Managerial Economics

Problem Set #5
Roses & Sugar
Solution

Part 1:

Section a:

The supply curve for an individual firm is simply the portion of marginal cost schedule for the individual firm that lies above the average cost curve.

To find the marginal cost curve, we differentiate the total cost function for the firm:

$$MC(q) = C'(q) = 0.5q + 0.5$$

Recall that the first step in finding the optimal production level for a firm in a competitive market is setting marginal revenue, or price, equal to marginal cost. Thus when p is equal to marginal cost, the firm is producing the profit-maximizing level of output:

$$p = 0.5q + 0.5 \Rightarrow q = 2p - 1$$

So we have a rule that, for any p, gives the profit-maximizing level of output.

But we must also deal with the second step: making sure that revenue at the optimal output level is sufficient to cover the fixed costs. This will be true at each point where p is greater than the average cost of production. Another way to say this is that profits will be positive anywhere that marginal cost is above average cost. Hence the firm supply curve is the portion of the marginal cost curve above average cost.

We know that marginal cost intersects average cost when average cost is at a minimum. Thus one way to find the point of intersection is simply to find the minimum of the average cost curve. Since $C(q) = 0.25q^2 + 0.5q + 36$, the average cost function $AC(q) = 0.25q + 0.5 + 36/q$. To minimize this function:

$$AC'(q) = \frac{dAC(q)}{dq} = 0.25 - \frac{36}{q^2}$$

Setting this equal to zero finds the first order condition:

$$0 = 0.25 - \frac{36}{q^2} \Rightarrow 0.25 = \frac{36}{q^2}$$

Thus the average cost function reaches its minimum when q is equal to 12. At this point, we can use either MC(q) or AC(q) to find that price is equal to 6.5.

Thus the supply curve for the individual grower is $q = 2p - 1$ when $p \geq 6.5$ and zero if $p < 6.5$. This supply curve is shown graphically in figure 1.

Section b:

The problem states that the industry consists of 100 identical growers. We can find the industry supply by summing up the individual firms' supply curves:

$Q = q1 + q2 + \ldots + q100 = 100 * (2p - 1) = 200p - 100$ if $p \geq 6.5$

Section c:

We find the equilibrium by solving for a price which sets quantity demanded equal to quantity supplied:

$$Q_d = Q_s$$
$$2400 - 50p = 200p - 100$$
$$250p = 2500$$
$$p = 10$$

The supply and demand curves are shown in figure 2.

Section d:

Once more, we look for a price at which quantity supplied equals quantity demanded:

$$Q_d = Q_s$$
$$6150 - 50p = 200p - 100$$
$$250p = 6250$$
$$p = 25$$

Section e:

At a price of \$20, demand will exceed supply. This can be seen by plugging in a price of \$20 to the demand function ($Qd = 6150 - 50p = 6150 - 50(20) = 5150$) and the supply function ($Qs = 200p - 100 = 200(20) - 100 = 3900$). Figure 3 depicts this situation graphically.

Question 2:

Part a:

Each sugar cane plantation can produce five pounds per day, at a cost of \$0.05 per pound. Thus the firm supplies zero at prices below \$0.05, and five pounds at prices of \$0.05 and above. Figure 4 depicts this supply schedule.

Each farm has variable costs of $x^2 – 0.9x$ and fixed costs (per day) of 1. Thus the cost function may be written $C(x) = x^2 – 0.9x + 1$. We then modify this cost function to take into account the government subsidy of \$1 per pound produced. We consider this subsidy as a reduction in cost, so that:

$$C_1(x) = x^2 – 0.9x + 1 – (\$1 * x) = x^2 – 1.9x + 1$$

Thus the marginal cost of product is $MC_1(x) = 2x – 1.9$. To find the supply function, we set this equal to p and solve for x:

$$p = 2x – 1.9 \rightarrow x = 0.5p + 0.95$$

This line, however, also includes price-quantity combinations at which the firm would choose to shut down rather than produce. To find those combinations at which the firm would produce, we take the portion of the marginal cost schedule above the average cost function, $AC(x) = x – 1.9 + 1/x$.

