Investor Protection and Asset Pricing*

Rui Albuquerque† Neng Wang‡

September 7, 2004

Abstract
Corporations in most countries are run by controlling shareholders, who have substantially smaller cash flow rights than their control rights in the firm. This separation of ownership and control allows the controlling shareholders to pursue private benefits at the cost of outside minority investors by diverting resources away from the firm and distorting corporate investment and payout policies. We develop a dynamic stochastic general equilibrium asset pricing model that acknowledges the implications of agency conflicts through imperfect investor protection on security prices. We show that countries with weaker investor protection have lower market to book equity values, larger expected equity returns and return volatility, and higher interest rates. These predictions are consistent with empirical findings. We develop a new prediction that for relative risk aversion larger than unity countries with weaker investor protection have larger dividend yields. Finally, the utility cost of weak investor protection is shown to be economically large.

JEL Classification: G12, G31, G32, G34.
Keywords: asset prices, agency, corporate governance, investor protection.

---

*We thank Andrea Buraschi, Mike Barclay, Bob Hall, John Long, Ross Watts and seminar participants at ISCTE, Faculdade de Economia do Porto, Portuguese Catholic University, and the 2004 SED conference in Florence for comments.
†Simon School of Business, University of Rochester, Rochester, NY 14627. Email: albuquerque@simon.rochester.edu.
‡Columbia Business School, 3022 Broadway, Uris Hall 812, New York, NY 10027. Email: nw2128@columbia.edu.
1 Introduction

Separation of corporate control from ownership is one of the main features of modern capital markets. Among its many virtues, it allows the participation of small investors in the equity market, increasing the supply of funds, dissipating risks across the economy, and lowering the cost of capital for firms. Its biggest drawback is the agency conflict between corporate insiders who run the firm and can extract private benefits of control, and outside minority investors who have cash flow rights on the firm, but no control rights (e.g. Berle and Means (1932) and Jensen and Meckling (1976)). This agency conflict is the focus of a voluminous body of research in corporate finance. Recurrent corporate scandals constitute a stern reminder of the existence of these conflicts and of the private benefits exploited by insiders even in the least suspicious markets.

Following Shleifer and Vishny (1997) and La Porta et al. (1998), a large empirical literature has firmly established the existence of large shareholders in many corporations around the world (La Portal et al. (1999)). These shareholders have much larger control rights within the firm compared with their cash flow rights as they obtain effective control through dual class shares, pyramid ownership structures, or cross ownership (Bebchuk et al. (2000)). With the separation of control from ownership, controlling shareholders have the incentives to expropriate outside minority shareholders. This conflict of interests is at the core of agency conflicts in most countries and is only partially remedied by regulation aimed at protecting minority or outside investors. There is considerable empirical evidence that stock market prices reflect the magnitude of these private benefits derived by controlling shareholders. Firm value increases with the extent of protection of minority investors, and with the stock ownership of controlling shareholders. While it is intuitive that weak investor protection lowers equity prices, the effect of investor protection on equity returns and the interest rate is less obvious conceptually.

In this paper, we provide a model to study the effect of imperfect investor protection on equilibrium asset pricing. We depart from traditional production (investment) based equilibrium asset pricing models in two important ways. First, we acknowledge that the controlling shareholder makes the firm investment decisions in his own interest, which naturally differ from firm value maximization. Second, we embed the separation of ownership and control into an equilibrium asset pricing model in which both the controlling shareholder and outside investors optimize their consumption and asset allocations.

---

1 For empirical work see La Porta et al. (1999, 2002), Claessens et al. (2002), and Baek et al. (2004). For theoretical work see La Porta et al. (2002), Shleifer and Wolfenzon (2002), and Lan and Wang (2004). See La Porta et al. (2000b) for a survey of the investor protection literature.
Hence, the equilibrium asset prices affect the investment and payout decisions of the controlling shareholder through his preference to smooth consumption over time and, in turn, these investment and payout decisions also affect the equilibrium asset prices.

The controlling shareholder chooses consumption and the risk-free asset holdings in addition to firm investment and payout policies to maximize his lifetime utility. The trade-offs associated with the corporate investment decision in our model differs from the standard value-maximizing one. First, the controlling shareholder’s marginal benefit to investment is his *private* marginal return to capital, net of the cost of extracting these benefits and net of depreciation. The controlling shareholder’s private return to capital is higher than the observed public return to capital, and in the model, as in the data (see Barclay and Holderness (1989)), the level of private benefits increases with firm size.\(^2\) Both of these features contribute to an overinvestment incentive by the controlling shareholder relative to a world with perfect investor protection, in line with Jensen’s (1986) free-cash flow and empire-building hypothesis.\(^3\) Second, the marginal cost to investment has two components, one being the traditional cost due to postponing consumption and the other resulting from an increased volatility in consumption. Motivated by empirical evidence, we assume that capital accumulation is stochastic with the volatility of shocks increasing in the level of investment. Intuitively, more investment generates on average more output in the future, but it also yields more volatile output. Therefore, our model predicts that weak investor protection generates overinvestment and a higher mean growth rate and volatility of output in the economy.

Minority investors solve an intertemporal consumption and portfolio choice problem à la Merton (1971) by taking dividends and security prices as given. Recall that under imperfect investor protection the controlling shareholder extracts private benefits from the firm’s assets in place. This reduces the value attached by minority shareholders to assets in place relative to the historical cost of these assets, i.e., Tobin’s \(q\) is lower under imperfect investor protection. Consistent with the empirical evidence cited above, improvements in investor protection alleviate the agency conflicts, reduce overinvestment, increase payouts, and increase the value of assets in place, i.e., Tobin’s \(q\).

Our main result is that the excess equity return is affected in equilibrium by the degree of investor protection. Weaker investor protection increases the riskiness of the stock to minority investors and thus the risk premium they charge to hold the shares. In

\(^2\)Intuitively, the controlling shareholder in charge of a conglomerate is more likely to fly private jets than the one heading a small firm.

\(^3\)For evidence on overinvestment see, for example, Lang, Stulz, and Walking (1991) and Harford (1999).
the model, the risk premium reflects the price attached by the minority investors to the uncertainty associated with the economy’s single shock to capital. A positive shock leads to a higher stock return and a higher dividend payment. Minority investors’ consumption increases and their marginal utility declines. This negative correlation between stock returns and marginal utility of consumption is larger in absolute terms when investor protection is weaker and the investment rate is higher. This is because the value of this correlation in equilibrium is tied to the volatility of the output, and a higher investment rate makes output more volatile. Thus, the risk premium is larger.

The model prediction on excess equity returns is consistent with the empirical evidence. Daouk, Lee, and Ng (2004) document that improvements in their index of capital market governance are associated with lower equity risk premia. Using the cross-country data on excess returns in Campbell (2003), we find that civil law countries, those with weaker investor protection (La Portal et al. (1998)), have a higher average excess equity returns than common law countries. Harvey (1995), Bekaert and Harvey (1997), and Bekaert and Urias (1999) show that emerging markets display higher volatility of returns and larger equity risk premia. Similarly, Erb et al. (1996) find that expected returns, as well as volatility, are higher when country credit risk is higher. Since emerging market economies have on average weaker corporate governance, this empirical evidence lends further support to our theory.

The model also predicts that countries with weaker investor protection not only have overinvestment, but also have higher interest rates. Overinvestment (associated with weak investor protection) implies a larger output in the future and intertemporal consumption smoothing motivates agents to finance current consumption by borrowing, leading to a higher current equilibrium interest rate. But, overinvestment also makes capital accumulation more volatile and implies a stronger precautionary saving effect, thus pushing down the current equilibrium interest rate. The former effect dominates for low values of the investment-capital ratio, implying that interest rates are higher under weaker investor protection. Using the interest rate data in Campbell (2003), we find that civil law countries, those with weaker investor protection (La Portal et al. (1998)), have higher average interest rates than common law countries.

The effect of investor protection on dividend yield depends on the elasticity of intertemporal substitution. When the elasticity of intertemporal substitution is smaller than unity, as most estimates indicate, the substitution effect is dominated by the income effect. Thus, the lower interest rate that results from stronger investor protection

If investors have logarithmic utility functions, the dividend yield is equal to the agents’ subjective
gives rise to a smaller demand for current consumption and thus a lower dividend yield, *ceteris paribus*.

Our model also quantifies how valuable a better corporate governance is to minority investors. A calibration of the model to the United States and South Korea indicates that minority investors in each country would be willing to give up a substantial part of their wealth to move to a world with perfect investor protection, *ceteris paribus*.

**Related Theoretical Literature**

Our model is cast in an agency-based asset pricing framework. This is in contrast with the majority of asset pricing models, which are constructed for pure exchange economies (Lucas (1978) and Breeden (1979)). Our approach also contrasts with the existing literature linking asset prices to physical investment decisions. Cox, Ingersoll and Ross (CIR) (1985) provide a theory of the equilibrium term structure of interest rates based on firm’s value maximization. Cochrane (1991) links the marginal rate of transformation to the cost of capital. These production-based asset pricing models abstract away from agency conflicts and hence do not generate any predictions on asset returns across countries that would result from variation in the quality of corporate governance. We incorporate the effect of agency costs on equilibrium asset prices. Obviously, our model also relates to the heterogeneous-agent equilibrium asset pricing literature.5

The papers more closely related to ours are Dow et al. (2003) and Gorton and He (2003). Dow et al. (2003) develop a model in which the manager has an empire building preference as in Jensen (1986) and wants to invest all of the firm’s free cash flow if possible. As a result, the shareholder needs to use some of the firm’s resources to hire auditors to constrain the manager’s empire building incentives. Unlike their paper, our paper is motivated by the empirical observation that managers in most countries around the world are often controlling shareholders who themselves have cash flow rights in the firm and trade off the gains from pursuing private benefits with the cost of decreasing their share of firm value. The critical determinant of this trade-off is the extent of investor protection, as convincingly documented by the large empirical research indicated above. Our model thus allows us to derive predictions on the cross-sectional relationship between asset prices and investor protection. A second major difference between Dow et al. (2003) and our paper is that Dow et al. (2003) assume that all firm claimants are discount rate and hence is unaffected by investor protection. This reflects the myopic nature of investors with logarithmic utility.

---

5 Asset pricing models with investor heterogeneity have mostly been worked out in the paradigm of endowment economies. Studies have analyzed heterogeneity in preferences, endowments, and beliefs. See Campbell (2003) for a recent survey.
identical. Thus, the manager partly decides on the cash flow paid to shareholders, but has no role in affecting the equilibrium discount factor. The equilibrium marginal rate of substitution of the representative consumer in their paper is determined by setting the representative consumer’s consumption to the dividend. Unlike their model, our paper explicitly incorporates the consumption and asset allocation decisions by both the controlling shareholders and outside minority investors. This brings us to the third main difference between the two models. In our model, the corporate investment decisions of the controlling shareholder are affected by the equilibrium security prices: the consumption and asset allocations decisions of all agents affect the equilibrium prices which then determine the willingness of the controlling shareholder to smooth consumption through the capital accumulation and payout policies of the firm.

Gorton and He (2003) present a model with heterogeneous investors where trading in the firm’s equity between the manager and the outside investors (and not a compensation contract) is used by the latter to give incentives to the manager to exert effort in production. The model is cast in a setup where unobservable managerial effort gives rise to a moral hazard problem. Intuitively, a higher equity share owned by the manager leads to higher effort and, conditionally, higher mean output. Hence, firm value increases. However, and in contrast with our model, there is a tendency for a higher interest rate (with higher managerial equity share), because the higher output being produced in the future generates a desire to increase consumption today. This outcome can only be an equilibrium if the current interest rate rises. In addition, in our model we allow trading in the risk-free asset and in any derivative security between the controlling shareholder and minority investors, whereas this is not possible in Gorton and He (2003). Shleifer and Wolfenzon (2002) and Castro, Rui, Clementi, and MacDonald (2004) also predict that countries with better investor protection have higher interest rates. Neither paper addresses properties of other asset prices.

