Costs of Production and Profit Maximizing Production: 3 examples.

In this handout, we analyze costs and profit maximizing output decisions by looking at three different possible costs structures. Three different examples will be used to illustrate: all the relevant cost concepts in section I, and the profit maximizing output choices in section II.

I. Costs of Production

We are analyzing the costs associated with the production of output of a particular firm in a particular market. To fix ideas, you might want to think to the costs faced by TV Listing Magazine in providing the town of Cheyenne with an arbitrary number of copies. Remember that we are only focusing on the avoidable costs, i.e., on the costs directly related to the task of producing magazines in Cheyenne. In all the examples, we exploit the following equality:

\[
\text{Total (avoidable) costs} = \text{fixed (avoidable) costs} + \text{variable (avoidable) costs}
\]

In formula:

\[ C(Q) = FC + VC(Q), \]

where Q denotes the units of output produced, FC the fixed cost, VC(Q) the variable cost associated with the production of Q units of output and C(Q) the total cost associated with the production of Q units of output.

Also, remember that:

Average Total Cost = Total Cost/Output = Average Fixed Cost + Average Variable Cost;

in formula:

\[ AC(Q) = AFC(Q) + AVC(Q);\]
\[ \text{AVC}(Q) = \frac{VC(Q)}{Q} \text{ and } AFC(Q) = \frac{FC}{Q} \]

Finally the marginal cost evaluated at Q units of output, MC(Q), is the cost generated by the production of an extra unit of output.

Example 1: Simplest conceivable cost structure (e.g., TV Listing Magazines)

Description of the cost structure:

The firm can produce at most 100 units of output per year, i.e., capacity = 100. In order to produce, it must incur in a fixed cost of $100 (per year); FC = $100. The variable cost per unit of output is $1 up to capacity
Since it costs $1 of variable inputs to produce one unit of output, it costs $Q of variable inputs to produce Q units of output. Hence:

\[ VC(Q) = Q \text{ and } AVC(Q) = 1, \text{ for each } Q \leq 100 \]

Since the fixed cost is $100, it follows that

\[ AFC(Q) = \frac{100}{Q}, \text{ for each } Q \leq 100. \]

Hence:

\[ AC(Q) = \left(\frac{100}{Q}\right) + 1, \text{ for each } Q \leq 100. \]

Observe that as Q approaches 0, the AFC(Q) (and, therefore, the AC(Q)) becomes very, very large (100/Q goes to infinity). Moreover, when the firm is producing at capacity, i.e., Q = 100, the AC(100) = $2.

Finally suppose that you are producing Q units, Q < 100, in order to add an extra unit you incur an additional cost of $1. Hence:

\[ MC(Q) = AVC(Q) = 1. \]
**Example 2:**
In order to visualize this situation, think about problem set 1. There, the bank could use 3 different sources of funds in order to “produce” loans. However, the cost associated to each source was different and, moreover, the first two sources (checking and savings deposits) were limited in quantity.

**Description of the cost structure:**
The firm can produce at most 200 units of output per year, i.e., capacity = 200. In order to do that, it must incur in a fixed cost whose annual value is $100; i.e., FC=$100. The firm has to pay $1 per unit in order to produce output up to a capacity of 100. For each unit produced above 100, the per unit cost is $4 up to a capacity of 200.

Suppose that the firm is producing Q units, $ Q \leq 100. Since it pays $1 per unit in variable costs, the variable cost of producing Q units is:

$$ VC(Q) = Q, \text{ for } Q \leq 100, $$

and the total cost is

$$ C(Q) = Q+100, \text{ for } Q \leq 100 $$

Suppose now that the firm is producing Q units, $ Q>100. Then, the firm pays $1 per unit in variable cost for the first 100 units and $4 per unit for the remaining (Q-100) units. Hence, the variable cost of producing Q units is

$$ VC(Q) = 100*1+(Q-100)*4 = 4Q-300, \text{ for } 100 < Q < 200, $$

and the total cost is

$$ C(Q)=FC+VC(Q)=100+4Q-300=4Q-200, \text{ for } 100 < Q < 200. $$

**Summarizing:**

$$ C(Q) = 100 +Q, \text{ if } Q \leq 100 $$

$$ C(Q) = 4Q - 200, \text{ if } 100 < Q \leq 200 $$

Therefore, the average cost is:

$$ AVC(Q) = 1 + (100/Q), \text{ if } Q \leq 100 $$

$$ AVC(Q) = 4 - 200/Q, \text{ if } 100 < Q \leq 200, $$

and the average variable cost is:

$$ AVC(Q) =VC(Q)/Q = 1, \text{ if } Q \leq 100,$$
AVC = 4 - (300/Q), if 100 < Q ≤ 200.