To find this region, we set the two function equal and solve for x:

$$AC(x) = MC(x)$$
$$x – 1.9 + 1/x = 2x – 1.9$$
$$x + 1/x = 2x$$
$$1/x = x$$
$$x = 1$$

Since $MC(1) = 2(1) – 1.9 = 0.1$, the firm will produce when $p > 0.10$. Thus the supply schedule is $x = 0.5p + 0.95$ when $p > 0.1$ and zero otherwise. This schedule is shown in figure 5. The supply curve for all 100 farms is obtained by summing up the production at each price level on each farm. Using an upper case X to denote the supply of all farms:

$$X = 100x = 100(0.5p + 0.95) = 50p + 95 \text{ for } p \geq 0.10$$

To trace the industry supply curve for sugar, think about the market price gradually rising from zero. At price above zero but below \$0.05, there is no production either on the beet farms or cane plantations. Once the price reaches \$0.05, production begins on the plantation. Each of twenty plantations produces five pounds, a total of 100 pounds. As the price moves above \$0.05 towards \$0.06 the farms will not find it profitable to operate so the supply remains fixed at 100 pounds. Only when the price reaches \$0.10 do the farms begin to operate. At the price of \$0.10 each farm produces one pound, so all 100 farms produce 100 pounds. As the price increases about \$0.10, the production of the farms increases as per the supply schedule derived above. The industry supply curve for sugar is shown in figure 6.

Part b:

Figure 7 depicts the industry supply curve and the market demand curve. The intersection of the schedules is on the vertical region of the supply curve between prices of \$0.05 and \$0.10. The quantity supplied in this region is 100. (More than this won't be supplied at price below \$0.10.) The plantations could sell their product at \$0.05, but they could clearly raise the price a bit without lessening demand. In fact, the price at which the market would demand 100 pounds is found using the demand curve: $Q_d = 109 - 100p$. Substituting 100 for Q_d and solving for p:

$$100 = 109 - 100p \rightarrow p = 0.09$$

Thus the plantations can charge \$0.09 per pound, sell their entire output and thus earn (as an industry) of \$9. Since they produce at a cost of \$0.05 per pound, total cost is \$5 and profit \$4. Since the beet farmers do not produce, the cost of the subsidy program, and the beet farmers profit, is zero.

Part c:

Figure 8 shows the industry supply curve and the new demand schedule. Finding the intersection algebraically can be tricky, given the non-linearity in the supply schedule. Since the price is clear about \$0.1, we can simply assume that the first hundred pounds is produce by the cane plantations. Then the industry supply is effectively $Q(p) = 100 + 50p + 95 = 195 + 50p$. The equilibrium can be found by setting $Q_d = Q_s$:

$$Q_d = Q_s$$
$$240 - 100p = 195 + 50p$$
$$45 = 150p$$
$$p = 0.3$$

Substituting back into the demand curve, $Q_d = 240 - 100p = 240 - 100(0.30) = 210$. The cane plantations produce 100 pounds of this total and earn revenue of \$30. Their total cost is only \$5, so their profits total \$25. The beet farms sell the remaining 110 pounds and thus earn revenue of \$33. Each individual farm produces 1.1 pounds, so the costs (less subsidy) for each individual farm are: $C(1.1) = x^2 - 1.9x + 1 = 1.1^2 - 1.9(1.1) + 1 = 0.12$. Thus total industry costs are \$12 and industry profits \$33 - \$12 = \$21. The cost of the subsidy program (to the government) is \$110 per day.