The remainder of the paper is organized as follows. Section 2 presents the model and states the main theorem. Section 3 characterizes the equilibrium outcome and agent’s optimality conditions in detail. Section 4 presents the perfect investor protection benchmark and section 5 gives the model’s main predictions on interest rates, equity prices and returns. Section 6 provides a calibration and supplies quantitative predictions of the model. Section 7 concludes. The appendix contains the technical details and proofs for the propositions in the paper.
2 The Model

The economy is populated by two types of agents: controlling shareholders and identical minority investors. Each controlling shareholder operates a firm. All firms and their controlling shareholders are assumed to be identical and subject to the same shocks. Both types of agents have infinite horizons and time is continuous. Without loss of generality, we only need analyze the decision problems for a representative controlling shareholder and a representative outside minority investor. Let the total mass of both controlling shareholders and minority investors be unity.

Next, we describe the consumption and production sides of the economy, and the objectives and choice variables of both the controlling shareholder and the minority investors.

2.1 Setup

Production and Investment Opportunities. The firm is defined by a production technology. Let $K$ be firm’s capital stock process. We assume that $K$ evolves according to

$$dK(t) = (I(t) - \delta K(t))dt + \epsilon I(t) dZ(t),$$  
(1)

where $\epsilon > 0$, $\delta > 0$ is the depreciation rate, $Z(t)$ is a Brownian process, and $I(t)$ represents the firm’s gross investment. Gross investment is given by

$$I(t) = hK(t) - D(t) - s(t)hK(t).$$  
(2)

Gross investment $I$ equals gross output $hK$, minus the sum of dividends $D$, and the private benefits extracted by the controlling shareholder $shK$.\(^6\)

The production function has constant returns to scale and the capital accumulation process is stochastic with shocks proportional to gross investment $I$. This modelling device gives great tractability and has the simple interpretation that capital accumulation is part of the production process. It is also similar to the production process adopted in Cox, Ingersoll, and Ross (1985).\(^7\) The magnitude of private benefits extracted by the

\(^6\)As the controlling shareholder is sometimes the entrepreneur that started the firm it is likely that the value of productivity $h$ partly represents the controlling shareholder’s human capital.

\(^7\)Our choice of stochastic capital accumulation gives great tractability in our model. In Albuquerque and Wang (2004), we model production in a more conventional way by specifying that shocks affect output directly. The main results and intuition obtained using the two specifications are similar.
controlling shareholder depends on the degree of investor protection which we discuss below.\(^8\)

Next, we discuss the controlling shareholder’s objective and his decision variables. We are motivated by the large amount of empirical evidence around the world in delegating the firm’s decision making to the controlling shareholder.

**Controlling Shareholder.** The controlling shareholder is risk averse and has lifetime utility over consumption plans given by

\[
E \left[ \int_0^\infty e^{-\rho t} u(C_1(t))dt \right],
\]

where \(C_1\) denotes the flow of consumption of the controlling shareholder, and the period utility function is given by

\[
u(C) = \begin{cases} \frac{1}{1-\gamma} (C^{1-\gamma} - 1) & \gamma \geq 0, \gamma \neq 1 \\ \log C & \gamma = 1 \end{cases}.
\]

The rate of time preference is \(\rho > 0\), and \(\gamma\) is the coefficient of relative risk aversion. Throughout the paper, we use the subscripts “1” and “2” to index variables for the controlling shareholder and the minority investor, respectively.

The controlling shareholder owns a fixed fraction \(\alpha < 1\) of the firm’s shares. This ownership share gives him control over the firm’s investment and payout policies. In real economies, control rights generally differ from cash flows rights: a fraction of votes higher than that of cash flow rights can be obtained by either owning shares with superior voting rights, through ownership pyramids, cross ownership, or by controlling the board. We refer readers to Bebchuk et al. (2000) for details on how control rights can differ from cash-flow rights.\(^9\) For now, we treat \(\alpha\) as constant and non-tradable. This assumption is consistent with La Porta et al. (1999) who argue that the controlling shareholder’s

\(^8\)An alternative interpretation to our production formulation is that \(K\) is total productive capital stock; the sum of the firm’s tangible and intangible assets. The amount \(eI(t)dZ(t)\) then gives the value of intangible assets produced contemporaneously to the investment expenditures \(I\). This new formulation of the dynamics of capital is meant to capture the idea that “[f]irms produce productive capital by combining plant, equipment, new ideas, and organization” (Hall (2001)). Hall (2001) argues that securities markets record in their valuation of a firm not only the increases in physical capital but also the increases in intangible capital. This formulation of our model contrasts with previous production based asset pricing models that rely exclusively on the connection of firm returns to physical capital returns (e.g. CIR (1985) and Cochrane (1991)).

\(^9\)Giving all the control rights to a controlling shareholder is in line with evidence provided in La Porta et al. (1999) who document for many countries that the control of firms is often heavily concentrated in the hands of a founding family.
ownership share is extremely stable over time, but is not needed. In section 3.3, we allow the controlling shareholder to optimize over his ownership stake and show that the no trade outcome is indeed an equilibrium.\footnote{It is possible to endogenize the decision for the initial share ownership of the controlling shareholder. This is done in Shleifer and Wolfenzon (2002) in a static model and Lan and Wang (2004) in a dynamic setting. In these models weaker investor protection leads to a more concentrated ownership, \textit{ceteris paribus}.}

We assume that the controlling shareholder can only invest his wealth in the risk-free asset. Let $W_1$ denote the controlling shareholder’s tradable wealth. The risk-free asset holdings of the controlling shareholder are $B_1(t) = W_1(t)$. We assume that the controlling shareholder’s initial tradable asset holding is zero, in that $W_1(0) = 0$. Therefore, the controlling shareholder’s tradable wealth $W_1(t)$ evolves as follows:

$$dW_1(t) = [r(t)W_1(t) + M(t) - C_1(t)] \, dt,$$

where $M(t)$ is the flow of goods which the controlling shareholder obtains from the firm, either through dividend payments $\alpha D(t)$ or through private benefits:\footnote{See Barclay and Holderness (1989) for early work on the empirical evidence in support of private benefits of control. See also Dyck and Zingales (2004) and Bae, Kang and Kim (2002).}

$$M(t) = \alpha D(t) + s(t)hK(t) - \Phi(s(t), hK(t)).$$

Private benefits of control are modeled as a fraction $s(t)$ of gross output $hK(t)$, with $h > 0$ being the productivity of capital. Expropriation is costly to both the firm and the controlling shareholder and, \textit{ceteris paribus}, for the controlling shareholder pursuing private benefits is more costly when investor protection is stronger. If the controlling shareholder diverts a fraction $s$ of the gross revenue $hK(t)$, then he pays a cost

$$\Phi(s, hK) = \frac{\eta}{2}s^2hK.$$ 

The cost function (7) is increasing and convex in the fraction $s$ of gross output that the controlling shareholder diverts for private benefits. The convexity of $\Phi(s, hK)$ in $s$ guarantees that it is more costly to divert increasingly large fractions of private benefits. For the remainder of the paper, we use the word “stealing” to mean “the pursuit of private benefits by diverting resources away from firm.” The cost function (7) also assumes that the cost of diverting a given fraction $s$ of cash from a larger firm is assumed to be higher, because a larger amount $shK$ of gross output is diverted. That is, $\partial \Phi(s, hK) / \partial K > 0$. But, the total cost of stealing the same \textit{level} $shK$ is lower for a larger firm than for a smaller firm. This can be seen by re-writing the cost of stealing as $\Phi(s, hK) = \eta (shK)^2 / (2hK)$. 

10 It is possible to endogenize the decision for the initial share ownership of the controlling shareholder. This is done in Shleifer and Wolfenzon (2002) in a static model and Lan and Wang (2004) in a dynamic setting. In these models weaker investor protection leads to a more concentrated ownership, \textit{ceteris paribus}.
Following La Porta et al. (2002), we interpret the parameter $\eta$ as a measure of investor protection. A higher $\eta$ implies a larger marginal cost $\eta shK$ of diverting cash for private benefits. In the case of $\eta = 0$, there is no cost of diverting cash for private benefits and the financing channel breaks down, because investors anticipate no payback from the firm after they sink their funds. As a result, *ex ante*, no investor is willing to invest in the firm. In contrast, in the limiting case of $\eta = \infty$, the marginal cost of pursuing a marginal unit of private benefit is infinity and minority shareholders are thus fully protected from expropriation. We will show later that in this case in the equilibrium we analyze the incentives of the controlling shareholder are perfectly aligned with those of the minority investors.

In summary, the objective for the controlling shareholder is to maximize his life-time utility defined in (3) and (4), subject to the firm’s capital stock dynamics given in (1)-(2), the controlling shareholder’s wealth accumulation dynamics (5)-(6), the cost function (7) for the controlling shareholder to pursue his private benefits, and the transversality condition specified in the appendix. In solving his optimization problem, the controlling shareholder chooses $\{C_1(t), s(t), I(t), K(t), D(t), W_1(t) : t \geq 0\}$ and takes the equilibrium interest rate process $\{r(t) : t \geq 0\}$ as exogenously given.

Let $D$ and $K$ be the dividend and firm’s capital stock process chosen by the controlling shareholder. Without loss of generality, we may write both the dividend and capital stock processes as follows:

\[
\begin{align*}
    dD(t) &= \mu_D(t)D(t)dt + \sigma_D(t)D(t)dZ(t), \\
    dK(t) &= \mu_K(t)K(t)dt + \sigma_K(t)K(t)dZ(t),
\end{align*}
\]

where the drift processes $\mu_D$ and $\mu_K$, and the volatility processes $\sigma_D$ and $\sigma_K$ are chosen by the controlling shareholder.

**Financial Assets.** Outside minority investors trade equity shares on the firm. While the controlling shareholder chooses the dividend stream, the price of the firm’s stock is determined in equilibrium by rational minority investors. We write the equilibrium stock price process as follows:

\[
    dP(t) = \mu_P(t)P(t)dt + \sigma_P(t)P(t)dZ(t),
\]

where $\mu_P$ and $\sigma_P$ are the equilibrium drift and volatility processes for stock returns.

In addition to firm stock traded by minority investors, there is also a risk-free asset available in zero net supply. Both minority investors and the controlling shareholder may
trade the risk-free asset. Let $r(t)$ be the short term interest rate paid on this risk-free asset. We determine $r$, $\mu_P$ and $\sigma_P$ simultaneously in equilibrium in Section 3.

**Minority Investors.** Minority investors have preferences with the same functional form as (3) and (4). They jointly own $(1 - \alpha)$ of the firm’s shares and can sell or buy of these shares in competitive markets with other minority investors at the equilibrium price $P(t)$. They can also invest in the risk-free asset earning interest at the equilibrium interest rate. Each minority investor’s optimization problem is a standard consumption-asset allocation problem in the spirit of Merton (1971). Unlike Merton (1971), in our model, both the stock price and the interest rate are endogenously determined in equilibrium.

Each minority investor accumulates his wealth as follows:

$$dW_2(t) = [r(t)W_2(t) - C_2(t) + \omega(t)W_2(t)\lambda(t)]\,dt + \sigma_P(t)\omega(t)W_2(t)dZ(t), \quad (11)$$

where $\lambda(t)$ is the excess stock return inclusive of dividend payments $D(t)$, in that $\lambda(t) \equiv \mu_P(t) + D(t)/P(t) - r(t)$, and $\omega(t)$ is the fraction of wealth invested in the firm’s stock. We use the subscript ‘2’ to denote variables chosen by minority investors, when it is necessary to differentiate the corresponding variables for the controlling shareholder. For example, in the wealth accumulation equation (11), $C_2(t)$ is the flow of consumption of the minority investor. The risk-free asset holdings of the minority investors are $B_2(t) = (1 - \omega(t))W_2(t)$. Finally, each minority investor’s initial wealth is $W_2(0) > 0$.

