THE ECONOMIC AND POLICY CONSEQUENCES OF CATASTROPHES*

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Abstract: What is the likelihood that the U.S. will experience a devastating catastrophic event over the next few decades that would substantially reduce the capital stock, GDP and wealth? And how much should society be willing to pay to reduce the probability or impact of a catastrophe? We show how answers to these questions can be inferred from economic data. We provide a framework for policy analysis which is based on a general equilibrium model of production, capital accumulation, and household preferences. Calibrating to economic and financial data provides estimates of the annual mean arrival rate of shocks and their size distribution, as well as investment, Tobin's q, and the coefficient of relative risk aversion. We use the model to calculate the tax on consumption society would accept to limit the maximum size of a catastrophic shock, and the cost to insure against its actual impact.

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1 Introduction.

What is the likelihood that the U.S. will experience a devastating catastrophic event over the next few decades? And how much should society be willing to pay to limit the possible impact of such an event? By "catastrophic event," we mean something national or global in scale that would substantially reduce the capital stock and/or the productive efficiency of capital, thereby substantially reducing GDP, consumption, and wealth. Examples might include a nuclear or biological terrorist attack (far worse than even 9/11), a highly contagious "mega-virus" that spreads uncontrollably, a global environmental disaster, or a financial and economic crisis on the order of the Great Depression.[1] We show how the probability and possible impact of such an event can be inferred from the behavior of economic and financial variables such as investment, interest rates, and equity returns. We also show how our framework can be used to estimate the amount society should be willing to pay to reduce the probability of a catastrophic event, or to insure against its actual impact should it occur.

An emerging literature has used historical data to estimate the likelihood and expected impact of catastrophic events.[2] Examples include Barro (2006, 2009), Barro and Ursúa (2008), and Barro, Nakamura, Steinsson, and Ursúa (2009).[3] These studies, however, are limited in two respects. First, many of the included disasters are manifestations of three global events — the two World Wars and the Great Depression. Second, the possible catastrophic events that we think are of greatest interest today have little or no historical precedent

[1]Readers who are incurable optimists or have limited imaginations should read Posner (2004), who provides more examples and argues that society fails to take these risks seriously enough, and Sunstein (2007). For a sobering discussion of the likelihood and possible impact of nuclear terrorism, see Allison (2004).

[2]The roots of this literature go back to the observation by Rietz (1988) that low-probability catastrophes might explain the equity premium puzzle first noted by Mehra and Prescott (1985), i.e., could help reconcile a relatively large equity premium (5 to 7%) and low real risk-free rate of interest (0 to 2%) with moderate risk aversion on the part of households. Weitzman (2007) has shown that the equity premium and real risk-free rate puzzles could alternatively be explained by "structural uncertainty" in which one or more key parameters, such as the true variance of equity returns, is estimated through Bayesian updating.

[3]In related work, Gourio (2008) modeled an exchange economy with recursive preferences and disasters that have limited duration. He found that the effect of recoveries on the equity premium could be positive or negative, depending on the elasticity of intertemporal substitution. Gabaix (2008) and Wachter (2008) showed that a time-varying disaster arrival rate could explain the high volatility of the stock market (in addition to the equity premium and real risk-free rate).

— there are no data, for example, on the frequency or impact of nuclear or biological terrorist attacks. Or consider the forty-year period beginning around 1950 and ending with the breakup of the Soviet Union, during which one potential catastrophic event dominated all others: the possibility of a U.S.-Soviet nuclear war. The Department of Defense, the RAND Corporation and others studied the likelihood and potential impact of such an event, but there was no historical precedent on which to base estimates.

We take a different approach from earlier studies and ask what event arrival rate and impact distribution are implied by the behavior of basic economic and financial variables. We do not try to estimate the characteristics of catastrophic events from historical data on drops in consumption or GDP, nor do we use the estimates of others. Instead, we develop an equilibrium model of the economy that incorporates catastrophic shocks to the capital stock, and that links the first four moments of equity returns, along with economic variables such as consumption, investment, interest rates, and Tobin's q, to parameters describing the characteristics of shocks as well as behavioral parameters such as the coefficient of relative risk aversion and elasticity of intertemporal substitution. We can then determine the characteristics of catastrophes as a calibration *output* of our analysis. In effect, we are assuming that these characteristics are those perceived by firms and households, in that they are consistent with the data for key economic and financial variables.[4]

Our framework also provides a tool for policy analysis. For example, how much should society be willing to pay to reduce or limit the impact of a catastrophic event? To measure "willingness to pay" (WTP), we calculate the maximum permanent percentage tax rate that society should be willing to accept in order to eliminate the possibility of a catastrophic shock, or reduce the maximum possible impact of such as shock. We also show how our framework can be used to calculate the equilibrium price of insurance against catastrophic risk, and we compare the use of insurance to the cost of reducing or eliminating risk.

In the next section we lay out a parsimonious model with an AK production technology,

[4]In related work, Russett and Slemrod (1993) used survey data to show how beliefs about the likelihood of nuclear war affected savings behavior, and argue that such beliefs can help explain the low propensity to save in the U.S. relative to other countries. Also, see Slemrod (1990) and Russett and Lackey (1987).

adjustment costs (which we show are crucial), and shocks that arrive unpredictably. Each shock destroys a random fraction of the capital stock. We treat as catastrophic those shocks that reduce the capital stock by a "large" amount, e.g., something more than 10 or 15 percent. We explain how the model's calibration yields information about the characteristics of shocks, as well as important behavioral parameters, and we show how all of the parameters of the model can be identified on a block-wise basis. Proceeding in stages, we show (1) how the variance, skewness, and kurtosis of equity returns identifies the parameters that characterize both unpredictable shocks and continuous fluctuations in the capital stock; (2) how the equity risk premium can then be used to identify the coefficient of relative risk aversion; (3) how, given these parameters, the risk-free interest rate then identifies the elasticity of intertemporal substitution and/or the rate of time preference; and (4) how the consumption-investment ratio and the real growth rate of GDP then determine the marginal propensity to consume, Tobin's q, and investment. We also explain how the calibrated model can be used to determine the equilibrium price of insurance against catastrophic risk.

To calibrate the model, we use data for the U.S. economy and financial markets over the period 1947 through 2008. Section 3 presents our calibration results and discusses their implications for the characteristics of shocks and for behavioral parameters. Section 3 also shows the implications of the model for the price of insurance against catastrophes of various sizes, and demonstrates the importance of adjustment costs. Section 4 discusses the application of our framework to policy analysis. In particular, we calculate the maximum permanent tax on consumption that society would accept to reduce or eliminate the impact of catastrophic shocks. Section 5 concludes.

2 Framework.

In this section we lay out the building blocks of a simple general equilibrium model, and then explain how the model is solved.

2.1 Building Blocks.

Preferences. We use the Duffie and Epstein (1992) continuous-time version of Epstein-Weil-Zin (EWZ) preferences, so that a representative consumer has homothetic recursive preferences given by:[5]

$$V_t = \mathcal{E}_t \left[\int_t^{\infty} f(C_s, V_s) ds \right] \ , \tag{1}$$

where

$$f(C, V) = \frac{\rho}{1 - \psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\gamma)V)^{\omega}}{((1-\gamma)V)^{\omega - 1}} \ . \tag{2}$$

Here ρ is the rate of time preference, ψ the elasticity of intertemporal substitution (EIS), γ the coefficient of relative risk aversion, and we define $\omega = (1 - \psi^{-1})/(1 - \gamma)$. Unlike time-additive utility, recursive preferences as defined by eqns. (1) and (2) disentangle risk aversion from the EIS. Note that with these preferences, the marginal benefit of consumption is $f_C = \rho C^{-\psi - 1}/[(1-\gamma)V]^{\omega - 1}$, which depends not only on current consumption but also (through V) on the expected trajectory of future consumption.

If $\gamma = \psi^{-1}$ so that $\omega = 1$, we have the standard constant-relative-risk-aversion (CRRA) expected utility, represented by the additively separable aggregator:

$$f(C, V) = \frac{\rho C^{1-\gamma}}{1 - \gamma} - \rho V. \tag{3}$$

One of the questions we address is whether γ is close to ψ^{-1}, so that the simple CRRA utility function is a reasonable approximation for modeling purposes. More generally, we examine how equilibrium allocation and pricing constrains the model's parameters, including the EIS and the coefficient of relative risk aversion.

Production. Aggregate output has an AK production technology:

$$Y = AK \ , \tag{4}$$

where A is a constant that defines productivity and the capital stock K is the sole factor of production. The AK model is widely used, in part because it generates balanced growth in

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[5]Epstein and Zin (1989) and Weil (1990) developed homothetic non-expected utility in discrete time, which separates the elasticity of intertermporal substitution from the coefficient of relative risk aversion.

equilibrium. In our specification, K is the *total* stock of capital; it includes physical capital as traditionally measured, but also human capital and firm-based intangible capital (such as, patents, know-how, brand value, and organizational capital).

Shocks to the Capital Stock. We assume that discrete downward jumps to the capital stock ("shocks") occur as Poisson arrivals with a mean arrival rate λ. There is no limit to the number of these shocks; the occurrence of a shock does not change the likelihood of another, and in principle shocks can occur frequently.[6] When a shock does occur, it permanently destroys a stochastic fraction $(1 - Z)$ of the capital stock K, so that Z is the remaining fraction. (For example, if a particular shock destroyed 15 percent of capital stock, we would have $Z = .85$.) We assume that Z follows a well-behaved probability density function (pdf) $\zeta(Z)$ with $0 \leq Z \leq 1$. By well-behaved, we mean that the moments $\mathcal{E}(Z^n)$ exist for $n = 1$, $1 - \gamma$, and $-\gamma$. As we will see, these are the only moments of Z that are relevant for our analysis.

As we will show in Section 3 when we discuss the calibration of the model, shocks occur frequently, but for most shocks losses are small. We consider catastrophes to be shocks for which the drop in the capital stock is sufficiently large, e.g., more than 10 or 15 percent. Using our calibration, we will see that the model predicts that catastrophic shocks are rare.

The capital stock is also subject to ongoing continuous fluctuations. These continuous fluctuations, along with small jumps, can be thought of as the stochastic depreciation of capital. Large shocks, on the other hand, are interpreted as (rare) catastrophic events.

Investment and Capital Accumulation. Letting I denote aggregate investment, the capital stock K evolves as:

$$dK_t = \Phi(I_t, K_t)dt + \sigma K_t dW_t - (1 - Z)K_t dJ_t \ . \tag{5}$$

Here the parameter σ captures diffusion volatility, W_t is a standard Brownian motion process, and J_t is a jump process with mean arrival rate λ that captures discrete shocks; if a jump

[6]Stochastic fluctuations in the capital stock have been widely used in the growth literature with an AK technology, but unlike the existing literature, we examine the economic effects of shocks to capital that involve discrete (catastrophic) jumps. See Jones and Manuelli (2005) for a survey of endogenous growth models with with a stochastic AK technology.

occurs, K falls by the random fraction $(1 - Z)$. The adjustment cost function $\Phi(I, K)$ captures effects of depreciation and costs of installing capital. Because installing capital is costly, installed capital earns rents in equilibrium so that Tobin's q, the ratio between the market value and the replacement cost of capital, exceeds one. We assume $\Phi(I, K)$ is homogeneous of degree one in I and K and thus can be written as:

$$\Phi(I, K) = \phi(i)K \ , \tag{6}$$

where $i = I/K$ and $\phi(i)$ is increasing and concave. Unlike other models of catastrophes, we explicitly account for the effects of adjustment costs on equilibrium price and quantities.[7]

For simplicity, we use a quadratic adjustment cost function, which can be viewed as a second-order approximation to a more general one:

$$\phi(i) = i - \tfrac{1}{2}\theta i^2 - \delta \ . \tag{7}$$

Catastrophic Risk Insurance. We will use our model allows us to determine the equilibrium premium for catastrophic risk insurance. In order to make our analysis of insurance as general as possible, we introduce *catastrophic insurance swaps* (CIS) for shocks of every possible size as follows. These swaps are defined as follows: a CIS for the survival fraction in the interval $(Z, Z + dZ)$ is a swap contract in which the buyer makes a continuum of payments $p(Z)dZ$ to the seller and in exchange receives a lump-sum payoff if and only if a shock with survival fraction in $(Z, Z + dZ)$ occurs. That is, the buyer stops paying the seller if and only if the defined catastrophic event occurs and then collects one unit of the consumption good as a payoff from the seller. Note the close analogy between our CIS contracts and the widely used credit default swap (CDS) contracts. Unlike typical pricing models for CDS contracts, however, ours is a general equilibrium model with an endogenously determined risk premium.

