funds

|                            | 1982:1-1996:12 |           | 1982:1-1988:12 |           | 1989:1-1996:12 |           |
|----------------------------|----------------|-----------|----------------|-----------|----------------|-----------|
|                            | 60% stocks     |           | 60% stocks     |           | 60% stocks     |           |
|                            | 100% stock     | 40% bonds | 100% stock     | 40% bonds | 100% stock     | 40% bonds |
| EW CTAs Break-Even Return  | 6.16%          | 6.51%     | 7.38%          | 7.52%     | 5.03%          | 5.49%     |
| Average Return             | 23.16%         | 23.16%    | 34.32%         | 34.32%    | 13.32%         | 13.32%    |
| VW CTAs Break-Even Return  | 6.63%          | 6.92%     | 7.93%          | 8.01%     | 5.37%          | 5.99%     |
| Average Return             | 13.80%         | 13.80%    | 18.00%         | 18.00%    | 10.08%         | 10.08%    |
| EW Pool Break-Even Return  | 5.86%          | 6.23%     | 6.77%          | 6.76%     | 5.37%          | 5.95%     |
| Average Return             | 18.36%         | 18.36%    | 28.68%         | 28.72%    | 9.36%          | 9.36%     |
| VW Pool Break-Even Return  | 5.85%          | 6.20%     | 7.34%          | 7.64%     | 4.29%          | 4.71%     |
| Average Return             | 16.68%         | 16.68%    | 19.80%         | 19.80%    | 13.92%         | 13.92%    |
| EW Funds Break-Even Return | 7.25%          | 7.53%     | 8.27%          | 8.35%     | 6.51%          | 7.05%     |
| Average Return             | 9.84%          | 9.84%     | 14.04%         | 14.04%    | 6.24%          | 6.24%     |
| VW Funds Break-Even Return | 7.27%          | 7.67%     | 8.24%          | 8.48%     | 6.66%          | 7.29%     |
| Average Return             | 8.64%          | 8.64%     | 10.08%         | 10.08%    | 7.56%          | 7.56%     |

RS - Randomly-Selected, single-CTA, pool, fund, hedge fund, or fund of hedge fund portfolios.

EW - Equally-Weighted Market Portfolio; VW - Value-Weighted Market Portfolio.

stocks - S&P500 (large-cap)

bonds - Long-term Corporate Bonds



Table 8  
Optimal Portfolio Allocations, 1982-1996  
12-month rule for CTAs, 5-month rule for pools, and 6-month rule for funds

