

## **A Duopoly Example.**

Consider an industry with two firms. Firms are identical and produce an homogenous product. Firms have to select outputs (capacity) in order to maximize profits. Each firm knows its own total cost of production, the total cost of production of the competitor and the industry demand.

We analyze two different scenarios:

- (i) one-shot scenario, i.e., the life of the industry lasts one period
- (ii) repeated scenario, i.e., the life of the industry lasts several periods.

The following data are known by both firms and describe the industry situation:

- 1)  $p = 140 - (Q_1 + Q_2)$  (industry demand)
- 2)  $TC_1 = 20Q_1$  (total cost of firm 1),
- 3)  $TC_2 = 20Q_2$  (total cost of firm 2).

Observe that the industry price, equation 1, depends on the output of both firms. This feature has two implications: a) since the profits of each firm depend on the price, they depend on the choice of the competitor (strategic interaction), b) in order to establish profit maximizing decisions, each firm has to guess what the competitor will do.

### **1. One shot case.**

We analyze and compare two different situations. In the first, firms compete strategically. In order to maximize their profits, they guess and take into account what the competitor does (Cournot - Nash). In the second, firms collude and coordinate their actions by forming a Cartel.

#### **1.1. Cournot-Nash Competition.**

We know that firms with market power maximize profits by selecting a level of output at which their marginal revenue is equal to their marginal costs. However, contrary to what we have seen up to now, in a situation of strategic interaction, the marginal revenue of a firm depends on the competitors choices.

We divide the analyses in three steps. In the first, we compute the marginal revenue curves. In the second, the profit maximizing outputs. In the third, equilibrium price and profits.

##### **Step 1: Compute the marginal revenues for both firms.**

Start with firm 1. The total revenues of firm 1,  $R_1$ , are:

$$R_1 = pQ_1 = (140 - (Q_1 + Q_2))Q_1 = 140Q_1 - Q_1^2 - Q_2Q_1.$$

The marginal revenue of firm 1,  $MR_1$ , is just the derivative of the total revenues with respect to  $Q_1$ . Hence

$$5) \quad MR_1 = 140 - 2Q_1 - Q_2 \quad (\text{Marginal revenue of firm 1}).$$

Similarly, the total revenues of firm 2,  $R_2$  are:

$$R_2 = pQ_2 = (140 - (Q_1 + Q_2))Q_2 = 140Q_2 - Q_1Q_2 - Q_2^2.$$

Hence, the marginal revenue for firm 2,  $MR_2$ , is:

$$6) \quad MR_2 = 140 - 2Q_2 - Q_1, \quad (\text{Marginal revenue of firm 2})$$

**Step 2: Compute the profit maximizing outputs for both firms.**

To start with observe that equations 2) and 3) imply that  $MC_1 = MC_2 = 20$ . Start with firm 1. Profit maximization for both firms entails selecting an output at which the marginal revenue equates the marginal cost. Hence for firm 1,  $MR_1 = MC_1$  implies by equation 4):

$$140 - 2Q_1 - Q_2 = 20 \quad (\text{Marginal revenue of firm 1})$$

or equivalently

$$6) \quad Q_1 = (120 - Q_2)/2 \quad (\text{Reaction function of firm 1, } RF_1)$$

Equation 6) is called a reaction function: it sets the profit maximizing value of firm 1's output,  $Q_1$ , for any value of the competitor output,  $Q_2$ .

Using the same argument, for firm 2,  $MR_2 = MC_2$  implies by equation 4)

$$140 - 2Q_2 - Q_1 = 20 \quad (\text{Marginal revenue of firm 2})$$

or equivalently

$$7) \quad Q_2 = (120 - Q_1)/2 \quad (\text{Reaction function of firm 2})$$

Hence, for given choices of the competitors, both firms use their reaction functions to set profit maximizing quantities of their outputs. However, the game is simultaneous and firms do not observe the actions selected by the competitor. Therefore, how are they going to guess the competitor output? Each firm knows that everybody in the market will try to maximize profits or, equivalently, that every firm will use its own reaction function. Both firms can compute the solution  $(Q_1^*, Q_2^*)$  to equations 6 and 7.