[7] Homogeneous adjustment cost functions are analytically tractable and have been widely used in the q theory of investment literature. Hayashi (1982) showed that with homogeneous adjustment costs and perfect capital markets, marginal and average q are equal. Jermann (1998) integrated this type of adjustment costs into an equilibrium business cycle/asset pricing model.

2.2 Competitive Equilibrium.

Our model can be solved as a social planning problem, but we want to assert that the result is equivalent to a decentralized competitive equilibrium with complete markets. That is, we assume that the following securities can be traded at each point in time: (i) a risk-free asset, (ii) a claim on the value of capital of the representative firm, and (iii) insurance claims for catastrophes with every possible recovery fraction Z.

Because we allow for jumps in the capital stock, market completeness requires that agents can trade these insurance claims. But note that as with the risk-free asset, in equilibrium the demand for these insurance claims is zero. Although no trading of the risk-free asset or insurance claims will occur in equilibrium, we allow for the possibility of trading so that we can determine the equilibrium prices. In a representative agent model like ours, this "zero demand" result is a natural consequence. With heterogeneous agents (differing, e.g., in preferences, endowments, or beliefs), there will be no trading in general. but some agents will be buyers and some sellers of these assets. However, the net demand for the risk-free and insurance assets will be zero (as implied by market clearing.

We define the recursive competitive equilibrium as follows: (1) The representative consumer dynamically chooses investments in the risk-free asset, risky equity, and various CIS claims to maximize utility as given by eqns. (1) and (2). These choices are made taking the equilibrium prices of all assets and investment/consumption goods as given. (2) The representative firm chooses the level of investment that maximizes its market value, which is the present discounted value of future cash flows, using the equilibrium stochastic discount factor. (3) All markets clear. In particular, (i) the net supply of the risk-free asset is zero; (ii) the demand for the claim to the representative firm is equal to unity, the normalized aggregate supply; (iii) the net demand for the CIS of each possible recovery fraction Z is zero; and (iv) the goods market clears, i.e., $I_t = Y_t - C_t$ at all $t \geq 0$.

These market-clearing conditions are standard. When all markets are available for trading by investors and firms, the prices of claims such as the risk-free asset and CIS claims are at levels implying zero demand in equilibrium. With these conditions, we can invoke the welfare theorem to solve the social planner's problem and obtain the competitive equilibrium allocation, and then use the representative agent's marginal utility to price all assets in the economy. We emphasize that CIS insurance markets are crucial to dynamically complete the markets. This is a fundamental difference from models based purely on diffusion processes without jump risk.

We next summarize the solution of the model via the social planner's problem, leaving details to Appendix A. A separate appendix, available from the authors on request, derives the decentralized competitive market equilibrium and shows that it yields the same solution.

2.3 Model Solution.

The Hamilton-Jacobi-Bellman (HJB) equation for the social planner's allocation problem is:

$$0 = \max_{C} \left\{ f(C,V) + \Phi(I,K)V'(K) + \frac{1}{2}\sigma^2 K^2 V''(K) + \lambda \mathcal{E}\left[V(ZK) - V(K)\right] \right\} , \qquad (8)$$

where $V(K)$ is the value function and the expectation is with respect to the density function $\zeta(Z)$ for the survival fraction Z. We have the following first-order condition for I:

$$f_C(C,V) = \Phi_I(I,K)V'(K) \ . \qquad (9)$$

The left-hand side of eqn. (9) is the marginal benefit of consumption and the right-hand side is its marginal cost, which equals the marginal value of capital $V'(K)$ times the marginal efficiency of converting a unit of the consumption good into a unit of capital, $\Phi_I(I,K)$. With homogeneity, we have $\Phi_I(I,K) = \phi'(i)$.

We will show that the value function is homogeneous and takes the following form:

$$V(K) = \frac{1}{1-\gamma}\,(bK)^{1-\gamma} , \qquad (10)$$

where b is a coefficient determined as part of the solution. Let $c = C/K = A - i$. (Lower-case letters in this paper express quantities relative to the capital stock K.) Appendix A shows that b is related to the equilibrium level of the investment-capital ratio, i^*, by:

$$b = (A - i^*)^{1/(1-\psi)} \left(\frac{\rho}{\phi'(i^*)} \right)^{-\psi/(1-\psi)} \ . \qquad (11)$$

The equilibrium i^* can then be found as the solution of the following non-linear equation:

$$A - i = \frac{1}{\phi'(i)}\left[\rho + (\psi^{-1} - 1)\left(\phi(i) - \frac{\gamma\sigma^2}{2} - \frac{\lambda}{1-\gamma}\mathcal{E}\left(1 - Z^{1-\gamma}\right)\right)\right] . \tag{12}$$

Note that in equilibrium, the optimal investment-capital ratio $I/K = i^*$ is constant.

Consider the special case of no adjustment costs, for which our adjustment cost function of eqn. (7) becomes $\phi(i) = i - \delta$, where δ can be interpreted as the expected rate of stochastic depreciation. It is straightforward to show that in this case

$$i = \delta + \psi\left[A - \delta - \rho + (\psi^{-1} - 1)\left(\frac{\gamma\sigma^2}{2} + \frac{\lambda}{1-\gamma}\mathcal{E}\left(1 - Z^{1-\gamma}\right)\right)\right] . \tag{13}$$

Investment in this special case depends on $A - \delta - \rho$, so that the model cannot separately identify A, δ, and ρ.[8] In contrast, the introduction of adjustment costs in our model lets us separate the effects of A from those of δ and the subjective discount rate ρ, in addition to generating rents for capital, which implies $q \neq 1$.

Equilibrium capital accumulation in our model is given by:

$$dK_t/K_t = \phi(i^*)dt + \sigma dW_t - (1 - Z)dJ_t \ , \tag{14}$$

where i^* is the solution of eqn. (12). Let g denote the expected growth rate *conditional on no jumps*. Note that by setting $dJ_t = 0$ in eqn. (14), $g = \phi(i^*)$. The expected growth rate *inclusive of jumps* is then

$$\overline{g} = \phi(i^*) - \lambda\mathcal{E}(1 - Z) \, , \tag{15}$$

where the second term is the expected decline of the capital stock due to jumps.

Appendix A shows that the solution to the social planner's problem yields a goods-market clearing condition and first-order conditions for the consumer and the producer:

$$i = A - c \tag{16}$$

$$q = \frac{1}{\phi'(i)} = \frac{1}{1 - \theta i} \tag{17}$$

$$c/q = \rho + (\psi^{-1} - 1)\left(g - \frac{\gamma\sigma^2}{2} - \frac{\lambda}{1-\gamma}\mathcal{E}\left(1 - Z^{1-\gamma}\right)\right) \tag{18}$$

[8]This is an important drawback of an AK model without adjustment costs, such as in Barro (2009).

Eqn. (16) is simply an accounting identity that equates saving and investment. Eqn. (17) is a first-order condition for producers. Re-writing it as $\phi'(i)q = 1$, it equates the marginal benefit of an extra unit of investment (which at the margin yields $\phi'(i)$ units of capital, each of which is worth q) with its marginal opportunity cost (1 unit of the consumption good).

The left-hand side of eqn. (18) is the consumption-wealth ratio, c/q. In equilibrium, c/q is the marginal propensity to consume (MPC) out of wealth, and it is also the dividend yield, because consumption in equilibrium is totally financed by dividends, and total wealth is given by the market value of equity. (Note that the entire capital stock is marketable and its value is qK.) Eqn. (18) is a first-order condition for consumers. Multiplying both sides by q, it equates consumption (normalized by the capital stock) to the marginal propensity to consume times q, the marginal value of a unit of capital. What drives the MPC, c/q? Looking at the right-hand side of the equation, if $\psi = 1$, wealth and substitution effects just offset each other, and $c/q = \rho$, the rate of time preference. More generally, if $\psi < 1$, the wealth effect is stronger than the substitution effect, and hence the MPC increases with the growth rate g and decreases with risk aversion and volatility. The opposite holds if $\psi > 1$.

This equilibrium resource allocation has the following implications for the risk-free interest rate r and the equity risk premium rp:

$$r = \rho + \psi^{-1}g - \frac{\gamma(\psi^{-1}+1)\sigma^2}{2} - \lambda\mathcal{E}\left[\left(Z^{-\gamma}-1\right) + \left(\psi^{-1}-\gamma\right)\left(\frac{1-Z^{1-\gamma}}{1-\gamma}\right)\right] \tag{19}$$

$$rp = \gamma\sigma^2 + \lambda\mathcal{E}\left[(1-Z)\left(Z^{-\gamma}-1\right)\right] \tag{20}$$

Eqn. (19) for the interest rate r is a generalized Ramsey rule. If $\psi^{-1} = \gamma$ so that preferences simplify to CRRA expected utility, and if there were no stochastic changes in K, the deterministic Ramsey rule $r = \rho + \gamma g$ would hold. In our model there are two sources of uncertainty; continuous stochastic fluctuations in K and discrete shocks (i.e., jumps in K). The third term in eqn. (19) captures the precautionary savings effect under recursive preferences of continuous fluctuations in K, and the last term adjusts for shocks. Note that the first term in the square brackets is the reduction in the interest rate due to shocks under expected utility (and does not depend on ψ). The second term gives the additional effects of shock risk for non-expected utility; when $\psi^{-1} < \gamma$, the risk of shocks further increases the equilibrium interest rate from the level implied by standard CRRA utility.

Eqn. (20) describes the equity risk premium, rp. The first term on the RHS is the usual risk premium in diffusion models, and the second term is the increase in the premium due to jumps in K. When a jump occurs, $(1 - Z)$ is the fraction of loss, and $(Z^{-\gamma} - 1)$ is the percentage increase in marginal utility from that loss, i.e., the price of risk. The jump component of the equity risk premium is given by λ times the expectation of the product of these two random variables. Note that the fraction of loss and the increase of marginal utility are positively correlated, which substantially contributes to the risk premium. (In the limiting case where the loss is close to 100%, the increase in marginal utility approaches infinity.) Also note that the risk premium depends only on the coefficient of risk aversion, and does not depend on the EIS or rate of time preference.

The model can also be used to determine the equilibrium price of catastrophic risk insurance. We will examine the price of insurance in the next section when we discuss the calibration of the model, after specifying the distribution $\zeta(Z)$ for Z.

3 Calibration.

This section explains our calibration procedure. We begin by specifying the probability distribution for the survival fraction Z, and we show how this distribution simplifies the model and also yields identifying conditions on the second, third, and fourth moments of equity returns. Those conditions along with the other equations of the model can be used to identify the various parameters. We describe the data used to obtain values for the model's inputs, and we present a baseline calibration and additional sensitivity calibrations. We turn next to the pricing of catastrophic risk insurance, and show insurance premia for different size losses. Lastly, we turn to the role of adjustment costs and compare our results with those of Barro (2009). This helps to show the importance of adjustment costs and the implications of certain parameter choices.

