Knowing What Others Know: Coordination Motives in Information Acquisition

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Two types of information acquisition

Passive learning - Agents are endowed with signals, learn as an unintended consequence or observe prices/ quantities.

Morris and Shin (2002), Angeletos and Werning (2005), Hellwig, Mukherji and Tsyvinski (2005), ect.

Active learning - Agents choose information to observe. Choices are not strategic.

Sims (2003), Reis (2006), ect.

What information will strategic agents choose to learn? How will this effect economic outcomes?

Main idea

Coordination (substitutability) in actions produces coordination (substitutability) in information choices.

Why is this result important? Does it get us back to square one?

1. Conditions for multiple equilibria change. Depends on:
    - Are information choices discrete or continuous?
    - Is information public or private?
2. Equilibrium switches move covariances, not levels.
    - Actions can only covary with what agents observe.
3. It explains why some applied models find information herding, while others predict specialization.

Outline

- A strategic game with many agents - beauty contest.
  3 types of information choices.
  1. Discrete choice.
  2. Continuous choice over the precision of a private signal.
  3. Continuous choice over the precision of a public signal.
- Dynamic price-setting and planning model (Reis, 2006).
  - How does each theoretical insight change price dynamics?

A Beauty Contest Game

- Continuum of agents. Each agent sets a_i to minimize

$$EL(a_i, a, s) = E\left[(1-r)(a_i - s)^2 + r(a_i - a)^2\right]$$

where $a = \int a_i di$. Exogenous state variable: $s \sim \mathcal{N}\left(y, \tau_s^{-1}\right)$.

- Under full information, optimum is $a_i = (1-r)s + ra$.

- Three cases:

$$\begin{aligned} r &> 0: && \text{Strategic complements, optimal } a_i \text{ increasing in } a. \\ r &= 0: && \text{No interaction, optimal } a_i \text{ independent of } a. \\ r &< 0: && \text{Strategic substitutes, optimal } a_i \text{ decreasing in } a. \end{aligned}$$

Order of Events

1. Natures draws $s \sim \mathcal{N}\left(y, \tau_s^{-1}\right)$.
2. Each agent receives private signal $x_i \sim \mathcal{N}\left(s, \tau_x^{-1}\right)$.
3. Agents decide whether to pay $C > 0$ to acquire additional information:
   - private signal $w_i \sim \mathcal{N}\left(s, \tau_w^{-1}\right)$
   - common signal $z \sim \mathcal{N}\left(s, \tau_z^{-1}\right)$
4. Agents choose a_i.

Equilibrium

An equilibrium is:

- a probability of acquiring information $\lambda^* \in [0,1]$ that minimizes $EL + \lambda C$,
- a decision rule $a_U(x_i)$ that minimizes EL for agents who remain uninformed,
- a decision rule $a_I(x_i, w_i, z)$ that minimizes EL for agents who acquire information.

The Main Result

- Define *Value of information* $B(\lambda) = EL_U(\lambda) - EL_I(\lambda)$.
- **Proposition:** Suppose $\max\{\tau_w, \tau_z\} > 0$. Then:

$$
\begin{aligned}
r &> 0 \iff B'(\lambda) > 0 \text{ for all } \lambda \in [0,1] \\
r &= 0 \iff B'(\lambda) = 0 \text{ for all } \lambda \in [0,1] \\
r &< 0 \iff B'(\lambda) < 0 \text{ for all } \lambda \in [0,1]
\end{aligned}
$$

Why substitutability?

Information value is a difference in conditional variances:

$$
\begin{aligned}
B(\lambda) &= V_U\left((1-r)s + ra\right) - V_I\left((1-r)s + ra\right) \\
&= (1-r)^2\left[V_U(s) - V_I(s)\right] \\
&\quad +2(1-r)r\left[cov_U(s,a) - cov_I(s,a)\right] + r^2\left[V_U(a) - V_I(a)\right]
\end{aligned}
$$

- First term: variance reduction in s – direct effect.
- Second term: covariance risk from aggregate action and the state, dominant strategic effect. Substitutability means $r < 0$.
- Third term: aggregate action uncertainty, strategic effect.

High λ raises cov(a,s), creates less payoff uncertainty, lowers information value, if $r < 0$.

Why does substitutability matter?

- Delivers a new interpretation of Grossman and Stiglitz (1980).
  - Old logic: The value of information falls as more investors buy it because it is revealed through the asset's price.
  - New insight: Information is a strategic substitute because investors do not want to buy assets that others buy - strategic substitutability in actions is sufficient.
- Buying different information and taking different actions can have very different aggregate outcomes.
  Vives (1984), Van Nieuwerburgh and Veldkamp (2005)
- If $r < 0$, there is always a unique, possibly mixed equilibrium.

