Some Unpleasant General Equilibrium Implications of Executive Incentive Compensation Contracts

John B. Donaldson† Natalia Gershun‡ Marc P. Giannoni§
Columbia University Pace University Columbia University
CEPR and NBER

December 10, 2010

Abstract

We consider a simple variant of the standard real business cycle model in which shareholders hire a self-interested executive to manage the firm on their behalf. An optimal compensation contract is derived which aligns the interests of shareholders and manager to achieve a first-best allocation of resources. When employed in a decentralized context the optimal contract is convex in the aggregate firms’ dividend for a wide class of model parameters. When compensation is convex in the firm’s own dividend, a given increase in the firm’s output generated by an additional unit of physical investment results in a more than proportional increase in the manager’s income. Incentive contracts of this form can easily result in an indeterminate general equilibrium, one in which business cycles are driven solely by self-fulfilling fluctuations in the manager’s expectations that are unrelated to the economy’s fundamentals. Arbitrarily large fluctuations in macroeconomic variables may result.

JEL Classification: E32, J33

Keywords: Delegation, executive compensation, indeterminacy and instability

---

*We thank the Editor, the Associate Editor and an anonymous referee for very constructive and insightful comments.

†Columbia Business School, Uris Hall, 3022 Broadway, New York, NY 10027.
‡Lubin School of Business, 1 Pace Plaza, New York, NY 10038.
§Columbia Business School, Uris Hall, 3022 Broadway, New York, NY 10027; e-mail: mg2190@columbia.edu; corresponding author.
1 Introduction

Informed by principal-agent theory, corporate boards have sought to develop managerial compensation packages with stronger links between pay and performance. Stock options, in particular, are perceived as providing one such link, and have emerged as the single largest component of U.S. executive compensation. According to Hall and Murphy (2002), “in fiscal 1999, 94% of S&P 500 companies granted options to their top executives. Moreover, the grant-date value of stock options accounted for 47% of total pay for S&P 500 CEOs in 1999.” CEOs of the largest U.S. companies frequently receive annual stock option awards that are on average larger than their salaries and bonuses combined.1,2 Executive options contracts represent a particular instance of a highly non-linear convex style contract.3

In this paper we demonstrate that convex executive pay practices, within the context of the separation of ownership and control in the modern corporation, may have dramatic, adverse business cycle consequences. In particular, we show that convex compensation contracts, which are broadly typical of U.S. CEO pay practices, may give rise to generic sunspot equilibria in otherwise standard dynamic stochastic general equilibrium (DSGE) modeling frameworks. Sunspot equilibria (indeterminacy) formalize the notion that expectations not grounded in fundamentals may lead to behavior by which they are fulfilled. These equilibria may involve arbitrarily large fluctuations in macroeconomic variables even though production is characterized by constant returns to scale at the social as well as private level. As such, convex managerial compensation contracts provide an entirely new mechanism by which indeterminacy may arise in real (non-monetary) economies. An

1 See Jensen and Murphy (1990); also, Shleifer and Vishny (1997) and Murphy (1999).
2 More recently, corporate boards have expanded the range of incentive instruments to include other forms of equity based incentive pay (e.g., direct stock grants). This has gone hand-in-hand with a steady reduction in the salary component of executive pay. For March 2007, Mercer Consulting reports that equity related incentive pay represents 2/3 of total compensation (average of 1000 largest U.S. firms by sales). Cash salary compensation represents only 19%. (For both statistics see www.mercer.com, “Study of CEO Compensation Trends,” May 15th, 2008.) Taken together, the various equity-related components remain, in value terms, highly convex functions of the standard measures of firm financial performance.
3 We need to clarify the sense of an options contract being convex. A single call option has a payoff at expiration \( c_T = \max \{0, q_T^p - E \} \) where \( T \) is the expiration date, \( q_T^p \) is the price of the underlying stock at expiration, and \( E \) is the exercise price. The payoff is piecewise linear and convex in the sense that if \( q_T^p \leq E \), \( c_T = 0 \), and if \( q_T^p > E \), \( c_T = (q_T^p - E) \), the latter being representable as a line with unit slope over its region of definition.

A portfolio of \( N \) call options would have a diagonal payoff line that is much steeper (the slope would, in fact, be "\( N \)"). In this sense the payoff to \( N \) options is "more convex" than the payoff to one option: increases in \( q_T^p \) above \( E \) have a much greater monetary benefit to the owner of the calls. When a CEO is given a grant of 1,000,000 options the diagonal line becomes nearly vertical and convexity in the above sense becomes enormous.
even more disturbing observation is that convex contracts may lead, under certain parameter con-
figurations, to non-stationary behavior. Practically speaking this means that convex contracts may
induce the self-interested manager to adopt investment policies that drive his firm’s equilibrium
capital stock to zero.⁴

Our focus on convex contracts derives from the fact that these contracts arise endogenously in
our model context: the optimal contract, optimal in the sense of generating the first best allocation,
is convex in the firms’ aggregate free cash flow (dividends). This feature is necessary to align the
incentives of managers and shareholders who are assumed to have different elasticities of inter-
temporal substitution (equivalently, to be differentially risk averse) and thus different preferences
for intertemporal consumption allocations.⁵ While the optimal contract in this model is convex in
aggregate dividends for a wide range of parameter values, it does not, per se, generate equilibrium
indeterminacy. We show, however, that equilibrium indeterminacy can easily arise in the face of
small deviations from the optimal contract. By small deviations we mean that the contract is con-
vex in the firm’s own dividend and involves a slightly higher degree of convexity than is required
to perfectly align incentives, or involves a fixed salary component that is added to the manager’s
incentive compensation, features that characterize actual compensation contracts. Accordingly,
while we derive the optimal contract in the model to justify our interest in convex compensation
contracts in general, the focus of the analysis is on the equilibrium consequences of a general class
of convex compensation contracts which resemble contracts actually seen in practice. To keep the
analysis as simple as possible and to isolate the key source of equilibrium indeterminacy in the
model, we assume that both consumer-worker-shareholders and managers have the same informa-
tion. This assumption stands in contrast with standard principal-agent theory that focuses either
on the effects of hidden information as to the manager’s type or on the manager’s hidden actions
(effort). Our results apply more broadly, however, to contexts where consumer-shareholder-workers

⁴Financial firms seem especially prone to lavishly convex compensation practices. We are reminded of the financial
crises surrounding the collapse of LTCM. In the year preceding its bankruptcy, the partners took the deliberate deci-
sion to reduce the firm’s capital, as a device for maximizing returns. More recently (2008) highly convex managerial
compensation at various investment banks was observed in parallel with their bankruptcy. We view these compensa-
tion contracts as highly convex to either the firm’s stock price or its free cash flow (or distributions to investors in
the case of hedge funds).

⁵To say it differently, such a feature helps to align the stochastic discount factors of the managers and consumer-
worker-shareholders.
and managers have differing information regarding the economy’s state variables.\footnote{This particular information asymmetry is accommodated in Danthine and Donaldson (2010). From a macroeconomic perspective it is most relevant asymmetry.} We eschew this added generality for two reasons: First, it allows us to focus exclusively on the indeterminacy generating mechanism which is the same in either context. Second, for the model presented here, the presence or absence of indeterminacy is related only to the manager, his information and elasticity of intertemporal substitution, and the terms of the contract given to him. Nothing else is relevant. Further augmenting the model to allow for hidden information regarding the manager’s "type" or actions, while likely providing additional justifications for offering a convex contract, would not, per se, affect the presence or absence of indeterminacy.\footnote{Let us be absolutely clear. There is no unobservable effort decision on the manager’s part in our model there is no issue as to his concealed "type." The "Revelation Principle" thus does not pertain to our model. Rather, the information asymmetry, should we have elected to include it, concerns the inability of the shareholder-workers to infer the economy’s state variables, in particular, its productivity shock. These are known only to the manager. If the manager is given the optimal contract, a first-best allocation will also be achieved under the information asymmetry. This contract is identical to the one presented in the full information equilibrium we emphasize.}

Sunspot equilibria have previously been studied in one-sector dynamic equilibrium models with external effects or monopolistic competition coupled with some degree of increasing returns.\footnote{Schmitt-Grohe (1997) compares four prominent models with these features.} In one-sector models with increasing returns, if all agents simultaneously decide, based on a non-fundamental belief shock, to increase their investment in an asset above the level associated with the initial equilibrium, the rate of return on that asset tends to increase, justifying the higher level of investment and validating agents’ beliefs. The main objection to this literature has been its empirical implausibility due to the relatively large increasing returns required to sustain the sunspot equilibria. Empirical estimates, furthermore, suggest that aggregate returns to scale seem to be constant, if not decreasing.\footnote{See Basu and Fernald (1997) and Laitner and Stolyarov (2004).} In Wen’s (1998) one-sector production model with variable capacity utilization of the capital stock, indeterminacy occurs with a significantly lower level of increasing returns. But the ability of the representative agent to alter capacity utilization in response to shock realization produces counterfactually smooth, in fact almost constant, aggregate consumption.

Indeterminacy can also arise in models with multiple production sectors if returns to scale differ at the social and private levels. Benhabib and Farmer (1996) demonstrate that indeterminacy occurs in a two-sector model with small, sector-specific external effects and very mild increasing returns.
at the aggregate level, while private returns are constant. Perli (1998) obtains similar results by introducing home production. The argument against multi-sector models with aggregate increasing returns is that they imply a convex-to-the-origin production possibility frontier at the social level, which means that sectoral aggregate supply curves are negatively sloped. In Benhabib, Meng and Nishimura (2000) multiple equilibria arise in multisector economies with constant social returns in all sectors combined with minor external effects in some sectors. To generate indeterminacy in this latter model, decreasing returns are necessary at the level of private firms with the implication that firms earn positive profits. Some kind of fixed cost is then needed to forestall potential new entrants.

In multi-sector models, increasing the relative price and the output of the capital good can also lead to an increase of its marginal product if the production of the capital good is relatively more capital intensive. When combined with market distortions and external effects, the rise in the capital stock may not be enough to offset the initial increase of its marginal product. Both the stock and the marginal product of capital rise simultaneously, mimicking the effect of increasing returns in a one-sector model.

In addition, models of endogenous firm entry and exit decisions and models with a variable degree of competition can also give rise to equilibrium indeterminacy (Jaimovich, 2007). In monetary models, an indeterminate equilibrium can arise in the case that monetary policy is conducted by following, e.g., an interest-rate rule not satisfying the so-called Taylor principle (see, e.g., Clarida, Galí and Gertler (2000), Woodford (2003), or, alternatively, in the case that a sufficiently large fraction of households do not participate in asset markets (Bilbiie (2008)).

In contrast, the model considered in this paper does not need aggregate increasing returns, a difference between social and private returns to scale, a variable degree of competition, or monetary phenomena to generate multiple equilibria. In our economy with delegated management and a convex executive compensation contract, the wedge between the actual return on capital and the return on capital as experienced by the manager is at the heart of the indeterminacy result. The power (degree of convexity) of the performance portion of the executive compensation contract tends to magnify the effective rate of return on capital from the manager’s perspective. As a result, the expectation of a high return on capital may increase the income of the manager next period to
such an extent that consumption smoothing considerations dictate a diminished level of investment today, thereby fulfilling the high return expectation. Nevertheless, our analytical and numerical results reveal that the degree of contract convexity required for indeterminacy is very low, especially so relative to a standard call options style incentive contract.

An outline of the paper is as follows: Section 2 describes the model, characterizes the optimal state-contingent plan, and presents a family of compensation contracts that are consistent with that optimal plan. It provides the theoretical basis for the analysis of convex contracts within the assumed model context. Section 3 considers a decentralized version of this economy in which managers are compensated with similarly convex contracts, analyzes the equilibrium and details the precise circumstances under which equilibrium indeterminacy and instability arise. Section 4 provides an overview of the methodology by which equilibrium is computed numerically and applies it to the study of the economy’s business cycle characteristics. Section 5 concludes.

2 Convex Contracting in a General Equilibrium Model of Delegated Management

We focus on the context of a self-interested manager and the consumer-shareholder-workers on whose behalf he undertakes the firm’s investment and hiring decisions in light of his compensation contract. We assume that there exists a continuum of measure 1 of identical consumer-worker-shareholders who consume a single good and supply homogenous labor services to a continuum of measure 1 of identical firms.10 The consumer-shareholder-workers delegate the firm’s management to a measure $\mu \in [0, 1]$ of managers who receive sufficient compensation to be willing to oversee the firm. We assume that once the manager agrees to operate the firm, he commits to it for the indefinite future. We further assume full information in the sense that both the consumer-worker-shareholder (principal) and the manager (agent) know the realization of all present and past variables.11 Since

---

10 We suppose that an equal number of workers participate in each firm.
11 Delegating to the managers the firm’s investment and hiring decisions makes sense even in an environment of full information if, say, there is a cost to shareholders of gathering and voting on the preferred investment and hiring plans (there would be unanimity). This cost could also be associated with information acquisition and borne either by a small measure of delegated managers or all the shareholder-workers. Under the optimal contract, whether the shareholders make the decision themselves or the managers behave in a self-directed way in light of their contract, the equilibrium allocation is the same.
there is no distortion in this economy, the first best can potentially be achieved. In the benchmark model, we suppose that the managers have no access to financial markets, so that, in equilibrium, they consume all of their income in each period. Guided by their compensation contract, they seek to smooth their consumption over time by undertaking the firm’s investment and hiring decisions; otherwise, they have no opportunity to borrow or lend. In Section 3 and Appendix D, we extend the model to allow managers to buy or sell riskless bonds. This is done to show that the results to be presented here are not sensitive to the assumption that managers are excluded from financial markets.

We start by describing the environment surrounding the consumer-shareholder-worker, the firm and the manager. We then characterize the Pareto optimal (first best) plan and determine a compensation contract for the manager that induces him to manage the firm in a manner that precisely replicates that optimal equilibrium. In Section 3, we consider a decentralized version of the model and characterize the set of equilibria when the manager’s compensation is given by contracts of the proposed form, yet where parameter values differ marginally from those associated with the optimal contract. Such contracts are shown to generate sunspot equilibria.12

2.1 The Model

2.1.1 The Representative Consumer-Worker-Shareholder

The representative consumer-worker-shareholder chooses processes for per-capita consumption $c_t^c$, the fraction of the time endowment, $n_t^s$, he wishes to work and selects his next period’s investment in one period risk-free discount bonds, $b_{t+1}^s$, and equity holdings $z_{t+1}^e(f)$ in firm $f$, to maximize his expected life-time utility

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{(c_t^c)^{1-\eta_s}}{1-\eta_s} - B \frac{(n_t^s)^{1+\zeta}}{1+\zeta} \right) \right]$$  \hspace{1cm} (1)$$

subject to his budget constraint

$$c_t^c + \int_0^1 q_t^e(f) z_{t+1}^e(f) df + q_t^b b_{t+1}^s = \int_0^1 (q_t^e(f) + d_t(f)) z_t^e(f) df + w_t n_t^s + b_t^s,$$  \hspace{1cm} (2)$$

12This model generalizes and refines one found in Danthine and Donaldson (2008 a,b).
in all periods. The subjective discount factor $\beta$ satisfies $0 < \beta < 1$, the parameter $\eta_s$ denotes the representative shareholder’s relative risk aversion coefficient ($0 < \eta_s < \infty$) and $\zeta$ is the inverse of the Frisch elasticity of labor supply ($0 \leq \zeta < \infty$). Note that in the case of $\zeta = 0$, the utility function in (1) reduces to the indivisible labor utility specification of Hansen (1985), while we obtain the case of fixed labor supply when $\zeta \to \infty$. Each consumer-worker-shareholder is endowed with one unit of time; the parameter $B > 0$ determines in part the fraction of that time endowment devoted to work. In (2), $w_t$ denotes the competitive wage rate, $q^b_t$ is the bond price, $q^c_f (f)$ is the share price of firm $f$, and $d_t (f)$ its associated dividend. We assume that each household holds a fully diversified portfolio of shares of each firm in equal proportions. We furthermore assume non-negativity constraints on all of the above variables except for $b^e_t$ which we suppose is larger than a (negative) lower bound guaranteeing that debt can be repaid in all states of the world.