3.1 The Distribution for Shocks.

The solution of the model presented above applies to any well-behaved distribution for recovery Z. We assume that Z follows a power distribution over (0,1) with parameter $\alpha > 0$:

$$\zeta(Z) = \alpha Z^{\alpha-1} \;\; ; \;\; 0 \leq Z \leq 1 \; , \tag{21}$$

so that $\mathcal{E}(Z) = \alpha/(\alpha+1)$. Thus a large value of α implies a small expected loss $\mathcal{E}(1-Z)$.

The distribution given by eqn. (21) is general. If $\alpha = 1$, Z follows a uniform distribution. For any $\alpha > 0$, eqn. (21) implies that $-\ln Z$ is exponentially distributed with mean $\mathcal{E}(-\ln Z) = 1/\alpha$. Eqn. (21) also implies that the inverse of the remaining fraction of the capital stock follows a Pareto distribution with density function $\alpha(1/Z)^{-\alpha-1}$ with $1/Z > 1$. The Pareto distribution is fat-tailed and often used to model extreme events.

The power distribution for Z given in (21) simplifies the solution of the model. We need three moments of Z, namely $\mathcal{E}(Z^n)$ where $n = 1$, $1-\gamma$, and $-\gamma$. Eqn. (21) implies

$$\mathcal{E}(Z^n) = \alpha/(\alpha+n) \; , \tag{22}$$

provided that $\alpha + n > 0$. Since the smallest relevant value of n is $-\gamma$, we require $\alpha > \gamma$, which ensures that the expected impact of a catastrophe is sufficiently limited so that the model admits an interior solution for any level of risk aversion γ. Thus $\mathcal{E}(1-Z) = 1/(\alpha+1)$ is the expected loss if an event occurs, and $\mathcal{E}(Z^{-\gamma}-1) = \gamma/(\alpha-\gamma)$ is the expected percentage increase in marginal utility from the loss; both are decreasing in α.

3.2 Equity Returns.

The distribution for Z given by eqn. (21) can be used to obtain moment conditions on equity returns. Recall that $-\ln Z$ is exponentially distributed with mean $\mathcal{E}(-\ln Z) = 1/\alpha$. Thus $\mathcal{E}((\ln Z)^2) = 2/\alpha^2$ and $\mathcal{E}((\ln Z)^3) = -6/\alpha^3$.

Because the equilibrium value of Tobin's q is constant, the value of the firm, $Q = qK$, follows the same stochastic process (with the same drift and volatility) as the capital stock K. Also, in equilibrium the dividend yield is constant, so only capital gains contribute to second and higher order moments of stock returns. Therefore, the variance, skewness, and kurtosis for logarithmic equity returns over the time interval $(t, t+\Delta t)$ equal the corresponding moments for $\ln K_{t+\Delta t}/K_t$. Let $\mathcal{V}, \mathcal{S}$, and $\mathcal{K}$ denote the variance, skewness, and kurtosis, respectively, for equity returns. We show in Appendix B that they are given by:

$$\mathcal{V} = \Delta t \left(\sigma^2 + 2\lambda/\alpha^2\right) \tag{23}$$

$$\mathcal{S} = \frac{1}{\sqrt{\Delta t}} \frac{-6\lambda/\alpha^3}{(\sigma^2 + 2\lambda/\alpha^2)^{3/2}} \tag{24}$$

$$\mathcal{K} = 3 + \frac{1}{\Delta t} \frac{24\lambda/\alpha^4}{(\sigma^2 + 2\lambda/\alpha^2)^2} \tag{25}$$

Here Δt is the frequency with which returns are measured. In our case returns are measured monthly and are in monthly terms; because all variables are expressed in annual terms for purposes of our calibration, $\Delta t = 1/12$.

Using eqn. (22), the expected growth rate that includes possible jumps is

$$\overline{g} = g - \frac{\lambda}{\alpha + 1}\,. \tag{26}$$

Eqn. (22) can also be used to simplify eqns. (18), (19), and (20), which now become:

$$\frac{c}{q} = r + rp - \overline{g} \tag{27}$$

$$r = \rho + \psi^{-1} g - \frac{\gamma(\psi^{-1}+1)\sigma^2}{2} - \lambda \left[\frac{(\psi^{-1}-\gamma)(\alpha-\gamma) + \gamma(\alpha-\gamma+1)}{(\alpha-\gamma)(\alpha-\gamma+1)}\right] \tag{28}$$

$$rp = \gamma\sigma^2 + \lambda\gamma \left[\frac{1}{\alpha-\gamma} - \frac{\alpha}{(\alpha+1)(\alpha+1-\gamma)}\right] \tag{29}$$

Recall that in equilibrium the consumption-wealth ratio c/q is equal to the dividend yield. Eqn. (27) is essentially a Gordon growth formula; it states that the expected return on equity $(r + rp)$ equals the dividend yield (c/q) plus the expected growth rate $\overline{g}$ (inclusive of jumps).

3.3 Identification.

With eqns. (16), (17), and (23) to (29) we can identify the key parameters and variables of the model. To do this we use the following inputs: the variance, skewness, and kurtosis of equity returns, the real risk-free rate r and equity premium rp, the output/capital ratio Y/K, the consumption/investment ratio c/i, and the per capita expected real growth rate $\overline{g}$.

We discuss the data and calculation of these inputs below. The identification of the model is easiest to see in steps.

First, given the variance, skewness and kurtosis for equity returns, we use eqns. (23) to (25) to calculate λ, α and σ. Thus the three parameters that govern stochastic changes in the capital stock are all determined by the second and higher moments of equity returns.

Second, given these three parameters, we use eqn. (29) for the equity risk premium equation to calculate the coefficient of relative risk aversion, γ. Thus γ is determined by the cost of equity capital relative to the risk-free rate.

Third, we can use eqn. (28) for the risk-free rate to identify either the rate of time preference ρ or the EIS ψ. Except for the special case of expected utility, where $\psi = 1/\gamma$, our parsimonious model does not allow us to separately identify these two parameters. Instead we use eqn. (28) to obtain ψ as a function of the discount rate ρ, and then consider a range of "reasonable" values for ψ and the implications for ρ.

Lastly, we use the equations for the real side of the model to identify the remaining variables and parameters. We calculate the productivity parameter A directly; it is just the average output/capital ratio (with the capital stock broadly defined to include physical, human, and intangible capital). Then, given c/i, eqn. (16) determines both c and i. Finally, given the expected growth rate $\overline{g}$, eqn. (18) determines q, and eqn. (17) determines the adjustment cost parameter θ.

The identification of the model can also be seen in terms of equations and unknowns. We have a total of 8 equations: (16), (17), and (23) through (29). We use these equations to identify 8 parameters and variables: the parameters λ, α and σ that govern stochastic changes in K, the behavioral parameters γ and either ψ or ρ, and the economic variables c, i and q.

3.4 Baseline Calibration.

Ours is an equilibrium model, so its calibration should be based on data covering a time period that is long and relatively stable. We therefore use data for the U.S. economy from 1947 to 2008 to construct average values of the output-capital ratio Y/K, the consumption-

Table 1: Summary of Baseline Calibration

Calibration inputs (Annual rates)	Symbol	Value	Calibration outputs	Symbol	Value
output-capital ratio	A	0.113	diffusion volatility	σ	0.1355
consumption-investment ratio	c/i	2.84	mean arrival rate	λ	0.734
real expected growth rate	$\overline{g}$	0.02	distribution parameter	α	23.17
EIS	ψ	1.5	expected loss	$\mathcal{E}(1-Z)$	0.0414
risk-free interest rate	r	0.008	average q	q	1.548
equity risk premium	rp	0.066	coefficient of risk aversion	γ	3.066
stock return variance	$\mathcal{V}$	0.0211	rate of time preference	ρ	0.0498
stock return skewness	$\mathcal{S}$	-0.1156	adjustment cost parameter	θ	12.025
stock return kurtosis	$\mathcal{K}$	0.1374	consumption-capital ratio	c	0.0836

investment ratio C/I, the real risk-free rate r, and the expected real growth rate $\overline{g}$. We calculate the equity risk premium rp and second, third, and fourth moments of equity returns using monthly data for the real total value-weighted return on the S&P500. As discussed in Appendix D, our measure of the capital stock includes physical capital, estimates of human capital, and estimates of firm-based intangible capital (e.g., patents, know-how, brand value, and organizational capital). Thus, we obtain a measure of the productivity parameter $A = Y/K$ consistent with the AK production technology of eqn. (4). Our measure of investment (and GDP) includes investment in firm-based intangible capital, and we assume that investment in human capital occurs through education and is part of consumption.

Table 1 summarizes the inputs used in the baseline calibration, and the calibration outputs. Note that we obtain a value of 3.1 for the coefficient of relative risk aversion, which is well within the consensus range. Recall that we cannot separately identify ψ and ρ (except for the special case of expected CRRA utility), so in Table 1 we set $\psi = 1.5$, which yields a value of just under .05 for the rate of time preference ρ.[9] Also, our estimate of q is about 1.55, which is close to the value of 1.43 obtained by Riddick and Whited (2009), who used

[9] Estimates of ψ in the literature vary considerably, ranging from the number we obtained to values as high as 2. Bansal and Yaron (2004) argue that the elasticity of intertemporal substitution is above unity and use 1.5 in their long-run risk model. Attanasio and Vissing-Jørgensen (2003) estimate the elasticity to be above unity for stockholders, while Hall (1988), using aggregate consumption data, obtains an estimate near zero. The Appendix to Hall (2009) provides a brief survey of estimates in the literature.

firm-level Compustat data for 1972 to 2006.[10]

We obtain a mean arrival rate λ of 0.734 for the jump process and a value for the distributional parameter α of 23.17. These numbers imply that a shock occurs about every 1.4 years on average, with a mean loss $\mathcal{E}(1-Z) = 1/(\alpha+1)$ of only about 4%. Thus most shocks are small, and could be viewed as part of the "normal" fluctuation in the capital stock. What about larger, "catastrophic" shocks? For the power distribution specified in (21), given that the jump occurs, the probability that the loss from a shock will be a fraction L or greater, i.e., the probability that $Z \leq 1-L$, is $(1-L)^{\alpha}$. Thus the probability that the loss will be at least 10% is $.90^{23.17} = .087$, at least 15% is .023, and at least 20% is .006.

Table 2 reports the probability that at least one shock causing a loss larger than L will occur over a given time span T. Using the Poisson distribution property, the probability of one or more shocks with loss larger than L occurring over time span T is

$$\Pr(T, L) = 1 - \exp\left[-\lambda T \int_0^{1-L} \zeta(Z)dZ\right] = 1 - \exp\left[-\lambda T(1-L)^{\alpha}\right] . \tag{30}$$

For example, if we consider as catastrophic a shock for which the loss is 15% or greater, the annual likelihood of such an event is $(.85)^{\alpha}\lambda = .017$. This implies substantial risk; for example, the probability that *at least one* catastrophe (with a loss of 15% or greater) will occur over the next 50 years is $1 - e^{-.017\times 50} = .57$.

