Slow Boom, Sudden Crash

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Abstract

Many asset markets exhibit slow booms and sudden crashes. This pattern is explained by an endogenous flow of information. In the model, agents undertake more economic activity in good times than in bad. Economic activity generates public information about the state of the economy. If the economic state changes when times are good and information is abundant, asset prices adjust quickly and a sudden crash occurs. When times are bad, scarce information and high uncertainty slow agents' reactions as the economy improves; a gradual boom ensues. Data from U.S. and emerging credit markets support the theory.

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Gradual booms and sudden crashes are a ubiquitous feature of financial markets. In the four months following the 1994 peso crisis, Mexican lending rates rose more than fourfold (21% to 91%). It took 33 months for loan rates to return to their pre-crisis level. During the financial crises in Thailand, Russia and Indonesia, crashes were shorter and sharper than any boom of equal magnitude. In almost every emerging market, bond yields and lending rates exhibit this type of asymmetry.

A model where information flows increase as asset values rise, or as interest rates fall, can explain this asymmetry. Consider investors who choose whether or not to invest in a risky asset. The probability of a positive return is the unobserved state of the economy. Investing generates information about this state. When investors believe the state is good (high probability of positive return), many investments generate a large sample of observations. If the state switches to bad, investors quickly deduce that the state has changed. Investment plummets and interest rates soar. When the state is bad and switches to good, the prevailing low investment levels create a small data set problem. Greater uncertainty slows reactions to the state change. The result is a gradual boom. Although the model is narrowly about a lending market, the idea of endogenous information extends to other asset markets with asymmetry such as equity, venture capital, and foreign investment. The conclusion discusses some of these alternate interpretations.

The contribution of the model is to introduce an endogenous rate of information flow that varies with the level of economic activity. This information mechanism endogenously generates unconditional asymmetry in lending rate and investment changes.

To judge the asymmetry of the model and the data, I use two measures: time-irreversibility and skewness. A stochastic process is time-reversible if the probability of starting at any point x and moving to any y is the same as the probability of starting at y and moving to x. Reversing a process like lending rates that has large increases and gradual declines would produce slow increases and sudden declines. Because reversing lending rates changes their properties, they are time-irreversible. Similarly, many gradual decreases and occasional large increases produce a distribution of changes with many small negative observations and a few positive outliers. The fat right tail in this distribution is measured as positive skewness.

Theoretical results show that endogenous information generates time-irreversibility and unconditional asymmetry: large, one-period interest rate increases that are more likely than equally large decreases. In contrast, a benchmark model with a constant information flow is shown to generate symmetric changes. Quantitative results show that endogenous information produces a magnitude of asymmetry comparable to that in emerging market bonds. Time-reversibility tests reject symmetry in the data and the model, but fail to reject for the constant learning model, even when bad signals are more informative than good ones. Furthermore, the model's skewness lies within the 95% confidence interval of the data. Finally, a cross-section examination of interest rate dynamics in three corporate lending markets supports an indirect prediction of the model that the riskiest markets should be the most asymmetric.

The literature on crashes includes papers on overreactions and papers on asymmetry. Herding models, ([19], [2], [4], [7]) portfolio insurance models of stock market crashes [13], and information-based models of bubbles [1] produce big market reactions to small changes in fundamentals. In this sense they are complements to this paper, which produces asymmetry but no overreaction. Coupling endogenous information with an overreaction theory would produce crashes that are faster than booms, with only small changes in fundamentals.

Chamley and Gale [9] model asymmetry. Their work shares the feature that investment generates information. Irreversibility of investment delays the resumption of growth after a downturn. Delay produces a distribution of investment changes with many observations near zero, but tails that are symmetric. This theory focuses on precisely the asymmetry in tail events: booms and crashes. Furthermore, if firms with larger irreversible investments take out larger loans, then Chamley and Gale would predict that lending rates on large loans are the most asymmetric. In the data, the largest corporate loans have the least lending rate asymmetry [section 4.5]. This finding is at odds with the irreversible investment theory but supportive of endogenous information.

1 The Model

There is a credit market with M perfectly competitive lenders and a finite number, $N < M$, of entrepreneurs who are potential borrowers. Both lenders and entrepreneurs are infinitelylived, risk-neutral profit maximizers. Each period, lenders can either invest one indivisible unit of capital in a risk-free bond which pays a return of $(1 + r)$ next period, or they can loan capital to an entrepreneur borrower. The risk-free rate is exogenous and constant. If an entrepreneur borrows and the venture succeeds, the lender receives $(1 + \rho)$ next period. If the venture fails, the lender gets back nothing. The market lending rate is endogenous and depends on the expected rate of default.

Each period, an entrepreneur can borrow one unit of capital to start up a new venture or can work for a fixed wage. Entrepreneur borrowers know what their profit will be if the venture succeeds. If successful, borrower i receives an exogenously determined investment payoff $v_i \,\epsilon\, (\underline{v}, \bar{v})$ that is constant over time. These bounds are chosen so that an entrepreneur with an investment payoff in the interval could change a decision based on observed information. This assumption excludes uninteresting actors who would always invest or never invest. Section 4.5 relaxes this assumption and discusses its effects.

If unsuccessful, an entrepreneur who borrows receives zero profit. Borrowers also face an opportunity cost of undertaking a business venture: they forego a riskless wage income w.

The probability of success for all new ventures is the same and depends on an unobserved state variable, $\omega \,\epsilon\, \{\omega_g, \omega_b\}$. The state changes with a small probability λ each period. If the market is in the good state, ω_g, then the probability of a venture succeeding and a loan being repaid is θ_g. In a bad state, that probability is θ_b $(\theta_g > \theta_b)$. By assumption, all agents begin with common beliefs. Furthermore, all successes and failures are publicly observable, so agents will always have common beliefs about the probability μ of being in the good state. Throughout the paper, I use the subscript t on μ and θ to denote beliefs about the probability of being in the good state and of a venture succeeding in period t, conditional on all information observed before the start of period t.[1]

The ordering of events in a period is as follows:

1. All agents enter the period with beliefs, μ_t.

2. Lenders set interest rates, and borrowers decide whether or not to accept a loan and

[1]More precisely, if f_t is the set of all signals observed during period t, and $\mathcal{F}_t$ is the filtration generated by $\{\mu_1, f_1, \ldots, f_{t-1}\}$, then μ_t and θ_t are $\mathcal{F}_t$-adapted.

invest in a venture.[2]

3. All lenders not matched with borrowers invest in the risk-free security. All borrowers who decide not to invest are matched with fixed-wage jobs.

4. The outcomes of all ventures are publicly observed, and all payoffs are received.

5. The state changes with probability λ, and beliefs are updated.

2 Equilibrium

Definition of Equilibrium

A subgame perfect equilibrium, for a given initial belief μ_0 is a sequence of borrowing decisions by borrowers $\{b_{it}\}$, interest rates chosen by lenders $\{\rho_{jt}\}$, funded ventures $\{n_t\}$, updated beliefs about the probability of being in a good state $\{\mu_t\}$, and the probability of success of a venture $\{\theta_t\}$ for all agents, as shown below.

1. **Borrowers**. Given a set of available lending rates J_t, borrower i maximizes expected profit by choosing whether to borrow and which lender to borrow from.

$$\max_{b_{it}\epsilon\{0,1\}, j\epsilon\{1,\ldots,M\}} b_{it}\theta_t(v_i - (1+\rho_{jt})) + (1-b_{it})w \tag{1}$$

2. **Lenders**. Given strategies of the other lenders, lender j chooses an interest rate ρ_{jt} to maximize his expected profit

$$\max_{\rho_j} l(\rho_{jt}, \rho_{-jt})\theta_t(1+\rho_{jt}) + (1-l_{jt})r \tag{2}$$

where $l_{jt} = 1$ if a borrower decides to borrow from lender j in period t and 0 otherwise.

3. **Number of Funded Ventures**. This number of ventures is equal to the number of

[2] In equilibrium, borrowers will be indifferent between loans from different lenders. However, off the equilibrium path, a market-clearing device may be needed. Auctioning loans to the bidder willing to pay the highest interest rate will suffice. In reality, capital is often allocated through an auction mechanism, such as the uniform price auction used for U.S. Treasury bills.

borrowers who decide to take out a loan.

$$n_t = \sum_{i=1}^{N} b_{it} \tag{3}$$

4. **Beliefs**. All agents observe the number of successes and failures during period t and form posterior beliefs μ_t^P, using Bayes' Law. If s is the number of successes observed out of n funded ventures in period t, then the formula for posterior beliefs is

$$\mu_t^P = \frac{C_s^n \theta_g^s (1-\theta_g)^{n-s} \mu_t}{C_s^n \theta_g^s (1-\theta_g)^{n-s} \mu_t + C_{n-s}^n \theta_b^s (1-\theta_b)^{n-s} (1-\mu_t)}. \tag{4}$$

where $C_s^n = n!/((n-s)!s!)$. Since the number of combinations C_s^n and C_{n-s}^n are equal, they drop out of the equation. These posterior beliefs are converted to next period's beliefs by adjusting for the probability of a state transition

$$\mu_{t+1} = (1-\lambda)\mu_t^P + \lambda(1-\mu_t^P). \tag{5}$$

The relationship between the belief about the current state and the expected probability of success of a venture is as follows:

$$\theta_{t+1} = \mu_{t+1}\theta_g + (1-\mu_{t+1})\theta_b. \tag{6}$$

This definition of equilibrium considers agents to be adaptive learners and implicitly rules out active experimentation. In other words, agents hold their current beliefs fixed when deciding future expected payoffs. This assumption is made for computational simplicity. However, since information is a public good in this setting, any experimentation would suffer from a free-rider problem, and its effect would be negligible.