Given  $Q_2^*$ , firm 1 will maximize its profit by choosing  $Q_1^*$  and, given  $Q_1^*$ , firm 2 will maximize its profit by choosing  $Q_2^*$ . In other words, the pair  $(Q_1^*, Q_2^*)$  constitutes a Nash equilibrium: no firm has an incentive to take unilateral deviations.

In order to compute the pair  $(Q_1^*, Q_2^*)$ , we need to solve equations 6 and 7. However, a simple observation will simplify the computations. The two firms are identical and, therefore, it must be that  $Q_1^* = Q_2^*$ . (more precisely, 6 and 7 are linear equations symmetric in the indexes 1 and 2). Substituting the equation  $Q_1^* = Q_2^*$  into equation 6 (or 7) we get:

$$Q_1^* = (120 - Q_1^*)/2$$

which together with  $Q_1^* = Q_2^*$  implies

$$Q_1^* = Q_2^* = 40$$

Hence, substituting  $Q_1^* = Q_2^* = 40$  in the demand expression (equation 1)), we get

$$p^* = 140 - (Q_1^* + Q_2^*) = 60$$

Profits are:

$$A_1^* = p^* Q_1^* - 20 Q_1^* = 60 \cdot 40 - 20 \cdot 40 = 1,600$$

$$A_2^* = p^* Q_2^* - 20 Q_2^* = 60 \cdot 40 - 20 \cdot 40 = 1,600$$

## 1.2 Cartel

Suppose that both firms agree to form a cartel. The goal of the cartel is to set the industry output at a level that maximizes industry profits. A rule governing the cartel behavior specifies how the industry output and profits must be shared among the cartel members. In our example, things are very simple. So we can safely assume that firms equally share both profits and production. Let  $Q$  denote the industry output. Profits maximization of the industry implies that the industry marginal revenue must equate the industry marginal cost. Since by 1) demand is  $p = 140 - Q$ , the industry revenues are

$$R = pQ = (140 - Q)Q = 140Q - Q^2.$$

The industry marginal revenue,  $MR$ , is just the derivative of the industry total revenues with respect to  $Q$ . Hence:

$$MR = 140 - 2Q$$

Since both firms have the same constant marginal (and average) cost, the marginal cost of the industry,  $MC$ , is equal to 20 (by equation 2) and 3)). Therefore,  $MR = MC$  implies

$$140 - 2Q = 20$$

which implies that the profit maximizing industry output,  $Q^C$ , is

$$Q^C = 60$$

Substituting  $Q^C = 60$  into the demand (equation 1) we get that

$$p^C = 140 - Q^C = 140 - 60 = 80.$$

Each firm will therefore produce (and sell) half of the industry output (i.e.,  $Q_1^C = Q_2^C = 30$ ) realizing profits equal to:

$$\begin{aligned} A_1^C &= p^C Q_1^C - 20 Q_1^C = 80 \cdot 30 - 20 \cdot 30 = 1,800, \\ A_2^C &= p^C Q_2^C - 20 Q_2^C = 80 \cdot 30 - 20 \cdot 30 = 1,800. \end{aligned}$$

### Stability of the cartel

When acting according to the Cournot - Nash prediction, firms realize profits equal to \$1,600. When they form a cartel, they both realize profits equal to \$1,800. Since the goal of both firms is profit maximization, should we conclude that the existence of the Cartel is the only sensible prediction of the one shot situation? The answer to this question is negative. The cartel is intrinsically unstable: both firms have a unilateral incentive to produce above the cartel agreement (i.e., above 30). In different words, the cartel agreement does not constitute a Nash equilibrium and it is, therefore, vulnerable to unilateral deviations.