Figure 1

Individual Grower's Supply Curve

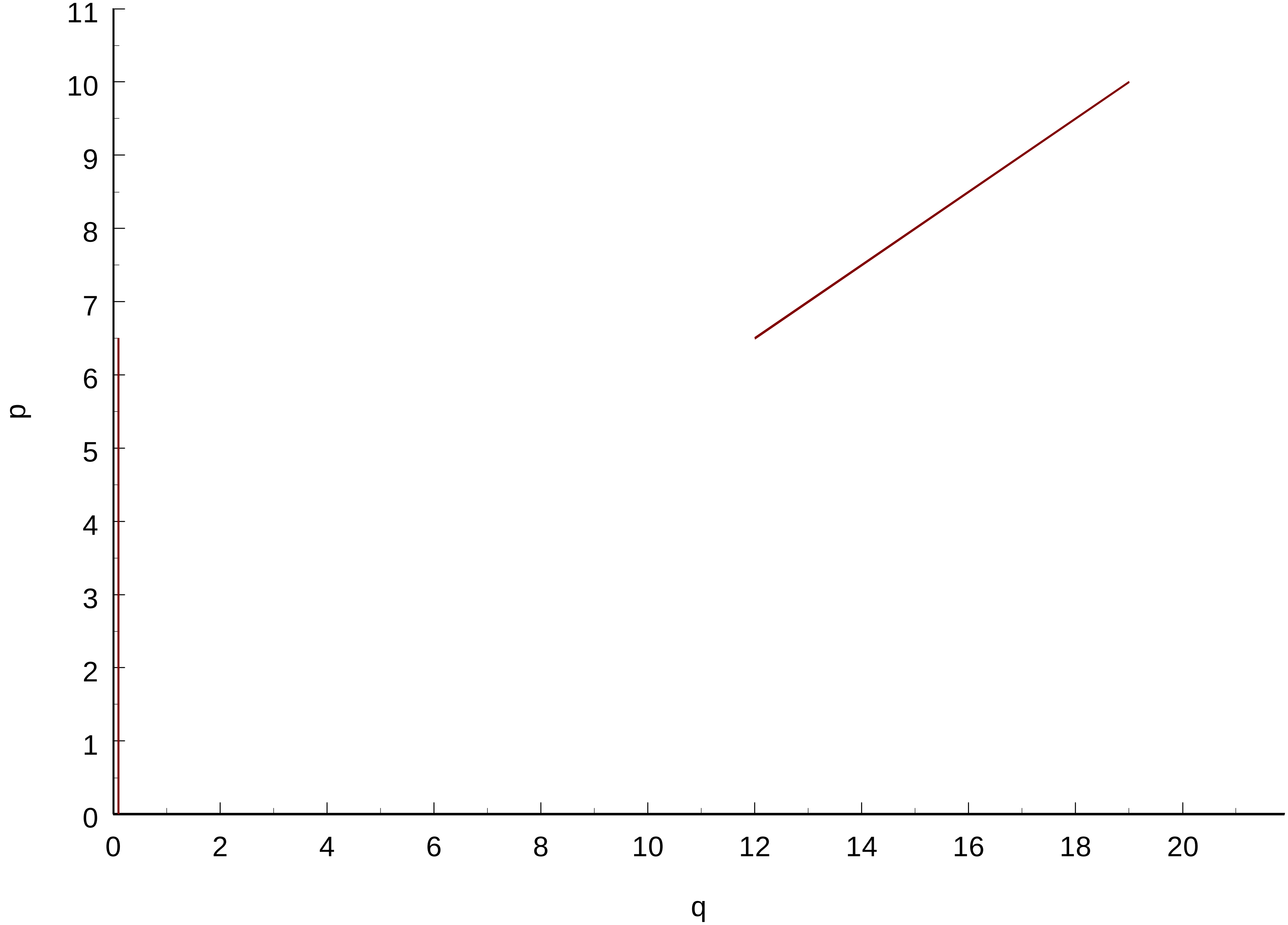


Figure 2

Market Equilibrium

