

Examples on Monopoly and Third Degree Price Discrimination

This hand out contains two different parts. In the first, there are examples concerning the profit maximizing strategy for a firm with market power that cannot price discriminate (Monopoly problem). The second part contains examples of third degree price discrimination.

Part I: The Monopoly Problem:

A firm is facing a decreasing demand curve for its product. The problem of the firm is to select the best point on its demand curve, i.e., a price-quantity pair that maximizes its economic profit. It is evident that, since each point on the demand curve specifies both quantity and price, we can think about the profit maximization either in terms of selecting a profit maximizing output or selecting a profit maximizing price. We find easier (given what we have done in perfect competition) to think in terms of quantity choice.

Summarizing: the firm has knowledge of the following data : I) Demand curve for its own product, II) Cost curve indicating the economic costs of producing any given level of output.

In order to find the profit maximizing output, Q^M , and the associated price, p^M , the firm goes through the following steps:

Step 1: Compute the Marginal Revenue curve (MR) from the demand curve and the Marginal Cost curve (MC) from the cost curve;

Step 2: Compute the profit maximizing output, Q^M , by solving the equation $MR(Q^M)=MC(Q^M)$.

Step 3: Compute p^M , the “right price” for Q^M (i.e., the price at which buyers are willing to buy Q^M units of output) by substituting Q^M in the demand curve.

The next two examples illustrate this procedure. In the third example, the firm is facing a production capacity constraint, i.e., it cannot produce more than a certain amount. The presence of a capacity constraint slightly modifies step 3.

Example 1:

Data:

- (1) $Q = 100 - p$; (demand curve for firm output)
- (2) $C(Q) = 1,000 + 20Q$; (cost curve)

Step 1: Compute the MR and MC curves

We start by computing the marginal revenue curve. The marginal revenue is the increase in revenues generated by the sale (the production) of an extra unit of output. If we think about infinitesimal units, the marginal revenue can be thought as the derivative with respect to Q of the total revenue, TR.

Evidently, the total revenue is:

$$TR(Q) = pQ.$$

However, in order to compute the marginal revenue we need to express the total revenues just in terms of output and take the derivative with respect to Q . Hence, in order to do that we go through the following steps:

i) Express the price in terms of output.

Solve the demand curve, equation (1), in terms of price. From (1), we get

$$(3) \quad p = 100 - Q.$$

Equation (3) is called the “inverse demand curve”, since it expresses prices in terms of quantity (while the demand curve expresses quantities in terms of prices).

ii) Express the TR curve in terms of output

Substitute (3) in the total revenue expression and get

$$TR(Q) = pQ = (100 - Q) \cdot Q = 100Q - Q^2$$

iii) Compute the Marginal Revenue

Just take the derivative of TR with respect to Q and get:

$$(4) \quad MR(Q) = dTR(Q)/dQ = 100 - 2Q$$

MC - curve

As we already know the MC-curve is just the derivative with respect to Q of the cost curve, equation (2). Hence:

$$(5) \quad MC(Q) = dC(Q)/dQ = 20$$

Step 2: Compute the profit maximizing output Q^M

Equate the marginal revenue curve, equation (4), to the marginal cost curve, equation (5), and get:

$$100 - 2Q = 20 \quad \text{or} \quad Q^M = 40$$

Step 3 :Compute p^M

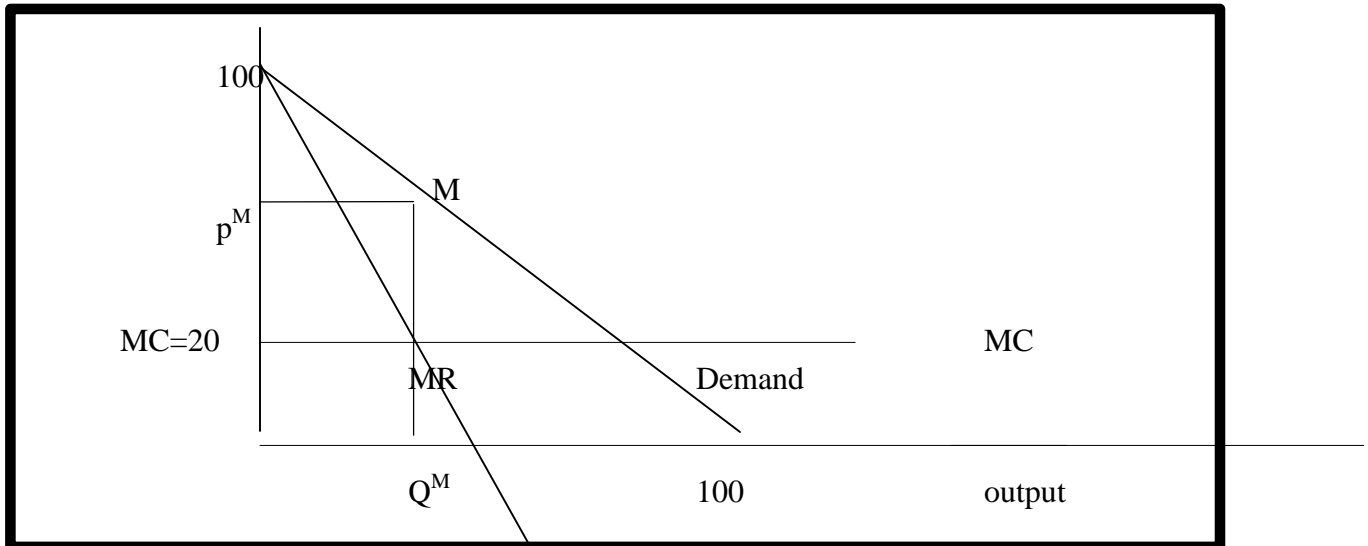
Substitute $Q^M = 40$ into the demand curve, equation (2), and get:

$$40 = 100 - p \quad \text{or} \quad p^M = 60$$

At this point we can easily compute the optimal economic profits realized by the firm:

$$\text{profits} = p^M Q^M - C(Q^M) = 40 \cdot 60 - 1,000 - 20 \cdot 40 = 600.$$

The picture below illustrates graphically the determination of p^M , Q^M .



Example 2:

In Example 2, the firm is facing the same demand curve of example 1, but a more complex cost structure.

Data:

(1) $Q = 100 - p$; (demand curve for firm output)

(2') $C(Q) = 100 + 20Q + Q^2$; (cost curve)

Step 1: Compute the MR and MC curves

Since the demand curve is identical to the one in example 1, the MR curve is expressed by equation (4) of the previous example. In order to compute the MC-curve, take the derivative with respect to Q of the cost curve, equation (2'). Hence:

(5') $MC(Q) = dC(Q)/dQ = 20 + 2Q$

Step 2: Compute the profit maximizing output Q^M

Equate the MR, equation (4) to the MC, equation (5'), and get:

$$100 - 2Q = 20 + 2Q \quad \text{or} \quad Q^M = 20$$

Step 3: Compute p^M

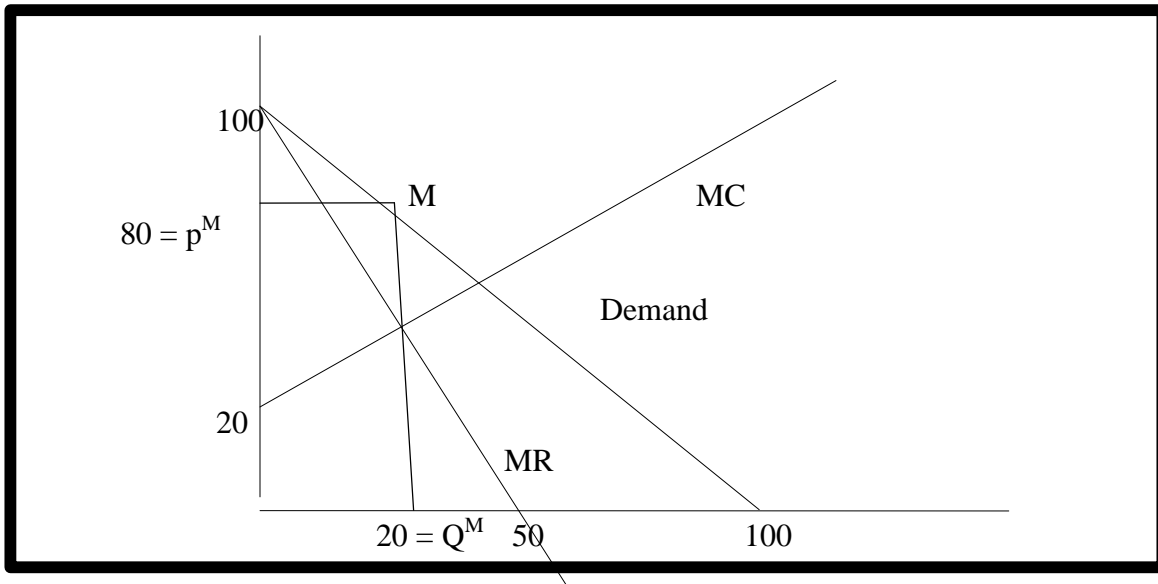
Substitute $Q^M = 20$ into the demand curve, equation (1), and get:

$$20 = 100 - p \quad \text{or} \quad p^M = 80$$

At this point we can easily compute the economic profits realized by this firm

$$\text{profits} = p^M Q^M - C(Q^M) = 80 \cdot 20 - 100 - 20 \cdot 20 - (20)^2 = 700.$$

The picture below illustrates graphically the determination of p^M , Q^M .



Example 3: Capacity Constraint

In this example we use the same cost and demand curve of example 1. However we introduce a capacity constraint. Remember that in example 1, the profit maximizing output is $Q^M = 40$.

Will the profit maximizing output change if the firm has a productive capacity greater than 40? The answer is obviously no. Without capacity constraints the firm, by producing Q^M , realizes profits higher than profits associated to any other output level. Since with a capacity constraint higher than 40, the firm can still achieve maximum profits by producing $Q^M = 40$, it will keep doing so.

Suppose now that the capacity constraint is less than $Q^M = 40$, for instance it is 30. What will the profit maximizing output be in this case? Remember that it is worth producing each additional unit for which $MR > MC$. Since both demand and cost curves are

identical to example 1, the MR and MC curves are the ones indicated in equations (4) and (5) of example 1. Hence, the difference between MR and MC is given by

$$MR(Q) - MC(Q) = 100 - 2Q - 20 = 80 - 2Q.$$

Observe that $MR(Q) > MC(Q)$ for each output $0 < Q < 40$. Hence, if the firm finds it worthwhile to produce, it will produce at capacity, i.e., $Q' = 30$. By substituting $Q' = 30$, in the demand curve, we get

$$30 = 100 - p \quad \text{or} \quad p' = 70.$$

The profits associated to $Q' = 30$ and $p' = 70$ are:

$$\text{Profits} = p'Q' - C(Q') = 70 \cdot 30 - 1,000 - 20 \cdot 30 = 500.$$

Finally, observe that since profits are positive the firm will find it worthwhile to produce. How would you have changed your answer if the profits were negative?

Remark (If this is confusing ignore it).

In cases, problem sets and classes we will be always dealing with linear demand curves, i.e., with demand curves whose graph is a straight line. The purpose of this remark is to compute the MR curve associated to any demand curve. Any demand curve has the form:

$$(1) \quad Q = A - Bp; \quad (\text{demand curve})$$

where both A and B are strictly positive numbers. For instance if $A = 100$ and $B = 1$, we get back the demand curve of the previous examples.

