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# Market timing, investment, and risk management ☆

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ABSTRACT

The 2008 financial crisis exemplifies significant uncertainties in corporate financing conditions. We develop a unified dynamic $q$-theoretic framework where firms have both a precautionary-savings motive and a market-timing motive for external financing and payout decisions, induced by stochastic financing conditions. The model predicts (1) cuts in investment and payouts in bad times and equity issues in good times even without immediate financing needs; (2) a positive correlation between equity issuance and stock repurchase waves. We show quantitatively that real effects of financing shocks may be substantially smoothed out as a result of firms' adjustments in anticipation of future financial crises.



## 1. Introduction

The financial crisis of 2008 and the European debt crisis of 2011 are fresh reminders that corporations at times face substantial uncertainties about their external financing conditions. Recent studies show dramatic changes in firms' financing and investment behaviors during these crises. For example, Ivashina and Scharfstein (2010) find aggressive credit line drawdowns by firms for precautionary reasons. Campello, Graham, and Harvey (2010) and Campello, Giambona, Graham, and Harvey (2011) show that more financially constrained firms planned deeper cuts in investment and spending, burned more cash, drew more credit from banks, and engaged in more asset sales during the crisis.

Rational firms could plausibly adapt to fluctuations in financing conditions by hoarding cash, postponing or

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bringing forward investments, timing favorable market conditions to raise more funds than they really need, or hedging against unfavorable market conditions. Recently, there has been much empirical work on the corporation's cash holdings.[1] Yet, very little theoretical research tries to answer the following related questions. How should firms change their financing, investment, and risk management policies during a period of severe financial constraints? How should firms behave when facing the threat of a future financial crisis? What are the overall real effects of changes in financing conditions when firms can prepare for future shocks through cash and risk management policies?

We address these questions in a quantitative model of corporate investment, financing, and risk management for firms facing stochastic financing conditions. Our model builds on the recent dynamic frameworks by Decamps, Mariotti, Rochet, and Villeneuve (2011) and Bolton, Chen, and Wang (2011, henceforth BCW), mainly by adding stochastic financing opportunities. The five main building blocks of the model are (1) a constant returns-to-scale production function with independently and identically distributed (i.i.d.) productivity shocks and convex capital adjustment costs as in Hayashi (1982), (2) stochastic external financing costs, (3) constant cash-carrying costs, (4) risk premia for productivity and financing shocks, and (5) dynamic hedging opportunities. The firm optimally manages its cash reserves, financing, and payout decisions by following a state-dependent optimal double-barrier policy for issuance and payout, combined with continuous adjustments of investment, cash accumulation, and hedging between the issuance and payout barriers.

The main results of our analysis are as follows. First, during a financial crisis, to avoid extremely high external financing costs, the firm optimally cuts back on investment, delays payout, and, if needed, engages in asset sales, even if the productivity of its capital remains unaffected. This is especially true when the firm enters the crisis with low cash reserves. These predictions are consistent with the stylized facts about firm behavior during the recent financial crisis.

Second, during favorable market conditions (a period of low external financing costs), the firm could time the market and issue equity even when no immediate need exists for external funds. Such behavior is consistent with the findings in Baker and Wurgler (2002), DeAngelo, DeAngelo, and Stulz (2010), Fama and French (2005), and Huang and Ritter (2009). We thus explain firms' investment, saving, and financing decisions through a combination of stochastic variations in the supply of external financing and firms' precautionary demand for liquidity. We also show that, due to market timing, investment can be decreasing in the firm's cash reserves. The reason is that the market timing option together with the fixed external financing costs can cause firm value to become locally convex in financial slack. This local convexity also implies that it could be optimal for the firm to engage in speculation rather than hedging to increase the value of the market timing option.

Third, along with the timing of equity issues by firms with low cash holdings, our model predicts the timing of cash payouts and stock repurchases by firms with high cash holdings. Just as firms with low cash holdings seek to take advantage of low costs of external financing to raise more funds, firms with high cash holdings are inclined to disburse their cash through stock repurchases when financing conditions improve. This result is consistent with the finding of Dittmar and Dittmar (2008) that aggregate equity issuances and stock repurchases are positively correlated. They point out that the finding that increases in stock repurchases tend to follow increases in stock market valuations contradicts the received wisdom that firms engage in stock repurchases because of the belief that their shares are undervalued. Our model provides a simple and plausible explanation for their finding: improved financing conditions raises stock prices and lowers the precautionary demand for cash buffers, which in turn can result in more stock repurchases by cash-rich firms.