Each minority investor chooses $\{C_2(t), W_2(t), \omega(t) : t \geq 0\}$ to maximize his lifetime utility function subject to his wealth accumulation dynamics (11), and the transversality condition specified in the appendix. In solving this problem, the minority investor takes as given the equilibrium dividend process, the firm’s stock price, and the interest rate.

### 2.2 Definition and Existence of Equilibrium

We are now ready to define an equilibrium in our economy and state the theorem characterizing the equilibrium.

**Definition 1** An equilibrium in which the interest rate $r$ is constant has the following properties:

(i) $\{C_1(t), s(t), I(t), K(t), D(t), W_1(t) : t \geq 0\}$ solve the controlling shareholder’s problem for the given interest rate $r$;

(ii) $\{C_2(t), W_2(t), \omega(t) : t \geq 0\}$ solve the minority investor’s problem for given interest rate $r$, and stock price and dividend payout stochastic processes $\{P(t), D(t) : t \geq 0\}$;
(iii) The risk-free asset market clears, in that
\[ B_1(t) + B_2(t) = 0, \quad \text{for all } t; \]
(iv) The stock market clears for minority investors, in that
\[ (1 - \alpha) P(t) = \omega(t) W_2(t), \quad \text{for all } t; \]
(v) The consumption goods market clears, in that
\[ C_1(t) + C_2(t) = (1 - \alpha) D(t) + M(t), \quad \text{for all } t. \]

Note that a difficulty with our model is the presence of heterogeneous investors. In general, with heterogeneous investors, agents keep track of the wealth distribution in the economy \((W_1(t), W_2(t))\) besides the level of physical capital invested in the firm \(K(t)\). In our model though this problem is greatly simplified. First, in all equilibria with a constant interest rate, the tradable part of the controlling shareholder’s wealth \(W_1(t) = 0\). Second, the wealth of the minority investors is proportional to \(K(t)\). This feature significantly reduces the dimensionality of the problem from three state variables to one. The theorem to be introduced below completely characterizes the equilibrium.

Before we present the main theorem, we state the assumptions that are needed for our conjectured equilibrium.

**Assumption 1** \( h > \rho + \delta (1 - \gamma) \).

**Assumption 2** \( 1 - \alpha < \eta \).

**Assumption 3** \( 2(\gamma + 1) [(1 + \psi) h - \rho - \delta (1 - \gamma)] \epsilon^2 \leq \gamma [1 + (1 + \psi) h \epsilon^2]^2 \).

**Assumption 4** \( (1 - \phi) h > i \).

**Assumption 5** \( \rho + (\gamma - 1) (i - \delta) - \gamma (\gamma - 1) i^2 \epsilon^2 / 2 > 0 \).

Assumption 1 states that the firm is sufficiently productive and thus investment will be positive for risk-neutral firms under perfect investor protection. Assumption 2 ensures agency costs exist and lie within the economically interesting and relevant region. Assumptions 3 and 4 ensure positive investment and dividends, respectively. Assumption 5 gives rise to finite and positive Tobin’s \( q \) and dividend yield. While we have described the intuition behind these assumptions, obviously we cannot take the intuition and implications of these assumptions in isolation. These assumptions jointly ensure that the equilibrium exists with positive and finite net private benefits, investment rate, dividend, and Tobin’s \( q \).
Theorem 1. Under Assumptions 1-5, there exists an equilibrium with the following properties. The outside minority investors hold no risk-free asset \((B_2(t) = 0)\), and only stock \((\omega(t) = 1)\). Minority investors’ consumption equals their entitled dividends:

\[ C_2(t) = (1 - \alpha) D(t). \]

The controlling shareholder holds no risk-free asset \((B_1(t) = 0)\). He steals a constant fraction of gross revenue, in that

\[ s(t) = \phi \equiv \frac{1 - \alpha}{\eta}. \] (12)

The controlling shareholder’s consumption \(C_1(t)\), firm’s investment \(I(t)\), and firm’s dividend payout \(D(t)\) are all proportional to firm’s capital stock \(K(t)\), in that \(C_1(t)/K(t) = m\), \(I(t)/K(t) = i\), \(D(t)/K(t) = d\), where

\[ m = \alpha [(1 + \psi) h - i] > 0, \]

\[ i = \frac{1 + (1 + \psi) h \epsilon^2}{(\gamma + 1) \epsilon^2} \left[ 1 - \sqrt{1 - \frac{2(\gamma + 1) \epsilon^2 ((1 + \psi) h - \rho - \delta (1 - \gamma))}{\gamma [1 + (1 + \psi) h \epsilon^2]^2}} \right] > 0, \] (13)

\[ d = (1 - \phi) h - i > 0, \] (14)

and \(\psi\) is a measure of agency costs and is given by

\[ \psi = \frac{(1 - \alpha)^2}{2\alpha \eta}. \] (15)

The equilibrium dividend process (8), the stock price process (10), and the capital accumulation process (9) all follow geometric Brownian motions, with the same drift and volatility coefficients, in that \(\mu_D = \mu_P = \mu_K = i - \delta\) and \(\sigma_D = \sigma_P = \sigma_K = \epsilon \sigma\), where \(i\) is the constant equilibrium investment-capital ratio given in (14). The equilibrium firm value is also proportional to the firm’s capital stock, in that \(P(t) = qK(t)\), where the coefficient \(q\), known as Tobin’s \(q\), is given by

\[ q = \left(1 + \frac{1 - \alpha^2}{2\eta \alpha d} h \right)^{-1} \frac{1}{1 - \gamma \epsilon^2 i}. \] (17)

The equilibrium interest rate is given by

\[ r = \rho + \gamma \mu_D - \frac{\sigma_D^2}{2\gamma} (\gamma + 1). \] (18)
The parameter $\psi$ given in (16) summarizes the relevance of investor protection and the controlling shareholder’s cash-flow rights in the firm on investment and payout decisions. In particular, $\psi$ is a decreasing function of the cost of stealing $\eta$, and of the equity share of the controlling shareholder $\alpha$.

In equilibrium, financial and real variables—price $P$, dividend $D$, controlling shareholder’s consumption $C_1$ and wealth $W_1$, firm investment $I$, minority investor’s consumption $C_2$ and wealth $W_2$—are all proportional to the firm’s capital stock $K$. That is, in our model, the economy grows stochastically on a balanced path. In order to deliver such an intuitive and analytically tractable equilibrium, the following assumptions or properties in the model are useful: (i) a constant return to scale production and capital accumulation technology specified in (1); (ii) optimal “net” private benefits linear in the firm’s capital stock (arising from the assumptions that the controlling shareholder’s benefit of stealing is linear in $s$ and his cost of stealing is quadratic in $s$); (iii) the controlling shareholder and the minority investors have preferences that are homothetic with respect to the firm’s capital stock. We think the key intuition and results of our model are robust to various generalizations. Since the economy is on a balanced growth path, in the remainder of the paper we focus primarily on scaled variables such as the investment-capital ratio $i$ and the dividend-capital ratio $d$.

In the next section, we prove Theorem 1, present the derivations of equilibrium prices and quantities, and highlight the intuition behind the construction and solution methodology of the equilibrium.

3 Equilibrium Characterization

The natural and direct way to solve for the model’s equilibria in our economy is to solve the controlling shareholder’s consumption and production decisions and the minority investor’s consumption and asset allocation problem for a general price process and to aggregate up the demands for the stock, the risk-free asset, and the consumption good. However, this approach is technically quite complicated and analytically not tractable. The controlling shareholder’s optimization problem is one with both incomplete markets consumption-savings problem and a capital accumulation problem with agency costs. We know from the voluminous consumption-savings literature that there is no analytically tractable model with constant relative risk aversion utility (Zeldes (1989)). If even solving a subset of such an optimization problem is technically difficult, we naturally anticipate the joint consumption and production optimization problem for the controlling
shareholder to be intractable, not to mention finding the equilibrium fixed point.

Here we adopt the alternative approach by directly conjecturing, and then verifying, the equilibrium allocations and prices. Specifically, we conjecture an equilibrium in which the interest rate is constant and there is no trading of the risk-free asset and then show that such an equilibrium satisfies all the optimality and market clearing conditions. We start with the controlling shareholder’s optimization problem.

### 3.1 The Controlling Shareholder’s Optimization

We first conjecture that the controlling shareholder holds zero risk-free assets in equilibrium, in that \( B_1(t) = 0 \), for all \( t \geq 0 \). Therefore, his consumption is given by \( C_1(t) = M(t) \), where \( M(t) \) is given in (6).\(^{12}\) We will then show that under this conjecture, the rate \( r \) that satisfies the controlling shareholder’s optimality condition is equal to the equilibrium interest rate given in (18), presented in Theorem 1. In order to demonstrate that our conjectured interest rate is the equilibrium one, we also need verify that the optimality condition for the minority investors under the conjectured interest rate implies zero demand for the risk-free asset. We verify this later in the section.

Recall that the only tradable asset for the controlling shareholder in this economy is the risk-free asset. Therefore, together with our conjectured equilibrium demand for the risk-free asset by the controlling shareholder, we may equivalently write the controlling shareholder’s optimization problem as the following resource allocation problem:

\[
J_1(K_0) = \max_{D,s} E \left[ \int_0^\infty e^{-\rho t} u(M(t)) dt \right],
\]

subject to the firm’s capital accumulation dynamics (1)–(2), the cost of stealing (7), and the transversality condition specified in the appendix.

The controlling shareholder’s optimal payout decision \( D \) and “stealing” decision \( s \) solve the following Hamilton-Jacobi-Bellman equation:\(^{13}\)

\[
0 = \sup_{D,s} \left\{ \frac{1}{1 - \gamma} \left( M^{1-\gamma} - 1 \right) - \rho J_1(K) + (I - \delta K) J_1'(K) + \frac{\epsilon^2}{2} I J_1''(K) \right\}. \quad (19)
\]

The first-order conditions with respect to dividend payout \( D \) and cash diversion \( s \) are

\[
M^{-\gamma} \alpha - \epsilon^2 I J_1''(K) = J_1'(K), \quad (20)
\]

\[
M^{-\gamma} (hK - \eta shK) - \epsilon^2 I J_1''(K) hK = J_1'(K) hK. \quad (21)
\]

\(^{12}\) An auxiliary condition is that the initial wealth of the controlling shareholder is all in equity.

\(^{13}\) We verify the solution and provide technical regularity conditions in the appendix.
In (20) and (21), the controlling shareholder trades off the marginal benefits of receiving higher current cash flows, attainable either via a higher dividend payout as in (20), or more stealing as in (21), against the marginal costs of doing so. Higher current cash flows to the controlling shareholder provide two benefits. One is higher current utility, which is a standard result in any consumption Euler equation. This incremental gain in current utility associated with higher dividends and more stealing is given by the first term on the left-hand sides of (20) and (21), respectively. More interestingly, higher current cash flows to the controlling shareholder also increase his value function by reducing the volatility of capital stock accumulation, as seen from the second term on the left-hand side of both (20) and (21). The intuition for this volatility effect comes from (i) the concavity of the value function due to risk aversion and (ii) that the dividend payout reduces investment, which in turn lowers the volatility of capital accumulation. The marginal costs of higher current cash flows are losses of marginal values from investing that amount in the firm. The right-hand sides of (20) and (21) are the controlling shareholder’s marginal costs of paying out one more unit of dividends and stealing one additional fraction of gross output, respectively.

Solving (20) and (21) gives a constant solution for the stealing function, in that $s(t) = \phi \equiv (1 - \alpha)/\eta$. Intuitively, stealing is higher when investor protection is worse (lower $\eta$) and the conflicts of interests are bigger (smaller $\alpha$).

