Investing in a relationship

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A principal can make an investment anticipating a repeated relationship with an agent, but the agent may appropriate the returns through ex post bargaining. I study how this holdup problem and efficiency depend on the contracting environment. When investment returns are observable, informal contracts ex post can be more efficient than formal contracts, as they induce higher investment ex ante: the principal invests not only to generate direct returns, but also to improve relational incentives. Unobservability of returns increases the principal’s ability to appropriate the returns but reduces her ability to improve incentives. The optimal information structure depends on bargaining power.

1. Introduction

A firm invests in plant and equipment. In subsequent negotiations with the workers it hires, the firm is vulnerable to having some of the return from those investments captured by workers in the form of higher wages (i.e., the firm can be held up). Anticipating this, the firm will have diminished investment incentives – as has been remarked upon by a number of authors. On the other hand, the relation between firm and workers is ongoing and, moreover, likely involves various aspects that don’t lend themselves to formal contracts. Rather, cooperation with regard to those aspects is sustained by relational contracting – promises to cooperate today are credible because of the breakdown in the relationship tomorrow that follows either side’s reneging. In order to induce the parties to care enough about tomorrow that they cooperate today, the firm may wish to increase its investment. This article explores that idea – which arises in a variety of different economic settings – and finds that reliance on relational contracting has important implications for the holdup problem.

I focus on two questions. First, if workers can observe the returns to the firm’s investment, how does the use of relational contracts ex post affect investment incentives ex ante and the overall efficiency of the relationship? Second, what are the effects of the returns to the investment being unobservable by the workers? In practice, workers may observe the firm’s new equipment but may not be able to perfectly assess the actual cost savings or observe the firm’s net profits.

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1 See Williamson (1975); Klein, Crawford, and Alchian (1978); Grout (1984); and Tirole (1986) for classic analyses, and Schmitz (2001) and Che and Sákovics (2008) for surveys. More recent work is discussed below.
Furthermore, firms may influence the extent to which this information is available, for example, through the use of audits or privacy rights. Previous work has noted that controlling the flow of information can be a useful tool in dealing with the holdup problem (see Gul, 2001). Here, I show that the optimal control of information depends on the nature of incentive contracts and the parties’ bargaining powers.

The model considers a risk-neutral principal and agent who can trade for infinitely many periods. Before trade starts, the principal makes an investment that generates stochastic, fully persistent returns in each period in which the parties trade, but which has no value outside the relationship. Contracting is not feasible at this stage. After the investment is made, one of the parties receives the bargaining power. In each subsequent period, that party makes a take-it-or-leave-it offer to the other, specifying a fixed wage and a discretionary performance-contingent payment for the agent. If the offer is accepted, the agent privately chooses effort, which determines his performance. The principal’s output in each period depends on the returns to the initial investment and the agent’s performance in that period.

As is well known, the set of subgame-perfect (constrained) Pareto-optimal equilibria in repeated games can be large. This is especially problematic for the study of bargaining: absent sufficient structure, the allocation of bargaining power is meaningless. Following a large strand of the literature, I thus focus on a subset of the Pareto-optimal equilibria by placing restrictions on what parties can do in every contingency. As in Halac (2012), I assume the relationship may end if a party reneges on a promised payment but remains on the Pareto-optimal frontier otherwise. Thus, if information is complete, the party who makes the offers appropriates all the rents, including the returns to the investment.

I first show that when the returns to the initial investment are observable, the inability to formally contract on performance leads to higher investment, and possibly to higher overall efficiency. The logic is simple. Ex post, given a sunk investment, a relational contract induces an inefficiently low level of effort by the agent – whereas incentives would be first-best under contractible performance, relational incentives must be self-enforcing and are thus limited by the value of the relationship. Now, ex ante, this inefficiency in the provision of effort incentives helps reduce the inefficiency caused by the holdup problem in the provision of investment incentives – because investment increases the expected value of the relationship, a higher investment benefits the principal (if she makes the offers) not only by increasing expected investment returns, but also, indirectly, by relaxing the enforcement constraint and increasing effort. Hence, the principal invests more when incentives are relational and, if investment returns are sufficiently important, higher total surplus than under formal contracts results.

I next study the effects of investment returns being unobservable. Suppose the agent does not observe whether the principal’s investment has been highly successful and generates high returns in each period, so the principal is a “high type,” or the investment has been less successful and generates low returns, so the principal is a “low type.” I show that, unless the future is heavily discounted, the agent cannot induce the principal types to separate when he makes the offers, as the high type wants to pretend to be a low type to appropriate future rents. Unobservability of returns then eliminates the holdup problem: the agent cannot capture the returns to the principal’s investment when he is unable to observe the realized returns. However, when incentives are

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2 Preinvestment contracting may be particularly difficult when the investment is in new technologies. For an interesting example, see Duhigg and Bradsher (2012) on how a Chinese company became Apple’s supplier after investing in glass screens for the iPhone.

3 This is the simplest noncooperative bargaining game that, in a setting of complete information, is equivalent to the generalized Nash bargaining solution.

4 That is, I postulate that a failure to honor a promised payment may lead to conflict and breakup, whereas an unexpected investment decision, an unexpected offer, or a rejection, is not a credible excuse to walk away or impose similar punishments. See Section 5 for a discussion.

5 A point made by MacLeod and Malcomson (1989) and Levin (2003), among others.

6 That is, investment acts as a form of Williamson (1983) hostage giving: by investing, the principal gives a hostage (higher relationship value) as a means of committing to not renege on promises to make payments.
related, unobservability also weakens, and may eliminate, the effects of investment on effort: the agent’s effort responds less to an increase in the value of the relationship when the agent is unable to observe the realized value. I show that if the agent’s bargaining power is high (i.e., the agent is likely to make the offers), the first effect dominates and efficiency increases when returns are unobservable. Instead, if the principal’s bargaining power is high, the latter effect dominates and efficiency goes down.

Table 1 summarizes the results. I compare investment and efficiency when incentives are relational versus in the benchmark case of contractible performance, and depending on whether the returns to the principal’s investment are observable or unobservable. A main insight concerns the effects of private information. Consider the employment example. If contracting is formal, firms want to “hide” their successes from workers, and indeed the use of privacy rights results in higher firm investment and total surplus. However, compensation is often based on hard-to-verify aspects of performance, such as workers’ contribution to client satisfaction, innovation, and leadership. Furthermore, even when written contracts are used, these are often relational insofar as parties honor them only because of the value of their ongoing relationships; see Hermalin, Li, and Naughton (2013) for a discussion. Table 1 shows that, unlike under formal contracting, audits and disclosure rules can be optimal when incentive contracts are informal. Specifically, if labor unions are not too strong, disclosure rules can benefit firms and workers. One implication is that, ceteris paribus, unions will more likely be able to access firm information and obtain wage increases when contracting is relational.

The results also show that for a given firm, different policies may be optimal for different relationships. For example, a technology startup company engages in relational contracting with employees and suppliers but is likely to be in a weaker bargaining position against the latter. The company will invest more efficiently in these relationships if employees have the ability to assess the value to the investments, whereas suppliers lack this knowledge. Moreover, the company may bias its investments toward products or projects that are particularly visible to employees, but which can be hidden from others.

Related literature. Several articles study how the holdup problem depends on the information structure. Gul (2001) shows that the holdup problem disappears when investment is unobservable and the noninvesting party makes frequently repeated offers. Different from the
model below, Gul assumes that returns are a deterministic function of investment, so what is un-
observable in his model is the investment itself. Hermalin and Katz (2009) and Skrzypacz (2004)
consider an observable investment with stochastic, unobservable returns like I do; however, the
former considers a static framework and the latter focuses on specific distortions that arise when
investment affects the support of the distribution of returns. More importantly, none of these
articles considers a principal-agent relationship nor studies relational contracting.

Within the relational contracts literature, Levin (2003) characterizes optimal relational
contracts under different forms of nonpersistent private information, including moral hazard,
as featured in the model below. Halac (2012) studies relational contracting when parties also
have persistent private information about outside payoffs. The model is related to the setup with
unobservable returns considered here; importantly, however, that model assumes an exogenous
distribution of types, whereas here, I allow for an upfront investment through which the principal
endogenously chooses that distribution. To my knowledge, noncontractible upfront investments
in long-term relationships and their interaction with ex post effort constraints has only been
studied in Ramey and Watson (1997, 2001). Unlike this article, this work focuses on the effects of
market frictions in matching markets and does not consider a principal-agent setting nor private
information about investment returns.

Also related are a number of articles that study investment incentives in relational contracting
settings. Garvey (1995); Baker, Gibbons, and Murphy (2002); Halonen (2002); Blonski and
Spagnolo (2007); and Halonen and Pafilis (2013) analyze the effects of different ownership
structures, as in the classic studies of Grossman and Hart (1986) and Hart and Moore (1990),
but when specific investments are made in each period of a repeated relationship. Relational
contracts are used to sustain these ongoing investments, and property rights matter because they
affect parties’ incentives to renege. Unlike this literature, this article is concerned with a one-time
investment that takes place before the relationship starts, and which is thus not part of a relational
agreement. I do not consider the allocation of ownership but study how investment is affected
by future incentive constraints and the information structure.

