Blocks, Liquidity, and Corporate Control

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ABSTRACT

The paper develops a simple model of corporate ownership structure in which costs and benefits of ownership concentration are analyzed. The model compares the liquidity benefits obtained through dispersed corporate ownership with the benefits from efficient management control achieved by some degree of ownership concentration. The paper reexamines the free-rider problem in corporate control in the presence of liquidity trading, derives predictions for the trade and pricing of blocks, and provides criteria for the optimal choice of ownership structure.

The recent incomplete contracting approach in corporate finance has considerably improved our understanding of how small firms determine their capital structure. The basic setting considered in this line of research is one where a founder-manager seeks funding from one or several financiers. The main premise is that the founder-manager, in her dealings with the financiers, is primarily concerned with maintaining her private benefits of control. For small firms these are often quite large relative to the financial returns. Thus, for a small firm the problem of determining the financial structure often reduces to the problem of how to obtain funding while giving away as little control as possible to the financiers. Of course, most financiers insist on some form of protection, so that the final compromise reached in most financial contracts for small firms is one resembling a debt contract (or a venture capital contract), which protects the founder-manager’s control as long as the firm is performing adequately.¹

This perspective for small firms does not extend naturally to large firms because the private benefits of control of the managers for large firms are likely to be small relative to the firm’s monetary returns, so that protection of these benefits is not an overriding consideration. Moreover, large firms

¹ For a recent discussion of this literature see Hart (1995).
tend to have more dispersed ownership and therefore less effective investor control. If anything, the main control problem for large firms seems to be how to get investors or shareholders to exert more control. Indeed, this is the problem on which the recent literature on corporate governance has focused.\textsuperscript{2}

Interestingly, there is little variation across nations in the capital structure of small firms, but for large firms the problem of inducing more effective investor control has been tackled very differently. In broad terms, there have been two different responses. In the United States and the United Kingdom primarily, share-ownership in large firms is widely dispersed, turnover is high, and investor control is often exerted through the threat of takeovers. In other countries, share-ownership is more concentrated, turnover is much lower, and control is exerted, if at all, by the largest shareholders (or debtholders). Three countries that exemplify this latter type of financial system are Germany, France, and Japan.\textsuperscript{3}

When evaluating the differences of these two types of systems, it is important to realize that the issue is not whether ownership concentration per se is desirable or not. The issue rather is how often and at what points in a firm’s life ownership should be concentrated. One alternative for the design of ownership structure is persistent ownership concentration, wherein a large blockholder is expected to exercise control of management continuously; the other is ownership dispersion, with reliance on secondary market trading to create concentration whenever necessary for intervention in managerial decision making. This paper develops a framework in which the costs and benefits of the two systems—dispersed ownership combined with a takeover mechanism on the one hand, and large controlling blocks (or "noyaux durs"), on the other—can be evaluated. The benefits of dispersion are mainly greater market liquidity and better risk-diversification.\textsuperscript{4}

Several empirical studies have documented that the liquidity of a stock is larger the larger the market capitalization of the firm. The explanations usually given for this finding relate in one way or another to the presence of transactions costs, which limit market participation.\textsuperscript{5} Thus, for example, if market makers face set-up costs, the number of market makers dealing in a stock will be increasing—and, therefore, liquidity will be increasing—in the market capitalization of that stock. Similarly, if market participants face information asymmetries, the number of investors willing to invest in infor-

\textsuperscript{2} Surveys of recent research on corporate governance can be found in Kojima (1995) and Shleifer and Vishny (1997). For a discussion of the recent policy debate on corporate governance see also The Economist (1994).

\textsuperscript{3} For a description of the main differences between the financial systems of the United States and the United Kingdom, on the one hand, and of Germany, France, and Japan, on the other, see Franks and Mayer (1994) and Kojima (1995).

\textsuperscript{4} The model we develop focuses on the liquidity benefits of dispersion. We abstract from risk-diversification issues mostly to keep the model tractable. For an analysis of the impact of risk sharing on corporate control see Admati, Pfleiderer, and Zechner (1994).

\textsuperscript{5} See, for example, Demsetz (1968), Merton (1987), Pagano (1989), and Allen and Gale (1994).
information acquisition in a particular stock will be increasing in the anticipated
gains from trade and, hence, in the stock’s market capitalization.

This argument immediately implies that when a firm decides to set up a
controlling block, it reduces the number of shareholders who can participate
in the trading of the firm’s stock and, therefore, it effectively reduces the
market capitalization, and hence the liquidity, of its stock. This is the prin-
cipal mechanism relating concentration and liquidity in our model.

Besides limited market participation by potential shareholders, the model
builds on two other key assumptions. First, in the spirit of Shleifer and
Vishny (1986), it is assumed that some degree of ownership concentration
improves control of management and, therefore, can increase firm value.
There is a large body of cross-sectional evidence in support of this assump-
tion, but there are also well-known counterexamples. These include cases of
large owners who have managed their companies for very long periods (John-
son et al. (1985)), or of management-affiliated blockholders who allow man-
agement to dilute share value away from other shareholders (Wruck (1989),
Slovin and Sushka (1993)). We are aware of these cases, but we do not con-
side them in this paper.

Second, it is assumed that by exercising control a shareholder incurs net pri-
cate costs. This assumption may appear more restrictive than the first one, in
light of some of the empirical evidence on the pricing of corporate control trans-
actions. Barclay and Holderness (1989), for example, find that large blocks of
shares mostly trade at a premium, which, they argue, results from private ben-
fits accruing to the blockholder. As we discuss in Section II, however, the em-
pirical evidence on block pricing reflects several different elements, of which
the costs of controlling management are only one. Indeed, our main justifi-
cation for focusing on the costs rather than the benefits of control is empirical.
If private benefits of control were, in general, more important than private costs,
corporate governance would be a relatively minor problem; the real problem
would be to select among the bidders for large blocks of shares who volunteer
to control management. This is not the typical situation we observe in the United
States, to which the above-mentioned studies refer—the importance of block-
holders with private benefits of control notwithstanding.

Building on these assumptions, our model yields several simple and inter-
esting insights. In particular, the analysis shows that a firm is more likely to
choose a dispersed ownership structure, other things being equal, if there is
more active trading in its secondary market and if regulation facilitates take-
overs as a means to acquire control. Complementing this, we find that greater
liquidity trading in the secondary market by itself facilitates takeovers by
reducing free riding. These observations shed some new light on the issue of

6 For publicly traded firms in the United States, see, for example, Wruck (1989) or Hertzel
and Smith (1993). For evidence on French and German listed companies, see Franks, Mayer,

7 Barclay and Holderness (1989), in view of the sizeable number of discounts in their sample,
“suggest that block ownership involves private costs as well as private benefits.”
why the United States and the United Kingdom have such different capital and governance structures in large firms from those observed in Germany, France, or Japan. In particular, in the United States and the United Kingdom the takeover mechanism is much less restricted, there are stricter legal restrictions on institutional equity holdings, and secondary market participation by institutional investors is greater and more frequent. All of these features favor ownership dispersion in our model.