2.1.2 The Firms

On the production side, there is a continuum of measure one of identical, competitive firms. Firm $f \in [0, 1]$ produces output via a standard constant returns to scale Cobb-Douglas production function:

$$y_t (f) = (k_t (f))^\alpha (n_t (f))^{1-\alpha} e^{\lambda_t}$$  \hspace{1cm} (3)

with two inputs – capital, $k_t (f)$, and labor, $n_t (f)$ – and the current level of technology $\lambda_t$; the latter is assumed to be common to all firms and to follow a stationary process which we denote by $\lambda_{t+1} \sim dG (\lambda_{t+1}; \lambda_t)$. The evolution of the firm’s capital stock, $k_t (f)$, follows:

$$k_{t+1} (f) = (1 - \Omega) k_t (f) + i_t (f), \quad k_0 (f) = k_0 \text{ given},$$  \hspace{1cm} (4)

where $i_t (f)$ is the period $t$ investment of firm $f$ and $\Omega$, $0 < \Omega < 1$, the depreciation rate. The firm’s dividend, $d_t (f)$, is in turn given by the free cash flows of the firm

$$d_t (f) = y_t (f) - w_t n_t (f) - i_t (f) - \mu g^m_t (f)$$  \hspace{1cm} (5)
which amounts to the firm’s income minus its wage bill, its investment expenditures and the managers’ compensation $\mu g_t^m (f)$. Here, $g_t^m (f)$ denotes the per-capita compensation of the managers of firm $f$, and $\mu \in (0, 1]$ denotes the measure of such managers working in firm $f$. In what follows we view managers as acting collegially, and thus use the words "manager" and "managers" interchangeably.

### 2.1.3 The Manager

At date $0$, the representative manager of firm $f$ decides whether or not to manage the firm. If he elects not to manage the firm, he receives a constant stream of reservation utility $\bar{u}^m$ in each period. If he does decide to manage the firm, he chooses processes for his consumption $c_t^m (f)$, hiring decisions $n_t (f)$, investment in the physical capital stock $k_t (f)$, and dividends $d_t (f)$ to maximize his expected utility

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( c_t^m (f) \right)^{1-\eta_m} \right]$$

subject to the production function (3), the capital accumulation equation (4), the dividend equation (5), and the constraints

$$c_t^m (f) \leq g_t^m (f)$$

$$c_t^m (f), n_t (f), k_t (f), y_t (f) \geq 0.$$

The manager thus chooses to operate the firm provided that his participation constraint

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( c_t^m (f) \right)^{1-\eta_m} \right] \geq \frac{\bar{u}^m}{1-\beta}$$

is also satisfied.

Managers need not be risk neutral nor have the same degree of relative risk aversion, $\eta_m$, as the consumer-worker-shareholders. For simplicity, we assume managers do not receive hourly wages and that the labor-leisure trade-off is therefore irrelevant for them.

The manager’s budget constraint (7) states that his consumption can be no larger than his compensation, $g_t^m (f)$. As noted earlier, we assume that the manager does not participate in the
capital markets. It is realistic to presume the manager is banned from trading the equity issued by the firm he manages. Not only does this rule protect shareholders from insider trading, but it also prevents the manager from using financial markets to mitigate the force of his contract. Restrictions on the ability of the executives to assume short positions in the stock of their own firms, or to adjust their long positions, are commonplace. It is more controversial however to assume that the manager cannot take a position in the risk free asset, although this assumption is common in the partial equilibrium contracting literature. We maintain this assumption here, though we relax it in Appendix D, and show there that our main results are, in fact, reinforced when managers are allowed to trade risk-free assets.

2.2 The Optimal Plan

The optimal plan specifies the optimal allocation of consumption, employment, investment, and payments at all dates and in all states of the world. It can be characterized by maximizing the consumer-shareholder-worker’s utility (1) subject to the constraints (2)–(5), (7) at all dates and in all states, and the manager’s participation constraint (8) in all states. Noting that (7) and (8) must hold with equality in an optimal equilibrium, and combining (3) and (4) with (5), we can express the Lagrangian for this problem as

\[
\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left( \frac{(c^e_t)^{1-\eta_s}}{1-\eta_s} - B (n^s_t)^{1+\zeta} \right) \right. \\
+ \Lambda_{1t} \left[ \int_0^1 (q^e_t (f) + d_t (f)) z^e_t (f) df + w_t n^s_t + b^s_t - c^s_t - \int_0^1 q^e_t (f) z^e_{t+1} (f) df - q^b_t b^s_{t+1} \right] \\
+ \int_0^1 \Lambda_{2t} (f) \left[ (k_t (f))^\alpha (n_t (f))^{1-\alpha} e^{\lambda t} - w_t n_t (f) - k_{t+1} (f) + (1 - \Omega) k_t (f) - \mu g^m_t (f) - d_t (f) \right] df \\
+ \int_0^1 \mu \Lambda_{3t} (f) [g^m_t (f) - c^m_t (f)] df \\
+ \int_0^1 \mu \Lambda_4 (f) \left[ \frac{(c^m_t (f))^{1-\eta_m}}{1-\eta_m} - \bar{u}^m \right] df \right\},
\]

where \( \{ \Lambda_{1t}, \Lambda_{2t} (f), \mu \Lambda_{3t} (f) \}_{t=0}^{\infty} \) are the Lagrange multipliers associated respectively to the constraints (2), (5), (7) at all dates and in all states and \( \mu \Lambda_4 (f) \) is the multiplier associated with constraint (8).

9
The necessary and sufficient first-order conditions with respect to \( c_t^e, n_t^e, z_{t+1}^e (f), b_{t+1}^e \) imply

\[
\Lambda_{1t} = (c_t^e)^{-\eta_e} \tag{10}
\]

\[
\Lambda_{1t} w_t = B (n_t^e)^\kappa \tag{11}
\]

\[
q_t^e (f) = E_t \left[ \beta \frac{\Lambda_{1t+1}}{\Lambda_{1t}} \left( q_{t+1}^e (f) + d_{t+1} (f) \right) \right] \tag{12}
\]

\[
q_t^b = E_t \left[ \beta \frac{\Lambda_{1t+1}}{\Lambda_{1t}} \right]. \tag{13}
\]

Similarly, the necessary and sufficient first-order conditions with respect to \( k_{t+1} (f), n_t (f), c_t^m (f), d_t (f) \) imply

\[
\Lambda_{2t} (f) = E_t \left\{ \beta \Lambda_{2t+1} (f) \left[ 1 - \Omega + \alpha y_{t+1} (f) / k_{t+1} (f) \right] \right\} \tag{14}
\]

\[
w_t = (1 - \alpha) y_t (f) / n_t (f) \tag{15}
\]

\[
(c_t^m (f))^{-\eta_m} = \frac{\Lambda_{3t} (f)}{\Lambda_{4} (f)} \tag{16}
\]

\[
\Lambda_{2t} (f) = \Lambda_{1t} z_t^e (f). \tag{17}
\]

The first-order condition with respect to the optimal compensation \( g_t^m (f) \) yields

\[
\Lambda_{2t} (f) = \Lambda_{3t} (f). \tag{18}
\]

Note that formulation (9) suggests a Pareto allocation implemented, in equilibrium, by the market system.

### 2.2.1 The optimal (equilibrium) allocation

The optimal equilibrium is a set of processes \( \{ c_t^e, c_t^m (f), n_t^e, n_t (f), b_t^e, z_t^e (f), i_t (f), y_t (f), k_t (f), w_t, g_t^b, q_t^e (f), d_t (f), g_t^m (f) \} \) and Lagrange multipliers \( \{ \Lambda_{1t}, \Lambda_{2t} (f), \Lambda_{3t} (f) \} \), and \( \Lambda_4 (f) \) such that:

1. The first-order conditions (10)–(18) are satisfied together with the constraints (2)–(5), (7)–(8), all holding with equality, and the transversality conditions: \( \lim_{t \to \infty} \beta^t (c_t^m (f))^{-\eta_m} k_{t+1} (f) = 0 \), for any given initial \( k_0 \) common to all firms.
2. The labor, goods and capital markets clear: \( n^s_t = n_t; \) \( y_t = c^s_t + \mu c^m_t + i_t \) where we denote by \( n_t, y_t, c^m_t, i_t \) the aggregates over all firms, i.e., \( n_t \equiv \int_0^1 n_t \, df \), and so on; investors hold all outstanding equity shares normalized to one for each firm, \( z^s_t (f) = 1 \), and all other assets (one period bonds) are in zero net supply, \( b^s_t = 0 \). \(^{13}\)

Since all firms and hence all managers face the same constraints and solve the same problem, they all make the same decisions. We may thus without loss of generality characterize the optimal equilibrium in terms of aggregate variables only, replacing all variables indexed by \( f \) with their aggregate counterpart (e.g., \( y_t (f) = y_t \equiv \int_0^1 y_t (f) \, df \) and so on for all \( f \)). It is noteworthy that the equilibrium just described is the Pareto-efficient or first-best equilibrium. The same allocation can in fact also be obtained by a traditional central planner problem whereby the consumer-shareholder-worker’s utility (1) is maximized subject to the constraints (3), (4), the goods market clearing condition \( y_t = c^s_t + \mu c^m_t + i_t \) and the manager’s participation constraint (8).

In equilibrium, it follows from (10), (17) and (18) that \[ \Lambda_{1t} = \Lambda_{2t} = \Lambda_{3t} = (c^s_t)^{-\eta_s}. \]

This “multiple equality,” together with (16), implies, furthermore, that the marginal utilities of consumption of the consumer-shareholder-worker and of the manager must be proportional in the optimal equilibrium, i.e.,

\[ (c^m_t)^{-\eta_m} = \Lambda_4^{-1} (c^s_t)^{-\eta_s} \tag{19} \]

at all dates and in all states, where \( \Lambda_4 \) depends, among other factors, on the manager’s reservation utility \( \bar{u}^m \). Equation (19) guarantees perfect risk sharing between the consumer-worker-shareholder and the manager. The remaining equilibrium conditions can be written as follows

\[ (c^s_t)^{-\eta_s} w_t = B n^e_t \tag{20} \]

\[ q^e_t = E_t \left[ \beta (c^e_{t+1})^{-\eta_s} (q^e_{t+1} + d_{t+1}) \right] \tag{21} \]

\(^{13}\)Note that by Walras law, if the labor and capital markets clear: \( n^s_t = n_t; \) \( z^s_t (f) = 1 \), \( b^s_t = 0 \), then the goods market clears as well. In fact, adding on both sides the consumer-shareholder-worker’s budget constraint (2) and \( \mu \) times the manager’s budget constraint (7), and using (5) to replace \( d_t (f) \), we obtain \( y_t = c^s_t + \mu c^m_t + i_t \) in equilibrium.
\[ q_t^b = E_t \left[ \beta \left( \frac{c_{t+1}^s}{c_t^s} \right)^{-\eta_s} \right] \]  
\[ (c_t^s)^{-\eta_s} = E_t \left[ \beta \left( c_{t+1}^s \right)^{-\eta_s} r_{t+1} \right] \]  
\[ w_t = (1 - \alpha) y_t / n_t \]  
\[ c_t^s = w_t n_t + d_t \]  
\[ c_t^m = g_t^m \]  
\[ d_t = y_t - w_t n_t - i_t - \mu g_t^m \]  
\[ y_t = k_t^\alpha n_t^{1-\alpha} e^{\lambda_t} \]  
\[ k_{t+1} = (1 - \Omega) k_t + i_t, \quad k_0 \text{ given} \]  
\[ \frac{g^m}{1 - \beta} = E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( c_t^m \right)^{1-\eta_m} \right] \]  

where

\[ r_t \equiv 1 - \Omega + \alpha y_t / k_t \]  

defines the rate of return of investment in physical capital.

We next define the contract and subsequently, the contracting equilibrium that replicates the allocation characterized in equations (19) through (31).

### 2.2.2 An optimal contract

Having characterized the set of conditions resulting in the optimal equilibrium, we now seek to determine a compensation function \( g^m() \) for each manager that implies an equilibrium outcome consistent with all equilibrium conditions stated above. The main purpose of the contract is to align the manager’s interests with those of the consumer-worker-shareholders who hire him.\(^{14}\) We consider the general family of contracts

\[ g^m (w_t n_t, d_t, d_t (f)) = A + \varphi \left[ (\delta w_t n_t + d_t)^\gamma + d_t (f) - d_t \right]^\vartheta \]  

\(^{14}\)This means that the manager chooses the same investment and labor functions as the shareholder-workers would choose if they themselves managed the firm.
with constant coefficients $A \geq 0$, $\varphi \geq 0$, $0 \leq \delta \leq 1$, $\gamma > 0$ and $\theta > 0$. The expression $w_t n_t$ denotes the equilibrium aggregate wage bill; and $d_t$ the equilibrium aggregate dividend; $\delta$ represents the relative compensation weight applied to the wage bill vis-a-vis the dividend and $\varphi$ the overall compensation scale parameter. Parameters $\gamma$ and $\theta$ determine the degree of the convexity of the compensation contract. Contracts of the form (32) are optimal within our model context, provided appropriate restrictions on the values of the parameters $A, \delta, \varphi, \gamma, \theta$ are imposed. This assertion is summarized in the two theorems detailed below.

**Theorem 1** A compensation contract of the form (32) with $A = 0, 0 < \varphi = \Lambda_1^{1/\eta_m}, \delta = 1, \gamma = \eta_s/\eta_m > 0$ and $\theta = 1$ is optimal, in the sense that it results in the optimal plan. If, in addition, the consumer-worker-shareholders are more risk-averse than the managers, $\eta_s > \eta_m$, then $\gamma > 1$ and this optimal contract provides managerial compensation that is convex in the sum of the aggregate wage bill and aggregate dividends.

**Proof.** See Appendix A.\(^{15}\)

This theorem states that a contract of the form

$$g^m(w_t n_t, d_t, d_t(f)) = \Lambda_4^{1/\eta_m}(w_t n_t + d_t)^{\eta_s/\eta_m} + (d_t(f) - d_t)$$

is optimal. Since all firms are identical, $d_t(f) = d_t$ for all $f$, in equilibrium, each manager’s compensation reduces to

$$g^m(w_t n_t, d_t, d_t(f)) = \Lambda_4^{1/\eta_m}(w_t n_t + d_t)^{\eta_s/\eta_m}.$$ 

To get some intuition for the optimality of such a contract, recall that the manager’s compensation needs to guarantee (19), i.e., that the marginal utilities of the manager and of the consumer-worker-shareholder remain proportional at all dates and in all states. Combining (19), (25) and (26), we obtain

$$g^m_t = c^m_t = \Lambda_4^{1/\eta_m}(c_t^*)^{\eta_s/\eta_m} = \Lambda_4^{1/\eta_m}(w_t n_t + d_t)^{\eta_s/\eta_m}.$$

\(^{15}\)In the proof of Theorem 1 and (to follow) Theorem 2, we rely on the characterization of the decentralized equilibrium found in Section 3.
which coincides with (19). The proposed compensation contract is thus socially optimal in equilibrium \((d_t(f) = d_t \text{ for all } f)\), as it perfectly aligns the marginal utilities of the manager and the consumer-shareholder-workers.

Note that in this economy, the consumer-worker-shareholders do not confront any moral-hazard problem vis-à-vis the manager, as they both have the same (full) information set. Yet, even in this context, a convex contract (where \(\eta_s > \eta_m\)) may be required to align, properly, the marginal utilities of the consumer-worker-shareholders and of managers in all states of the world, as per (19). The basic intuition underlying Theorem 1 may be summarized as follows: in order for the delegated manager to select the investment and hiring plans preferred by the consumer-worker-shareholders, he must (i) be given an income stream with the same stochastic characteristics and (ii) he must be equally sensitive to these same income variations.\(^{16}\) By making equilibrium compensation depend on \(w_t n_t + d_t\), the first of these requirements is satisfied, given (25). By raising this quantity to the power \(\eta_s/\eta_m\), the marginal utility of the manager is made proportional to that of the consumer-shareholder-worker.

For instance, if \(0 \leq \eta_m < \eta_s\), then the consumer-worker-shareholders ideally offer a convex compensation contract to the relatively risk-loving manager, in order to counteract the manager’s weak concavity in preferences. In contrast, if \(0 \leq \eta_s < \eta_m\), contract concavity effectively induces the manager to behave in a more risk-averse fashion. The constant \(\varphi > 0\) is chosen to satisfy the manager’s participation constraint and is determined by the welfare weights assigned to the two agents in the Pareto formulation.\(^{17}\) The last term in (33) in turn guarantees that the manager makes the optimal intertemporal decisions, with physical capital investment the same as in the optimal plan.

More generally, in dynamic stochastic general equilibrium models such as the one considered here, dividends (i.e., free cash flows) are countercyclical. This fact induces the risk-averse manager to smooth out the firm’s investment series much more than the consumer-worker-shareholders find optimal. To do otherwise would force the manager into a circumstance of very low consumption during cyclical upturns when investment is high. The convexity of the contract overcomes the

\(^{16}\)With homogeneous utility and constant return to scale production, the manager will make the same investment decisions irrespective of the scale of his income stream.

\(^{17}\)Formulation (9) assigns equal weights.
aforementioned disincentive and induces the manager to adopt a much more strongly pro-cyclical investment plan.