Equilibrium Outcomes

Since lenders are perfectly competitive, they make no profit in equilibrium. The no-profit condition implies that all loans accepted by borrowers have the common interest rate ρ_t, where ρ_t satisfies

$$\theta_t(1+\rho_t) = 1+r. \tag{7}$$

Since all lenders charge identical interest rates, the choice of lending rates becomes irrelevant. The only choice left is to borrow or not. The solution is a cutoff rule in the payoff v_i. An entrepreneur borrows money to start a venture in period t if

$$\theta_t(v_i - (1+\rho_t)) \geq w. \tag{8}$$

Another way to interpret this condition is that borrowers and lenders enter into a contract if and only if the expected joint surplus of that project is positive. The contract payoffs are the Nash bargaining outcome where borrowers have all the negotiating power.

A venture's expected probability of success enters into the borrowing rule in two ways. A higher θ increases the expected payoff of borrowing, and it decreases the market interest rate, ρ. Since both of these effects make borrowing more attractive, the number of loans that are extended is increasing in θ.

Substituting the equilibrium lending rate ρ in inequality [8], results in the following condition under which entrepreneur i will take out a loan:

$$v_i > \frac{1}{\theta_t}(1+r+w). \tag{9}$$

The risk-free rate plus the wage is the opportunity cost to investing. An entrepreneur invests if the expected payoff ($\theta_t v_i$) is greater than the opportunity cost. The true lending rate ρ_t varies in the probability of success θ_t. However, this is exactly offset by the fact that the loan is only repaid if the venture succeed, which also happens with probability θ_t.

This inequality allows us to solve for the payoff bounds $\underline{v}$ and $\bar{v}$. An entrepreneur who is certain he was in the good state in the previous period would believe that he was in the good state with probability $(1-\lambda)$ today and that the probability of success was $\theta_g(1-\lambda)+\theta_b\lambda$. This is the most optimistic he can possibly be. With payoff $\underline{v}$, the most optimistic entrepreneur possible would be indifferent between borrowing or working for a fixed wage. With payoff $\bar{v}$, the most pessimistic possible entrepreneur would also be indifferent.

$$[\underline{v}, \bar{v}] = \left[\frac{1+w+r}{\theta_g(1-\lambda)+\theta_b\lambda}, \frac{1+w+r}{\theta_g\lambda+\theta_b(1-\lambda)}\right]. \tag{10}$$

A key variable is the number of ventures funded each period, n_t, because it is also the number of signals observed about the current state. Since n_t only depends on μ_t and fixed parameters, it can be expressed as a function $n(\mu)$, which counts the number of v_i's greater than the cutoff value in [9].

$$n(\mu_t) = \sum_{i \epsilon \{1,2,\ldots,N\}} \mathbf{1}_{\{v_i > \frac{1+w+r}{\mu_t\theta_g+(1-\mu_t)\theta_b}\}}. \tag{11}$$

This function is non-decreasing; there are more signals per period when the expected probability of the good state is higher. To see this, let $\tilde{v}$ denote the right hand side of the inequality in [11]. $\frac{\partial \tilde{v}}{\partial \mu} = -(\theta_g - \theta_b)\frac{1+w+r}{(\mu_t\theta_g+(1-\mu_t)\theta_b)^2}$. By assumption, $\theta_g > \theta_b$ and $w, r > 0$, so the derivative is negative. For any v_i, and $\mu' > \mu$, if the inequality holds for μ, it must hold for μ'. Thus, the number of v_i that satisfy the inequality for μ' must be at least as large as $n(\mu)$.

To fully solve this model analytically, one would have to write out a function that took in the current state variables and produced the possible outcomes in the following period. However, the changing number of signals complicates the writing of a general function for every possible state. There would have to be a separate function for each $n \epsilon \{0, 1, \ldots, N\}$. Since most markets involve a large number of actors, writing out an explicit solution is intractable.

3 Analytical Results

This model can generate lending rate changes that are time-irreversible and have an asymmetric unconditional distribution where the probability of large interest rate increases is higher than the probability of equally-sized decreases. In contrast, when the information flow is constant, interest rate changes are time-reversible and symmetric, for any interest rate function of beliefs. Since the lending rate and the unobserved state form a joint Markov process, standard procedure would be to derive the distribution that is a fixed point of the transition function. Because that function is nonalgebraic, there is no analytic method for computing its fixed point. Therefore, the proof relies on a time-reversal argument to establish symmetry properties without deriving the distribution itself.

I impose one additional assumption in the theory to isolate the endogenous information effect: Good and bad signals are equally informative, meaning that the probability of success in good states equals the probability of failure in bad states. ($\theta_g = 1 - \theta_b$). When examining the model's quantitative predictions, this assumption will be relaxed.

The first set of results establish symmetry of the constant learning benchmark model. The only change is that N signals are observed every period, regardless of investment choices. Each period, agents are able to observe what the outcome of all investments would have been, even if they were not undertaken. The number of signals is not dependent on the state of the economy. In this *constant learning model*, the symmetric nature of Bayes' law makes booms and crashes the same.

Since the time-reverse of every up is a down and vice-versa, one way to think about whether ups and downs are the same in a stochastic process is to ask if the process is the same forward and backward in time. Time-reversibility means that the probability of starting at price x and moving to price y, equals the probability of the reverse movement, starting at y and moving to x: $P[p_t = x, p_{t+1} = y] = P[p_t = y, p_{t+1} = x]$.

Proposition 1 *In a constant information economy with equally informative signals, lending rates $\{\rho_t\}$ are a time-reversible process.*

All proofs are in the appendix.

Time-reversibility and the unconditional symmetry of changes are inexorably linked. For any transition that results in an increase in price, there is an equal probability event that results in the same size decrease in price. Realize that the probability of any size change in price $f(\Delta p)$ is just a sum over all transitions such that $y = x + \Delta p$. If the probability of any same-size increase and decrease in the price are the same ($f(\Delta p) = f(-\Delta p)$), then the distribution of price changes is symmetric.

Proposition 2 *In a constant information economy with equally informative signals ($\theta_b = 1 - \theta_g$), changes in lending rates have a symmetric unconditional distribution.*

Propositions 1 and 2 hold generally for any asset price that is a function of beliefs. The reason is that if two sequences have identical stochastic processes (in this case, forwards and reverse-time beliefs), then any function of those sequences must also be identical processes. This is useful because it demonstrates why the form of preferences, the type of asset, or any other modification to the pricing function is not relevant to the discussion of asymmetric changes.

The previous discussion established symmetry of constant information models, as a benchmark against which to compare the asymmetry of the endogenous information model. With endogenous information, lending rates are time-irreversible and their distribution is asymmetric.

Proposition 3 *In the endogenous information economy with equally informative signals, beliefs* $\{\mu_t\}$ *are a time-irreversible process.*

Beliefs are time-irreversible because the change in beliefs in the good state is, on average, not as large as the reverse movement in the bad state. Figure 1 illustrates this point. If the economy starts with beliefs at μ_G and $\omega = G$, beliefs are likely to increase the following period (because μ is a submartingale in the good state) to a level such as μ_x. The size of that change will be affected by the number of signals observed during the period, $n(\mu_G)$.

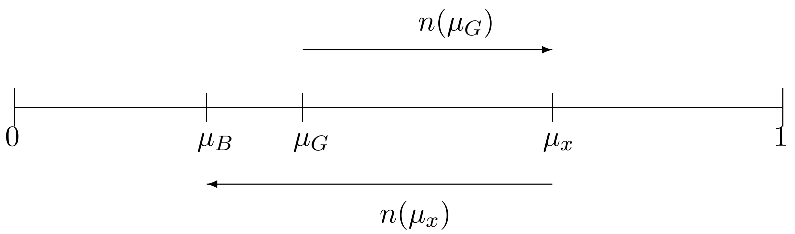

Figure 1: Asymmetric changes in beliefs in an endogenous information economy

Now, consider the economy starting with beliefs μ_x and $\omega = B$. Beliefs are likely to fall the next period and the size of the change will be affected by the number of signals observed during the period, $n(\mu_x)$. Recall that n is increasing in μ so that $n(\mu_x) > n(\mu_G)$. And since expected size of the movement in beliefs is increasing in n, the expected size of the downwards movement in beliefs is larger than the expected upwards movement. Not only are expected changes increasing in n_t, but also the largest possible changes in beliefs are increasing in n_t. If the largest possible downward change in beliefs is larger in magnitude than the largest upward changes, then the largest one-period lending rate increases will be bigger than the largest declines.

Proposition 4 *In the endogenous information economy with equally informative signals, let* $\hat{p}$ *be the largest increase and* $\underline{p}$ *be the largest decrease in prices that occurs with strictly positive probability.*

1. *For an asset whose price is any strictly decreasing function of beliefs* $p(\cdot) : [0,1] \to \Re$, $|\hat{p}| > |\underline{p}|$.
2. *For an asset whose price is any strictly increasing function of beliefs* $p(\cdot) : [0,1] \to \Re$, $|\hat{p}| < |\underline{p}|$.