Conjecture that the controlling shareholder’s value function $J_1(K)$ is given by

$$J_1(K) = \frac{1}{1 - \gamma} \left( A_1 K^{1-\gamma} - \frac{1}{\rho} \right),$$

where the coefficient $A_1$ is given in the appendix. We verify this conjecture by solving the Hamilton-Jacobi-Bellman equation (19) and the associated first-order conditions (20)-(21) in the appendix. We show that the controlling shareholder’s consumption-capital ratio $M(t)/K(t)$, the investment-capital ratio $I(t)/K(t)$, and the dividend-capital ratio $D(t)/K(t)$ are all constant and are given by (13), (14), and (15), respectively.

The next proposition states the main properties of investment.

**Proposition 1** The equilibrium investment-capital ratio decreases with investor protection $\eta$ and the controlling shareholder’s cash flow rights $\alpha$, in that $di/d\eta < 0$ and $di/d\alpha < 0$, respectively.

This result is quite intuitive. Consider the trade-offs faced by the controlling shareholder in making the firm’s investment decision. The marginal costs to investing result
from postponing consumption and from the increased volatility of consumption through the volatility of capital accumulation. None of these depend directly on the corporate control frictions we highlight. The marginal benefits to investing, net of depreciation, come from the productivity of capital, which is given by $h$, but also from the private benefits net of the cost of extracting these benefits. Because the marginal net private benefits are proportional to productivity $h$, the total future discounted benefit to the controlling shareholder from investing one additional unit today can be summarized by $h$ adjusted for by the agency cost frictions (described by $\psi$). Hence, the presence of weak corporate governance increases the flow of net benefits to the controlling shareholder from investment, and thus generates overinvestment relative to the perfect investor protection benchmark.

Overinvestment as a consequence of weak corporate governance is a story that we think fits well the evidence in many emerging market economies but also in developed economies. Jensen’s (1986) free cash flow hypothesis is that managers are empire-builders if left unconstrained. Harford (1999) documents that US cash-rich firms are more likely to attempt acquisitions, but that these acquisitions are value decreasing measured by either stock return performance or operating performance.14 Pinkowitz, Stulz, and Williamson (2003) document that, after controlling for the demand for liquidity, one dollar of cash holdings held by firms in countries with poor corporate governance is worth much less to outside shareholders than if in countries with better corporate governance. For emerging market economies the evidence also abounds. Prior to the 1997 East Asian crisis, the countries in East Asia that suffered the crisis were running significant current account deficits, putting the borrowed money into questionable local investments. Burnside et al. (2001) use Thailand and Korea as examples of countries that borrowed significant amounts in foreign currency at low interest rates to lend locally at higher rates benefitting from a fixed exchange rate regime and from a government bailout policy. The volume of non-performing loans was already at 25 percent of GDP for Korea and 30 of GDP for Thailand prior to 1997! China is yet another example of a country with very large amounts of non-performing loans in the banking sector fruit of a government that tirelessly dumps cash in inefficient state owned enterprises. Allen et al. (2002) show that China has had consistent high growth rates since the beginning of the economic reforms in the late 1970s, even though its legal system is not well developed and law enforcement is poor. Our paper argues that the incentives for the controlling shareholders to overin-

14See also the earlier papers by Lang, Stulz, and Walking (1991) and Blanchard, López-de-Silanes, and Shleifer (1994).
vest can at least partly account for China’s high economic growth despite weak investor protection.\footnote{A sizable portion of China’s economy is state-owned enterprises. While we do not formally model state-owned enterprises in this paper, in practice these state-owned enterprises are not much different than the firms with controlling shareholders as described in our model. The cash flow rights of the managers come from their regular pay, which in general depends on firm performance, and the control rights come from appointment of the manager by the government. The managers in these firms are often government officials or their affiliates.}

In our model overinvestment arises because of the pursuit of private benefits by the controlling shareholder. This is likely to be a dominant issue for larger firms. There is a parallel line of research in corporate finance that highlights the role of costly external financing (Hubbard (1998)). That literature aims mostly at explaining the behavior of growth and exit of small firms, and highlights the role of underinvestment in production. We view these two lines of research complementary to each other.

We now return to our model’s implication on the controlling shareholder’s problem. We still need verify that the controlling shareholder’s consumption rule (13) and the equilibrium interest rate (18) are consistent with the implication that his optimal risk-free asset holding is indeed zero. This can be done by showing that the interest rate implied by the marginal utility of the controlling shareholder when \( C_1(t) = M(t) \) is the equilibrium one. The controlling shareholder’s marginal utility is given by \( \xi_1(t) = e^{-\rho t} C_1(t)^{-\gamma} \). In equilibrium, \( \xi_1(t) = e^{-\rho t} m^{-\gamma} K(t)^{-\gamma} \). Applying Ito’s lemma gives the following dynamics for \( \xi_1(t) \):

\[
\frac{d\xi_1(t)}{\xi_1(t)} = -\rho dt - \gamma \frac{dK(t)}{K(t)} + \frac{e^{2t^2}}{2} \gamma (\gamma + 1) dt.
\]

In order for \( \xi_1 \) to be the equilibrium stochastic discount factor, the drift of \( \xi_1 \) needs to equal \(-r \xi_1\). This equilibrium restriction and (22) together gives the equilibrium interest rate in (18). We refer the reader to Section 5.1 below for a discussion of the properties of the equilibrium interest rate.

Next, we turn to the minority investor’s optimization problem and his equilibrium security valuation.

\subsection*{3.2 Minority Investors’ Optimization}

Minority investors trade two securities: the stock and the risk-free asset. Each minority investor faces a standard consumption and asset allocation problem. The minority investor accumulates his wealth by either investing in the risky asset (firm asset) or the
risk-free asset. His wealth accumulation process is given by

\[ dW_2(t) = [r(t)W_2(t) - C_2(t) + \omega(t)W_2(t)\lambda(t)] dt + \sigma \omega(t) W_2(t) dZ(t), \]

where \( \lambda(t) = \mu_P(t) + D(t)/P(t) - r(t) \) is the equilibrium risk premium and \( \omega \) is the fraction of wealth invested in the risky asset. Under the conjecture that both equilibrium risk premium and equilibrium interest rate are constant, we conjecture that the minority investor’s value function as follows:

\[ J_2(W) = \frac{1}{1 - \gamma} \left( A_2 W^{1-\gamma} - \frac{1}{\rho} \right), \]

where \( A_2 \) is the coefficient to be determined in the appendix. We obtain the following standard consumption function and asset allocation solutions

\[ C_2(t) = \frac{\rho - r(1 - \gamma)}{\gamma} - \frac{\lambda^2(1 - \gamma)}{2\gamma^2\sigma_P^2} W(t), \]

\[ \omega(t) = \omega = -\frac{J_2(W)}{W J_2''(W)} \frac{\lambda}{\gamma} \sigma_P^2 = \frac{\lambda}{\gamma\sigma_P^2}. \]

In the proposed equilibrium, the minority investor only holds stock (\( \omega = 1 \)) and no risk-free asset. Hence, the equilibrium excess stock return must satisfy

\[ \lambda = \gamma \sigma_P^2 = \frac{\gamma^2}{2}. \]

The first equality is the usual result (e.g. Lucas (1978) and Breeden (1979)) that the equity premium commanded by investors to hold the stock is the product of the price of risk, given by the investor’s coefficient of relative risk aversion, and the quantity of risk, as given by the infinitesimal variance of the stock return. The last equality states that the standard deviation of equity returns is proportional to the investment-capital ratio. A higher investment rate gives rise to a larger volatility of output and equity prices.

In equilibrium, with zero risk-free asset holdings, the minority investor’s consumption is \( C_2(t) = (1 - \alpha) D(t) \). We apply Ito’s lemma to the minority investor’s marginal utility, \( \xi_2(t) = e^{-\rho t} C_2(t)^{-\gamma} = e^{-\rho t} [(1 - \alpha) dK(t)]^{-\gamma} \), to obtain the following dynamics of \( \xi_2(t) \):

\[ \frac{d\xi_2(t)}{\xi_2(t)} = -\rho dt - \gamma \frac{dK(t)}{K(t)} + \frac{\sigma^2}{2} \gamma (\gamma + 1) dt. \]

Because \( \xi_2 \) is the equilibrium stochastic discount factor, the drift of \( \xi_2 \) needs to equal \(-r\xi_2\), where \( r \) is the equilibrium interest rate. This equilibrium restriction and (25) together give the equilibrium interest rate in (18). Importantly, the implied equilibrium
interest rate by the controlling shareholder’s $\xi_1$ and the minority investor’s $\xi_2$ are equal. We thus verify that, like the controlling shareholder, minority investors find it optimal not to trade the risk-free asset at the equilibrium interest rate (18).

It remains to be shown that the price process (10) appropriately constructed is an equilibrium one for equity trading among minority investors, and generates a constant excess stock return. Using the minority investor’s marginal utility, we may obtain the per share price of the stock by dividing the discounted value of total dividends paid to minority investors by the number of shares $(1 - \alpha)$:

$$P(t) = \frac{1}{1 - \alpha} E_t \left[ \int_{t}^{\infty} \frac{\xi_2(s)}{\xi_2(t)} (1 - \alpha) D(s) ds \right] = q K(t),$$

where Tobin’s $q$, also known as the firm’s market-to-book value, is given by (17). Tobin’s $q$ is positive for $1 - e^{2 \gamma} > 0$, which holds under Assumption 5. With constant $q$ and dividend-capital ratio $d$, in equilibrium, it is straightforward to show that the drift coefficients for dividend, stock price, and capital stock are all the same, in that $\mu_D = \mu_P = \mu_K = i - \delta$, and the volatility coefficients for dividend, stock price, and capital stock are also the same, in that $\sigma_P = \epsilon i$. Constant risk premium $\lambda$ is an immediate implication of constant $\mu_P$, constant dividend-capital ratio $d$, and constant equilibrium risk-free interest rate.

### 3.3 Equity Trading Between the Controlling Shareholder and Minority Investors

We have so far exogenously assumed that the controlling shareholder cannot trade equity with the minority investors. In this section, we extend our model by allowing both the controlling shareholder and outside minority investors to trade equity. We show that in equilibrium both the controlling shareholder and outside minority investors rationally choose not to trade with each other. The key in our analysis is to identify a free-rider situation similar to the free-rider problem identified in Grossman and Hart (1980) in the corporate takeover context. Lan and Wang (2004) propose such a free rider argument between the risk-neutral controlling shareholder and risk-neutral outside minority investors. Here, we apply the free-rider argument to risk averse agents.\(^{16}\)

The key insight behind our proof for the no-trade result is that the controlling shareholder is unable to enjoy any surplus generated from increasing firm value (via a more

\(^{16}\)To the best of our knowledge, this is the first paper that identifies a free-rider problem with risk-averse agents.
concentrated ownership structure.) The crucial assumption is that the controlling shareholder cannot trade anonymously. The inability to trade anonymously is realistic. For example, in almost all countries, the insiders need to file a report before selling or buying their own firm’s shares. We now provide the formal argument for the no-trade result.

Let $\alpha$ be the controlling shareholder’s current ownership in the firm. Suppose the controlling shareholder considers the possibility of increasing his ownership from $\alpha$ to $\alpha'$, if it is in his interest to do so. With a slight abuse of notation, let $P_{\alpha'}$ and $P_{\alpha}$ denote the equilibrium equity price (resulting from competitive trading), when the controlling shareholder’s ownership is $\alpha'$ and $\alpha$, respectively. Because a higher ownership concentration gives a better incentive alignment, investors rationally anticipate $P_{\alpha'} > P_{\alpha}$, for $\alpha' > \alpha$ (see equation 26 above and Proposition 3 below). Obviously, the controlling shareholder will not buy any shares at prices above $P_{\alpha'}$. Moreover, we show in the appendix, that at most he is willing to pay $P_{\alpha}$.

