# Knowing What Others Know: Coordination Motives in Information Acquisition

## Additional Notes

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## 1 Proof of Equilibrium Uniqueness in the Beauty Contest Game

This section shows that the equilibrium of the action game in section 1 of the main text is unique. It does that by adapting an argument first made Angeletos and Pavan (2007, propositions 1 and 3) to our environment. The idea of the proof is that there is a social planner problem such that every equilibrium of our model is also a solution to this planning problem. The planning problem is strictly convex, meaning that it has a unique minimum. Since the planning problem has a unique solution and every equilibrium is a solution to the planning problem, the equilibrium of the model must be unique.

We begin by setting up some notation for the proof. We let $\hat{p}(\cdot)$ denote the candidate equilibrium function characterized by equation (4) in the main text, and will make use of the fact that $s = \mathbf{b}'\omega$. We let $F(\omega)$ denote the prior distribution of $\omega$, with density $f(\omega)$. We let $\mu$ denote the distribution of the agents' information choices, and $\phi(X\mathbf{z}|\omega)$ the distribution of observed signals, conditional on the state $\omega$. Together, $\mu$ and $\phi$ determine the distribution $\mathcal{F}(\mathcal{I}|\omega)$of information sets $\mathcal{I} = (\chi, X\mathbf{z})$, conditional on the state $\omega$. The agents' posterior beliefs conditional on $\mathcal{I}$ are defined by the pdf

$$\hat{\phi}(\omega|\mathcal{I}) = \frac{\phi(X\mathbf{z}|\omega) f(\omega)}{\int_{\hat{\omega}} \phi(X\mathbf{z}|\hat{\omega}) \, dF(\hat{\omega})}.$$

**Proposition 1** *Let $\mathcal{P}$ denote the set of functions $p$ for which*

$$p(\mathcal{I}) = \int_{\omega} \left\{ (1-r)\,\mathbf{b}'\omega + r \int_{\mathcal{I}'} p(\mathcal{I}') \, d\mathcal{F}(\mathcal{I}|\omega) \right\} \hat{\phi}(\omega|\mathcal{I}) \, d\omega$$

*for all but a zero measure of types. Then, $\tilde{p} \in \mathcal{P}$, if and only if $\tilde{p} = \hat{p}$, almost everywhere.*

That is, up to a measure zero perturbation, the equilibrium strategies are uniquely characterized by equation (4) in the main text.

**Proof:** Define the functional

$$\mathcal{L}(p) = \int_\omega \int_\mathcal{I} \left(p(\mathcal{I}) - \mathbf{b}'\omega\right)^2 d\mathcal{F}(\mathcal{I}|\omega)\, dF(\omega) - r\int_\omega \left(\int_\mathcal{I} p(\mathcal{I})\, d\mathcal{F}(\mathcal{I}|\omega) - \mathbf{b}'\omega\right)^2 dF(\omega)$$

Step 1 shows that $\mathcal{L}(p)$ is strictly convex in $p$, which implies that if $\tilde{p}_1, \tilde{p}_2 \in \arg\min_p \mathcal{L}(p)$, $\tilde{p}_1 = \tilde{p}_2$ almost everywhere. Step 2 shows that $\mathcal{P} = \arg\min_p \mathcal{L}(p)$. But then, $\tilde{p}_1, \tilde{p}_2 \in \mathcal{P}$ implies $\tilde{p}_1 = \tilde{p}_2$ almost everywhere, and the proposition then follows from noting that $\hat{p} \in \mathcal{P}$.