Finally, this article relates to previous work on the interaction between formal and informal
contracts. Baker, Gibbons, and Murphy (1994) and Schmidt and Schnitzer (1995) show that the
possibility of writing formal contracts can render optimal relational contracts infeasible because
it improves a party’s payoff after reneging. In Bernheim and Whinston (1998), when some aspects
of performance are noncontractible, parties can benefit from leaving other aspects unspecified
as well, because that gives them recourse if a party reneges. That is, additional discretion facil-
itates punishment, thus lowering parties’ reneging payoffs. In my model, the ability to formally
contract on performance can also lower efficiency, though not because it increases parties’
incentives to renege, but because it lowers parties’ incentives to invest in the relationship. Note
also that in these articles, relaxing the enforcement constraint is always beneficial; instead, here I
show that it may cause investment and efficiency to fall.

Section 2 presents the model. Section 3 considers a benchmark with contractible perform-
ance. Section 4 studies relational incentive contracts when investment returns are observable
and when they are unobservable. Section 5 analyzes complementarities between investment and
effort and discusses the restrictions on strategies. Section 6 concludes.

2. The model

 Setup. Consider a risk-neutral principal (she) who can trade with a risk-neutral agent (he)
for an infinite number of periods. Both parties have the same discount factor \( \delta \in (0, 1) \). Before
the relationship starts, the principal makes an investment that generates returns in each period in
which the parties trade. Contracting is not feasible at this stage. The timing is as follows.

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\(^7\) See MacLeod (2007) and Malcomson (2013) for surveys.

\(^8\) Like investment in this article, ownership structure in this literature is also an *ex ante* choice.
• \( t=0 \): Principal invests, future returns are determined (observable/unobservable). Independent of the agent’s action, in any future period in which the principal and agent trade, the principal receives a return \( r \in \{e, \mathcal{F}\}, \mathcal{F} > e \). The value of \( r \) is a consequence of the principal’s investment at this point in time. It is without loss of generality to view that investment as the principal choosing the probability, \( p \), that \( r = \mathcal{F} \). Let the principal’s cost of choosing \( p \) be \( k(p) \), where \( k(\cdot) \) is a twice differentiable function exhibiting the properties \( k(0) = k'(0) = 0, k''(p) > 0 \) for all \( p \in [0,1] \), and \( (1 - \delta)k'(1) > \delta(\mathcal{F} - r) \). In other words, the cost and marginal cost of no investment are zero, the investment technology exhibits strictly decreasing returns to scale, and it is not surplus maximizing to guarantee \( r = \mathcal{F} \). Although the principal’s investment is observable, the value of \( r \) may or may not be observable. The two cases are considered in Section 4.

• \( t=1 \): Party \( i \in \{A, P\} \) receives the bargaining power. At the beginning of period \( t = 1 \), the allocation of bargaining power is exogenously determined for all future periods \( t = 1,2, \ldots \). With probability \( \lambda \in [0,1] \), the principal receives the bargaining power, and with probability \( 1 - \lambda \), the agent does. The superscript \( i \in \{A, P\} \) indicates the party who has received the bargaining power.

• \( t=1,2,\ldots \): Party \( i \) makes offer, \( A \) chooses effort, output is realized, payments are made. In each period \( t = 1,2,\ldots \), the party with the bargaining power makes a take-it-or-leave-it offer to the other party. That party accepts or rejects. Rejection means the parties get their no-trade payoffs, \( \overline{\mathcal{F}} \) and \( \overline{\pi} \) for principal and agent, respectively. Note that no trade means the principal does not receive \( r \) that period. If the offer is accepted, then the agent chooses a probability, \( e \in [0,1] \), that performance is \( \overline{\mathcal{F}} \). With probability \( 1 - e \), performance is \( q < \overline{\mathcal{F}} \). The agent’s private cost is \( c(e) \), where \( c(\cdot) \) is a twice differentiable function exhibiting the properties \( c(0) = c'(0) = 0, c''(e) > 0 \) for all \( e \in [0,1] \), and \( c'(1) > \overline{\mathcal{F}} - q \). The agent’s performance is observable to him and the principal, but not to a third party (i.e., \( q \) is not verifiable). The principal receives output \( y = r + q \) and the parties make the payments, which are described below.\(^7\)

It is worth noting that the noncooperative bargaining game above leads to the generalized Nash bargaining solution if information is complete. That is, for the case in which investment returns are observable, the results are unchanged if one assumes that in each period the parties split the gains from trading with each other with shares \( \lambda \) and \( 1 - \lambda \). Specifying a bargaining game, however, is necessary to deal with the case of incomplete information.

An offer is a compensation package for the agent: a fixed wage, \( w_i \), and a performance-contingent bonus, \( b_i \), where, without loss of generality, \( b_i \in \{b_i, \overline{b}_i\} \) corresponding, respectively, to \( q, \overline{\mathcal{F}}; b_i \leq 0 \) and \( \overline{b}_i \geq 0 \). When not confusing, a contract \( \{w, b\} \) is simply denoted by \( b \). The fixed wage is formally enforced but, because performance is nonverifiable, the bonus is not. If \( b_i > 0 \), the principal has the decision whether to honor or renege on the bonus payment at the end of period \( t \); if \( b_i < 0 \), the agent has this decision. (There is no limited liability.) Total compensation is denoted by \( W_i \), where \( W_i = w_i + b_i \), if contingent payments are honored and \( W_i = w_i \), if they are not. The principal’s payoff is the difference between output and the agent’s payment.

The ex post per-period expected surplus at \( t = 1, 2, \ldots \), given realized return \( r \) and effort \( e \), is \( s(r, e) \equiv \mathbb{E}[y - c|r, e] = r + \mathbb{E}[q - c|e] \). I assume that effort is essential but high returns are not: \( \max_s s(e, e) > \mathcal{F} + \overline{u} > s(\mathcal{F}, 0) \).\(^9\) I also assume that the whole investment must be made in period \( t = 0 \), before a trading relationship is started.\(^11\)

\(^7\) To simplify the analysis, I assume that investment increases the productivity of effort without affecting its marginal product. This allows for a cleaner characterization, as first-best effort and investment are then independent. See Section 5 for the case where investment and effort are complements.

\(^9\) Assuming that high returns are not essential is consistent with, and important to understand, more general settings where returns are not binary (where it is reasonable to assume that trade is profitable for more than one return realization).

\(^11\) This assumption captures two ideas: first, an initial investment is necessary for trade to be feasible; second, investment is not fully divisible and once an investment is chosen, adjustments are more costly.
I multiply expected lifetime payoffs by \((1 - \delta)\) to express them as per-period averages. The principal’s and agent’s expected payoffs at any time \(t = 1, 2, \ldots\) are, respectively,

\[
\pi_t = (1 - \delta)\mathbb{E} \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left\{ \chi_{\tau}(y_{\tau} - W_{\tau}) + (1 - \chi_{\tau})\pi \right\}
\]

and

\[
u_t = (1 - \delta)\mathbb{E} \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left\{ \chi_{\tau}(W_{\tau} - c(e_{\tau})) + (1 - \chi_{\tau})\nu \right\},
\]

where \(\chi_{\tau} \in \{0, 1\}\) denotes, respectively, no trade or trade at time \(\tau\). The principal and agent’s payoffs at time 0 are \(\pi_0 = \pi - k(p)\) and \(u_0 = \pi\), respectively. The expected surplus is \(S_t = \pi_t + u_t\).

**Equilibrium concepts.** In the first subsection of Section 4, where investment returns are observable and thus information is complete, I characterize Pareto-optimal contracts using the concept of perfect public equilibrium (PPE); see Fudenberg, Levine, and Maskin (1994). In the second subsection, I study a setting where investment returns are not observable by the agent, so \(r \in \{r, \bar{r}\}\) is the principal’s type and the agent forms a belief about this type. I characterize Pareto-optimal contracts in this setting using the concept of perfect public Bayesian equilibrium (PPBE), which is the natural extension of PPE for dynamic Bayesian games.

A PPBE is a set of public strategies and posterior beliefs such that the strategies form a Bayesian Nash equilibrium in every continuation game given the posterior beliefs, and the beliefs are updated according to Bayes’ rule whenever possible. Following Fudenberg and Tirole (1991), I require that beliefs be updated using Bayes’ rule not only on the equilibrium path but also in continuation games reached with zero probability. Because Bayes’ rule does not apply when beliefs are degenerate, an assumption such as Assumption 1 is needed (see Rubinstein, 1985).

**Assumption 1.** If, at any point, the agent’s posterior belief assigns probability one to a given type, then his beliefs continue to do so, no matter what happens.