To analyze this point more clearly, suppose that firm 2 credibly commits itself to produce, according to the Cartel prescription, an output  $Q_2^C = 30$ . What is, given the output choice of firm 2, the profit maximizing output of firm 1? From the previous analysis, we know that firm 1 computes the profit maximizing output utilizing its own reaction function. Therefore, the profit maximizing output,  $Q_1$ , is obtained by substituting  $Q_2^C = 30$  into firm 1 reaction function, equation 7):

$$Q_1 = (120 - Q_2^C)/2 = (120 - 30)/2 = 45$$

The market price is computed by substituting  $Q_1 + Q_2^C$  into the demand equation (equation 1).

$$p = 140 - (Q_2^C + Q_1) = 140 - (30 + 45) = 65.$$

Profits are equal to:

$$\underline{A}_1 = \underline{p} Q_1^C - 20 Q_1^C = 65*45 - 20*45 = 2,025 ,$$

$$\underline{A}_2 = \underline{p} Q_2^C - 20 Q_2^C = 80*30 - 20*30 = 1,350.$$

If firm 1 breaks the cartel producing 45, instead of 30, it realizes profits equal to \$2,025, while if it stays in the cartel, it realizes profits equal to \$ 1,800. It follows that firm 1 has a unilateral incentive to break the cartel. Of course this is also true for firm 2. Therefore, cartel will never be an outcome of this game.

By simplifying a bit, the situation can be represented as a very simple simultaneous game, where each firm has just two actions, according to the following table:

Firm1 \ firm2	RF <sub>2</sub>	Cartel
RF <sub>1</sub>	1,600 \ 1,600	2,025 \ 1,350
Cartel	1,350 \ 2,025	1,800 \ 1,800

As usual, the first number represents firm 1 profit and the second firm 2. Firm 1 selects rows, firm 2 columns.

It is immediately apparent that RF is, for both firms, a dominant strategy. Hence, (RF<sub>1</sub>, RF<sub>2</sub>) is the only Nash equilibrium of the game. This is a Prisoners Dilemma type of game. The pair of actions (Cartel, Cartel) provides higher payoffs (relative to the Nash equilibrium payoffs). However, both players have an incentive to deviate unilaterally by playing their dominant strategies.

## 2. Repeated Game

Are the conclusion of the one-shot situation going to change once we increase the length of the industry life to several periods? What is the prediction of the theory about this situation of strategic interaction? Will cartel prevail as an equilibrium outcome of the repeated game?

These are the questions that we analyze in this section. It turns out that the conclusions depends on whether we assume or not the existence of a last date for the industry life. Hence, we are going to look at two scenarios : I) the industry life lasts T period; II) the industry life lasts for ever.

### 2.1 Finite Industry Life.

The life of the industry lasts several periods. Each period, the two firms play the simultaneous one shot game described by the table at the end of the previous section. Firms know the actions selected in the past by themselves and the opponents. For sake of simplicity, suppose that the discount factor is one so that firms wish to maximize the (undiscounted) sum of each period profits. In other words, each period, each firm chooses simultaneously and without observing the choice of the opponent whether to play

RF or Cartel. Then, in each period, the firm collects profits according to its action and the action of the opponent.

It is natural to conjecture that a situation where firms interact for several periods provides a powerful incentive to the formation of a Cartel. Unfortunately, this conjecture is not correct (from the point of view of the theory).

The easiest way to understand the problem (I believe) is to start studying the two period game. We proceed by backward induction. What is going to happen in the second, last period of the game? Since the game lasts only two periods, both firms are in the last period in the same situation of the one shot game previously analyzed. We know that the only (Nash equilibrium) outcome in this case is (RF,RF). Now go back to the first period. Both firms know that independently of what they play today, tomorrow they will both select (RF,RF). In other words, the actions selected today will not affect the decisions of tomorrow. But, then, even in the first period we are in the same situation of the one shot game and therefore both firms will choose the dominant action RF.