More generally, our analysis shows that both types of ownership structure—concentrated or dispersed—can be optimal, depending on the characteristics of the firm and the environment in which it operates. Our comparative statics results show that higher average liquidity demand by investors, lower costs of controlling management, higher potential benefits from correcting managerial failures, and higher transactions costs for secondary market trading all favor a dispersed ownership structure, and vice versa for concentration. Our model is set up to apply in principle to both listed and unlisted firms. Concerning unlisted firms, it is therefore interesting to note that the above characteristics hold—at least to a first approximation—for law and accounting partnerships, which are well-known examples of private firms with dispersed ownership structures (see, e.g., Hansmann (1995)).

For publicly listed companies, where empirical data are best, our model provides a new explanation for why equity block purchase agreements usually are priced at sizeable discounts, but trades of existing blocks are not. Our explanation is that single shares and blocks, when traded, must reflect the anticipated costs of corporate control equally, because by arbitrage, one type of seller cannot be treated differently from another. Anticipating that he will not be fully compensated for holding the block in future trading, however, a block purchaser will demand a discount at the issue stage.

There is a large literature related to several aspects of our model. This literature almost exclusively is concerned with large listed companies, whereas the argument of the present paper applies to privately held firms as well. A closely related paper, by Pagano and Röell (1998), considers a privately held firm with a large shareholder which needs to increase its capital base by bringing in new shareholders. The firm has the option of remaining private and introducing another set of large shareholders, or going public and getting a new set of small shareholders. The benefit of the former option is that the firm does not have to pay listing costs. The downside is that there may be excessive monitoring by large shareholders (as in Burkart, Gromb, and Panunzi (1997)) and the stock is less liquid.

Merton (1987) introduces asymmetric information in the standard CAPM to obtain a model in which investors only trade in the limited number of shares

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8 In the United States, actively managed pension funds hold shares for an average of only 1.9 years (Kojima (1995)), and on the NYSE, for example, institutions account for 72 percent of trading volume.

9 See Barclay and Holderness (1989) and Hertzel and Smith (1993) for descriptions of the empirical puzzle.
about which they have information. His model gives rise to the type of limited market participation we consider in the present paper. In fact, our model of secondary market trading can be interpreted as a reduced form of Merton’s model for one single stock with an additional emphasis on corporate control issues.

Admati et al. (1994) take a similar view to ours on the benefits of large shareholder intervention, and ask to what extent these benefits can be realized by risk-averse investors who optimize their portfolio choice. Kahn and Winton (1998) argue that market liquidity can undermine effective control by a large shareholder by giving him excessive incentives to speculate rather than monitor. In contrast, Holmström and Tirole (1993) argue that insider trading by a large shareholder provides incentives to monitor management but results in a higher cost of capital for the firm.

Recent papers that are related to our analysis of the free-rider problem are Zingales (1995) and Gromb (1993), who show how free riding by small shareholders can be used to increase the ex ante value of the firm. Also, Bebchuk (1994) considers the problem of the trading mechanism of controlling blocks when large shareholders obtain both monetary and private benefits of control; he contrasts the equal opportunity rule (according to which minority shareholders can participate in the sale of a block on the same terms as large ones) and the market rule (where no distinction is drawn between shareholders according to the size of their holdings).

In interesting historical studies, Coffee (1991), Bhide (1993), and, more comprehensively, Roe (1994) argue that political considerations have helped shape the development of U.S. capital markets toward higher liquidity and lower investor activism, thus providing evidence for one manifestation of the trade-off studied in our paper. We come back to Bhide’s study in Section III below.

Finally, it is worth mentioning a recent study by La Porta et al. (1996), who take quite a different perspective on the observed international differences in ownership concentration. They regress ownership concentration on several measures of law and law enforcement in different countries and argue that the lack of adequate minority shareholder protection in Germany, France, and Japan relative to the United States (and the United Kingdom) may be a cause for the differences in financial systems. We shall discuss more of the relevant empirical literature in the main body of the paper.

The organization of the rest of this paper is as follows: Section I describes our model. Section II analyzes the two polar forms of ownership structure relevant in the present model, and Section III evaluates the trade-off between the two. Section IV concludes with some remarks about the scope of the analysis.

I. The Model

Consider a firm that operates for two time-periods, $t = 1, 2$. At time $t = 0$, the original owners want to sell the firm and issue shares, which give rights to the firm’s future returns. We assume that because of fixed trading costs these claims will not be traded in arbitrarily fine units, so that the effective
number of shares, $M$, is finite. $M$ is a constant to the model, determined by the costs of trading.

There are a large number of potential investors in the firm’s shares. To simplify our analysis, we assume that investors are sufficiently wealthy to buy up the whole firm if they wish.\textsuperscript{10} Each investor $i$ can invest in a safe, liquid asset whose net rate of return is normalized to zero, or in shares of the firm. If there are $n$ investors each buying $h_{0}^{i}$ shares at $t = 0$, the initial ownership structure is $(h_{0}^{1}, \ldots, h_{0}^{n})$, with $\sum_{i=1}^{n} h_{0}^{i} = M$. We denote the price of the firm at the issue date $t = 0$ by $V_{0}$.

Even if investors can afford to buy the whole firm they may not want to, for liquidity reasons. Following Gorton and Pennacchi (1990) and others, we model investors’ demand for liquidity as follows. Each investor has liquidity needs in the future, which arise in either period $1$ or $2$. At time $t = 0$, the timing of these needs is unknown to everybody, and all investors have the following identical expected utility from consumption in periods $t = 1$ and $t = 2$:

$$u(c_{1}, c_{2}) = qc_{1} + (1 - q)c_{2}. \quad (1)$$

When investors buy shares, they do not know exactly when they will want to consume. If they want to consume early (which happens with probability $q$), then they have to sell their shares; if they want to consume later, they may buy more shares. The consumption shocks of investors are assumed to be independent. Thus, even though investors are identical ex ante, they generally have different liquidity needs ex post, and $qV_{0}$ can be interpreted as average liquidity demand by existing shareholders. We shall refer to investors who want to consume early as “impatient” and to those who want to wait until period $t = 2$ as “patient.” The preferences specified in (1) provide the simplest possible specification of liquidity preferences.\textsuperscript{11}

The trading of shares following the liquidity shocks in $t = 1$ typically depends on the institutional framework in which the firm operates. The specific structure of trading is largely irrelevant for our analysis, and we choose the simplest: We assume that after preference shocks have been realized, aggregate liquidity demand becomes publicly known and the following three-stage bidding game is played:

1. Patient investors simultaneously make unrestricted offers for shares.
2. If there are two or more identical offers, one is selected at random.
3. All owners can tender their shares to the selected buyer at the offered price.

\textsuperscript{10} Introducing wealth constraints is conceptually not difficult, but technically somewhat complicated, and does not change our main results. We discuss wealth constraints as we go along in the analysis and in the conclusion.