Even under the requirements of PPE and PPBE, the set of Pareto-optimal equilibria is very large. Inefficient levels of trade as well as no-trade outcomes can be supported in equilibrium, and thus be used by the parties as “punishments” for different behaviors. I concentrate on a subset of the Pareto-optimal equilibria by imposing restrictions on strategies and, hence, on these punishments.

**Assumption 2.** If a party reneges on a payment at time \(t\), the relationship ends with some probability \(1 - \gamma_{t+1} > 0\) at time \(t + 1\), and continues on the Pareto-optimal frontier with probability \(\gamma_{t+1}\). If no party reneges, the relationship continues on the Pareto-optimal frontier with probability one.

In Section 5, I show that the first part of this assumption is without loss of generality and discuss the consequences of relaxing the second part. Importantly, in a setting where neither formal nor relational contracting is feasible at the investment stage, Assumption 2 ensures that investment incentives cannot be provided through variation in continuation payoffs and, under complete information, the party who makes the offers appropriates all the rents.

**First best.** Suppose that preinvestment contracting is feasible and the investment return \(r\) and performance \(q\) are verifiable. Maximizing the total expected surplus at time \(t = 0\) gives that the first-best levels of investment and effort are \(p^{fb}\) and \(e^{fb}\), where

\[
1 - \frac{\delta}{\delta} k'(p^{fb}) = \bar{r} - r, \quad (1)
\]

\[c'(e^{fb}) = \bar{q} - q.\]
3. Contractible performance benchmark

As a benchmark, suppose that the agent’s performance in each period is verifiable. The results of this section are in the same spirit as those in Gul (2001).

Given verifiability, the parties use a formally enforced contract to provide effort incentives. Under risk neutrality, there exists a simple contract that implements first-best effort \( e^{fb} \); for example, this contract specifies bonus payments \( b = 0 \) and \( \bar{b} = \bar{q} - q \). The proposed fixed wage depends on which party receives the bargaining power and on whether total output – namely, the returns to the initial investment – can be observed by the agent.

Suppose first that the agent observes the realized returns. Then regardless of which party \( i \in \{A, P\} \) receives the bargaining power, this party captures all the surplus, including the returns to the initial investment. Hence, given a sunk investment \( p \) and realized return \( r \), the principal proposes \( w^{ofb} = \bar{u} - E[b^{of} - c|e^{fb}] \) if she makes the offers, and the agent proposes \( w^{o\hat{f}A} = E[y - b^{of}|r, e^{fb}] - \pi \) if he makes the offers, where the superscript “\( \hat{f} \)” denotes observable-able investment returns and formal incentive contracting. Given this, the principal’s investment decision problem at time \( t = 0 \) is:

\[
\max_p \lambda \delta \left[ ps(\bar{r}, e^{fb}) + (1 - p)s(\bar{r}, e^{fb}) \right] - (1 - \delta)k(p).
\]

The first-order condition is:

\[
\frac{1 - \delta}{\delta} k'(p^{of}) = \lambda(\bar{r} - \bar{r}). \tag{2}
\]

Although the principal always pays the cost of investment, she captures the returns only when she obtains the bargaining power. Comparing equations (1) and (2), it thus follows that for \( \lambda < 1 \), the principal invests at an inefficiently low level. That is, a holdup problem arises if \( ex \ post \), the agent may be able to appropriate the gains from the principal’s sunk investment.

Suppose next that the agent cannot observe the returns to the principal’s investment, so \( r \in \{r, \bar{r}\} \) is the principal’s type. The agent observes the investment and thus knows that the principal is a high type \( (r = \bar{r}) \) with probability \( \bar{p} \) and a low type \( (r = r) \) with probability \( 1 - \bar{p} \). As above, if the principal makes the offers, she captures all the surplus; the question is whether the agent can also capture all the surplus when he makes the offers. Note that to appropriate the investment returns, the agent must induce the principal types to separate.

One can show that if the agent induces separation of types, he does so via rejection: one type of principal rejects, whereas the other accepts the agent’s offer.\(^{12}\) Suppose then that the agent proposes contract \( (w^A, b^A) \) with expected payment \( E[W|e^{fb}] \) and the principal’s type is fully revealed through rejection. Because the low type has a lower relationship value than the high type, it must be the low type who rejects, whereas the high type accepts. This means that if the offer is rejected, the agent proposes \( (w^A, b^A) \) with \( E[W^A|e^{fb}] = E[y|\bar{r}, e^{fb}] - \pi \) from then on. If the offer is accepted, the agent proposes \( (w^{ofA}, b^{ofA}) \) with \( E[W^{ofA}|e^{fb}] = E[y|\bar{r}, e^{fb}] - \pi \) from then on. Thus, the low type will indeed reject only if

\[
(1 - \delta)E[y - W^A|\bar{r}, e^{fb}] + \delta \pi \leq \pi,
\]

and the high type will indeed accept only if

\[
(1 - \delta)E[y - W^A|\bar{r}, e^{fb}] + \delta \pi \geq \pi + \delta(\bar{r} - r).
\]

Combining these two conditions, full separation of types through rejection is feasible only if

\[
\delta \leq \frac{1}{2}.
\]

This condition is also necessary for partial separation through rejection. In particular, if this condition does not hold, an equilibrium where the high type mixes between accepting and rejecting

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\(^{12}\) This follows from the fact that if the agent offers a menu of contracts, an equilibrium where the two types separate by accepting different contracts does not exist. See the Appendix.
and the low type rejects does not exist.\footnote{An equilibrium where the high type accepts and the low type mixes between accepting and rejecting is not feasible either, as the high type would want to reject.} A simple Coase-conjecture-like argument (Coase, 1972) shows why. Under such strategies, after observing a number of rejections, the agent’s belief becomes low enough that, if the principal again rejects, the agent optimally proposes \(w^{o,A}, b^{o}\) from then on. However, then, when that belief is reached, the high type rejects any offer that the low type rejects, so the agent proposes \(w^{o,A}, b^{o}\) at that point. Continuing with this reasoning gives that the agent proposes \(w^{o,A}, b^{o}\) in all periods.

Therefore, unless the future is heavily discounted, the agent cannot induce separation of types when he makes the offers: the high type has incentives to mimic the low type and reject in the present to enjoy rents in the future. As a consequence, the high type always keeps her rents and the holdup problem disappears. Going back to Table 1 in the Introduction, we thus obtain the results corresponding to the first line.

**Proposition 1.** Suppose that performance is contractible. Investment and total surplus are strictly below first-best if the investment returns are observable and the agent has positive bargaining power (i.e., \(\lambda < 1\)), whereas they are first-best if the investment returns are unobservable by the agent and the parties are sufficiently patient (i.e., \(\delta > 1/2\)).

### 4. Relational incentives

- Assume hereafter that the agent’s performance in each period is observable but not verifiable, so incentives for effort must be self-enforcing.

#### Observable investment returns.

Suppose first that the agent can observe the returns to the principal’s investment. It follows from Levin (2003) that an optimal self-enforcing contract is stationary: given realized return \(r\) and party \(i\) has the bargaining power then, on the equilibrium path, \(e_i = e(r), (b_i, \bar{b}_i) = (b_i(r), \bar{b}_i(r))\), and \(w_i = w_i(r)\) every period. Thus, the principal and agent’s expected payoffs in any period are

\[
\pi_i(r) = E[y - W^i(q, r)|r, e(r)] \quad \text{and} \quad u_i(r) = E[W^i(q, r) - c|r, e(r)].
\]

For the compensation schedule to be self-enforcing, neither party can wish to renege on a promised payment. Because, here, no party ever reneges in equilibrium, it is without loss to assume that a default leads to termination of the relationship with certainty, which is the worst punishment (Abreu, 1988). A self-enforcing contract then satisfies

\[
\delta \pi_i(r) \geq (1 - \delta) \bar{b}_i(r) + \delta \pi,
\]

\[
\delta u_i(r) \geq -(1 - \delta) b_i^r(r) + \delta u.
\]

Depending on the bargaining power distribution, the fixed wage is adjusted and slack transferred from one constraint to the other. The two conditions above can then be combined into a single enforcement constraint, \((E N|r)\). The optimal contract maximizes expected surplus subject to an incentive compatibility constraint for effort and \((E N|r)\). For \(r \in \{\underline{r}, \bar{r}\}\),

\[
\max_{e \in \lambda, [\underline{b}_r, \bar{b}_r]} s(r, e) = E[y - c|r, e(r)]
\]

subject to \(e(r) \in \arg \max_e e(\bar{b}_r - \bar{b}(r)) - c(e)\),

\[
\frac{\delta}{1 - \delta}(s(r, e) - \bar{\pi} - \bar{u}) \geq \bar{b}_i(r) - \bar{b}_i(r),
\]

\((I C_A|r)\)

\((E N|r)\)
The solution is denoted by $e^w_r$, $b^w_r$, where “or” denotes observable investment returns and relational incentive contracting. Note that, unlike under contractible performance, here the first-best level of effort may not be implementable, as bonus payments are discretionary and hence the enforcement constraint must be satisfied. The enforcement constraint is tighter the lower the discount factor and the higher the parties’ outside options. For the analysis to be interesting, I assume that for $r \in [r^l, r^h]$, parameters are such that $(E_\pi| r)$ binds but some bonus scheme is enforceable. Given that the party making the offers will capture all the surplus, the optimal contract implements an effort $e^w_r$ (below first-best level) by specifying

$$b^w_r = \frac{\delta}{1-\delta} \tilde{\lambda}(s(r, e^w_r) - \overline{\pi} - \overline{w}),$$

$$b^w_r = -\frac{\delta}{1-\delta}(1 - \tilde{\lambda})(s(r, e^w_r) - \overline{\pi} - \overline{w}),$$

where $\tilde{\lambda}$ is equal to one if the principal has the bargaining power and zero otherwise. The wage is $w^w_r = \overline{w} - E[b^w_r - c|e^w_r]$ if the principal has the bargaining power and $w^w_r = E[y - b^w_r| r, e^w_r] - \overline{\pi}$ if the agent does.