Corollary 1 *The unconditional distribution of changes in interest rates in the endogenous learning economy is asymmetric:* $f(\Delta\rho) \neq f(-\Delta\rho)$ *for some* $\Delta\rho$. *The largest interest rate increase is larger in absolute value than the largest decrease.*

These results are all statements about *unconditional asymmetry.* Within the asymmetry literature, there are also papers on *conditional asymmetry.* For example, Veronesi [18] generates asset price booms that start gradually and build speed and crashes that start suddenly and lose momentum. In such a model, beliefs are a time-reversible process: They are just as likely to move in either direction along a beliefs-price curve. Thus, any steep region of the curve that could produce large downward price movements must be just as likely to produce a same-size upward move. This paper proves that no matter what the shape of the pricing curve in a setup like Veronesi's, the price changes will still be *unconditionally symmetric.*

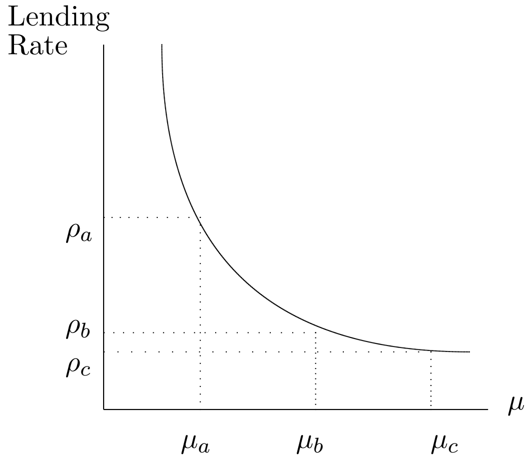

Figure 2: The relationship between changes in beliefs and changes in lending rates

To illustrate the difference between constant and endogenous learning and their relationship with conditional and unconditional asymmetry, consider the following example in Figure 2. The relationship between beliefs and the lending rate is given by the pricing rule in equation [7]. Suppose that, the probability of beliefs moving from μ_c to μ_b is the same as beliefs moving from μ_a to μ_b. This produces conditional asymmetry in lending rate changes despite symmetry in beliefs because the nonlinear relationship between ρ and μ means that equal size upward and downward changes in beliefs do not produce equal size changes in lending rates. The increase in interest rate, $(\rho_b - \rho_c)$, will be much smaller than the equally likely decrease, $(\rho_b - \rho_a)$. This is the type of asymmetry that Veronesi [18] discusses.

However, the fact that constant-information beliefs are time-reversible means that the unconditional probability of moving from ρ_a to ρ_b is the same as moving from ρ_b to ρ_a. So, the probability of this large interest rate decrease is the same as the equally large increase. Veronesi's model and the overreaction models are unconditionally symmetric because upward belief movements are just as likely as downward movements.

The assumption that signals are equally informative ($\theta_b = 1 - \theta_g$) is restrictive. Relaxing this assumption produces asymmetry in the constant learning model. For example, if bad signals contained more information than good ones ($\theta_g > 1 - \theta_b$), then beliefs would have larger but less frequent jumps down than up. A variety of models employ this mechanism.

Boldrin and Levine's [5] asymmetric spread of new technologies, Zeira's [20] [21] learning about market capacity, and Chalkley and Lee's [8] noise traders all generate bad signals that are more informative than good signals. Similarly, Campbell and Hentchel's [6] GARCH dividends make bad news more extreme than good news. To distinguish these theories from endogenous learning, we need to look at the data. Section 4.3 calibrates an asymmetric signal model and shows that it cannot match the asymmetry in the data. Furthermore, section 4.5 shows that variation in asymmetry across markets is inconsistent with an asymmetric signal explanation.

4 Quantitative Results

One way to determine if the model's explanation for asymmetry is plausible is to ask: with reasonable parameter values, can the model produce the magnitude of the asymmetry we see in the data? This section compares simulated lending rates from the model to yields on emerging market corporate bonds. The data is a monthly panel of dollar-denominated bond price indices calculated by Salomon Brothers for 13 emerging markets, between 1994 and 2000.[3] The reason for looking at emerging markets is that information flow should be volatile for the model effects to be strong. In a stable market, such as the U.S. corporate bond market, where information is always abundant, the asymmetry effect would be weaker. Section 4.5 formulates this prediction within the model and tests it with a cross-section of lending data.

4.1 Asymmetry in Emerging Market Bonds

Time-Reversibility

Theoretical results showed that, when signals are equally informative, the constant information model is time-reversible, while the endogenous information model is not. I begin by documenting irreversibility in emerging market bond yields.

[3] To convert the price indices into measures of yields that can be compared to the model, use a first-order Taylor approximation to percentage changes in yields: $\frac{\Delta P_t}{P_t} = -M \frac{\Delta(1+\rho_t)}{(1+\rho_t)}$. where P_t is the bond price, ρ_t is the yield, and M is a constant equal to 100 times the bond's Macaulay duration. Following Ramsey and Rothman [15], I take log differences of the bond yield series before computing the test statistic to ensure that the series is stationary. To be consistent, I use log differences of model yields as well.

To test for time-reversibility, Ramsey and Rothman [15] use a measure based on bicovariance: $TR = E[\rho_{t-k}^2 \rho_t] - E[\rho_{t-k} \rho_t^2]$. In a time-reversible series, moving from ρ_{t-k} to ρ_t has the same transition probability as moving backwards in time from ρ_t to ρ_{t-k}. Therefore, it doesn't matter which element is squared. The two expectations $E[\rho_{t-k}^2 \rho_t]$ and $E[\rho_{t-k} \rho_t^2]$ should be equal. Hinich (1982) developed a bicoherence statistic that uses weighted bicovariances at all lag lengths k, which has higher power.[4] I modify Hinich's statistic to test the joint hypothesis that all 13 countries' bond yields are time-reversible. (See appendix B.)

The test rejects the null hypothesis of time reversibility with greater than 99% confidence. (See table 1.) Looking at each country series individually, time-reversibility can be rejected with 95% confidence in every country.

Skewness

The higher moments of the distribution of lending rate changes are intertwined with time-reversibility. Theory predicts that bond yield changes should have larger positive outliers than negative outliers. This corresponds to positive skewness in the distribution of yield changes. Figure 6 shows a histogram of percentage changes in the monthly implied yields.[5] The fat tail on the right side, which indicates positive skewness in the yields, (skewness = 2.90) is clearly discernable. Looking at the country series separately, 12 of 13 have positive skewness. Bootstrap analysis reveals that the probability of negative skewness is 4.1% and the standard deviation of the skewness statistic is 1.59.

A potential data problem is the presence of one extreme outlier: the 1998 financial crisis in Russia. While the Russia crisis captures the type of sudden state change the model describes, it is worrisome if the results depend on a single event. However, when the Russia observation is removed, the skewness remains positive at 1.41. Furthermore, daily data, which has twenty times as many observations and no obvious outliers, also has positive skewness, significant at the 5% level. Lastly, changes in lending rates for a panel of 71 countries between 1977-99

[4]This assertion is based on my own Monte Carlo studies of asymmetry test statistics on stochastic processes with various forms of time-irreversibility.

[5]To convert the bond price data into implied yields, an estimate of the bonds' Macaulay duration is needed. It is inferred from the data by comparing the variances of the percentage yield changes in the data and the model and scaling the data by the square root of their ratio. Since skewness is invariant in a constant multiplier, positive skewness in yield changes corresponds to negative skewness in bond price changes, of equal magnitude.

have a skewness of 1.74.[6]

4.2 Calibration

Choosing parameter values in such a stylized model is not straightforward. The previous assumption that $\theta_g = 1 - \theta_b$ is no longer in force. To estimate a venture's conditional probability of success, yearly default rates on U.S. speculative grade bonds are split into two subsamples: recession years and boom years.[7] The reason for selecting U.S speculative grade bonds is that default rate data for emerging markets is not available. Since emerging market bonds are likely to be riskier than U.S. corporate bonds, the riskiest grade of U.S. bonds provides the closest match. The default rate in recession years was 5%, whereas the default rate in non-recession years was 3%. Therefore, $\theta_b = 0.95$ and $\theta_g = 0.97$ for the benchmark model.

For the probability of a state transition, λ, world GDP data guides the estimate. For each of the country years listed in the Penn World Tables, boom (bust) labels are assigned to country years with positive (negative) growth of real GDP per capita. An average of 36.5 months between state changes implies a 0.027 monthly probability of a transition.[8]

N, is the largest potential number of independent observable signals. Interpreting N as a number of firms or investments would overstate the number of signals because highly correlated outcomes reduce signals' information content. If every U.S. firm produced an independent signal, forecast errors for macroeconomic aggregates would be negligible. Rather, N measures information. If N is high, agents' beliefs can change quickly. To measure this speed, one can look at the speed of price adjustments: the number of periods for the price series to cross from its top quintile to its bottom quintile, and vice-versa. $N = 25$ is the integer that makes the average of all these upward and downward crossing times closest in the data and in the simulated model (13 periods).[9] An alternative way to calibrate N

[6] Data is from International Financial Statistics (3791 observations). Skewness is computed for the log changes of average lending rates.

[7] Default rate data is from Moody's. (www.moodysqra.com/research/mdr.asp) Years of business cycle downturns are defined as years that have at least six months lying after the peak and before the next trough of a business cycle. Peak and trough dates are the NBER business cycle dates. Data spans 1970-1999.

[8] Restricting the data to emerging markets or using NBER recession dates to calculate the transition probability for the U.S. yields similar results.

[9] This is a simulated method of moments exercise with one parameter and the moment being the expected crossing time.

would be to estimate how many independent signals would generate the same amount of information as all emerging market bond yields.[10] For the 13 countries in the bond data, 95% of the variation in the data is explained by 2principal components; 99% of the variation is explained by the first 4 components. These results support the idea that the number of independent signals observed in these markets is actually quite small.