Finally suppose that the firm is producing Q units of output, Q < 100. In order to produce an extra unit the firm must pay $1 of variable costs. Hence,

\[ MC(Q) = 1, \text{ for each } Q < 100. \]

However, if the firm is producing Q, 100 ≤ Q < 200, the extra unit requires $4 of variable inputs. Hence,

\[ MC(Q) = 4, \text{ for each } 100 ≤ Q < 200. \]

Observe that when Q < 100,

\[ MC(Q) = 1 < AC(Q) = 1 + 100/Q \]

and therefore, the AC(Q) is declining for 0 < Q < 100. However, when Q > 100,

\[ MC(Q) = 4 > AC(Q) = 4 - (200/Q) \]

and the AC(Q) is increasing for Q > 100. In fact, the AC reaches its minimum exactly at Q = 100 where it is equal to $2.

Finally, observe that AVC(Q) = 4 - (300/Q) and MC(Q) = 4, for Q > 100. This shows that it might be very well true (depending on the technologies) that marginal costs and variable costs are different.

See the graph below.
Example 3: Textbook technology.

The firm is facing the following total cost function:

\[ C(Q) = 100 + Q + Q^2 \]

We are going to find: i) the average cost curve, ii) the marginal cost curve, and iii) the minimum average cost and its corresponding output.
i) The average total cost is:

\[ AC(Q) = \frac{C(Q)}{Q} = \frac{100}{Q} + 1 + Q. \]

ii) The marginal cost is the derivative of the total cost with respect to Q:

\[ MC(Q) = \frac{dC(Q)}{dQ} = 1 + 2Q \]

iii) The average cost reaches its minimum at output level \( Q^* \) where \( MC(Q^*) = AC(Q^*) \). Hence, \( Q^* \) solves the following equation:

\[ \frac{100}{Q} + 1 + Q = 1 + 2Q \]

or

\[ Q^2 = 100. \]

Therefore, \( Q^* = 10 \). The minimum average cost is \( AC(10) = 21 \). For \( Q < 10 \), the \( MC(Q) \) is less than the \( AC(Q) \) and the average cost is declining. For \( Q > 10 \), the \( AC(Q) > MC(Q) \) and the average cost is increasing.
II. Profit maximizing output decisions and the supply curve of the firm.

A firm is analyzing the possibility of entering in a market. The market price is given and is denoted by p. The firm can supply any quantity of output without affecting the market price. Hence the firm must give answers to two questions: a) Should it enter the market or not? B) If entry takes place, what is the profit maximizing output to be produced?

Bear in mind that the goal of the firm is to maximize its economic profits and that, therefore, only avoidable costs have to be considered in this analysis. An answer to a) and b) for all conceivable market prices defines the supply curve of the firm.

In this section, we look at this problem by analyzing the three cost structures of the previous section.

It will be useful to remember the following simple observations:

i) the firm decides to produce positive output if and only if this decision generates positive economic profits. Otherwise, the optimal choice is to stay out, i.e., to not engage in any production activity;
ii) by definition, the economic profits generated by the production of Q units of output are equal to the profits per unit times the number of units:

\[ \text{profits} = pQ - C(Q) = (p - AC(Q))Q = (\text{profit per unit}) \times (\text{number of units}) \]

These two observations imply that the firm will decide to produce if and only if the market price allows for positive profits per unit, i.e., only if \( p > AC(Q) \), for some level of output Q. However, the price is above the average cost of production of some output level Q if and only if \( p > \text{min AC} \). Hence, in order to decide whether to enter the market or not, it suffices to compare the market price to the minimum average cost of production. If \( p > \text{min AC} \), enter, if \( p < \text{min AC} \), stay out.
(In the perverse case \( p = \text{min AC} \), the firm is indifferent between two choices: a) produce at \( Q^* \), the output corresponding to the minimum average cost, and realize 0 economic profits, b) do not enter the market and, again, realize zero economic profits.)

Example 1

If the market price is below 2 (the min AC), the firm does not enter the market. However, if \( p > 2 \), the firm will find entry and production profitable. The problem is to find the profit maximizing output for \( p > 2 \).

It is useful to start from an arbitrary level and to perform marginal adjustments. Hence, say that, for \( p > 2 \), the firm is considering the production of Q units of output, Q < 100. Is this choice optimal?
Consider the possibility of increasing the production by one unit. How is this choice going to change economic profits? Or, more precisely, what are the changes in revenues and the changes in costs generated by the production of this extra unit?