Complementarity in Information Acquisition ($r > 0$)

- High λ raises $\mathrm{cov}(a, s)$, creates more payoff uncertainty, increases information demand.
- **Theorem:** *If $r > 0$, there exists a non-empty interval $\left[\underline{C}, \overline{C}\right]$, s.t. if $C \in \left[\underline{C}, \overline{C}\right]$, both $\lambda = 0$ and $\lambda = 1$ are equilibria in information acquisition.*

... but multiple equilibria depend on discrete choice set.

- Note: Changes in λ change $\mathrm{cov}(a, s)$. Equilibrium switches will alter the covariance of the state and aggregate choice variable.

Continuous Private Information Choice

Same set-up as before, but:

- All public information is in exogenous priors $\tau_z = 0$.
- Choose precision of private signal τ_x, independent across agents. Interpretation: private research and analysis.
- Cost of information $C(\tau_x)$ increasing, convex, twice differentiable.

Symmetric equilibrium: Value τ_x^*, s.t. if all other agents choose a private signal precision τ_x^*, it is optimal for each individual agent to do so as well.

A Unique Equilibrium

- Equilibrium precision τ_x^* equates marginal cost and benefit:

$$C'\left(\tau_x^*\right) = - \left.\frac{\partial EL\left(\tau_x, \tau_x^*\right)}{\partial \tau_x}\right|_{\tau_x = \tau_x^*} = \left(\frac{1-r}{\left(1-r\right)\tau_x^* + \tau_s}\right)^2$$

- Best response relation increasing in aggregate τ_x^*, when $r > 0$ (complements), decreasing when $r < 0$ (substitutes).
- With complementarity, $EL\left(\tau_x, \tau_x^*\right)$ is decreasing, convex in τ_x.

Convexity → *unique equilibrium!*

Public Information Choice: Newspaper Model

Same set-up as before, but:

- Private information is exogenous $\tau_x \geq 0$.
- There is a sequence of i.i.d. signals $\{z_n\}_{n=1}^{\infty}$, $z_n \sim \mathcal{N}\left(s, \delta^{-1}\right)$. Focus on small δ. Like words in a newspaper.
- Choose N. Observe first N signals: $\{z_n\}_{n=1}^{N}$.
- Information cost $C\left(\tau_z\right) = C\left(N\delta\right)$, increasing and convex.

- **Symmetric equilibrium:** An N^* s.t. if others choose N^* signals, it is optimal.

Multiple Equilibria with Public Information

Suppose others observe N^* signals, agent chooses N.

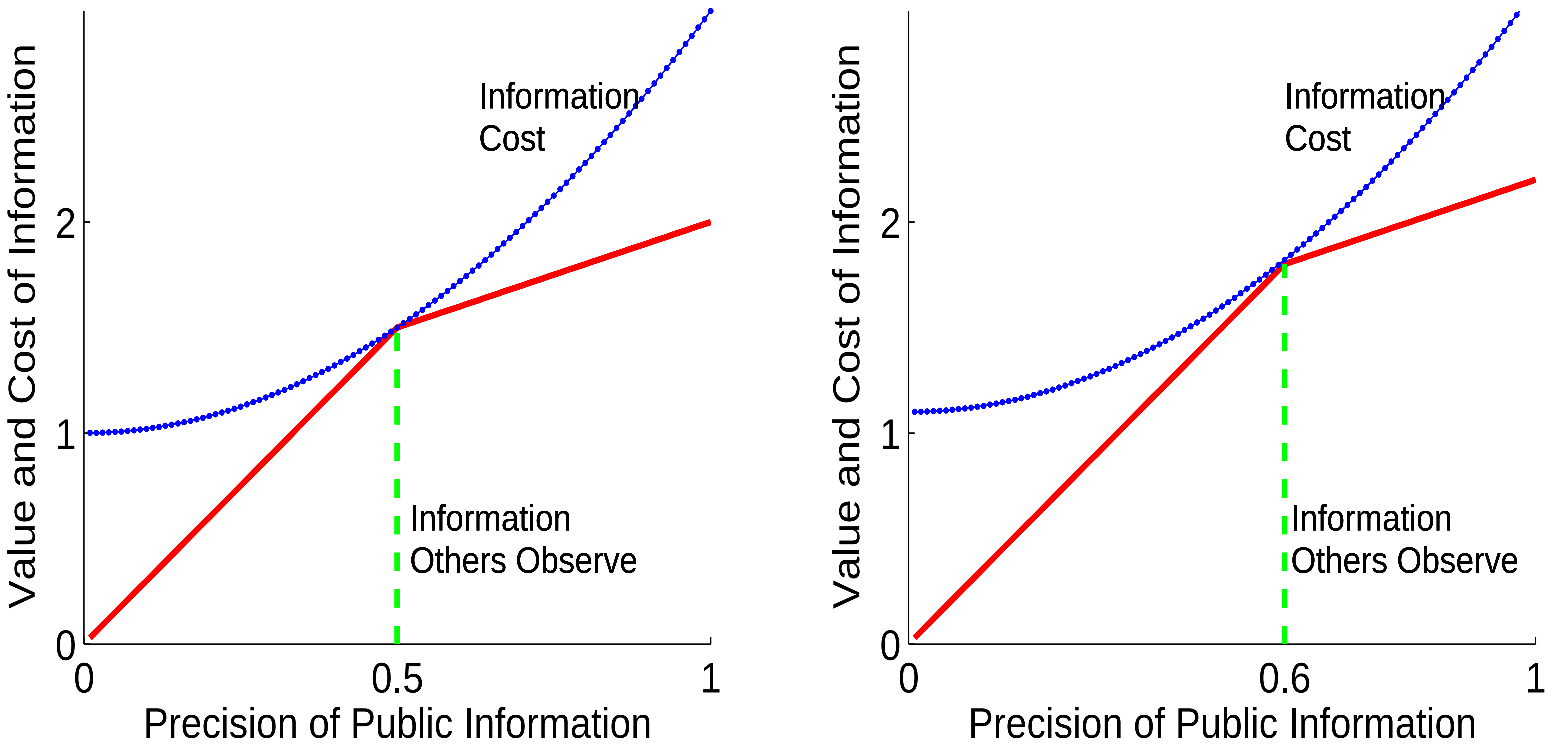
- If $N > N^*$, additional signals raise precision of his *private* information.
- If $N < N^*$, additional signals are *public*.