The entrepreneur's payoffs v_i have no data counterpart. They will be distributed normally with mean in the center of the interval $[\underline{v}, \bar{v}]$ where beliefs can affect investment choices. The variance is such that payoffs are in that interval with 95% probability. The remaining parameters ($r = 0.42\%, w = 1$) only affect the scale of the lending rate; skewness is invariant in scale.

4.3 Simulation Results

A calibrated endogenous information model can come close to the asymmetry in the data. However, a model with constant information flow and asymmetric signals does not generate sufficient asymmetry to reject time-reversibility or to produce skewness of the magnitude in the data.

Time-Reversibility

The time-reversibility test statistic is sensitive to the amount of data being tested. Therefore, simulated data series are 1053 observations long, to match the length of the data. As predicted, the endogenous information model and the data exhibit significant time-irreversibility. The constant learning model, even with asymmetric signal quality, does not (table 1).

The difference between ups and downs that time-irreversibility measures is observable in the time series of lending rates [figure 3]. Increases in lending rates (crashes) are steeply sloped. Equal size decreases in lending rates (booms) are more gradual.

Skewness

The model's analog to the data's bond yield skewness is the skewness of the percentage changes in lending rates, $(\ln(\rho_t) - \ln(\rho_{t-1}))$. Ten thousand repeated simulations, each with

[10]Thanks to Robert Lucas for suggesting this idea.

Series	χ^2 stat	Deg. Freedom	Prob(Time-Reversible)
Endogenous Information	122*	36	0.00
Constant Information	30	36	0.77
Emerging Mkt. Bond Yields	3876*	468	0.00

Table 1: Time-reversibility (bicoherence) statistics are chi-squared under the null hypothesis of time-reversibility. $\star$ indicates rejection of time-reversibility at 95% level.

10,000 periods, produces an average skewness estimate of 2.35, with a Monte Carlo standard error of 0.06. This skewness value lies well within the 95% confidence interval of the data.

Figure 4 shows a histogram of the log changes in lending rates. The positive skewness is clearly visible as a large right tail. One strange feature of the histogram is the truncation near -0.4. This is a result of successes being likely in both states. Having all ventures succeed is common and reduces lending rates by a maximum of 0.4%. Observing many venture failures is rare and causes lending rates to react more. Figure 5 is a histogram of percentage changes in interest rates in a simulation with 12 states.[11] Having a richer state space alleviates the truncation problem.

A constant information version of the model where N signals are observed every period produces a skewness of 0.93. The reason this skewness is not zero is because bad signals are more informative than good ones. However, asymmetric signal quality alone is insufficient to generate the quantity of asymmetry in the data.

A related question is whether asymmetric transition probabilities could account for skewness. Using Hamilton's [11] estimates of business cycle transition probabilities, and symmetric signals[12] skewness for the constant learning model is 0.02. With asymmetric signals and transitions, skewness rises to 1.17. Transition probabilities generate so little skewness because they drive beliefs slowly toward 1/2. This can be seen in the way λ enters the updating

[11]The states used to generate this graphs had the following probabilities of success: $\theta_g \ \epsilon \ \{.8, .9, .95, .955., .96, .965, .97, .975, .98, .985, .99, .995\}$. Estimating conditional probabilities for so many states is not possible, given the small size of the data set on default rates (30 observations and 12 parameters). Therefore, parameters were chosen to be close to the observed success rates and on the basis of their ability to replicate the data.

[12]Transition probabilities adapted to monthly data are $P[\omega_g|\omega_b] = 0.09$ and $P[\omega_b|\omega_g] = 0.03$. Symmetric signals are $\theta_g = 1 - \theta_b = 0.53$.

equations [4] and [5]. Any reasonable transition probability is incapable of generating the extreme changes that skewness measures weight most heavily.

Variance and Kurtosis

Examining conditional variance of the model reveals a surprising feature: The variance of lending rate is smaller in good states, when rates are low. This finding is consistent with observations from the stock market showing that volatility is higher when prices are lower (see e.g. [10], [3]). The reason for the lower variance is that more information makes beliefs more accurate and less variable.

The model also generates excess kurtosis, a well-known property of financial asset returns [14]. With two states, kurtosis (K=10) is much lower than that in the data (K=41), because of the two-state truncation problem. However, with 12 states, kurtosis (K=30) is within the 95% confidence interval of the data.

4.4 Sensitivity Analysis and Comparative Statics

The sensitivity analysis varies ones baseline parameter at a time, and measures skewness of the simulated lending rate changes. For a wide range of parameter values, the model consistently produces significant positive skewness in interest rate changes. High and low values of the parameters are chosen to be as extreme as possible without making learning trivial or impossible. Skewness-neutral parameters r, μ_0, and w, remain at benchmark values.

Table 2 highlights the model's comparative statics. When the state changes frequently (λ high), beliefs revert faster to their unconditional mean. Because mean reversion is symmetric, stronger mean reversion equalizes learning between high and low investment markets, reducing asymmetry. Likewise, when signals are very noisy, symmetric state changes have a larger relative effect on beliefs, reducing asymmetry.

The high number of borrowers is chosen to be $4,000$ because when $N > 4,000$ posterior beliefs become numerically indistinguishable from zero or one.[13] Varying N illustrates two

[13]The model can accommodate larger N if signals become more noisy. For any N , there is a pair of conditional probabilities θ_g, θ_b such that learning is not too "too fast." To see the basis for this argument, examine equation [4], multiply the number of observed successes s and the number of funded ventures n by a factor γ. Than take the limit $lim_{\gamma\to\infty}\mu_t^P$. Realize that this converges to either 0 or 1, unless $\theta_g \to \theta_b$. If the θ's converge at just the right speed, it is possible for μ_t^P to be stable.

model variation	skewness
Calibrated model	2.35
Constant Information model	0.93
Thick market $N = 4,000$	4.43
Thin market $N = 2$	4.97
Volatile state $\lambda = 0.2$	1.57
Stable state $\lambda = 0.001$	4.86
Noisy signals $\theta_g = .51$, $\theta_b = .49$	0.29
Clear signals $\theta_g = .99$, $\theta_b = .5$	3.67
Uniformly distributed investment payoffs $\{v_i\}$	1.60
Emerging Market Bond Yields	2.90

Table 2: Sample skewnesses of simulated lending rate changes and yield changes of emerging market bonds.

competing effects: First, more agents generating more signals makes beliefs rise faster from a low level. This dampens skewness. Second, the difference between the number of signals observed in good and bad times widens. Increasing information asymmetry boosts skewness.

Changing the distribution of investment payoffs, v, from normal to uniform increases the density of payoffs close to $\bar{v}$. As beliefs begin to rise from near zero, entrepreneurs start to borrow at a faster rate. Because the mapping from μ to n is steeper when μ is low, escaping from a low information trap is faster; booms are more sudden, and asymmetry is less.

4.5 High and Low Risk Markets

Developed-country lending markets do not exhibit as much asymmetry in their interest rate movements as riskier emerging markets. This section explains why endogenous information produces more asymmetry in riskier markets, and tests the prediction within a country using a panel of U.S. lending data.

This exercise also helps to differentiate an endogenous information theory from an exogenous asymmetric signal quality theory. In very risky markets where bad outcomes are common, bad signals will have the least information content. In low-risk markets where bad signals are rare, the observation of one bad signal carries a large information content. If the source of asymmetry is more informative bad signals, then low-risk markets, where bad signals are the most informative, should have the most asymmetry. This cross-sectional analysis shows that the opposite is true.

Predictions of the Model

The theory presented so far is primarily a story about high-risk markets, meaning markets where a large fraction firms will fail in bad times. If most firms in a market are not at risk and will continue to operate in good and bad times, then only a small amount of extra information will be observed in booms. To model a low-risk market, the model will be altered to include a number of actors who receive publicly observable signals every period, no matter what their beliefs are. This constant flow of information reduces asymmetry by making the number of signals and the ability to learn more similar in good and bad states.

Another way of interpreting this setup is that there are a number of entrepreneurs in this economy who have venture payoffs, v_i's, that lie above the interval where actions depend on beliefs. Thus, they always borrow, generating more signals every period. [14] The setup of the model is the same with one exception: If n_t is the number of entrepreneurs who borrow at t, the number of signals observed is $n_t + I$. I is the number of riskless firms.

The results in Table 3 suggest that lower risk markets are less likely to exhibit the positive skewness in interest rates that this model predicts. The benchmark model ($I = 0$) is a high-risk market. 100% of firms are at risk. When $I = 25$, the market is medium-risk: 50% of firms are at risk. Finally, $I = 250$ is a low-risk market where less than 10% of firms are at risk. This is the type of market in which large, successful U.S. companies operate. Skewness in this type of market is very low.

What is important about adding more signals each period is not the increased amount of information, but the convergence in the quantity of information available across states.

[14] The effect of allowing v to fall below the interval would only be to reduce the number of active borrowers, N. Since agents with v_i's below $\underline{v}$ never invest and never generate information, they are irrelevant.

	Model		*Data*			
	Permanent Signals	Skewness	Loan Size	Risk Premium	Risk Rating	Skewness
High Risk	$I = 0$	2.35	Less than $ 100,000	4.21	3.4	**1.12** (0.28)
Medium Risk	$I = 25$	1.13	$100,000 - $1 million	3.36	3.1	**0.48** (0.24)
Low Risk	$I = 250$	0.58	Greater than $1 million	1.50	2.6	**-0.23** (0.22)

Table 3: Model: Simulation results (50,000 periods) for endogenous information model with a varying number of permanent signals. Data: Descriptive statistics for quarterly U.S. lending rate spreads (risk premia) over the federal funds rate. Bootstrap standard errors for skewness in parentheses.