| 1982:1-1996:12            |       |       |       |       |       |       |       |       |       |              |       |       |       |       |       |       |       |       |       |       |
|---------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Unconstrained             |       |       |       |       |       |       |       |       |       | Constrained* |       |       |       |       |       |       |       |       |       |       |
| Managed Futures           | w/o   | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9            | w/o   | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     |
| EW CTAs                   |       | 0.29  |       |       |       |       |       | 0.29  |       | 0.29         |       | 0.27  |       |       |       |       |       | 0.27  |       | 0.27  |
| EW Private Pools          |       |       | 0.21  |       |       |       |       | 0.00  |       | 0.00         |       |       | 0.27  |       |       |       |       | 0.00  |       | 0.00  |
| EW Public Funds           |       |       |       | 0.06  |       |       |       | 0.00  |       | 0.00         |       |       |       | 0.06  |       |       |       | 0.00  |       | 0.00  |
| VW CTAs                   |       |       |       |       | 0.14  |       |       |       | 0.00  | 0.00         |       |       |       |       | 0.17  |       |       |       | 0.00  | 0.00  |
| VW Private Pools          |       |       |       |       |       | 0.28  |       |       |       | 0.28         | 0.00  |       |       |       |       | 0.27  |       |       | 0.27  | 0.00  |
| VW Public Funds           |       |       |       |       |       |       | 0.02  |       | 0.00  | 0.00         |       |       |       |       |       | 0.02  |       |       | 0.00  | 0.00  |
| Standard Assets           |       |       |       |       |       |       |       |       |       |              |       |       |       |       |       |       |       |       |       |       |
| S&P500 (large-cap)        | 0.26  | 0.19  | 0.21  | 0.24  | 0.22  | 0.20  | 0.25  | 0.19  | 0.20  | 0.19         | 0.45  | 0.45  | 0.45  | 0.45  | 0.45  | 0.45  | 0.45  | 0.45  | 0.45  | 0.45  |
| Long-term Corp. Bonds     | 0.31  | 0.05  | 0.08  | 0.29  | 0.20  | 0.04  | 0.30  | 0.05  | 0.04  | 0.05         | 0.17  | 0.09  | 0.09  | 0.17  | 0.17  | 0.09  | 0.17  | 0.09  | 0.09  | 0.09  |
| Inter-term Gov't Bonds    | 0.43  | 0.47  | 0.51  | 0.41  | 0.44  | 0.48  | 0.42  | 0.47  | 0.48  | 0.47         | 0.20  | 0.08  | 0.08  | 0.20  | 0.10  | 0.08  | 0.20  | 0.08  | 0.08  | 0.08  |
| Long-term Gov't Bonds     | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00         | 0.14  | 0.07  | 0.07  | 0.08  | 0.07  | 0.07  | 0.12  | 0.07  | 0.07  | 0.07  |
| Russell 2000 (small-cap)  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00         | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  |
| Average Returns           | 12.9% | 15.4% | 13.6% | 12.7% | 12.8% | 13.5% | 12.8% | 15.4% | 13.5% | 15.4%        | 14.2% | 17.3% | 16.0% | 14.0% | 14.6% | 15.5% | 14.2% | 17.3% | 15.5% | 17.3% |
| Standard Deviations       | 7.1%  | 6.9%  | 6.4%  | 6.8%  | 6.5%  | 6.1%  | 7.0%  | 6.9%  | 6.1%  | 6.9%         | 9.3%  | 9.3%  | 9.3%  | 9.0%  | 9.1%  | 8.7%  | 9.2%  | 9.3%  | 8.7%  | 9.3%  |
| Sharpe Ratio              | 0.922 | 1.325 | 1.129 | 0.932 | 0.998 | 1.183 | 0.923 | 1.325 | 1.183 | 1.325        | 0.854 | 1.229 | 1.084 | 0.908 | 0.962 | 1.102 | 0.902 | 1.229 | 1.102 | 1.229 |
| Change                    | 43.7% | 22.5% | 1.1%  | 8.2%  | 28.3% | 0.1%  | 43.7% | 28.3% | 43.7% |              | 43.9% | 26.9% | 6.3%  |       | 12.6% | 29.0% | 5.6%  | 43.9% | 29.0% | 43.9% |
| Sub-Period 1982:1-1988:12 |       |       |       |       |       |       |       |       |       |              |       |       |       |       |       |       |       |       |       |       |
| Unconstrained             |       |       |       |       |       |       |       |       |       | Constrained* |       |       |       |       |       |       |       |       |       |       |
| Managed Futures           | w/o   | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9            | w/o   | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     |
| EW CTAs                   |       | 0.26  |       |       |       |       |       | 0.26  |       | 0.26         |       | 0.27  |       |       |       |       |       | 0.27  |       | 0.27  |
| EW Private Pools          |       |       | 0.20  |       |       |       |       | 0.00  |       | 0.00         |       |       | 0.27  |       |       |       |       | 0.00  |       | 0.00  |
| EW Public Funds           |       |       |       | 0.09  |       |       |       | 0.00  |       | 0.00         |       |       |       | 0.09  |       |       |       | 0.00  |       | 0.00  |
| VW CTAs                   |       |       |       |       | 0.13  |       |       |       | 0.00  | 0.00         |       |       |       |       | 0.17  |       |       |       | 0.00  | 0.00  |
| VW Private Pools          |       |       |       |       |       | 0.21  |       |       |       | 0.21         | 0.00  |       |       |       |       | 0.27  |       |       | 0.27  | 0.00  |
| VW Public Funds           |       |       |       |       |       |       | 0.03  |       | 0.00  | 0.00         |       |       |       |       |       | 0.00  |       | 0.00  | 0.00  | 0.00  |
| Standard Assets           |       |       |       |       |       |       |       |       |       |              |       |       |       |       |       |       |       |       |       |       |
| S&P500 (large-cap)        | 0.15  | 0.10  | 0.12  | 0.12  | 0.11  | 0.12  | 0.14  | 0.10  | 0.12  | 0.10         | 0.45  | 0.45  | 0.45  | 0.45  | 0.45  | 0.45  | 0.45  | 0.45  | 0.45  | 0.45  |
| Long-term Corp. Bonds     | 0.18  | 0.00  | 0.00  | 0.13  | 0.05  | 0.00  | 0.17  | 0.00  | 0.00  | 0.00         | 0.17  | 0.09  | 0.09  | 0.17  | 0.17  | 0.09  | 0.17  | 0.09  | 0.09  | 0.09  |
| Inter-term Gov't Bonds    | 0.67  | 0.64  | 0.68  | 0.65  | 0.71  | 0.67  | 0.66  | 0.64  | 0.67  | 0.64         | 0.20  | 0.08  | 0.08  | 0.18  | 0.10  | 0.08  | 0.20  | 0.08  | 0.08  | 0.08  |
| Long-term Gov't Bonds     | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00         | 0.14  | 0.07  | 0.07  | 0.07  | 0.07  | 0.07  | 0.14  | 0.07  | 0.07  | 0.07  |
| Russell 2000 (small-cap)  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00         | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  |
| Average Returns           | 14.1% | 18.8% | 16.5% | 13.9% | 14.1% | 14.7% | 13.9% | 18.8% | 14.7% | 18.8%        | 16.0% | 21.3% | 19.8% | 15.9% | 16.7% | 17.4% | 16.0% | 21.3% | 17.4% | 21.3% |
| Standard Deviations       | 7.0%  | 7.3%  | 6.7%  | 6.4%  | 6.2%  | 6.1%  | 6.7%  | 7.3%  | 6.1%  | 7.3%         | 10.8% | 11.1% | 11.1% | 10.4% | 10.7% | 10.5% | 10.8% | 11.1% | 10.5% | 11.1% |
| Sharpe Ratio              | 0.940 | 1.539 | 1.337 | 0.987 | 1.057 | 1.167 | 0.945 | 1.539 | 1.167 | 1.539        | 0.783 | 1.297 | 1.166 | 0.863 | 0.915 | 1.000 | 0.844 | 1.297 | 1.000 | 1.297 |
| Change                    | 63.7% | 42.2% | 5.0%  | 12.4% | 24.1% | 0.5%  | 63.7% | 24.1% | 63.7% |              | 65.5% | 48.8% | 10.2% |       | 16.8% | 27.7% | 7.7%  | 65.5% | 27.7% | 65.5% |
| Sub-Period 1989:1-1996:12 |       |       |       |       |       |       |       |       |       |              |       |       |       |       |       |       |       |       |       |       |
| Unconstrained             |       |       |       |       |       |       |       |       |       | Constrained* |       |       |       |       |       |       |       |       |       |       |
| Managed Futures           | w/o   | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9            | w/o   | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     |
| EW CTAs                   |       | 0.39  |       |       |       |       |       | 0.39  |       | 0.00         |       | 0.27  |       |       |       |       |       | 0.27  |       | 0.00  |
| EW Private Pools          |       |       | 0.18  |       |       |       |       | 0.00  |       | 0.00         |       |       | 0.19  |       |       |       |       | 0.00  |       | 0.00  |
| EW Public Funds           |       |       |       | 0.00  |       |       |       | 0.00  |       | 0.00         |       |       |       | 0.00  |       |       |       | 0.00  |       | 0.00  |
| VW CTAs                   |       |       |       |       | 0.18  |       |       |       | 0.00  | 0.00         |       |       |       |       | 0.19  |       |       |       | 0.00  | 0.00  |
| VW Private Pools          |       |       |       |       |       | 0.48  |       |       |       | 0.48         | 0.48  |       |       |       |       | 0.27  |       |       | 0.27  | 0.27  |
| VW Public Funds           |       |       |       |       |       |       | 0.00  |       | 0.00  | 0.00         |       |       |       |       |       | 0.03  |       | 0.00  | 0.00  | 0.00  |
| Standard Assets           |       |       |       |       |       |       |       |       |       |              |       |       |       |       |       |       |       |       |       |       |
| S&P500 (large-cap)        | 0.47  | 0.25  | 0.41  | 0.47  | 0.41  | 0.22  | 0.47  | 0.25  | 0.22  | 0.22         | 0.50  | 0.45  | 0.45  | 0.50  | 0.45  | 0.45  | 0.48  | 0.45  | 0.45  | 0.45  |
| Long-term Corp. Bonds     | 0.46  | 0.10  | 0.26  | 0.46  | 0.31  | 0.00  | 0.46  | 0.10  | 0.00  | 0.00         | 0.17  | 0.09  | 0.17  | 0.17  | 0.17  | 0.09  | 0.17  | 0.09  | 0.09  | 0.09  |
| Inter-term Gov't Bonds    | 0.07  | 0.18  | 0.14  | 0.07  | 0.08  | 0.21  | 0.07  | 0.18  | 0.21  | 0.21         | 0.20  | 0.08  | 0.08  | 0.20  | 0.08  | 0.08  | 0.20  | 0.08  | 0.08  | 0.08  |
| Long-term Gov't Bonds     | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00         | 0.09  | 0.07  | 0.07  | 0.09  | 0.07  | 0.07  | 0.08  | 0.07  | 0.07  | 0.07  |
| Russell 2000 (small-cap)  | 0.00  | 0.08  | 0.01  | 0.00  | 0.01  | 0.09  | 0.00  | 0.08  | 0.09  | 0.09         | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  |
| Average Returns           | 12.8% | 12.9% | 12.2% | 12.8% | 12.5% | 13.2% | 12.8% | 12.9% | 13.2% | 13.2%        | 12.9% | 13.7% | 12.7% | 12.9% | 12.8% | 13.8% | 12.8% | 13.7% | 13.8% | 13.8% |
| Standard Deviations       | 7.7%  | 6.1%  | 6.8%  | 7.7%  | 7.0%  | 5.6%  | 7.7%  | 6.1%  | 5.6%  | 5.6%         | 8.0%  | 7.2%  | 7.4%  | 8.0%  | 7.5%  | 6.9%  | 7.8%  | 7.2%  | 6.9%  | 6.9%  |
| Sharpe Ratio              | 0.979 | 1.250 | 1.021 | 0.979 | 1.031 | 1.423 | 0.979 | 1.250 | 1.423 | 1.423        | 0.964 | 1.182 | 1.010 | 0.964 | 1.021 | 1.256 | 0.964 | 1.182 | 1.256 | 1.256 |
| Change                    | 27.6% | 4.3%  | 0.0%  | 5.2%  | 45.4% | 0.0%  | 27.6% | 45.4% | 45.4% |              | 22.7% | 4.8%  | 0.0%  | 5.9%  | 30.3% | 0.1%  | 22.7% | 30.3% | 30.3% |       |