By comparison, using a sample of 24 (36) countries, Barro and Ursúa (2008) estimated λ as the proportion of years in which there was a contraction of real per capital consumption (or GDP) of 10% or more, and found λ to be 0.038 (for consumption and GDP). But for the U.S. experience (which corresponds to our calibration), there were only two contractions of consumption of 10% or more over 137 years (implying $\lambda = 0.015$), and five GDP contractions (implying $\lambda = 0.036$). Our estimate of λ uses equity market return moments and corresponds to the proportion of years for which there is a jump shock of any size. If we use a 10% or more loss as the threshold to define a catastrophe, the corresponding value of λ would be $(.734)(.90^{23.17}) = 0.064$, which is considerably larger than the Barro and Ursúa estimate.

[10]With measurement errors and heterogeneous firms, averaging firm-level data provides a more economically sensible estimate for the q of the representative firm than inferring q from aggregate data.

Table 2: PROBABILITY OF SHOCKS EXCEEDING L OVER HORIZON T.

Horizon T (Years)	1	10	20	40	50
$L = 0.10$	0.0619	0.4723	0.7215	0.9224	0.9591
$L = 0.15$	0.0169	0.1563	0.2882	0.4934	0.5726
$L = 0.20$	0.0042	0.0409	0.0801	0.1537	0.1883
$L = 0.25$	0.0009	0.0093	0.0185	0.0367	0.0457
$L = 0.30$	0.0002	0.0019	0.0038	0.0075	0.0094
$L = 0.35$	0.000034	0.0003	0.0007	0.0014	0.0017

Note: Each entry is the probability that at least one shock larger than L will occur during the time horizon T.

They also found an average contraction size (conditional on the 10% threshold, and for the international sample) of 0.22 for consumption and 0.20 for GDP. Using our results, the average contraction size conditional on a contraction greater than 10% is .137. Thus, compared to Barro and Ursúa, we find that shocks greater than 10% occur more frequently but on average are smaller in size.

Barro and Jin (2009) independently applied the same power distribution that we used in eqn. (21) to describe the size distribution for contractions. We obtained a value of the distribution parameter α as an output of our calibration; they estimated α for their sample of contractions. In our notation, their estimates of α were 6.27 for consumption contractions and 6.86 for GDP, implying a mean loss of about 14% for consumption and 13% for GDP.[11] However, they only considered contractions that were 10% or greater. As we explained above, applying our estimate of α (23.17) to losses of 10% or greater implies a mean contraction size of .137. This number is close to the Barro-Jin estimate, but note that we obtained it in a completely different way. Rather than use historical data on drops in consumptions or GDP, we found the mean contraction size as an output of our calibration.

[11] Eqn. (21) is the distribution for Z, the fraction of the remaining capital stock. It implies that $S = 1/Z$ has the distribution $f_S(s) = \alpha s^{-\alpha-1}$. Barro and Jin (2009) use data on S, conditioned on $S > 1.105$, to estimate α' for the distribution $f_S(s) = \alpha' s^{-\alpha'}$. Thus $\alpha = \alpha' - 1$.

3.5 Catastrophic Insurance Premium.

Our model solution also implies the equilibrium price of every possible insurance claim:

$$p(Z) = \lambda Z^{-\gamma}\zeta(Z) \tag{31}$$

where $\zeta(Z)$ is the probability density function for the recovery fraction Z, so that $\lambda\zeta(Z)$ is the conditional arrival intensity of a shock that destroys a fraction $(1-Z)$ of the capital stock. Eqn. (31) gives the payment rate that the CIS buyer must make to insure against a shock with loss fraction $(1-Z)$; should that shock occur, the buyer would receive one unit of the consumption good. Not surprisingly, the higher the arrival rate of a shock with survival fraction Z, $\lambda\zeta(Z)$, the higher the corresponding CIS payment. The multiplier $Z^{-\gamma}$ in eqn. (31) is the marginal rate of substitution between pre-jump and post-jump values, and measures the insurance risk premium; the higher is γ and the bigger is the loss (the lower is Z), the more expensive is the insurance.

Using eqn. (21) for the probability density function that governs the recovery fraction Z, we can calculate the cost of insuring against any particular risk. Recall that $\mathcal{E}(Z^n) = \alpha/(\alpha+n)$. Thus for each CIS with survival fraction Z, the required payment is:

$$p(Z) = \lambda\alpha Z^{\alpha-\gamma-1}\,. \tag{32}$$

For example, to obtain the cost of insuring against a shock that results in losing a fraction L or more of the capital stock (i.e., $1-Z \geq L$), the required payment per unit of capital is

$$\int_0^{1-L}(1-Z)p(Z)dZ = \lambda\alpha\left[\frac{(1-L)^{\alpha-\gamma}}{\alpha-\gamma} - \frac{(1-L)^{\alpha-\gamma+1}}{\alpha-\gamma+1}\right]. \tag{33}$$

Thus to obtain the required payment per unit of capital to insure against *any* size shock, just set $L=0$ in eqn. (33). Note that unlike the existing catastrophic insurance literature, we obtain the insurance premium in a general equilibrium setting. Also, observe from eqn. (33) that the CIS payment depends only on risk aversion γ, the parameters describing shocks, i.e., λ and α, and the lower bound L of the loss insurance. The CIS payment does not depend on the EIS ψ and the discount rate ρ, for example, because these parameters do not describe the characteristics of or attitudes toward risk.

Table 3: Loss Coverage and Components of Catastrophic Insurance Premia

Minimum loss covered, L	CIS	AF	CIS/AF
$L = 0.00$ (Full insurance)	0.4795	0.3633	1.320
$L = 0.05$	0.3429	0.2389	1.435
$L = 0.10$	0.1736	0.1049	1.655
$L = 0.15$	0.0734	0.0377	1.949
$L = 0.20$	0.0271	0.0116	2.331
$L = 0.25$	0.0089	0.0031	2.829
$L = 0.30$	0.0026	0.0007	3.484
$L = 0.35$	0.0007	0.00015	4.362
$L = 0.40$ (loss 40% or more)	0.0002	0.00003	5.564

Note: For each amount of loss coverage, CIS is the required annual insurance payment as a percent of consumption and AF is the actuarially fair payment. $L = 0.25$ means that only losses of 25% or more are covered.

Using our baseline calibration (which yielded $\gamma = 3.066$, $\lambda = 0.739$, and $\alpha = 23.17$) and eqn. (33), the annual CIS payment to insure against shocks of any size is about .040 per unit of capital, i.e., 4% of the capital stock. We have $A = .113$, so the total annual cost of the insurance would be $.040Y/.113 = .355Y$, i.e., about 35% of GDP, or about 48% of consumption.[12] How much of this very large annual CIS payment reflects the expected loss from a shock and how much is a risk premium? We first calculate the expected loss with no risk premium. The implied actuarially fair annual CIS payment is $\int_0^{1-L}(1-Z)\lambda\zeta(Z)dZ$, which can also be found by setting $\gamma = 0$ in eqn. (33). The "price" of risk is ratio of the annual CIS payment to the actuarially fair payment.

Table 3 summarizes both the CIS and actuarially fair payments (denoted by AF), both as a fraction of consumption, to cover losses of different amounts. If we treat as catastrophes shocks that result in losses of 15% or more, the annual payment to insure against such losses is over 7% of consumption — a substantial amount. If we restrict our definition of catastrophes to only include shocks that cause losses of 20% or more, the annual insurance payment is nearly 3% of consumption — still quite large. Note that the "price" of risk (the ratio of the CIS payment to the actuarially fair premium) increases with L, the lower bound

[12]Using $c + i = A = .113$ and $c/i = 2.84$ gives $C = .740Y = .0836K$.

of the loss fraction that is insured. For example, to insure only against catastrophes that generate a loss of 10% or more, the price of risk is about 1.7. But if insurance is limited to only those shocks causing losses of 25% or more (i.e. $L = 0.25$), the annual cost is just under 1% of consumption, while the actuarially fair rate is about 0.3% of consumption, implying a price of risk of about 2.8. The price of risk is higher in this case because the insurance is covering larger losses on average and insuring tail risk is expensive.

3.6 The Role of Adjustment Costs.

How important are adjustment costs? To address this question and do welfare calculations, we use the quadratic adjustment cost function given by eqn. (7). In our baseline calibration, the resulting value of θ is 12.03, which is determined by eqn. (17): $q = 1/\phi'(i) = 1/(1-\theta i)$. In our calibration, $q = 1.55$, $i + c = A$, and $c/i = 2.84$, which pins down $\theta = 12.03$.

To explore the role of adjustment costs, we first review Barro's (2009) results and then add adjustment costs to his model. Based on historic "consumption disasters," Barro estimated λ to be .017. He set $\gamma = 4$, and using an empirical distribution for consumption declines, estimated the three moments $\mathcal{E}(Z)$, $\mathcal{E}(Z^{1-\gamma})$, and $\mathcal{E}(Z^{-\gamma})$. He also set $\psi = 2$, $\rho = .052$, $\sigma = .02$, and $A = .174$. (Because there are no adjustment costs, only $A - \rho$ can be identified in Barro's model.)

The first row of the top panel of Table 4 shows this calibration of Barro's model; there are no adjustment costs so capital is assumed to be perfectly liquid and $q = 1$. The calibration gives a sensible estimate of the risk-free rate r and risk premium rp, but yields a consumption-investment ratio of only 0.38, whereas the actual ratio is about 3. The rest of the top panel shows how the results change as the adjustment cost parameter θ in eqn. (7) is increased. The experiment here is to hold the structural parameters for both preferences and the technology fixed, change only θ, and then re-solve for the new equilibrium price and quantity allocations. First, as we increase θ, investment becomes more costly, so i falls and $c = A - i$ increases. Additionally, q increases because installed capital now earns higher rents in equilibrium. When $\theta = 8$, both c/i and q roughly match the data. However, r falls below -3%. Basically, given Barro's parameter choices (particularly γ and the productivity parameter A) along

Table 4: Effects of Adjustment Costs

(1) Barro (2009) Parameters: $\gamma = 4$, $\psi = 2$, $\rho = .052$, $\sigma = .02$, $A = .174$, $\lambda = .017$, $\mathcal{E}(Z) = .71$, $\mathcal{E}(Z^{1-\gamma}) = 4.05$, $\mathcal{E}(Z^{-\gamma}) = 7.69$					
θ	i	c	c/i	r	q
0	**0.126**	**0.048**	**0.381**	**.011**	**1.00**
4	0.062	0.112	1.806	-.025	1.33
8	0.038	0.136	3.579	-.036	1.43
12	0.027	0.147	5.512	-.041	1.47
20	0.017	0.157	9.234	-.045	1.51
(2) Our Parameters: $\gamma = 3.066$, $\psi = 1.5$, $\rho = .0498$, $\sigma = .1355$, $A = .113$, $\lambda = .734$, $\mathcal{E}(Z) = .9586$, $\mathcal{E}(Z^{1-\gamma}) = 1.098$, $\mathcal{E}(Z^{-\gamma}) = 1.153$					
θ	i	c	c/i	r	q
0	0.0764	0.0366	0.479	.0428	1.00
4	0.0543	0.0587	1.080	.0241	1.28
8	0.0388	0.0742	1.911	.0137	1.45
12.03	**0.0294**	**0.0836**	**2.84**	**0.008**	**1.55**
20	0.0196	0.0934	4.77	.002	1.64

with the exogenous inputs for λ and the moments of Z, the model cannot match all of the basic economic facts, even allowing for adjustment costs.

The bottom panel of Table 4 shows results using our baseline calibration, but varying the value for θ. (The boldface row corresponds to our baseline calibration where $\theta = 12.03$.) As θ increases, the cost of investing increases, so investment becomes less attractive relative to consumption, and thus i falls. With $\psi = 1.5 > 1$, the substitution effect dominates the wealth effect, and hence r also falls. As a result of this drop in the cost of capital, q increases. Thus for a calibrated model to match the data, adjustment costs are crucial.

