Web Appendix:
Relational Contracts and the Value of Relationships

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I. Equilibrium concepts

This section defines the equilibrium concepts used in the paper.

Given party $i$’s offer $b^i$, let $d^j \in \{0,1\}$ denote party $j$’s decision to accept or reject. Let $h_t = (b_t^i, d_t^j, y_t, W_t^i)$ denote the public outcome at time $t$, and $h^t = (h_0, ..., h_{t-1})$ the public history up to time $t$. A public strategy for a type-$\theta$ principal is a triple $\sigma_{\theta t} = (g_\theta(h^t, b), a_\theta(h^t, b), k_\theta(h^t, b^i))$, where $g_\theta$ is the probability with which she offers contract $b$ (when she is the contract offerer), $a_\theta$ is the probability with which she accepts contract $b$ (when the offeree), and $k_\theta$ is the probability with which she honors the contract (i.e., pays the agent when $y = \bar{y}$). A public strategy for the agent is analogously defined as $\sigma_{A t} = (g_A(h^t, b), a_A(h^t, b), c(h^t, b^i), k_A(h^t, b^i))$, where $c$ is the agent’s effort choice.

Let $g_\theta(h^t, b) \equiv g_\theta$, $a_\theta(h^t, b) \equiv a_\theta$, $k_\theta(h^t, b^i) \equiv k_\theta$. A PPBE is a quintuple $(\sigma_\ell, \sigma_h, \sigma_A, \mu, \phi)$ such that Assumptions 1 and 2 are satisfied and

1. $\sigma_\ell$, $\sigma_h$, and $\sigma_A$ are mutual best responses for all $t$ and $h^t$,
2. $\mu(p|b^P) = \frac{p g_\ell}{p g_\ell + (1-p)g_h} \forall b^P$ s.t. $g_\theta > 0$ for some $\theta$,
3. $\mu(p|b^A) = \frac{p a_\ell}{p a_\ell + (1-p)a_h} \forall b^A$ s.t. $a_\theta > 0$ for some $\theta$,
4. $\mu(p|\text{reject } b^A) = \frac{p(1-a_\ell)}{p(1-a_\ell) + (1-p)(1-a_h)} \forall b^A$ s.t. $a_\theta < 1$ for some $\theta$,
5. $\phi(\mu(p)|w^j + \bar{y}) = \frac{\mu(p|b^i)k_\ell^j}{\mu(p|b^i)k_\ell^j + (1-\mu(p|b^i))k_h^j} \forall b^j$ s.t. $k_\theta^j > 0$ for some $\theta$,
6. $\phi(\mu(p)|w^j) = \frac{\mu(p|b^i)(1-k_\ell^j)}{\mu(p|b^i)(1-k_\ell^j) + (1-\mu(p|b^i))(1-k_h^j)} \forall b^j$ s.t. $k_\theta^j < 1$ for some $\theta$.

A WMPBE is a PPBE where strategies are weak Markov and beliefs Markov as defined in the text. To define the weak Markov strategies formally, let the parties make a decision to continue or end the relationship at the beginning of each period $t$. Denote the probabilities with which a type-$\theta$ principal and the agent decide to continue by $\gamma_{\theta t}$ and $\gamma_{A t}$. Let $\Gamma_t = 1$ if the principal and agent’s observed decisions
at time $t$ are both to continue, and $\Gamma_t = 0$ otherwise. A weak Markov strategy for a type-$\theta$ principal is $\sigma^{wm}_{\theta t} = (\gamma_{\theta}(\mu_t, h_{t-1}), g_{\theta}(\Gamma_t, \mu_t, b), a_{\theta}(\Gamma_t, \mu_t, b), k_{\theta}(\Gamma_t, \mu_t, b'))$, and for the agent it is $\sigma^{wm}_{At} = (\gamma_{A}(\mu_t, h_{t-1}), (g_{A}(\Gamma_t, \mu_t, b), a_{A}(\Gamma_t, \mu_t, b), e(\Gamma_t, \mu_t, b'), k_{A}(\Gamma_t, \mu_t, b'))$.

II. Consequences of default, rejection, and unexpected offers

This section states and proves the results discussed in Section IVB of the paper.

