

**Managerial Economics**  
**Problem Set #4**  
**(The Rock Collector)**  
**Solution**

Part 1:

We apply the standard two-step decision procedure for firms operating in competitive markets: First, we find the profit-maximizing level of output. Then we determine whether the revenue from producing the profit-maximizing level of output is sufficient to cover the costs of production.

To find the profit-maximizing level of output, we set marginal revenue equal to marginal cost. In a competitive market, marginal revenue is simply the market price of \$7.

To find marginal cost, we need to first derive a total cost function, the sum of fixed and variable cost functions. Fixed costs include the \$300 per day opportunity cost of not working as a consultant, plus the \$12.50 per day access fee. So total fixed costs are \$312.50. We are given average variable costs as a function of quantity produced. To derive total variable costs, we simply multiply this average variable cost function by the quantity produced:

$$VC(q) = AVC(q) * q = (2.5 + 0.005q)q = 2.5q + 0.005q^2$$

Thus the total cost function is:

$$C(q) = FC + VC(q) = 312.50 + 2.5q + 0.005q^2$$

And marginal cost is:

$$MC(q) = \frac{dC(q)}{dq} = 2.5 + 0.01q$$

With all the pieces in hand, it is now simple to find  $q^*$ :

$$7 = 2.5 + 0.01q$$
$$q^* = 450$$

We now know that the firm maximizes profits, given that it produces at all, when  $q = 450$ . Now we need to determine whether it makes sense to produce at all. This is the second step of the process and is most easily done by comparing average revenue to average cost.

To derive the average cost, we start with the cost function and simply divide by the quantity produced:

$$AC(q) = \frac{C(q)}{q} = \frac{312.50}{q} + 2.5 + 0.005q$$

Now plugging in  $q = 450$ :

$$AC(450) = \frac{312.50}{450} + 2.5 + 0.005(450) = 5.44$$

Since the average cost (per unit) of collecting 450 units is less than the average revenue (per unit), total costs are less than total revenue. The entrepreneur should collect 450 rocks per day.

Part 2:

The basic procedure remains as in part 1; only the number change. Step 1 involves setting marginal revenue equal to marginal cost:

$$\begin{aligned} 4.99 &= 2.5 + 0.01q \\ q^* &= 249 \end{aligned}$$

Now, as a second step, we need to see whether average cost exceeds average revenue (\$4.99) when output is 249:

$$AC(249) = \frac{312.50}{249} + 2.5 + 0.005(249) = 5.00$$

Since average cost exceeds average revenue, the entrepreneur does better by not producing, even at the optimal level of 249 units.

Part 3:

Before the first day of the year, no costs are sunk. If the price is expected to remain at \$7, the entrepreneur might reasonably calculate the daily fixed cost by allocating the annual fixed cost over the number of workdays in the year, which is about 250. Thus under these perfectly reasonable assumptions, the entrepreneur should behave as in part 1.

Part 4:

Now the entire \$3125 represents a sunk cost, which should play no further role in decision making. Ignoring this fee, we once again apply the two steps: The first step is just as in part 2, as fixed costs do not in any way enter into marginal cost functions. So the optimal level of output is again 249 units.

The second step requires recalculation of the *ex post* average cost function, ignoring the \$3125 annual (or \$12.50 daily) access fee:

$$AC_{ExPost}(q) = \frac{300}{q} + 2.5 + 0.005q$$

Taking output equal to 249:

$$AC_{ExPost}(249) = \frac{300}{249} + 2.5 + 0.005(249) = 4.94$$

Now average cost (*ex post*) is less than average revenue. So the entrepreneur will collect 249 rocks per day.

Part 5:

In part 2, we consider all costs *ex ante*. In part 4, we consider only those costs which are avoidable *ex post*. Thus, in part 4, we find lower average and total cost. So where total profits, the difference between total revenue and total cost, are negative in part 2, these are positive in part 4. Again the key difference is that in part 2 we consider costs *ex ante* while in part 4 we consider costs *ex post*.

Part 6:

As long as the entrepreneur pays the \$3125 fee on the first day of the year, that money is sunk and therefore irrelevant to further decision-making. Thus it makes no difference whether the unexpected fall in rock prices occurs on the second or last day of the year.