To summarize, these results illustrate some important differences between our approach and that of Barro (2009): (1) By construction, $q = 1$ in models with no adjustment costs because physical capital is *assumed* to be perfectly liquid, which is inconsistent with the data. Our model captures this important feature of the illiquidity of physical capital and the empirical fact that $q \neq 1$. (2) Models with no adjustment costs cannot separately identify the effect of the productivity coefficient A from the effects of the rate of time preference ρ and the expected rate of depreciation δ. In our model, A has a distinct effect on the investment-capital ratio i that differs from the effects of ρ, so that A and ρ can be separately identified. (3) As noted earlier, Barro's AK model generates an unrealistically high investment-capital ratio; our model does not because adjustment costs make investment more expensive.

4 Policy Consequences.

We now turn to the second question raised in the first paragraph of this paper: What is society's willingness to pay to reduce the probability or likely impact of catastrophic events? Our measure of WTP is the maximum *permanent* consumption tax rate τ that society would be willing to accept if the resulting stream of government revenue could finance whatever activities are needed to permanently limit the maximum loss from any catastrophic shock that might occur to some level $\widehat{L}$. Of course it might not be feasible to limit the maximum loss to $\widehat{L}$ with a tax of that size, or it might be possible to achieve this objective with a smaller tax. In effect, we are examining the "demand side" of public policy — society's "reservation price" (maximum tax) for attaining this policy objective.

4.1 Willingness to Pay.

We want to determine the effect of a permanent consumption tax. Given investment I_t and output Y_t, households pay $\tau(Y_t - I_t)$ to the government and consume the remainder:

$$C_t = (1-\tau)(Y_t - I_t) \ . \tag{34}$$

Suppose that a costly technology exists that could ensure that any shock that occurs would lead to a loss no greater than $\widehat{L} = (1-\widehat{Z})$. That is, the technology would permanently change the recovery size distribution $\zeta(Z)$ to a truncated distribution, given by

$$\widehat{\zeta}(Z;\widehat{Z}) = \frac{\alpha Z^{\alpha-1}}{\int_{\widehat{Z}}^{1} \alpha Z^{\alpha-1} dZ} = \frac{\alpha}{1-\widehat{Z}^{\alpha}} Z^{\alpha-1} \quad ; \ \ \widehat{Z} \le Z \le 1 \ . \tag{35}$$

Here, $\widehat{Z}$ is the minimal level of recovery Z.

Using this truncated distribution, we obtain the optimal investment-capital ratio $\hat{i}$ as the solution of the following equation:

$$A - \hat{i} = \frac{1}{\phi'(\hat{i})}\left[\rho + (\psi^{-1}-1)\left(\phi(\hat{i}) - \frac{\gamma\sigma^2}{2} - \frac{\lambda}{1-\gamma}\widehat{\mathcal{E}}\left(1 - Z^{1-\gamma}\right)\right)\right] \ , \tag{36}$$

where $\widehat{\mathcal{E}}$ is the expectation with respect to the truncated distribution $\widehat{\zeta}(Z)$.

How much would households be willing to pay the government to finance such a technology? Consider two options: (1) the status quo with no taxes and the original recovery size distribution $\zeta(Z)$; and (2) paying a permanent consumption tax at rate τ to adopt the new technology which changes the distribution $\zeta(Z)$ to $\widehat{\zeta}(Z)$ given by eqn. (35). Households would be indifferent if and only if the following condition holds:

$$\widehat{V}(K;\tau) = V(K;0) \ , \tag{37}$$

where $\widehat{V}(K;\tau)$ is the household's value function given by eqns. (76) and (81) with a consumption tax rate τ, and with the optimal investment-capital ratio $\hat{i}$ for the truncated distribution given by eqn. (36). In Appendix C, we show that this condition implies that:

$$\widehat{b}(\tau) = (1-\tau)\widehat{b}(0) = b(0) \ , \tag{38}$$

where $\widehat{b}(0)$, given by eqn. (82), is the coefficient in the value function $\widehat{V}(K;0)$ when there is no tax but the distribution for Z is truncated, as given by eqn. (35). Thus to eliminate the possibility of catastrophic shocks with losses greater than $\widehat{L} = (1-\widehat{Z})$, households would be willing to pay a consumption tax at the constant rate

$$\tau = 1 - \frac{b(0)}{\widehat{b}(0)} \ . \tag{39}$$

For the household, a permanent tax at rate τ is equivalent to giving up a fraction τ of the capital stock. This is because the tax is non-distortionary. The tax is proportional to output, so households' after-tax consumption is lowered by the same fraction as the tax rate in all states and in all future periods. Thus households' intertemporal marginal rate of substitution, which determines the equilibrium interest rate and the pricing of risk, remains unchanged for any give rate of tax τ. (Although equilibrium pricing and resource allocations are the same with or without a tax, they are not the same for different distributions for Z, i.e., for the truncated versus non-truncated distribution.) Likewise, the total value of capital (including the taxes paid to the government) is unchanged, and investment is unchanged, for any given tax rate τ. A fraction τ of ownership is simply transferred from households

Table 5: WTP Tax Calculations

Note: $A = .113$, $r = .008$, $c/i = 2.84$, $rp = .066$, $\mathcal{V} = .0211$, $\mathcal{S} = -0.1116$, $\mathcal{K} = 0.137$, $g = .020$, $q = 1.55$			
Maximum	Maximum Tax Rate τ		
Loss	$\psi = 1.0$	$\psi = 1.5$	$\psi = 2.0$
0	.474	.522	.560
0.05	.298	.316	.329
0.10	.154	.159	.162
0.15	.066	.067	.067
0.20	.024	.024	.024
0.25	.008	.008	.008
0.30	.002	.002	.002
0.35	.001	.001	.001

to the government. This key result follows from the recursive homothetic preferences and equilibrium property that the economy is on a stochastic balanced growth path.[13]

4.2 Tax Calculations.

Table 5 shows the maximum permanent tax rate on consumption that society would accept to limit the maximum loss from a jump shock to various levels. The tax rates are shown for three different values of the EIS ψ, 1.0, 1.5, and 2.0. (Recall that given ρ we can pin down ψ, but we cannot pin down *both* ρ and ψ.) The first row, for which the maximum loss is zero, gives the tax that society would pay to eliminate *all* jump shocks. That tax rate is very large, on the order of 50%. But even if we were to limit the impact of shocks to a loss no greater than 15%, the warranted tax is substantial — close to 7% per year (forever). Shocks causing losses greater than 25% or 30% are very rare, so the tax is much lower.

To a first-order approximation, the CIS premium for insuring against losses above a particular percentage is close to the maximum consumption tax society would pay to eliminate the possibility of such losses. For example, the CIS premium for insuring against losses above 15% is 7.34% of consumption, which is only slightly larger than the corresponding consump-

[13]We have focused on policies that would limit the maximum loss from a shock, but the results also apply if the tax is used to reduce the likelihood of a shock of any size.

tion tax rate. Eliminating or reducing catastrophic risk, however, is fundamentally different from purchasing insurance since the latter is a zero NPV financial transaction, with no gain in value. Using tax proceeds to reduce the consequences of catastrophic shocks is a different way to manage aggregate risk than purchasing insurance because the former changes real economic activities (consumption and investment) while the latter simply transfers resources from one party to the other depending on whether the insured event takes place. Using tax proceeds to reduce the consequences of catastrophic shocks would be a positive NPV project and thus welfare enhancing if it could be done at a cost lower than the WTP.

For example, suppose a technology existed such that at an annual cost of 2% of consumption, the maximum loss L from shocks could be capped at 15%. As can be seen from Table 5, consumers could then gain approximately a $6.7\% - 2\% = 4.7\%$ increase in certainty-equivalent consumption units. In this case, the benefit of a reduction in catastrophic risk (by capping the loss at 15%) would clearly outweigh the cost.

Note that because our cost-benefit analysis is a general equilibrium one, it is fundamentally different from the standard cost-benefit approach in which an NPV is calculated treating input prices and the cost of capital as exogenous. The standard approach is adequate for evaluating the construction of a new bridge, because while the bridge involves a change in cash flows, there is no change in the cost of capital (i.e., the pricing kernel). But when the "project" involves a major change in the economy (e.g., reducing catastrophic risk), prices as well as cash flows change, so the "project" can only be evaluated by comparing its cost to its WTP, as we have done above. Our model of a production economy with adjustment costs is a natural framework for general equilibrium cost-benefit analysis.

4.3 Welfare Decomposition.

Our analysis is related to the "cost of business cycle" literature. Beginning with Lucas (1987), much of that literature has found the welfare gains from eliminating business cycle risk to be low. An exception is Barro (2009), who found the welfare gains from eliminating "conventional" business cycle risk — i.e., continuous fluctuations in output — to be very low, but the gains from eliminating disaster risk (jump risk in our model) to be substantial.

Table 6: WELFARE ANALYSIS

Change from Baseline	Maximum Tax Rate τ
Note: $A = .113$, $r = .008$, $c/i = 2.84$, $rp = .066$, $g = .020$, $\mathcal{V} = .0211$, $\mathcal{S} = -0.1116$, $\mathcal{K} = 0.137$, $q = 1.55$, $\psi = 1.5$. Unless otherwise indicated, $\sigma = .1355$, $\lambda = 0.734$, $\theta = 12.025$	
$\lambda = 0$	.522
$\sigma = 0$	.441
$\theta = 0$	.289
$\lambda = \sigma = 0$	.786
$\sigma = \theta = 0$	.730
$\lambda = \theta = 0$	.804
$\lambda = \sigma = \theta = 0$	.986

For comparison to the earlier literature, we use our model to evaluate these welfare gains.

In our model, we measure welfare gains in terms of WTP, i.e., the maximum consumption tax society would pay to change the economy such that various risks are reduced or eliminated. We therefore calculate the maximum tax rate to eliminate continuous fluctuations in the capital stock, i.e., to make $\sigma = 0$, leaving jump risk unchanged; to eliminate jump risk (i.e., make $\lambda = 0$), leaving σ unchanged; and to make $\sigma = \lambda = 0$. We also want to determine the welfare implications of adjustment costs (missing in Barro's analysis), so we repeat these calculations but also setting the adjustment cost parameter $\theta = 0$. Finally, we calculate the welfare gain from eliminating adjustment costs, but leaving both σ and λ unchanged.

The results are shown in Table 6. The first row, for $\lambda = 0$, corresponds to the first row of Table 5, and shows a willingness to pay a consumption tax of over 50% to eliminate jump risk. However, the willingness to pay to eliminate continuous fluctuations ($\sigma = 0$), leaving jump risk unchanged, is almost as large — a 44% tax. This is in stark contrast to the results of Barro (2009), Lucas (1987) and others. There are two reasons for this. First, earlier studies use the 2 to 2.5% standard deviation of log changes in consumption as the value of σ that describes business cycle risk, whereas we estimate σ to be 13.5% based on stock return data. Second, we include adjustment costs, which make stochastic fluctuations in the capital stock (whether continuous or jumps) more costly. Indeed, as the third row of

Table 6 shows, the willingness to pay to eliminate adjustment costs — leaving both σ and λ unchanged — is about 29%. The effects of adjustment costs can also be seen from rows 5 and 6. The WTP to eliminate both jump risk and adjustment costs is a tax rate of about 80%, which is much larger than a 52% tax rate society would pay to eliminate only jump risk. Similarly, the WTP to eliminate both diffusion risk and adjustment costs is a tax rate of 73%, as opposed to the 44% tax rate society would pay to eliminate only diffusion risk.