\textsuperscript{11} In particular, these preferences exhibit risk neutrality with respect to intraperiod consumption. We could have used a more general specification without, however, gaining much further insight. See Jacklin (1987) and von Thadden (1994) for a discussion of more general approaches to modeling liquidity preferences.
This mechanism implies maximum competition between potential buyers and resolves the coordination problem between identical bids which can cause existence problems. Furthermore, outside investors and existing owners bid for shares on equal terms.

Having defined liquidity needs and trading, we now turn to the firm’s return structure and, consequently, to the problem of corporate control. We assume that the firm has stochastic returns in \( t = 1 \) and \( t = 2 \) which are both determined and publicly observed in \( t = 1 \) and, for simplicity, assumed to be identical (all we use in the analysis is that first-period returns are informative about second-period returns). With probability \( \pi \) the firm earns high returns, \( y = R \), in both periods, and with probability \( 1 - \pi \) returns are low, normalized to \( y = 0 \). However, in the bad return state the firm can be reorganized, which generates an added value \( L, R > L > 0 \), in period \( t = 2 \).

The problem of corporate control arises from the fact that after the sale of the firm in \( t = 0 \), the firm is run by managers who always prefer continuation to reorganization. Therefore, reorganization can only be brought about by outside intervention, which typically costs time and resources. We model this by assuming that an owner who intervenes in the bad state improves firm value from 0 to \( L \) in \( t = 2 \), but incurs net private costs of \( C < L \). With efficient intervention, the firm’s value is, therefore, given by

\[
V^* = \pi 2R + (1 - \pi)(L - C). \tag{2}
\]

We denote per-share values by lowercase letters, e.g., \( r = R/M \), \( v^* = V^*/M \), etc. Because it is typically difficult to get a group of owners to act collectively in the common interest (Diamond (1984)), we assume that intervention costs cannot be shared among owners, but must be borne individually. This implies that only a single owner with a large enough stake in the firm will intervene in the bad state. In other words, intervention will occur only if its private gain exceeds its cost.

A final and key assumption, which makes the model nontrivial, concerns the secondary market for shares. As discussed in the introduction, market participation for any given stock is typically limited by information costs. In the present context this means that there are only a limited number of investors who are able to value and willing to buy shares from existing shareholders when trading at \( t = 1 \). We model this by assuming that outside investors (those who have not bought shares at \( t = 0 \)) are less willing to

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\(^{12}\) Because we assume below that an owner can invest a fixed cost in order to control management, the number of shares bought either in Walrasian trading or under standard Bertrand competition determines profits. Therefore, without a coordination stage such as 2, the payoff functions of market participants are not upper-hemicontinuous. The assumption that liquidity demand becomes known before trading simplifies the analysis without qualitative consequences.

\(^{13}\) These managers may well be among the owners of the firm. The important point is that their private interest in running the firm is stronger than the value of cash flow improvements achievable by reorganization.
intervene in the bad state and, therefore, are only ready to bid 0 in that state. Hence, in the bad state, they provide liquidity only at a discount. This assumption is stronger than necessary for our analysis, but it allows us to capture in a simple way the notion that selling out and “active investing” can be conflicting policies, with potentially different impacts on firm value.\(^\text{14}\)

We close the description of the model by recapitulating the timing of activities in the time line shown in Figure 1.

**II. Concentration versus Dispersion**

As noted previously, only an owner who internalizes the costs of control will intervene and force management to take the efficient continuation/reorganization decision in the bad return state. This is the case if and only if his shareholding \(h^t\) after trading in \(t = 1\) satisfies

\[
h^t l \geq C.
\]

Let \(T\) denote the smallest solution to this inequality, which we take for simplicity to be integer. It is worth pointing out that the fraction \(T/M\) necessary to internalize the cost of controlling management may be well below the threshold of 50 percent, or other majorities required formally for full corporate control. In the United States, for example, blocks of 10–20 percent, if established in a publicly traded company, already can be associated with significant share price increases due to improved corporate control.\(^\text{15}\) In fact,

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\(^{14}\) The notion of limited market participation can be modeled by different assumptions of different sophistication. We have chosen the simplest one. One alternative, as in Merton (1987), which gives slightly more complex formulas and the same results, would be to assume that only a limited number of investors can distinguish the good and the bad return states.

\(^{15}\) See, for example, Wruuck (1989) and Barclay and Holderness (1991). As these studies show, usually the share price response depends on the nature of the large blockholder.
if applied to publicly listed U.S. companies, our model should be restricted to blocks of this size or smaller. As Barclay and Holderness (1989) document for large block trades of NYSE and AMEX firms, block premia in such trades can be substantial for blocks of more than 25 percent, suggesting the existence of private benefits, but are smaller for blocks below 25 percent. Because our model focuses on the private costs of corporate control rather than on the benefits, the model should be restricted to smaller block sizes when applied to the group of firms studied by Barclay and Holderness (1989).

The key determinant of the trade-off between liquidity and control in our model is that achieving a certain level of concentration reduces the number of owners. Because outsiders only buy up shares at a discount in the bad state, this, in turn, reduces the number of trading partners in the secondary market who are ready to provide liquidity in all states. Hence, concentration of ownership, although it improves the incentives for the control of management, reduces valuable trading possibilities for all shareholders.

On the other hand, maximum dispersion of shares among initial owners increases trading opportunities among those who potentially will pay the full price for shares in the bad state, but may lead to a value-reducing lack of control because of ownership dispersion. Notice that dispersion in \( t = 0 \) does not necessarily imply that there is dispersion in \( t = 1 \), after liquidity trading. Indeed, one of the patient owners may buy enough shares in the bad state to assemble a stake that makes it profitable for her to intervene. Thus, the old owners, when selling out at \( t = 0 \), have two principal options: They create a controlling block of shares, hoping that it persists, or they disperse the initial ownership, which creates more trading opportunities in secondary trading, with the hope that a controlling block emerges when necessary in the secondary market.

In this section we consider these two alternatives in turn and provide pricing formulas for each of them. In the next, we determine under what circumstances either of these ownership structures dominates.

\section*{A. Concentrated Ownership}

Maximum dispersion of shares is obtained when there are as many owners as shares, \( M \). With a controlling block the maximum number of owners is \( M - T + 1 \), because at least one investor holds a block of at least \( T \) shares. Yet, when the firm creates a controlling block at date \( t = 0 \), a priori it is not clear that the initial controlling owner will actually control the manager at date \( t = 2 \) and enforce the efficient continuation/reorganization decision.

The reason is that an initial large owner may himself be impatient, in which case he is only concerned about selling his shares at the highest price. In this case it is possible in principle that he disperses his block in such a way that no large owner emerges ex post. However, under the trading structure considered here and under the assumption of no wealth constraints, concentration will necessarily persist in the secondary market. More generally, the reason for this result is the assumption of nonanonymous trading.
This assumption reflects the existence of stringent disclosure rules for publicly traded firms in most stock markets and the importance of private communication between owners in privately held firms.