Proposition: As $\delta \to 0$, there exist equilibria for all τ_z^*, s.t.

$$\left(\frac{1-r}{(1-r)\,\tau_x + \tau_s + \tau_z^*}\right)^2 \leq C'\left(\tau_z^*\right) \leq \left(\frac{1}{(1-r)\,\tau_x + \tau_s + \tau_z^*}\right)^2$$

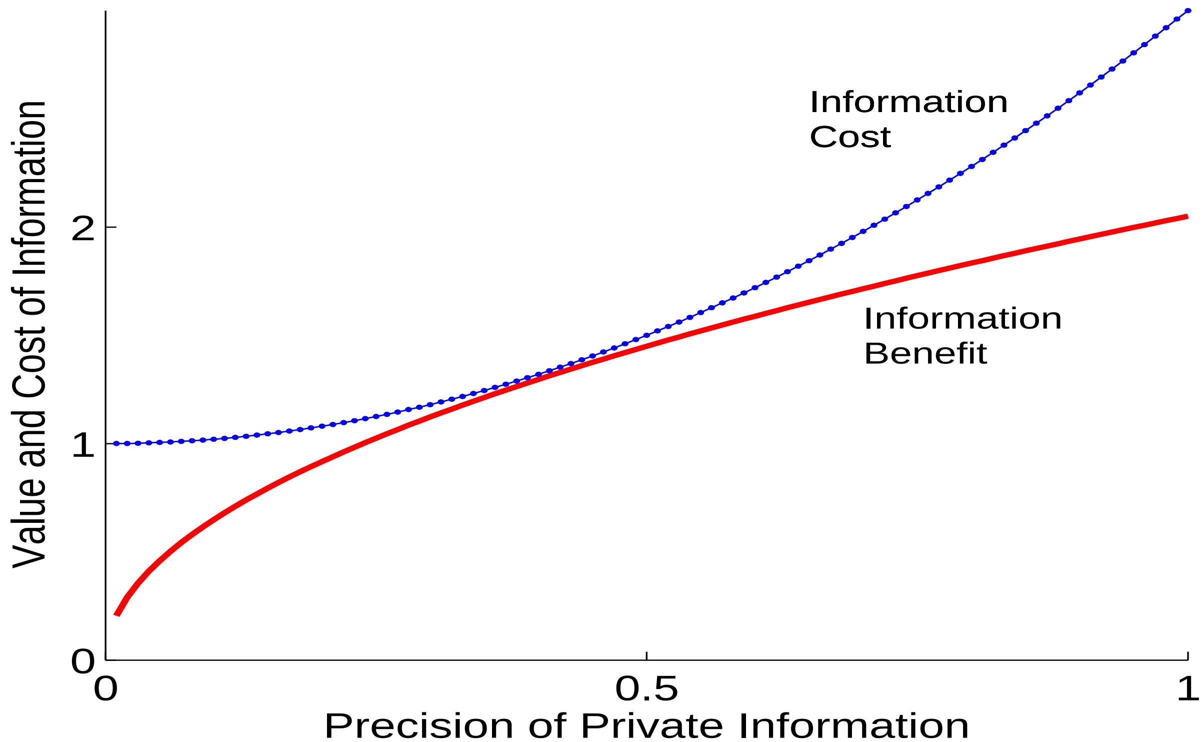
- When $r > 0$, marginal value of private information (left side) is less than marginal value of public information (right).
- When $r < 0$: No pure strategy equilibrium!