Another reason for the high willingness to pay to eliminate stochastic fluctuations (make σ, λ, or both zero) is that in our model changes in the capital stock are permanent. Percentage changes in the (endogenous) growth rate are iid, i.e., there is no mean reversion. Allowing for catastrophic shocks to be followed by recoveries (so that there is mean reversion in growth rate changes), e.g., as in Gourio (2008), would reduce the tax rates in Table 6.

5 Concluding Remarks.

We have provided a new framework for evaluating the characteristics of possible catastrophic events that are national or global in scale, calculating their implications for catastrophic risk insurance, and evaluating tax policies to limit their magnitude. Rather than use historical data on declines in consumption or GDP as others have done, we calculated event characteristics as calibration outputs from a general equilibrium model and used aggregate asset market data. Our framework provides a natural benchmark to quantitatively assess public policy; it fully incorporates general equilibrium quantity and price adjustments by the private sector in anticipation of a policy intervention. And unlike previous studies, our model also matches the production side of the economy such as the investment-capital ratio, and generates sensible estimates for Tobin's q and for the coefficient of relative risk aversion.

We calculated as a "willingness to pay" measure the permanent tax on consumption that society would accept to limit the maximum loss from a catastrophic shock. For our calibration, we found that a permanent tax of about 7% would be justified if the resulting revenues could be used to limit the impact of shocks to a loss no greater than 15% of the capital stock. Even if the objective was to limit losses to be no greater than 20 or 25%, a tax of one or two percent would be justified — a substantial tax given that it would be permanent. These results suggest that governments should devote greater resources to reducing the risk and potential impact of catastrophes, and thereby provide quantitative support to arguments along these lines made by Allison (2004), Posner (2004), and Parson (2007), among others. These arguments are especially important given the tendency of governments (with their short political cycles) to underestimate both the likelihood and possible impact of catastrophic events and the value of risk mitigation.[14]

An alternative to a tax is insurance. We have seen (compare Tables 3 and 5) that the annual payments required to insure against losses of, say, 15% to 25% are close to the corresponding tax rates. Insurance, however, is a zero-NPV transaction. In contrast, the use of tax revenues to eliminate or reduce the potential impact of catastrophes is especially attractive if the required revenue is less than the WTP, making the social NPV of the tax policy positive.

We estimated that the probability that over the next 50 years we will experience one or more shocks causing a loss of 15% or more of the capital stock exceeds 50%, and the probability of one or more shocks causing a loss of 25% or more is about 5%. (See Table 2.) These estimates apply to "generic" shocks, i.e., of an unspecified nature. For comparison, here are estimates of the likelihood and possible impact of some specific catastrophic shocks:

Global Warming: A consensus estimate of the increase in global mean temperature that would be catastrophic is about 7°C. A summary of 22 climate science studies surveyed by the Intergovernmental Panel on Climate Change (IPCC) in 2007 puts the probability of this occurring by the end of the century at around 5%. Weitzman (2009) argues that the probability distribution is fat-tailed, making the actual probability as high as 10%. What would be the impact of a catastrophic increase in temperature? Estimates of the effective reduction in (world) GDP range from 10% to 30%.[15]

Nuclear Terrorism: Various studies have assessed the likelihood and impact of the

[14] Our thanks to an anonymous referee for making this point.

[15] See, e.g., Pindyck (2012) and Stern (2007).

detonation of one or several nuclear weapons (with the yield of the Hiroshima bomb) in major cities. At the high end, Allison (2004) puts the probability of this occurring in the next *ten* years at about 50%! Others put the probability for the next ten years at 1 to 5%. For a survey, see Ackerman and Potter (2008). What would be the impact? Possibly a million or more deaths. But the main shock to the capital stock and GDP would be a reduction in trade and economic activity worldwide, as vast resources would have to be devoted to averting further events.

Megaviruses: Numerous authors view major pandemics and plagues (including bioterrorism) as both likely and having a catastrophic impact, but do not estimate probabilities. For a range of possibilities, see Byrne (2008). As with nuclear terrorism, the main shock to GDP would be a reduction in trade, travel, and economic activity worldwide.

Other Catastrophic Risks: Less likely, but certainly catastrophic, events include nuclear war, gamma ray bursts, an asteroid hitting the Earth, and unforeseen consequences of nanotechnology. For an overview, see Bostrom and Ćirković (2008).

In concluding, some caveats are clearly in order. Our model is intentionally simple and stylized. For example, we solved the social planner's problem for a representative firm with a simple AK production technology and adjustment costs, and a representative household with rational expectations. This is equivalent to a competitive equilibrium with a large number of identical firms and identical households, with the same production technology and preferences, so that we ignore heterogeneity among firms and households. And while our calibrations fit the basic economic aggregates, we do not statistically test the model. We also characterize catastrophic events in a simple way — a Poisson arrival with a constant mean arrival rate, and a permanent impact, the size of which is stochastic. On the other hand, these simplifications make the model highly tractable, and provide insight into the questions we raised in the Introduction. We believe that our main results and insights are robust to these simplifying assumptions.

Appendix

A. Solution of Model.

Substituting the conjectured value function (10) into the consumption FOC (9) yields:

$$\rho C^{-\psi^{-1}} \frac{1}{(bK)^{(1-\gamma)(\omega-1)}} = \phi'(i)(bK)^{-\gamma} b \ . \tag{40}$$

Simplifying and using $c = C/K$, we have

$$c = \left(\frac{\rho}{\phi'(i)}\right)^{\psi} b^{1-\psi} \ . \tag{41}$$

Substituting (41) back into the HJB eqn. (8) yields eqn. (12) for the optimal i^*.

From Duffie and Epstein (1992), the stochastic discount factor (SDF), $\{M_t : t \geq 0\}$, is

$$M_t = \exp\left[\int_0^t f_V(C_s, V_s)\, ds\right] f_C(C_t, V_t) \ . \tag{42}$$

From the equilibrium allocation results,

$$\begin{aligned} f_C(C,V) &= \phi'(i^*) b^{1-\gamma} K^{-\gamma} \,, && (43) \\ f_V(C,V) &= -h \,, && (44) \end{aligned}$$

where

$$h = -\frac{\rho(1-\gamma)}{1-\psi^{-1}} \left[\left(\frac{c^*}{b}\right)^{1-\psi^{-1}} \left(\frac{\psi^{-1}-\gamma}{1-\gamma}\right) - 1\right]. \tag{45}$$

Using the equilibrium relation between b and c^*, we can simplify the above as follows:

$$h = \rho + \left(\psi^{-1} - \gamma\right) \left[\phi(i) - \frac{\gamma\sigma^2}{2} - \lambda \mathcal{E}\left(\frac{1-Z^{1-\gamma}}{1-\gamma}\right)\right]. \tag{46}$$

Using Ito's lemma and the equilibrium allocation, we have

$$\frac{1}{M_{t-}} dM_t = -hdt - \gamma\left[\phi(i^*)dt + \sigma dW_t\right] + \frac{\gamma(\gamma+1)}{2}\sigma^2 dt + \left(Z^{-\gamma} - 1\right) dJ_t \ . \tag{47}$$

The equilibrium restriction that the expected rate of change of M_t must equal $-r_t$ implies the following formula for the equilibrium interest rate:

$$r = h + \gamma\phi(i^*) - \frac{\gamma(\gamma+1)\sigma^2}{2} - \lambda\mathcal{E}\left(Z^{-\gamma} - 1\right) \ . \tag{48}$$

Let $Q(K)$ denote the value of the capital stock and q denote Tobin's q. By homogeneity, $Q(K) = qK$. The equilibrium dividend is then $D_t = C_t$ for all t. The standard valuation methodology implies that $M_t D_t dt + d(M_t Q_t)$ has an instantaneous drift of zero. Using Ito's lemma and simplifying yields an equation for q:

$$\frac{c^*}{q} = \rho - \left(1-\psi^{-1}\right)\phi(i^*) + \frac{\gamma(1-\psi^{-1})\sigma^2}{2} + \frac{\lambda}{1-\gamma}\mathcal{E}\left[\left(\psi^{-1}-1\right)\left(Z^{1-\gamma}-1\right)\right]. \tag{49}$$

Using (41) and $q = 1/\phi'(i^*)$, we can write the above equation as:

$$b = \rho\left[1+\left(\frac{\psi^{-1}-1}{\rho}\right)\widehat{g}\right]^{\frac{1}{1-\psi}} q\ , \tag{50}$$

where $\widehat{g}$ is defined as follows,

$$\widehat{g} = g - \frac{\gamma\sigma^2}{2} - \frac{\lambda}{1-\gamma}\mathcal{E}\left(1-Z^{1-\gamma}\right). \tag{51}$$

The expected rate of return on equity is then

$$r^e = \rho + \psi^{-1}\phi(i^*) - \frac{\gamma(\psi^{-1}-1)\sigma^2}{2} + \lambda\mathcal{E}\left(Z-1\right) + \frac{\lambda}{1-\gamma}\mathcal{E}\left[\left(\psi^{-1}-1\right)\left(Z^{1-\gamma}-1\right)\right]. \tag{52}$$

Therefore, the aggregate risk premium rp is given by

$$rp = r^e - r = \gamma\sigma^2 + \lambda\mathcal{E}\left[(Z-1)\left(1-Z^{-\gamma}\right)\right] . \tag{53}$$

B. Moments of Equity Returns.

Here we derive eqns. (23), (24) and (25) for the moments of equity returns. Let $k = \ln K$ and let $F(t;x,k_t)$ denote the characteristic function: $F(t;x,k_t) = \mathcal{E}_t(e^{ixk_T})$. From (14),

$$dk_t = \widehat{\alpha}dt + \sigma dB_t - (1-z)dJ_t\,, \tag{54}$$

where $z = \ln Z$ and $\widehat{\alpha} = (\alpha - \frac{1}{2}\sigma^2)$. Then $F(t;x,k)$ satisfies the differential equation:

$$\widehat{\alpha}F_k + \frac{\sigma^2}{2}F_{kk} + F_t + \lambda\mathcal{E}\left[F(k+z) - F(k)\right] = 0\,. \tag{55}$$

We conjecture that

$$F(t,x,k) = \exp\left[ikG(t,x) + H(t,x)\right]. \tag{56}$$

Note that $G(T,x) = x$ and $H(T,x) = 0$. Substituting (56) for $F(t,x,k)$ into (55) gives

$$\widehat{\alpha}iG(t,x) - \tfrac{1}{2}\sigma^2 G(t,x)^2 + \left[ikG_t(t,x) + H_t(t,x)\right] + \lambda\mathcal{E}\left[e^{izG(t,x)} - 1\right] = 0\,. \tag{57}$$

For the term $ikG_t(t,x)$ we have $G_t(t,x) = 0$ and $G(T,x) = x$, so $G(t,x) = x$. For the remaining terms,

$$\widehat{\alpha}ix - \tfrac{1}{2}\sigma x^2 + H_t(t,x) + \lambda\mathcal{E}\left[e^{izx} - 1\right] = 0\,. \tag{58}$$

The solution is

$$F(t,x,k_t) = \exp\left[ixk + H(t,x)\right] , \tag{59}$$

where $H(t,x)$ is given by

$$H(t,x) = \left[\widehat{\alpha} ix - \tfrac{1}{2}\sigma^2 x^2 + \lambda \mathcal{E}\left(e^{izx} - 1\right)\right](T-t) . \tag{60}$$

Let $\nu_n = \mathcal{E}(z^n)$, and note that

$$\begin{aligned} H_x(t,0) &= i[\widehat{\alpha} + \lambda\mathcal{E}(z)](T-t) && (61)\\ H_{xx}(t,0) &= -(\sigma^2 + \lambda\nu_2)(T-t) && (62)\\ H_{xxx}(t,0) &= -i\lambda\nu_3(T-t) && (63)\\ H_{xxxx}(t,0) &= \lambda\nu_4(T-t) && (64)\end{aligned}$$