As long as trading is nonanonymous, any attempt by the large owner to unload his block of shares with several buyers would immediately be reflected in the price, which in turn would remove the incentive to disperse the block. Hence, if there is at least one patient shareholder, the large owner either sells his shares as a block or keeps them. In both cases the large block persists, and the efficient continuation/reorganization decision is taken. If none of the initial shareholders is patient, the block and all other shares must be sold to outsiders, in which case the problem of ex post concentration is irrelevant.

If the initial ownership structure consists of one large block and a fringe of small owners, ex post trading in shares has the following structure.

**Lemma 1:** Suppose the firm in \( t = 0 \) has \( M - T + 1 \) owners, one holding \( T \) shares and the others holding one share each. If in \( t = 1 \) there are at least two patient owners, then shares trade at the price

\[
b(I, r) = r
\]

in the good return state, and

\[
b(I, 0) = l - \frac{C}{T + I}
\]

in the bad state, where \( I \) is the number of impatient small owners. In the bad state, the block is preserved, and the blockholder’s ex post position is worth \( Tb(I, 0) \), regardless of whether he is patient or impatient.

**Proof:** In the good state, corporate control is not an issue and (3) holds trivially. Suppose that \( y = 0 \), the large owner is patient, and at least one small shareholder is patient, as well. Then bids by outsiders are irrelevant, because bidding among insiders alone will drive the price up beyond what outsiders are ready to pay (that is, 0).\(^{16}\) If a small patient shareholder wins the bidding with a bid of \( b \), the maximum number of shares he will attract is \( T + I \) (the block and all impatient shares), which will be worth to him \((T + I)(l - b) + l - C\). Hence, the optimal bid by the large shareholder, \( b(I, 0) \), keeps the small patient owners indifferent between bidding or not and satisfies:

\[
(T + I)(l - b(I, 0)) = C,
\]

\(^{16}\) We assume throughout that bidders do not use weakly dominated strategies, a standard assumption in auction theory.
which implies (4). By (5), the large owner’s profit from making this bid and 
attracting the impatient shares is the same as the profit from selling his 
stake at this price, namely $T b(I,0)$.

If the large owner is impatient and $I \leq M - T - 2$, the optimal bid by a 
small owner satisfies (5) by the same argument. Q.E.D.

As simple as it is, the lemma contains several interesting insights. First, 
the large block is not broken up in liquidity trading, except possibly for the 
extreme case of every owner being impatient. This is consistent with the 
empirical evidence. Barclay and Holderness (1989), for example, note that 
“the limited evidence available suggests that large blocks are seldom broken 
up.” Second, expected future intervention costs are partially borne by the 
small impatient shareholders. Single shares in the bad state trade at less 
than their ex post value, and patient shares are not traded at all.

Third, if in the bad state the large block is sold, it trades at the same price 
as single shares. This is due to the simple trading structure assumed in this 
model, which excludes the emergence of several blocks and essentially im-
poses a nonarbitrage condition on share trades of different size. In reality, 
things are more complex, as shown by the empirical evidence on premia on 
large block trades for publicly listed companies in the United States (most 
importantly, see Barclay and Holderness (1989)). These studies find large 
variations of premia and discounts and positive median block premia.\footnote{17} 
Furthermore, block size plays a role. When regressing block premia on block 
size with a sample split at 25 percent, Barclay and Holderness (1989) find a 
large and significant point estimate for block sizes above 25 percent, and a 
negative, though insignificant coefficient below 25 percent.\footnote{18} As noted 
previously, for this group of firms our focus on private costs, rather than ben-
efits of control, is therefore only justified when blocks are not too large.

In addition to the focus on private costs, the block trades considered in our 
model are seller-, and not buyer-, initiated. The available evidence (see, e.g., 
Holthausen, Leftwich, and Mayers (1987)) suggests that seller-initiated trades 
are accompanied by significantly lower, sometimes even negative, block pre-
mia. Note, however, that Holthausen et al. only consider blocks that are 
large in dollar terms, and typically not in percentage terms. We know of no 
study distinguishing between seller- and buyer-initiated trades of large per-
centage blocks. Yet, such differences seem to be plausible also for those blocks, 
given the wide variations of block discounts/premia documented by Barclay 

Hence, our model does not incorporate all elements of block pricing. It would 
be quite possible to include bargaining over block sales and buyer-initiated trades

\footnote{17}{For their sample of block trades of NYSE and AMEX firms between 1978 and 1982, Bar-
clay and Holderness (1989) find a median premium for large blocks of shares of 15.7 percent, 
and block discounts in 20 percent of the transactions.}

\footnote{18}{These estimates refer to block premia per share (their Table V, regression 4), which is the 
measure considered in Lemma 1.}
into the model (e.g., by introducing outsiders with random private benefits in \( t = 1 \), as in Grossman and Hart (1988)), which would generate positive average block premia.\(^{19} \) Yet, we have preferred to focus on the simpler model with only liquidity trading.

**Proposition 1:** The ex ante value of the firm with at least one large owner is maximized when there is exactly one large owner with a stake equal to \( T \) and all other owners hold only one share. The block is priced at a discount, and the firm’s ex ante value is

\[
V_0^c = 2\pi R + (1 - \pi)(1 - q^{M_T+1})(L - C). \tag{6}
\]

**Proof:** To have more than one large owner reduces liquidity and does not provide any improvement in the ex post reorganization/continuation decision. To have a single large owner with a stake strictly greater than \( T \) reduces liquidity and again does not improve ex post efficiency. This establishes that firm value is maximized by having exactly one large owner with a stake \( T \). By Lemma 1, the expected value of the large owner’s block, conditional on the bad state occurring, is

\[
v_B = (1 - q)q^{M_T}(L - C) + [1 - q^{M_T}(M - T + 1) + q^{M_T+1}(M - T)]Tl
- \sum_{i=0}^{M_T} \binom{M - T}{i} q^i(1 - q)^{M_T-i} \frac{TC}{T + i}
- (M - T)q^{M_T}(1 - q) \frac{TC}{M - 1} + q^{M_T} \frac{TC}{M}, \tag{7}
\]

and that of a single share

\[
v_S = (1 - q)q^{M_T}(L - C) - (M - T - 1)q^{M_T-1}(1 - q)^2 \frac{C}{M - 1}
- \sum_{i=0}^{M_T-3} \binom{M - T - 1}{i} q^{i+1}(1 - q)^{M_T-i-1} \frac{C}{T + i + 1}
+ [1 - (M - T - 1)q^{M_T-1}(1 - q) + (M - T)q^{M_T-1}(1 - q)^2
- q^{M_T-1}]l. \tag{8}
\]

\(^{19} \) An extension of our model in this direction would introduce a separate market for blocks and assume the possibility of pure (costless) diversion from minority shareholders to blockholders in the good state. Under appropriate parameter assumptions, in such a model the median block would trade at a premium and still command a discount at the issue stage.
Because $V_0 = v_B + (M - T)v_S$, (6) follows from (7) and (8) by direct computation. A further straightforward computation shows that

$$Tv_S - v_B = (T - 1)(1 - q)q^{M-T}(L - C) + \sum_{i=0}^{M-T} \left(1 - \frac{i}{M - T}\right)\binom{M - T}{i}q^i(1 - q)^{M-T-i} \frac{TC}{T + i} + q^{M-T}(1 - q) \frac{TC}{M - 1},$$

which is positive. Q.E.D.