Since the firm is producing an extra unit, it will be able to make an extra sale increasing by $p$ its revenues. However, the production of an extra unit increases costs by $MC(Q)$. In example 1, $MC(Q) = 1$, for all $Q < 100$. Hence the change in profits generated by the production of an extra unit of output is

\[
(\text{change in revenues} - \text{change in costs}) = p - 2 > 0.
\]

It follows that, if $p > 2$, every time $Q < 100$, the firm has an incentive to increase production. This implies that the firm should optimally produce at capacity.

Summarizing:

i) If $p < 2$, the profit maximizing choice is to not produce at all (or to stay out of the market);

ii) if $p > 2$, the firm produces at capacity.

i) and ii) entirely characterize the supply curve of the firm.

The blue thick line represents the supply curve of the firm.

**Example 2.**

In this case the min $AC = 2$. Therefore, the firm will not produce for any price below $2$, while it will for any $p > 2$. The problem is to find the profit maximizing output associated to any $p > 2$, i.e., to find the supply curve of the firm.
We analyze the problem by looking at three different price ranges:

i) Consider any market price $2 < p < 4$. What is optimal output at $p$? Remember that $MC(Q) = 1$, for $Q < 100$, while $MC(Q) = 4$, for $100 < Q < 200$. Should the firm produce less than 100 units? No. Why? By increasing output by one unit, the firm is going to increase revenues by $p$, while costs are going to increase by $MC(Q) = 1$ (since $Q < 100$). Therefore, the change in profits is equal to $p - MC(Q) = p - 1 > 0$. Therefore, it is never profit maximizing to produce below 100.

Should the firm produce above 100? No. Why? By reducing output by one unit, the firm is going to lose $p$ of revenues, while it will save $MC(Q) = 4$ of costs. This generates a change in profits equal to $-p + MC(Q) = 4 - p > 0$. Therefore, it is never profit maximizing to produce above 100 when $2 < p < 4$. It follows that the optimal output decision is $Q = 100$ for all price $p$, $2 < p < 4$.

ii) Suppose now that $p > 4$. What is the optimal output choice? Should the firm produce at less than capacity, i.e., $Q < 200$? No. Why? By increasing output by one unit, the firm increases revenues by $p$, while costs increase by $MC(Q)$. Therefore, the change in profits is equal to $p - MC(Q)$. If $Q < 100$, $p - MC(Q) = p - 1$, while if $100 < Q < 200$, $p - MC(Q) = p - 4$. In both circumstances since $p > 4$, the change in profits is positive and, therefore, the firm should increase production. It follows that $Q = 200$ is the optimal production level.

iii) Suppose now that $p = 4$. By point i), the firm will never stop production below 100 units and it is indifferent toward producing any amount between 100 and 200 units. Why? Each unit, in addition to the first 100, costs $MC(Q) = 4$ and yields $p = 4$ of revenues. The firm is neither making profits nor losses on any of these units and it is therefore indifferent about producing them.

Points i), ii) and iii) suffice to entirely characterize the supply curve of the firm. The thick blue curve in the picture below represents the supply curve of the firm.
Example 3:
Surprisingly, this is the simplest setting in which to characterize the supply curve of the firm. As already said, no production activity will take place for prices below the min AC, i.e., $p < 21$. For $p > 21$, (as discussed in class) the profit maximizing output satisfies the condition $p = MC(Q)$, i.e.:

\[(1) \quad p = 1 + 2Q.\]

Solving equation (1) in terms of $Q$ yields:

$$Q = \frac{(p - 1)}{2}.$$  

Summarizing, the supply curve of the firm is:

$$Q = \frac{(p - 1)}{2}, \text{ for } p \geq 21,$$

$$Q = 0, \text{ for } p \leq 21.$$

The supply of the firm is represented by the thick blues line:
Caveat: Remember that $C(Q) = FC + VC(Q)$. The fixed cost is independent of the level of output, while the variable cost depends on it. Moreover, if $Q = 0$, the variable cost is zero. However, since the fixed cost is independent of output, it is equal to $FC$ even when $Q = 0$. Does this mean that the firm has to pay the (avoidable) fixed costs even when it does not enter the market (or it does not produce at all)? The answer is no. The reason is that there is a subtle distinction between producing 0 and not producing at all (i.e., not entering the market). Think about the TV Listing magazine situation. If the firm enters the Cheyenne market, it needs plates, local data bases and so on. These costs are independent of how many copies of the magazines are produced weekly. If the firm produces just one copy per week it needs to pay all the fixed costs. However, if the firm stays out of Cheyenne, none of these costs are incurred. Hence, to be a bit pedantic, when we say that the supply of the firm is identically equal to zero for $p < \min AC$, we really mean that the firm stays out of the market, i.e., does not engage in any production activity.