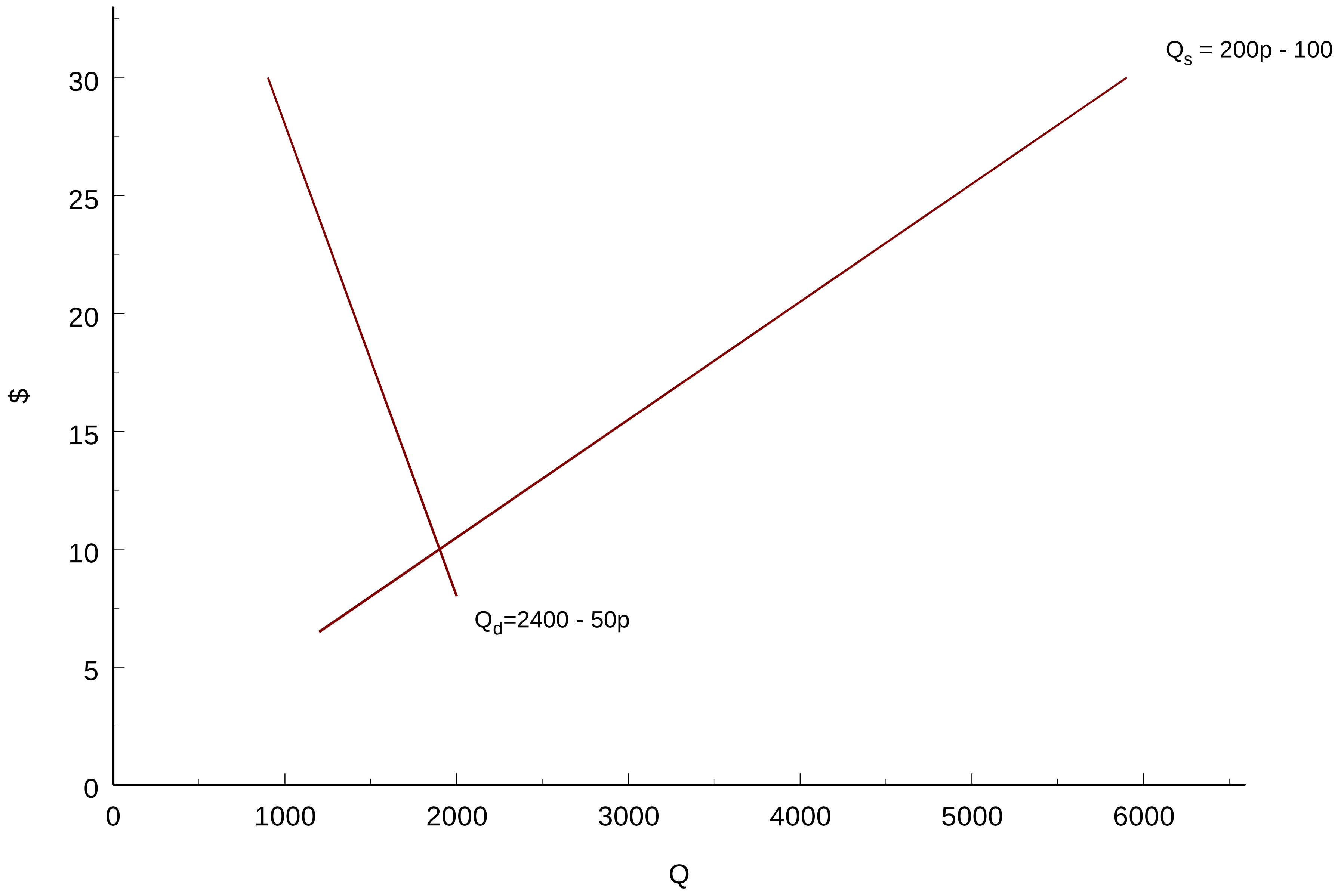


Figure 3

Market Disequilibrium (with Price Controls)