While Theorem 1 applies for any value of \( \mu \) (including \( \mu = 0 \)), the following Theorem states that in the limiting case where \( \mu = 0 \), one can find another contract that is also optimal, but that is convex in the firm’s own dividend.\(^{18}\)

**Theorem 2** Suppose that the measure of managers \( \mu = 0 \), and that \( \eta_s \) and \( \eta_m \) satisfy either 

\[
0 \leq \eta_s < 1, \quad 0 \leq \eta_m < 1, \quad \text{or} \quad 1 < \eta_s, \quad 1 < \eta_m.
\]

Then the contract of the form (32) with \( A = 0 \), 

\[
0 < \varphi = \Lambda^{1/\eta_m}, \quad \delta = 1, \quad \gamma = 1, \quad \text{and} \quad \theta = \frac{1-\eta_s}{1-\eta_m} > 0
\]

is optimal. If, in addition, \( |\eta_m - 1| < |\eta_s - 1| \), then \( \theta > 1 \) and this contract is convex in \( w_t n_t + d_t (f) \).\(^{19}\)

**Proof.** See Appendix B. \( \blacksquare \)

Typical contract-theoretic setups generally involve moral hazard associated with unobservable managerial effort in a context where effort is critical to production yet is provided only at a disutility to the manager. We have chosen to omit this feature for a number of reasons. First, recent events suggest that the effects of compensation agreements largely play themselves out via the firm’s investment decision, something that is front and center for the entirety of our analysis. It seems likely that the 2007–2008 Wall Street bankruptcies were not due to insufficient managerial effort, at least in the narrow sense of the word, but to the choice of inappropriate investment policies. Second, if a disutility of effort term were added to the manager’s period utility function, while also augmenting the production function with an “effort factor,” the optimal contract becomes one in which the manager receives all the firm’s net output \( (w_t n_t + d_t) \), and pays a fee to the shareholders for the privilege of managing the firm. The results we demonstrate below are not affected by the addition of an effort decision.

What forms do observed managerial contracts actually have and do they resemble our theoretical constructs? As pointed out by Bolton and Dewatripont (2005, p. 157), “in most cases a manager’s compensation package in a listed company comprises a salary, a bonus related to the firm’s profits in the current year, and stock options (or other related forms of compensation based on the firm’s

---

\(^{18}\)A less general version of this optimal contract is considered in Danthine and Donaldson (2008a).

\(^{19}\)The form of the contract is altered as optimal risk sharing, equation (19), does not apply under the measure zero assumption.
share price). [...] In other words, the manager’s remuneration can broadly be divided into a ‘safe’ transfer (the wage), a short term incentive component (the bonus), and a long term incentive component (the stock option).”

The type of contracts we consider here have this flavor. For instance if θ = 2, the contract (32) can be rewritten as

\[
g^m (w_t n_t, d_t, d_t (f)) = A + \varphi \left[ (\delta w_t n_t + d_t)^{2\gamma} + 2 (\delta w_t n_t + d_t)^\gamma (d_t (f) - d_t) + (d_t (f) - d_t)^2 \right]
\]

where \(A + \varphi (\delta w_t n_t + d_t)^{2\gamma}\) denotes a constant wage payment, \(2\varphi (\delta w_t n_t + d_t)^\gamma (d_t (f) - d_t)\) denotes a variable ‘bonus’ component, that is proportional to the firm’s dividend (or free cash flow), and \(\varphi (d_t (f) - d_t)^2\) approximates an ‘option’ component.

Note that when focusing on first-order approximations, as we do in the next section, the equity price \(q^e_t\) can be substituted for the dividend without loss of optimality (although the coefficient \(\varphi\) may need to be modified accordingly). To a first-order approximation, we may express the incentive component in terms of the manager’s own firm stock price instead of the dividend.

How large is the degree of convexity in the typical compensation contract? Gabaix and Landier (2008) carefully study the link between firms’ total market value (debt plus equity) and total compensation for the 1,000 highest paid CEOs in the U.S., over the period from 1992 to 2004. Their compensation measure includes the following components: salary, bonus, restricted stock grants and Black-Scholes values of stock options granted. Using panel regressions, they find that the elasticity of CEO compensation to the firms’ total market value is slightly above 1 (see their Table 2). While they do not formally reject an elasticity of 1 at the 5% confidence level, the point estimates lie above 1 in all specifications and are in some cases significantly larger than 1, at the 10% confidence level. Using the more aggregated compensation index of Jensen, Murphy, and Wruck (2004), which is based on all CEOs included in the S&P 500, they estimate that an increase of 1% in the mean of the largest 500 firms’ asset market values increases CEO compensation by 1.14% on average in the 1970-2003 sample (see their Table 3). Their Figure 1 suggests that this elasticity

---

20 One qualification to the general thrust of these remarks is the fact that convex contracting of the options related sort applies only to public companies. These account for roughly 50% of aggregate business capital in the U.S. (see McGrattan and Prescott (2007)) not the 100% our model implicitly assumes.
is significantly larger in the 1990-2000 period. While we will focus our analysis on moderate levels of contract convexity, it is important to note that this convexity can easily be very large when the compensation involves many call options.21

3 Generalized Equilibrium Implications of Delegated Management

With Theorems 1 and 2 in mind, we next explore, in a fully decentralized setting, the general equilibrium implications of economies where the manager is paid according to (32). In particular, we admit a wider range of contract parameters than those guaranteeing the Pareto optimum. Our goal is to demonstrate that marginal departures from the optimal contracts can easily generate sunspot equilibria by which we mean business cycle fluctuations driven by self-fulfilling managerial expectations. While optimal contracts do not themselves create indeterminacy or instability (a result confirmed in Section 3.6), contracts with positive fixed payments or slightly excessive convexity easily can. It is to this latter family of contracts that Section 3 is addressed. We begin by defining our decentralized equilibrium context.

More specifically, we consider a version of the model in which the consumer-worker-shareholders choose their own consumption, labor supply, bonds and equity holdings and delegate the operations of the firm to a manager. The measure $\mu$ of managers of the firm $f \in [0, 1]$ in turn choose their own streams of consumption, physical investment, hiring, dividend payouts, and are paid according to the pre-specified compensation contract of the form (32) discussed above.

3.1 Decentralized equilibrium

The consumer-worker-shareholders chooses $\{c_i^t, n_t^s, z_{t+1}^t, b_{t+1}^t\}$ to maximize his utility function (1) subject to his budget constraint (2). This yields the first-order necessary conditions (10)–(13).

Each manager in turn decides whether to manage the firm or, instead, to receive his reservation

21 To put our claim in perspective, consider a standard call options contract where the degree of convexity is measured, using the Black-Scholes call valuation formulae, by gamma ($\Gamma$). To award a manager a call option on his firm’s stock is directly analogous to granting him a compensation contract of the form (32) with $\theta > 1$. Typically, the strike price of an options award is set equal to the then-prevailing stock price. The gamma of a long position in a call option is always positive and reaches a maximum under this circumstance (for given volatility, time to expiration, etc.). Furthermore, as a function of the firm’s stock price, an award of, say, 100,000 call options yields an overall contract convexity 100,000 times that of an individual call’s gamma. We can imagine overall contract convexity becoming extremely large.
utility. The measure \( \mu \) of managers who work in firm \( f \) choose \( \{ k_{t+1} (f) , n_t (f) , c_t^n (f) , d_t (f) \} \) to maximize their utility (6) subject to the restrictions (3)–(5), (7) and the compensation contract (32). We will assume that \( \varphi \) is sufficiently large for each manager’s participation constraint (8) to be satisfied. The Lagrangian for the problem of the representative manager in firm \( f \) can be expressed as

\[
\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left( \frac{(c_t^n (f))^{1-\eta_m}}{1-\eta_m} \right) \right. \\
+ \Lambda_{2t} (f) \left[ (k_t (f))^{\alpha} (n_t (f))^{1-\alpha} e^{\lambda_t} - w_t n_t (f) - k_{t+1} (f) + (1 - \Omega) k_t (f) \right] \\
- \mu \left( A + \varphi [(\delta w_t n_t + d_t)^\gamma + d_t (f) - d_t]^\theta \right) - d_t (f) \\
\left. + \Lambda_{3t} (f) \left[ A + \varphi [(\delta w_t n_t + d_t)^\gamma + d_t (f) - d_t]^\theta - c_t^n (f) \right] \right\}.
\]

The necessary first-order conditions with respect to \( c_t^n (f) , n_t (f) , k_{t+1} (f) , d_t (f) \) imply

\[
\Lambda_{3t} (f) = (c_t^n (f))^{-\eta_m} \tag{34}
\]

\[
w_t = (1 - \alpha) y_t (f) / n_t (f) \tag{35}
\]

\[
\Lambda_{2t} (f) = E_t \{ \beta \Lambda_{2t+1} (f) [1 - \Omega + \alpha y_{t+1} (f) / k_{t+1} (f)] \} \tag{36}
\]

\[
\Lambda_{2t} (f) = \Lambda_{3t} (f) \frac{x_t (f)}{1 + \mu x_t (f)} \tag{37}
\]

for all \( f \), where

\[
x_t (f) \equiv \frac{\partial g^m (w_t n_t, d_t, d_t (f))}{\partial d_t (f)} = \theta \varphi [(\delta w_t n_t + d_t)^\gamma + d_t (f) - d_t]^\theta - 1. \tag{38}
\]

The variable \( x_t (f) \) will turn out to be critical. It denotes the marginal contribution to the manager’s compensation of increasing the firm’s dividend by one unit.

Since all firms are identical, all managers face the same constraints, solve the same problem, and thus make the same decisions. It follows that \( c_t^n (f) = c_t^n \equiv \int_0^1 c_t^n (f) \, df \), \( y_t (f) = y_t \equiv \int_0^1 y_t (f) \, df \) and so on for all \( f \). As a consequence, the same constraints and first-order conditions hold without the index \( f \). Using (34) and (37) to solve for the Lagrange multipliers \( \Lambda_{2t} (f) \) and \( \Lambda_{3t} (f) \), we can
rewrite (36) as:

\[
\frac{(c_t^m)^{-\eta_m} x_t}{1 + \mu x_t} = E_t \left[ \beta (c_{t+1}^m)^{-\eta_m} \frac{x_{t+1}}{1 + \mu x_{t+1}} r_{t+1} \right]
\]

(39)

where \(r_t\) is again defined as in (31), and

\[
x_t = \theta \varphi^{1/\theta} (c_t^m - A)^{\frac{\theta-1}{\sigma}}
\]

(40)

using (38), the contract (32) and the fact that \(c_t^m = g_t^m\).

The decentralized equilibrium is then a set of processes \(\{c_t^m, c_t^s, n_t, n_t^f, b_t^s, z_t^e, i_t, y_t, k_t, r_t, w_t, q_t^h, q_t^e, d_t, x_t\}\) such that:

1. The first-order conditions (10)–(13), (31), (35), (39), (40) are satisfied together with constraints (2)–(5), (7)–(8) all holding with equality, and the transversality condition:

\[
\lim_{t \to \infty} \beta^t (c_t^m)^{-\eta_m} k_{t+1} = 0, \text{ for any given initial } k_0.
\]

2. The labor, goods and capital markets clear: \(n_t^s = n_t\); \(y_t = c_t^s + \mu c_t^m + i_t\); investors hold all outstanding equity shares, \(z_t^e = 1\), and all other assets (one period bonds) are in zero net supply, \(b_t^e = 0\).

For the general family of contracts (32), the equilibrium conditions can be written as follows.

Consumption of the manager and of the consumer-worker-shareholders depends on labor income and dividends\(^{22}\)

\[
c_t^m = A + \varphi (\delta w_t n_t + d_t)^{\gamma \theta}
\]

\[
c_t^s = w_t n_t + d_t,
\]

where dividends, in turn, relate to income and investment according to

\[
d_t = y_t - w_t n_t - i_t - \mu \left( A + \varphi (\delta w_t n_t + d_t)^{\gamma \theta} \right).
\]

---

\(^{22}\)The first expression is obtained from (7) and contract (32), recognizing that in equilibrium \(d_t (f) = d_t\), while the second expression follows from (2), noting that \(b_t^e = 0\) and \(z_t^e = 1\) in equilibrium.
The production function yields

\[ y_t = k_t^{\alpha} n_t^{1-\alpha} e^{\lambda_t}, \]

so that the real wage and the return on capital, \( r_t \), are given, respectively, by

\[ w_t = (1 - \alpha) \left( \frac{y_t}{n_t} \right), \quad \text{and} \]
\[ r_t = \alpha \left( \frac{y_t}{k_t} \right) + 1 - \Omega. \]

The intratemporal first-order condition for the shareholder-worker’s optimal consumption-leisure decision is

\[ (c_t^*)^{-\eta} w_t = B n_t^\zeta. \]

While the above equations are all a-temporal, the equations determining the model’s intertemporal dynamics are the capital accumulation equation

\[ k_{t+1} = (1 - \Omega) k_t + i_t \]

and the Euler equation for the optimal intertemporal allocation of the manager’s consumption\(^{23}\)

\[ (c_t^m)^{-\eta_m} \frac{x_t}{1 + \mu x_t} = E_t \left[ \beta \left( c_{t+1}^m \right)^{-\eta_m} \frac{x_{t+1}}{1 + \mu x_{t+1}} r_{t+1} \right] \]

where \( x_t \) is given in (40) and satisfies

\[ x_t = \theta \varphi^{1/\theta} \left( c_t^m - A \right)^{\theta-1}. \]

Again, we assume that \( \varphi \) is large enough for the manager’s participation constraint (8) to be satisfied, a restriction that does not affect the results to follow in any way. These equations, and their log-linearized counterparts, given below, form the basis of the analysis to follow.\(^{24}\)

\(^{23}\) This equation determines the optimal intertemporal allocation of the manager’s consumption even though managers do not have access to financial markets. The application of the optimal contract is effectively a substitute for security trading.

\(^{24}\) Note that equations (21) and (22) determine asset prices \( q_t^e \) and \( q_t^b \). We omit these equations here as asset prices are residual (non-state) variables.
3.2 Approximating the Decentralized Equilibrium around the Deterministic Steady State

As we now show, the degree of convexity of the manager’s contract has first order effects on the equilibrium dynamics. In particular, the convexity of the contract is crucial to determining whether the general equilibrium is unique or whether it exists at all. A global analysis of the existence and uniqueness of the general equilibrium of this model is beyond the scope of this paper. Instead, we focus here on the local analysis of the equilibrium dynamics around the deterministic steady state in the face of small enough exogenous disturbances. Assuming the exogenous variable \( \lambda_t \) is bounded, and denoting the steady-state value of a variable with an overhead bar and the log-deviations from that steady-state value with a hat, \( \hat{\cdot} \), we can characterize the model’s approximate dynamics by the following log-linearized equilibrium conditions:

\[
\begin{align*}
\hat{c}^m_t &= \Xi \left[ \delta \omega \hat{y}_t + (1 - \omega) \hat{d}_t \right], \quad \text{where} \quad \omega \equiv \frac{\bar{w} \bar{n}}{\bar{w} \bar{n} + \bar{d}}, \quad \Xi \equiv \frac{\gamma (1 - A/\bar{c}^m)}{\delta \omega + 1 - \omega} > 0 \quad (41) \\
\hat{c}_t^s &= \omega \hat{y}_t + (1 - \omega) \hat{d}_t \quad (42) \\
\Omega \hat{k}_{yt} &= \left( \alpha - \mu \frac{\bar{c}^m}{\bar{y}} \Xi \delta \omega \right) \hat{y}_t - \left( \frac{\bar{d}}{\bar{y}} + \mu \frac{\bar{c}^m}{\bar{y}} (1 - \omega) \right) \hat{d}_t \quad (43) \\
\hat{y}_t &= \hat{\lambda}_t + \alpha \hat{k}_t + (1 - \alpha) \hat{n}_t \quad (44) \\
\hat{w}_t &= \hat{y}_t - \hat{n}_t \quad (45) \\
\hat{r}_t &= (1 - \beta (1 - \Omega)) \left( \hat{y}_t - \hat{k}_t \right) \quad (46) \\
\hat{\omega}_t &= \eta_s \hat{c}_t^s + \zeta \hat{n}_t \quad (47) \\
\hat{\kappa}_{t+1} &= (1 - \Omega) \hat{k}_t + \Omega \hat{n}_t, \quad (48)
\end{align*}
\]

and the log-linearized Euler equation for the manager’s consumption:

\[
\hat{c}_t^m = E_t \hat{c}_{t+1}^m - \psi^{-1} E_t \hat{r}_{t+1} \quad (49)
\]

25 The steady state in this economy is defined as the solution to the following set of equations: \( \bar{c}^m = A + \varphi (\delta \bar{w} \bar{n} + \bar{d})^{\varphi}; \quad \bar{w} \bar{n} = (1 - \alpha) \bar{y}; \quad \bar{y} = \bar{k}^{1 - \alpha}; \quad \Omega \bar{k} = \bar{r}; \quad \bar{x} = \theta \varphi^{1/\theta} (\bar{c}^m - A)^{(\theta - 1)/\theta}; \quad \bar{r} = \alpha \bar{y} \bar{k} + 1 - \Omega; \quad \beta^{-1} = \bar{r}; \quad \bar{c}^s = \bar{w} \bar{n} + \bar{d} \text{ and } (\bar{c}^s)^{-\alpha} \bar{w} = B \bar{n}^{1 - \alpha} \text{ and } \bar{y} = \bar{x} + \bar{r} + \mu \bar{c}^m. \)

26 Equation (49) is obtained after linearizing (40), and substituting for \( \hat{x}_t = \left( \frac{\theta - 1}{\theta} \right) \frac{\bar{c}^m - \bar{c}_t^m}{(\bar{c}^m)^{\theta - 1}}. \)
where
\[ \psi \equiv \eta_m - \frac{\theta - 1}{\theta} \left( 1 - \frac{1}{A/c_m} \right) \frac{\mu_x}{(1 + \mu_x)} \]  
(50)
corresponds to the manager’s coefficient of relative risk aversion adjusted for features of the incentive contract, such as its degree of convexity \( \theta \), and the fraction \( A/c_m \) of the manager’s compensation that is fixed \((0 \leq A/c_m < 1)\). As we will see below, \( \psi \) will turn out to be a key coefficient for the model’s dynamics.