Let us turn to the minority investor’s decision problem. Consider the decision of a minority investor $j$ facing a buy order from the controlling shareholder at price $P_{\alpha}$. If sufficient shares are tendered to the controlling shareholder by other minority investors at any acceptable price to the controlling shareholder (which is obviously lower than $P_{\alpha'}$), then the deal will go through even if investor $j$ does not sell. As a result, investor $j$ enjoys a price appreciation and obtains a higher valuation by free riding on other investors. Because each minority investor is infinitesimal and not a pivotal decision maker, the free-rider incentive implies no trade in equilibrium. The appendix contains a detailed and formal proof for this argument.17

Before delving into the details on the relationship between investor protection and asset returns, we first analyze equilibrium for the no agency cost setting. This neoclassical setting (with no agency cost) serves naturally as the benchmark against which we may quantify the effect of imperfect investor protection on asset prices and returns.

## 4 Benchmark: Perfect Investor Protection

This section summarizes the main results on both the real and financial sides of an economy under perfect investor protection.

---

17The free-rider argument developed here breaks down if the controlling shareholder instead of buying a small number of shares offers to buy the remaining outstanding shares. In this case, however, suppose that the controlling shareholder finances his acquisition by borrowing in the bond market. In a more general framework, one that incorporates weak investor protection in the bond market, these bonds would be bought at a premium possibly large enough to offset the gain from buying all the shares from the minority investors, and no trade would occur. We thank John Long for making this point.
When investor protection is perfect, the cost of diverting resources away from the firm is infinity, even if the controlling shareholder diverts a negligible fraction of the firm’s resources. Therefore, the controlling shareholder rationally decides not to pursue any private benefits and maximizes the present discounted value of cash flows using the unique discount factor in the economy. That is, there are no conflicts of interest between the controlling shareholder and the outside minority investors. Our model is then essentially a neoclassical production based asset pricing model, similar to Cox, Ingersoll, and Ross (CIR) (1985). We will highlight the main differences between our model and CIR later in this section.

The controlling shareholder chooses the first-best investment level \( I^*(t) = i^*K^*(t) \), where investment-capital ratio \( i^* \) is obtained from (14) by letting \( \eta \to \infty \) and is

\[
i^* = \left[ \frac{1 + he^2}{(\gamma + 1)e^2} \right] \left[ 1 - \sqrt{1 - \frac{2(\gamma + 1)e^2(h - \rho - \delta(1 - \gamma))}{\gamma(1 + he^2)^2}} \right].
\]

Starred variables ("*") denote the equilibrium values of the variables under perfect investor protection. >From Proposition 1 we know that there is overinvestment under weak investor protection, \( i^* < i \).

Tobin’s \( q \) under this first-best benchmark is given by

\[
q^* = \frac{1}{1 - \varepsilon^2 \gamma i^*} \geq 1.
\]

Before analyzing the stochastic case \( (\varepsilon > 0) \), we briefly sketch the model’s prediction when capital accumulation is deterministic \( (\varepsilon = 0) \). It is easy to show that without volatility in the capital accumulation equation (1), Tobin’s \( q \) is equal to unity.\(^{18}\) This is implied by no arbitrage when capital accumulation is deterministic and incurs no adjustment cost and production function has the constant return to scale property.

The key prediction of our model on the real side under perfect investor protection is that Tobin’s \( q \) is larger than unity, when capital accumulation is subject to shocks \( (\varepsilon > 0) \). That is, the value of installed capital is larger than that of to-be-installed capital. The intuition is as follows. In our model, the net change of capital stock over a fixed time interval \( \Delta t \) is subject to both a net investment \( (I - \delta K) \Delta t \), but also innovations \( \varepsilon I \Delta Z(t) \) that are proportional to the level of gross investment. That is, new investment introduces uncertainty into the capital accumulation process. This production risk is systematic and thus must be priced in equilibrium. As a result, risk averse investors view it as costly.

\(^{18}\) The first-best investment-capital ratio when \( \varepsilon = 0 \) is given by \( i^* = [h - \rho - \delta (1 - \gamma)] / \gamma \), by applying L’Hôpital’s rule to (27).
to adjust capital stock in equilibrium. This in turn drives a wedge between the price of uninstalled capital and the price of installed capital. We call this channel the production risk channel.

The main difference between our model and the CIR model is Tobin’s $q$, or equivalently stated, the price of installed capital. In CIR, the production technology is also constant return to scale. However, in their model, the volatility of output does not depend on the level of gross investment. Therefore, the price of capital in CIR is equal to unity. Dow et al. (2003) incorporate the manager’s empire-building incentive into the neoclassical production-based asset pricing framework such as CIR, retaining the feature that the price of capital is equal to unity.

Because the outside minority investor and the controlling shareholder have the same utility functions, and markets are effectively complete in the perfect investor protection case, we naturally expect that both the controlling shareholder and outside minority investors to hold no risk-free asset in equilibrium and invest all of their wealth in the risky asset in equilibrium. The minority investors’ and the controlling shareholder’s consumption plans are equal to their respective entitled dividends, in that $C_2^* (t) = (1 - \alpha) D^* (t)$, and $C_1^* (t) = \alpha D^* (t)$, and $D^* (t) = d^* K^* (t)$, with the first-best dividend-capital ratio given by $d^* = h - i^*$. The equilibrium interest rate under perfect investor protection, $r^*$, is given by (18), associated with the first-best investment-capital ratio $i^*$. Equation (18) indicates that the interest rate $r^*$ is constant and is determined by the following three components: (i) the investor’s subjective discount rate $\rho$, (ii) the net investment rate $(i - \delta)$, and (iii) the precautionary saving motive. In a risk-neutral world, the interest rate must equal the subjective discount rate in order to clear the market. This explains the first term. The second term captures the economic growth effect on the interest rate. A higher net investment rate $(i - \delta)$ implies more resources are available for consumption in the future and thus pushes up demand for current consumption relative to future consumption. To clear the market, the interest rate must increase. This effect is stronger when the agent is less willing to substitute consumption inter-temporally, which corresponds to a lower elasticity of intertemporal substitution $1/\gamma$.\textsuperscript{19} The third term captures the precautionary savings effect on interest rate determination. A high net investment rate increases the

\textsuperscript{19}In expected utility framework, elasticity of inter-temporal substitution is equal to the inverse of the coefficient of relative risk aversion. In a recursive utility such as Epstein-Zin utility, the elasticity of intertemporal substitution and coefficient of risk aversion may be partially disentangled. In that recursive utility framework, the coefficient for the growth-investment term will be the inverse of the elasticity of intertemporal substitution.
riskiness of firm’s cash flows, and thus makes agents more willing to save. This preference for precautionary savings reduces current demand for consumption and lowers the interest rate, *ceteris paribus*.

In this benchmark case, the equilibrium stock price $P^*$ is given by a geometric Brownian motion (10) with drift $\mu^*_p = i^* - \delta$ and volatility $\sigma^*_p = i^* \epsilon$.

Next, we analyze how different degrees of investor protection affect asset prices and returns.

## 5 Equilibrium Asset Returns

We first analyze the equilibrium interest rate and then turn to the stock return.

### 5.1 Risk-Free Rate

The next proposition relates the interest rate under imperfect investor protection to that in the benchmark case.

**Proposition 2** Worse investor protection or lower share of equity held by the controlling shareholder are associated with a higher risk-free interest rate if and only if $1 > \epsilon^2 (\gamma + 1) i$. Specifically, the interest rate in an economy with imperfect investor protection is higher than that under perfect investor protection if and only if $1 > \epsilon^2 (\gamma + 1) i$.

Changes in the degree of investor protection produce two opposing effects on the equilibrium interest rate. Both effects result from investment being higher under weaker investor protection. First, because of the effect of economic growth on the interest rate, higher investment implies larger output in the future and intertemporal consumption smoothing makes the agent willing to finance his current consumption by borrowing, leading to a higher current equilibrium interest rate. Second, higher investment makes capital accumulation more volatile and implies a stronger precautionary saving effect, thus pushing down the current equilibrium interest rate, *ceteris paribus*. The proposition illustrates that the growth effect dominates the precautionary effect if and only if $1 > \epsilon^2 (\gamma + 1) i$, that is in the region where the equilibrium interest rate increases with the investment rate. As demonstrated in the appendix this condition is satisfied for sufficiently low $\epsilon$, $h$, or $\psi$, and holds in all our calibrations below. It implies that the growth effect dominates and interest rates are higher under weaker investor protection. The cross-country interest rate data on eleven developed countries in Campbell (2003) suggests that civil law countries, those with weaker investor protection, have higher interest
rates than common law countries. The average interest rate on his sample of common
law countries is 1.89 percent, statistically smaller than the 2.35 percent average interest
rate on his sample of civil law countries.

We now turn to equilibrium valuation from both the controlling shareholder and the
minority investor’s perspective.

5.2 Firm Valuation and Returns

Controlling Shareholder’s Shadow Equity Valuation. Even though the controlling
shareholder cannot trade firm equity with outside minority investors, the controlling
shareholder nonetheless has a shadow value for equity. Let \( \hat{P}(t) \) denote this shadow price of equity for the controlling shareholder. We may compute \( \hat{P}(t) \) as follows:

\[
\hat{P}(t) = \frac{1}{\alpha} E_t \left[ \int_t^\infty e^{-\rho(s-t)} \frac{M(s)^{1-\gamma}}{M(t)^{-\gamma}} ds \right] = \frac{1}{1 - e^{2i\gamma}} K(t).
\]

The equilibrium shadow market-to-book value of the firm to the controlling shareholder or shadow Tobin’s \( q \) is therefore given by

\[
\hat{q} = \frac{1}{1 - e^{2i\gamma}}.
\]

We note that the shadow value \( \hat{q} \) is higher than \( q^* \), Tobin’s \( q \) under perfect investor protection. The intuition is as follows. The controlling shareholder distorts the capital accumulation decision in pursuit of his private benefits and thus obtains a shadow value for the firm higher than \( q^* \). By revealed preference, the controlling shareholder could set the investment-capital ratio to \( i^* \) and steal nothing \( s = 0 \), which would imply \( \hat{q} = q = q^* \). Therefore, by choosing \( s > 0 \), the controlling shareholder’s decisions \( (i > i^*) \) must imply that his valuation \( \hat{q} > q^* \). We next turn to the minority investor’s valuation.

Minority Investors’ Valuation. Theorem 1 shows that the equilibrium price for firm equity is proportional to the capital stock and is given by \( P(t) = qK(t) \), where \( q \) measures Tobin’s \( q \) also known as the market-to-book ratio. The next proposition characterizes the monotonic relationship between \( q \) and investor protection.

Proposition 3 Tobin’s \( q \) increases with investor protection, in that \( dq/d\eta > 0 \), and increases with the controlling shareholder’s cash flow rights, in that \( dq/d\alpha > 0 \).

Proposition 3 demonstrates that the model is consistent with the evidence offered in La Porta et al. (2002), Gompers et al. (2003), and Doidge (2004) on the relationship
between firm value and investor protection. The model also predicts that firm value increases with the controlling shareholder’s ownership $\alpha$. This incentive alignment effect due to higher cash flow rights is consistent with empirical evidence in Claessens et al. (2002) on firm value and cash flow ownership, and with the evidence for Korea in Baek et al. (2004) where it is found that non-chaebol firms experienced a smaller reduction in their share value during the East Asian crisis.

The intuition for a monotonically increasing firm value in investor protection relies on the fact that private benefits of control are extracted from installed capital. The possibility of extracting private benefits lowers the value of installed capital relative to new capital goods, which is to say the firm’s market-to-book ratio. This agency channel is not the only mechanism at work. Tobin’s $q$ also varies with the production risk channel. By the production risk channel, the higher $\eta$ (or lower $\alpha$) leads to lower investment rates and lower volatility of installed capital and dividends. This reduces the wedge between installed capital and new capital goods, i.e. $q$. Proposition 3 shows that the agency channel dominates.