Using k_t and k interchangeably, the characteristic function of $k_T - k_t$ is

$$F^*(t,k,x) = \mathcal{E}_t\left(e^{ix(k_T - k_t)}\right) = e^{-ixk}F(k,x) = e^{H(t,x)} . \tag{65}$$

Let m_n denote the nth (uncentered) moment of $k_T - k_t$. Then

$$m_n = \mathcal{E}_t(k_T - k_t)^n = \frac{1}{i^n}\frac{\partial^n F^*}{\partial x^n}\bigg|_{x=0} . \tag{66}$$

We have

$$im_1 = \frac{\partial F^*}{\partial x}\bigg|_{x=0} = H_x(t,0) = i(T-t)[\widehat{\alpha} + \lambda\mathcal{E}(z)] , \tag{67}$$

which implies $m_1 = (T-t)(\widehat{\alpha} + \lambda\nu_1)$. Similarly,

$$\begin{aligned} i^2 m_2 = \frac{\partial^2 F^*}{\partial x^2}\bigg|_{x=0} &= H_x(t,0)^2 + H_{xx}(t,0) && (68)\\ &= i^2 m_1^2 + \left[-\sigma^2 + \lambda\mathcal{E}(i^2 z^2)\right](T-t) && (69)\end{aligned}$$

which implies $m_2 = m_1^2 + (\sigma^2 + \lambda\nu_2)(T-t)$. Therefore, the conditional variance of $k_T - k_t$ is

$$m_2 - m_1^2 = (\sigma^2 + \lambda\nu_2)(T-t) . \tag{70}$$

Similarly,

$$i^3 m_3 = \frac{\partial^3 F^*}{\partial x^3}\bigg|_{x=0} = H_{xxx}(t,0) + 3H_{xx}(t,0)H_x(t,0) + H_x(t,0)^3 , \tag{71}$$

so $m_3 = 3(m_2 - m_1^2)m_1 + m_1^3 + \lambda\nu_3(T-t)$, and the conditional skewness of $k_T - k_t$ is

$$\frac{m_3 - (3(m_2 - m_1^2)m_1 + m_1^3)}{(m_2 - m_1^2)^{3/2}} = \frac{1}{\sqrt{T-t}}\left[\frac{\lambda\nu_3}{(\sigma^2 + \lambda\nu_2)^{3/2}}\right] . \tag{72}$$

For the fourth moment,

$$i^4 m_4 \quad = \frac{\partial^4 F^*}{\partial x^4}\bigg|_{x=0} = \tag{73}$$
$$H_{xxxx}(t,0) + 4H_{xxx}(t,0)H_x(t,0) + 6H_{xx}(t,0)H_x(t,0)^2 + 3H_{xx}(t,0)^2 + H_x(t,0)^4\,,$$

which implies $m_4 = \lambda\nu_4(T-t) \; + 4m_3m_1 - 6m_2m_1^2 + 3m_1^4 + 3(m_2 - m_1^2)^2$. Therefore, the conditional kurtosis of $k_T - k_t$ is

$$\frac{m_4 - (4m_3m_1 - 6m_2m_1^2 + 3m_1^4)}{(m_2 - m_1^2)^2} = 3 + \frac{1}{T-t}\left[\frac{\lambda\nu_4}{(\sigma^2 + \lambda\nu_2)^2}\right]. \tag{74}$$

For the excess kurtosis, subtract 3 from the RHS of (74). Eqns. (23) to (25) follow directly, with $\Delta t = T - t$. Note that with annual data and returns measured on an annual basis, $\Delta t = T - t = 1$. With monthly data and annualized returns, $\Delta t = T - t = 1/12$.

C. Consumption Tax.

We will show that in our model a permanent consumption tax is non-distortionary, so the social planner's solution coincides with the competitive market equilibrium. We thus proceed by solving the social planner's problem.

With a tax and a truncated distribution whose probability density function is given by eqn. (35), the first-order condition (FOC) for consumption in the planner's problem is

$$(1-\tau)f_C(C,V) = \phi'(i)\widehat{V}'(K)\,. \tag{75}$$

Note that the original non-truncated distribution $\zeta(\,\cdot\,)$ is a special case with $\widehat{Z} = 0$.

Consider a tax to permanently limit the maximum loss from any catastrophic shock that might occur to some level $\widehat{L}$. We conjecture that $\widehat{V}(K;\tau)$, the value function for given values of τ, has the homothetic form:

$$\widehat{V}(K;\tau) = \frac{1}{1-\gamma}\left(\widehat{b}(\tau)K\right)^{1-\gamma}, \tag{76}$$

where $\widehat{b}(\tau)$ measures certainty-equivalent wealth (per unit of capital) when consumption is permanently taxed at rate τ. Let $\widehat{V}(K;0)$ and $\widehat{b}(0)$ denote the corresponding quantities in the absence of a tax as in Section 2. Let $\widehat{c} = (1-\tau)(A-\widehat{i})$ denote the after-tax consumption-capital ratio with truncated distribution (35). Substituting $\widehat{V}(K;\tau)$ given by (76) into the FOC (75) yields:

$$\widehat{c} = \left(\frac{(1-\tau)\rho}{\phi'(\widehat{i})}\right)^{\psi} \widehat{b}(\tau)^{1-\psi}\,. \tag{77}$$

Substituting (77) for $\widehat{i} = A - \widehat{c}/(1-\tau)$ into the Bellman eqn. (8) and simplifying, we can write the equilibrium consumption-capital ratio $\widehat{c}^*(\tau)$ as:

$$\widehat{c}^*(\tau) = \frac{1-\tau}{\phi'(\widehat{i}^*)}\left[\rho + (\psi^{-1} - 1)\left(\phi(\widehat{i}^*) - \frac{\gamma\sigma^2}{2} + \frac{\lambda}{1-\gamma}\widehat{\mathcal{E}}\left(1 - Z^{1-\gamma}\right)\right)\right]. \tag{78}$$

Using the identity $\widehat{c}^* = (1-\tau)\left(A - \widehat{i}^*\right)$, the optimal investment-capital ratio i^* solves:

$$A - \widehat{i}^* = \frac{1}{\phi'(\widehat{i}^*)}\left[\rho + (\psi^{-1} - 1)\left(\phi(\widehat{i}^*) - \frac{\gamma\sigma^2}{2} + \frac{\lambda}{1-\gamma}\widehat{\mathcal{E}}\left(1 - Z^{1-\gamma}\right)\right)\right]. \tag{79}$$

Rewriting (77) and using $\widehat{i}^*$ from solving (79), we have the following equation for $\widehat{b}(\tau)$:

$$\widehat{b}(\tau) = (\widehat{c}^*)^{1/(1-\psi)}\left[\frac{(1-\tau)\rho}{\phi'(\widehat{i}^*)}\right]^{-\psi/(1-\psi)}. \tag{80}$$

Therefore,

$$\widehat{b}(\tau) = (1-\tau)^{1/(1-\psi)}(A - \widehat{i}^*)^{1/(1-\psi)}(1-\tau)^{-\psi/(1-\psi)}\left[\frac{\rho}{\phi'(\widehat{i}^*)}\right]^{-\psi/(1-\psi)} = (1-\tau)\widehat{b}(0),$$

where the last equality follows from (79), in that $\widehat{i}$ is independent of tax rate τ. We have shown that:

$$\widehat{b}(\tau) = (1-\tau)\widehat{b}(0)\ , \tag{81}$$

where

$$\widehat{b}(0) = (A - \widehat{i}^*)^{1/(1-\psi)}\left[\frac{\rho}{\phi'(\widehat{i}^*)}\right]^{-\psi/(1-\psi)} \tag{82}$$

and the equilibrium i^* (without a tax) solves (79) with the truncated distribution given by (35). That is, the aggregate investment-capital ratio, aggregate output, and the aggregate capital stock all remain unchanged with respect to taxes, but not distribution $\widehat{\zeta}(\,\cdot\,)$.

D. Data and Inputs to Calibration.

Unless otherwise indicated, National Income and Product data are from the Dept. of Commerce (www.bea.gov/national/nipaweb), data on fixed reproducible assets are from the Federal Reserve's Flow of Funds (www.federalreserve.gov/releases/), data on T-Bill rates and the CPI are from the Federal Reserve, and returns on the S&P 500 are from Robert Shiller (www.econ.yale.edu/˜ shiller). The data are for the period January 1947 to December 2008. Inputs to the calibration are measured or calculated as follows. (All data and calculations are in a spreadsheet available from the authors on request.)

Capital Stock. Our measure of the total capital stock K_T has three components: physical capital K_P, human capital K_H, and intangible capital held by firms K_I. Physical capital, from the Fed's Flow of Funds data, consists of fixed reproducible assets, including those held by federal and state and local governments. To estimate the stock of human capital, we use an approach suggested by Mankiw, Romer, and Weil (1998), and take the wage premium (the average wage minus the minimum wage) to be the return to human capital. We also assume that physical and human capital earn the same rate of return. Thus the total annual return to human capital is the wage premium as a fraction of the average wage (about .60 on average) times total compensation of employees. To get the rate of return on physical capital, we use total capital income (corporate profits including the capital consumption adjustment, i.e., gross of depreciation, plus rental income, plus proprietors' income) as a fraction of the stock of physical capital. That rate of return (about 7%) is used to capitalize the annual return to human capital.[16] For the stock of intangible capital, we use a weighted average of McGrattan and Prescott's (2005) estimates of the intangible capital stock as a fraction of GDP for 1960–69 and 1990–2001. The result is $K_I = .68Y$. Given annual values for K_P, K_H, and K_I, we calculate annual values for $A = Y/K_T$ and use the average value of A (0.121) as an input to our calibration.

Investment. We need total investment, inclusive of investment in intangible capital, to measure the consumption-investment ratio C/I. In equilibrium, investment in intangible capital is given by $I_I = (\delta_I + g)K_I$, where δ_I is the depreciation rate for intangible capital and g is the real GDP growth rate (.02). The BEA's estimate of the depreciation rate on R&D is 11%, but McGrattan and Prescott (2005) argue that this rate is too high for most non-R&D intangible capital. McGrattan and Prescott (2010) estimate the depreciation rate for intangible capital to be 8%, which is the rate we use. Thus $I_I = .10K_I = .068Y$.[17] Adding this to investment in physical capital yields a consumption-investment ratio of 2.84. We assume here that investment in human capital occurs through education and is thus part of measured consumption.

Real Risk-Free Interest Rate and Equity Returns. For the real risk-free rate, we use monthly data on the nominal 3-month T-bill rate net of the percentage increase in the CPI for all items. Averaging over the (annualized) monthly numbers yields $r = 0.008$. For the equity risk premium, we use the monthly total value-weighted return (capital gains plus dividends) on the S&P500, from CRSP, and subtract the nominal 3-month T-bill rate. Averaging over the annualized monthly numbers yields $rp = 0.066$. For the variance, skewness, and kurtosis of equity returns, we again use the monthly total return on the S&P500, net of the percentage increase in the CPI.

Real GDP Growth Rate. We use real GDP and population data from the Bureau of Economic Analysis to compute the annual growth rate of real per-capita GDP. Averaging over these annual growth rates yields $\overline{g} = .020$.

[16]For comparison, we also calculated the stock of human capital using the results in Jones, Manuelli, Siu, and Stacchetti (2005), who estimated investment in human capital as a fraction of GDP. Assuming the depreciation rates for human and physical capital are the same, the equilibrium stocks will be in proportion to the investment levels. We obtained similar results (to within 15%) for the stock of human capital.