Step 1: $\mathcal{L}(p)$ is strictly convex for all $p$. For arbitrary functions $p_1$ and $p_2$, and $\alpha \in (0,1)$, notice that $\alpha p_1 + (1-\alpha) p_2 = p_1 + (1-\alpha)\Delta = p_2 - \alpha\Delta$, where $\Delta = p_2 - p_1$. After some algebra, one obtains:

$$\begin{aligned}
& \mathcal{L}(\alpha p_1 + (1-\alpha) p_2) - \alpha\mathcal{L}(p_1) - (1-\alpha)\mathcal{L}(p_2) \\
= {} & \alpha\left[\mathcal{L}(p_1 + (1-\alpha)\Delta) - \mathcal{L}(p_1)\right] + (1-\alpha)\left[\mathcal{L}(p_2 - \alpha\Delta) - \mathcal{L}(p_2)\right] \\
= {} & -\alpha(1-\alpha)\int_\omega \left\{\int_\mathcal{I} (\Delta(\mathcal{I}))^2 d\mathcal{F}(\mathcal{I}|\omega) - r\left(\int_\mathcal{I} \Delta(\mathcal{I})\, d\mathcal{F}(\mathcal{I}|\omega)\right)^2\right\} dF(\omega) \\
= {} & -\alpha(1-\alpha)\int_\omega \left\{Var(\Delta(\mathcal{I})|\omega) + (1-r)\left[\mathbb{E}(\Delta(\mathcal{I})|\omega)\right]^2\right\} dF(\omega) \leq 0,
\end{aligned}$$

where $\mathbb{E}(\Delta(\mathcal{I})|\omega) = \int_\mathcal{I} \Delta(\mathcal{I})\, d\mathcal{F}(\mathcal{I}|\omega)$ and $Var(\Delta(\mathcal{I})|\omega) = \int_\mathcal{I} (\Delta(\mathcal{I}))^2 d\mathcal{F}(\mathcal{I}|\omega) - \left[\mathbb{E}(\Delta(\mathcal{I})|\omega)\right]^2$.

Moreover, the last inequality is strict, whenever $p_1(\mathcal{I}) \neq p_2(\mathcal{I})$ for a positive measure of $\mathcal{I}$'s, implying that for any $p_1, p_2 \in \arg\min_p \mathcal{L}(p)$, $p_1(\mathcal{I}) = p_2(\mathcal{I})$ for almost every $\mathcal{I}$.

Step 2: $\mathcal{P} = \arg\min_p \mathcal{L}(p)$. For arbitrary $p(\cdot)$ and $\delta(\cdot)$ and a scalar $t$,

$$\begin{aligned}
\mathcal{L}(p + t\delta) - \mathcal{L}(p) &= t^2 A(\delta) + 2tB(p,\delta) \\
\text{where } A(\delta) &= \int_\omega \left\{\int_\mathcal{I} (\delta(\mathcal{I}))^2 d\mathcal{F}(\mathcal{I}|\omega) - r\left(\int_\mathcal{I} \delta(\mathcal{I})\, d\mathcal{F}(\mathcal{I}|\omega)\right)^2\right\} dF(\omega) \\
B(p,\delta) &= \int_\omega \left\{\int_\mathcal{I} \delta(\mathcal{I})\left(p(\mathcal{I}) - \mathbf{b}'\omega\right) d\mathcal{F}(\mathcal{I}|\omega)\right. \\
& \quad \left. -r\int_\mathcal{I} \delta(\mathcal{I})\left(\int_{\mathcal{I}'} p(\mathcal{I}')\, d\mathcal{F}(\mathcal{I}'|\omega) - \mathbf{b}'\omega\right) d\mathcal{F}(\mathcal{I}|\omega)\right\} dF(\omega)
\end{aligned}$$

Clearly, $A(\delta) > 0$, for all $\delta(\cdot)$ for which $\int_{\mathcal{I}} (\delta(\mathcal{I}))^2 \, d\mathcal{F}(\mathcal{I}|\omega) > 0$ (i.e. that are different from zero for a positive measure of types). Then, for any pair $(p, \delta)$, $\mathcal{L}(p + t\delta)$ is minimized at $t^* = -B^2(p, \delta)/A(\delta)$, and $\mathcal{L}(p + t^*\delta) = \mathcal{L}(p) - B^2(p, \delta)/A(\delta)$. Therefore, $\tilde{p} \in \arg\min_p \mathcal{L}(p)$ if and only if $B(\tilde{p}, \delta) = 0$, for every $\delta(\cdot)$.