Thus, the liquidity cost of having a large owner at date $t = 0$ is measured entirely in terms of the implied reduction in the number of owners from $M$ to $M - T + 1$. The simplicity of the expression for $V_0$ comes from the assumption of no wealth constraints. With such constraints, an additional cost of creating a large block is that this block may have to be split up for some realizations of liquidity shocks, thus eliminating the efficiency gains relating to corporate control.

The fact that the block is priced at a discount at the issue date may seem surprising because, by (3) and (4), the shares will trade at their ex post value in the good state and in the bad state impatient owners will subsidize the blockholder for the future monitoring costs. Because everyone has the same liquidity preferences ex ante, one might expect the costs to be borne equitably at the issue date, and therefore to observe no block discount. This is not the case, however, for two reasons—both of which are reflected in equation (9). First, as shown in Lemma 1, small impatient shareholders only bear part of the expected monitoring costs ex post. For the remaining part of these costs, the large blockholder must be compensated ex ante. This corresponds to the second term on the right-hand side of (9).

The second component of the block discount is due to an inherent public goods problem in the design of ownership structure. From the point of view of the individual investor, a minimum ownership stake in $t = 0$ is desirable, because this maximizes his liquidity benefits without impairing corporate control ex post. This feature of free riding on liquidity provision is quite general; it manifests itself in the present model through the fact that every owner, regardless of his stake, has the same chance of benefiting from liquidity trading ex post. In order to persuade an investor to take on more than the one share necessary to obtain this benefit, such shares must receive an additional discount, which is given by the first term on the right-hand side of (9). It is in this sense that the blockholder must be compensated for reduced liquidity in the present model.

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20 See, for example, Bhattacharya and Gale (1987) for an analysis of the public goods problem in liquidity provision in interbank lending.
Empirically, the discounting of blocks at the issue date is well documented. Wruck (1989) finds small but significant discounts for block equity placements of NYSE and AMEX firms.\textsuperscript{21} Consistent with the prediction of Proposition 1, she suggests that the pricing reflects compensation for reduced liquidity and for "factors other than reduced liquidity."

In a similar vein, for a sample of private equity placements also covering OTC firms, Hertzel and Smith (1993) report mean discounts of 20.1 percent. They, too, "think it unlikely that a pure illiquidity effect can explain the magnitude of the discounts we find," and suggest as an additional explanation, in the spirit of Myers and Majluf (1984) and slightly different from our explanation, costly monitoring by the blockholder at the issue date. In a quite different institutional context, Molin (1996) reports a mean discount of 15.9 percent on private equity placements on the Stockholm Stock Exchange. As another possible application of the above argument, it has been pointed out to us that holding companies in Belgium and Germany generally trade at a discount to the value of their portfolios. We do not have the data to verify this systematically. If this is true, however, it provides another manifestation of the discount established in Proposition 1.\textsuperscript{22}

\textit{B. Dispersed Ownership}

When there is no concentration at date $t = 0$, the main question is whether an owner who is large enough to control management emerges after trading in the secondary market in the bad return state. The main factor preventing the emergence of a controlling block at date $t = 1$ is free riding by small patient owners. Small patient owners are willing to sell their shares to a large owner only at the ex post value of these shares. Hence, if in the bad state a controlling block were to emerge with probability $1$, then small patient owners would only be willing to sell their shares at a price $l$. But no bidder would offer to buy shares at that price in order to build a controlling block because he needs to be compensated for the intervention cost $C$. For this reason, the emergence of a controlling block in secondary trading is not guaranteed. There is, however, an important factor favoring the emergence of a controlling block: liquidity selling by the impatient owners. To determine when and how a controlling block emerges, we need to study in detail the response of individual owners to buy offers in secondary trading.

The key observation here is that a small owner considering a selling decision is aware that the decision will have an effect on the overall probability of success of a tender offer. Owners in our model are not atomistic; thus,

\textsuperscript{21} For reasons we do not know, Wruck (1989) measures the discount with respect to the market price one day before the announcement date. Given that average abnormal returns from day $-1$ to day 0 alone in her sample are 1.9 percent, her numbers underestimate the block discount as more naively defined.

\textsuperscript{22} Because holding companies typically provide real control services, their case is different from the more controversial case of closed-end mutual funds (see, e.g., Lee, Shleifer, and Thaler (1991)).
even if there are many owners, each owner is aware that he may be pivotal with some probability. As is well known from the theory of takeovers (see Kovenock (1984), Bagnoli and Lipman (1988), and Holmström and Nalebuff (1992)), when owners are pivotal there are many Nash equilibria in such tender games. However, one important feature distinguishes our model from this literature: Owners differ according to their impatience to consume. As in the case of concentrated shareholding discussed above, impatient owners will tender their shares with probability 1. But the best response of patient owners is not necessarily unique.

Our problem differs in another way from the existing literature in that we have competition between identical potential bidders as soon as there are at least two patient owners. Thus, in our model the equilibrium tender offer at date \( t = 1 \) must be such that the owner making the offer is indifferent between remaining a small patient owner and attempting to build a controlling interest. As will be shown below, in the bad state this adds a binding restriction on trading.

In the good state, regardless of liquidity demand, shares trade at their ex post value, \( r \), and their allocation is irrelevant for the value of the firm. In the bad state, however, because of the need for intervention, we have to distinguish two cases of liquidity trading: one where the number of impatient owners is at least \( T - 1 \), and the other where it is strictly less than \( T - 1 \).

In the first case equilibrium is easy to characterize. If all owners are impatient, they sell out to an arbitrary outsider at the competitive price 0. If there is exactly one patient owner, he also bids 0 (or slightly more) and acquires the firm at a discount reflecting his insider position. More interestingly, if the number of impatient owners, \( I \), satisfies \( T - 1 \leq I \leq M - 2 \), a controlling block can still be built solely with impatient shares, and therefore emerges with probability 1. Each patient owner, by remaining small, gets \( l \) in the bad state. Because the successful bidder makes a bid strictly below the ex post value of the share, all patient owners hold on to their shares. On the other hand, all impatient owners sell. Thus, if the outstanding bid is \( b \), the new controlling owner gets \( l(I + 1) - C - bI \). The equilibrium bid is then given by \( l(I + 1) - C - bI = l \), which implies

\[
b(I,0) = l - \frac{C}{I}.
\]

The equilibrium bid \( b(I,0) \) has exactly the same structure as the one in the case of concentrated shareholding, equation (4). Whereas with concentrated shareholding the controlling stake exists ex ante and is known to persist, here such a stake does not exist, but is known to emerge with certainty in the bad state. Notice that the controlling interest emerges despite free riding by all the patient owners. Ironically, the intervention cost is entirely borne by the impatient owners. In effect, impatient owners subsidize one patient owner to exercise control in the firm in the future, so that they can sell out
in the present. This yields an interesting twist to the famous thesis of “exit, voice, and loyalty” (Hirschman (1970)) as possible responses to organizational failures. Here, owners wishing to sell out can indeed choose the “exit” option. However, although the firm’s future does not interest them anymore, they have to pay the price of future corporate control; “exit” is costly.