3.3 Indeterminacy: The Intuition

To understand how self-fulfilling fluctuations may arise in this economy, it is important to note that in equation (49), the coefficient \( \psi^{-1} \) represents the elasticity of intertemporal substitution for the manager’s consumption in response to changes in the manager’s personal rate of return on investment, i.e., the effective rate of return from the manager’s point of view. That rate of return represents not only the additional output generated by another unit of investment in physical capital, but also the additional compensation distributed to the manager as a result of that additional unit of output. As indicated in (50), the degree of convexity of the incentive contract, \( \theta \), is a key determinant of the manager’s intertemporal elasticity of substitution, and that for \( \theta \) sufficiently larger than 1, \( \psi^{-1} \) may even be negative. As argued below, a negative \( \psi \) can imply an indeterminate equilibrium, so that economic fluctuations may result from self-fulfilling manager’s expectations.

For comparison purposes, let us first explore the case where \( \theta = 1 \), a linear contract, so that \( \psi = \eta_m \). For these circumstances equation (49) reduces to the same log-linearized consumption Euler equation as would be obtained for the standard representative agent problem:

\[ \hat{c}_t = E_t \hat{c}_{t+1} - \eta_m^{-1} \hat{\rho}_{t+1} \] 

Here date \( t \) consumption responds negatively to increases in the expected rate of return (for given expected future consumption), and the response coefficient is the elasticity of intertemporal substitution. Assume that the manager suddenly expects a higher rate of return on capital next period, \( \hat{\rho}_{t+1} \), than would be justified by fundamentals. In this case \((E_t \hat{c}_{t+1}^m - \hat{c}_t^m)\) must increase or \( \hat{c}_t^m \) must get smaller, which can occur only if the agent saves more and so simultaneously

---

27 Equations (41) through (50) omit approximation error terms of second order or smaller.
must increase. This can only happen by having the manager invest in future capital stock so that \( \hat{k}_{t+1} \) also increases. The increase in the capital stock causes the marginal product of capital to drop (so that \( \hat{r}_{t+1} \) declines). As a result, expectations of a higher return on capital cannot be fulfilled and there is no supportable equilibrium indeterminacy.

In the case of a convex compensation function \( (\theta > 1) \), a given increase in the firm’s output generated by an additional unit of physical investment results in a more than proportional increase in the manager’s income. Let us consider the manager’s contract with convexity sufficiently larger than 1 to guarantee that \( \psi \) is negative. In that case, suppose that the manager has the belief (unrelated to fundamentals) that his own personal return will be “high” next period. The perception of a high income next period will lead him – in the interest of consumption smoothing – to consume more today, and thus to reduce his investment today. The lower investment leads to a higher rate of return on capital which confirms the manager’s belief of a high personal rate of return.

In general, the larger the convexity of the compensation contract, the more likely \( \psi^{-1} \) is to be negative, and hence the more likely the manager will increase his consumption and lower investment in the firm in response to his belief of an increase in the rate of return. It follows that the more convex the executive contract, the more likely the general equilibrium to be indeterminate, so that business cycles can be driven by self-fulfilling fluctuations in the manager’s expectations. In contrast, with low convexity in the manager’s contract \( (\theta \leq 1) \), the increase in the return on capital is smaller from the manager’s perspective. In this case, the contract works against the manager’s “personal returns expectations” being fulfilled.

The mechanism works similarly to the combination of constant returns to scale at the firm level but increasing returns at the economy-wide level, although in our case, production is generated with constant returns to scale. The appearance of equilibrium indeterminacy when \( \psi \) becomes negative relates also to Bilbiie (2008), who obtains an indeterminate equilibrium in a monetary model, when the fraction of households who are excluded from asset markets is sufficiently large. In that case, the economy-wide elasticity of intertemporal substitution in consumption similarly becomes negative. However, in contrast to Bilbiie (2008), our mechanism does not require households to be excluded from asset markets.\(^{28}\)

\(^{28}\)While we assume that managers do not trade securities, this assumption is not critical for our result. In fact,
An inspection of (50) also reveals that the lower the manager’s risk aversion, \( \eta_m \), the more likely \( \psi \) is to be negative and hence the more likely equilibrium indeterminacy will arise \( (\partial \psi / \partial \eta_m = 1 > 0) \).

In the extreme case of a risk-neutral manager \( (\eta_m = 0) \), any convexity of the incentive contract \( (\theta > 1) \), no matter how small, implies a negative coefficient \( \psi \), and hence can result in an indeterminate equilibrium. Similarly, for any convex contract \( (\theta > 1) \), the larger the constant salary component of the executive contract, \( A \), the more likely \( \psi \) will be negative, and, again, the more likely equilibrium indeterminacy can arise. By making the manager’s compensation less volatile, the higher fixed salary component reduces the magnitude of the incentives part of the contract necessary to generate indeterminacy. Below, we formalize this intuition.

### 3.4 Indeterminacy and Instability in the Case of Fixed Labor Supply: An Analytical Characterization

We now determine the regions of the parameter space in which the model dynamics around the deterministic steady state yields (i) a unique bounded equilibrium, (ii) an indeterminate equilibrium so that an infinite number of bounded equilibria are consistent with the model’s equations, or (iii) no bounded equilibrium so that the model’s dynamics can only result in explosive paths. To derive analytical results, we consider a specification of (1) with a fixed labor supply \( (\zeta \to \infty) \). It can be demonstrated numerically that none of the results presented here are affected by this assumption. In particular, the region of equilibrium indeterminacy is independent of the value of \( \zeta \).

After combining the linearized equilibrium conditions (41)–(50), the model’s local dynamics can be summarized by the following two equations:

\[
E_t \hat{c}_t^{m+1} = \hat{c}_t^{m} - \psi^{-1} (1 - \alpha) (1 - \beta (1 - \Omega)) \hat{k}_{t+1} + exog_t \\
\hat{k}_{t+1} = B_21 \hat{c}_t^{m} + B_22 \hat{k}_t + exog_t
\]

(51) (52)

where \( exog_t \) denotes exogenous terms that depend on current and on expected future realizations when managers can trade riskless bonds (as in Appendix D) our results apply more generally. The magnitude of the measure of managers, \( \mu \), is also not material.
of the productivity shock $\lambda_t$, and\footnote{Equation (51) is obtained by combining (49) and (46), using (44) to substitute for $\hat{\psi}_t$, and noting from (47) that $\dot{\hat{n}}_t = 0$ in the case that $\zeta \to \infty$. To obtain (52), we first use (43) to express $\hat{i}_t$ as a function of $\hat{c}_t$, $\hat{k}_t$, and $\lambda_t$ exploiting (41) to solve for $\hat{d}_t$, and (44) to eliminate $\hat{y}_t$, and then combine the resulting expression with (48).}

\[ B_{21} = -\left( \frac{1}{(1-\omega)} \frac{\bar{d}}{k} + \frac{\hat{c}_m}{k} \right) < 0 \]
\[ B_{22} = \left( \Omega - (1-\Omega) \right) (1-\alpha) \delta (1-\Omega) (1-\alpha) + \alpha \beta^{-1} > 0. \]

Since $B_{22}$ is increasing in $\delta$, there exists a threshold

\[ \delta^* \equiv 1 - \frac{\beta^{-1} - 1}{(\beta^{-1} - 1 + \Omega) (1-\alpha)} < 1 \]

such that\footnote{Note in particular that when $\delta = 1$, $B_{22} = \beta^{-1} > 1$.}

\[ B_{22} < 1 \text{ if and only if } \delta < \delta^*. \]

The characterization of the regions of determinacy, indeterminacy and instability of the equilibrium resulting from the dynamic system (51)-(52) can be summarized as follows:

**Theorem 3** *In the case of a fixed labor supply ($\zeta \to \infty$), the linearized model admits:*

(i) an indeterminate equilibrium (i.e., a continuum of bounded solutions) if and only if the following conditions are jointly satisfied:

\[ \psi < \psi^* \equiv \frac{(1-\alpha) (1-\beta (1-\Omega)) B_{21}}{2 (B_{22} + 1)} < 0 \]
\[ \delta < \delta^*, \]

(ii) an unstable equilibrium (i.e., no bounded solutions) if and only if (54) holds and $\delta > \delta^*$,

(iii) a determinate equilibrium (i.e., a unique bounded solution) if and only if $\psi \geq \psi^*$.

**Proof.** See Appendix C. $\blacksquare$

A sufficient condition for a unique bounded equilibrium is $\psi > 0$. This theorem states that indeterminacy or instability (i.e., no bounded solution) arises provided that $\psi$ is sufficiently negative. As we show below, this can easily occur with convex compensation contracts for the manager,
and even for moderate degrees of convexity. In this case, the equilibrium is indeterminate for \( \delta \) sufficiently small \( (\delta < \delta^*) \) and unstable for \( \delta > \delta^* \).

To have some sense of the relevance of these conditions, we calibrate the quarterly subjective discount factor, \( \beta \), at 0.99, the capital share of output, \( \alpha \), at 0.36 and the quarterly capital depreciation rate, \( \Omega \), at 0.025. These are values commonly used in the literature (see section 4.2 for more details on the model calibration.) The implied critical value \( \delta^* = 0.55 \). In addition, we assume that consumer-worker-shareholders have log utility in consumption, so that \( \eta_s = 1 \), and set the measure of managers, \( \mu \), at 0.05. While the percentage of executive, administrative, and managerial occupations in the labor force amounts to 17\%, we assume that less than a third of that makes actual hiring and investment decisions for the firm. (In any case, the particular value of \( \mu \) is not critical for the results discussed below). If we furthermore assume that consumer-shareholder-workers and managers have the same consumption to income ratio in steady state, the above calibration implies that \( \omega = 0.90 \), which corresponds very closely to the average ratio obtained in U.S. national income data over the period 1947-2009.\(^{31}\)

The departures from contract optimality assume a variety of forms. We allow the possibility that \( \delta < 1 \) (the relative magnitude of the aggregate wage bill may be too small relative to the aggregate dividend), \( A > 0 \) (there is a fixed wage, a feature absent under the optimal contract) and \( \theta \neq 1 \) (too much or too little overall contract convexity). We set \( \gamma \), the degree of convexity in the aggregate wage bill and aggregate dividends equal to 1, i.e., the optimal value in the case that all agents have the same degree of risk aversion. It is important to note however that choosing different values for this parameter would not change any of our results in regards to the region of equilibrium determinacy.

Figure 1 represents the regions of determinacy, indeterminacy and instability for various values of \( \eta_m \) and the parameters characterizing the incentive contract, i.e., \( \theta, \delta \) and \( A/\bar{\sigma}^m \). The boundary for the region of determinacy in the \( (A/\bar{\sigma}^m, \theta) \) space remains largely the same for the different values of \( \delta \) represented in the two columns of Figure 1. With \( \delta \) sufficiently low, the model exhibits local indeterminacy when the convexity of the contract \( \theta \) rises. For \( \delta = 1 \), even a mildly convex

---

\(^{31}\)The ratio of employees’ compensation over the sum of employees’ compensation and corporate profits (after corporate tax, with IVA and CCAdj) amounts to 0.91.
managerial contract ($\theta > 1$) can lead to an explosive general equilibrium in the economy.

To get some intuition for this result, suppose that we start at the steady state and that there is no fundamental shock, i.e., $\lambda_t = 0$ for all $t$.

**Case 1: determinacy ($\psi > \psi^*$)**

Suppose that the manager’s compensation contract is linear or concave ($\theta \leq 1$) and that agents observe an unexpected sunspot $\nu_0$ shock at date 0, which leads managers to consume more than in steady state ($c^m_0 > 0$). Since the initial capital stock is fixed at $\bar{k}$, the linearized equation for the capital stock, (52), implies that the capital stock in the next period will need to decrease ($\hat{k}_1 < 0$) because managers now consume more and invest less in physical capital. According to (51), high consumption by managers at date 0 and lower capital stock at date 1, leads to even higher managerial consumption in period 1 if $\psi$ is large enough. This, in turn, causes a further drop in the capital stock and an increase in consumption by managers in period 2: $E_t \hat{c}^m_2 > E_t \hat{c}^m_1 > c_0$, and so on. In the case of $\psi > \psi^*$ the presence of a sunspot is inconsistent with a bounded equilibrium. Therefore, if a bounded equilibrium exists, it must be unique.

**Case 2: indeterminacy ($\psi < \psi^* < 0$ and $\delta < \delta^*$)**

Suppose instead that $\psi < \psi^* < 0$ (which occurs with a sufficiently convex compensation contract) and agents again observe an unexpected sunspot $\nu_0$ at date 0, which leads managers to consume more at that time. Again, since the initial capital stock is fixed at its steady state value, equation (52) implies that the capital stock must decrease.

But with $\psi < \psi^*$ equations (51) and (52) imply that the increase in managerial consumption at date 0 combined with the lower capital stock at date 1, leads the manager to eventually consume less and accumulate more capital so that the economy reverts back to the steady state. This process leads to a stationary path for the manager’s consumption and for the capital stock. If $\psi < \psi^*$, sunspot shocks are therefore consistent with a bounded equilibrium and there exists an infinite number of such bounded equilibria satisfying the model’s restrictions, including some equilibria with arbitrary large fluctuations, as $\nu_t$ itself can be arbitrarily large.

**Case 3: instability ($\psi < \psi^*$ and $\delta > \delta^*$)**
Suppose again that $\psi < \psi^*$ and that $\delta$ exceeds its critical value so that $B_{22} > 1$. Consider a productivity or sunspot shock at date 0, which leads managers to invest less. Again, the capital stock at date 1 must fall below its steady state value. According to equation (51), if $\psi < \psi^*$ the manager’s consumption in period 1 tends to be lower than in period 0, which according to equation (52), tends to bring the future capital closer to its steady state value. However, with $B_{22} > 1$, the date 1 deviation of the capital stock is amplified. It follows that the capital stock embarks on an explosive (or implosive) trajectory, eventually exiting any neighborhood of the steady state. This, in turn, results in an explosive evolution of the manager’s consumption. Hence the model admits no bounded solution.

3.5 Indeterminacy and Instability for General Labor Supply

We use a numerical solution to show that the regions of determinacy remain the same when the labor supply is elastic although the split between indeterminacy and instability regions depends on the value of the Frisch elasticity of labor supply $\zeta^{-1}$ as well as the fraction $\delta$ of executive compensation related to aggregate labor income. Figures 2 and 3 show determinacy, indeterminacy, and instability regions in the $(A/c^m, \theta)$ space, for different values of $\zeta$. In Figure 2, we set $\zeta^{-1} = 0.5$ to match the Frisch elasticity of labor supply often found in microeconomic studies, while in Figure 3, $\zeta^{-1} \rightarrow \infty$ consistently with the labor supply found in Hansen (1985). Once again, with sufficiently low $\delta$, the convex executive compensation contract can easily generate an indeterminate equilibrium. Instead when $\delta$ is sufficiently large, the convex contract results in an explosive equilibrium dynamics.

3.6 General Equilibrium and Optimal Incentive Contracts

The discussions in Sections 3.1–3.5 refer to the general family of incentive contracts of the form (32), and argue that the general equilibrium can be indeterminate or explosive if the contract convexity, $\theta$, is excessive or if the fix payment, $A$, is sufficiently large. A natural question is how close an optimal contract would be to these regions of equilibrium indeterminacy or instability. As we now show, while the optimal incentive contract would result in a unique stable equilibrium, slightly more generous incentive contracts could easily result in undesirable outcomes.