If there are more signals observable each period, no matter what beliefs are, the ability to learn in the bad state catches up to learning in the good state. As information becomes more symmetric, skewness in interest rates falls.

Empirical Results From High and Low Risk Markets

This section uses U.S. commercial and industrial loan rates for small, medium and large loans to test if high risk markets exhibit more asymmetry than low risk markets.[15] The reason for being interested in loan size is that it is related to risk. Large, stable companies are more likely to qualify for large loans of over $1 million. Riskier firms take out smaller loans of less than $100,000. Loan risk ratings and risk premia confirm this relationship.

The fact that the skewness rises with loan risk supports the endogenous information explanation for asymmetry. Another interesting feature of these results is that the magnitude of the skewness in the small loan data approximately matches the medium risk model (1.12 and 1.13) and the medium loan data skewness approximately matches the high risk model (0.48 and 0.58). These results provide some insight as to how much information is available about U.S. lending markets.

[15] The data are from the Federal Reserve's survey of terms of business lending. The risk rating scale ranges from 1 for minimal risk loans, to 5 for special mention (high risk) loans. Risk ratings are weighted by the loan amount and averaged across loans. A complete description of risk categories is available from the Banking Analysis section of the Federal Reserve Board.

5 Conclusion

What is crucial for endogenous information asymmetry is that rate of information flow is correlated with market optimism. In this model, more investments are undertaken in hot markets. A larger number of investments generates a larger quantity of signals and more information. Another mechanism would be better signal quality when investment is high. Or perhaps the economy produces the same number and same quality of signals all the time, but more of those signals are transmitted in the form of news items when the market is hot.

One avenue for future research is to model market incentives for information transmission that would generate faster information flows about more profitable firms. The following example illustrates the idea. An investor in a small firm has difficulty obtaining firm information. News about a firm's success trickles through the market. A lack of media coverage leads to slow information dissemination and a slow boom. Five years later, the firm is a market leader. Any new produce release or management change is broadcast in real time on MSNBC. The cost of borrowing is much lower because the firm has become less risky. If profits soared, the news would be learned quickly. However, because the lending rate was already close to the risk-free rate, it wouldn't fall by much. If something bad happened to this firm, its cost of future borrowing would rise within minutes – a sudden crash.

One of the drawbacks of the model presented here is that its simplicity makes it difficult to directly match the model to data. Van Nieuwerburgh and Veldkamp [17] show that learning asymmetry emerges in a dynamic general equilibrium model, with idiosyncratic production shocks and agents who learn about the technology level. The source of the information asymmetry is a higher signal to noise ratio when production is high. Substituting a variable signal quality for the varying number of signals used in this paper makes more precise calibration possible.

The ideas in this paper can also be applied to demand for consumption goods. For example, consumers learn about the risk of side effects of a new drug (such as Fen Phen) by hearing about the experiences of numerous other consumers who have tried the drug. The more popular the drug, the faster consumers will learn if there are problems. A variation of this model with random unobserved changes in quality could predict slow growth and big crashes in demand for new goods. The same logic could explain asymmetric venture capital in a market of uncertain profit potential, or the asymmetric firm entry and exit in an industry.

Finally, the assumption of public signals can be relaxed to apply the idea to settings where signals are more likely to be private. If actions partially reveal private signals, as in herding models, then more private signals would generate more informative actions and more information in booms. As long as the flow of information is connected to beliefs about the state, and signals can be transmitted, however imperfectly, to market participants, then learning and asset price movements should be unconditionally asymmetric.

References

[1] F. Allen, S. Morris, H.S. Shin, Beauty Contests, Bubbles and Iterated Expectations in Asset Markets, Cowles Foundation working paper, 2003.

[2] A. Banerjee, A Simple Model of Herd Behavior, *Quart. J. Econ.*, 107:3 (1992), 797-817.

[3] G. Bekaert, G. Wu, Asymmetric Volatility and Risk in Equity Markets, *Rev. Finan. Stud.*, 13:1 (2000), 1-42.

[4] S. Bikhchandani, D. Hirshleifer, I. Welch, A Theory of Fads, Fashion, Custom, and Cultural Change as Information Cascades, *J. Polit. Economy*, 100 (1992), 992-1026.

[5] M. Boldrin, D. Levine, Growth Cycles and Market Crashes, *J. Econ. Theory*, 96 (2001), 13-39.

[6] J. Campbell, L. Hentschel, No News is Good News: An Asymmetric Model of Changing Volatility in Asset Returns, *J. Finan. Econ.*, 31 (1992), 281-318.

[7] A. Caplin, J. Leahy, Business as Usual, Market Crashes, and Wisdom After the Fact, *The Amer. Econ. Rev.*, 84:3 (1994), 548-565.

[8] M. Chalkley, I.H. Lee, Learning and Asymmetric Business Cycles, *Rev. Econ. Dynam.*, 1 (1998), 623-45.

[9] C. Chamley, D. Gale, Information Revelation and Strategic Delay in a Model of Investment, *Econometrica*, 62:5 (1994), 1065-1085.

[10] J. Cox, S. Ross, The Valuation of Options for Alternative Stochastic Processes, *J. Finan. Econ.*, 3:1-2 (1976), 145-66.

[11] J. Hamilton, A New Approach to the Econometric Analysis of Nonstationary Times Series and the Business Cycle, *Econometrica*, 57 (1989), 357-84.

[12] M. Hinich, Testing for Gaussianity and Linearity of a Stationary Time Series, *Journal of Time Series Analysis*, 3 (1982), 169-76.

[13] C. Jacklin, A. Kleidon, P. Pfleiderer, Underestimation of Portfolio Insurance and the Crash of October 1987, *Rev. Finan. Stud.*, 5:1 (1992), 35-63.

[14] A. Pagan, The Econometrics of Financial Markets, *J. Empirical Finance*, 3:1 (1996) 15-102.

[15] J. Ramsey, P. Rothman, Time Irreversibility and Business Cycle Asymmetry, *J. Money, Credit, Banking*, 28:1 (1996), 1-21.

[16] N. Stokey, R. Lucas, "Recursive Methods in Economic Dynamics," Harvard University Press, Cambridge, MA, 1989.

[17] S. Van Nieuwerburgh, L. Veldkamp, Learning Asymmetries in Real Business Cycles, NYU Stern Working Paper, 2003.

[18] P. Veronesi, Stock Market Overreactions to Bad News in Good Times: A Rational Expectations Equilibrium Model, *Rev. Finan. Stud.*, 12:5 (1999), 975-1007.

[19] I. Welch, Sequential Sales, Learning and Cascades, *J. Finance*, 47:2 (1992), 695-732.

[20] J. Zeira, Informational Cycles, *Rev. Econ. Stud.*, 61 (1994), 31-44.

[21] J. Zeira, Informational Overshooting, Booms and Crashes, *J. Monet. Econ.*, 43 (1999), 237-257.

A Appendix

Throughout the proofs, let $\bar{\mu}$ denote beliefs in an environment where the number of signals observed each period, n, is constant. Furthermore, let $\theta \equiv \theta_g = 1 - \theta_b$.

A.1 Proposition 1: Time-Reversibility

Step 1: In a constant learning economy with equally informative signals, the stochastic process for beliefs in the good state is the time-reverse of the process for beliefs in the bad state, $P[\bar{\mu}_{G,t+1} = x | \bar{\mu}_{G,t} = y] = P[\bar{\mu}_{B,t} = x | \bar{\mu}_{B,t+1} = y] \quad \forall x, y \in [0, 1]$.

Consider belief changes from y to x that are realized with positive probability in the G economy. Each must have an associated number of observed success signals z that produced this posterior belief. The relationship between x, y, and z is given by Bayes' Law:

$$y = \frac{\theta^z (1-\theta)^{n-z} x}{\theta^z (1-\theta)^{n-z} x + (1-\theta)^z \theta^{n-z} (1-x)} \tag{12}$$

Likewise, for any change from x to y in the B economy, there must be a number of observed venture successes, v, that produces the posterior belief x.

$$x = \frac{\theta^v (1-\theta)^{n-v} y}{\theta^v (1-\theta)^{n-v} y + (1-\theta)^v \theta^{n-v} (1-y)} \tag{13}$$

Suppose that z solves equation (12) for a given x, y, then $v = n - z$ solves equation (13). To verify this, substitute $n - z$ for v in equation (13) and substitute equation (13) in to (12) and show that they reduce to an identity.

$$x = \frac{\theta^{n-z} (1-\theta)^z y}{\theta^{n-z} (1-\theta)^z y + (1-\theta)^{n-z} \theta^z (1-y)}. \tag{14}$$

$$y = \frac{\frac{(1-\theta)^n \theta^n y}{\theta^{n-z}(1-\theta)^z y + (1-\theta)^{n-z}\theta^z (1-y)}}{\frac{(1-\theta)^n \theta^n y}{\theta^{n-z}(1-\theta)^z y + (1-\theta)^{n-z}\theta^z (1-y)} + \frac{(1-\theta)^n \theta^n y}{\theta^{n-z}(1-\theta)^z y + (1-\theta)^{n-z}\theta^z (1-y)}} \tag{15}$$

Cancelling terms results in

$$y = \frac{y}{y + (1-y)} = y \tag{16}$$

If C_z^n is the number of combinations of z elements from a set of size n, the formula for the transition probabilities is then

$$P[\bar{\mu}_{G,t+1} = y | \bar{\mu}_{G,t} = x] = \begin{cases} C_z^n \theta^z (1-\theta)^{n-z} & \text{if } \exists\, z \in \{0, 1, \ldots, n\} \text{ s.t. equation 12 holds} \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

$$P[\bar{\mu}_{B,t+1} = x | \bar{\mu}_{B,t} = y] = \begin{cases} C_v^{n-v} \theta^v (1-\theta)^v & \text{if } \exists\, v \in \{0, 1, \ldots, n\} \text{ s.t. equation 13 holds.} \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

When, $z = n - v$, these two probabilities will be equal. This proves the proposition for the case where $P[\bar{\mu}_{U,t+1} = y|\bar{\mu}_{U,t} = x] > 0$.