Fourth, we show that a greater likelihood of deterioration in the financing conditions leads to stronger cash hoarding incentives. With a higher probability of a crisis occurring, firms invest more conservatively, issue equity sooner, and delay payouts to shareholders in good times. Consequently, firms' cash inventories rise, investment becomes less sensitive to changes in cash holdings, and the ex post impact of financing shocks on investment is much weaker. This effect is quantitatively significant. When we raise the probability of a financial crisis within a year from 1% to 10%, the average reduction in a firm's investment-capital ratio following the realization of the shock drops from 6.6% to 1.8%. Furthermore, this reduced investment response is in large part due to the firm cutting back investment in the good state in preparation for the crisis. These findings provide important new insights on the transmission mechanism of financial shocks to the real sector and helps us interpret empirical measures of the real effects of financing shocks. In particular, it shows that judging the real effect of financial crisis by measuring the changes of investment following the crisis can be misguided.

Fifth, due to the presence of aggregate financing shocks, the firm's risk premium in our model has two components: a productivity risk premium and a financing risk premium. Both risk premia change substantially with the firm's cash holdings, especially when external financing conditions are poor. Quantitatively, the financing risk premium is significant for firms with low cash holdings, especially in a financial crisis, or when the probability of a financial crisis is high. However, due to firms' precautionary savings, the financing risk premium is low for the majority of firms as they are able to avoid falling into a low cash trap. Moreover, our model predicts that idiosyncratic cashflow risk affects a firm's cost of capital. Firms facing higher idiosyncratic risk optimally hold more cash on average, which lowers their beta and expected returns. This result highlights that the endogeneity of cash

[1] See Almeida, Campello, and Weisbach (2004), Dittmar and Mahrt-Smith (2007), Dittmar and Dittmar (2008), Bates, Kahle, and Stulz (2009), Riddick and Whited (2009), among many others, on the empirical relevance and potential explanations of corporate cash holding policies.

holdings is key to understanding the cross-sectional relation between cash holdings and returns.

Our analysis reveals that first-generation static models of financial constraints are inadequate to explain how corporate investment responds to changing financing opportunities. Static models, such as Fazzari, Hubbard, and Petersen (1988), Froot, Scharfstein, and Stein (1993), and Kaplan and Zingales (1997), cannot explain the effects of market timing on corporate investment, because these effects cannot be captured by a permanent exogenous change in the cost of external financing or an exogenous change in the firm's cash holdings in the static setting. Market timing effects can only appear when there is a finitely lived window of opportunity of getting access to cheaper equity financing. More recent dynamic models of investment with financial constraints include Gomes (2001), Hennessy and Whited (2005, 2007), Riddick and Whited (2009), and BCW, among others. However, all these models assume that financing conditions are time-invariant.

Our work is also related to two other sets of dynamic models of financing. First, DeMarzo, Fishman, He, and Wang (2012) develop a dynamic contracting model of corporate investment and financing with managerial agency, by building on Bolton and Scharfstein (1990) and using the dynamic contracting framework of DeMarzo and Sannikov (2006) and DeMarzo and Fishman (2007b).[2] These models derive optimal dynamic contracts and corporate investment with capital adjustment costs. Second, Rampini and Viswanathan (2010, 2011) develop dynamic models of collateralized financing, in which the firm has access to complete markets but is subject to endogenous collateral constraints induced by limited enforcement.

Our paper is one of the first dynamic models of corporate investment with stochastic financing conditions. We echo the view expressed in Baker (2010) that equity supply effects are important for corporate finance. While we treat changes in financing conditions as exogenous in this paper, the cause of these variations could be changes in financial intermediation costs, changes in investors' risk attitudes, changes in market sentiment, or changes in aggregate uncertainty and information asymmetry. Stein (1996) develops a static model of market timing and Baker, Stein, and Wurgler (2003) empirically test this model.[3] To some extent, our model can be viewed as a dynamic formulation of Stein (1996), in which a rational manager behaves in the interest of existing shareholders in the face of a market that is subject to potentially irrational changes of investor sentiment. The manager then times the market optimally and issues equity when financing conditions are favorable. This interpretation assumes that markets underreact to the manager's timing behavior, causing favorable financing conditions to persist.