---

32 The interpretation of the experiments being undertaken in Figures 2 and 3 is exactly the same as for Figure 1.
As stated in Theorem 1, the optimal contract requires that \( A = 0, \delta = 1, \gamma = \eta_s/\eta_m, \) and \( \theta = 1. \) It is convex in the sum of the aggregate wage bill and aggregate dividends if \( \eta_s > \eta_m. \) While this optimal contract, indicated by a * in Figures 1, 2, and 3 lies in the region of determinacy, we can easily obtain equilibrium indeterminacy or instability if the contract specifies larger values of the fix payment \( A, \) or a higher convexity \( \theta \) in the sum of aggregate wages and the individual firm’s dividend, or a lower weight \( \delta \) on the aggregate wage bill. The risk of entering obtaining such undesirable equilibria is especially pronounced if the manager’s degree of risk aversion \( \eta_m \) is low.

By Theorem 2, there exists also another optimal contract, in the limiting case that \( \mu = 0. \) This contract requires that \( A = 0 \) and \( \theta = \frac{1-\eta_s}{1-\eta_m}. \) In Figure 4, the two asymptotic-to-the-vertical and boldfaced curves capture this latter relationship in the \((\eta_m, \theta)\) plane, for two different choices of \( \eta_s \) on either side of 1.\(^{33}\) In the special case that \( \eta_m = \eta_s \neq 1, \) the convexity of the optimal contract is \( \theta = 1. \) Instead, if \( \eta_m = \eta_s = 1, \) so that both agents have log preferences, Appendix B informs us that the optimal contract convexity can be any value \( \theta \geq 0, \) as represented by the vertical line. Conditional on shareholder risk aversion, \( \eta_s, \) these lines effectively determine the optimal contract convexity given the manager’s coefficient of relative risk aversion, \( \eta_m. \)

In contrast, the shaded regions in Figure 4 present the parameter combinations for which indeterminacy or instability will arise. Essentially, these are parameter configurations which satisfy \( \psi < \psi^*, \) where \( \psi \) is again defined in (50). In particular, the dark-shaded region, defined by \( \{ (\eta_m, \theta) : \psi \equiv \eta_m - \frac{\theta-1}{\theta} \frac{1}{(1-A/\bar{c})^{(1+\mu \bar{\theta})}} < \psi^*; A = 0, \mu = 0 \} \) is the region of indeterminacy or instability in the case that the fixed payment is set at the value of the optimal contract, \( A = 0. \) As mentioned above, indeterminacy or instability may arise provided that the contract is sufficiently convex and the manager’s risk aversion is sufficiently low. Notice that this region does not intersect with either of the boldfaced curve representing the optimal contracts, nor would it for any other choice of \( \eta_s. \) While the optimal contract does not per se lead to equilibrium indeterminacy or instability, a slightly more generous compensation in terms of higher contract convexity would could easily result in such bad outcomes. While we assume full information in this model, these outcomes could as well arise in an extension of the model where consumer-worker-shareholders, who

\(^{33}\)More specifically, the set-theoretic representations of the left and right boldfaced curves are respectively: \( \{ (\eta_m, \theta) : \theta = \frac{1-\eta_s}{1-\eta_m}, \eta_s = 0.5 \} \) and \( \{ (\eta_m, \theta) : \theta = \frac{1-\eta_s}{1-\eta_m}, \eta_s = 2 \}. \)
determine the contract form and its parameters, know their own coefficient of relative risk aversion but mistakenly over-estimate the manager’s true degree of risk aversion. For example, suppose that the true $\eta_s$ and $\eta_m$ satisfy $\eta_s = \eta_m = 0.5$, so that the optimal contract parameters are $A = 0$ and $\theta = 1$. If the shareholders counterfactually estimate $\eta_m = 0.8$, then they will choose contract parameters $A = 0$ and $\theta = \frac{1-\eta_s}{1-\eta_m} = 2.5$. Relative to the true $\eta_m$ which guides the manager’s actions, this choice of $\theta$ leads to indeterminacy or explosive equilibria as $\psi = \eta_m - \frac{\theta-1}{\theta} = 0.5 - \frac{15}{25} < 0$.

The larger shaded region (comprising the dark and light gray regions) similarly represents the region of indeterminacy or instability, but assuming a positive fraction of the manager’s compensation in the form of a fixed payment. Specifically, it assumes $A/\sigma^m = 0.5$. As Figure 4 makes clear, for any $\eta_m$, the minimal magnitude of $\theta$ necessary for indeterminacy is strictly less than in the $A/\sigma^m = 0$ case. Note also that for a wide range of manager risk aversion, $\eta_m$, the choice of convexity $\theta$ in the optimal contract leads to indeterminacy or instability when $A/\sigma^m = 0.5$. A larger positive fixed payment thus allows indeterminacy to arise for a much larger set of parameter configurations. We are also reminded that the presence or absence of indeterminacy is related only to the manager and the terms of his contract: nowhere does $\eta_s$ enter into the definition of the region of indeterminacy. This observation follows from the fact that the manager alone determines the firm’s investment decision in the delegated management economy.

In light of these observations, we find intriguing the recent decision of some financial institutions to alter the compensation structure for their managing directors by increasing the share of fixed compensation while retaining some of the contract convexity. For instance on May 23, 2009, the Wall-Street Journal reported: “Morgan Stanley Boosts Salaries as Its Bonuses Are Limited [...] Under the changes, managing directors will see about 25% to 30% of their overall compensation come from their base salary, up from about 15% to 20%.” Within the context of our model, and without a corresponding reduction in the convexity of the bonus portion of their compensation, such actions increase the likelihood of sunspot equilibria, if other firms act similarly.

### 3.7 Bond Trading

In Appendix D, we generalize the model of Sections 2 and 3 to allow the manager and the shareholder-workers to trade a one period default free bond in zero net supply. The question
is whether this generalization compromises our indeterminacy/instability results in a material way. In fact, it does not: the regions of indeterminacy/instability presented in Figures 1–4 generally expand when bond trading is introduced; that is, indeterminacy is more likely to arise in the sense that a wider class of contract parameters (misspecified relative to the Pareto optimal contract) will admit it (see Figures 5, 6 and 7). The intuition for this result is straightforward: the opportunity for bond trading does not diminish the manager’s expectation of a high personal return under a convex compensation contract. Viewing his contract as an "asset" whose expected return is both very high and risky, the presence of the bond provides an alternative safe "asset" by which this future income risk can be reduced. As the cases presented in Appendix D suggest, this fact encourages him to consume even more and further reduce his investment when confronted with a positive expectational shock. Appendix E, furthermore, confirms that contract (32), in conjunction with parameter choices specified in Theorem 1, remains optimal even if the structure of the economy is generalized to admit managerial bond trading.

4 Computing Equilibria

Our numerical work is guided by three questions: (1) Are sunspot shocks fully harmonious with a productivity shock in the sense of the addition of the former not compromising the overall model’s performance with regard to replicating the basic stylized facts of the business cycle? (2) In conjunction with a standard technology shock, does the addition of a sunspot shock in fact enhance the explanatory power of the model in the context, e.g., of labor market volatility? Lastly, (3) is it possible to generate a business cycle with the observed properties on the basis of sunspot shocks alone? If this is the case it becomes difficult to separate out the sources of business cycle fluctuations. For those skeptical of the notion of a productivity disturbance as an economic driver, such a result diminishes their significance, at least in our neoclassical context. It has the less attractive implication, however, of suggesting that future macroeconomic volatility may not be forecastable since it may in part be determined by pure belief shocks.
4.1 Numerical Strategy

The linearized rational expectations model presented in Section 3, can be rewritten in the canonical form:

\[ \Gamma_0 (\vartheta) s_t = \Gamma_1 (\vartheta) s_{t-1} + \Psi (\vartheta) \varepsilon_t + \Pi (\vartheta) \tau_t \]  

(55)

where the model parameters are collected in the vector \( \vartheta = [\beta, \eta_A, B, \eta_M, \mu, \theta, \varphi, \delta, A, \alpha, \Omega, \rho, \sigma] \), with \( \rho \) and \( \sigma \) denoting, respectively, the persistence and the standard deviation of the exogenous shock, and \( s_t \) representing the vector of the model’s endogenous variables:

\[ s_t = \begin{bmatrix} \hat{c}_t^s, \hat{c}_t^m, \hat{k}_t, \hat{n}_t, \hat{i}_t, \hat{d}_t, \hat{y}_t, \lambda_t, E_t\hat{n}_{t+1}, E_t\hat{c}_{t+1}^m, E_t\hat{d}_{t+1}, E_t\lambda_{t+1} \end{bmatrix}^\prime. \]

Finally, \( \tau_t \) denotes the vector of rational expectations forecast errors:

\[ \tau_t = \begin{bmatrix} (\hat{n}_t - E_{t-1}\hat{n}_t), (\hat{c}_t^m - E_{t-1}\hat{c}_t^m), (\hat{d}_t - E_{t-1}\hat{d}_t), (\lambda_t - E_{t-1}\lambda_t) \end{bmatrix}^\prime, \]

and \( \varepsilon_t \) represents innovations to exogenous productivity disturbances.

The model is solved using the solution algorithm developed by Sims (2000) as adapted to sunspot equilibria by Lubik and Schorfheide (2003). In the case of indeterminacy, in addition to the fundamental technology shock, the manager observes an exogenous sunspot shock, \( \nu_t \), which influences dynamics of the key macroeconomic variables. Consistency with rational expectations requires that the sunspot is i.i.d. with \( E_{t-1}\nu_t = 0 \).

Because of the linear structure of the model, the forecast errors for the next period labor, level of technology, dividend, and manager’s consumption can be expressed as function of two sources of uncertainty: the technology shock and the sunspot

\[ \tau_t = \Phi_1 \varepsilon_t + \Phi_2 \nu_t \]

where \( \Phi_1 \) and \( \Phi_2 \) have dimension \( 4 \times 1 \). The solution algorithm of Sims (2000) explicitly constructs a mapping from shocks to the expectation errors in (55). As shown above, when \( \delta \) is sufficiently small, our model has at least one stable solution. If \( \Phi_1 \) is uniquely determined by the parameters
and \( \Phi_2 = 0 \), the model has a unique solution. This is the case of determinacy in which the propagation mechanism of the technology shocks is uniquely determined. Thus, sunspots do not affect equilibrium allocations; neither do they induce fluctuations. If \( \Phi_1 \) is not uniquely determined by the parameters \( \vartheta \) and \( \Phi_2 \) is different from zero, the equilibrium is indeterminate. In this case, sunspot shocks can be interpreted as shocks to endogenous forecast errors. Detailed technical conditions for indeterminacy are developed in Lubik and Schorfheide (2003). In our model, it is the subset of \( \vartheta \) related to the manager’s compensation and risk aversion that is critical for indeterminacy; this is given by \( \vartheta_{\nu} = [\eta_m, \theta, \delta, \Lambda] \).

### 4.2 Calibration

We calibrate the model in two steps by first dividing the set of parameters described in vector \( \vartheta \) into two groups. The values for the first group correspond to their point estimates, obtained using data from the National Income and Product Accounts (NIPA). As is customary in real business cycle literature, we simulate the model at a quarterly frequency. We establish the quarterly subjective discount factor, \( \beta \) at 0.99, yielding a steady state risk free rate of return of 4% per year. The capital share of output, \( \alpha \), is chosen to be 0.36 (see Cooley and Prescott (1995) for a discussion) and the quarterly capital depreciation rate, \( \Omega \), fixed at 0.025. All three values are in line with empirical estimates and the values commonly used by the literature (see for instance Christiano and Eichenbaum (1992), Campbell (1994), Jermann (1998), King and Rebelo (1999), and Boldrin, Christiano and Fisher (2001)). Following earlier justification, the measure of managers, \( \mu \), is established at \( \mu = 0.05 \).

The remaining parameters in this subset principally concern the consumer-worker-shareholder. In particular, following Hansen (1985) and many others, we choose his coefficient of the intertemporal elasticity of substitution in consumption to be \( \eta_s = 1 \) and the inverse of the Frisch elasticity of labor supply to be \( \zeta = 0 \). Time allocation studies (e.g., Ghez and Becker (1975) and Juster and Stafford (1991)) estimate that the average time devoted to market activities in the U.S. is equal to one third of discretionary time. Consequently, we choose \( B = 2.85 \) so that the steady state value of labor, \( \pi_T \), is equal to one-third of the time endowment. With these parameter choices, our model is directly analogous to the representative agent construct of Hansen (1985) – a long
standing benchmark in the real business cycle literature.

The log of the technology process is of the form:

\[ \lambda_t = \rho \lambda_{t-1} + \varepsilon_t. \]

Several studies have found that this process is highly persistent (see Prescott (1986) for details). Accordingly, we choose the value of the persistence parameter, \( \rho \), in the AR(1) process equal to 0.95. When the model economy is driven by the technology shocks alone, we select the volatility, \( \sigma_\varepsilon \), to match the empirical standard deviation of departures from trend output in the U.S. data (1.81%). The i.i.d. sunspot shock, if present, is distributed \( N(0; \sigma_v) \). As with the shock to productivity, we choose the standard deviation \( \sigma_v \) to match the volatility of output, \( \sigma_y \), observed in the data, a common practice in the literature on indeterminacy when the model’s fluctuations result from sunspots shocks only.

Our focus is exclusively on executive incentive contracts of the form (32) in equilibrium. Following Jensen, Murphy and Wruck (2004), the salary component \( \varphi = \chi \mu \) is fixed equal to one half of the overall steady state managerial compensation. In Section 4.4 we present some sensitivity analysis with respect to changes in this parameter.

Precise a priori knowledge for the second group of parameters is unavailable. This subset of parameters, denoted by \( \vartheta_1 \), includes the manager’s elasticity of intertemporal substitution in consumption, \( \eta_m \), the share of the aggregate wage bill in the manager’s contract, \( \delta \), and the contract convexity parameter, \( \theta \). When the model economy incorporates technology shocks and the sunspot shocks simultaneously, there is no obvious way to estimate the individual variances of these shocks or their correlation. Vector \( \vartheta_1 \) thus also includes \( \sigma_\varepsilon, \sigma_v, \) and \( \rho_{\varepsilon v} \).\(^{34}\)

Following Jermann (1998) and others, we choose the parameter values for the second group in a way that maximizes the model’s ability to replicate certain business cycle moments. Let \( \vartheta_1 \) denote the vector of remaining model parameters to calibrate, \( \vartheta_1 = [\eta_m, \delta, \theta, \sigma_\varepsilon, \sigma_v, \rho_{\varepsilon v}] \), and \( g_T \) denote

\(^{34}\)To our knowledge, there is no reliable procedure to estimate the properties of the sunspot shock process from the data. Farmer and Guo (1995) and Salyer and Sheffrin (1998) try to identify sunspot shocks from rational expectations residuals that are left unexplained by exogenous shocks to fundamentals. But this technique is sensitive to model’s misspecification and cannot distinguish between actual sunspots and missing fundamentals. Lubik and Schorfheide (2004) develop an econometric technique to assess the quantitative importance of equilibrium indeterminacy in dynamic stochastic general equilibrium models, such as ours.
the set of data moments to match. In our case, \( g_T \) includes the standard deviations of output, total consumption, investment, and labor, and contemporaneous correlations of consumption and labor with output characteristic of the U.S. economy. We calibrate \( \theta_1 \) as the solution that minimizes the following criterion:

\[
J(\theta_1) = \left[ g_t - f(\theta_1) \right]^T \Sigma^{-1} \left[ g_T - f(\theta_1) \right]
\]

where \( f(\theta_1) \) is the vector of moments implied by the model for a given realization of \( \theta_1 \), \( \Sigma \) is a weighting matrix and \( g_T \) the vector of the point estimates of the target moments, computed using the data. The matrix \( \Sigma \) is a diagonal matrix with the standard errors of the estimates in \( \theta_1 \) on the main diagonal. This calibration procedure thus minimizes a weighted average of the moment deviations.

Using the full set of parameters chosen as per above, the model’s optimal policy functions were computed in the manner described in Section 4.1, and artificial time series generated accordingly. These series were then detrended using the Hodrick-Prescott filter, and the model’s business cycle statistics computed from their detrended components. We evaluated the criterion \( J(\theta_1) \) for the following regions of parameter values: \( n_m \in [0.00, 1.00], \delta \in [0.01, 1.00], \theta \in [0.5, 5], \sigma_\varepsilon \in [0.005, 0.05], \sigma_\nu \in [0.005, 0.1], \) and \( \rho_{\varepsilon\nu} \in [-1, 1] \) with a partition norm equal to .01. The choice of regions for the first three parameters guarantees that the Baseline Model supports multiple equilibria.\(^{35}\) Table 1 summarizes the results of the full calibration procedure.