It is clear that, given a sunk investment $p$ and realized return $r$, the inability to formally contract on performance reduces the ex post surplus that the parties can generate. However, how does this affect the principal’s ex ante investment decision? At $t = 0$, the principal solves:

$$\max_p \lambda \delta\left[ps(\overline{r}, e^w_r) + (1 - p)s(\overline{r}, e^w_r)\right] - (1 - \delta)k(p).$$

The first-order condition is:

$$\frac{1 - \delta}{\delta}k'(p^w) = \lambda \left[\overline{r} - r + \left(\overline{E}[q - c|e^w_r] - \overline{E}[q - c|e^w_r]\right)\right]. \quad (3)$$

As in the case of contractible performance, the principal captures the benefits from her investment only if she receives the bargaining power, with probability $\lambda$. However, here the marginal benefit of investment has two components. First, as above, there is a direct effect: a higher investment increases expected future returns. Second, there is now also an indirect effect: by increasing expected returns, investment increases the expected future value of the relationship, which relaxes the expected enforcement constraint and causes expected incentives and effort, and thus expected performance net of effort costs, to increase. (This effect is shown by the term in parentheses on the right-hand side of equation (3), which is strictly positive.) Thus, comparing (2) and (3), the principal always invests more when the agent’s performance is noncontractible than when it is contractible. In fact, comparing with equation (1), we see that the principal will invest more than the first-best level if the indirect effect of investment is sufficiently large (i.e., $\lambda\left(\overline{E}[q - c|e^w_r] - \overline{E}[q - c|e^w_r]\right) > (1 - \lambda)(\overline{r} - r)$).

The difference in total surplus relative to the contractible performance benchmark is:

$$S^w - S^c = \delta \left[ p^w \overline{E}[q - c|e^w_r] + (1 - p^w)\overline{E}[q - c|e^w_r] - \overline{E}[q - c|e^w_r]\right]$$

$$+ \left[\delta(p^w - p^c)(\overline{r} - r) - (1 - \delta)k(p^w) - k(p^c)\right].$$

The first term in square brackets is the difference in surplus due to different effort levels. Because effort is inefficiently low when incentives are relational, this term is negative. The second term in square brackets is the difference in surplus due to different investment levels. Because investment is higher when incentives are relational, this term may be positive. Moreover, if the returns to investment are sufficiently important, this term may more than compensate for the first one, in which case total surplus is higher under relational contracts.

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14 This term may also be negative; this can occur if $p^w > p^c$. 

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Proposition 2. Suppose that the investment returns are observable. Investment is higher when performance is noncontractible (incentives are relational) than when it is contractible. Moreover, there exist parameters when neither side has absolute bargaining power (i.e., when $0 < \lambda < 1$) such that total expected surplus is also higher.

The results correspond to the first column of Table 1 and are illustrated with an example in Figure 1. This proposition says that if preinvestment contracting is infeasible, agency relationships can be more efficient in environments where effective, verifiable measures of performance are not available, or legal institutions and thus formal contract enforcement are weak. Parties will then rely on relational contracts, where the constraints in the provision of effort incentives \textit{ex post} increase the parties’ incentives to invest in the relationship \textit{ex ante}.

Proposition 2 also implies a substitution between investment in monitoring and investment in the relationship. That is, suppose that the principal can choose \textit{ex ante} either to invest in a monitoring technology (organizational design, more generally) that facilitates formal enforcement of incentive contracts, or to invest in the productive relationship, thus increasing relational enforcement of incentive contracts. No further investments can be made once trade starts, so the monitoring technology is fixed thereafter. Proposition 2 shows that investing in the relationship can yield greater surplus than investing in monitoring.

The relational contracts literature stresses that efficiency depends on how binding the enforcement constraint is. In particular, the higher the discount factor and the lower the parties’ outside options, the less tight this constraint and the higher the levels of effort and total surplus that can be sustained (see, e.g., Levin, 2003). A corollary of Proposition 2 is that this result may be reversed when \textit{ex ante} investments are important. Intuitively, as the enforcement constraint is

\footnote{The figure is based on the parametric assumptions given in the proof of Proposition 2.}

\footnote{As in the case of productive investments, investing in monitoring \textit{ex post} may be difficult and parties may then face higher implementation costs.}
relaxed and relational incentives become more efficient *ex post*, the effect of investment on effort becomes smaller, so the principal’s incentives to invest fall.

**Corollary 1.** Suppose that the investment returns are observable. Consider a change in parameters that relaxes the enforcement constraint and thus increases efficiency given a sunk investment (e.g., a decline in the parties’ outside options). There exist parameters such that investment and overall efficiency fall following this change.

An illustrative example is provided in Figure 2.\(^{17}\)

□ **Unobservable investment returns.** Consider next the case in which the gains from the principal’s investment are unobservable by the agent. The realized return \(r \in \{r, \bar{r}\}\) is now the principal’s type. Given the investment, the agent knows that the principal is a high type \((r = \bar{r})\) with probability \(p\) and a low type \((r = r)\) with probability \(1 - p\).\(^{18}\)

This setting is arguably more complex.\(^{19}\) As shown above, when incentives are relational, effort depends on how valuable the relationship is. Now, when the investment returns are unobservable by the agent, the value of the relationship is not commonly known; hence, as I show next, effort depends on whether information about the value of the relationship is revealed and how this revelation occurs.

I relegate some parts of the formal analysis to the Appendix. Here, I focus on how the informational asymmetry, by affecting continuation play, influences the principal’s investment,\(^{17}\) The figure is based on the parametric assumptions given in the proof of Corollary 1.\(^{18}\) Assume throughout this section that \(k'(1)\) is high enough that \(p < 1\).\(^{19}\) Halac (2012) considers a similar setting, where the principal’s outside option is her private information. An important difference is that, here, there is an initial investment stage and a potential holdup problem. The principal’s investment determines the probability that her type is high, whereas this probability is exogenous in Halac (2012).
and how overall efficiency compares to the case of observable returns. The next lemma, which follows from the results in Halac (2012), simplifies the analysis.

**Lemma 1.** Suppose that the investment returns are unobservable by the agent. If separation of types occurs in a Pareto-optimal equilibrium, it occurs through a rejection if the agent has the bargaining power and through a default if the principal does. No separation occurs through the principal’s choice of contract.

Given Lemma 1, the case where the agent receives the bargaining power is analogous to that described in Section 3 for contractible performance. To induce separation of types, the agent must propose a contract that the high type is willing to accept whereas the low type rejects. However, as shown, if the discount factor is high enough, the high type wants to pretend to be a low type to capture future rents, and separation of types is not feasible. As a result, the holdup problem disappears. However, when incentives are relational, this also has implications for the agent’s effort: if the agent can capture only the value of the relationship with the low type when he makes the offers, incentives in that case must correspond to the low type’s symmetric-information incentives \( b_{e^r} \), so effort in each period is \( e_{e^r} \).

**Lemma 2.** Suppose that the investment returns are unobservable by the agent and the agent has the bargaining power. If the parties are sufficiently patient (i.e., \( \delta > 1/2 \)), the agent cannot induce separation of types and \( \{w_{e^r}, b_{e^r}\} \) is implemented in every period.

Consider next the case where the principal receives the bargaining power. Here, in contrast with the case above, it is the low type who wants to misrepresent her type – she wants to pretend to be a high type to provide high-powered relational incentives to the agent.

The principal’s proposed contract may induce pooling or separation of types. Note that (given Assumption 2) the principal can always choose to act as if her type were low and obtain a payoff \( \pi^P(r|b_{e^r}^x) \equiv s(r, e_{e^r}^x) - \overline{u} \), regardless of the agent’s beliefs. Hence, if no separation is induced, \( \{w_{e^r}^p, b_{e^r}^p\} \) is implemented in each period, as this is the best the principal can do under no information revelation. Moreover, if separation is induced, both types’ payoffs must be above \( \pi^P(r|b_{e^r}^x) \), so the separating equilibrium Pareto-dominates the pooling one.