La Porta et al. (2002) and Shleifer and Wolfenzon (2002) provide a theoretical explanation for the decline in $q$ resulting from worse investor protection that relies on a static analysis and risk neutrality. Lan and Wang (2004) extend the analysis to a dynamic equilibrium analysis with risk neutral entrepreneurs and outside minority investors. Ours is the first paper to explain this empirical evidence while computing firm value in a dynamic stochastic general equilibrium asset pricing model with risk averse agents.

We next turn to the dividend yield. Let $y$ be the equilibrium dividend yield, in that $y = D/P = d/q$. We solve for the dividend yield using the following equilibrium excess return relationship:

$$\lambda + r = \gamma \sigma_D^2 + r = \mu_D + y,$$

(29)

where the first equality uses $\lambda = \gamma \sigma_D^2$ in equilibrium and $\mu_P = \mu_D$. Using the equilibrium interest rate formula (18), we have the following expression for the dividend yield:

$$y = \rho + \mu_D (\gamma - 1) - \frac{\gamma (\gamma - 1) \sigma_D^2}{2}.$$

(30)

**Proposition 4** Dividend yield decreases (increases) with the degree of investor protection if and only if $\gamma > 1$ ($\gamma < 1$).

We note that both Tobin’s $q$ and the dividend-capital ratio $d$ monotonically increase in investor protection $\eta$. The later is consistent with the evidence in La Porta et al. (2000a). However, the dividend yield $y$, which is given by the ratio between dividend-capital ratio
$d$ and Tobin’s $q$, may either increase or decrease in investor protection, depending on $\gamma$. This reflects the model’s prediction that the percentage change of dividend-capital $d$, $d \log d/d\eta$, and the percentage change of Tobin’s $q$, $d \log q/d\eta$, in general are different. We show that whether dividend yield decreases or increases in investor protection depends on whether $\gamma > 1$ or not.

The reason the effect on the dividend yield of investor protection depends on the elasticity of intertemporal substitution$^{20}$ is that better investor protection leads to lower risk-adjusted discount rates thus changing the willingness of investors to save. To understand this mechanism, consider first the case in which the dividend yield does not depend on investor protection. If agents have logarithmic utility functions, the dividend yield is equal to the agent’s subjective discount rate $\rho$, directly implied by (30). This reflects the myopic nature of logarithmic utility agents. When $\gamma > 1$, the elasticity of intertemporal substitution is less than unity. The income effect is stronger than the substitution effect. Then, the result of lower rates (netting the income and substitution effects) is to motivate a decrease in current consumption, which translates into a lower dividend yield (note that in equilibrium $C_2/W_2 = y$). Whether $\gamma$ is interpreted as the risk aversion coefficient or the inverse of the elasticity of inter-temporal substitution, empirical estimates of $\gamma$ are in general larger than unity.$^{21}$ Therefore, with a plausible estimate of $\gamma > 1$, the model predicts a higher dividend yield in countries with weaker investor protection and higher interest rates.

The next proposition gives our main results on equilibrium returns. It indicates that the valuation effects of investor protection are not restricted to price level effects as previously studied, or the dividend yield, but have implications for discount rates as well.

**Proposition 5** *Expected return inclusive of dividends, return volatility $\sigma_P$, and risk premium $\lambda$, all decrease in investor protection $\eta$ and ownership $\alpha$.*

The proposition shows that the rate of excess equity returns is affected in equilibrium by changes in the degree of investor protection. The intuition is quite simple. Weaker investor protection increases the riskiness of the stock to minority investor and thus the risk premium they charge to hold its shares. To see this reasoning in more detail note that the equilibrium equity risk premium is given by

$$\lambda = \gamma \sigma_P^2 = \gamma \epsilon_i^2 \epsilon^2 .$$

---

$^{20}$The elasticity is equal to the inverse of the coefficient relative risk aversion.

$^{21}$See Hansen and Singleton (1982), for example.
The risk premium reflects the price attached by the minority investors to the uncertainty associated with the economy’s single factor (i.e. $Z(t)$). A positive shock to capital (i.e. $dZ(t) > 0$) leads to a higher stock return and a higher dividend payment. Minority investors’ consumption increases and their marginal utility declines. This negative correlation between stock returns and marginal utility of consumption is larger in absolute terms when investor protection is weaker and the investment rate is higher. This is because the value of this correlation in equilibrium is tied to the volatility of the capital stock, and a higher investment rate makes the existing capital stock more volatile. Thus, the risk premium is larger.\(^{22}\)

There is evidence in support of Proposition 5. Daouk, Lee, and Ng (2004) create an index of capital market governance which captures differences in insider trading laws, short-selling restrictions, and earnings opacity. They model excess equity returns using an international capital asset market model which allows for varying degrees of financial integration. Consistent with proposition 5, they show that improvements in their index of capital market governance are associated with lower equity risk premia.

The cross-country data in Campbell (2003) indicates that civil law countries, those with weaker investor protection, have higher excess equity returns than common law countries. The average excess equity return on his sample of common law countries is 4.12 percent, smaller than the 6.97 percent average excess equity return on his sample of civil law countries.

Harvey (1995), Bekaert and Harvey (1997), and Bekaert and Urias (1999) show that emerging markets display higher volatility of returns and larger equity risk premia. Bekaert and Harvey (1997) correlate their estimated conditional stock return volatilities with financial, microstructure, and macroeconomic variables and find some evidence that countries with lower country credit ratings, as measured by Institutional Investor, have higher volatility. Erb et al. (1996) show that expected returns, as well as volatility, are higher when country credit risk is higher. Since emerging market economies and countries with worse credit ratings have on average weaker corporate governance, this empirical evidence lends further support to our theory.

The minority investors in our model behave much like the investors in a traditional consumption capital asset pricing model augmented to include a production sector. However, minority investors are not the ones choosing the investment rate and in fact they will be faced with too much capital accumulation and demand for savings. These predic-

\(^{22}\)As indicated in Proposition 4, not all of the excess returns come necessarily from higher capital accumulation (as a result of overinvestment) and subsequent price appreciation. If $\gamma < 1$, the dividend yield is higher with worse investor protection.
tions differ from production models where the minority investors are the ones choosing the capital accumulation path and can use the investment rate to smooth out business cycle fluctuations. In these other models, as in the benchmark model described above, the volatility of dividends is smaller and the economy’s risk premium is smaller. Hence the model generates a higher risk premium than do traditional neoclassical asset pricing models with production like the CIR (1985) model.

Note that, because equity pricing in the model is done by outside minority investors, the relevant consumption data to feed into the risk premium calculations is that of minority investors and not aggregate consumption. Our approach is thus similar to Mankiw and Zeldes (1991) who focus on consumption data of a smaller sample of stockholders. Relative to Mankiw and Zeldes, our approach allows us to compute the risk premium directly by working with production (investment) data, and avoid dealing with consumer data, which generally produces very noisy estimates. Specifically, our model predicts that, for equal risk aversion γ and volatility parameter ε, the percentage difference in equity premia between any two countries should be of the same order of magnitude as the percentage difference in squared investment-capital ratios.

Naturally, the disagreement in valuation between the controlling shareholder and outside minority investors approaches zero as investor protection increases, because \( q \to q^* \) and \( \hat{q} \to q^* \) as \( \eta \to \infty \). In the case of perfect investor protection the controlling shareholder is homogeneous to the minority investors and investment and dividend policies chosen by the former coincide with what the later would do.

Despite the disagreement between minority investors and the controlling shareholder on the firm’s market-to-book value under imperfect investor protection, they agree on expected returns. The instantaneous “shadow” return to the controlling shareholder is

\[
\frac{d\hat{P}(t)}{\hat{P}(t)} + \frac{(M(t)/\alpha) dt}{\hat{P}(t)} = \left( i - \delta + \frac{m}{\alpha \hat{q}} \right) dt + \epsilon i dZ(t) = (\mu_P + y) dt + \sigma_P dZ(t).
\]

Therefore, the instantaneous “shadow” return is equal to \( \mu_P + y \), the expected stock return (including the dividend component) for outside minority investors. Intuitively, the economy grows stochastically on a balanced path. Both the controlling shareholder and outside minority investors share the same marginal valuation.

While we have focused on equity prices and returns and the risk-free rate, our model can be used to price financial securities with any given feature of cash flows, including equity options and futures. This is due to the fact that our model is one of effectively complete markets with an endogenously determined stochastic discount factor.
We now take our model’s implications and quantify the economic significance of imperfect investor protection on asset returns and the utility costs.

6 Quantitative Predictions

Our model is quite parsimonious in that it has only seven parameter values from both the production and investor side of the economy. The choice of parameters is done in one of two ways. Some parameters are obtained by direct measurements conducted in other studies. These include the risk aversion coefficient $\gamma$, the depreciation rate $\delta$, the rate of time preference $\rho$, and the equity share of the controlling shareholder $\alpha$. The remaining three parameters $(\eta, \epsilon, h)$ are picked so that the model matches three relevant moments in the data.

6.1 Calibration

We calibrate the model for the United States and South Korea. We start with the first set of parameters. We choose the coefficient of relative risk aversion to be 2, a commonly chosen level of risk aversion. The depreciation rate is set to an annual value of 0.07. The subjective discount rate is set to $\rho = 0.01$ based on empirical estimation results such as those reported in Hansen and Singleton (1982). We choose the share of firm ownership held by the controlling shareholder for the US to be $\alpha = 0.2$ and for Korea to be $\alpha = 0.18$ (La Porta et al. (2002)).

Turning now to the second set of parameters, we calibrate the investor protection parameter $\eta$, the volatility parameter $\epsilon$, and the productivity coefficient $h$ so that the model matches the following three moments of the data; (i) the ratio of private benefits to firm equity value taken from Dyck and Zingales (2004) (1.8 percent for the US and 15.7 percent for Korea); (ii) the annual standard deviation of detrended per capita real output growth (2 percent for the US and 4.3 percent for Korea); and (iii) the mean investment to GDP ratio (19.4 percent for the US and 32.2 percent for Korea). The data for all moments given in (ii) and (iii) refers to the period 1980-2000 and was obtained from the World Bank’s (2002) World Development Indicators database.

The resulting calibrated parameters are $(\epsilon, \eta, h) = (0.02, 50, 1.95)$ for the US and $(\epsilon, \eta, h) = (0.059, 8, 1.13)$ for Korea. For both countries these parameters imply that the

\footnote{To match the investment to GDP ratio in the data we construct a measure of GDP in the model given by $GDP = C_1 + C_2 + I$. Noting that the model has no labor input we adjust the investment to GDP ratio in the data by dividing it by the capital share of income (equal to 0.4 for the US and 0.32 for Korea).}
model matches (i) and (ii) exactly, while being slightly off on (iii): for the US the model produces an investment to GDP ratio of 17 percent while for Korea the model produces a ratio of 21.5 percent.

The calibrated stealing fraction \((\phi = (1 - \alpha) / \eta)\) is 1.4 percent for the US and 10.3 percent for Korea. This implies that the agency cost is approximately seven times higher for Korea: the cost of stealing as a fraction of gross output \((\Phi (s, hK) / hK = (1 - \alpha)^2 / 2\eta)\) is 0.6 percent for the US and 4.2 percent for Korea.

### 6.2 Results

We report numerical results for Tobin’s \(q\) and the risk premium. Each figure below contains four plots. The top plots contain the results for the US whereas the bottom plots contain the results for Korea. The two left plots give the model’s comparative statics when investor protection changes, whereas the two plots on the right describe the comparative statics when the equity share of the controlling shareholder changes.

**Tobin’s \(q\).** Consider the market valuation of minority investors and the implied market-to-book value. Figure 1 displays the model’s comparative statics. Recall from our discussion above that the effect on Tobin’s \(q\) of better investor protection comes from the combination of the the agency channel effect and the production risk channel effect. The former pushes \(q\) down whereas the later pushes \(q\) up. Figure 1 displays that the agency channel dominates (Proposition 3). A sufficiently large \(\eta\) or \(\alpha\) takes Tobin’s \(q\) closer to the benchmark case where we know it is larger than unity.