4.3 Quantitative Results of the Baseline Model

To calculate business cycle statistics, we computed averages of statistical quantities repeatedly for 500 sample paths each of 200 periods length. Summarizing model performance in this way is customary in the business cycle literature. This procedure does, however, tend to mask the sort of extreme behavior that might be associated with sunspot equilibria.

Table 2 considers several fundamental cases. In Panel B, we present business cycle statistics obtained from the version of the model parameterized as in Table 1 with technology shocks being

\(^{35}\)Note also that these regions include the parameter values that identify the optimal contract.
the single source of exogenous uncertainty. Nevertheless, the response of macro aggregates to this fundamental shock is indeterminate since the model, as parameterized, results in multiple equilibrium solutions. Panel C summarizes the model in which technology and sunspot shocks are both present with the indicated volatilities. In Panel D only the i.i.d. sunspot shock drives business cycle fluctuations. Panel A contains statistics estimated from the U.S. data, where available, and Panel E shows business cycle statistics from the Hansen (1985) indivisible labor model – a long-standing benchmark in the real business cycle literature. It also describes the analogous delegated management economy under the optimal contract for the indicated parameter values.

[Insert Table 2 about here]

Panel B (technology shock alone) easily respects the most basic stylized facts of the business cycle: investment is more volatile than output which is in turn more volatile than shareholder (and aggregate) consumption. Hours and investment are somewhat too smooth, however. Their relative standard deviations in the model are 0.24 and 1.91 respectively compared to 0.95 and 2.93 in the data. The Hansen (1985) model is more successful in replicating these particular statistics, but the model with the delegated manager is able to match the relative standard deviation of consumer-worker-shareholder’s (and total) consumption more successfully. In the Hansen (1985) model, consumption does not vary enough. This happens because the equilibrium wage rate is more variable in our model than in Hansen’s. When fluctuations in the model result from the technology shock alone, the manager’s consumption is very smooth, more so than the consumption of shareholders. The manager changes investment and hiring policies in response to these shocks very moderately because in this case he does not expect any extra return on capital (no sunspots). This version of the model also replicates contemporaneous correlations with output as well as the Hansen (1985) model.

In Panel C, we incorporate sunspot shocks in addition to technology shocks into the model. This is the Baseline case. Comparison of results from Panels B and C shows that with two shocks, the model with the delegated manager and the convex executive incentive contract is able to match, closely, the relative standard deviations of employment, investment and shareholders’ consumption. For all the major aggregates the same can be said of the cross-correlations with output. Being a
convex function of the dividend, which is itself a highly variable residual series in this version of the model, managerial consumption volatility appropriately exceeds that of the consumer-worker-shareholders.

Comparing the volatilities presented in Panel C with those obtained from U.S. data (Panel A), it would appear that sunspot shocks, when introduced into this production model setting, are fully harmonious with technology shocks (question 1). With the addition of sunspot shocks the model is also largely consistent with the stylized business cycle facts (question 2). In fact, the case with two shocks arguably does a better job of replicating the data than does the seminal paradigm of Hansen (1985) (Panel E). In particular, consumption volatility much more closely matches the data. The negative correlation of managerial consumption with output simply reflects the like correlation of its dividend base. Thus the answer to the second question, posed in the beginning of this section is also in the affirmative: sunspots can enhance the ability of the model to replicate salient business cycle facts.

Sunspot shocks alone (Panel D), however, give rise to a number of data inconsistencies. First, hours and investment are excessively volatile. Note that the volatility of the sunspot shock must more than double in order to compensate for the absence of technological uncertainty (Panel C vs. Panel D), if the required output volatility is to be maintained. This fact is not entirely surprising since sunspot shocks do not affect output directly, and thus must induce large responses in hours and investment in order to replicate $\sigma_y$ at the empirically observed $\sigma_y = 1.81\%$. As a result, $\sigma_i$ and $\sigma_n$ are high relative to their empirical counterparts. Dividends give rise to most of the variation in managerial compensation and these are countercyclical. As a result, managerial consumption is countercyclical as well. Being a residual after the wage bill and investment, the dividend, and thus managerial consumption, is also highly volatile.

The other major inconsistency is reflected in the negative correlation of consumer-worker-shareholder consumption with output. By implication, total consumption is negatively correlated with output as well. Sunspot equilibria, per se, seem to have manifestations that violate the notion of consumption as a normal good, at least in this case. Recall that a sunspot shock is essentially a rate of return on capital stock, and that a favorable sunspot shock induces very large procyclical responses in investment without output being itself simultaneously increased. In equilibrium, con-
sumption must therefore be countercyclical. Note that for all three of the considered cases contract convexity is a relatively modest $\theta = 3$.

4.4 Robustness Checks with Respect to Changes in Key Parameters

Table 3 explores the consequences of greater contract convexity, a larger salary component, and higher managerial risk aversion in the context of the Baseline case of Table 2, Panel C. We leave all other parameters (including standard deviations of shocks) unchanged (relative to the baseline case) to illustrate better the effect of changes in the three key parameters for indeterminacy on the ability of the model to replicate basic business cycle facts. The Table 3 cases illustrate indeterminate equilibria all of which provide reasonable replications of the statistical summary of the U.S. business cycle. It reflects the fact that indeterminacy is robust to a wide class of contract parameters and risk aversion levels for the manager.

[Insert Table 3 about here]

In Panel B (higher contract convexity with $\theta = 5$), the relative volatility of hours and investment is higher than in the baseline case (Panel A). Higher contract convexity encourages the manager to take greater advantage of a favorable shock by increasing his own consumption while reducing investment and the labor input, and vice versa with an unfavorable shock. This change leads to higher managerial consumption volatility and the reduced volatility of investment and hours as observed. Nevertheless, the degree of contract convexity does not dramatically alter the basic volatility and correlation structure of the various series.

Panel C in Table 3 explores the consequences of altering the level of the salary component. For the underlying parameters of $\eta_m = 0.25$ and $\theta = 3$, sensitivity analysis reveals that a minimum value of salary component share in the manager’s compensation $A/\sigma^m = 0.15$ appears to be necessary for indeterminate equilibria to arise. Increasing the magnitude of the fixed salary component increases the overall volatility in the economy once indeterminate equilibria are achieved. The effect is more pronounced in case of relative standard deviations of hours and investment. Increasing the weight of the fixed salary in the overall compensation package of the manager makes him effectively more risk tolerant and more willing to alter production plans in response to shocks. It also reduces his
own relative consumption volatility.

Panel D in Table 3 concerns the consequences of the enhanced managerial elasticity of intertemporal substitution in consumption (i.e., managerial risk aversion in case of the utility function (16)). Sensitivity analysis shows that the model economy exhibits local indeterminacy if the manager’s risk aversion coefficient does not exceed 0.49 with other parameters held at their baseline values.

As is evident, the influence of the increase in managerial risk aversion is very modest provided the equilibria are indeterminate. Indeed, for Panel D, not only are the stylized business cycle facts quite well replicated but the results seem relatively unaffected (relative to the benchmark) by the degree of managerial risk aversion (provided $\eta_m < 0.49$). Furthermore, in none of the cases presented in Table 3 is managerial consumption volatility particularly excessive, ranging only to roughly four times that of the average shareholder-worker.

In summary, the preceding cases inform us along a number of dimensions with regard to the three questions posed at the start of this section. First, the results from not only the baseline case with technology and sunspot shocks (Table 2, Panel C) but also many of the other cases suggest that these sources of uncertainty are fully compatible with one another, at least for this model framework. Our quantitative results suggest that one source of uncertainty can effectively be traded off against the other (with regard to the relative magnitudes of $\sigma_z$ and $\sigma_v$), a direct consequence being that a large increase in future sunspot volatility would not be inconsistent with past business cycle history as it is currently understood and measured.

With regard to our second question, standard business cycle models (e.g., Hansen, 1985) have difficulty replicating the relative volatility of hours. The addition of sunspot shocks appears to resolve this shortcoming.

Finally, within the realm of the simple construct we provide, it does not appear that convex-contract-induced sunspot equilibria, alone, can replicate the stylized facts of the business cycle. Shareholder consumption in these cases is negatively correlated with output and hours and investment exhibit excessive volatility. Such is the response to the third question posed.
5 Conclusion

The message of this paper is clear and direct: when confronted with compensation contracts which are mildly convex to the firm’s stock price or free cash flow, CEOs may well find it in their self-interest to adopt investment policies that lead to equilibrium indeterminacy or instability. As a result, the time path of the economy’s macroeconomic aggregates, as well as the executives’ compensation, at least with respect to their volatility, may bear little association to fundamentals. In this sense, convex CEO compensation contracts may substitute for technological increasing returns, a typical requirement of the earlier indeterminacy literature. Within a standard dynamic macroeconomic setup, these results appear to hold for a wide class of model parameters, at least for the compensation contracts studied here.

These results suggest that the early twenty-first century explosion in the incentive compensation among financial firms may have unforeseen consequences. We are only now beginning to see what these consequences are.
A Appendix A: Proof of Theorem 1

Section 3.1 describes the equilibrium conditions resulting from the model with a compensation contract (32). To prove Theorem 1 we simply need to show that in the case that the contract’s parameters take the values $A = 0$, $0 < \varphi = \Lambda^{1/\eta_m}$, $\delta = 1$, $\gamma = \eta_s/\eta_m > 0$ and $\theta = 1$, the equilibrium conditions reduce to expressions (19) - (31), which fully characterize the optimal plan. First, note that when the contract satisfies the restrictions just mentioned

$$
\begin{align*}
    c^m_t &= \Lambda^{1/\eta_m} (w_t n_t + d_t)^{\eta_s/\eta_m} \\
    c^s_t &= w_t n_t + d_t,
\end{align*}
$$

(A.1) (A.2)

so that the consumption of the manager and the shareholder satisfy the optimal risk sharing condition (19), (25) and (26). The variable $x_t$, given in (40), reduces to the constant

$$
x_t = \varphi = \Lambda^{1/\eta_m}
$$

so that the (39) reduces to

$$
(c^m_t)^{-\eta_m} = \beta E_t \left[ \left( c^m_{t+1} \right)^{-\eta_m} r_{t+1} \right].
$$

This, together with (A.1) and (A.2) above, yields the consumer-worker-shareholder’s optimal investment condition for the intertemporal allocation of consumption (23). All other conditions are identical to the remaining expressions characterizing the optimal plan, (20) - (22), (24), (27) - (31). It follows that this contract results in all equilibrium conditions characterizing the optimal plan being satisfied. This contract is thus optimal.

B Appendix B: Proof of Theorem 2

Section 3.1 describes the equilibrium conditions resulting from the model with a compensation contract (32). Suppose that the measure of managers $\mu = 0$, and that $\eta_s$ and $\eta_m$ satisfy either $0 \leq \eta_s < 1$, $0 \leq \eta_m < 1$, or $1 < \eta_s$, $1 < \eta_m$. To prove Theorem 2, we simply need to show that in the case that a contract of the form (32) has coefficients with values $A = 0$, $0 < \varphi = \Lambda^{1/\eta_m}$, $\delta = 1$, $\gamma = 1$, and $\theta = \frac{1-\eta_s}{1-\eta_m} > 0$, the equilibrium conditions reduce to expressions (20) - (31), characterizing the optimal plan. Note that in the case that $\mu = 0$ considered here, the optimal risk-sharing condition (19) need not be satisfied in an optimal equilibrium where there is a measure $\mu$ of managers. When the contract satisfies the restrictions just mentioned, we obtain

$$
\begin{align*}
    c^m_t &= \varphi (w_t n_t + d_t)^{1-\eta_s/\eta_m} \\
    c^s_t &= w_t n_t + d_t,
\end{align*}
$$

so that (25), (26) hold and

$$
(c^m_t) = \varphi (c^s_t)^{1-\eta_s/\eta_m}.
$$

(B.1)

The variable $x_t$, given in (40), reduces to

$$
x_t = \frac{1 - \eta_s \Lambda^{1-\eta_s} (c^m_t)^{\eta_m-\eta_s} (c^m_t)^{\eta_m-\eta_s}}{1 - \eta_m}. 
$$
so that equation (39) reduces to

\[
\left(c_t^m \right)^{\eta_m - \eta_s - \eta_m} = \beta E_t \left[ \left(c_{t+1}^m \right)^{\eta_m - \eta_s - \eta_m} r_{t+1} \right],
\]

when \( \mu = 0 \). Using (B.1) to replace \( c_t^m \), this expression reduces to the consumer-worker-shareholder’s optimal condition for intertemporal consumption allocation (23)

\[
(c_t^s)^{-\eta_s} = \beta E_t \left[ (c_{t+1}^s)^{-\eta_s} r_{t+1} \right].
\]

All other conditions are identical to the remaining expressions characterizing the optimal plan, (20)–(22), and (24)–(31). The proposed contract is thus optimal when \( \mu = 0 \).

## Appendix C: Proof of Theorem 3

To characterize the regions of determinacy, indeterminacy and instability, we rewrite the dynamic equations (51)–(52) in matrix form:

\[
A \begin{bmatrix} E_t c_t^m \\ \hat{k}_{t+1} \end{bmatrix} = B \begin{bmatrix} \hat{c}_t^m \\ \hat{k}_t \end{bmatrix} + C \lambda_t
\]

where

\[
A \equiv \begin{bmatrix} 1 & A_{12} \\ 0 & 1 \end{bmatrix}, \quad B \equiv \begin{bmatrix} 1 \\ B_{21} \quad B_{22} \end{bmatrix}
\]

and

\[
A_{12} = \psi^{-1}(1 - \alpha)(1 - \beta (1 - \Omega))
\]

\[
B_{21} = - \left( \frac{1}{(1 - \omega) \tilde{Z} \hat{k}} + \mu \frac{\tilde{c}_t^m}{\hat{k}} \right) < 0
\]

\[
B_{22} = (\beta^{-1} - (1 - \Omega))(1 - \alpha) \delta + (1 - \Omega)(1 - \alpha) + \alpha \beta^{-1} > 0.
\]

Note that \( \text{sign}(A_{12}) = \text{sign}(\psi) \).

Given that \( \hat{k}_t \) is a predetermined variable and \( \hat{c}_t^m \) is a nonpredetermined variable, the system admits a single bounded solution if and only if the eigenvalues \( \phi_1 \) and \( \phi_2 \) of the matrix \( M = A^{-1}B \) satisfy \( 0 \leq |\phi_1| < 1 < |\phi_2| \). The equilibrium is indeterminate if \( 0 \leq |\phi_1| < 1 \) and \( 0 \leq |\phi_2| < 1 \). There exists no bounded solution (and so there exist only explosive solutions) if \( 1 < |\phi_1| \) and \( 1 < |\phi_2| \).

It will be useful to appeal to the following proposition:

**Proposition 4** (Proposition 3.1 of Woodford (2003, p. 670)). Both eigenvalues of a 2 \times 2 matrix \( N \) lie outside the unit circle if and only if:

**either** (Case I):

\[
\text{det} (N) > 1 \quad \text{(C.1)}
\]

\[
\text{det} (N) - \text{tr} (N) + 1 > 0 \quad \text{(C.2)}
\]

\[
\text{det} (N) + \text{tr} (N) + 1 > 0 \quad \text{(C.3)}
\]

42
or (Case II):

\[
\begin{align*}
\det (N) - tr (N) + 1 & < 0 \\
\det (N) + tr (N) + 1 & < 0.
\end{align*}
\]

**C.1 Indeterminacy**

The system has an indeterminate equilibrium if and only if

\[
1 < |1/\phi_1| \text{ and } 1 < |1/\phi_2|,
\]

or equivalently if both eigenvalues of

\[
M^{-1} = \begin{bmatrix}
1 & A_{12} \\
-\frac{B_{21}}{B_{22}} & \frac{1}{B_{22}} (1 - A_{12} B_{21})
\end{bmatrix}
\]

lie outside the unit circle. Note that

\[
\det (M^{-1}) = 1/B_{22} \text{ and } tr (M^{-1}) = 1 + (1 - A_{12} B_{21}) / B_{22}.
\]

First suppose that \(\det (M^{-1}) = 1/B_{22}\) and \(tr (M^{-1}) = 1 + (1 - A_{12} B_{21}) / B_{22}\). Since \(B_{22} > 0\), we must have \(\det (M^{-1}) + tr (M^{-1}) + 1 = 2 \left(\frac{B_{22} + 1}{B_{22}}\right) - A_{12} \frac{B_{21}}{B_{22}} > 0\). So we cannot simultaneously satisfy both conditions (C.4)–(C.5) of Case II.