In light of Lemma 1, I consider only separation through default, where both types offer the same contract and one type reneges on the promised payment, whereas the other honors it. Also, for the purpose of this analysis, it is sufficient to consider separating equilibria in pure strategies, where the principal either honors or reneges with probability one and the relationship ends if she reneges. Suppose then that contract \( \{w^p, b^p\} \) is implemented and the principal’s type is fully revealed through default. Because the low type has a lower relationship value than the high type, it must be the low type who reneges. This means that if no default occurs, \( \{w_{e^r}^p, b_{e^r}^p\} \) is implemented from then on. Thus, the high type will indeed honor only if

\[
\delta \pi^P(r|b_{e^r}^x, 1) \geq (1 - \delta)b_{e^r}^p + \delta \pi, \tag{EN_r}
\]

and the low type will indeed renego only if

\[
\delta \pi^P(r|b_{e^r}^x, 1) \leq (1 - \delta)b_{e^r}^p + \delta \pi, \tag{DE_l}
\]

20 Note that when the agent has the bargaining power, no informal payments from the principal to the agent can be enforced; that is, \( \delta = 0 \) in any contract.

21 I discuss the case of mixed strategies in the Appendix.
where $\pi^p(\bar{r}|b^w, 1) = s(\bar{r}, e^w) - \bar{u}$ and $\pi^p(c|b^w, 1)$ is the low type’s continuation value when the agent believes that she is a high type with probability one. Note that, given such belief, the low type’s optimal strategy is to renege on $\bar{b}^w$ when performance is high, so that:

$$\pi^p(c|b^w, 1) = s(c, e^w) - \bar{u} + \left[ \frac{\delta e^w}{1 - \delta(1 - e^w)}(\bar{r} - c) \right].$$

The principal offers a contract that induces default only if her payoff is larger than her pooling payoff, $\pi^p(\bar{r}|b^w)$. Because the low type has a positive option to renege, her payoff under separation is always larger than $\pi^p(c|b^w)$ if the high type’s payoff is larger than $\pi^p(\bar{r}|b^w)$; hence, we can concentrate on the high type’s problem. Given $p$, the next program finds $b^p, e^p$ that maximize the high type’s expected payoff subject to an incentive constraint for effort (IC$_4$), the conditions for default (EN$_\pi$) and (DE$_e$), and the agent’s participation constraint, which is incorporated in the high type’s payoff below:

$$\max_{b^p, e^p} \pi^p(\bar{r}|b^p, p) = \frac{(1 - \delta)[s(\bar{r}, e^p) - \bar{u} - e^p(1 - p)\bar{b}^p] + \delta e^p\pi^p(\bar{r}|b^w, 1)}{1 - \delta(1 - e^p)}$$

subject to $e^p \in \arg\max_e e p \bar{b}^p - c(e)$, (IC$_4$)

$$\delta \pi^p(\bar{r}|b^w, 1) \geq (1 - \delta)\bar{b}^p + \delta \bar{r},$$

(EN$_\pi$)

$$\delta \pi^p(\bar{r}|b^w, 1) \leq (1 - \delta)\bar{b}^p + \delta \bar{r}.$$ (DE$_e$)

Let $b^{p*}, e^{p*}$ denote the solution. Then, a Pareto-optimal equilibrium that induces separation of types exists if and only if $\pi^p(\bar{r}|b^{p*}, p) \geq \pi^p(\bar{r}|b^w)$, or equivalently, $\delta > 1/2$:

$$(1 - \delta)(s(\bar{r}, e^{p*}) - s(\bar{r}, e^w)) + \delta e^{p*}(s(\bar{r}, e^w) - s(\bar{r}, e^w)) \geq (1 - \delta)e^{p*}(1 - p)\bar{b}^{p*}.$$ (IC$_4$)

This condition says that the principal induces separation of types if and only if the benefits outweigh the costs when she is a high type. The benefits come from the ability to provide stronger incentives when separation occurs. The costs are in the form of a compensation to the agent for the risk of default. As is intuitive, the benefits indeed exceed the costs if and only if the probability of a high type, $p$, is high enough.

**Lemma 3.** Suppose that the investment returns are unobservable by the agent and the principal has the bargaining power. There exists $\bar{p} \in (0, 1)$ such that if $p \geq \bar{p}$, the principal induces separation of types through default. If $p < \bar{p}$, no separation is induced and $\{w^{p, p}, b^w\}$ is implemented in every period.

Having solved for the continuation equilibria when the agent makes the offers (Lemma 2) and when the principal does (Lemma 3), I now turn to the principal’s initial investment decision. In what follows, I assume $\delta > 1/2$. The principal’s problem at time $t = 0$ is:

$$\max_p \delta[p \pi^p(\bar{r}|b^w, p) + (1 - p)\pi^p(c|b^w, p)] + (1 - \lambda)\delta p(\bar{r} - r) - (1 - \delta)k(p),$$

where “$ur$” denotes unobservable investment returns and relational incentive contracting. The analysis shows that if the agent receives the bargaining power, or if the principal does but the investment $p$ is low enough, no separation of types is induced. Hence, $\{w^{p, p}, b^w\} = \{w^{p, p}, b^w\}$ and the principal’s benefit from investing is $\bar{r} - r$. It follows that the principal invests at first-best level; however, because effort is $e^w < e^b$, surplus is strictly less than first-best. Moreover, note

---

22. Note that when the principal has the bargaining power, no informal payments from the agent to the principal can be enforced; that is, $b = 0$ in any contract.

23. This condition is derived here under the restriction of pure strategies. However, as discussed in the Appendix, the analysis can be extended to mixed strategies without affecting the qualitative results.
that effort corresponds to that under a known low type, $e_{\text{ur}}^*$, regardless of the realized return. Thus, unlike in the contractible performance benchmark, surplus under unobservable returns may be strictly less than under observable returns. This result depends on bargaining power: because the holdup problem is eliminated, unobservability increases efficiency when high bargaining power is on the agent’s side (i.e., when $\lambda$ is low), but, because the effects of investment on effort are also eliminated, unobservability lowers efficiency when high bargaining power is on the principal’s side (i.e., when $\lambda$ is high).

Lastly, Lemma 3 shows that if the principal receives the bargaining power and her investment $p$ is sufficiently high, separation of types is induced, with the low type reneging on a promised payment. In this case, the principal’s $\text{ex ante}$ expected payoff conditional on receiving the bargaining power, \( p\pi (\bar{r}|b_{\text{ur}}, p) + (1 - p)\pi (\bar{r}|b_{\text{ur}}, p) \), is:

\[
\frac{(1 - \delta)[ps(\bar{r}, e_{\text{ur}}^*) + (1 - p)s(\bar{r}, e_{\text{ur}}^*) - \bar{u}] + \delta e_{\text{ur}}^*[p(s(\bar{r}, e_{\text{ur}}^*) - \bar{u}) + (1 - p)\bar{u}]}{1 - \delta(1 - e_{\text{ur}}^*)}.
\]

From an $\text{ex ante}$ perspective, the rent that the principal makes when she is a low type and reneges is cancelled by the compensation for the risk of default that the principal must pay to the agent. Given an investment $p$, private information then reduces the principal’s $\text{ex ante}$ expected payoff conditional on making the offers by lowering effort when she is a high type and requiring inefficient termination when she is a low type. Note that the principal’s investment becomes optimal as her bargaining power increases; hence, it follows that if $\lambda$ is high enough, surplus is lower when returns are unobservable than when they are observable.

**Proposition 3.** Suppose that the investment returns are unobservable by the agent and $\delta > 1/2$. Total expected surplus is always strictly below the first-best level. Moreover, there exists a cutoff $\hat{\lambda} \in (0, 1)$ such that if $\lambda \leq \hat{\lambda}$, total surplus is higher than when the investment returns are observable, whereas if $\lambda > \hat{\lambda}$, total surplus is lower.

This proposition completes Table 1 by showing how the effects of private information depend on the contracting environment. Limiting the flow of information can be a useful tool to increase $\text{ex ante}$ investment incentives if a holdup problem is present and parties care enough about the future. However, limiting information can also hurt $\text{ex post}$ effort incentives and overall efficiency if commitment to the relationship is important. Proposition 3 shows that the net effect, and thus the optimal control of information, depends on the allocation of bargaining power.

## 5. Discussion

Below, I extend the results to a setting where investment and effort are complements and discuss the restrictions on strategies.

**Complementarities.** The model assumes that investment increases the productivity of effort but not its marginal product. This is consistent with many applications; for example, a firm’s investment that lowers the fixed costs of production, for any quantity produced, or a distributor’s investment that lowers the cost of transporting a manufacturer’s product, for any quality of such product.

An alternative formulation is one where investment increases the marginal product of effort. For instance, let output be $y = q(1 + r)$.\(^{24}\) Unlike above, first-best effort now depends on the investment returns, so a higher investment leads to higher effort even when performance is contractible. Yet though the analysis becomes more cumbersome, the main insights remain unchanged. When the investment returns are observable, informal contracting settings $\text{ex post}$ can increase efficiency by inducing higher investment $\text{ex ante}$.