With our calibrated baseline parameters, Tobin’s \(q\) is 0.92 for the US and 0.43 for Korea. While the level of Tobin’s \(q\) might be slightly lower than the estimates in Hall (2001) for the US, incorporating adjustment costs into our baseline model would raise the level of the calibrated Tobin’s \(q\) close to Hall’s estimates. More interestingly, we note that our model predicts Tobin’s \(q\) for the US to be 2.1 times larger than Tobin’s \(q\) for Korea. Our calibrated Tobin’s \(q\) ratio between the US and Korea is close to that reported in La Porta et al. (2002) of 2.8.

While the model may under-predict Tobin’s \(q\) for these countries, we also consider a robustness check. Suppose that the stealing fraction \(s\) is 10 times smaller for both the US and Korea. Namely, let \(s\) be a meager 0.14 percent and 1.03 percent for the US and Korea, respectively. Then the US Tobin’s \(q\) increases to 1 virtually at the first best level of 1.01, and the Korean Tobin’s \(q\) increases to 0.93. Even with this much smaller agency estimates, Tobin’s \(q\) for Korea is still 7.4 percent below the first best benchmark level.
This confirms that investor protection has a first-order effect on security prices. Next we quantify the effect of investor protection on the risk premium.

[Figure 1 here.]

**Risk Premium.** Recall that Proposition 5 shows that the benchmark case of perfect investor protection displayed a smaller risk premium than the imperfect investor protection case. Here, we investigate the quantitative significance of our mechanism. Figure 2 plots the ratio of the risk premium for a specific value of \( \eta \) or \( \alpha \) to the risk premium in the benchmark case (note that in the two plots on the right the benchmark level of the risk premium also changes with \( \alpha \)). The figure indicates that the US can lower its risk premium by almost 6 percent by moving to perfect investor protection, but South Korea can lower its equity premium in the stock market by almost 50 percentage points if it were to move to a world of perfect corporate governance. If the stealing fraction \( s \) in Korea was 10 times smaller, these gains would still be of 20 percent.

[Figure 2 here.]

The model-implied gains for South Korea are quite large. They compare quite reasonably to the empirically estimated gains on lower cost of equity capital from capital market liberalizations (see Stulz (1999) among others). Stulz (1999) suggests that capital market liberalizations can lead to lower domestic cost of capital because of diversification reasons and indirect improvements in corporate governance. However, the liberalization of the domestic capital market as an indirect way of improving investor protection generates only modest gains in the cost of capital relative to a more direct pursuit of changing local legislation and implementation of policies recommended here.

The level of the risk premium under our proposed baseline calibration is too small, but with CRRA preferences this is to be expected. We view our model as giving an indication that in relative terms, as investor protection changes the *changes* in the risk premium can be substantial. In our calibration, the Korean risk premium is 4 times larger than the US risk premium in the baseline case. When the stealing fraction is 10 times smaller than in the baseline case for both countries this number drops to 3. Using data from the World Development Indicators database, the *ex-post* excess equity return since 1989 in Korea is 12.5 percent, about 2 to 3 times that of the US.
6.3 The Cost of Imperfect Investor Protection

Here we explore the implications of imperfect investor protection on utility costs for outside investors. We measure utility costs in terms of equivalent variation by asking the following trade-off question for the outside minority investor. What fraction of his personal wealth is the outside minority investor willing to give up for a permanent improvement of investor protection from the current level $\eta$ to the benchmark (first-best) level of $\eta = \infty$? Let $(1 - \zeta)$ denote the fraction of his wealth that the minority investor is willing to give up for such a permanent increase in the quality of investor protection. Then, the minority investor is indifferent if and only if the following equality holds:

$$J_2^* (\zeta W_0) = J_2 (W_0),$$

where $J_2$ is the minority investor’s value function and $W_0$ is his initial wealth.\(^\text{24}\)

Since the minority investor’s wealth $W$ is proportional to the firm’s capital stock $K$ in equilibrium, $(1 - \zeta)$ is also the fraction of the capital stock that the minority investors own and are willing to give up, in exchange for better investor protection. Using the value function formula given in Section 3, we may calculate the cost of imperfect investor protection in terms of $\zeta$ and obtain: \(^\text{25}\)

$$\zeta = \left( \frac{y}{y^*} \right)^{1/(1-\gamma)} \frac{d}{d^*}. \tag{31}$$

**Proposition 6** The minority investors’ utility cost is higher under weaker investor protection, in that $d\zeta/d\eta > 0$. Naturally, for any $\eta < \infty$, $0 < \zeta < 1$.

Figure 3 plots $(1 - \zeta)$ against various levels of investor protection parameter $\eta$, holding ownership fixed in each of the two left panels, and plots $(1 - \zeta)$ against the controlling shareholder’s ownership $\alpha$, holding investor protection parameter $\eta$ fixed in the right plots. The results are quite striking. Investors are willing to give up a substantial part of their own wealth for a stronger investor protection. This is true even for the US whose

\(^{24}\)We use $J_2^*$ to denote the corresponding value function for minority investors under perfect investor protection.

\(^{25}\)By applying L’Hopital’s rule to (31) around $\gamma = 1$, we may obtain the formula for $\zeta$ for logarithmic utility. With some algebra, it can be verified that

$$\zeta = \frac{d}{d^*} \exp \left[ \frac{\left( \mu_D - \frac{1}{2}\sigma_D^2 \right) - \left( \mu^*_D - \frac{1}{2}\sigma^*_D^2 \right)}{\rho} \right].$$
minority investors are willing to give up 8 percent of their wealth to move to perfect investor protection. Even if the stealing fraction is 10 times smaller than the calibrated number for the US (i.e., \(s = 0.14\) percent), US investors are still willing to give up almost 1 percent of their wealth to have perfect investor protection. In Korea, minority investors are still willing to give up 8 percent of their wealth to have perfect investor protection when we calibrate the stealing fraction in Korea to be 1.03 percent of gross output. These benefits of increasing investor protection are economically large.

[Figure 3 here.]

7 Conclusions

Agency conflicts are at the core of modern corporate finance. The large corporate finance literature on investor protection has convincingly documented that corporations in most countries, especially those with weak investor protection, often have controlling shareholders. Controlling shareholders derive private benefits at the cost of outside minority shareholders, which means that firm value varies with investor protection regulations and enforcement.

Motivated by this vast literature, we construct a dynamic stochastic general equilibrium model in which the controlling shareholder makes all corporate decisions in his own interest and outside investors rationally formulate their asset allocation and consumption-saving decisions in a competitive way. Despite the heterogeneity between the controlling shareholder and outside investors, we are able to characterize the equilibrium in closed form. We show that the modeled agency conflicts lead to distorted corporate investment and payout policies which in turn affect asset prices. In equilibrium, however, asset prices affect the ability of the controlling shareholder to smooth consumption and in turn affect corporate investment decisions. This differentiates our work from previous asset pricing models based on endowment or production economies.

The model allows us to conveniently derive theoretical predictions on asset prices and returns. Among others, our model predicts that countries with weaker investor protection have lower firm value measured by Tobin’s \(q\), lower dividend payout ratio, more volatile stock returns, higher equilibrium interest rates, larger equity premia, and, for reasonable values of risk aversion, larger dividend yield. We show that the utility cost of weak investor protection is economically large. Our model suggests that strengthening investor protection has a significant effect on investor’s welfare.
In order to focus on how investor protection affects equilibrium asset prices and returns, we have chosen to study asset pricing for each country in isolation. Motivated by the empirical observation that currencies in countries with weaker investor protection experience larger depreciations during the East Asian financial crisis, Albuquerque and Wang (2004) generalize the current setup to a two-country world where the productivity shock follows a stationary regime switching process and analyzes the properties of asset prices including the exchange rate over the business cycle. Albuquerque and Wang (2004) show that investor protection has an economically significant effect on the equilibrium exchange rate that can explain the observed large depreciation of currencies in countries with weak investor protection.
Appendices

A Proofs

This appendix contains the proofs for the theorem and the propositions.

A.1 Proof of Theorem 1

The FOC (20) gives

\[ m^{-\gamma} \alpha = A_1 \left( 1 - \epsilon^2 i \gamma \right), \tag{A.1} \]

where \( m = M/K \) and \( i = I/K \) are the controlling shareholder’s equilibrium consumption-capital ratio, and the firm’s investment-capital ratio, respectively. Plugging the stealing function into (6) gives

\[ m = \alpha d + \frac{1 - \alpha^2}{2 \eta} h = \alpha \left( (1 - \phi) h - i + \frac{1 - \alpha^2}{2 \alpha \eta} h \right) \]

\[ = \alpha \left( (1 + \psi) h - i \right), \tag{A.2} \]

where \( d \) is the dividend-capital ratio. Plugging (A.1) and (A.2) into the HJB equation (19) gives

\[ 0 = \frac{1 - \gamma}{1 - \gamma} m^{1-\gamma} - \rho A_1 \frac{1}{1 - \gamma} + (i - \delta) A_1 - \frac{\epsilon^2}{2} i^2 \gamma A_1 \]

\[ = \frac{A_1}{1 - \gamma} \left( (1 + \psi) h - i \right) \left( 1 - \epsilon^2 \gamma i \right) - \rho \frac{A_1}{1 - \gamma} + (i - \delta) A_1 - \frac{\epsilon^2}{2} i^2 \gamma A_1. \]

The above equality implies the following relationship:

\[ ((1 + \psi) h - i) \left( 1 - \epsilon^2 \gamma i \right) = y, \tag{A.3} \]

where \( y \) is dividend yield and is given by

\[ y = \rho - (1 - \gamma) (i - \delta) + \frac{1}{2} \gamma (1 - \gamma) \epsilon^2 i^2. \tag{A.4} \]

We note that (A.3) and (A.4) automatically imply the following inequality for investment-capital ratio:

\[ i < \left( \epsilon^2 \gamma \right)^{-1}. \tag{A.5} \]

This above inequality will be used in proving propositions.

We may further simplify (A.3) and give the following quadratic equation for investment-capital ratio \( i \):

\[ \gamma \left( \frac{\gamma + 1}{2} \right) \epsilon^2 i^2 - \gamma \left[ 1 + (1 + \psi) h \epsilon^2 \right] i + (1 + \psi) h - (1 - \gamma) \delta - \rho = 0. \tag{A.6} \]
For $\gamma > 0$, solving the quadratic equation (A.6) gives

$$i = \frac{1}{\gamma(\gamma + 1)e^2} \left[ \gamma \left[ 1 + (1 + \psi) h e^2 \right] \pm \sqrt{\Delta} \right], \quad \text{(A.7)}$$

where

$$\Delta = \gamma^2 \left[ 1 + (1 + \psi) h e^2 \right]^2 \left[ 1 - \frac{2\gamma(\gamma + 1)e^2 ((1 + \psi) h - (1 - \gamma) \delta - \rho)}{\gamma^2 [1 + (1 + \psi) h e^2]^2} \right].$$

In order to ensure that investment rate given in (A.7) is a real number, we require that $\Delta > 0$, which is explicitly stated in Assumption 3. Next, we choose between the two roots for investment-capital ratio given in (A.7). We note that when $\epsilon = 0$, investment-capital ratio is

$$i = [(1 + \psi) h - (1 - \gamma) \delta - \rho] / \gamma,$$

as directly implied by (A.6). Therefore, by a continuity argument, for $\epsilon > 0$, the natural solution for the investment-capital ratio is the smaller root in (A.7) and is thus given by

$$i = \frac{1}{\gamma(\gamma + 1)e^2} \left[ \gamma \left[ 1 + (1 + \psi) h e^2 \right] - \sqrt{\Delta} \right]. \quad \text{(A.8)}$$

We may also solve for the value function coefficient $A_1$ and obtain

$$A_1 = \frac{m^{-\gamma} \alpha}{1 - e^{2i\gamma}} = \frac{m^{1-\gamma}}{y},$$

where $y$ is the dividend yield and is given by (A.4).