As usual, in order to compute the total revenue in terms of Q, solve equation (1) in terms of p:

$$(2) \quad p = A/B - Q/B \quad (\text{inverse demand curve})$$

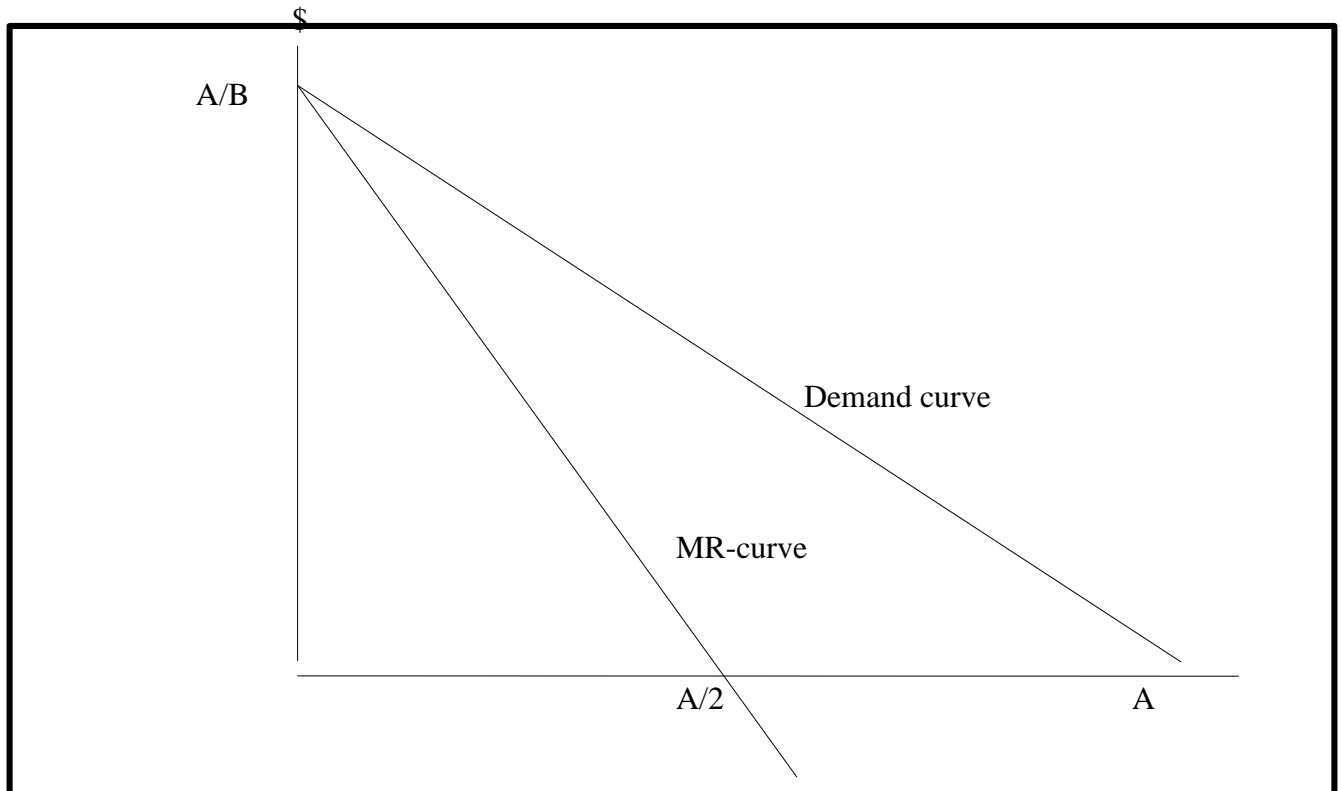
Substitute (2) in the total revenue expression and get:

$$TR(Q) = pQ = (A/B - Q/B)Q = (A/B)Q - Q^2/B$$

The MR curve is the derivative with respect to Q of the total revenue curve. It is, therefore:

$$MR(Q) = A/B - 2Q/B.$$

The picture below illustrates graphically the construction of the MR-curve. Observe that both the demand and the MR - curve intersect the price (or the y) axes at $p = A/B$. However, the demand curve intersects the quantity (or the x) axes at $Q = A$, while the MR- curve intersects it half way, at $Q = A/2$.