**Proposition A1.** If a Pareto-optimal equilibrium exists, there exists a Pareto-optimal equilibrium where, following default, the relationship ends with positive probability and continues on the Pareto-optimal frontier otherwise, and where the parties’ expected payoffs are the same.

**PROOF:**

Suppose that no default occurs in equilibrium. Then the worst punishment for default is optimal and terminating the relationship with probability one is without loss.

Suppose next that a default occurs in equilibrium. Then after default, $\ell$ and $h$’s continuation payoffs must be different; otherwise, both types would want to honor or both to renege, but then a default cannot occur in equilibrium. Thus, since $\ell$ and $h$ only differ in their outside options, it must be that a default is followed by a contract that involves no trade with strictly positive probability. Then assuming that the relationship ends with positive probability after default is without loss.

Finally, suppose there exists a Pareto-optimal equilibrium where, after default, the relationship ends with probability $1 - \gamma > 0$ and continues on an inefficient path of play with probability $\gamma$. Consider a second equilibrium where, after default, the relationship ends with probability $1 - \gamma' > 0$ and continues on an efficient path of play with probability $\gamma'$. Let $\gamma'$ be such that $h$’s continuation payoff after default is the same as in the first equilibrium. (It is straightforward to show that such $\gamma'$ exists.) Then $\ell$’s continuation payoff after default is lower than in the first equilibrium. Hence, the second equilibrium allows to implement the same or higher self-enforcing incentives as the first equilibrium while a default does not occur, and to lower the punishment for default for $h$ conditional on $\ell$’s enforcement constraint holding. The result follows.

**Proposition A2.** If an equilibrium is Pareto optimal under Assumptions 2 and 3, then it is Pareto optimal when Assumptions 2 and 3 are not imposed.

**PROOF:**

First, note that the first part of Assumption 2 is without loss by Proposition A1. Next, note that any equilibrium under Assumptions 2-3 is also an equilibrium when these assumptions are not imposed. Finally, suppose by contradiction that there exists a Pareto-optimal equilibrium under Assumptions 2-3 that is not
Pareto optimal when these assumptions are not imposed. Let the expected payoffs generated by this equilibrium be $u$, $\pi_\ell$, and $\pi_h$. Then it must be that, when Assumptions 2-3 are not imposed, there exists a Pareto-optimal equilibrium that (i) is not an equilibrium under Assumptions 2-3, and (ii) generates expected payoffs $u' \geq u$, $\pi'_\ell \geq \pi_\ell$, and $\pi'_h \geq \pi_h$, with at least one of these inequalities strict. Now (i) and (ii) imply that such an equilibrium must induce separation of types by either (a) prescribing inefficient play following a rejection by the principal or (b) prescribing inefficient play following an unexpected offer by the principal. But then, given that the continuation play following separation must be such that $h$ is willing to reject in case (a) and make an unexpected offer in case (b), it must be that at least one of the inequalities in (ii) is not satisfied. Contradiction.

**Proposition A3.** Suppose that the parties may end the relationship with positive probability after an unexpected offer. Then a contract-separating equilibrium exists for all $\lambda \in (0, 1]$.

**PROOF:**

Let the agent’s beliefs be $\mu(p_0|w_1, b_1) = 1$ for some contract $(w_1, b_1)$ and $\mu(p_0|w, b) = 0$ for any contract $(w, b) \neq (w_1, b_1)$. Let $b_1 = b_\ell$ and $w_1$ be such that $\pi^P_\ell(1, w_1, b_1) = r_\ell$. Suppose that the agent ends the relationship with probability one if the principal offers $(w, b) \neq (w_1, b_1)$. Then it is immediate that $\ell$ is indifferent between $(w_1, b_1)$ and $(w, b) \neq (w_1, b_1)$, while $h$ strictly prefers $(w, b) \neq (w_1, b_1)$. The claim follows.

**Proposition A4.** Regardless of the bargaining protocol and the restrictions on strategies, a separating equilibrium where trade occurs with probability one on the equilibrium path does not exist.

**PROOF:**

Suppose that trade occurs with probability one on the equilibrium path. Then since the two types differ only in their outside options, it must be that $\ell$ and $h$ take the same actions in equilibrium. But then no separation can occur. Contradiction.