If $P[\bar{\mu}_{U,t+1} = y|\bar{\mu}_{U,t} = x] = 0$, then $P[\bar{\mu}_{B,t} = y|\bar{\mu}_{B,t+1} = x] = 0$. If not, then some $z \neq n - v$ would solve (12) and (13), which contradicts the previous result.

Step 2: Suppose a state transition from ω_b to ω_g occurs at time t, and the beliefs at that date are $\mu_t = x$. Show that the following two conditional probabilities are equal for all s:

$$P[\mu_{t+s} = y, \mu_{t+s+1} = z|\omega_t = \omega_g, \mu_t = x] = P[\mu_{t-1-s} = y, \mu_{t-2-s} = z|\omega_{t-1} = \omega_b, \mu_t = x] \tag{19}$$

From lemma 2, we know that

$$P[\mu_{t+1} = y|\omega_t = \omega_g, \mu_t = x] = P[\mu_{t-1} = y|\omega_{t-1} = \omega_b, \mu_t = x] \tag{20}$$

holds for all n in the n-signal world. This means that it also holds for all s in any n-signal world without state changes ($\lambda = 0$, except at the one point where we assume that the state changes) in the following sense

$$P[\mu_{t+s} = y|\omega_t = \omega_g, \mu_t = x] = P[\mu_{t-1-s} = y|\omega_{t-1} = \omega_b, \mu_t = x]. \tag{21}$$

The reason this is true is that an s-period transition in an n-signal world identical to a 1-period transition in an (ns)-signal world when states don't change.

Now, consider allowing for state changes. Let Λ be the Markov transition matrix for the state $\Lambda = \begin{bmatrix} 1-\lambda & \lambda \\ \lambda & 1-\lambda \end{bmatrix}$. The probability $P[\omega_{t+s} = \omega_g] = \Lambda^s[1,1]$ and $P[\omega_{t-1-s} = \omega_b = \Lambda^s[2,2]$. Since Λ is symmetric, Λ^s is symmetric and $\Lambda^s[1,1] = \Lambda^s[2,2]$. Therefore, the probability of being in state ω_g at $t+s$ is the same as being in ω_b at $t-1-s$. Since the state probabilities are symmetric at each date, the probability of observing $\tilde{n}$ bad signals at date $t-1-s$ is still the same as the probability of observing $\tilde{n}$ good signals at date $t+s$. Therefore, the original proof of proposition 2 follows with relabelled probabilities to adjust for state changes, and equation 21 holds when $\lambda > 0$.

The fact that the probability of observing $\tilde{n}$ good signals at $t+s$ is the same as $\tilde{n}$ bad signals at $t-1-s$ also means that

$$P[\mu_{t+s+1} = z|\mu_{t+s} = y, \omega_t = \omega_g] = P[\mu_{t-2-s} = z|\mu_{t-1-s} = y, \omega_{t-1} = \omega_b]. \tag{22}$$

Multiplying (21) and (22) yields (19).

Step 3: Let ψ^s be the s-period transition kernel for beliefs. Show that ψ satisfies the conditions for weak convergence, meaning that

$$\lim_{s\to\infty} \psi^s(y|x) = \psi^\star(y) \ \ \forall\, x, y \,\epsilon\, [0,1]. \tag{23}$$

By Stokey and Lucas' (1989) Theorem 12.12, ψ has the weak convergence property if

1. ψ has the Feller Property
2. ψ is monotone
3. There $\exists\, c \,\epsilon\, [0,1]$ and an $s \geq 1$ s.t. $P[\mu_s \epsilon [c,1] | \mu_0 = 0] \geq \varepsilon$ and $P[\mu_s \epsilon [0,c] | \mu_0 = 1] \geq \varepsilon$.

Step 3.1: Show that ψ has the Feller Property.

Let $M(\mu, s)$ be the updated beliefs, given prior beliefs μ and s success signals out of n funded ventures. Define an operator T on any function f as

$$(Tf)(\mu) \equiv \int_0^1 f(\mu')\psi(\mu'|\mu)d\mu' \;\; \forall \mu \tag{24}$$

Then, ψ has the Feller Property if for any bounded, continuous function f, Tf is continuous in μ.

Since $\psi(\mu'|\mu)$ only takes a non-zero value where there is an $s \,\epsilon\, \{0, 1, \ldots, n\}$ such that $M(\mu, s) = \mu'$,

$$(Tf)(\mu) = \sum_{s=0}^{n} f(M(\mu, s)) * P(s) \tag{25}$$

By inspection of equation 4, we know that $M(\mu, s)$ is continuous for all $\mu \epsilon [0, 1]$ and all z. Since a composite function of two continuous functions is continuous, as is a linear combination of continuous functions, Tf is continuous in μ.

Step 3.2: Show that ψ is monotone.

ψ is monotone if for any bounded, continuous function f, Tf is also increasing. Realize that if $M(\mu, s)$ is increasing, then since $P(s) \geq 0$ for all z, then Tf will also be increasing.

To see that $M(\mu, s)$ is increasing in μ, examine equation 4, which gives the formula for the posterior belief, before adjusting for state changes. Realize that the posterior μ_t^P is increasing in μ_t iff $1/\mu_t^P$ is decreasing in μ_t.

$$\frac{1}{\mu_t^P} = 1 + \frac{C_{n-s}^n (1-\theta)^s \theta^{n-s} (1-\mu)}{C_s^n (1-\theta)^{n-s} \theta^s \mu} \tag{26}$$

Since all these terms are positive, the numerator is decreasing in μ, while the denominator is increasing in μ, which means that μ_t^P is increasing in μ_t. To get μ_{t+1}, use the formula

$$M(\mu, s) = \mu_{t+1} = \mu_t^P (1-\lambda) + (1-\mu_t^P)\lambda. \tag{27}$$

Because $\lambda < 1 - \lambda$, μ_{t+1} is increasing in μ_t^P and μ_t. Therefore, the monotonicity condition holds.

Step 3.3: Show that condition 3 holds.

Let $c = 1/2$ and $\varepsilon = (1-\theta)^{ns}$.

Start by showing that $\exists$ an s such that

$$P[\mu_s \epsilon [1/2, 1] | \mu_0 = 0] \geq (1-\theta)^{ns} \tag{28}$$

No matter what the unobserved state ω is, the probability of observing a good signal is always $\geq (1-\theta)$. So, the probability of observing ns good signals is always $\geq (1-\theta)^{ns}$. Now, it just remains to be shown that there is an s such that after ns consecutive good signals, $\mu_s \geq 1/2$.

If only good signals are observed, it is easy to verify that μ_s is a submartingale and that as $s \to \infty$, it converges to a limit $\mu^\star > 1/2$. By definition of a convergent sequence, for every η, there must be an s such that

$$|\mu^\star - \mu_S| < \eta \ \ \forall \, S \geq S \tag{29}$$

If $\eta = \mu^\star - 1/2$, then $\mu_S > 1/2 \ \forall \, S \geq s$. Therefore $\mu_s \geq 1/2$ and the condition holds.

Since all three conditions hold, the transition kernel for beliefs ψ converges to a unique unconditional distribution of beliefs as $s \to \infty$.

Step 4: Show that $P[\mu_{T+1} = z, \mu_T = y] = P[\mu_{S+1} = y, \mu_S = z]$ for all T, S.

The unobserved state process ω converges to a unique unconditional distribution $\lim_{s\to\infty} \Lambda^s = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$. So, the joint Markov process $[\mu, \omega]$ converges to a unique unconditional distribution. Taking the limit of both sides of equation 19 as $s \to \infty$ and applying weak convergence, produces the equality in unconditional probabilities.

Step 5: Show that $P[p_{T+1} = y, p_T = x] = P[p_{T+1} = x, p_T = y]$ for all T, and any function p.

For any p_{T+1}, p_T, there will be sets of beliefs $\mathcal{M}_{T+1}, \mathcal{M}_T$ of beliefs that correspond to the prices. Since all transitions between these belief sets are time-reversible, the price is as well.

$$\begin{aligned} P[p_{T+1} = y, p_T = x] &= \textstyle\sum_{m_T \, \epsilon \mathcal{M}_T} \sum_{m_{T+1} \, \epsilon \mathcal{M}_{T+1}} P[\mu_{T+1} = m_{T+1}, \mu_T = m_T] \\ &= \textstyle\sum_{m_T \, \epsilon \mathcal{M}_T} \sum_{m_{T+1} \, \epsilon \mathcal{M}_{T+1}} P[\mu_{T+1} = m_T, \mu_T = m_{T+1}] \\ &= P[p_{T+1} = x, p_T = y] \end{aligned} \tag{30}$$

A.2 Proposition 2: Symmetry of Constant Learning Model

In a constant learning economy with equally informative signals, an asset whose price is any function of beliefs $p(\cdot) : [0,1] \to \Re$, has unconditional probabililty of price changes $g(\cdot)$, such that $g(\Delta p) = g(-\Delta p)$, for all p.