Part 2: Third degree price discrimination: two examples.

We present two examples of third degree price discrimination. Only the first example was discussed in class. The second is a bit more technical.

The following scenario is common to both examples: A firm has successfully separated its customers into two groups. Customers cannot resale the product or switch from one group to the other.

The firm knows its own cost structure and the demand curves of both groups. Total costs depend on the total production (sales) and not on how total sales are divided among groups.

The firm has to decide the level of output to be sold to both groups and the prices to be charged. The goal is, as usual, profit maximization.

Example 1**Data**

The following equations describe the situation:

- 1) $Q_1 = 100 - p_1$ (demand of customers in group 1)
- 2) $Q_2 = 120 - .5p_2$ (demand of customers in group 2)
- 3) $TC = 2,000 + 20(Q_1 + Q_2)$ (total cost of production)

Step 1: Find the marginal revenues curves.

First, solve the demand equations for prices and get, from 1:

$$4) \quad p_1 = 100 - Q_1,$$

and from 2

$$5) \quad p_2 = 240 - 2Q_2.$$

Second, by exploiting equation 4), compute total revenue curve for group 1 ($TR_1(Q_1)$), i.e., the total revenue to the firm when Q_1 units are sold to group 1:

$$TR_1(Q_1) = p_1(Q_1) = (100 - Q_1)Q_1 = 100Q_1 - (Q_1)^2$$

and, by exploiting equation 5), total revenue curve for group 2 ($TR_2(Q_2)$):

$$TR_2(Q_2) = p_2(Q_2) = (240 - 2Q_2)Q_2 = 240Q_2 - 2(Q_2)^2.$$

Marginal revenues are obtained by taking the derivative of the total revenue curve with respect to output (or sales). Therefore, we get:

$$\begin{aligned} MR_1(Q_1) &= 100 - 2Q_1, \\ MR_2(Q_2) &= 240 - 4Q_2. \end{aligned}$$

Step 2: Find the optimal sales to the first, Q_1^ , and to the second, Q_2^* , group.*

To achieve profit maximization, the firm has to find sales to the first group, Q_1^* , and to the second group, Q_2^* , that satisfy the following conditions:

$$MR_1(Q_1^*) = MR_2(Q_2^*) = MC(Q_1^* + Q_2^*),$$

in words, the marginal revenues from the first group computed at optimal sales Q_1^* is equal to the marginal revenues from the second group computed at optimal sales Q_2^* and both marginal revenues are equal to the marginal cost of total production computed at $(Q_1^* + Q_2^*)$.

From the total cost expression (equation 3), remembering that marginal cost is the derivative of total cost with respect to total output $Q_1 + Q_2$, we get:

$$MC(Q_1 + Q_2) = 20.$$

Therefore, $MR_1(Q_1^*) = MC(Q_1^* + Q_2^*)$ translates into:

$$100 - 2Q_1^* = 20$$

which implies

$$Q_1^* = 40.$$

while, $MR_2(Q_2^*) = MC(Q_1^* + Q_2^*)$, becomes

$$240 - 4Q_2^* = 20$$

which implies

$$Q_2^* = 55.$$

Step 3: Find prices to be charged to the first, p_1^* , and to the second, p_2^* , group.

Optimal prices are easily found by substituting the volume of optimal sales into the demand curves. So, for group 1, substitute, $Q_1^* = 40$ into equation 1) (or, equivalently, equation 4)) and get

$$Q_1^* = 100 - p_1^* \quad \text{or} \quad 40 = 100 - p_1^*$$

which implies

$$p_1^* = 60.$$

For group 2, substitute, $Q_2^* = 55$ into equation 2) (or, equivalently, equation 5)) and get:

$$Q_2^* = 120 - .5p_2^* \quad \text{or} \quad 55 = 120 - p_2^*$$

which implies

$$p_2^* = 130.$$

Total profits are easily computed by using optimal values of sales and prices:

$$A^* = p_1^* Q_1^* + p_2^* Q_2^* - TC(Q_1^* + Q_2^*) = 60 \cdot 40 + 130 \cdot 55 - 2,000 - 20 \cdot 95 = 5650.$$