EW - Equally-Weighted Market Portfolio; VW - Value-Weighted Market Portfolio.

\* Constrained optimizations have the following restrictions on the weights: S&P500 - 45 to 65%, Long-term Corporate Bonds - 9 to 17%, Intermediate-term Government Bonds - 8 to 20%, Long-term Government Bonds - 7 to 19%, and the Russell 2000 - 4 to 8%. See Ibbotson, Siegal, and Love (1985).

Table 9  
Correlation Coefficients for CRB and MLM Indexes  
12-month rule for CTAs, 5-month rule for pools, and 6-month rule for funds

|                                    | 1982:1-1996:12 |           | 1982:1-1988:12 |           | 1989:1-1996:12 |           |
|------------------------------------|----------------|-----------|----------------|-----------|----------------|-----------|
|                                    | CRB Index      | MLM Index | CRB Index      | MLM Index | CRB Index      | MLM Index |
| EW CTAs                            | 0.13*          | 0.37*     | 0.17           | 0.37**    | 0.00           | 0.34**    |
| EW Private Pools                   | 0.06           | 0.35*     | 0.06           | 0.33**    | 0.04           | 0.37**    |
| EW Public Funds                    | 0.06           | 0.45*     | 0.12           | 0.48**    | -0.10          | 0.36**    |
| VW CTAs                            | 0.01           | 0.42*     | 0.02           | 0.46**    | -0.04          | 0.33**    |
| VW Private Pools                   | -0.03          | 0.36*     | -0.04          | 0.36**    | 0.00           | 0.36**    |
| VW Public Funds                    | 0.02           | 0.42*     | 0.07           | 0.46**    | -0.13          | 0.30**    |
| CRB Index                          | 1.00           | -0.04     | 1.00           | -0.08     | 1.00           | 0.03      |
| MLM Index                          | -0.04          | 1.00      | -0.08          | 1.00      | 0.03           | 1.00      |
| Common Stock Returns (S&P500)      | -0.04          | -0.09     | 0.01           | -0.05     | -0.16          | -0.17     |
| Long-term Corporate Bonds          | -0.21**        | 0.13*     | -0.21*         | 0.18      | -0.24**        | 0.01      |
| Intermediate-term Government Bonds | -0.21**        | 0.16**    | -0.19*         | 0.19*     | -0.28**        | 0.07      |
| Long-term Government Bonds         | -0.20**        | 0.16**    | -0.20*         | 0.21*     | -0.23**        | 0.03      |
| Treasury Bills                     | -0.05          | 0.21**    | -0.08          | 0.18      | -0.10          | 0.21**    |
| Russell 2000 (Small Cap. Index)    | 0.05           | -0.18**   | 0.04           | -0.12     | 0.06           | -0.28**   |

Correlations are computed using monthly returns.

EW - Equally-Weighted Market Portfolio; VW - Value-Weighted Market Portfolio.

\* significant at the 10% level.

\*\* significant at the 5% level.

Test statistic  $t(n-2) = r / ((1-r^2)/(n-2))^{0.5}$ .

For 1982-1996, the critical values at the 5% and 10% level are 1.9759 and 1.6551, respectively.

For 1982-1988, the critical values at the 5% and 10% level are 1.9886 and 1.6632, respectively.

For 1989-1996, the critical values at the 5% and 10% level are 1.985 and 1.6609, respectively.

\*\*\* The CRB Future Price Index returns plus the returns on Treasury bills.

Table 10  
Optimal Portfolio Allocations, 1982-1996  
12-month rule for CTAs, 5-month rule for pools, and 6-month rule for funds