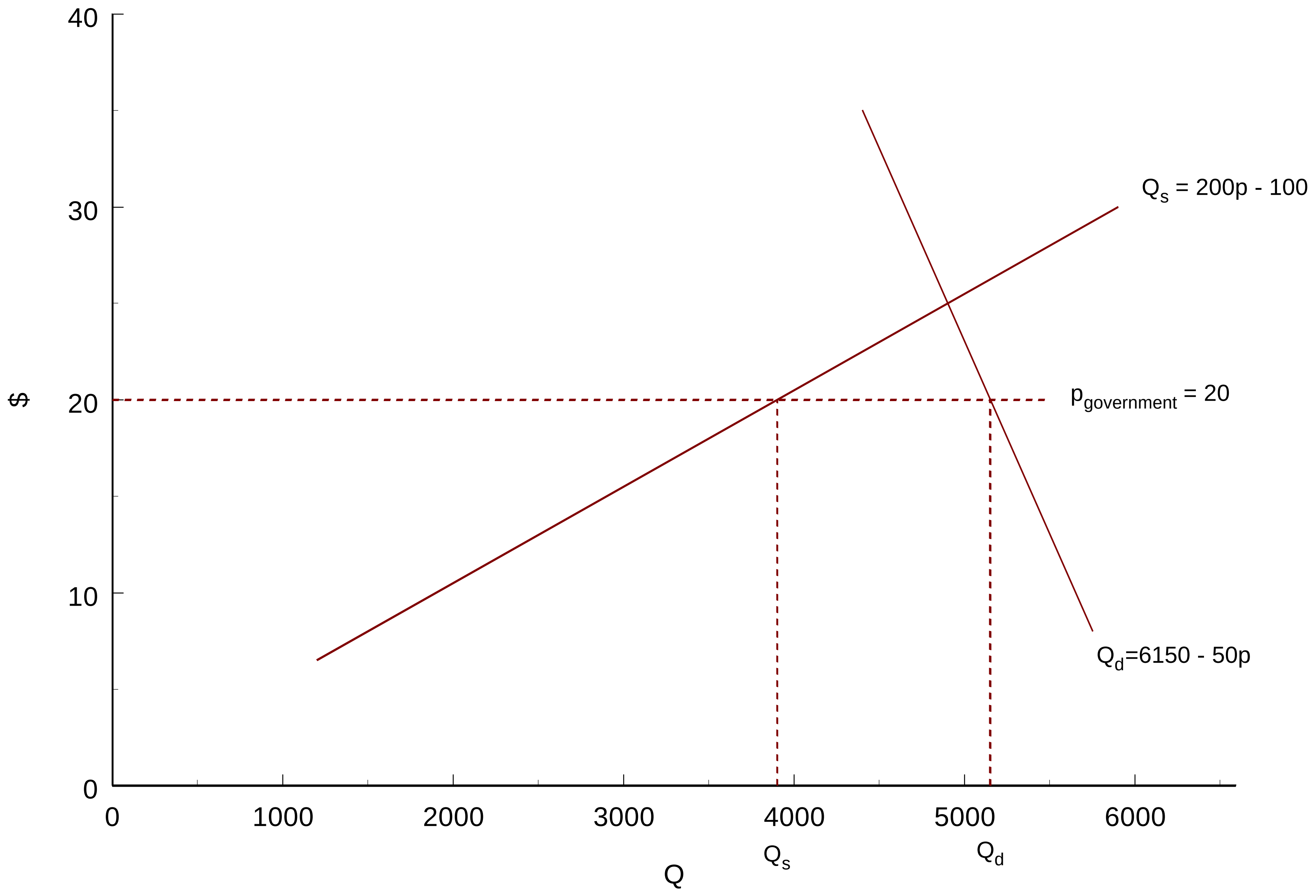


Figure 4

Sugar Cane Plantation Supply Schedule