\(^{24}\)Equivalently, let $y = qr$ but assume $r$ is strictly greater than zero.
Proposition 4. Consider a model where investment increases the marginal product of effort: \( y = q(1 + r) \). Suppose that the investment returns are observable. There exist parameters (with \( 0 < \lambda < 1 \)) such that total expected surplus is higher when performance is noncontractible (incentives are relational) than when it is contractible.

The effects of the returns to the investment being unobservable by the agent are also as shown above. The main difference is that, here, an agent with bargaining power may induce the principal types to separate either by accepting different contracts or through a rejection, and he may do so for a discount factor larger than one half. However, in either case, separation requires that the future be sufficiently discounted; otherwise, the high principal type wants to mimic the low type. Therefore, if the parties care enough about the future, unobservability of returns eliminates the holdup problem. At the same time, unobservability reduces the effects of investment on effort. The net effect depends on bargaining power.

Proposition 5. Consider a model where investment increases the marginal product of effort: \( y = q(1 + r) \). Suppose that the investment returns are unobservable by the agent and performance is noncontractible (incentives are relational). There exist cutoffs \( \delta \in (0, 1) \) and \( \lambda \in (0, 1) \) such that for \( \delta > \delta \), total surplus is higher than when the investment returns are observable if \( \lambda \leq \lambda \), whereas it is lower if \( \lambda > \lambda \).

\[ \Box \]

**Strategies.** The first part of Assumption 2 states that following default, the parties end the relationship with positive probability and continue on the Pareto-optimal frontier otherwise. This is without loss of generality: if a Pareto-optimal equilibrium exists, there exists a Pareto-optimal equilibrium that satisfies this assumption and gives the same expected payoffs to all the parties. This is immediate if no default occurs in equilibrium. Now suppose the principal’s type is unobservable by the agent and a default occurs in equilibrium. If trade continues with probability one, the two types of principal have the same incentives to renege; hence, assuming termination with some positive probability is without loss. Further, suppose that following default the relationship ends with probability \( 1 - \gamma \) and continues on an inefficient path with probability \( \gamma \). We can show that for some \( \gamma' \), there exists an equilibrium where, following default, the relationship ends with probability \( 1 - \gamma' \) and continues on an efficient path with probability \( \gamma' \), and where the parties’ expected payoffs are weakly higher.

The second part of Assumption 2 states that if no default occurs, the relationship remains on the Pareto-optimal frontier. This means that the parties cannot end the relationship or switch to an inefficient level of trade following an unexpected investment decision, an unexpected offer, or a rejection.\(^2\) This assumption is not without loss: absent this assumption, other Pareto-optimal equilibria can be supported. The main motivation for assuming Pareto-optimal play absent default is to consider situations where bargaining power matters. I am interested in situations where parties have to make investment decisions before they can negotiate with trade partners and where, importantly, their gains from the investment depend on the outcome of those future negotiations. An example is the firm that invests in production assets prior to contracting with workers, knowing that workers may later capture part of the benefits through wage bargaining. In these situations, parties’ *ex post* rents, and thus their *ex ante* incentives to invest, naturally depend on the allocation of bargaining power. However, in a repeated game where contracting is self-enforcing and Assumption 2 is not imposed, bargaining power needs not affect the distribution of rents: by threatening to cease trade following certain histories, any distribution can be sustained.\(^2\) In fact, if threats can be made contingent on past investment, parties can be given efficient incentives to

\(^{25}\) This type of restriction on punishments appears in models where parties can renegotiate after a disagreement. For example, Ramey and Watson (2002) and Miller and Watson (2013) make assumptions about the bargaining stage and the consequences of disagreement at that stage that are similar to those made here.

\(^{26}\) Hence, absent this assumption, the party who makes the offers does not necessarily capture all the rents when information is complete.
invest. This is unrealistic, however, as it is required that *ex post* parties punish decisions taken *ex ante* before their interaction had started.

6. Concluding remarks

*■* Parties often have to make specific, noncontractible investments before starting a trade relationship. This article has studied how investments and efficiency are affected by the relationship parties form in a general principal-agent framework. Going back to the employment example, I showed that if the returns to a firm’s investment are observable to the workers, informal, more restrictive contracting environments *ex post* lead to higher investment *ex ante*. The firm invests not only to generate direct returns, but also to increase its commitment to the relationship and relax contracting constraints. As a consequence, when investment is subject to a holdup problem, informal relationships can be more productive.

If firms have the right to withhold information about the gains from their investments, they may eliminate the possibility of holdup. Privacy rights, therefore, improve efficiency when labor unions are strong, and thus holdup problems are important. However, when contracting is relational, withholding information lowers efficiency if the firm’s bargaining power is high enough and thus commitment to the relationship is more important. As an implication, firms’ incentives may be biased toward investments that generate more or less visible returns, depending on the nature of incentive contracts and the allocation of bargaining power.

Appendix

The proofs of all results in the text are presented below.

**Proof of Proposition 1.** The first part of the proposition, referring to the case of observable investment returns, is immediate from the analysis in the text. For the second part, suppose that the investment returns are unobservable by the agent. I proceed by proving three claims.

**Claim 1.** Suppose that the agent receives the bargaining power. A PPBE where separation of types occurs through the principal’s choice of contract does not exist.

Suppose by contradiction that there exists a PPBE where, in some period $t$, the agent offers a menu of contracts and types $r$ and $p$ accept different contracts. Let $(w_1, b_1)$ be the contract accepted by $r$ and $(w_2, b_2)$ the contract accepted by $p$. Let $e_1, e_2$ be the effort levels that solve the agent’s incentive compatibility constraint, given $b_1, b_2$. After this period $t$, the continuation play is the symmetric-information equilibrium, with the agent offering contract $(w_2^d, b_2')$ following acceptance of $(w_1, b_1)$ and contract $(w_2^d, b_2')$ following acceptance of $(w_2, b_2)$. However, then if $E[q - W_2|e_1] > E[q - W_1|e_1]$, type $r$ wants to deviate to $(w_2, b_2)$, and if $E[q - W_1|e_1] \geq E[q - W_2|e_2]$, type $p$ wants to deviate to $(w_1, b_1)$. Contradiction.

The claim extends immediately to mixed strategies. If type $p$ mixes between $(w_1, b_1)$ and $(w_2, b_2)$, whereas type $r$ accepts $(w_1, b_1)$, the agent offers $(w_2^d, b_2')$ following acceptance of $(w_2, b_2)$. For $p$ to mix, it must thus be that $E[q - W_2|e_1] > E[q - W_1|e_1]$, but then $r$ wants to deviate to $(w_2, b_2)$. Finally, if $r$ mixes between $(w_1, b_1)$ and $(w_2, b_2)$, whereas $p$ accepts $(w_2, b_2)$, it must be that $E[q - W_1|e_1] = E[q - W_2|e_2]$, but then $p$ wants to deviate to $(w_1, b_1)$.

**Claim 2.** Suppose that the agent receives the bargaining power and $\delta > \frac{1}{2}$. The Pareto-optimal PPBE is pooling and implements $(w_2^d, b_2')$ in every period.

By Claim 1, separation of types through the principal’s choice of contract is not feasible. Consider separation of types through rejection. Immediate full separation through rejection is not feasible because, as shown in the text, any contract that induces type $r$ to reject also induces type $p$ to reject when $\delta > \frac{1}{2}$. Gradual separation through rejection is not feasible, either. First, note that a PPBE where $r$ mixes between accepting and rejecting and $p$ accepts with probability one does not exist, as again $r$ wants to reject. Consider then a PPBE where $p$ mixes between accepting and rejecting and $r$ rejects with probability one. Given a prior $p$ at time $t$, the agent offers some contract $(w^d(p), b^s(p))$. If the principal accepts, the agent learns that $r = p$ and offers $(w_2^d, b_2')$ from then on. If the principal rejects, the agent updates his belief. Then there exists a period $t'$ where the posterior $p'$ is low enough that the agent optimally offers $(w_2^d, b_2')$ from then on. However, then, at $t' = t' - 1$, $r$ rejects any offer that $r$ rejects. In turn, at $t'$, the agent optimally offers $(w_2^d, b_2')$. However, then, at $t'' = t' - 1$, $p$ rejects any offer that $r$ rejects. Continuing with this reasoning gives that separation is not feasible if immediate separation is not feasible.
Hence, the equilibrium must be pooling, and the Pareto-optimal equilibrium implements contract \(\{w^{opt}, b^{opt}\}\) in every period.