The conditional of interest rate changes is related to the distribution of interest rate levels in the following manner:

$$g(\Delta p) = \int_0^1 P[p_{T+1} = x + \Delta p, p_T = x] dx. \tag{31}$$

Using proposition 1,

$$= \int_0^1 P[p_{T+1} = x, p_T = x + \Delta p]dx \tag{32}$$

$$= g(-\Delta p). \tag{33}$$

A.3 Corollary: In a constant learning economy with equally informative signals, if the unconditional probability of a change in the lending rate is $g(\cdot)$, then $g(\Delta\rho_t) = g(-\Delta\rho_t)$, for all $\Delta\rho_t$.

Equation 7 shows that the lending rate ρ is a function of beliefs. By proposition 2, it follows that lending rate changes are symmetric.

A.4 Proof of Proposition 3: Time Irreversibility

Let $\bar{\psi}(x|y,g,n)$ be the probability of the transition from y to x when n signals are observed.
Suppose that the proposition is false: $\bar{\psi}(x|y,g,\hat{n}) = \bar{\psi}(x|y,g,\tilde{n})$ for some $\tilde{n} \neq \hat{n}$ and $\bar{\psi}(x|y,g,\hat{n}) > 0$.

If $\bar{\psi}(x|y,g,\hat{n}) > 0$, there must be a number of successes observed $z \in \{0,1,\ldots,\hat{n}\}$, such that updating beliefs y with z successes produces next period beliefs x. For the reverse transition to be possible, there must be a number of successes observed $u \in \{0,1,\ldots,\tilde{n}\}$, such that updating beliefs y with u successes produces next period beliefs x. Both processes will transition between x and y if the number of successes satisfies both

$$x = \frac{\theta^z(1-\theta)^{\hat{n}-z}y}{\theta^z(1-\theta)^{\hat{n}-z}y + (1-\theta)^z\theta^{\hat{n}-z}(1-y)} \tag{34}$$

and

$$x = \frac{\theta^u(1-\theta)^{\tilde{n}-u}y}{\theta^u(1-\theta)^{\tilde{n}-u}y + (1-\theta)^u\theta^{\tilde{n}-u}(1-y)}. \tag{35}$$

Equating the right sides of (34) and (35) and simplifying yields $(\frac{\theta}{1-\theta})^{\hat{n}-2z} = (\frac{\theta}{1-\theta})^{\tilde{n}-2u}$. For any $\theta \in (0,1)$, this can hold only if $\hat{n} - 2z = \tilde{n} - 2u$. If $\tilde{n}$ and $\hat{n}$ are given by (11) and z is determined by the transition x and y under consideration, then the last degree of freedom u is used up in getting the reverse transition to match x and y. However, there is one more condition that needs to be satisfied.

For each transition to be equally probable, the number of observed successes z and u that satisfy (34) and (35) must be equally probable:

$$\frac{\hat{n}!}{(\hat{n}-z)!z!}\theta^z(1-\theta)^{\hat{n}-z} = \frac{\tilde{n}!}{(\tilde{n}-u)!u!}\theta^u(1-\theta)^{\tilde{n}-u}. \tag{36}$$

Substituting $u = 1/2(\tilde{n} - \hat{n}) + z$ in (36) makes it clear that this condition is not satisfied for an arbitrary choice of $\tilde{n}$, $\hat{n}$, and z.

$$\frac{\hat{n}!((\tilde{n}-\hat{n})/2+z)!((\tilde{n}+\hat{n})/2-z)!}{(\hat{n}-z)!z!\tilde{n}!} = (\theta(1-\theta))^{(\tilde{n}-\hat{n})/2}. \tag{37}$$

Since the reverse transition will not occur with equal probability for a generic choice of x and y when $\tilde{n} \neq \hat{n}$, the process is time-irreversible.

A.5 Proof of Proposition 4: Asymmetry of changes in lending rates

Let $\hat{p} = \max\{\Delta p : g(\Delta p) > 0\}$, $\underline{p} = \min\{\Delta p : g(\Delta p) > 0\}$. Prove: $\hat{p} > -\underline{p}$ when $p(\mu)$ is decreasing.

If $\underline{p}$ occurs with positive probability, then there must be a set of $n(\mu_t)$ signals that generates $p(\mu_{t+1}) = p(\mu_t) + \underline{p}$. Since this is the largest possible downwards movement in price, and price is decreasing in μ, the set of signals must be $n(\mu_t)$ venture successes.

From proposition 1, we know that if $\mu_t = x$ and a set of n_t observed successes generates $\mu_{t+1} = x + \delta$, then the opposite set of signals (n_t observed failures) and $\mu_t = x + \delta$ will generate $\mu_{t+1} = x$. Recall that n is monotone increasing in beliefs: $n(x + \delta) > n(x)$. (This statement requires a weak assumption on the v_i's. They must have a rich enough distribution so that some borrower changes her investment decision in the event of the largest possible change in lending rates.) Since μ_{t+1} is strictly decreasing in the number of failures observed at time t, $n(x+\delta)$ failure signals and $\mu_t = x+\delta$ will produce $\mu_{t+1} = x-\eta < x$ with positive probability.

For the largest price decrease, $p(x+\delta)-p(x) < 0$, there is a set of signals observed with positive probability that generate a price increase $p(x - \eta) - p(x + \delta) > p(x) - p(x + \delta)$, of larger absolute value.

When p is increasing in μ, the largest increase in p will be generated by n_t observed successes. μ_t and $n(\mu_t)$ will produce a $\mu_{t+1} > \mu_t$. There will then be $n(\mu_{t+1}) > n(\mu_t)$ possible failure signals observed with positive probability that will produce a larger price decline than the increase.

A.6 Proof of Corollary 2

Since the interest rate is a strictly decreasing function of beliefs, by Proposition 2, the largest increase occurring with positive probability will be larger in absolute value, than the largest decrease. Since the model is stationary, the unconditional mean of price changes is zero. Therefore, $g(\Delta\rho) \neq g(-\Delta\rho)$ implies asymmetry of the distribution function g.

B Appendix: Bicoherence Test

Bicoherence is defined with respect to two frequencies ω_1 and ω_2. Let N be the length of the data series.

$$B(\omega_1, \omega_2) = \frac{S_{xxx}(\omega_1, \omega_2)}{(S_{xx}(\omega_1 + \omega_2)S_{xx}(\omega_1)S_{xx}(\omega_2))^{1/2}} \tag{38}$$

where S_{xxx} is the bispectrum

$$S_{xxx}(\omega_1, \omega_2) = \sum_{k=-N-1}^{N-1} \sum_{l=-N-1}^{N-1} E\{x_n x_{n+k} x_{n+l}\} \exp\{-2\pi(\omega_1 k + \omega_2 l)\} \tag{39}$$

and S_{xx} is the power spectrum

$$S_{xx}(\omega) = \sum_{k=-N-1}^{N-1} E\{x_n x_{n+k}\} \exp\{-2\pi\omega k\}. \tag{40}$$

The bicoherence of a time-reversible process is zero, at any frequency. Hinich (1982) shows that, under the null of time-reversibility, $B(\omega_1, \omega_2)^2$ is distributed chi-squared with two degrees of freedom. He then sums the squared bicoherence statistics over all sets of non-redundant frequencies to arrive at his test statistic

$$\tilde{B} = \sum B(\omega_1, \omega_2)^2 \sim \chi^2_{2p} \tag{41}$$

where p is the number of sets of frequencies.

To test the joint hypothesis that all 13 countries' bond yields are time-reversible, one can sum the test statistics for each country. Since the bicoherence statistic is distributed chi-square, the sum is also chi-square, with degrees of freedom equal to the sum over all degrees of freedom for each country.

Summing test statistics only results in a chi-squared joint test statistic if the percentage changes in bond yields are independent across countries. However, correlation of emerging market bond movements is a serious concern. To address this concern, I estimate the distribution of the joint test statistic by fitting a VAR to the 13 country time-series. The VAR generates 10,000 samples of data with the same length and panel size as the bond data. Since ARMA processes are time-reversible, the test statistics are distributed as they would be under the null hypothesis. The joint test statistic for the data lies in the 99th percentile of this distribution. Even accounting for cross-market correlations, the bond yields are significantly time-irreversible.

C Appendix: Multi-Period Projects

This section relaxes the assumption that all ventures are single period-lived projects. It shows that interest rates are still positively skewed, holding all other parameters fixed.

The reason this is an important robustness check is that the relative size of booms and crashes in the data can be sensitive to the time interval between observations. Since the data the model is being compared to is monthly data, ventures in the model would have to succeed or fail in one month. Since this is obviously unrealistic, showing that frequent observations on long-lived projects produces similar results is important.