| 1982:1-1996:12            |               |       |       |       |       |       |       |              |       |       |       |       |       |       |
|---------------------------|---------------|-------|-------|-------|-------|-------|-------|--------------|-------|-------|-------|-------|-------|-------|
| <u>Managed Futures</u>    | Unconstrained |       |       |       |       |       |       | Constrained* |       |       |       |       |       |       |
|                           | w/o           | 1     | 2     | 3     | 4     | 5     | 6     | w/o          | 1     | 2     | 3     | 4     | 5     | 6     |
| EW CTAs                   |               | 0.29  |       | 0.29  | 0.20  |       | 0.20  |              | 0.27  |       | 0.27  | 0.27  |       | 0.27  |
| EW Private Pools          |               | 0.00  |       | 0.00  | 0.00  |       | 0.00  |              | 0.00  |       | 0.00  | 0.00  |       | 0.00  |
| EW Public Funds           |               | 0.00  |       | 0.00  | 0.00  |       | 0.00  |              | 0.00  |       | 0.00  | 0.00  |       | 0.00  |
| VW CTAs                   |               |       | 0.00  | 0.00  |       | 0.00  | 0.00  |              |       | 0.00  | 0.00  |       | 0.00  | 0.00  |
| VW Private Pools          |               |       | 0.25  | 0.00  |       | 0.17  | 0.00  |              |       | 0.27  | 0.00  |       | 0.27  | 0.00  |
| VW Public Funds           |               |       | 0.00  | 0.00  |       | 0.00  | 0.00  |              |       | 0.00  | 0.00  |       | 0.00  | 0.00  |
| CRB Index**               |               | 0.01  | 0.08  | 0.01  |       |       |       |              | 0.00  | 0.00  | 0.00  |       |       |       |
| MLM Index                 |               |       |       |       | 0.26  | 0.32  | 0.26  |              |       |       |       | 0.00  | 0.00  | 0.00  |
| <u>Standard Assets</u>    |               |       |       |       |       |       |       |              |       |       |       |       |       |       |
| S&P500 (large-cap)        | 0.47          | 0.19  | 0.17  | 0.19  | 0.16  | 0.17  | 0.16  | 0.50         | 0.45  | 0.45  | 0.45  | 0.45  | 0.45  | 0.45  |
| Long-term Corp. Bonds     | 0.46          | 0.05  | 0.04  | 0.05  | 0.05  | 0.05  | 0.05  | 0.17         | 0.09  | 0.09  | 0.09  | 0.09  | 0.09  | 0.09  |
| Inter.-term Gov't Bonds   | 0.07          | 0.47  | 0.45  | 0.47  | 0.31  | 0.28  | 0.31  | 0.20         | 0.08  | 0.08  | 0.08  | 0.08  | 0.08  | 0.08  |
| Long-term Gov't Bonds     | 0.00          | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.09         | 0.07  | 0.07  | 0.07  | 0.07  | 0.07  | 0.07  |
| Russell 2000 (small-cap)  | 0.00          | 0.00  | 0.00  | 0.00  | 0.01  | 0.01  | 0.01  | 0.04         | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  |
| Average Returns           | 12.8%         | 15.3% | 12.8% | 15.4% | 14.4% | 12.9% | 14.4% | 12.9%        | 17.3% | 15.5% | 17.3% | 17.3% | 15.5% | 17.3% |
| Standard Deviations       | 7.7%          | 6.8%  | 5.5%  | 6.8%  | 5.9%  | 5.2%  | 5.9%  | 8.0%         | 9.3%  | 8.7%  | 9.3%  | 9.3%  | 8.7%  | 9.3%  |
| Sharpe Ratio              | 0.979         | 1.325 | 1.194 | 1.325 | 1.377 | 1.271 | 1.377 | 0.964        | 1.182 | 1.102 | 1.229 | 1.229 | 1.102 | 1.229 |
| Change                    |               | 35.3% | 21.9% | 35.3% | 40.6% | 29.8% | 40.6% |              | 22.6% | 14.4% | 27.5% | 27.5% | 14.4% | 27.5% |
| Sub-Period 1989:1-1996:12 |               |       |       |       |       |       |       |              |       |       |       |       |       |       |
| <u>Managed Futures</u>    | Unconstrained |       |       |       |       |       |       | Constrained* |       |       |       |       |       |       |
|                           | w/o           | 1     | 2     | 3     | 4     | 5     | 6     | w/o          | 1     | 2     | 3     | 4     | 5     | 6     |
| EW CTAs                   |               | 0.35  |       | 0.00  | 0.17  |       | 0.00  |              | 0.27  |       | 0.00  | 0.22  |       | 0.00  |
| EW Private Pools          |               | 0.00  |       | 0.00  | 0.00  |       | 0.00  |              | 0.00  |       | 0.00  | 0.00  |       | 0.00  |
| EW Public Funds           |               | 0.00  |       | 0.00  | 0.00  |       | 0.00  |              | 0.00  |       | 0.00  | 0.00  |       | 0.00  |
| VW CTAs                   |               |       | 0.00  | 0.00  |       | 0.00  | 0.00  |              |       | 0.00  | 0.00  |       | 0.00  | 0.00  |
| VW Private Pools          |               |       | 0.44  | 0.44  |       | 0.26  | 0.26  |              |       | 0.27  | 0.27  |       | 0.27  | 0.27  |
| VW Public Funds           |               |       | 0.00  | 0.00  |       | 0.00  | 0.00  |              |       | 0.00  | 0.00  |       | 0.00  | 0.00  |
| CRB Index**               |               | 0.07  | 0.05  | 0.05  |       |       |       |              | 0.00  | 0.00  | 0.00  |       |       |       |
| MLM Index                 |               |       |       |       | 0.47  | 0.39  | 0.39  |              |       |       |       | 0.05  | 0.00  | 0.00  |
| <u>Standard Assets</u>    |               |       |       |       |       |       |       |              |       |       |       |       |       |       |
| S&P500 (large-cap)        | 0.26          | 0.24  | 0.22  | 0.22  | 0.15  | 0.15  | 0.15  | 0.45         | 0.45  | 0.45  | 0.45  | 0.45  | 0.45  | 0.45  |
| Long-term Corp. Bonds     | 0.31          | 0.09  | 0.00  | 0.00  | 0.13  | 0.07  | 0.07  | 0.17         | 0.09  | 0.09  | 0.09  | 0.09  | 0.09  | 0.09  |
| Inter.-term Gov't Bonds   | 0.43          | 0.18  | 0.21  | 0.21  | 0.00  | 0.03  | 0.03  | 0.20         | 0.08  | 0.08  | 0.08  | 0.08  | 0.08  | 0.08  |
| Long-term Gov't Bonds     | 0.00          | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.14         | 0.07  | 0.07  | 0.07  | 0.07  | 0.07  | 0.07  |
| Russell 2000 (small-cap)  | 0.00          | 0.06  | 0.08  | 0.08  | 0.08  | 0.09  | 0.09  | 0.04         | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  | 0.04  |
| Average Returns           | 12.9%         | 12.2% | 12.7% | 12.7% | 11.5% | 12.0% | 12.0% | 14.2%        | 13.7% | 13.8% | 13.8% | 13.5% | 13.8% | 13.8% |
| Standard Deviations       | 7.1%          | 5.6%  | 5.3%  | 5.3%  | 4.3%  | 4.3%  | 4.3%  | 9.3%         | 7.2%  | 6.9%  | 6.9%  | 7.0%  | 6.9%  | 6.9%  |
| Sharpe Ratio              | 0.922         | 1.254 | 1.426 | 1.426 | 1.454 | 1.567 | 1.567 | 0.854        | 1.182 | 1.256 | 1.256 | 1.184 | 1.256 | 1.256 |
| Change                    |               | 36.0% | 54.7% | 54.7% | 57.7% | 69.9% | 69.9% |              | 38.4% | 47.1% | 47.1% | 38.7% | 47.1% | 47.1% |

EW - Equally-Weighted Market Portfolio; VW - Value-Weighted Market Portfolio.