Is this conclusion dependent on the assumption that the life of the industry lasts only two periods? Not at all. In fact say that the game is repeated  $T$  periods, with  $T = 10,000$  periods. Again, proceed by backward induction. What is going to happen in period  $T$ ? This is the last period, i.e., there is no future. Both firms play (RF,RF). Now go to period  $T-1$ . Firms choices at  $T-1$  will not affect what is going to happen at  $T$ . Therefore, in period  $T-1$  we are in a one shot game situation. (RF, RF) is the only possibility. Now go to period  $T-2$ . Both firms know that independently of what they do at  $T-2$ , both at  $T$  and at  $T-1$  the outcome will be (RF,RF). Evidently, we are again in a one-shot situation. Therefore, both firms choose (RF,RF). Proceed backward until the first period.  
Conclusion: in each period firms select (RF,RF).

### *Infinite Industry Life.*

The analysis so far developed is based on the fact that there exists a last period. Suppose that the life of the industry lasts for ever. The backward induction mechanism can not be applied. What is going to happen in this scenario?

Consider the following fictitious phone call exchanged by both firms (without being heard from the Justice Department) :

Firm 1: "Hello? This is firm 1".

Firm 2: " Firm 2 speaking."

Firm 1: "I have a proposal for you. Let us both play cartel for ever. However, if in any period, you cheat, by selecting the action RF, from the next period on, I will always choose RF. What do you say?"

Firm 2: " It sounds like a good idea. However, be aware that I will use the same strategy. If you cheat, from the next period on, I will play RF. "

Evidently, if both firms comply with the strategy outlined in the phone call, the cartel will last for ever. Are these strategies a Nash equilibrium? Or, equivalently, do firms have an

incentive to take unilateral deviations? Consider one firm, say firm 1. Also consider an arbitrary period. Let us analyze a unilateral deviation taken by firm 1. (Since the deviation is unilateral, firm 2 is complying to the strategy outlined in the phone call). If firm 1 plays Cartel realizes \$1,800 (firm 2 is playing Cartel), if it plays RF it realizes \$2,025.

Therefore, there is a short term gain of \$225. However, after this deviation, firm 2 will switch to RF (and so will firm 1). As a consequence of the deviation, firm 1 will realize \$1,600 losing each period and for ever \$200. Evidently, the long run losses are by far heavier than the short term gain. Hence, there is no room for unilateral deviations or, equivalently, the phone call strategies constitute a Nash equilibrium. Both firms play Cartel in each and every period of the game.

*Remark:* Although the phone call strategies are a Nash equilibrium, you might have noticed that there is an unpleasant aspect that makes them not credible. Say that firm 1 cheats, by selecting today RF. Firm 2 is supposed to “punish” the cheater by switching, from tomorrow on, to RF. However, by doing that firm 2 is also punishing itself: it goes from a payoff of 1,800 to a payoff of 1,600. There are credible and more sophisticated punishments that support the (Cartel, Cartel) outcome. If you want to know about them, come to talk to me.

*Conclusions:* You should be very careful in applying the theory results to real oligopolistic industries. A first point to take into account is the meaning of a finite industry life. Firms are run by managers that have a finite horizon. If the industry is stable (no entry from outsiders, no appearance of powerful substitute or of facts that change the nature of the industry) we may as well take the infinite version of the model to be the relevant one. However, we need to check one other important qualification. In order to punish the cheaters firms have to be able to detect deviations. This is possible only if there are few firms in the industry and if the demand is fairly stable. Historically, these seem to be the conditions that made cartel agreement effective in real industries. (Also, do not forget that collusive behavior is illegal in advanced capitalistic countries.) However, if the industry is unstable or facts emerge that signal that in the next future the industry structure is going to change, we must use the finite version of the model. Under these circumstances, as in the real world, Cartels break.