In the case where the number of impatient owners is smaller, \( I < T - 1 \), things are more complicated. In this case there are several Nash equilibria in the tender game between patient owners in the bad state; therefore, we shall follow the literature and focus on the unique symmetric mixed strategy equilibrium. Note that, because we suppose that there is maximum dispersion ex ante, each owner has only one share to sell at date \( t = 1 \). Thus, all patient owners are identical. Therefore, it is quite reasonable to suppose symmetric equilibrium behavior by patient owners.

We begin by introducing some notation. Suppose one patient owner makes an offer to buy shares in the bad state. Let \( P = M - I - 1 \) denote the number of the remaining patient owners. As indicated above, let \( m \) be the (symmetric) equilibrium probability of selling for each of these \( P \) owners. Let \( S \) denote the number of patient shares sold in equilibrium. \( S \) is a random variable on \( \{0, \ldots, P\} \) with density

\[
\sigma^P(S;m) = \binom{P}{S} m^S (1 - m)^{P-S}.
\]

Let \( K = T - I - 1 \geq 1 \) denote the minimum number of patient shares the offer has to attract in order to generate a controlling stake. Finally, let

\[
Q_K^P(m) = \sum_{s=K}^{P} \sigma^P(s;m)
\]

denote the equilibrium probability that the tender offer attracts at least \( K \) patient shares out of \( P \).

In equilibrium, each of the \( P \) potential sellers of shares must be indifferent between selling and not selling. This implies the following relationship between the outstanding bid, \( b \), and the selling probability \( m \in (0,1) \):

\[
b = Q_K^{P-1}(m) l,
\]  

(11)

where the left-hand side is the (sure) gain from selling and the right-hand side is the expected gain from holding on to the share. Given \( b \) and \( m \), the bidder’s expected profits are

\[
\Pi = \sum_{s=K}^{P} \sigma^P(s;m)((I + 1 + s)l - C) - \sum_{s=0}^{P} \sigma^P(s;m)(I + s)b.
\]

(12)

The crucial observation now is the following:
**Lemma 2:** For every bid satisfying (11) one has \( \Pi < 0 \).

The proof of Lemma 2 is in the Appendix. Lemma 2 implies that, whenever there are not sufficient impatient shares to guarantee the success of the tender offer, free riding makes the concentration of shares in the bad state impossible. Any bid that is high enough for patient owners to consider selling—as given by (11)—is too high to pay off for the bidder. Conversely, a bid that is sufficiently low to allow the bidder to recoup future intervention costs does not attract any patient shares. Hence, if \( I < T - 1 \), dispersion persists with probability 1 in the bad state, and the free-rider paradox, established by Grossman and Hart (1980) for the case of atomistic owners, prevails.

The intuition as to why there is such a stark contrast between the case of high and low liquidity trading is the following. If liquidity demand is high in the bad state, bidders can extract enough value from impatient owners to recoup the intervention costs (as shown in (10)). On the other hand, as soon as a bid needs patient shares to succeed, it has to pay these shares a premium to keep them from free riding. However, because the bidder cannot discriminate between patient and impatient owners, he has to pay this premium to the impatient owners, too, which makes the bid too expensive.\(^{23}\)

Lemma 2 now immediately implies:

**Proposition 2:** If the firm starts out with dispersed ownership in \( t = 0 \), concentration in the bad state obtains with probability

\[
\lambda = \lambda(q, T, M) = \sum_{i=T-1}^{M-1} \binom{M}{i} q^i (1 - q)^{M-i}.
\]

(13)

The ex ante value of the firm is given by

\[
V_0^d = 2\pi R + \lambda(1 - \pi)(L - C).
\]

(14)

Thus the control cost of having no large initial owner is measured entirely in terms of the probability that ownership fails to become concentrated when necessary ex post. As in the case of Proposition 1, all ex post distortions in liquidity trading wash out ex ante. If all owners or if too few owners are impatient, efficient corporate control fails in the bad return state, but for two different reasons. In the first case, no shareholder wants to take on the responsibility (and the costs) of controlling management, in the second case no one is able to do it because of excessive free riding by other shareholders.

In practice, of course, a controlling interest is not always built through a simple tender offer. If an owner decides to raise his ownership stake in re-

\(^{23}\) As in most models of free riding, allowing for bids that are conditional on obtaining 100 percent of the shares would resolve the problem formally. However, empirically this is not appealing.
response to perceived managerial failures, he may be able to buy out some other owners at lower prices, reflecting the imperfections of management control. However, through rising share prices and disclosure requirements the information will get into the market. The preceding analysis suggests that if information disseminates too quickly or if takeovers are very costly, owners will prefer to give up building a controlling interest altogether.24

III. Evaluating the Trade-off

The previous section has analyzed liquidity trading and firm value under two polar ownership structures. It is easy to see that ownership structures with a single block greater than $T$ or with more than one block of size $T$ are suboptimal in our model. Yet, it is less clear whether ownership structures with a block smaller than $T$ (which necessarily involve fewer owners than the fully dispersed structure) may be optimal. Relative to full dispersion such an ownership structure has the advantage of making every shareholder more pivotal in the tender game, thus reducing the free-rider problem of ownership concentration. On the other hand, it has the drawback of providing fewer trading opportunities and, therefore, fewer opportunities to concentrate ownership. A priori, it is unclear whether such a structure can give rise to concentration more often when it is needed (that is whether the resulting lambda is higher than in equation (13)).

We do not attempt to determine whether optimal ownership structures in our model can feature small blocks. Based on preliminary calculations and simulations we suspect, however, that ownership structures with a block of size less than $T$ are dominated either by full dispersion or by a structure with a block of size $T$. The point is that when control is the overriding concern, then even a small reduction in block size below $T$ involves a discrete upwards jump in costs of control loss, compared to only a marginal benefit of higher liquidity. Vice-versa, when liquidity is the primary concern (which means that large blocks cannot be optimal), the marginal benefit of a decrease in control loss is outweighed by the marginal cost of reduced liquidity.