We can rewrite $B(p, \delta)$ as

$$\begin{aligned}
& B(p,\delta) \\
= & \int_\omega \int_{\mathcal{I}} \delta(\mathcal{I})\, p(\mathcal{I})\, d\mathcal{F}(\mathcal{I}|\omega)\, dF(\omega) - \int_\omega \int_{\mathcal{I}} \delta(\mathcal{I}) \left[ (1-r)\,\mathbf{b}'\omega + r \int_{\mathcal{I}'} p(\mathcal{I}')\, d\mathcal{F}(\mathcal{I}'|\omega) \right] d\mathcal{F}(\mathcal{I}|\omega)\, dF(\omega) \\
= & \int_\omega \int_\chi \int_{\mathbf{z}} \delta(\mathcal{I})\, p(\mathcal{I})\, \phi(X\mathbf{z}|\omega)\, d\mathbf{z} d\mu(\chi)\, dF(\omega) \\
& - \int_\omega \int_\chi \int_{\mathbf{z}} \delta(\mathcal{I}) \left[ (1-r)\,\mathbf{b}'\omega + r \int_{\mathcal{I}'} p(\mathcal{I}')\, d\mathcal{F}(\mathcal{I}'|\omega) \right] \phi(X\mathbf{z}|\omega)\, d\mathbf{z} d\mu(\chi)\, dF(\omega) \\
= & \int_\chi \int_{\mathbf{z}} \delta(\mathcal{I})\, p(\mathcal{I}) \int_\omega \phi(X\mathbf{z}|\omega)\, dF(\omega)\, d\mathbf{z} d\mu(\chi) \\
& - \int_\chi \int_{\mathbf{z}} \delta(\mathcal{I}) \int_\omega \left[ (1-r)\,\mathbf{b}'\omega + r \int_{\mathcal{I}'} p(\mathcal{I}')\, d\mathcal{F}(\mathcal{I}'|\omega) \right] \phi(X\mathbf{z}|\omega)\, f(\omega)\, d\omega d\mathbf{z} d\mu(\chi)
\end{aligned}$$

Since $\hat{\phi}(\omega|\mathcal{I}) = \phi(X\mathbf{z}|\omega)\, f(\omega) / \int_{\hat{\omega}} \phi(X\mathbf{z}|\hat{\omega})\, dF(\hat{\omega})$, this last expression can be rewritten as

$$\begin{aligned}
B(p,\delta) &= \int_\chi \int_{\mathbf{z}} \delta(\mathcal{I}) \left\{ p(\mathcal{I}) - \int_{\hat{\omega}} \left[ (1-r)\,\mathbf{b}'\hat{\omega} + r \int_{\mathcal{I}'} p(\mathcal{I}')\, d\mathcal{F}(\mathcal{I}'|\hat{\omega}) \right] \hat{\phi}(\hat{\omega}|\mathcal{I})\, d\hat{\omega} \right\} \int_\omega \phi(X\mathbf{z}|\omega)\, dF(\omega)\, d\mathbf{z} d\mu(\chi) \\
&= \int_\omega \int_\chi \int_{\mathbf{z}} \left\{ p(\mathcal{I}) - \int_{\hat{\omega}} \left[ (1-r)\,\mathbf{b}'\hat{\omega} + r \int_{\mathcal{I}'} p(\mathcal{I}')\, d\mathcal{F}(\mathcal{I}'|\hat{\omega}) \right] \hat{\phi}(\hat{\omega}|\mathcal{I})\, d\hat{\omega} \right\} d\mathcal{F}(\mathcal{I}|\omega)\, dF(\omega)
\end{aligned}$$

Therefore, $\tilde{p} \in \mathcal{P}$ implies $B(\tilde{p}, \delta) = 0$ for all $\delta$, and $\tilde{p} \in \arg\min_p \mathcal{L}(p)$. For $\tilde{p} \notin \mathcal{P}$, setting

$$\delta(\mathcal{I}) = \tilde{p}(\mathcal{I}) - \int_\omega \left[ (1-r)\,\mathbf{b}'\omega + r \int_{\mathcal{I}'} p(\mathcal{I}')\, d\mathcal{F}(\mathcal{I}'|\omega) \right] \hat{\phi}(\omega|\mathcal{I})\, d\omega$$

yields $B(\tilde{p}, \delta) = \int_\omega \int_{\mathcal{I}} (\delta(\mathcal{I}))^2 \, d\mathcal{F}(\mathcal{I}|\omega)\, dF(\omega) > 0$, which implies $\tilde{p} \notin \arg\min_p \mathcal{L}(p)$.