Suppose that all ventures undertaken at time t either succeed or fail at time $t+s$. The true but unknown probability of success of the venture depends on the state variable ω_{t+s}. Based on information known at the beginning of time t, the expected probability of success s periods later is:

$$\begin{bmatrix} \mu_{t,t+s} \\ 1-\mu_{t,t+s} \end{bmatrix} = \begin{bmatrix} 1-\lambda & \lambda \\ \lambda & 1-\lambda \end{bmatrix}^s \cdot \begin{bmatrix} \mu_t & 0 \\ 0 & 1-\mu_t \end{bmatrix} \tag{42}$$

This is a simple linear transformation that moves all the belief measures closer to 1/2. Since the upper and lower bounds for $\mu_{t,t+s}$ are closer to 1/2, the relevant interval for investment payoffs is a strict subset of $(\underline{v}, \bar{v})$. So, holding the v_i's fixed, lengthening the time to project completion effectively reduces the number of borrowers, N, and results in more borrowers outside the relevant interval. Although the effect of v_i's outside the relevant interval is to reduce skewness, the effect of reducing N is ambiguous. Furthermore, since skewness is normalized by standard deviation cubed, the fact that lending rates are less volatile in this setting will amplify the skewness measure. Theoretically, the overall effect is unknown. However, as the simulation results show, the positive skewness in interest rates increases, coming very close to the value observed in the data.

Model	Skewness
Time To Build	2.95

Table 4: Simulated lending rate changes (50,000 periods). Time to build is 24 periods (two years).

The same strategy would allow pricing of assets with streams of payoffs, where each period's payoff depends on that period's state. As the number of periods in the future became large, the effect of the next period's payoff on skewness would converge to zero, and the overall skewness measure should converge. In this way, the result of this model that price changes are asymmetric could be extended to cover assets with infinite dividend streams.

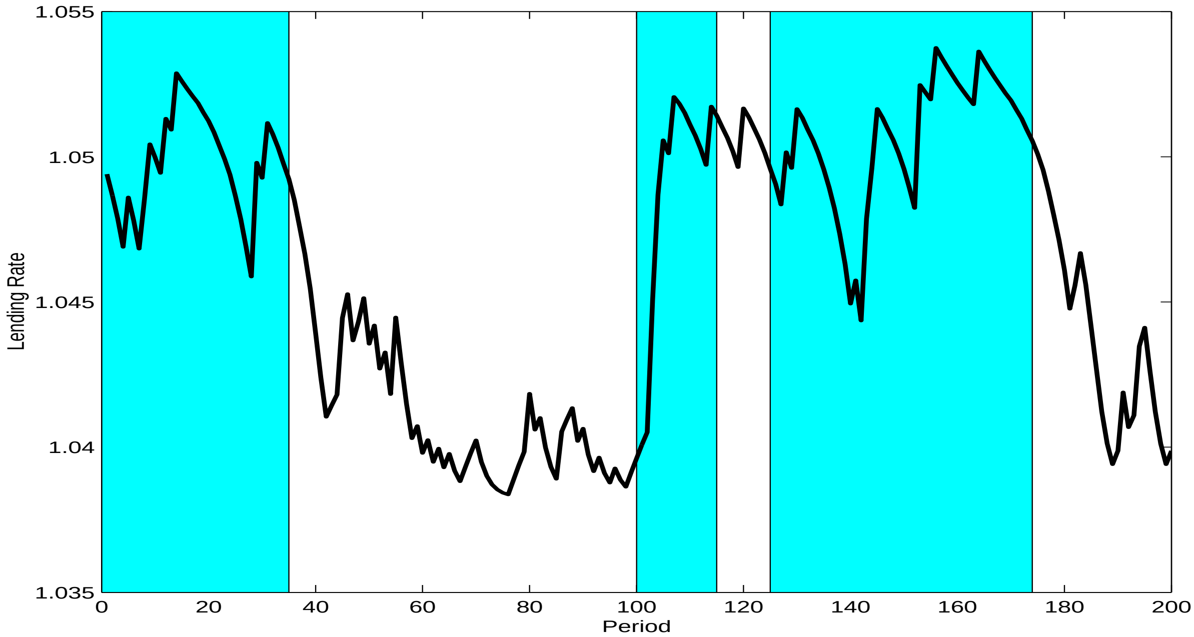

Figure 3: Time Series of Model Lending Rates. Shaded areas indicate times when the state is bad. Increases in lending rates are short and sharp. Decreases are more gradual.

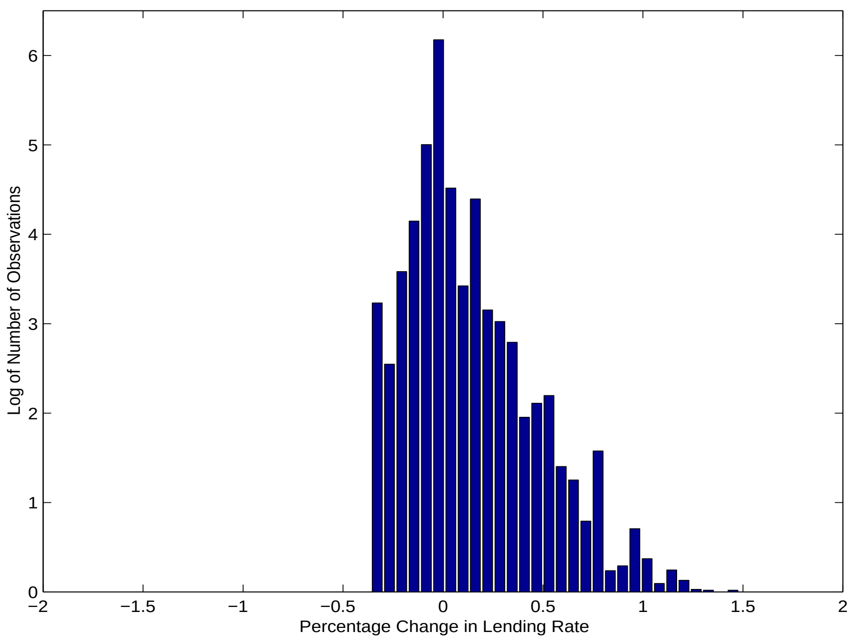

Figure 4: **Benchmark Model** - Histogram of Changes in Interest Rates
Skewness = 2.35 Kurtosis = 9.66

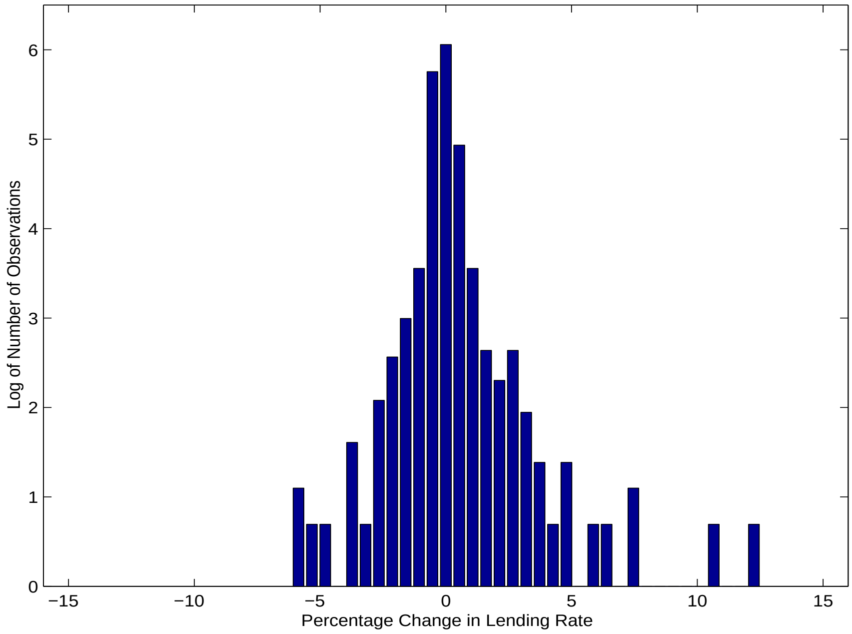

Figure 5: **Model with 12 States** - Histogram of Changes in Interest Rates.
Skewness = 2.35   Kurtosis = 29.8

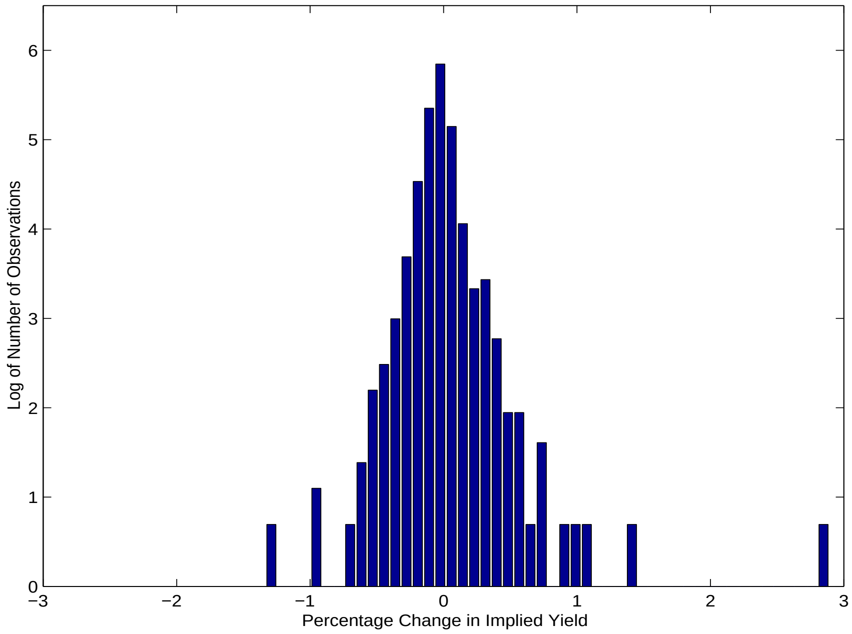

Figure 6: **Monthly Data** - Histogram of Changes in Emerging Market Bond Yields.
Skewness = 2.90   Kurtosis = 41.4