We thus have an indeterminate equilibrium if and only if all three conditions (C.1)–(C.3) of Case I are satisfied, i.e., if and only if

\[
0 < B_{22} < 1 \\
A_{12} \frac{B_{21}}{B_{22}} > 0 \\
2 \left(\frac{B_{22} + 1}{B_{22}}\right) - A_{12} \frac{B_{21}}{B_{22}} > 0.
\]

Given that \(B_{22}\) is positive and increasing in \(\delta\), the first condition is satisfied for \(\delta\) sufficiently small

\[
(\beta^{-1} - (1 - \Omega)) (1 - \alpha) \delta + (1 - \Omega) (1 - \alpha) + \alpha \beta^{-1} < 1
\]

or equivalently

\[
\delta < \delta^* \equiv \frac{1 - (1 - \Omega) (1 - \alpha) - \alpha \beta^{-1}}{(\beta^{-1} - (1 - \Omega)) (1 - \alpha)}. \quad (C.6)
\]

Given \(B_{22} > 0\) and \(B_{21} < 0\), the second condition is satisfied if and only if \(A_{12} < 0\), or equivalently if and only if

\[
\psi < 0. \quad (C.7)
\]

The third condition is in turn satisfied if and only if \(2 (B_{22} + 1) > \psi^{-1} (1 - \alpha) (1 - \beta (1 - \Omega)) B_{21}\) or equivalently if and only if

\[
\psi^{-1} \psi^* < 1 \quad (C.8)
\]

where

\[
\psi^* \equiv \frac{(1 - \alpha) (1 - \beta (1 - \Omega)) B_{21}}{2 (B_{22} + 1)} < 0.
\]

If (C.7) holds, then (C.8) can be rewritten as \(\psi < \psi^*\). It follows that conditions (C.7) and (C.8) jointly hold if and only if

\[
\psi < \psi^* < 0. \quad (C.9)
\]
To summarize, the model admits an indeterminate equilibrium if and only if (C.6) and (C.9) are jointly satisfied.

C.2 Instability

The system admits no bounded solution if and only if both eigenvalues of $M$ lie outside the unit circle. Note that $\det (M) = B_{22}$ and $tr (M) = 1 + B_{22} - A_{12}B_{21}$.

First suppose that $\det (M) - tr (M) + 1 = A_{12}B_{21} < 0$. Since $B_{22} > 0$, we must have $\det (M) + tr (M) + 1 = 2 (B_{22} + 1) - A_{12}B_{21} > 0$. So we cannot simultaneously satisfy both conditions (C.4)-(C.5) of Case II. Thus we have explosive solutions if and only if all three conditions (C.1)-(C.3) of Case I are satisfied, that is:

\[
\begin{align*}
B_{22} & > 1 \\
A_{12}B_{21} & > 0 \\
2 (B_{22} + 1) - A_{12}B_{21} & > 0.
\end{align*}
\]

The first condition is satisfied for $\delta$ sufficiently large

\[
\delta > \delta^*.
\]

(C.10)

Given $B_{21} < 0$, the second condition is satisfied if and only if $A_{12} < 0$, or equivalently if and only if (C.7) holds. The third condition is in turn satisfied if and only if (C.8) holds. The second and third conditions thus jointly hold if and only if (C.9) is satisfied. To summarize, the model admits no bounded solution if and only if (C.9) and (C.10) are jointly satisfied.

C.3 Determinacy

The system admits a unique bounded solution if and only if the equilibrium is neither indeterminate nor unstable, that is, if and only if $\psi \geq \psi^*$.

\[\square\]

D Appendix D: Generalized Model with Manager Participating on Financial Markets

In the model of Section 3, the managers have no access to financial markets. They may smooth their consumption over time by making investment decisions in the firm that result in the desired income pattern, but have no other opportunity to borrow or lend. We now extend this model to allow the managers to buy or sell riskless bonds and demonstrate that the results obtained in Section 3 are not sensitive to our assumption that managers are excluded from financial markets.

The model presented below is essentially identical to the one of Section 3 augmented with (i) the possibility for managers to access the bond market, and (ii) one technical change incorporated so that log-linear approximation techniques can, in this more general setting, still provide reliable conclusions pertaining to local determinacy or explosiveness of the general equilibrium. This technical change incorporates tiny time variation in the agents’ discount factors in order to retain stationarity of the bond holdings. In the absence of such time variation, when both managers and consumer-shareholder-workers have access the bond market, transitory productivity shocks...
will generally have a permanent effect on the agents’ consumption to the extent that one group of agents may borrow from the other group on impact and make interest payments forever.

The flow of ideas in this section is straightforward. First we characterize the competitive equilibrium under bond trading in an environment where managers are compensated according to the contract specified in (32) We then describe the sets of parameter combinations which lead to equilibrium indeterminacy/instability. These sets are presented in a manner directly analogous to the regions described in the prior figures 1–4.

Accordingly, we begin by detailing the consumer-worker and manager problems for this expanded setting.

D.1 The Consumer-Worker-Shareholder’s Problem

The representative consumer-worker-shareholder chooses processes for consumption \( c_t^s \), labor supply \( n_t^s \), bond holdings \( b_t^s \), and equity holdings \( z_t^e(f) \) to maximize his expected utility

\[
E_0 \left[ \sum_{t=0}^{\infty} \chi_{t-1} \left( \frac{(c_t^s)^{1-\eta_s}}{1-\eta_s} - B \frac{(n_t^s)^{1+\zeta}}{1+\zeta} \right) \right]
\]  

subject to his budget constraint

\[
c_t^s + \int q_t^b(f)z_{t+1}^e(f)df + q_t^b b_{t+1}^s = \int (q_t^b(f) + d_t(f)) z_t^e(f)df + w_t n_t^s + b_t^s.
\]  

As in Section 3, \( w_t \) denotes the competitive wage rate, \( q_t^b \) and \( q_t^e \), are respectively bond and equity prices, \( d_t \) denotes the equity security’s dividend. (We assume that each household is holding a fully diversified portfolio of shares of each firm in equal proportions so that the dividends \( d_t \) are aggregated over all firms.) Consistent with Ferrero, Gertler and Svensson (2009), the discount factor \( \chi_t \) evolves according to

\[
\chi_t = \beta_t \chi_{t-1}, \text{ with } \chi_{-1} = 1,
\]

where \( \beta_t \equiv \beta \left[ 1 + \phi \left( \log \bar{c} - \bar{\beta} \right) \right] / \left[ 1 + \phi \left( \log c_t - \bar{\beta} \right) \right],
\]

D.2 The Manager’s Problem

The manager of firm \( f \) chooses processes for his own consumption, \( c_t^m(f) \), hiring decisions, \( n_t(f) \), investment \( i_t(f) \) in the physical capital stock, \( k_t(f) \), dividends, \( d_t(f) \), and riskless bond holdings

\footnote{This formulation of the discount factor incorporates the stimulative effect on individual consumption of an increase in average consumption, as in Uzawa (1968). However the parameter \( \phi \) is calibrated to such a small value that this effect is negligible. As mentioned above, it merely serves as a technical device to guarantee a unique steady state in the case of incomplete financial markets across groups of agents. One can alternatively obtain a such a unique steady state by assuming a constant discount factor \( \beta \), but introducing a debt-elastic interest rate premium in the budget constraints (D.2) and (D.5) below, as in Benigno (2001), Kollmann (2002), Schmitt-Grohe and Uribe (2003), and Justiniano and Preston (2010).}
$b^m_t (f)$ to maximize her discounted expected utility

$$E_0 \left[ \sum_{t=0}^{\infty} \chi_{t-1} c^m_t (f)^{1-\eta_m} \right]$$

subject to

$$c^m_t (f) + q^b_t b^m_{t+1} (f) \leq g^m_t (f) + b^i_t (f)$$

$$d_t (f) = y_t (f) - w_t n_t (f) - i_t (f) - \mu g^m_t (f)$$

$$y_t (f) = (k_t (f))^\alpha (n_t (f))^{1-\alpha} e^{\lambda t}$$

$$k_{t+1} (f) = (1-\Omega) k_t (f) + i_t (f), \quad k_0 (f) \text{ given.}$$

The manager’s budget constraint (D.5) states that her consumption and the value of her newly purchased bonds can be no larger than her compensation, $g^m_t (f)$, plus the value of the bond holdings in the prior period. Equation (D.6) states that dividends, $d_t (f)$, are given by the free cash flows of the firm. Finally (D.7) specifies the firm’s production technology, while (D.8) describes the firm’s capital accumulation. The manager chooses to operate the firm provided that

$$E_0 \left[ \sum_{t=0}^{\infty} \chi_{t-1} (c^m_t)^{1-\eta_m} \right] \geq \frac{\pi^m}{1-\beta}.$$

### D.3 The Decentralized Equilibrium

We next define the decentralized equilibrium in which the consumer-shareholder-worker chooses his own consumption, labor supply, bonds and equity holdings and in which the manager of firm $f \in [0,1]$ chooses her own stream of consumption, makes decisions about physical investment, hiring, and dividend payouts, and is paid according to the following pre-specified compensation contract:

$$g^m_t (w_t n_t, d_t, d_t (f)) = A + \varphi \left[ \delta (w_t n_t + d_t)^\gamma + d_t (f) - d_t \right]$$

for non-negative coefficients $A, \delta, \varphi, \gamma$ and $\theta$. The relevant contract is identical to (32).

The consumer-shareholder-worker chooses $c^s_t, n_t, z_{t+1} (f), b^s_{t+1}$ to maximize his utility function (D.1) subject to his budget constraint (D.2) and (D.3). This yields the first-order necessary conditions

$$\Lambda_{1t} = (c^s_t)^{-\eta_s}$$

$$\Lambda_{1t} w_t = B (n_t^z)^\zeta$$

$$q^s_t (f) = E_t \left[ \beta_t \Lambda_{1t+1} (q^s_{t+1} (f) + d_{t+1} (f)) \right]$$

$$q^b_t = E_t \left[ \beta_t \Lambda_{1t+1} \right]$$

which are identical to (10)–(13), except for $\beta_t$ being allowed to vary.

Each manager in turn decides whether to manage the firm or instead whether receive her reservation utility. The measure $\mu$ of managers who work in firm $f$ chooses $c^m_t (f), n_t (f), k_{t+1} (f), b^m_{t+1} (f), d_t (f)$ to maximize their utility (D.4) subject to the restrictions (D.5)–(D.8) and the compensation contract (D.10). We will assume that the compensation is sufficiently generous for each man-
The necessary first-order conditions with respect to $c^m_t(f), n_t(f), b^m_{t+1}(f), k_{t+1}(f), d_t(f)$ are

$$\Lambda_{3t}(f) = (c^{m}_t(f))^{-\eta_m} \sum_{t=0}^{\infty} \chi_{t-1} \left( \frac{\left(c^{m}_t(f)\right)^{1-\eta_m}}{1-\eta_m} \right) + \Lambda_{2t}(f) \left[ (k_t(f))^\alpha (n_t(f))^{1-\alpha} e^{\lambda_t} - w_t n_t(f) - k_{t+1}(f) + (1-\Omega) k_t(f) \right] - \mu \left( A + \varphi [ (\delta w_t n_t + d_t)\gamma + d_t(f) - d_t]^{\theta} - d_t(f) \right) + \Lambda_{3t}(f) \left[ A + \varphi [(\delta w_t n_t + d_t)\gamma + d_t(f) - d_t]^{\theta} + b^m_t(f) - c^m_t(f) - d_t b^m_{t+1}(f) \right].$$

The necessary first-order conditions with respect to $c^m_t(f), n_t(f), b^m_{t+1}(f), k_{t+1}(f), d_t(f)$ are

$$\Lambda_{3t}(f) = (c^{m}_t(f))^{-\eta_m} \sum_{t=0}^{\infty} \chi_{t-1} \left( \frac{\left(c^{m}_t(f)\right)^{1-\eta_m}}{1-\eta_m} \right) + \Lambda_{2t}(f) \left[ (k_t(f))^\alpha (n_t(f))^{1-\alpha} e^{\lambda_t} - w_t n_t(f) - k_{t+1}(f) + (1-\Omega) k_t(f) \right] - \mu \left( A + \varphi [ (\delta w_t n_t + d_t)\gamma + d_t(f) - d_t]^{\theta} - d_t(f) \right) + \Lambda_{3t}(f) \left[ A + \varphi [(\delta w_t n_t + d_t)\gamma + d_t(f) - d_t]^{\theta} + b^m_t(f) - c^m_t(f) - d_t b^m_{t+1}(f) \right].$$

$$\Lambda_{2t}(f) = E_t \left\{ \beta_t \Lambda_{3t+1}(f) \right\} \frac{x_t(f)}{1 + \mu x_t(f)}$$

for all $f$, where $x_t(f)$ is again defined as in (38), so that

$$x_t(f) = \theta \varphi \left[ (\delta w_t n_t + d_t)^{\gamma} + d_t(f) - d_t]^{\theta-1}. \right.$$

### D.3.1 Equilibrium definition

The decentralized equilibrium is a set of processes $\{c^e_t, c^m_t, n_t^e, n_t^m, b^e_t, b^m_t, z^e_t, z^m_t, i_t, y_t, k_t, r_t, w_t, q^b_t, q^e_t, d_t, \beta_t, \chi_{t-1}\}$ such that:

1. The first-order conditions (D.11–D.20) are satisfied together with the constraints (D.2), (D.3), (D.5)–(D.8), (31), $x_t = \beta_t \chi_{t-1}$, all holding with equality, and the transversality condition: $\lim_{t \to \infty} \chi_{t-1} u^m_t(c^m_t) k_{t+1} = 0$, for any given initial $k_0$, and for $\chi_{-1} = 1$.

2. The labor, goods and capital markets clear: $n^e_t = n_t$; $y_t = i_t + c^e_t + \mu c^m_t$; and investors hold all outstanding equity shares, $z^e_t = 1$, and all other assets (one period bonds) are in zero net supply, $b^e_t + \mu b^m_t = 0$.

By imposing market clearing relationships on the necessary and sufficient first order conditions detailed above, we can also fully characterize the equilibrium as the set of processes
\{c_t^e, c_t^m, n_t^e, n_t^m, b_t^e, b_t^m, z_t^e, i_t, y_t, k_t, r_t, w_t, q_t^b, q_t^e, d_t, \beta_t\} which satisfy:

\[(c_t^e)^{-\eta_e} w_t = B (n_t)^{\xi} \]
\[q_t^e = E_t \left[ \beta \left( \frac{c_t^{e+1}}{c_t^e} \right)^{-\eta_e} \left( q_{t+1}^e + d_{t+1} \right) \right] \]
\[q_t^b = E_t \left[ \beta \left( \frac{c_t^{e+1}}{c_t^e} \right)^{-\eta_m} \right] \]
\[q_t^b = E_t \left[ \beta \left( \frac{c_t^{e+1}}{c_t^e} \right)^{-\eta_m} \right] \]
\[r_t \equiv 1 - \Omega + \alpha y_t / k_t \]
\[w_t = (1 - \alpha) y_t / n_t \]
\[c_t^e = w_t n_t + d_t - \mu \left( b_t^m - q_t^b b_{t+1}^m \right) \]
\[c_t^m = A + \varphi (\delta w_t n_t + d_t)^\gamma \theta + b_t^m - q_t^b b_{t+1}^m \]
\[d_t = y_t - w_t n_t - i_t - \mu \left( A + \varphi (\delta w_t n_t + d_t)^\gamma \theta \right) \]
\[y_t = k_t^{a_0} n_t^{1-a} e^{\lambda t} \]
\[k_{t+1} = (1 - \Omega) k_t + i_t \]
\[\beta_t \equiv \beta \left[ 1 + \phi \left( \log c_t - \bar{\varphi} \right) \right] / \left[ 1 + \phi \left( \log c_t - \bar{\varphi} \right) \right] \]
\[(c_t^m)^{-\eta_m} = E_t \left[ \beta_t \left( c_t^{m+1} \right)^{-\eta_m} \left( \frac{1 + \mu x_t}{1 + \mu x_{t+1}} \right) \frac{x_{t+1}}{x_t} r_t+1 \right] \]
\[x_t = \theta \varphi (\delta w_t n_t + d_t)^\gamma (\theta - 1) \]

where \(k_0\) is given, and where again we assume that the manager’s compensation is large enough for his participation constraint (D.9) to be satisfied.