## 2 General Equilibrium Foundations for Planning Model

In this appendix, we derive micro-foundations for our dynamic planning and price adjustment model from a fully specified dynamic general equilibrium model. On the household side, our formulation follows the continuous time model Golosov and Lucas (2007). On the firm side however, there are distinct differences, as Golosov and Lucas generate nominal rigidities through menu costs of price adjustment, whereas here, they arise from the firms' cost of planning.

Time is continuous and infinite. There is a measure 1 continuum of different intermediate goods, indexed by $i \in [0, 1]$, each produced by one monopolistic firm using labor as the unique input into production. There is a final consumption good, which is produced by a perfectly competitive final goods sector using the continuum of intermediates according to a Dixit-Stiglitz CES technology with constant returns to scale. On the consumption side, there is an infinitely-lived representative household, who purchases the final consumption good and supplies labor to the intermediate firms. Finally, there is a complete set of markets for contingent nominal bonds. Markets are open continuously, and firms continuously adjust prices, but they only update their information infrequently.

**Money Supply Process:** The Logarithm of nominal money supply follows an exogenous Brownian Motion with no drift, and a diffusion parameter $\sigma$:

$$d\log M_t = \mu dt + \sigma dZ_t$$

nominal money injections take the form of lump sum taxes or transfers to the representative household.

**Representative Household:** The representative household's preferences are defined over the final consumption good, labor supply, and real balances $\left\{C_t, n_t, M_t^D/P_t\right\}_0^\infty$,

$$U_0 = \mathbb{E}_0\left\{\int_0^\infty e^{-\rho t}\left[\frac{C_t^{1-\varepsilon}}{1-\varepsilon} - \delta n_t + \log\left(\frac{M_t^D}{P_t}\right)\right]dt\right\} \tag{1}$$

where $\rho$ denotes the discount rate, $P_t$ the price of the final consumption good, and $M_t^d$ the demand for nominal balances.

Let $Q_t$ denote the process for the shadow price of nominal cash flows, so that an earnings stream $\{D_t\}_0^\infty$ is valued as $\mathbb{E}_0\left[\int_0^\infty Q_t D_t dt\right]$. The household's budget constraint is then

$$M_0 \geq \mathbb{E}_0\left[\int_0^\infty Q_t\left(P_t C_t + R_t M_t^D - W_t n_t - \Pi_t\right)dt\right] \tag{2}$$

where $\Pi_t$ indicates the income from nominal profits and lump sum money transfers, and $R_t$ denotes the nominal interest rate, which is implicitly defined by $Q_t = e^{R_t dt}\mathbb{E}_t\left(Q_{t+dt}\right)$. The term $R_t M_t^d$ thus represents the opportunity cost of holding nominal balances. The household chooses processes $\left\{C_t, n_t, M_t^D/P_t\right\}_{t=0}^\infty$ to maximize (1) subject to (2), taking as given the process $\{Q_t, P_t, W_t, R_t, \Pi_t\}_{t=0}^\infty$.

**Final Good Producers:** A large number of final goods producers uses the intermediate goods to produce the final output according to a constant returns to scale technology, which is given by the CES aggregator

$$C_t = \left[\int_0^1 \left(c_t^i\right)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}. \tag{3}$$

Final goods producers maximize profits, taking as given the market prices of intermediate and final goods. For a total demand $Y_t$ of the final good by the household, a final goods price $P_t$, and input prices $P_t^i$, the demand for intermediate good $i$ by the final good sector is

$$c_t^i = c\left(P_t^i\right) = C_t \left(\frac{P_t^i}{P_t}\right)^{-\theta}. \tag{4}$$