[2] DeMarzo and Fishman (2007a) study optimal investment dynamics with managerial agency in a discrete time setting.

[3] Using a panel of international data, Birru (2012) finds that market-wide mispricing can mitigate underinvestment through its effect on the cost of capital, and links market-wide mispricing to the macroeconomy.

Finally, in contemporaneous work, Hugonnier, Malamud, and Morellec (2012) also develop a dynamic model with stochastic financing conditions. They model investment as a growth option, while we model investment as in Hayashi's $q$-theory framework. In addition, they model the window of financing opportunities via a Poisson process. When such an opportunity arrives, the firm has to decide immediately whether to raise funds or not. Thus, the duration of the financing opportunity in their model is instantaneous. In our model, the finite duration of financing states is important for generating market timing. The two papers share the same overall focus but differ significantly in their modeling approaches, thus complementing each other.

The remainder of the paper proceeds as follows. Section 2 sets up the model. Section 3 presents the model solution. Section 4 investigates the quantitative results. We then study the impact of market timing in a good state on investment, financing, and payout (Section 5), risk and return (Section 6), and hedging (Section 7). Section 8 concludes.

## 2. The model

We consider a financially constrained firm facing stochastic investment and external financing conditions. Specifically, we assume that the firm can be in one of two (observable) states of the world, denoted by $s_t = G,B$. In the two states, the firm faces potentially different financing and investment opportunities. Over a short time interval $\Delta$, the state switches from $G$ to $B$ (or from $B$ to $G$) with a constant probability $\zeta_G \Delta$ (or $\zeta_B \Delta$).

### 2.1. Production technology

The firm employs capital and cash as the only factors of production. We normalize the price of capital to one and denote by $K$ and $I$ the firm's capital stock and gross investment, respectively. As is standard in capital accumulation models, the capital stock $K$ evolves according to

$$dK_t = (I_t - \delta K_t)\,dt, \quad t \geq 0, \tag{1}$$

where $\delta \geq 0$ is the rate of depreciation.

The firm's operating revenue is proportional to its capital stock $K_t$ and is given by $K_t\,dA_t$, where $dA_t$ is the firm's productivity shock over time increment $dt$. We assume that

$$dA_t = \mu(s_t)\,dt + \sigma(s_t)\,dZ_t^A, \tag{2}$$

where $Z_t^A$ is a standard Brownian motion and $\mu(s) \equiv \mu_s$ and $\sigma(s) \equiv \sigma_s$ denote the drift and volatility in state $s$, respectively. In the remainder of this paper, we use the notation $x_s$ to denote a state-dependent variable $x(s)$ whenever there is no ambiguity.

The firm's operating profit $dY_t$ over time increment $dt$ is then given by

$$dY_t = K_t\,dA_t - I_t\,dt - \Gamma(I_t, K_t, s_t)\,dt, \quad t \geq 0, \tag{3}$$

where $K_t\,dA_t$ is the firm's operating revenue, $I_t\,dt$ is the investment cost over time $dt$, and $\Gamma(I_t, K_t, s_t)\,dt$ is the additional adjustment cost that the firm incurs in the investment process.

Following the neoclassical investment literature (Hayashi, 1982), we assume that the firm's adjustment cost is homogeneous of degree one in $I$ and $K$. In other words, the adjustment cost takes the form $\Gamma(I,K,s)= g_s(i)K$, where $i$ is the firm's investment capital ratio ($i=I/K$) and $g_s(i)$ is a state-dependent function that is increasing and convex in $i$. We further assume that $g_s(i)$ is quadratic:

$$g_s(i)=\frac{\theta_s(i-\nu_s)^2}{2}, \tag{4}$$

where $\theta_s$ is the adjustment cost parameter and $\nu_s$ is a constant.[4] This assumption makes the analysis more tractable, but our main results do not depend on the specific functional form of $g_s(i)$. Our model allows for state-dependent adjustment costs of investment. For example, in bad times, assets are often sold at a deep discount (see Shleifer and Vishny, 1992; Acharya and Viswanathan, 2011), which can be captured in this model by making $\theta_s$ large when financing conditions are tough.