\* Constrained optimizations have the following restrictions on the weights: S&P500 - 45 to 65%, Long-term Corporate Bonds - 9 to 17%, Intermediate-term Government Bonds - 8 to 20%, Long-term Government Bonds - 7 to 19%, and the Russell 2000 - 4 to 8%. See Ibbotson, Siegal, and Love (1985).

\*\* The CRB Future Price Index returns plus the returns on Treasury bills.

Appendix 1  
Relationship of Performance to Size: Alternative Exclusion Rules\*

| Commodity Trading Advisors |   |                         |           |          |                        |           |          |                        |           |          |
|----------------------------|---|-------------------------|-----------|----------|------------------------|-----------|----------|------------------------|-----------|----------|
|                            |   | All Data 1980:1-1996:12 |           |          | 12 Rule 1980:4-1996:12 |           |          | 28 Rule 1981:8-1996:12 |           |          |
|                            |   | SIZE                    | Mon. Ret. | Yr. Ret. | MV                     | Mon. Ret. | Yr. Ret. | MV                     | Mon. Ret. | Yr. Ret. |
| Smallest Q                 | 1 | 0.15                    | 4.39%     | 52.68%   | 0.23                   | 3.35%     | 40.16%   | 0.37                   | 2.10%     | 25.26%   |
|                            | 2 | 0.70                    | 2.94%     | 35.22%   | 1.05                   | 2.24%     | 26.88%   | 1.72                   | 1.74%     | 20.91%   |
|                            | 3 | 2.39                    | 2.25%     | 27.02%   | 3.62                   | 2.21%     | 26.55%   | 6.05                   | 1.43%     | 17.14%   |
|                            | 4 | 9.51                    | 1.73%     | 20.76%   | 12.94                  | 1.34%     | 16.03%   | 19.60                  | 1.10%     | 13.15%   |
| Largest Qu                 | 5 | 70.34                   | 1.18%     | 14.14%   | 81.62                  | 0.94%     | 11.28%   | 106.50                 | 1.00%     | 11.98%   |
| S-L                        |   |                         | 3.21%     | 38.53%   |                        | 2.41%     | 28.89%   |                        | 1.11%     | 13.28%   |
| t-stat                     |   |                         | (8.07)    |          |                        | (5.95)    |          |                        | (3.04)    |          |
| Private Pools              |   |                         |           |          |                        |           |          |                        |           |          |
|                            |   | All Data 1983:2-1996:12 |           |          | 5 Rule 1983:6-1996:12  |           |          | 12 Rule 1984:1-1996:12 |           |          |
|                            |   | SIZE                    | Mon. Ret. | Yr. Ret. | MV                     | Mon. Ret. | Yr. Ret. | MV                     | Mon. Ret. | Yr. Ret. |
| Smallest Q                 | 1 | 0.17                    | 1.44%     | 17.34%   | 0.17                   | 1.22%     | 14.60%   | 0.17                   | 1.07%     | 12.89%   |
|                            | 2 | 0.59                    | 1.62%     | 19.43%   | 0.61                   | 1.62%     | 19.43%   | 0.63                   | 1.62%     | 19.44%   |
|                            | 3 | 1.34                    | 1.76%     | 21.13%   | 1.40                   | 1.60%     | 19.22%   | 1.49                   | 1.48%     | 17.76%   |
|                            | 4 | 3.50                    | 1.32%     | 15.89%   | 3.70                   | 1.50%     | 17.95%   | 3.98                   | 1.43%     | 17.22%   |
| Largest Qu                 | 5 | 31.42                   | 1.35%     | 16.23%   | 33.18                  | 1.34%     | 16.13%   | 36.96                  | 1.51%     | 18.15%   |
| S-L                        |   |                         | 0.09%     | 1.10%    |                        | -0.13%    | -1.53%   |                        | -0.44%    | -5.26%   |
| t-stat                     |   |                         | (0.21)    |          |                        | (-0.31)   |          |                        | (-0.99)   |          |
| Public Funds               |   |                         |           |          |                        |           |          |                        |           |          |
|                            |   | All Data 1981:3-1996:12 |           |          | 6 Rule 1981:8-1996:12  |           |          | 12 Rule 1984:1-1996:12 |           |          |
|                            |   | SIZE                    | Mon. Ret. | Yr. Ret. | MV                     | Mon. Ret. | Yr. Ret. | MV                     | Mon. Ret. | Yr. Ret. |
| Smallest Q                 | 1 | 0.52                    | 0.81%     | 9.74%    | 0.51                   | 0.76%     | 9.11%    | 0.50                   | 0.81%     | 9.76%    |
|                            | 2 | 1.67                    | 0.74%     | 8.93%    | 1.62                   | 0.76%     | 9.07%    | 1.54                   | 0.67%     | 8.10%    |
|                            | 3 | 4.05                    | 0.86%     | 10.34%   | 3.97                   | 0.91%     | 10.90%   | 3.76                   | 0.93%     | 11.10%   |
|                            | 4 | 10.61                   | 0.74%     | 8.94%    | 10.59                  | 0.76%     | 9.13%    | 10.31                  | 0.86%     | 10.27%   |
| Largest Qu                 | 5 | 49.26                   | 0.79%     | 9.54%    | 48.79                  | 0.73%     | 8.76%    | 47.71                  | 0.82%     | 9.85%    |
| S-L                        |   |                         | 0.02%     | 0.20%    |                        | 0.03%     | 0.35%    |                        | -0.01%    | -0.09%   |
| t-stat                     |   |                         | (0.07)    |          |                        | (0.14)    |          |                        | (-0.04)   |          |

\*Alternative rules used to correct for self-selection bias are to exclude the first 5, 6, 12, or 28 months of returns.

SIZE - Average dollars (in millions) under management by funds in quintile, over time.

S-L - Smallest quintile mean return minus largest quintile mean return.

Quintiles are formed on a monthly basis according to the previous month's dollars under management. In addition, quintiles are formed only when there are at least 15 CTAs, pools, or funds in a quintile.