Public Information → Multiple Equilibria



Public information, in excess of what others observe, is private.
Kink in marginal value → multiplicity.

Private Information → Unique Equilibrium



Private information is complementary. The complementarity is not strong enough to generate multiplicity.

Summary of theory results

- Coordination (substitution) in actions generates coordination (substitution) in information. True in a wide range of environments and information structures.
- Discrete information choices generate multiple equilibria.
- Public information has a fundamental discontinuity in its marginal value. Makes multiple equilibria possible.
- Changes in information outcomes change the covariance of aggregates.

How do each of these insights matter for applied models?

A Costly Planning Model (Reis 2006)

- Firms choose price p_t^i to minimize losses:

$$\sum_{t=0}^{\infty} \beta^t \left(p_t^i - p_t^*\right)^2$$

- p_t^* is "target price" (a full-info optimum):

$$p_t^* = (1 - r)\, m_t + r p_t$$

- p_t is average price of all firms, m_t is nominal money supply.
- m_t is a random walk:

$$m_t = m_{t-1} + \varepsilon_t; \qquad \varepsilon_t \sim \mathcal{N}\left(0, \sigma^2\right)$$

- To 'plan': Pay C to observe $\{m_\tau\}_{\tau=0}^t$, $\{p_\tau\}_{\tau=0}^t$.

Staggered Equilibria With Strategic Planning

- *Staggered planning* - Agents plan every T periods. A fraction $1/T$ of agents plans each period.

Proposition: *There exists a staggered planning equilibrium with planning horizon T, if and only if*

$$\sigma^2 \sum_{\tau=1}^{T-1} \frac{1-\beta^\tau}{1-\beta} \left(\frac{1-r}{1-r\frac{\tau}{T}} \right)^2 \leq C \leq \sigma^2 \sum_{\tau=1}^{T} \frac{1-\beta^\tau}{1-\beta} \left(\frac{1-r}{1-r\frac{\tau}{T}} \right)^2$$

- If $r > 0$, potentially multiple equilibria with different horizons.
- if $r \leq 0$, unique equilibrium, possibly with mixed strategies.

Synchronized Equilibria With Strategic Planning

- *Synchronized planning* - Agents plan every T periods, all in the same period.

Proposition: Sufficient Condition for a staggered planning equilibrium for all T:

$$(1-r)^2\,\sigma^2\,\frac{T^2}{4} \leq C \leq \sigma^2$$

- If $r > 0$, potentially multiple equilibria with different horizons.
- If $r < 0$, then $T < 2$ (updating every period) needed to satisfy the condition. No truly synchronized equilibrium.

Why Multiple Equilibria?

1. Choices are discrete because time is discrete.
   - In a continuous time model, there is a unique staggered planning equilibrium.
2. Price information is like newspaper information.
   - In continuous time, multiple synchronized equilibria persist.
   - Information observed at a time when others don't observe it is private. Information that everyone observes at the same time is public.

What Does This Mean For Pricing Models?

- Information complementarity is a new source of price persistence.

$$p_t = \sum_{s \in \{t, t-1, \ldots\}}^{\infty} \frac{(1-r)\Lambda_{t,s}}{1 - r\Lambda_{t,s}} \varepsilon_s$$

- We need to look to data to infer what information price-setters might be acquiring.
- This can explain parameter instability in inflation forecasts. Exchange rates? Equity markets?
- A new solution to an old multiple equilibria problem? Continuous choice variables *and* private signals. (Mackowiak and Wiederholt, 2006)

Using Uncertainty to Restore Uniqueness

- Private information in priors does not eliminate multiplicity. Information choices depend on second moments. These are common knowledge.
- Heterogeneity in beliefs about second moments could work.
- With continuous aggregate data, second moments can usually be deduced instantaneously and perfectly. First moments can't.
- Multiple information equilibria are easier to sustain than multiple action equilibria.

Conclusions

- Complementarity in actions means complementarity in information choices. Likewise for substitutability.
- This simple idea has important consequences.
  - The nature of the equilibria change.
  - The conditions for multiple equilibria change.
- Results provide guidance in building future models of information choice.