### D.3.2 Approximating the Decentralized Equilibrium around the Deterministic Steady State

We assume that in the steady state, consumer-shareholder-workers and managers hold no nominal bonds (\(\hat{b}^m = 0\)). As a result, the steady state in this economy is the same as the model of Section 3. Denoting the steady state value of a variable with an overhead bar, it is defined as the solution to the following set of equations: \(\bar{c}^m = A + (\varphi \delta \bar{w} \bar{n} + \varphi \bar{d})^\gamma \theta \); \(\bar{w} \bar{n} = (1 - \alpha) \bar{y} \); \(\bar{y} = \bar{k}^{a_0} \bar{n}^{1-a} \); \(\Omega \bar{k} = \bar{r} \); \(\bar{x} = \varphi \theta (\delta \bar{w} \bar{n} + \bar{d})^\gamma (\theta - 1) \); \(\bar{r} = \alpha \bar{y} / \bar{k} + 1 - \Omega \); \(\beta^{-1} = \bar{r} \); \(\bar{c}^e = \bar{w} \bar{n} + \bar{d} \) and \((\bar{c}^e)^{-\eta_e} \bar{w} = B \bar{n}^{\xi} \). Denoting the log-deviations from this steady state value with a \(^\circ\), and defining \(\hat{b}_t^m \equiv \hat{b}_t^m / \bar{c}^m \), we can approximate the model’s dynamics by the following log-linearized equilibrium conditions:

\[\bar{c}_t^e \equiv \hat{c}_t^e = \Xi \left[ \delta \omega \hat{y}_t + (1 - \omega) \hat{d}_t \right] + \hat{b}_t^m - \beta \hat{b}_{t+1}^m \]
\[\bar{c}_t^m = \omega \hat{y}_t + (1 - \omega) \hat{d}_t - \mu \frac{\bar{c}_t^m}{\bar{c}_t^e} \left( \hat{b}_t^m - \beta \hat{b}_{t+1}^m \right) \]
\[\Omega \frac{\bar{k}}{\bar{y}} \hat{i}_t = \left( \alpha - \mu \frac{\bar{c}_t^m}{\bar{y}} \Xi \delta \omega \right) \hat{y}_t - \left( \frac{\bar{d}_t}{\bar{y}} + \mu \frac{\bar{c}_t^m}{\bar{y}} (1 - \omega) \right) \hat{d}_t \]

48
\[
\begin{align*}
\dot{y}_t &= \lambda_t + \alpha \dot{k}_t + (1 - \alpha) \dot{n}_t \\
\dot{w}_t &= \dot{y}_t - \dot{n}_t \\
\dot{r}_t &= (1 - \beta (1 - \Omega)) \left( \dot{y}_t - \dot{k}_t \right) \\
\dot{w}_t &= \eta_t \dot{c}_t^s + \zeta \dot{n}_t \\
\dot{\beta}_t &= -\tilde{\phi} \left( \dot{c}_t^s + \mu \frac{\bar{c}_t^m}{c_t^m} \right) \\
\dot{q}_t &= \dot{\beta}_t - \eta_s E_t \dot{c}_t^s + \eta_s \dot{c}_t^s \\
\dot{q}_t &= \dot{\beta}_t - \eta_m E_t \dot{c}_t^m + \eta_m \dot{c}_t^m \\
\dot{x}_t &= \left( \frac{\theta - 1}{\theta} \right) \frac{\bar{c}_t^m}{c_t^m - A} \left( \dot{c}_t^m - \dot{\bar{n}}_t + \beta \dot{\bar{n}}_{t+1} \right) \\
\dot{k}_{t+1} &= (1 - \Omega) \dot{k}_t + \Omega \dot{n}_t
\end{align*}
\]
and the log-linearized Euler equation for the manager’s consumption
\[
-\eta_m \dot{c}_t^m = E_t \left[ \dot{\beta}_t - \eta_m \dot{c}_t^m + \frac{1}{1 + \mu \bar{c}_t} (\dot{x}_{t+1} - \dot{\bar{x}} + \dot{r}_{t+1}) \right],
\]
where
\[
\omega \equiv \frac{\bar{w} \bar{n}}{\bar{w} \bar{n} + \bar{d}}, \quad \Xi \equiv \frac{\gamma \theta (1 - A/c_t^m)}{\delta \omega + 1 - \omega} > 0, \quad \tilde{\phi} \equiv \frac{\phi c_t^s / \bar{c}}{1 + \phi (\log \bar{c} - \bar{v})} > 0.
\]

As in Section 3, the set of equations above will form the basis for our characterization of the regions of indeterminacy.

### D.3.3 Regions of indeterminacy

Figures 5, 6, and 7 present the regions of indeterminacy/instability for the same specific scenarios, respectively, that underlie Figures 1, 2, and 3 with the addition of bond trading by the managers. In all cases the contract form remains as in (32). Subject to this contract form, the regions identify sets of parameter combinations that give rise to indeterminacy/instability, as before.

### E Is Bond Trading Important?

Under the optimal contract, the resulting equilibrium is a Pareto optimum and thus represents an allocation that cannot be improved upon for both managers and shareholder-workers. The presence of voluntary bond trading between agents can neither improve upon nor detract from the Pareto allocation. In other words, the optimal contract (32) in conjunction with parameter choices specified in Theorem 1, remains optimal even if the structure of the economy is generalized to admit bond trading. The task of this Appendix is to verify this claim formally. We do so by first deriving the solution to the planner’s problem (the complete markets solution), and then demonstrating that the equilibrium allocation under the optimal contract and zero bonds trading concides with it. In this sense optimal contracting can be viewed as a substitute for complete markets and thus more than a substitute for bond trading.
The planner’s problem is as follows:

$$\max_{\{c_t^m, c_t^s, n_t^s, k_{t+1}\}} E_0 \left( \sum_{t=0}^{\infty} \beta^t \left[ \phi \mu \left( c_t^m \right)^{1-\eta_m} + \left( c_t^s \right)^{1-\eta_s} - B \left( n_t^s \right)^{1-\zeta} \right] \right)$$

s.t.

$$c_t^s + \mu c_t^m + k_{t+1} - (1 - \Omega) k_t = y_t = k_t^\alpha (n_t^s)^{1-\alpha} \lambda_t.$$  

It is well known that the necessary and sufficient first order conditions for the above problem are:

$$\phi \left( c_t^m \right)^{-\eta_m} = \left( c_t^s \right)^{-\eta_s}$$  \quad (E.1)

$$B \left( \eta_t \right)^{-\zeta} = \left( c_t^s \right)^{-\eta_m} k_t^\alpha (n_t^s)^{-\alpha} \lambda_t$$  \quad (E.2)

$$(c_t^m)^{-\eta_m} = \beta E_t \left[ \left( c_{t+1}^m \right)^{-\eta_m} \left\{ \alpha k_{t+1} (n_{t+1}^s)^{1-\alpha} \lambda_{t+1} + (1 - \Omega) \right\} \right]$$  \quad (E.3)

Together with the constraints

$$\mu c_t^m + c_t^s + i_t = y_t, \text{ and}$$  \quad (E.4)

$$k_{t+1} = (1 - \Omega) k_t + i_t,$$  \quad (E.5)

these five equations completely and uniquely determine the real Pareto allocation. Given the optimal contract form and parameter values and no-bond-trading, we will next demonstrate that the equilibrium characterization (D.21)–(D.34) guarantees these exact same relationships. Its real allocation must thus be identical.

By (D.26) and (D.21), (E.2) is satisfied. By (D.26), (D.33) and (D.34), and recognizing that for the optimal contact, \( \theta = 1 \) so that \( x_t = \varphi \), (D.33) is identical to (E.3). By construction (E.5) and (D.31) are identical. By (D.27), (D.28) and the definition of dividends, (D.29), (E.4) is guaranteed. Lastly, for the optimal parameter choices \( (A = 0, \theta = 1, \gamma = \frac{\Omega}{\eta_m}) \) and no bond trading \( (b_t^m = b_t^s = 0) \), (D.27) and (D.28) guarantee optimal risk sharing, (E.1).

In this setting, therefore, optimal contracting substitutes for market completeness, and thus for bond trading as well. For contracts parameterized non-optimally, however, bond trading may expand the parameter set leading to indeterminacy/instability.

References


### Table 1 – Parameter Choices: Baseline Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital share of output</td>
<td>$\alpha$</td>
<td>0.36</td>
</tr>
<tr>
<td>Subjective discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Capital depreciation rate</td>
<td>$\Omega$</td>
<td>0.025</td>
</tr>
<tr>
<td>Shareholders’ elasticity of intertemporal substitution in consumption</td>
<td>$\eta_s$</td>
<td>1</td>
</tr>
<tr>
<td>Disutility of labor parameter</td>
<td>$B$</td>
<td>2.86</td>
</tr>
<tr>
<td>Inverse of the Frisch elasticity of labor supply</td>
<td>$\zeta$</td>
<td>0</td>
</tr>
<tr>
<td>Persistence of the technology process</td>
<td>$\rho$</td>
<td>0.95</td>
</tr>
<tr>
<td>Measure of managers</td>
<td>$\mu$</td>
<td>0.05</td>
</tr>
<tr>
<td>Fraction of the salary component in the managerial contract</td>
<td>$A/\tau_m$</td>
<td>0.5</td>
</tr>
<tr>
<td>Manager’s elasticity of intertemporal substitution in consumption</td>
<td>$\eta_m$</td>
<td>0.25</td>
</tr>
<tr>
<td>Percentage of the aggregate wage in the managerial contract</td>
<td>$\delta$</td>
<td>0.2</td>
</tr>
<tr>
<td>Convexity of the managerial contract</td>
<td>$\theta$</td>
<td>3</td>
</tr>
<tr>
<td>Weighting on the variable compensation component</td>
<td>$\varphi$</td>
<td>4.894</td>
</tr>
<tr>
<td>Standard deviation of the technology shock</td>
<td>$\sigma_\epsilon$</td>
<td>0.0107</td>
</tr>
<tr>
<td>Standard deviation of the sunspot shock</td>
<td>$\sigma_v$</td>
<td>0.0570</td>
</tr>
<tr>
<td>Correlation coefficient between the technology shock and the sunspot</td>
<td>$\rho_{\epsilon v}$</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 2 — Business Cycle Statistics<sup>(i)</sup>

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Panel A</th>
<th>Panel B&lt;sup&gt;(ii)&lt;/sup&gt;</th>
<th>Panel C</th>
<th>Panel D</th>
<th>Panel E</th>
<th>Hansen</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Hansen</td>
<td></td>
</tr>
<tr>
<td>I. Standard Deviations (in percent)&lt;sup&gt;(iii)&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Hansen</td>
<td></td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>1.81</td>
<td>1.81</td>
<td>1.81</td>
<td>1.81</td>
<td>1.81</td>
<td></td>
</tr>
<tr>
<td>$\sigma_k/\sigma_y$</td>
<td>0.35</td>
<td>0.18</td>
<td>0.30</td>
<td>0.56</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>$\sigma_n/\sigma_y$</td>
<td>0.95</td>
<td>0.24</td>
<td>0.77</td>
<td>1.56</td>
<td>0.77</td>
<td></td>
</tr>
<tr>
<td>$\sigma_i/\sigma_y$</td>
<td>2.93</td>
<td>1.91</td>
<td>3.24</td>
<td>5.83</td>
<td>3.01</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{e/}\sigma_y$</td>
<td>0.75</td>
<td>0.76</td>
<td>0.73</td>
<td>0.61</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\epsilon s}/\sigma_y$</td>
<td>NA</td>
<td>0.13</td>
<td>3.98</td>
<td>8.37</td>
<td>NA</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\epsilon}$</td>
<td>NA</td>
<td>1.22</td>
<td>1.07</td>
<td>NA</td>
<td>0.0073</td>
<td></td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>NA</td>
<td>5.70</td>
<td>11.90</td>
<td>NA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>II. Contemporaneous Correlations with Output</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Hansen</td>
<td></td>
</tr>
<tr>
<td>$\rho_{k,y}$</td>
<td>0.06</td>
<td>0.20</td>
<td>0.24</td>
<td>0.38</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>$\rho_{n,y}$</td>
<td>0.88</td>
<td>0.99</td>
<td>0.64</td>
<td>0.98</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td>$\rho_{i,y}$</td>
<td>0.80</td>
<td>1.00</td>
<td>0.86</td>
<td>0.99</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>$\rho_{\epsilon, y}$</td>
<td>0.88</td>
<td>1.00</td>
<td>0.64</td>
<td>-0.86</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>$\rho_{\epsilon s,y}$</td>
<td>NA</td>
<td>-0.03</td>
<td>-0.46</td>
<td>-0.97</td>
<td>NA</td>
<td></td>
</tr>
</tbody>
</table>

<sup>(i)</sup> All parameters, where applicable, are as in Table 1
Table 3 — Simulations with Alternative Parameterizations of the Managerial Contract\(^{(i)}\), \(^{(ii)}\)

<table>
<thead>
<tr>
<th>Panel A</th>
<th>Panel B</th>
<th>Panel C</th>
<th>Panel D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>High contract convexity</td>
<td>Large salary component</td>
<td>High manager’s risk aversion</td>
</tr>
<tr>
<td>$\theta = 5$</td>
<td>$A/c^m = 0.7$</td>
<td>$\eta_m = 0.45$</td>
<td></td>
</tr>
</tbody>
</table>

**Standard Deviations**

<table>
<thead>
<tr>
<th></th>
<th>Panel A</th>
<th>Panel B</th>
<th>Panel C</th>
<th>Panel D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_y$</td>
<td>1.81</td>
<td>1.68</td>
<td>2.04</td>
<td>1.82</td>
</tr>
<tr>
<td>$\sigma_{k/y}$</td>
<td>0.30</td>
<td>0.24</td>
<td>0.37</td>
<td>0.31</td>
</tr>
<tr>
<td>$\sigma_{n/y}$</td>
<td>0.77</td>
<td>0.54</td>
<td>1.00</td>
<td>0.77</td>
</tr>
<tr>
<td>$\sigma_{i/y}$</td>
<td>3.24</td>
<td>2.63</td>
<td>3.90</td>
<td>3.23</td>
</tr>
<tr>
<td>$\sigma_{c^m/y}$</td>
<td>0.73</td>
<td>0.77</td>
<td>0.72</td>
<td>0.73</td>
</tr>
<tr>
<td>$\sigma_{c^m/y}$</td>
<td>3.98</td>
<td>4.30</td>
<td>3.55</td>
<td>3.97</td>
</tr>
<tr>
<td>$\sigma_{z}$</td>
<td>1.07</td>
<td>1.07</td>
<td>1.07</td>
<td>1.07</td>
</tr>
<tr>
<td>$\sigma_{v}$</td>
<td>5.70</td>
<td>5.70</td>
<td>5.70</td>
<td>5.70</td>
</tr>
</tbody>
</table>

**Contemporaneous Correlations with Output**

<table>
<thead>
<tr>
<th></th>
<th>Panel A</th>
<th>Panel B</th>
<th>Panel C</th>
<th>Panel D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{k,y}$</td>
<td>0.24</td>
<td>0.23</td>
<td>0.27</td>
<td>0.24</td>
</tr>
<tr>
<td>$\rho_{n,y}$</td>
<td>0.69</td>
<td>0.68</td>
<td>0.74</td>
<td>0.69</td>
</tr>
<tr>
<td>$\rho_{i,y}$</td>
<td>0.86</td>
<td>0.87</td>
<td>0.87</td>
<td>0.86</td>
</tr>
<tr>
<td>$\rho_{c^m,y}$</td>
<td>0.64</td>
<td>0.85</td>
<td>0.35</td>
<td>0.64</td>
</tr>
<tr>
<td>$\rho_{c^m,y}$</td>
<td>-0.46</td>
<td>-0.30</td>
<td>-0.62</td>
<td>-0.46</td>
</tr>
</tbody>
</table>

\(^{(i)}\) Except for those indicated, all parameters assume Table 1 values.

\(^{(ii)}\) These parameter choices lead steady states which differ from that of the Benchmark.
Figure 1: Regions of determinacy (white), indeterminacy (light gray) and instability (dark gray) in the case of fixed labor supply (Frisch elasticity $\zeta^{-1} \rightarrow 0$). In all cases $\beta = .99$, $\alpha = .36$, $\mu = .05$, $\gamma = 1$, $\Omega = .025$ (these define a critical value of $\delta^* = .55$), and $\eta_s = 1$. Departures from optimal contract design include the possibility that $\delta < 1$, $A > 0$, and $\theta \neq 1$.

Figure 2: Regions of determinacy (white), indeterminacy (light gray) and instability (dark gray) in the case of variable labor supply (Frisch elasticity $\zeta^{-1} = 0.5$). All other parameter values are as in Figure 1, as well as the admissible departures from optimal contract design.
Figure 3: Regions of determinacy (white), indeterminacy (light gray) and instability (dark gray) with Hansen (1985) labor supply. (Frisch elasticity $\zeta^{-1} \to \infty$). All other parameter values and admissible departures from optimal contract design are as in Figures 1 and 2.

Figure 4: Regions of indeterminacy and instability, and optimal contract parameter combinations; $\mu = 0$ is assumed (Theorem 2 applies).
Figure 5: Analogue of Figure 1 but with the admission of bond trading.

Figure 6: Analogue of Figure 2 but with the admission of bond trading.
Figure 7: Analogue of Figure 3 but with the admission of bond trading.