Appendix 2  
Attrition Rates, 1980-1996

|                 | CTAs     |            |            | Private Pools |           |            | Public Funds |           |            |
|-----------------|----------|------------|------------|---------------|-----------|------------|--------------|-----------|------------|
|                 | All Data | Exclude 12 | Exclude 28 | All Data      | Exclude 5 | Exclude 12 | All Data     | Exclude 6 | Exclude 12 |
| <u>Averages</u> |          |            |            |               |           |            |              |           |            |
| 1980-96         | 11.84%   | 12.84%     | 12.85%     | 7.66%         | 7.68%     | 7.97%      | 5.85%        | 6.07%     | 6.39%      |
| 1980-88         | 8.67%    | 9.30%      | 8.93%      | 1.10%         | 1.13%     | 1.30%      | 0.16%        | 0.18%     | 0.23%      |
| 1989-96         | 15.41%   | 16.38%     | 16.28%     | 15.04%        | 15.05%    | 14.65%     | 12.25%       | 12.68%    | 12.55%     |
| 1989-92         | 11.29%   | 12.38%     | 13.11%     | 11.49%        | 11.21%    | 10.82%     | 8.07%        | 8.04%     | 8.04%      |
| 1993-96         | 19.53%   | 20.37%     | 19.46%     | 18.59%        | 18.90%    | 18.47%     | 16.43%       | 17.33%    | 17.07%     |

\* Alternative rules used to correct for self-selection bias are to exclude the first 5, 6, 12, or 28 months of returns.

Appendix 3  
Survivorship Bias  
12-month rule for CTAs, 5-month rule for pools, and 6-month rule for funds

| Year         | Average<br>Monthly<br>Returns(%) | Number<br>of<br>Monthly Obs. | Average<br>Monthly<br>Returns(%) | Number<br>of<br>Monthly Obs. | Average<br>Monthly<br>Returns(%) | Number<br>of<br>Monthly Obs. |
|--------------|----------------------------------|------------------------------|----------------------------------|------------------------------|----------------------------------|------------------------------|
|              | CTAs                             |                              | Survived CTAs                    |                              | Defunct CTAs                     |                              |
| 1989         | 1.40                             | 2,788                        | 1.57                             | 973                          | 1.30                             | 1,815                        |
| 1990         | 2.35                             | 3,225                        | 3.09                             | 1,151                        | 1.94                             | 2,074                        |
| 1991         | 1.15                             | 3,818                        | 1.64                             | 1,442                        | 0.86                             | 2,376                        |
| 1992         | 0.67                             | 4,597                        | 1.12                             | 1,926                        | 0.34                             | 2,671                        |
| 1993         | 1.06                             | 5,331                        | 1.53                             | 2,498                        | 0.66                             | 2,833                        |
| 1994         | 0.37                             | 5,847                        | 0.71                             | 3,103                        | -0.01                            | 2,744                        |
| 1995         | 1.08                             | 5,629                        | 1.68                             | 3,760                        | -0.11                            | 1,869                        |
| 1996         | 0.89                             | 5,389                        | 1.09                             | 4,731                        | -0.50                            | 658                          |
| Average      | 1.12                             |                              | 1.55                             |                              | 0.56                             |                              |
| Monthly Bias | 0.43                             |                              |                                  |                              |                                  |                              |
| Annual Bias  | 5.17                             |                              |                                  |                              |                                  |                              |
|              | Pools                            |                              | Survived Pools                   |                              | Defunct Pools                    |                              |
| 1989         | 0.63                             | 1,700                        | 1.39                             | 633                          | 0.17                             | 1,067                        |
| 1990         | 1.97                             | 1,999                        | 3.03                             | 739                          | 1.34                             | 1,260                        |
| 1991         | 0.48                             | 2,216                        | 0.96                             | 914                          | 0.14                             | 1,302                        |
| 1992         | 0.05                             | 2,436                        | 0.70                             | 1,094                        | -0.48                            | 1,342                        |
| 1993         | 0.99                             | 2,547                        | 1.60                             | 1,270                        | 0.39                             | 1,277                        |
| 1994         | -0.06                            | 2,542                        | 0.35                             | 1,477                        | -0.63                            | 1,065                        |
| 1995         | 1.01                             | 2,342                        | 1.33                             | 1,703                        | 0.17                             | 639                          |
| 1996         | 1.15                             | 2,180                        | 1.36                             | 1,938                        | -0.53                            | 242                          |
| Average      | 0.78                             |                              | 1.34                             |                              | 0.07                             |                              |
| Monthly Bias | 0.56                             |                              |                                  |                              |                                  |                              |
| Annual Bias  | 6.74                             |                              |                                  |                              |                                  |                              |
|              | Funds                            |                              | Survived Funds                   |                              | Defunct Funds                    |                              |
| 1989         | 0.34                             | 1,890                        | 0.01                             | 711                          | 0.53                             | 1,179                        |
| 1990         | 1.54                             | 2,350                        | 2.14                             | 866                          | 1.19                             | 1,484                        |
| 1991         | 0.58                             | 2,711                        | 1.01                             | 1,040                        | 0.31                             | 1,671                        |
| 1992         | -0.07                            | 2,938                        | 0.21                             | 1,309                        | -0.29                            | 1,629                        |
| 1993         | 0.82                             | 3,488                        | 1.26                             | 1,794                        | 0.36                             | 1,694                        |
| 1994         | -0.59                            | 3,747                        | -0.34                            | 2,245                        | -0.97                            | 1,502                        |
| 1995         | 0.80                             | 3,910                        | 1.04                             | 2,881                        | 0.14                             | 1,029                        |
| 1996         | 0.66                             | 3,784                        | 0.79                             | 3,355                        | -0.30                            | 429                          |
| Average      | 0.51                             |                              | 0.76                             |                              | 0.12                             |                              |
| Monthly Bias | 0.25                             |                              |                                  |                              |                                  |                              |
| Annual Bias  | 3.05                             |                              |                                  |                              |                                  |                              |

Monthly Bias = "Survived" minus "All."

Annual Bias = Monthly Bias multiplied by 12.