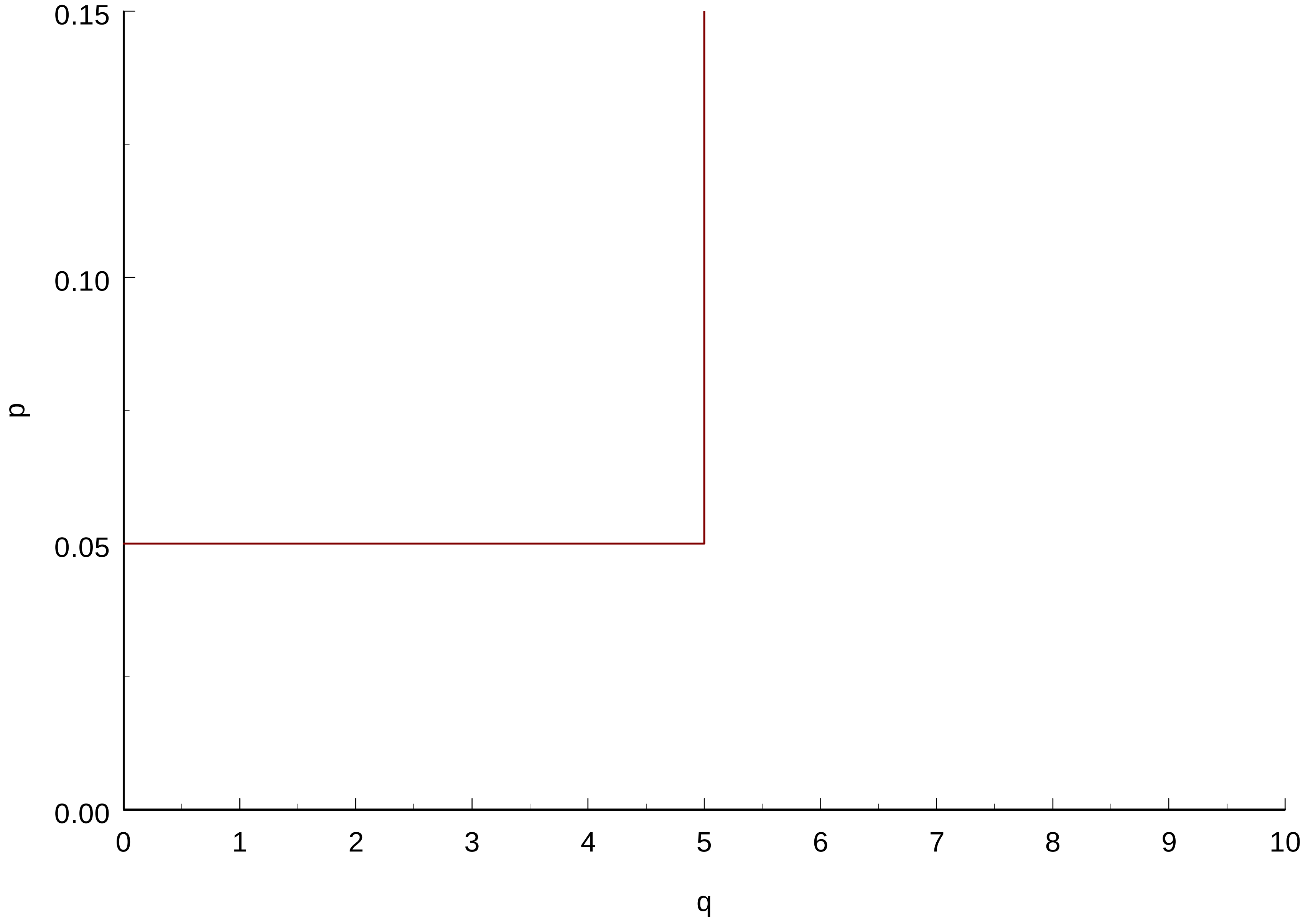


Figure 5

Sugar Beet Farm Supply Schedule