The final goods price $P_t$ is given by the Dixit-Stiglitz aggregator

$$P_t = \left[\int_0^1 \left(P_t^i\right)^{1-\theta} di\right]^{\frac{1}{1-\theta}}. \tag{5}$$

**Intermediate Good Producers:** Each intermediate good is produced by a single monopolist firm, using labor $l_{it}$ as an input, according to

$$y_{it} = A l_{it}^{\alpha}, \tag{6}$$

for some $A > 0$, and $\alpha \leq 1$. Firms' nominal profits in period $t$, not including planning costs, are their price times quantity sold, minus wages ($W_t$) times labor:

$$\pi_{it} = P_t^i c\left(P_t^i\right) - W_t \left(c\left(P_t^i\right)/A\right)^{1/\alpha}. \tag{7}$$

Each firm faces a fixed labor cost $F$, if they decide to update their information or "plan". The firm then chooses its process of prices $\left\{P_t^i\right\}_0^\infty$, and a process of planning dates $D_i(t)$, where $dD_i(t) = 1$ if the firm decides to plan at date $t$, and $dD_i(t) = 0$ otherwise, to maximize its expected discounted profits

$$\mathbb{E}_0^i \left[\int_0^\infty Q_t \pi_{it} dt - F \int_0^\infty Q_t W_t dD_i(t)\right] \tag{8}$$

taking as given the processes $\{Q_t, P_t, W_t, Y_t\}_0^\infty$, and its date-0 expectations $\mathbb{E}_0^i[\cdot]$.

**Market equilibrium:** An equilibrium is characterized by processes $\left\{Q_t, P_t, W_t, R_t; n_t, C_t, M_t^D\right\}_0^\infty$ for the aggregate variables and $\left\{D_i(t), P_t^i\right\}_0^\infty$, for each $i$, that solve the firms' and household's optimization problem, and clear goods and labor markets: At every date $t$, $M_t^D = M_t$, $C_t = Y_t$ (in the firm's problem), and labor supply $n_t$ equals the total labor demand for production and planning purposes. The following proposition summarizes the solution to the representative household problem:

**Proposition 2** *There exists a market equilibrium, in which the following conditions hold:*

*(i) Nominal interest rates are constant:* $R_t = R = \rho + \mu - 1/2\sigma^2$.

*(ii) The nominal wage rate is proportional to $M_t$: $W_t = \delta R M_t$*

*(iii) Real demand is given by $C_t = \left(\frac{RM_t}{P_t}\right)^{1/\varepsilon}$.*

*(iv) The state-price process is $Q_t = 1/(R\chi) \cdot e^{-\rho t}(M_t)^{-1}$, where $\chi$ denotes the Lagrange multiplier on the Household's budget constraint.*

**Proof.** The household's first-order conditions w.r.t. $C_t$, $M_t$ and $n_t$ satisfy $e^{-\rho t}C_t^{-\varepsilon} = \chi P_t Q_t$, $e^{-\rho t}M_t^{-1} = R_t Q_t$ and $e^{-\rho t}\delta = \chi W_t Q_t$. From these three conditions, *(ii)*, *(iii)* and *(iv)* follow immediately.

We therefore just need to show that the equilibrium nominal interest rate is indeed constant. $R_t$ satisfies $Q_t = e^{R_t dt}\mathbb{E}_t(Q_{t+dt})$. Using the FOC for $M_t$, we have $\mathbb{E}_t(Q_{t+dt}/Q_t) = e^{-\rho dt}\mathbb{E}_t(R_t M_t/(R_{t+dt}M_{t+dt}))$. We conjecture (and verify) that $R_t = R$ indeed solves this condition: In that case, $\mathbb{E}_t(R_t M_t/(R_{t+dt}M_{t+dt})) = \mathbb{E}_t(M_t/M_{t+dt}) = e^{-\mu dt + 1/2\sigma^2 dt}$. Therefore the condition for $R_t$ becomes $1 = e^{R_t dt}e^{-\rho dt}e^{-\mu dt+1/2\sigma^2 dt}$, or, after taking logs, $R_t = \rho + \mu - 1/2\sigma^2$ ■

These properties follow from our assumptions that *(i)* the disutility of labor is linear, *(ii)* preferences for real balances are logarithmic, and *(iii)* nominal spending shocks follow a Brownian motion (without mean reversion). Since these properties do not rely in any way on the exact form of the labor demand or individual pricing processes on the intermediate firm's side, they directly apply also to our model in which there are planning, instead of price adjustment costs.