Appendix 4  
CTAs: Returns and Standard Deviations  
Exclude First 28 Months

| Year    | Number<br>of<br>CTAs | One-CTA<br>Portfolio* | Equal-Weighted Market<br>Portfolio of CTAs |                       |                  | Value-Weighted Market<br>Portfolio of CTAs |                   |                       |                  |
|---------|----------------------|-----------------------|--|-----------------------|------------------|--|-------------------|-----------------------|------------------|
|         |                      | Standard<br>Deviation | Monthly<br>Return                          | Standard<br>Deviation | Annual<br>Return | Number<br>of<br>CTAs                       | Monthly<br>Return | Standard<br>Deviation | Annual<br>Return |
| 1980    | 0                    | 0                     | NA   | NA                    | NA               | 0  | NA                | NA                    | NA               |
| 1981    | 30**                 | 10.90                 | NA   | NA                    | NA               | 28**                                       | NA                | NA                    | NA               |
| 1982    | 40                   | 11.18                 | 1.81                                       | 6.31                  | 21.72            | 38   | 1.19              | 7.25                  | 14.24            |
| 1983    | 52                   | 13.24                 | 1.79                                       | 7.98                  | 21.46            | 49   | 0.00              | 6.44                  | 0.01             |
| 1984    | 56                   | 13.39                 | 1.59                                       | 7.55                  | 19.08            | 51   | 1.83              | 8.94                  | 22.01            |
| 1985    | 68                   | 12.17                 | 2.09                                       | 4.91                  | 25.06            | 66   | 2.01              | 6.48                  | 24.12            |
| 1986    | 94                   | 13.29                 | 1.38                                       | 4.99                  | 16.52            | 91   | -0.52             | 6.55                  | -6.20            |
| 1987    | 126                  | 14.77                 | 3.94                                       | 6.16                  | 47.28            | 122  | 3.60              | 5.44                  | 43.20            |
| 1988    | 166                  | 16.24                 | 2.33                                       | 8.75                  | 27.96            | 159  | 1.25              | 6.35                  | 14.95            |
| 1989    | 179                  | 11.62                 | 1.12                                       | 4.70                  | 13.44            | 174  | 0.35              | 4.92                  | 4.20             |
| 1990    | 205                  | 9.54                  | 2.29                                       | 3.29                  | 27.48            | 201  | 1.96              | 3.46                  | 23.54            |
| 1991    | 231                  | 9.26                  | 0.76                                       | 4.12                  | 9.17             | 228  | 1.24              | 5.11                  | 14.87            |
| 1992    | 266                  | 6.97                  | 0.36                                       | 2.91                  | 4.28             | 262  | 0.18              | 4.14                  | 2.20             |
| 1993    | 331                  | 6.62                  | 0.95                                       | 2.33                  | 11.35            | 324  | 1.13              | 2.58                  | 13.55            |
| 1994    | 365                  | 8.15                  | 0.31                                       | 2.24                  | 3.74             | 355  | -0.22             | 2.32                  | -2.68            |
| 1995    | 352                  | 7.84                  | 1.13                                       | 2.34                  | 13.50            | 343  | 1.04              | 2.70                  | 12.43            |
| 1996    | 321                  | 7.87                  | 0.97                                       | 2.99                  | 11.60            | 319  | 1.10              | 3.38                  | 13.25            |
| 1982-96 |                      | 9.83                  | 1.52                                       | 5.08                  | 18.26            |  | 1.08              | 5.29                  | 12.96            |
| 1982-88 |                      | 14.25                 | 2.13                                       | 6.60                  | 25.62            |  | 1.34              | 6.72                  | 16.08            |
| 1989-96 |                      | 8.35                  | 0.99                                       | 3.16                  | 11.83            |  | 0.85              | 3.63                  | 10.20            |
| 1989-92 |                      | 9.31                  | 1.13                                       | 3.76                  | 13.56            |  | 0.93              | 4.37                  | 11.16            |
| 1993-96 |                      | 7.69                  | 0.84                                       | 2.43                  | 10.08            |  | 0.76              | 2.74                  | 9.12             |

Monthly Return: Arithmetic average of monthly returns during the year.

Standard Deviation: Standard deviation of monthly returns during the year.

Annual Return: Monthly return multiplied by 12.

\* The return on a one-CTA randomly-selected portfolio is identical to the return on an equally-weighted portfolio of CTAs.

The standard deviation of one-CTA portfolio returns is the standard deviation on all possible one-CTA portfolio returns.

\*\* Monthly returns start from 1981:8.

## ENDNOTES

1. Managed commodity funds are subject to some government regulation and oversight. Public funds must register with the Commodity Futures Trading Commission (CFTC) and the Securities and Exchange Commission (SEC). CPO's, if they accept "public funds," also are required to register with the CFTC and the SEC. A CPO is not considered to have accepted "public funds" if it does not have more than 499 investors in the pool and does not have more than 35 "unaccredited" investors. An "accredited" investor is one with a net worth of at least \$1 million or an annual income of more than \$200,000 for at least two consecutive years. Finally, CTA's must register with the CFTC and are subject to regulation by the CFTC.
2. These data are provided by Managed Account Reports (MAR), which receives monthly performance information from participating CTAs, pools, and funds. While the database is quite large, it does not include all CTAs, pools, or funds in existence. MAR relies on voluntary reporting. Although CTAs and pools do not have a mandatory reporting requirement, they have an incentive to report their performance to MAR because of the marketing benefits associated with MAR's publicizing their performance.
3. In reporting monthly returns net of fees, it is necessary to adopt an accounting convention to account for fees. In some cases, funds themselves accrue the fees over the relevant months, and the data reported to MAR reflect those fee accruals. In other cases, MAR revises reported monthly returns by spreading the fees over the relevant months.
4. According to the Commodity Pool Operator and Trading Advisor Regulations promulgated by the Commodity Futures Trading Commission (CFTC) under the Commodities Exchange Act as Amended, CTAs are required to report in their disclosure documents at least three years of prior trading performance (five years starting on August 25, 1995). The regulations do not require the disclosure of proprietary trading performance. However, if any proprietary trading performance is disclosed, all such trading must be disclosed. Typically, a newly-registered CTA has little or no pre-registration customer trading performance and therefore has little else to show but its proprietary trading history.
5. Edwards and Ma (1988) show that the pre-registration returns are significantly higher than the post-registration returns of public funds, which suggests that a self-selection bias exists.
6. Non-survivors are defined as CTAs, pools, or funds, which began operations at any time subsequent to March, 1980, but ceased operations at any time prior to year-end 1996. Survivors are defined as CTAs, pools, and funds still operating at year-end 1996. The data consist of about the same number of fund-month observations for surviving and non-surviving funds (59,328 vs. 60,153), so that the performance of non-survivors is well represented in the data.
7. This size effect does not exist for pools and funds, probably because large pools and funds can use small CTAs as well as large CTAs.
8. First-reporting dates are reported by MAR for seventy to ninety percent of CTAs, pools, and funds.
9. The respective mean number of pre-reporting months are sixteen for CTAs, eight for pools, and nine for public funds. Medians are used rather than means because mean values are sensitive to a few extreme observations.
10. In the analyses in Appendices 2 and 3, non-surviving funds are defined as CTAs, pools, or funds



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that discontinue operations at any time prior to year-end 1996. The annual “attrition rate” is computed as the proportion of funds that existed at the beginning of a year that no longer exist (or are surviving) at the end of the year. CTAs, pools, and funds may cease to exist because of poor performance or because they voluntarily dissolve or go out of business. However, it is probable that most cease to exist because of poor performance.