In this section, we discuss how the choice between the two structures analyzed in the last section is influenced by the characteristics of the firm and its environment. Comparison of the expression for $V_0^{c}$ and $V_0^{d}$ in equations (6) and (14), respectively, shows immediately that $V_0^{c} > V_0^{d}$ if and only if

$$1 - q^{M - T + 1} - \lambda > 0.$$  

(15)

This simple condition is a concise description of the trade-off between the costs and benefits of the two ownership structures. If ownership is dispersed, there is a larger group of investors to trade with, but only a reduced

24 Note that Lemma 2 and Proposition 2 do not depend on our assumption that outsiders are less willing to control management than existing owners. This is because the problem is caused by free riding of existing owners. If there are more than $M - T + 1$ patient owners, a tender offer must fail regardless of the number of potential bidders.
chance, measured by $\lambda$, of efficient corporate control, due to free riding. Under concentration, corporate control is provided by the large owner, but because of reduced trading opportunities the owners benefit from it only with a reduced probability, $1 - q^{M-T+1}$. As (15) shows, the trade-off depends only on three variables: average liquidity demand $q$, the total number of shares $M$, and the efficient controlling stake, $T = (C/L)M$. Other variables, in particular $\pi$ and $R$, determinants of the firm’s cash flow, do not enter. This is a consequence of our assumption that the costs and benefits of ownership concentration only accrue in bad cash flow states. If ownership structure matters in all states, the comparison becomes more complicated.

We first turn to the question of how ownership structure is affected by the investors’ average demand for liquidity, $q$.

**Proposition 3:** There is a critical value $\bar{q} = \bar{q}(T,M) \in (0,1)$ such that concentrated ownership in $t = 0$ is optimal for $q < \bar{q}$, and dispersed ownership is optimal for $q > \bar{q}$.

**Proof:** Ignoring the dependence of $\lambda$ on $M$ for the moment and writing $\lambda = \lambda(q,T)$, we have $\lambda(0,T) = \lambda(1,T) = 0$. By straightforward calculation the derivative of $\lambda$ with respect to $q$ is

$$\lambda_q(q,T) = M \left( \frac{M-1}{T-2} \right) q^{T-2} (1 - q)^{M-T+1} - M q^{M-1}.$$ 

Denote the left-hand side of (15) by $g(q,T)$. We have $g(0,T) = 1$, $g(1,T) = 0$, and

$$g_q(q,T) = -(M - T + 1) q^{M-T} - \lambda_q(q,T),$$

(16)

hence, $g_q(1,T) > 0$. A further straightforward calculation shows that $g_q(q,T) = 0$ implies $g_{qq}(q,T) > 0$. Hence, $g$ as a function of $q$ can have only local minima. By the above, it therefore has exactly one zero, $\bar{q}$, in $(0,1)$. Q.E.D.

At one level, Proposition 3 is intuitive and straightforward: When $q$ is large there is a high value of liquidity (which favors dispersion), and when $q$ is low there is a low value of liquidity (which favors concentration). However, this only concerns the demand for liquidity. The less obvious part of Proposition 3, which our analysis brings out, is that this is also a supply phenomenon. By the assumption of no wealth constraints and maximum ex post competition for shares, the supply of liquidity is a constraint only when all shareholders want to sell out simultaneously in the bad state. This happens with probability $(1 - \pi)q^n$ where $n$ is the number of initial shareholders. With the remaining probability of $1 - (1 - \pi)q^n$, shareholders get the full ex post value of their shares in both return states—their shares are fully liquid. If $q$ is large, this probability is relatively small, so that increasing it by increasing $n$ is relatively valuable. On the other hand, if $q$ is small, lacking liquidity is relatively less likely and, therefore, reducing $n$ to achieve better corporate control is relatively more valuable.
Proposition 3 shows, in particular, that the existence of a large block in a firm's capital structure can increase as well as reduce firm value. This is because, at the optimum, establishing a block reconciles as well as possible the liquidity needs of investors with the need for corporate control. Away from the optimum, the costs of having a large blockholder may, of course, exceed the gains. This provides a different explanation for the observed variability of stock price reactions to block transactions than the one given by Barclay and Holderness (1989, 1991) and others. There, different stock price reactions are explained by differences in the ability and willingness of blockholders to control management on behalf of the other shareholders. In our model, control by blockholders always has a positive externality on other shareholders, but establishing a block has an opportunity cost that may outweigh the benefit. Of course, these two explanations are mutually compatible.

Proposition 3 implies that if investors are patient, ownership concentration is more likely. This implication is consistent with the empirical observation that shareholdings in Germany and France are more long term than those in the United Kingdom and the United States (e.g., Kojima (1995)). Comparing the ownership structures of French or German publicly listed firms with those in the United States or the United Kingdom, ownership concentration is indeed one of the most striking differences: In the United Kingdom, 16 percent of the largest quoted firms have an owner with a stake of at least 25 percent, against 85 percent in Germany and 79 percent in France (Franks and Mayer (1994)).

We now turn to the relative cost of shareholder intervention. This cost, measured by the size of the efficient intervention stake $T$, is influenced by two parameters of the model, the actual cost of intervening, $C$, and the benefit brought about by intervention, $L$.

**Proposition 4:** When $T$ increases, the threshold level of average liquidity demand, $\bar{q}(T,M)$, increases; hence, concentration is more likely to be efficient.

**Proof:** We have, for $T \leq (M + 1)/2,$

$$g(q, T + 1) - g(q, T) = (1 - q)q^{T-1}(\left(\frac{M}{T-1}\right)(1 - q)^{M-T} - q^{M-2T+1}) \tag{17}$$

It is not difficult, but lengthy, to verify that this expression is positive if $g(q, T) = 0$. Furthermore, from the proof of Proposition 3, $g_q(\bar{q}(T, M), T) < 0$. This implies the desired result. The case $T > (M + 1)/2$ is similar. Q.E.D.

Proposition 4 states that if intervention becomes relatively more costly ($T$ increases), concentration becomes more valuable at the margin. In other words, when average liquidity demand $q$ is such that investors are indifferent between concentration or dispersion, an increase in the relative cost of intervention makes concentration more attractive than dispersion. Cross-sectionally, one should therefore observe more concentration of ownership if the relative costs of intervention are higher.
This prediction corresponds well to the findings of Demsetz and Lehn (1985) about ownership concentration in large U.S. firms, and also casts some new light on their theoretical arguments. They, too, argue that the costs of monitoring are an important explanatory variable for ownership concentration. They proxy these costs, which are directly unobservable, by the volatility of the exogenous factors influencing corporate performance. The reason for this choice of proxy is that “disentangling the effects of managerial behavior on firm performance from the corresponding effects of these other, largely exogenous factors is costly” (Demsetz and Lehn (1985)). Empirically, they indeed find that firms with higher earnings and stock price volatility tend to have a more concentrated ownership structure. At the same time, however, they predict that ownership concentration should also depend positively on the “control potential,” that is, the potential gain from controlling management. Because they measure this variable again by earnings and stock price volatility, they find support for this hypothesis. Our analysis suggests that this might be too quick a conclusion, driven by the fact that Demsetz and Lehn (1985) do not model all the costs of concentration. Indeed, in our model, $L$ can be considered to be a good proxy for the “control potential”: the higher $L$, the larger the gain from intervention in the low return state. Proposition 4 now implies that, with $C$ held constant, increasing $L$ should, on average, decrease concentration. Why this discrepancy? The reason is that an increased benefit from control does not only render ex ante concentration more attractive, but also makes it easier to achieve ex post, by lowering the stake necessary for efficient intervention. Because ex ante concentration has costs in terms of liquidity, it follows that ownership dispersion, with its reliance on ex post trading to achieve intervention, becomes more attractive.