**Claim 3.** Suppose that \(\beta > \frac{1}{2}\). Then investment and total expected surplus are first-best.

By Claims 1 and 2, the agent cannot induce separation of types when he receives the bargaining power. Thus, at time \(t = 0\), the principal solves

\[
\max_p \delta \pi(p) \left(1 - \delta \right) k(p).
\]

The solution is \(p = p^{opt}\) and the total expected surplus is thus

\[
S = \delta \left[p^{opt} \pi(p^{opt}) + (1 - p^{opt}) s(r, e^{opt})\right] - \left(1 - \delta \right) k(p^{opt}) = S^{opt}.
\]

**Proof of Proposition 2.** The first part follows from comparison of equations (2) and (3). For the second part, consider the following example: \(\beta = \frac{1}{2}, r \in [0, \frac{1}{2}], q \in [0, \frac{1}{2}], \pi = \frac{1}{2}, r = 0, k(p) = \frac{1}{2}, c(e) = \frac{1}{2}\). First-best investment and effort are \(p^{opt} = \frac{1}{2}, e^{opt} = \frac{1}{2}\). Under contractible performance, \(p^{opt} = \frac{1}{2}, e^{opt} = \frac{1}{2}\). When performance is noncontractible, \(e^{opt} = \frac{1}{2}\), \(e^{opt} = \frac{1}{2}\), and \(p^{opt} = \frac{1}{2}\) if \(\lambda \leq \frac{1}{2}\), \(p^{opt} = 1\) otherwise. One can immediately verify that \(S^{opt} > \max S\) for \(\lambda \in [0.52, 0.86]\). Figure 1 depicts \(S^{opt}\), \(S^{opt}\), and \(S^{opt}\) for \(\lambda \in [0.4, 1]\).

**Proof of Corollary 1.** Consider the example in the proof of Proposition 2 with \(r = \frac{1}{2}, \frac{1}{15}\). Consider lowering \(\pi\) from \(\pi = \frac{1}{2}\) to \(\pi = \frac{1}{2} - \frac{1}{15}\). For \(\lambda = 0.6\), Figure 2 shows that this change lowers total surplus.

**Proof of Lemma 1.** Suppose that the investment returns are unobservable by the agent. I proceed by proving four claims.

**Claim 1.** Suppose that the agent receives the bargaining power. If separation of types occurs in a PPBE, it occurs through a rejection.

This claim follows from the fact that separation of types through the principal’s choice of contract is not feasible. The proof is analogous to the one above when performance is contractible (Claim 1 in the proof of Proposition 1, where here, Assumption 2 ensures that continuation play after separation is given by the symmetric-information equilibria) and is thus omitted.

**Claim 2.** Suppose that the principal receives the bargaining power. A PPBE where full separation of types occurs through the principal’s choice of contract does not exist.

Suppose by contradiction that there exists a PPBE where types \(r\) and \(F\) offer different contracts in some period \(t\). Let \(\{w_1, b_1\}\) be the contract offered by \(r\) and \(\{w_2, b_2\}\) the contract offered by \(F\). Denote by \(\pi^F(\cdot, b, p)\) the payoff to type \(r\), given that the principal has the bargaining power, the implemented bonus is \(b\), and the agent’s belief is \(p\). By Assumption 2, after this period \(t\), the continuation play is the symmetric-information equilibrium, with payoff to the principal \(\pi^F(\cdot, e^{opt} - \pi)\). Let \(\rho\) be the belief that solve the agent’s incentive compatibility constraint given \(b_1, b_2\), and the agent’s beliefs. Suppose first that both \(\{w_1, b_1\}\) and \(\{w_2, b_2\}\) are such that the agent accepts, in equilibrium both types honor the payments and the relationship ends if a party reneges. Then,

\[
\pi^F(\cdot|b_1, 0) = (1 - \delta)(\pi + E[q - W^F|e_1]) + \delta s(\pi, e^{opt} - \pi),
\]

\[
\pi^F(\cdot|b_2, 1) = (1 - \delta)(\pi + E[q - W^F|e_2]) + \delta s(\pi, e^{opt} - \pi).
\]

Now if type \(F\) deviates to \(\{w_1, b_1\}\), she obtains

\[
\pi^F(\cdot|b_1, 0) = (1 - \delta)(\pi + E[q - W^F|e_1]) + \delta s(\pi, e^{opt} - \pi) = \pi^F(\cdot|b_1, 0) + (\pi - \pi),
\]

and if type \(F\) deviates to \(\{w_2, b_2\}\), she obtains

\[
\pi^F(\cdot|b_2, 1) = (1 - \delta)(\pi + E[q - W^F|e_2])
\]

\[
+ \max \left(\delta \pi^F(\cdot|e^{opt} - \pi, e_2) + (1 - e_2) \pi^F(\cdot|e^{opt} - \pi, 1), \right)
\]

where \(\pi^F(\cdot|e^{opt} - \pi, 1)\) is the continuation value of type \(r\) when the agent’s posterior belief is one. As shown in the text,

\[
\pi^F(\cdot|e^{opt} - \pi, 1) = s(\pi, e^{opt} - \pi) + \left(\frac{\delta e^{opt}}{1 - \delta(1 - e^{opt})}(\pi - \pi)\right).
\]
Note that \( \pi \) gains from the option of being able to renege. Hence,
\[
\pi'(\pi(b_2, 1) > \pi'(\pi(b_2, 1) - (\tau - \pi)). \quad \text{(A2)}
\]

A PPBE where full separation occurs through the principal’s contract offer exists if and only if for \( \delta \in (0, 1) \), we have that \( \pi'(\pi(b_1, 0) > \pi'(\pi(b_2, 1)) \) and \( \pi'(\pi(b_2, 1) > \pi'(\pi(b_1, 0)) \). However, equations (A1) and (A2) imply that one of these two incentive compatibility constraints is always violated. Contradiction.

Suppose next that \( \{w_1, b_1\} \) or \( \{w_2, b_2\} \) are both such that the agent rejects. If \( \{w_1, b_1\} \) is rejected, \( \pi \) has incentives to deviate to \( \pi'_{b''} \), which (by Assumption 2) is always accepted and gives \( \pi \) a continuation value at least as high as \( \{w_1, b_1\} \). If \( \{w_2, b_2\} \) is rejected, the argument above is strengthened because the difference between \( \pi'(\pi(b_1, 1)) \) and \( \pi'(\pi(b_2, 1)) - (\tau - \pi) \) increases.

**Claim 3.** Suppose that the principal receives the bargaining power. A PPBE implementing contracts \( b' \) and \( b'' < b' \), where partial separation of types occurs through the principal’s choice of contract, exists only if there exists a PPBE implementing \( b' \) where separation occurs only through default.

Step 1. Suppose there exists a PPBE where contracts \( b', b'' \) are implemented and partial separation occurs through the principal’s contract offer. Suppose no type reneges in equilibrium. Then it must be that \( b' = b'' = b''_{2w} \); otherwise, regardless of beliefs, some type would want to deviate. However, then no separation occurs. Contradiction.

Step 2. Suppose there exists a PPBE where contracts \( b', b'' \) are implemented and partial separation occurs through the principal’s contract offer. Suppose \( \pi \) offers contract \( b'' \) and \( \tau \) mixes between \( b' \) and \( b'' \). Let \( \rho' \) be the agent’s belief when contract \( b'' \) is offered and \( \rho'' \) when contract \( b'' \) is offered. Then in equilibrium, \( \pi'(\pi(b'', \rho'') \geq s(\pi', \pi'' - \tau) \) (otherwise, \( \tau \) would deviate to \( b''_{2w} \), and thus, \( b'' > b''_{2w} \). Now because \( \rho'' = 0, \pi'(\pi(b'', \rho'') < s(\pi', \pi'' - \tau) = s(\pi', \pi' - \tau) \leq \pi'(\pi(b'', \rho'') \leq \pi'(\pi(b', \rho'') - (\tau - \pi) \). Therefore, then type \( \pi \) is not willing to mix between \( b' \) and \( b'' \). Contradiction.

Step 3. By Steps 1 and 2, in any PPBE with partial separation through the principal’s contract offer, \( \pi \) offers some contract \( b'' \); \( \tau \) mixes between \( b' \) and some \( b'' < b' \), \( \pi \) reneges on \( b' \) with positive probability, and \( \tau \) honors \( b' \) and \( b'' \) with probability one. Now note that if \( \pi \) offers \( b' \) with probability \( \rho_e \in (0, 1), \rho'' = \rho_e \rho_0 + (1 - \rho)) < p \), and thus given a probability with which \( \pi \) honors the contract \( \rho_e \| 0, 1 \), the probability that \( b'' \) is honored is \( p'' = (1 - p) \rho_e < \rho_0 \geq (1 - p) \rho_e < e(b', \rho'') < e(b', \rho''). \) However, then \( \pi''(r(b', \rho'') < \pi''(r(b', \rho'')) \). Therefore, if there exists a PPBE where both types offer \( b'' \) given belief \( \rho' \), there exists a PPBE where both types offer \( b'' \) given belief \( \rho'' \).

**Claim 4.** Suppose that the principal receives the bargaining power. If separation of types occurs in a Pareto-optimal PPBE, it occurs only through default.

By Claim 2, we only need to consider a PPBE with separation through default and a PPBE with partial separation through the principal’s contract offer. Suppose a PPBE with partial separation through the principal’s contract offer exists. Then by Claim 3, a PPBE where separation occurs only through default (with both types offering the same contract) also exists. Moreover, it is immediate from the proof of Claim 3 (Step 3) that a PPBE where separation occurs exclusively through default Pareto-dominates the PPBE where separation also occurs through the principal’s contract offer. The claim follows.