¿From here on, we will assume that equilibrium nominal wages, state prices and real demand are governed by the above processes. We focus on the intermediate firm's pricing and planning problem.

**Pricing and Planning Decisions:** The updating decisions take place as described in the main text. In what follows, we will use the same notation for information sets and expectations. Substituting the state price process, the real demand, and the nominal wage rate into the intermediate firm's profit function, the firms' period-$t$ profits, not including information costs, and valued at the price of nominal cash flows $Q_t$, are

$$Q_t \pi\left(P_t^i; M_t, P_t\right) = e^{-\rho t}\left[\left(\frac{RM_t}{P_t}\right)^{1/\varepsilon - 1}\left(\frac{P_t^i}{P_t}\right)^{1-\theta} - \frac{\delta}{A^{1/\alpha}}\left(\frac{RM_t}{P_t}\right)^{1/(\alpha\varepsilon)}\left(\frac{P_t^i}{P_t}\right)^{-\theta/\alpha}\right]. \tag{9}$$

Therefore, the full model's counterpart to the firm's reduced form objective (equation 9 of the main text) is given by

$$\mathbb{E}_0^i\left\{\int_0^\infty e^{-\rho t}\left[\left(\frac{RM_t}{P_t}\right)^{1/\varepsilon-1}\left(\frac{P_t^i}{P_t}\right)^{1-\theta} - \frac{\delta}{A^{1/\alpha}}\left(\frac{RM_t}{P_t}\right)^{1/(\alpha\varepsilon)}\left(\frac{P_t^i}{P_t}\right)^{-\theta/\alpha}\right]dt - \delta F\int_0^\infty e^{-\rho t}dD_i(t)\right\}. \tag{10}$$

with the average price $P_t$ given by (5). With this structure, our equilibrium definition from the main text applies identically.

**Full-Information Price** Under full information, the optimal price in period $t$ is

$$P_t^* = \left( \frac{\delta}{A^{1/\alpha}} \frac{\theta/\alpha}{\theta - 1} \right)^{\frac{1}{1-\theta+\theta/\alpha}} \cdot (RM_t)^{1-r} P_t^r, \tag{11}$$

$$\text{where} \quad r = 1 - \frac{1 + \varepsilon^{-1} (1-\alpha)/\alpha}{1 + \theta (1-\alpha)/\alpha}. \tag{12}$$

We normalize $A$ so that the initial constant term is equal to 1. Taking logarithms, we find an expression for $\log P^* (t)$, which mirrors equation (10) from the main text:

$$\log P_t^* = (1-r) \log (RM_t) + r \log P_t.$$

Moreover, (5) is approximated by $\log P_t = \int_0^1 \log P_t^i di$.

**Second-order approximation:** We conclude this appendix by showing that the reduced-form formulation considered in the main text is a second-order approximation to the full general equilibrium formulation considered here. We take a constant (first term) and subtract from it the firm's objective (8). Maximizing (8) is equivalent to minimizing

$$\mathbb{E}_0^i \left[ \int_0^\infty Q_t \left( \pi \left( P^* (t) ; M_t, P_t \right) - \pi \left( P_t^i ; M_t, P_t \right) \right) dt + F \int_0^\infty Q_t W_t dD_i (t) \right]. \tag{13}$$

The last integral term represents information costs. Substituting in the formulas for $W_t$ and $Q_t$ from proposition 1, it becomes $-\delta F \int_0^\infty e^{-\rho t} dD_i (t)$.