Finally, we look at how ownership structure varies with the number of shares, $M$, which we have argued reflects the transactions costs of trading. Because the comparative statics become more difficult, we only do them asymptotically.

**Proposition 5:** For small values of $M$, both ownership structures, dispersion or concentration, can be optimal, depending on the other parameters. If $M$ tends to infinity, concentration always dominates.

**Proof:** Because $C < L$, $q^{M−T+1} \to 0$ and $\lambda$ is bounded away from 1 for $M \to \infty$. Hence, (15) holds for $M$ large enough. For $M = 2$ (which necessitates $T = 2$) the left-hand side of (15) becomes $(1 − q)(1 − 2q)$, whose sign depends on $q$. Q.E.D.

Proposition 5 states that if shares can be arbitrarily finely divided, concentration of ownership is always preferable to dispersion. This is intuitive, because in this case even under ownership concentration the total number of owners $(1 − C/L)M + 1$, can be made large, thus achieving good liquidity without costs in terms of corporate control. This result confirms a conjecture offered by Bhide (1993) in a study of corporate governance issues of publicly traded U.S. companies. Bhide suggests that combining a thick secondary market for shares with the presence of a large controlling block can poten-
tially solve the corporate governance problem. Our analysis shows that, in the framework of our model, this is correct, if shares are arbitrarily divisible. On the other hand, if the transactions costs of trading are taken into account, the denomination of shares is typically bounded below by the costs of transacting and thick secondary markets can no longer be taken for granted. In fact, secondary market liquidity then is one element in the design of corporate ownership structure, and more generally of capital market structure, which is subject to the trade-off analyzed in this paper.

IV. Conclusion

The present paper has developed a simple framework in which to analyze costs and benefits of ownership concentration, if the efficiency of corporate control and secondary market liquidity are both taken into account. An important assumption has been that, in some circumstances, the number of investors holding the firm's stock can be important for secondary market trading. This assumption, which relies on the idea that firms are not monitored perfectly by the market, is a convenient simplification of the general insight that market participation is limited by transaction costs.

A second simplifying assumption of the analysis has been to ignore wealth constraints. This assumption allows us to obtain simple expressions for the pricing of shares in ex post trading, because in each contingency owners have enough wealth to pay the ex post value of all impatient shares. If this is not the case, ex post competition becomes more complicated. However, under reasonable pricing mechanisms the two important qualitative features of ex post ownership remain unchanged: With ex ante concentration the controlling block, and hence efficient corporate control, persists in bad states of nature, whereas with ex ante dispersion a controlling block fails to emerge if liquidity trading is low. For our analysis to hold, this is all that counts (cf. Propositions 1 and 2), because ex post distortions and redistributions wash out ex ante by symmetry.

The model has one unrealistic implication if one takes the usual interpretation of \( t = 2 \) as the "long run" seriously. This is that in the long run there is a tendency for ownership to be concentrated. This implication results from the assumption that the firm is put up for sale once and trade afterwards can only result in more concentration, not less. This assumption ignores the fact that if ownership has been concentrated for some time in response to bad firm performance, its owners may well agree to disperse ownership again, thus restarting the cycle of the present paper. Incorporating this feature into a dynamic model of ownership change yields an alternating structure of dispersion and concentration, which is more appealing empirically. We consider such a structure in Bolton and von Thadden (1998) in the context of a model of overlapping generations of investors.

To conclude the paper, it is worth noting that the general problem raised in this paper does not only apply to equity financing. Conceptually, the liquidity-control trade-off is also relevant to debt financing. Thus, one of the benefits of bank financing is that, with a sufficiently large stake, a bank
may be willing to monitor the firm and guide managerial decisions. Of course, because the bank itself is not a major shareholder it may not take the ex post efficient continuation decision. Nevertheless, even if the bank does not necessarily act in the interest of stockholders, it is likely that its interests are more in line with those of other debt holders. However, for other debt holders, the cost of having a large bank is some lack of liquidity. This is why bond financing may sometimes be preferred to bank financing. An interesting question concerning bond financing then is why there does not seem to be a parallel for bonds to the notion of a controlling interest for stocks. Is this due to regulatory restrictions? Is it due to the fact that with most bonds bondholders are unlikely to ever be in a position where they can exercise control? These are questions for further research.

**Appendix: Proof of Lemma 2**

Denote by

$$E_K^P(m) = Q_K^P(m) E[S|S \geq K] = \sum_{s=K}^{P} s \sigma^P(s;m)$$

the expected number of patient shares including and above the threshold level that are sold in equilibrium. Using the fact that $C = iT$, expected profits from bidding $b$, given by (11) in the main text, then are

$$\Pi = lE_K^P(m) - lKQ_K^P(m) - (1 + mP)b. \quad (A1)$$

Notice that

$$(1 - m)Q_{K-1}^P(m) = \sum_{s=K}^{P} \left(1 - \frac{s}{P}\right) \binom{P}{s} m^s (1 - m)^{P-s} = Q_K^P(m) - \frac{1}{P} E_K^P(m).$$

(A2)

Inserting (A2) into (11) and all into (A1) yields

$$\Pi = \frac{l}{1-m} \left[ \frac{M - 1}{P} E_K^P(m) - (m(M - T) + T - 1)Q_K^P(m) \right]. \quad (A3)$$

Denote the term in brackets in (A3) by $F(m)$. We have

$$Q_{K-1}^P(m) = Q_K^{P-1}(m) + \binom{P-1}{K-1} m^{K-1}(1 - m)^{P-K}.$$
Therefore,

\[ F'(m) = ((M - 1)K - P(T - 1) - mP(M - T)) \times \left( \frac{P - 1}{K - 1} \right)^{m-1} \left( 1 - m \right)^{P-K} - \frac{F(m)}{1 - m}. \]  \hspace{1cm} (A4)

Remembering that \( P = M - I - 1 \) and \( K = T - I - 1 \), we have for \( m > 0 \),

\[ (M - 1)K - P(T - 1) - mP(M - T) < 0. \]  \hspace{1cm} (A5)

Equations (A4), (A5), and the fact that \( F(0) = 0 \) imply that \( F(m) < 0 \) for \( m \in (0,1) \). Q.E.D.

REFERENCES


Franks, Julian, Colin Mayer, and Luc Rennebog, 1995, The role of large share stakes in poorly performing companies, Working paper, LBS.


Kovenock, Dan, 1984, A note on takeover bids, Note, Purdue University.


Myers, Stewart, and Nicholas Majluf, 1984, Corporate financing and investment decisions when firms have information that investors do not have, *Journal of Financial Economics* 13, 187–221.