Remark 1: Use this example to understand the ideas behind price discrimination: in order to maximize profits, the firm is charging \$60 to first group and \$130 to the second for each unit of the product. Although, the price to the second group is more than twice the price to the first it would be a mistake to conclude that the firm should increase sales to the second group. Why? The key indicators are the marginal revenues. In order to maximize profits, the firm equates the marginal revenues, not the prices, of the different groups. We can look at the problem from a different point of view. We have defined “cost of sale” as the difference between price and marginal revenue. In the previous example the marginal revenue of the first group computed at $Q_1^* = 40$ is $MR(Q_1^*) = 100 - 2Q_1^* = 20$. Since $p_1^* = 60$, the cost of sale of the first group is $p_1^* - MR_1(Q_1^*) = 60 - 20 = 40$. For the second group, the marginal revenue computed at $Q_2^* = 55$ is $MR(Q_2^*) = 240 - 4Q_2^* = 240 - 220 = 20$, which makes the cost of sale of the second group equal to $p_2^* - MR_2(Q_2^*) = 130 - 20 = 110$. Hence, the firm charges a higher price to the second group because its cost of sale is higher.

Example 2.

This example has not been presented in class. The only complication introduced with respect to the first example is in the total cost:

$$TC = 2,000 + (Q_1 + Q_2)^2.$$

The marginal cost is the derivative of the total cost with respect to total output Q_1+Q_2 and it is therefore equal to

$$MC = 2(Q_1+Q_2).$$

Therefore, contrary to the first example where $MC = 20$, in this case MC depends on total sales. This creates some (mild) complications in solving the problem.

Demand for both groups are identical to example 1. Therefore, we can skip the first step and go directly to the second.

Step 2: Find the optimal sales to the first, Q_1^* , and to the second, Q_2^* , group.

To achieve profit maximization, the firm has to find sales to the first group, Q_1^* , and to the second group, Q_2^* , that satisfy the following conditions:

$$MR_1(Q_1^*) = MR_2(Q_2^*) = MC(Q_1^* + Q_2^*),$$

Therefore, $MR_1(Q_1^*) = MC(Q_1^* + Q_2^*)$, translates into

$$100 - 2Q_1^* = 2(Q_1^* + Q_2^*),$$

or, equivalently,

$$6) \quad Q_1^* = 25 - Q_2^*/2$$

while, $MR_2(Q_2^*) = MC(Q_1^* + Q_2^*)$, becomes

$$240 - 4Q_2^* = 2(Q_1^* + Q_2^*),$$

or, equivalently,

$$7) \quad 6Q_2^* = 240 - 2Q_1^*.$$

Substitute equation 6 into equation 7 and get :

$$6Q_2^* = 240 - (50 - Q_2^*)$$

or

$$Q_2^* = 210/5 = 42.$$

Now substitute $Q_2^* = 42$ into equation 6 and get

$$Q_1^* = 25 - 42/2 = 4.$$

Step 3: Find prices to be charged to the first, p_1^* , and to the second, p_2^* , group.

Optimal prices are easily found by substituting the volume of optimal sales into the demand curves. So, for group 1, substitute, $Q_1^* = 4$ into equation 1) (or, equivalently, equation 4)) and get

$$Q_1^* = 100 - p_1^* \quad \text{or} \quad 4 = 100 - p_1^*$$

which implies

$$p_1^* = 96.$$

For group 2, substitute, $Q_2^* = 42$ into equation 2) (or, equivalently, equation 5)) and get

$$Q_2^* = 120 - .5p_2^* \quad \text{or} \quad 42 = 120 - .5p_2^*$$

which implies

$$p_2^* = 156.$$

Total profits are easily computed by using optimal values of sales and prices:

$$A^* = p_1^* Q_1^* + p_2^* Q_2^* - TC(Q_1^* + Q_2^*) = 96*4 + 156*42 - 2,000 - (42+4)^2 = 5650.$$