[17]This is within the range of the Corrado et al. (2005) estimates of investment in intangible capital.

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Appendix E. Decentralized Market Solution.
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Here, we provide the decentralized market equilibrium solution. First, we find the representative consumer's optimal consumption, portfolio choice and CIS demand. Second, we turn to firm value maximization taking prices as given. Finally, we conjecture and verify equilibrium prices and resource allocation.

Consumer Optimality. Let X denote the consumer's total marketable wealth and π the fraction allocated to the market portfolio. For catastrophe with recovery fraction in $(Z, Z+dZ)$, $\xi_t(Z)X_t dt$ gives the total demand for the CIS over time period $(t, t+dt)$. The total CIS premium payment in the time interval $(t, t+dt)$ is then $\left(\int_0^1 \xi_t(Z)p(Z)dZ\right) X_t dt$.

We conjecture that the cum-dividend return of the market portfolio is given by

$$\frac{dQ_t + D_t dt}{Q_{t-}} = \mu dt + \sigma dW_t - (1-Z)dJ_t\,, \tag{83}$$

where μ is the expected return on the market portfolio (including dividends) but without the effects of catastrophic risk (and will be determined in equilibrium). When a catastrophe occurs, the consumer's wealth changes from X_{t-} to X_t as follows:

$$X_t = X_{t-} - (1-Z)\pi_{t-}X_{t-} + \xi_{t-}(Z)X_{t-}\,. \tag{84}$$

The consumer's wealth accumulation is then given by

$$\begin{aligned} dX_t &= r\left(1-\pi_{t-}\right)X_{t-}dt + \mu\pi_{t-}X_{t-}dt + \sigma\pi_{t-}X_{t-}dW_t - C_{t-}dt \\ &\quad - \left(\int_0^1 \xi_{t-}(Z)p(Z)dZ\right)X_{t-}dt + \xi_{t-}(Z)X_{t-}dJ_t - (1-Z)\pi_{t-}X_{t-}dJ_t\,. \end{aligned} \tag{85}$$

The first four terms in (85) are standard in classic portfolio choice problems (with no insurance or catastrophes). The last three terms capture the effects of catastrophes on wealth accumulation. The fifth term is the total CIS premium paid before any catastrophe. The sixth term gives the CIS payments by the seller to the buyer when a catastrophe occurs. The last term is the loss of consumer wealth from exposure to the market portfolio.

The HJB equation for the consumer in the decentralized market setting is given by[18]

$$0 = \max_{C,\pi,\xi(\,\cdot\,)} \left\{ f(C,J) + \left[rX\left(1-\pi\right) + \mu\pi X - \left(\int_0^1 \xi(Z)p(Z)dZ\right)X - C\right] J'(X) \right.$$

[18] In writing the HJB equation (8), we use the result that the "normalized" aggregator as defined and derived by Duffie and Epstein (1992) applies to our setting with both jumps and a diffusion. See Benzoni, Collin-Dufresne, and Goldstein, "Explaining Asset Pricing Puzzles Associated with the 1987 Market Crash," Columbia Univ. working paper, 2010.

$$+\frac{1}{2}\sigma^2\pi^2X^2J''(X)+\lambda\mathcal{E}\left[J(X-(1-Z)\pi X+\xi(Z)X)-J(X)\right]\Big\} \ . \quad (86)$$

The FOCs for consumption C, market portfolio allocation as a fraction π of total wealth X, and the CIS demand $\xi(Z)$ for each Z are respectively:

$$f_C(C,J) = J'(X) \quad (87)$$

$$(\mu-r)XJ'(X) = -\sigma^2\pi X^2J''(X)+\lambda\mathcal{E}\left[(1-Z)J'(X-(1-Z)\pi X+\xi(Z)X)\right] \quad (88)$$

$$0 = -Xp(Z)J'(X)+\lambda X\left[J'(X-(1-Z)\pi X+\xi(Z)X)\right]f_Z(Z)\,. \quad (89)$$

The last FOC follows from the point-by-point optimization in (86) for the CIS demand and hence it holds for all levels of Z. Now conjecture that the consumer's value function is

$$J(X)=\frac{1}{1-\gamma}(uX)^{1-\gamma}\,, \quad (90)$$

where u is a constant to be determined. Using the consumption FOC (87) and the conjectured value function (90), we obtain the following linear consumption rule:

$$C=\rho^{\psi}u^{1-\psi}X\,. \quad (91)$$

Imposing the equilibrium outcome in which (1) $\pi=1$; (2) $\xi(Z)=0$ for all Z; and (3) the consumer's wealth equals the total value of the market portfolio, $X=Q$, we obtain:

$$0 = (\mu-r)J'(Q)+\sigma^2QJ''(Q)-\lambda\mathcal{E}\left[(1-Z)J'(ZQ)\right] \quad (92)$$

$$p(Z) = \lambda\frac{J'(ZQ)}{J'(Q)}f_Z(Z) \quad (93)$$

Using these equilibrium conditions, we can simplify the HJB equation as follows:

$$\begin{aligned}0 \;=\; & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{\rho}{u}\right)^{\psi-1}-1\right]u^{1-\gamma}X^{1-\gamma}+\left(\mu-\rho^{\psi}u^{1-\psi}\right)(uX)^{1-\gamma}-\frac{\gamma}{2}\sigma^2(uX)^{1-\gamma}\\ & +\lambda\mathcal{E}\left[Z^{1-\gamma}-1\right]\frac{1}{1-\gamma}(uX)^{1-\gamma}\end{aligned} \quad (94)$$

Eqn. (91) implies $c=\rho^{\psi}u^{1-\psi}q$ under the equilibrium condition $X=Q=qK$. Substituting $c=\rho^{\psi}u^{1-\psi}q$ into (94), we obtain

$$0 = \frac{1}{1-\psi^{-1}}\left(\frac{c}{q}-\rho\right)+\left(\mu-\frac{c}{q}\right)-\frac{\gamma}{2}\sigma^2+\lambda\mathcal{E}\left[Z^{1-\gamma}-1\right]\frac{1}{1-\gamma} \quad (95)$$

Firm Value Maximization. We assume financial markets are perfectly competitive and M-M holds. While the firm can hold financial positions (e.g., CIS contracts), equilibrium pricing implies that there is no value in doing so. We can thus ignore financial contracts and only focus on investment I when maximizing firm value, which is independent of financing. Taking the unique stochastic discount factor (SDF) implied by the equilibrium consumption process as given, the firm maximizes its value by choosing I to solve:

$$\max_{I} \mathcal{E}\left[\int_0^\infty \frac{M_s}{M_0}\left(AK_s - I_s\right) ds\right], \tag{96}$$

subject to capital accumulation, the production technology, and the transversality condition.

Using the homogeneity property of our model, we conjecture that the SDF is given by a geometric Brownian motion with constant drift, constant volatility and proportional jump for each possible recovery fraction Z, i.e.

$$dM_t = -rM_{t-}dt - \eta M_{t-}dW_t + M_{t-}\left[\left(Z^{-\gamma}-1\right)dJ_t - \lambda\mathcal{E}\left(Z^{-\gamma}-1\right)dt\right]. \tag{97}$$

The second and the third terms capture diffusion and catastrophic risk respectively. Both terms are martingales. Note that to make the catastrophe term a martingale, we must subtract the expected change of M due to all possible catastrophes. Finally, the first term gives the equilibrium drift of M, which must be $-rM_t$ from the no-arbitrage condition.

No arbitrage implies the drift of $M_t(AK_t - I_t)dt + d\left(M_tQ_t\right)$ is zero. From Ito's Lemma we have the following dynamics for $Q(K)$:

$$dQ(K) = \left(\Phi(I,K)Q_K + \frac{1}{2}Q_{KK}\sigma^2K^2\right)dt + \sigma KQ_KdW_t + \left(Q(ZK) - Q(K)\right)dJ_t\,. \tag{98}$$

Again using Ito's Lemma, we have

$$\begin{aligned}&M_{t-}(AK-I)dt + M_{t-}\left(Q_K\Phi(I,K)dt + \frac{1}{2}\sigma^2K^2Q_{KK}dt\right) + Q\left[-r-\lambda\mathcal{E}\left(Z^{-\gamma}-1\right)\right]M_{t-}dt\\ &-\eta M_{t-}\sigma KQ_Kdt + \lambda\mathcal{E}\left(Z^{-\gamma}Q(ZK)-Q(K)\right)M_{t-}dt = 0\,.\end{aligned} \tag{99}$$

Simplifying the above, we have

$$\begin{aligned}\left[r+\lambda\mathcal{E}\left(Z^{-\gamma}-1\right)\right]Q(K) \;&=\; (AK-I) + Q_K\left(\Phi(I,K)-\eta\sigma K\right) + \frac{1}{2}\sigma^2K^2Q_{KK}\\ &\quad +\lambda\mathcal{E}\left(Z^{-\gamma}Q(ZK)-Q(K)\right).\end{aligned} \tag{100}$$

The FOC with respect to investment is therefore

$$1 = \Phi_I(I,K)Q_K\,. \tag{101}$$

Using the homogeneity assumption, we conjecture that firm value is $Q(K) = qK$, where Tobin's q is to be determined. We can thus simplify (100) as follows:

$$\left[r+\lambda\mathcal{E}\left(Z^{-\gamma}-1\right)\right]q = (A-i) + q\left(\phi(i)-\eta\sigma\right) + \lambda\mathcal{E}\left(Z^{1-\gamma}-1\right)q\,. \tag{102}$$

The equilibrium dynamic for firm value Q_t is then given by

$$dQ_t = gQ_{t-}dt + \sigma Q_{t-}dW_t - (1-Z)\, Q_{t-}dJ_t \,. \tag{103}$$

where $g = \phi(i)$ is the expected growth without the effects of catastrophes.

The FOC (101) can be simplified as follows:

$$q = \frac{1}{\phi'(i)} \,. \tag{104}$$

Market Equilibrium. We now verify that the conjectured prices and quantities are consistent with equilibrium market outcomes, and replicate the six key equations (16)-(31) in the text. First, eqn. (16) follows immediately from the goods market clearing condition, $Y = C + I$, and the homogeneity property. Second, eqn. (17) is the FOC for the producer under homogeneity. Third, we obtain eqn. (18) for consumption by comparing the dynamics for firm value on the consumer and firm sides, (83) and (103), to obtain the restriction:

$$\mu = \phi(i) + \frac{c}{q} \,. \tag{105}$$

The expected rate of return (without catastrophes) is $\phi(i)$ plus the dividend yield, which is also the consumption-wealth ratio. Substituting (105) into (95) gives eqn. (18).

Fourth, using the equilibrium consumption and evaluating the SDF via (42), we obtain the equilibrium interest rate r given by (19) and the equilibrium market price of diffusion risk $\eta = \gamma\sigma$. Note that the implied interest rate is also consistent with eqn (102).

Fifth, simplifying (92), we have the following result:

$$0 \;=\; (\mu - r) - \gamma\sigma^2 - \lambda\mathcal{E}\left[Z^{-\gamma}(1-Z)\right] \,. \tag{106}$$

Adding the expected loss due to the catastrophic risk, we obtain the following formula for the equity risk premium rp:

$$rp = \mu + \lambda\mathcal{E}(1-Z) - r = \gamma\sigma^2 + \lambda\mathcal{E}\left[(1-Z)\left(Z^{-\gamma} - 1\right)\right] \,, \tag{107}$$

which is eqn. (20). Finally, substituting (90) into (93) gives the CIS insurance premium $p(Z)$ of eqn. (31). We have verified that the conjectured equilibrium is indeed consistent with the social planner's solution.