11. Prior to 1990 (when MAR changed ownership) the MAR database consisted of only the largest 25 CTAs. Smaller CTAs, and CTAs who went out of business, were not included in the data. MAR used the unpublished performance records maintained by the previous owners, including the performance records of both surviving and non-surviving CTAs not included in the published database, to “backfill” the database for early years. In backfilling the data, however, it is likely that some non-surviving CTAs were inadvertently excluded from the database.

12. Prior studies, based on fewer observations, also find a survivorship bias. Fung and Hsieh (1997) find that reported annual CTA returns in the 1980’s could be inflated by as much as 3.48 percentage points because of the failure to include non-surviving CTAs in the data. Schneeweis, Spurgin, and McCarthy (1996, p. 768) report that for an EWMP’s of CTAs “... including nonsurviving CTAs ... would have reduced annual returns by approximately 1.0-2.5%, increased standard deviation by 1.2-1.4%, and reduced the Sharpe ratios from 16 to 27% ...” For a discussion of survivorship bias, see Brown, Goetzmann, and Ross (1995).

13. In other words, in an EWMP each month funds are taken from last month's winners and given to last month's losers to maintain an equally-weighted portfolio. In a VWMP winners get larger and therefore receive larger allocations.

14. An alternative way to measure fund performance is to examine portfolio returns over some investment horizon longer than a month (such as a year). For example, if the assumed investment horizon were a calendar year, each CTA, pool or fund in the portfolio could be given an equal amount of funds at the beginning of a calendar year and this allocation maintained during the entire year. Unlike EWMP and VWMP returns, therefore, this procedure would not require a rebalancing of funds each month, and would capture the compounding effects during the course of the year. The disadvantage of this approach is that measured returns are highly dependent both on the specific investment horizon selected and the starting date of that horizon. To avoid this arbitrariness, this procedure is not used in this paper.

15. Sharpe ratios are a reasonable measure of risk-adjusted returns for commodity funds because their returns are typically uncorrelated with the returns on traditional asset classes (such as on stocks and corporate bonds), so that there is little “systematic” risk.

16. New entrants and non-surviving CTAs during the year are included in calculating the standard deviation. The annual standard deviation for an EWMP of CTA returns is the standard deviation of EWMP monthly returns multiplied by the square root of 12.

17. While excluding the first twenty-eight months of CTA returns results in lower average EWMP returns for CTAs, average VWMP returns for CTAs are not affected very much because small CTAs receive a much lesser weight in a VWMP than in an EWMP. (Compare Table 2 with Appendix 4.)

18. Use of the 28-month exclusion rule would result in a considerably lower ranking for an EWMP of CTAs because of lower CTA returns. See Appendix 4. As discussed earlier, however, there is reason to believe that use of the 28-month rule results in a downward bias in CTA returns because of the small-CTA effect.

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19. This finding is consistent with earlier studies. Elton, Gruber and Rentzler (1987, 1990) find that public funds perform poorly relative to stocks and bonds during the 1979-88 period. Irwin, Krukemyer and Zulauf (1992) find that under no scenario does the Sharpe ratio of a public fund investment exceed the Sharpe ratio of a stock or bond investment during the 1979-89 period. Earlier studies by Lintner (1983) and Irwin and Brorsen (1985) find some evidence that funds make good stand-alone investments, but those studies examine a relatively small number of funds for only a few years during the early 1980's.

20. The performance of a VWMP of pools is insensitive to the exclusion rule used: it ranks first among all investments in 1989-96 no matter which rule is used to exclude early returns data. Results are available upon request.

21. See Burr (1994) and Mattlin (1991).

22. It has been shown that every risk-averse investor -- regardless of the degree of his risk-aversion -- will be made better off by adding a new asset to his or her portfolio if the inclusion of that asset shifts the efficient frontier upward and/or to the left.

23. See Elton, Gruber and Rentzler (1987).

24. Break-even returns for randomly-selected, one-CTA, pool, and fund portfolios are not shown because there is no obviously correct way to compute the relevant correlations between the returns on those investments and the other financial assets.

25. An objective of maximizing the portfolio Sharpe ratio implicitly assumes a specific investor risk-preference function. Other implicit assumptions are that there can be riskless borrowing and lending at the same rate and that short sales are impossible.

26. Irwin, Krukemyer and Zulauf (1992) argue that constraining the portfolio allocations reduces the estimation error when solving an optimal portfolio problem. See also Frost and Savarino (1988).

27. See Ibbotson, Siegel and Love (1985). The ranges are 45.5 to 64.3 percent for large-cap common stocks, 4.3 to 7.3 percent for small-cap stocks, 8.9 to 19.8 percent for intermediate-term Government bonds, 7.1 to 19.0 percent for long-term government bonds, and 9.9 to 17.0 percent for long-term corporate bonds.

28. Of all the commodity fund investments, only public funds receive a zero allocation in some time periods and for some portfolios. In 1989-96, an EWMP of funds receives a zero allocation in both unconstrained and constrained portfolios, and a VWMP of funds receives a zero allocation in the unconstrained portfolio and only a three percent allocation in a constrained portfolio. Elton, Gruber and Rentzler (1987, 1990), and Irwin, Krukemyer and Zulauf (1992) also find that adding public commodity funds to a diversified portfolio does not enhance performance. See also Lintner (1983), Baratz and Eresian (1986, 1990), Orr (1987), Peters (1989), and Oberuc (1990).

29. Because a futures index is a fully-collateralized investment and no funds (other than margin) are required to purchase futures positions, all investment funds earn the Treasury bill rate. The commodities in the index are Cocoa, Coffee, Copper, Corn, Cotton, Crude Oil, Gold, Heating Oil, Hogs, Live Cattle, Lumber, Orange Juice, Platinum, Pork Bellies, Soybeans, Soybean Meal, Soybean Oil, Sugar, Unleaded Gasoline, and Wheat. No fees are deducted from reported CRB returns. Investors can directly trade CRB futures contracts.

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**30. The MLM index is produced and sold by Mount Lucas Management. It consists of equally-weighted futures positions in Australian Dollars, British Pounds, Canadian Dollars, Coffee, Copper, Corn, Cotton, Crude Oil, German Marks, Gold, Heating Oil, Japanese Yen, Live Cattle, Natural Gas, Silver, Soybean, Soybean Meal, Soybean Oil, Sugar, Swiss Franc, 5-Year Treasury Notes, 10-Year Treasury Notes, Treasury Bonds, Unleaded Gasoline, and Wheat. Reported MLM returns are net of fees, which Mount Lucas Management claims are approximately 50 basis points a year per invested dollar.**

**31. Billingsley and Chance (1996) suggests that fewer than ten CTAs or pools are needed to achieve most of the benefits that are attained by including a diversified investment in commodity funds in a diversified asset portfolio.**