**Proof of Lemma 2.** Suppose the investment returns are unobservable by the agent, \( \delta > \frac{1}{2} \), and the agent has the bargaining power. By Claim 1 in the proof of Lemma 1, separation of types must involve a rejection. By an analysis analogous to that in the text for contractible performance, immediate full separation through rejection is not feasible. Moreover, the argument in the proof of Proposition 1 shows that gradual separation through rejection is not feasible, either. Hence, separation of types cannot be induced, and the agent optimally offers the symmetric-information contract of the low type, \( \{w_{b''}, b''_{2w}\} \), in every period.

**Proof of Lemma 3.** Suppose that the investment returns are unobservable by the agent. I proceed by proving two claims.

**Claim 1.** Suppose that the principal receives the bargaining power. A pooling PPBE always exists, and any such PPBE implements \( \{w_{b''}, b''_{2w}\} \) in all periods.

For existence, consider this PPBE: the agent’s belief upon the principal offering \( b''_{2w} \) is \( \rho \), his belief upon the principal offering \( b' \neq b''_{2w} \) is zero, and his belief upon the principal offering \( b''_{2w} \) and then making any payment \( W \) is \( \rho \); the principal offers \( b''_{2w} \) (with fixed wage \( w_{b''} \)) in every period; the agent accepts and chooses \( e_{b''} \); both types always honor; if the principal reneges, the relationship ends. Clearly, beliefs are consistent, the agent’s participation and effort decisions are optimal, and the principal’s payment decision is optimal. To see that offering \( b''_{2w} \) is optimal for the principal, note that for \( r \in (\pi', \pi) \), \( \pi'(r(b'_{2w}) = s(r, e_{b''}) - \pi, \pi'(r(b', \rho) < s(r, e_{b''}) - \pi \) for any \( b' < b''_{2w} \) because the agent then chooses \( e' < e_{b''} \), and \( \pi'(r(b', \rho) < s(r, e_{b''}) - \pi \) for any \( b'' > b'' \) because the agent rejects, or accepts and chooses \( e' = 0 \). Finally, because both types follow the same strategy, no information is revealed.
To show that any pooling PPBE implements $h^\pi_r$, note that such PPBE cannot implement $b^r > h^\pi_r$; regardless of beliefs (given Assumption 2), both types can increase their expected payoffs by deviating to $h^\pi_r$. Further, any PPBE that implements $b^r > h^\pi_r$ induces separation. Otherwise, either both types always renge or both always honor. However, if both types renge, the agent exerts no effort, so $b^r$ cannot be optimal. And, as $b^r > h^\pi_r$, $\zeta$ optimally renge.

**Claim 2.** Suppose that the principal receives the bargaining power. There exists $\tilde{p} \in (0, 1)$ such that a PPBE that induces separation of types exists if and only if $p \geq \tilde{p}$.

The analysis of separating equilibria in the text can be extended to allow for mixed strategies. In such equilibria, $\zeta$ initially mixes between honoring and renge, until in some finite period $r$ she renge with probability one, and the relationship ends with positive probability if the principal renge. A program similar to the one presented in Section 4 (but allowing for mixed strategies) can be derived, and such program gives a necessary and sufficient condition for the existence of a PPBE with separation through default, which is analogous to the condition given in the text. It then follows directly from inspection of these conditions that there exists $\tilde{p} \in (0, 1)$ such that the conditions hold if and only if $p \geq \tilde{p}$.

The reader is referred to Proposition 2 in Halac (2012) for details.

Finally, by Lemma 1 and Claim 1, we only need to consider PPBE where separation occurs only through default and the pooling PPBE implementing $h^\pi_r$. By Claim 2, there exists $\tilde{p} \in (0, 1)$ such that if $p \geq \tilde{p}$, a separating PPBE exists, which also implies that both types prefer this PPBE to the pooling PPBE. If $p < \tilde{p}$, a PPBE where separation occurs through a default does not exist, which (by Claim 3 in the proof of Lemma 1) implies that the pooling PPBE is unique, and thus Pareto-optimal.

### Proof of Proposition 3

The first part of the proposition is immediate. For the second part, suppose first that $p < \tilde{p}$, with $\tilde{p}$ defined in Lemma 3. Then $S^r - S^0$ is equal to

$$\delta \left[ p^r \left( E[E(q - c] - E[q - c]) - (1 - \delta) \left( k(p^r) - k(p^\beta) \right) \right) \right],$$

where $p^r$ is given by condition (1). For $\lambda > 0$, $p^\lambda \to 0$, and thus $S^r - S^0 < 0$. Note also that if $p^r = p^\beta$, then $S^r - S^0 > 0$. However, for $\lambda$ high enough, $p^\lambda$ maximizes $S^r$, and thus it must be that $S^r - S^0 > 0$.

Suppose next that $p \geq \tilde{p}$ and a Pareto-optimal separating equilibrium when the principal makes the offers implements $e^\pi_r$. Assume without loss $\bar{p} = 0$. Then $S^r - S^0$ is equal to

$$\frac{\bar{p}}{1 - p^r} \left[ 1 + \delta \left( p^r s_1(\alpha_r, e^\pi_r) - (1 - \delta) \left( k(p^r) - k(p^\beta) \right) \right) \right].$$

It is immediate that for $\lambda$ low enough, $S^r - S^0 < 0$. Also, note that if $p^r = p^\alpha$, with $p^\alpha \geq \tilde{p}$, then $S^r - S^0 > 0$. However, for $\lambda$ high enough, $p^\lambda$ maximizes $S^r$, and thus it must be that $S^r - S^0 > 0$.

### Proof of Proposition 4

Consider the following example: $\delta = \frac{1}{2}$, $r \in (0, \frac{1}{2})$, $\eta \in (0, \frac{1}{2})$, $\pi = \frac{1}{3}$, $\pi = 0$, $k(p) = \frac{\pi}{\eta}$, $c(\eta) = \frac{\delta}{2}$. First-best investment and effort are $p^\beta = \frac{\pi}{\eta}$, $e^\pi = 1$, $e^\beta = \frac{1}{2}$. Under contractible performance, $p^r = \frac{\eta}{2} \lambda$, $e^r = e^\beta$. When performance is noncontractible, $e^r = \frac{\eta}{2} \left( 1 - \frac{1}{2} \right)$, $e^\pi = \frac{1}{2}$. Then $s_1(\eta, e^\pi) = \frac{\eta}{2} (1 + \frac{1}{2}) - (e^\pi)^2$, $s_1(\eta, e^\pi) = 5/32$, and $p^r = \min \left( \frac{1}{2} s_1(\eta, e^\pi) - s_1(\eta, e^\pi), \lambda, 1 \right)$. One can verify that $S^r > S^\pi$ for $\lambda \in [0.57, 0.85]$.

### Proof of Proposition 5

Suppose that the agent received the bargaining power. It is easy to see that if the agent cannot induce separation of types through rejection, he cannot induce separation of types through the principal’s choice of contract (i.e., through $\eta$ accepting a different contract than $\xi$), as rejection maximizes the difference between the two types’ payoffs. Consider then the conditions for rejection. Suppose that the agent proposes a contract $\{w_\eta, h_\pi\}$ with expected payment $E[W^\eta | e^\xi]$ and the principal’s type is fully revealed through rejection. Because $\xi$ has a lower value of the relationship than $\eta$, it must be $\xi$ who rejects, whereas $\eta$ accepts. Thus, if the offer is rejected, the agent proposes $\{w_\eta, h_\pi\}$ from then on, where $h_\pi$ solves the program in Section 4 with the new expected function (and $r = \xi$) and $w_\eta$ is such that $E[W^\eta | e^\eta] = E[y | e^\xi] - E[y | e^\eta]$. If the offer is accepted, the agent proposes $\{w_\eta, h_\pi\}$, where $w_\eta$ is such that $E[W^\eta | e^\eta] = E[y | e^\eta]$. Hence, type $\xi$ will indeed reject only if

$$(1 - \delta) E[y - W^\eta | e^\xi] + \delta \pi \leq \pi,$$

and type $\eta$ will indeed accept only if

$$(1 - \delta) E[y - W^\eta | e^\eta] + \delta \pi \geq \pi + \pi (E[y | e^\xi] - E[y | e^\eta]).$$

Combining these two conditions, full separation of types through rejection is feasible only if

$$\delta \frac{\pi}{1 - \delta} \leq E[q | e^\eta].$$

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Note that $\mathbb{E}[q|\omega]\geq 0$, and thus there exists $\overline{\delta} \in (0, 1)$ such that this condition is violated for $\delta > \overline{\delta}$. Finally, a similar argument as the one used in the proof of Proposition 1 shows that gradual separation through rejection is not feasible if immediate full separation through rejection is not feasible. The rest of the analysis is analogous to that in the main text and thus omitted.

References