Using a second-order Taylor expansion of (9) in the first term, we have:

$$\begin{aligned} & e^{\rho t} Q_t \left( \pi \left( P_t^i ; M_t, P_t \right) - \pi \left( P_t^* ; M_t, P_t \right) \right) \\ = & \left( \frac{RM_t}{P_t} \right)^{1/\varepsilon - 1} \left( \frac{P_t^*}{P_t} \right)^{1-\theta} \left[ 1 - \left( \frac{P_t^i}{P_t^*} \right)^{1-\theta} \right] - \frac{\delta}{A^{1/\alpha}} \left( \frac{RM_t}{P_t} \right)^{1/(\alpha\varepsilon)} \left( \frac{P_t^*}{P_t} \right)^{-\theta/\alpha} \left[ 1 - \left( \frac{P_t^i}{P_t^*} \right)^{-\theta/\alpha} \right] \\ = & \, g (M_t, P_t) \left\{ 1 - \left( \frac{P_t^i}{P_t^*} \right)^{1-\theta} - \frac{\theta - 1}{\theta/\alpha} \left[ 1 - \left( \frac{P_t^i}{P_t^*} \right)^{-\theta/\alpha} \right] \right\} \end{aligned}$$

where

$$g (M_t, P_t) = \left( \frac{RM_t}{P_t} \right)^{\frac{1/\varepsilon - \theta}{\alpha + \theta - \alpha\theta}}$$

The linearized first-order condition tells us that under full information, $R_t M_t = P_t$. Therefore, if the shocks are small, the economy is close to the full-information economy and $g (M_t, P_t) \approx 1$.

Defining $x_t^i = \log P_t^i - \log P_t^*$ and using a second-order Taylor expansion around $x_t^i = 0$, we have

$$\begin{aligned} 1-\left(\frac{P_t^i}{P_t^*}\right)^{1-\theta} - \frac{\theta-1}{\theta/\alpha}\left[1-\left(\frac{P_t^i}{P_t^*}\right)^{-\theta/\alpha}\right] &= 1-e^{(1-\theta)x_t^i} - \frac{\theta-1}{\theta/\alpha}\left[1-e^{-\theta/\alpha x_t^i}\right] \\ &\approx \frac{\theta-1}{2}\left[1-\theta+\theta/\alpha\right]\left(x_t^i\right)^2 \end{aligned}$$

Thus, the firm's objective is approximated by

$$\mathbb{E}_0^i\left\{\int_0^\infty e^{-\rho t}\frac{\theta-1}{2}\left[1-\theta+\theta/\alpha\right]\left(\log P_t^i - \log P_t^*\right)^2 dt - \delta F\int_0^\infty e^{-\rho t} dD_i\left(t\right)\right\}. \tag{14}$$

This mirrors the objective in the planning model (equation 4 of the paper), once we define the planning cost as $2\delta F/\left((1-\theta+\theta/\alpha)(\theta-1)\right)$.

**Comparative Statics** Finally, we examine the relationship between the structural parameters and two key parameters in the reduced-form model of the main text that determine price rigidity and updating frequency. One is $r$, the complementarity in price-setting, defined in equation (12). There are three underlying structural parameters that determine complementarity. First, relative risk aversion $\epsilon$ increases complementarity ($\partial r/\partial\epsilon > 0$). Second, the elasticity of substitution $\theta$ increases complementarity ($\partial r/\partial\theta > 0$). Finally, the rate of diminishing marginal returns to labor $\alpha$ increases complementarity iff $\theta > 1/\epsilon$.

The second key determinant of updating frequency is the cost of information. In this micro-founded model, that cost is a labor cost and therefore varies over time with the wage. The planning cost is increasing in $\alpha$ and decreasing in $\theta$, as long as there are diminishing returns $\alpha < 1$ and elasticity $\theta > 1$. Therefore, if $\theta > 1/\epsilon$, then increases in $\alpha$ increase complementarity and increase the planning cost, both of which make updating less frequent and prices more sticky. But the effect of changes in elasticity $\theta$ are ambiguous because they increase complementarity but decrease planning costs.

## References


[1] Angeletos, George-Marios and Alessandro Pavan (2007), "Policy with Dispersed Information," working paper.

[2] Golosov, Mike, and Robert Lucas (2007), "Menus Menu Costs and Phillips Curves," *Journal of Political Economy*, 115, 171-199.