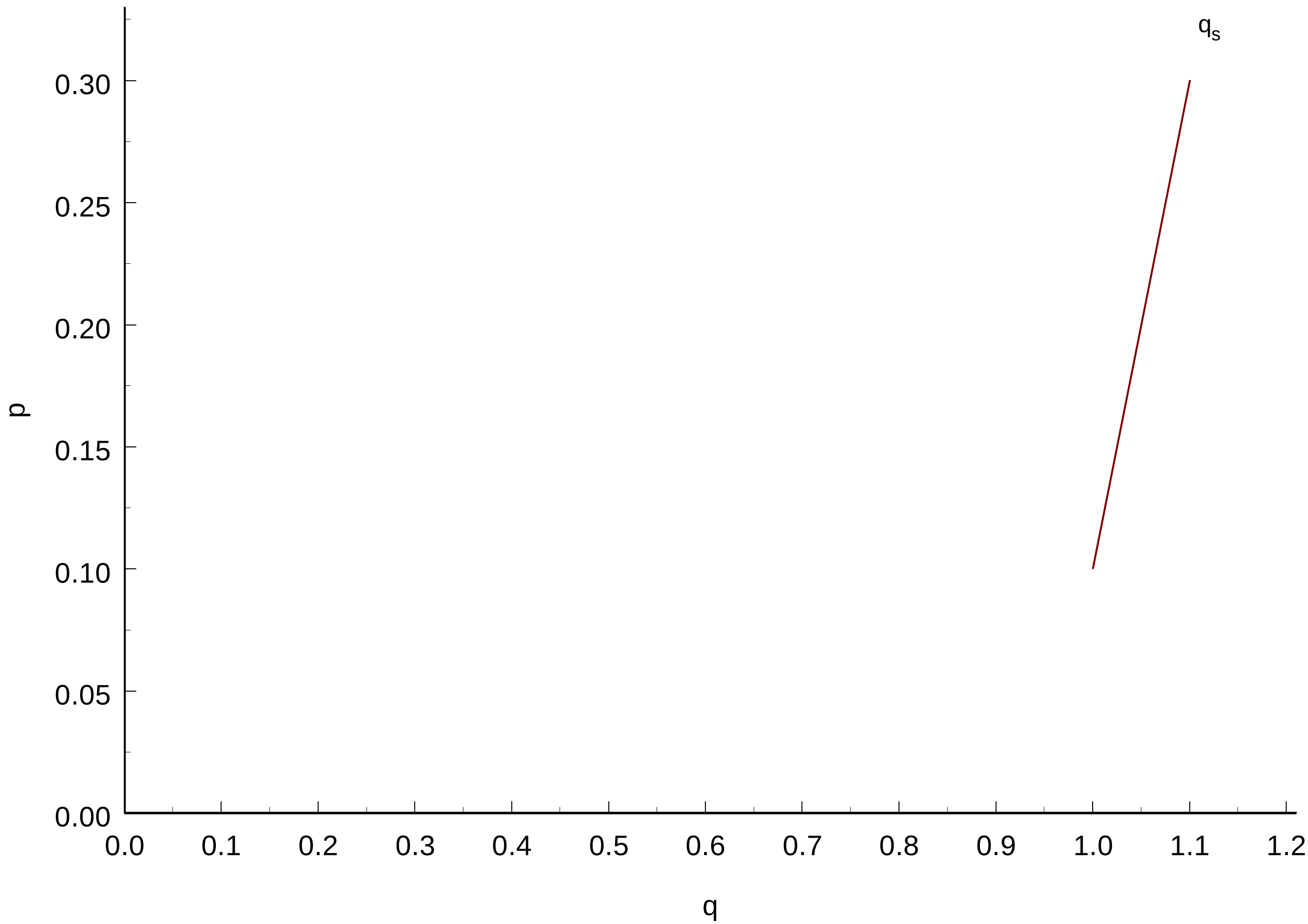
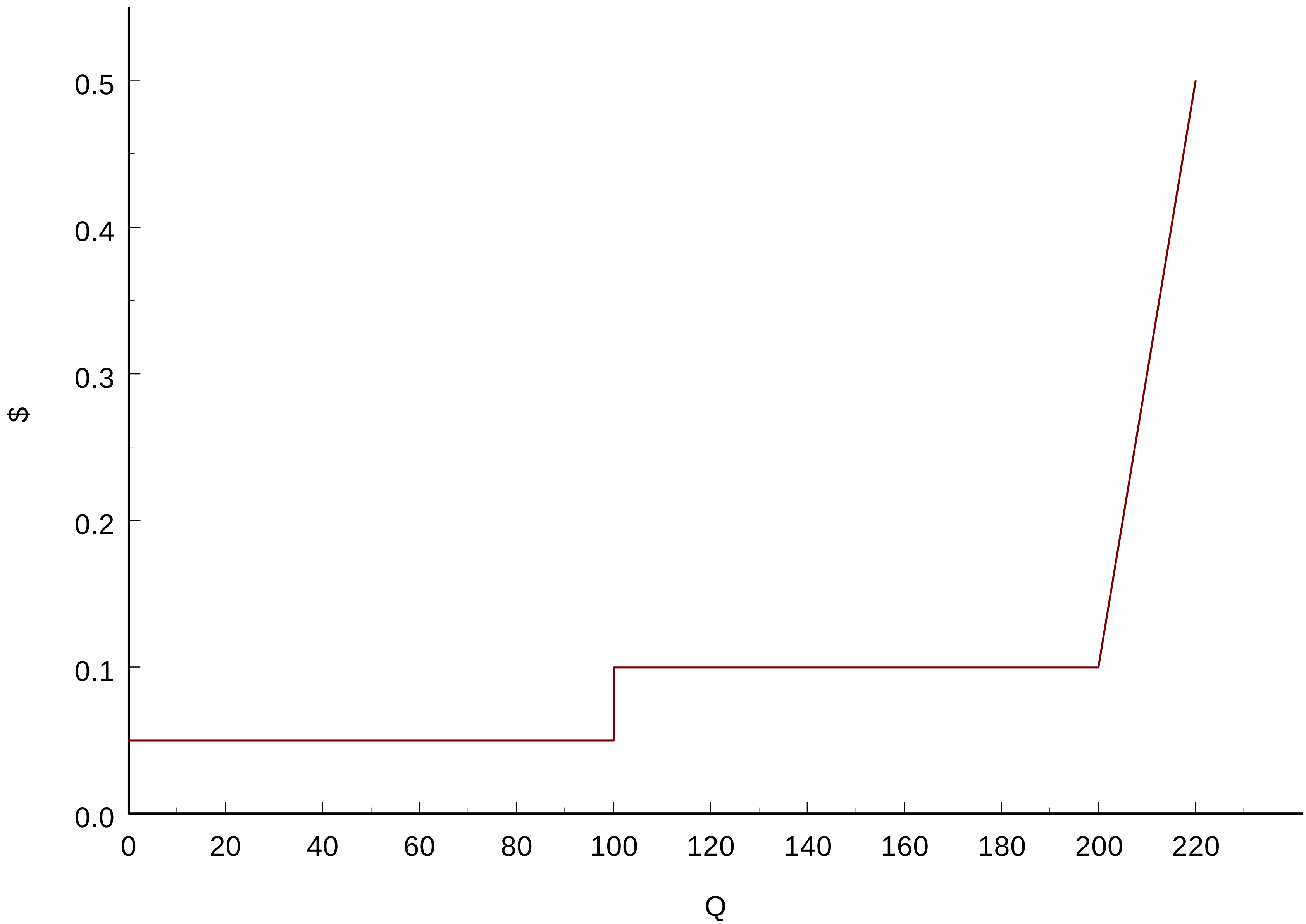