Next, we check the transversality condition for the controlling shareholder:

$$\lim_{T \to \infty} E \left( e^{-\rho T} | J_1(K(T)) | \right) = 0. \quad \text{(A.9)}$$

It is equivalent to verify $\lim_{T \to \infty} E \left( e^{-\rho T} K(T)^{1-\gamma} \right) = 0$. We note that

$$E \left( e^{-\rho T} K(T)^{1-\gamma} \right) = E \left[ e^{-\rho T} K_0^{1-\gamma} \exp \left( (1 - \gamma) \left( i - \delta - \frac{\epsilon^2 i^2}{2} \right) T + eiz(T) \right) \right]$$

$$= e^{-\rho T} K_0^{1-\gamma} \exp \left[ (1 - \gamma) \left( i - \delta - \frac{\epsilon^2 i^2}{2} + \frac{1 - \gamma}{2} \epsilon^2 i^2 \right) T \right]. \quad \text{(A.10)}$$

Therefore, the transversality condition will be satisfied if $\rho > 0$ and dividend yield is positive ($y > 0$), as stated in Assumption 5.

Now, we turn to the optimal consumption and asset allocation decisions for the controlling shareholder. The transversality condition for the minority investor is

$$\lim_{T \to \infty} E \left( e^{-\rho T} | J_2(W(T)) | \right) = 0. \quad \text{(A.11)}$$
Recall that in equilibrium, the minority investor’s wealth is all invested in firm equity and thus his initial wealth satisfies \( W_0 = (1 - \alpha) qK_0 \). Since the minority investor’s wealth dynamics and the firm’s capital accumulation dynamics are both geometric Brownian motions with the same drift and volatility parameters, it is immediate to note that the transversality condition for minority investor is also met if and only if dividend yield \( y \) is positive, as stated in Assumption 5. Moreover, we may verify that the minority investor’s value function is given by

\[
J_2(W_0) = E \left[ \int_0^\infty e^{-\rho t} \frac{1}{1-\gamma} \left( [(1 - \alpha) dK(t)]^{1-\gamma} - 1 \right) dt \right] = \frac{1}{1-\gamma} \left( [(1 - \alpha) dK_0]^{1-\gamma} \frac{1}{y} - \frac{1}{\rho} \right) = \frac{1}{1-\gamma} \left( W_0^{1-\gamma} \frac{1}{y^\gamma} - \frac{1}{\rho} \right), \tag{A.12}
\]

where the second line uses (A.10). Thus, the value function coefficient \( A_2 \) is given by \( A_2 = 1/y^\gamma \). In Section 6.3, we use the explicit formula for the minority investor’s value function \( J_2(W_0) \) to calculate the utility cost of imperfect investor protection.

### A.2 Proof For the Free-Rider Argument in Section 3.3

We elaborate on the details of how our free-rider argument gives rise to a constant ownership structure over time. We use \( J_1(K; \alpha) \) to denote the explicit dependence of the controlling shareholder’s value function on his ownership \( \alpha \). Using the envelope theorem, we have

\[
\frac{d}{d\alpha} J_1(K; \alpha) = E \left[ \int_1^\infty e^{-\rho(s-t)} M(s)^{-\gamma} D(s) ds \left| K(t) = K \right. \right] = A_1 K^{1-\gamma} \frac{d}{dK}, \tag{A.13}
\]

where the last equality uses the functional form of the value function. The derivative in (A.13) describes the increase in the controlling shareholder’s lifetime utility due to a marginal increase of his ownership. This is not his monetary valuation because the controlling shareholder is risk averse (\( \gamma > 0 \)). To derive his monetary valuation, or willingness to pay, we first note that in equilibrium, the controlling shareholder’s stock market wealth is proportional to the firm’s capital stock, in that \( W = \alpha \hat{q} K \), where \( \hat{q} \) is the controlling shareholder’s shadow Tobin’s \( q \) given in Section 5. Using the chain rule, we thus have

\[
\frac{d}{dK} J_1(K; \alpha) = \alpha \hat{q} \frac{d}{dW} J_1 \left( \frac{W}{\alpha \hat{q}} ; \alpha \right) = \alpha \hat{q} \frac{d}{dW} J_1(K; \alpha). \]
Dividing $dJ_1/d\alpha$ by the marginal value function $dJ_1(W)/dW$ gives the controlling shareholder’s willingness to pay for the incremental unit of the newly acquired shares:

$$\frac{d}{d\alpha}J_1(K; \alpha) = \frac{A_1K^{1-\gamma} \frac{d}{dW} \frac{W}{\alpha q}}{\frac{1}{\alpha q} A_1 \frac{W}{\alpha q}} = \alpha \hat{q} \frac{d}{dW} K = qK = P_\alpha,$$

where the next to last equality uses the relationship between Tobin’s $q$ and $\hat{q}$, and $P_\alpha$ is the time $t$ price per share set by minority shareholders. Note that $\hat{P}_\alpha$ as given in Section 5 represents the value of the existing shares for the controlling shareholder and is different from his willingness to pay as given by $P_\alpha$ when acquiring additional shares.

The free rider problem is now apparent. If the equilibrium is for all minority shareholders to sell at $P_\alpha$ then by deviating from this equilibrium, an infinitesimal investor can gain because trading with other minority investors after the trade with the controlling shareholder has taken place yields a higher valuation $P_{\alpha'}$. This higher valuation results from a higher $q$ due to a higher equity share of the controlling shareholder (see Proposition 3). Finally, note that selling by the controlling shareholder is not desirable because he stands to lose the private benefits, but minority investors would also not buy at $P_\alpha$ because trade that occurs in the next instant will be already at a lower price.

### A.3 Proofs for propositions

#### Proof of Proposition 1

Define

$$f(x) = \frac{\gamma (\gamma + 1)}{2} \epsilon^2 x^2 - \left[1 + (1 + \psi) h \epsilon^2\right] \gamma x + (1 + \psi) h - \rho - \delta (1 - \gamma). \quad (A.14)$$

Note that $f(i) = 0$, where $i$ is the equilibrium investment rate and the smaller of the zeros of $f$. Also, $f(x) < 0$ for any value of $x$ between the two zeros of $f$ and is greater than or equal to zero elsewhere. Now,

$$f \left( \gamma^{-1} \epsilon^{-2} \right) = \frac{1 - \gamma}{2 \gamma \epsilon^2} - \rho - \delta (1 - \gamma).$$

Therefore, $f \left( \gamma^{-1} \epsilon^{-2} \right) < 0$, if and only if Assumption 5 is met. Hence, under Assumption 5, $i < \gamma^{-1} \epsilon^{-2}$. Also, under Assumption 1, $f(0) = (1 + \psi) h - \rho - \delta (1 - \gamma) > 0$ which implies that $i > 0$.

Abusing notation slightly use (A.14) to define the equilibrium investment rate implicitly $f(i, \psi) = 0$. Taking the total differential of $f$ with respect to $\psi$, we obtain

$$\frac{di}{d\psi} = \frac{1}{\gamma \left(1 - \gamma \epsilon^2 i\right)} \frac{h (1 - \gamma \epsilon^2 i)}{\left(1 + \psi \left(h - i\right) \epsilon^2\right)}.$$
At the smaller zero of \( f, \ i < \gamma^{-1}e^{-2}. \) Together with \((1 + \psi) h - i > (1 - \phi) h - i = d > 0,\) implies that \( di/d\psi > 0.\)

**Proof of Proposition 2** Differentiate (18) with respect to the agency cost parameter \( \psi \) to get:

\[
\frac{dr}{d\psi} = \gamma \left[ 1 - e^2 (\gamma + 1) i \right] \frac{di}{d\psi}
\]

and note that \( di/d\psi > 0. \) Hence, the interest rate is lower when investor protection improves if and only if \( 1 > e^2 (\gamma + 1) i, \) or using (A.8), if and only if,

\[
\gamma > 2 [(1 + \psi) h - (\gamma + 1) ((1 - \gamma) \delta + \rho)] e^2.
\]

This inequality is true always if \((1 + \psi) h - (\gamma + 1) ((1 - \gamma) \delta + \rho) < 0,\) and otherwise it holds for sufficiently low \( \epsilon, h, \) or \( \psi.\)

**Proof of Proposition 3** We prove the proposition for investor protection. The case for the equity share of the controlling shareholder is then immediate. Use the expression for the dividend yield in (30) to express Tobin’s \( q \) as the ratio between dividend-capital ratio \( d \) and dividend yield \( y. \) Differentiating \( \log q \) with respect to investor protection gives

\[
\frac{dq}{d\eta} = \frac{1}{y} \left[ \frac{y}{d\eta} - \frac{dy}{d\eta} - \left( \frac{d}{y} \right) \frac{dy}{d\eta} \right]
= \frac{1}{y} \left[ \frac{y}{d\eta} - \frac{dy}{d\eta} - q \left( \frac{\gamma - 1}{\gamma - 1} \frac{dy}{d\eta} - \gamma (\gamma - 1) e^2 i \frac{dy}{d\eta} \right) \right]
= \frac{1}{y} \left[ \frac{1 - \alpha}{\eta^2} h - \frac{di}{d\eta} \left( 1 + \frac{1 - \alpha^2}{2\eta\alpha d} h \right)^{-1} \left( \frac{1 - \alpha^2}{2\eta\alpha d} h + \gamma \right) \right] > 0,
\]

where the inequality uses \( \gamma > 0 \) and \( di/d\eta < 0.\)

**Proof of Proposition 4** Differentiate the dividend yield with respect to \( \psi \) to get:

\[
\frac{dy}{d\psi} = \frac{di}{d\psi} \left( \gamma - 1 \right) \left( 1 - \gamma e^2 i \right) \leq 0 \text{ iff } \gamma \leq 1,
\]

and note that the agency cost parameter \( \psi \) decreases with both investor protection and \( \eta \) and ownership \( \alpha.\)
Proof of Proposition 5  Weaker investor protection or lower share of equity held by the controlling shareholder both lead to a higher agency cost parameter $\psi$. Proposition 1 shows that a higher $\psi$ leads to more investment hence higher volatility of stock returns $\sigma_P^2 = \epsilon^2 i^2$ and higher expected excess returns $\lambda = \gamma \sigma_P^2$. Finally, we get that the change in total returns when investor protection changes depends on $\gamma$. From (29):
\[
\frac{d(\gamma \epsilon^2 i^2 + r)}{d\psi} = \gamma (\epsilon^2 i + 1 - \epsilon^2 i \gamma) \frac{di}{d\psi},
\]
which is strictly positive under Assumption 5. Expected returns are higher with weaker investor protection or lower share of equity held by the controlling shareholder.

Proof of Proposition 6  Differentiating $\log \zeta$ with respect to $\eta$ gives
\[
\frac{d \log \zeta}{d \eta} = \frac{d \log d}{d \eta} + \frac{1}{1 - \gamma} d \log y
\]
\[
= \frac{d \log d}{d \eta} + \frac{1}{1 - \gamma} y \left( (\gamma - 1) \frac{di}{d\eta} - \gamma (\gamma - 1) \epsilon^2 i \frac{di}{d\eta} \right)
\]
\[
= \frac{d \log d}{d \eta} - \frac{di}{d\eta} y (1 - \gamma^2 i).
\]
Using $1 - \gamma \epsilon^2 i > 0$ and $di/d\eta < 0$, the results reported in Proposition 1, and $d \log d/d \eta > 0$, implies $d \zeta/d \eta > 0$. 

References


Figure 1: Tobin’s $q$. 
Figure 2: Ratio of the risk premium under imperfect investor protection to the risk premium in the benchmark case of perfect investor protection.
Figure 3: Utility cost of imperfect investor protection, $1 - \zeta$ (percent of wealth).