Figure 6

Industry Supply Schedule



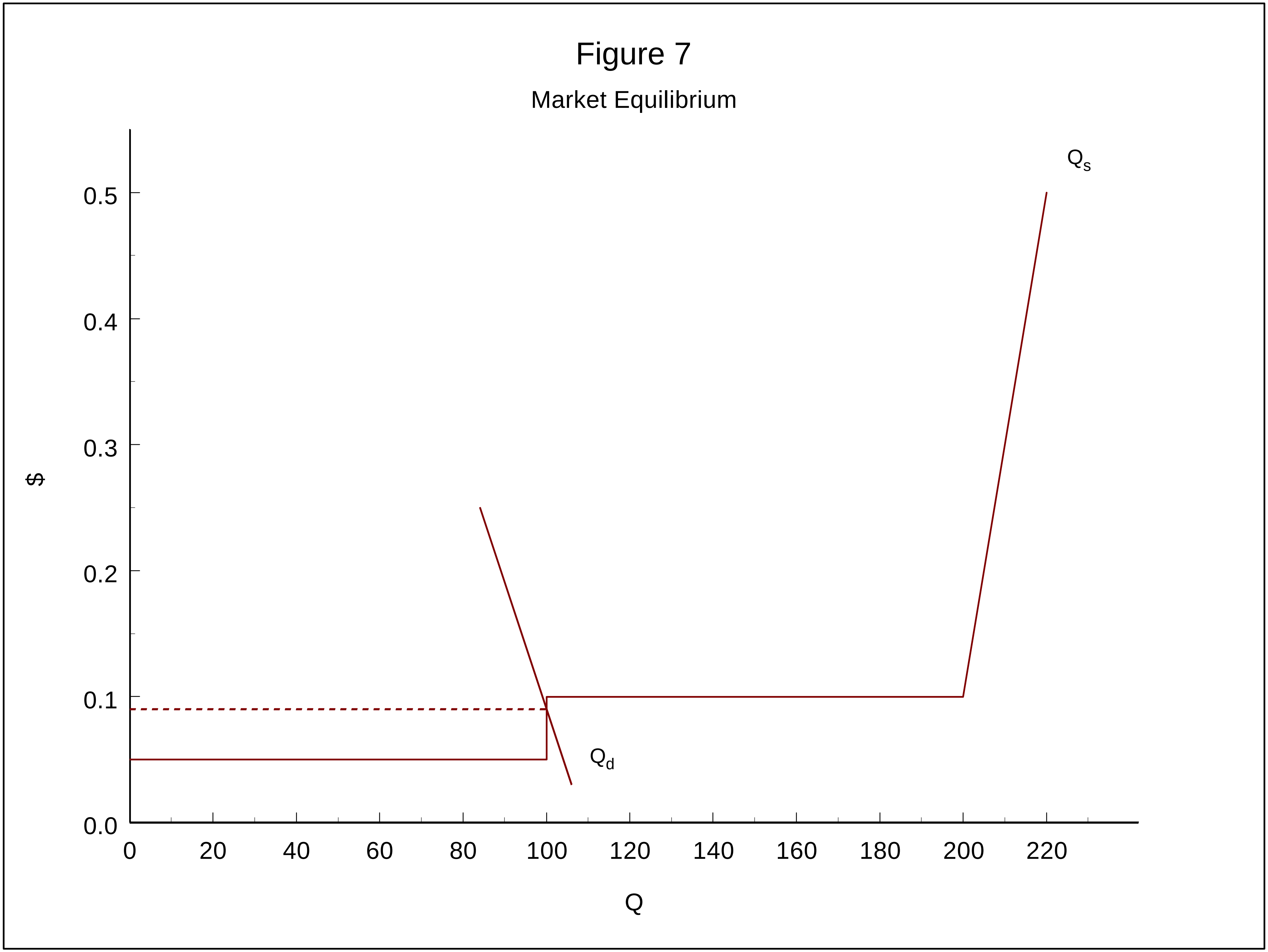


Figure 7
Market Equilibrium
$
0.0
0.1
0.2
0.3
0.4
0.5
0
20
40
60
80
100
120
140
160
180
200
220
Q
Qs
Qd