Finally, the firm can liquidate its assets at any time and obtain a liquidation value $L_t$ that is also proportional to the firm's capital stock $K_t$. Specifically, the liquidation value is $L_t = l_s K_t$, where $l_s$ denotes the recovery value per unit of capital in state $s$.

The production technology in our model is essentially the same as the ones used in BCW, DeMarzo, Fishman, He, and Wang (2012) in an optimal dynamic contracting setting, and Wang, Wang, and Yang (2012) for a model of entrepreneurship dynamics. Our model also allows for the drift, the volatility, and the adjustment cost functions to be regime dependent. As in these papers, the homogeneity property in our model allows us to study the impact of stochastic financing conditions on corporate investment, external financing, and liquidity hoarding in an analytically tractable framework.

### 2.2. Stochastic financing opportunities

The firm may choose to raise external equity financing at any time.[5] When doing so, it incurs a fixed as well as a variable cost of issuing stock. The fixed cost is given by $\phi_s K$, where $\phi_s$ is the fixed cost parameter in state $s$. We take the fixed cost to be proportional to the firm's capital stock $K$, which ensures that the firm does not grow out of its fixed costs of issuing equity. This assumption is also analytically convenient, as it preserves the homogeneity of the model in the firm's capital stock $K$. Besides the fixed cost $\phi_s K$, the firm incurs a variable cost $\gamma_s > 0$ for each incremental dollar it raises.

We denote by $H$ the process for the firm's cumulative external financing (so that $dH_t$ denotes the net proceeds from external financing over time $dt$), by $X$ the firm's cumulative issuance costs (so that $dX_t$ denotes the financing costs to raise net proceeds $dH_t$ from external financing), and by $U$ the firm's cumulative nondecreasing payout process to shareholders (so that $dU_t$ is the payout over time $dt$).

The financing cost to raise net proceeds $dH_t$ under our assumptions is given by $dX_t = \phi_{s_t} K_t 1_{\{dH_t>0\}} + \gamma_{s_t}\, dH_t$. If the firm runs out of cash ($W_t=0$), it needs to raise external funds to continue operating, or its assets will be liquidated. If the firm chooses to raise external funds to continue operating, it must pay the financing costs specified above. The firm could prefer liquidation if the cost of financing is too high relative to the continuation value. We denote by $\tau$ the firm's stochastic liquidation time.

Distributing cash to shareholders could take the form of a special dividend or a share repurchase.[6] The benefit of a payout is that shareholders can invest the proceeds at the market rate of return and avoid paying a carry cost on the firm's retained cash holdings. We capture this carry cost by assuming that cash inside the firm earns a below-market riskless return, with the difference denoted by $\lambda > 0$.[7]

We can then write the dynamics for the firm's cash $W$ as follows:

$$dW_t = (K_t\, dA_t - (I_t + \Gamma(I_t,K_t,s_t))\, dt) + (r(s_t)-\lambda)W_t\, dt + dH_t - dU_t, \tag{5}$$

where $r(s) \equiv r_s$ is the risk-free interest rate in state $s$. The first term in (5) is the firm's cash flow from operations $dY_t$ given in (3); the second term is the return on $W_t$ (net of the carry cost $\lambda$); the third term $dH_t$ is the net proceeds from external financing; and the last term $dU_t$ is the payout to investors. Note that (5) is a general financial accounting equation, in which $dH_t$ and $dU_t$ are endogenously determined by the firm.

The homogeneity assumptions embedded in the production technology, the adjustment cost, and the financing costs allow us to deliver our key results in a parsimonious and analytically tractable framework. Adjustment costs might not always be convex and the production technology could exhibit long-run decreasing returns to scale in practice, but these functional forms substantially complicate the formal analysis.[8]

### 2.3. Systematic risk and the pricing of risk

Our model has two different sources of systematic risk: (1) a small diffusion shock to productivity, and (2) a

[4] In the literature, common choices of $\nu_s$ are either zero or the rate of deprecation $\delta$. While the former choice implies zero adjustment cost for zero gross investment, the latter choice implies a zero adjustment cost when net investment is zero.

[5] For simplicity, we only consider external equity financing as the source of external funds for the firm. We leave the generalization to include debt financing for future research.

[6] We cannot distinguish between a special dividend and a share repurchase, as we do not model taxes. However, a commitment to regular dividend payments is suboptimal in our model. We also exclude any fixed or variable payout costs so as not to overburden the model.

[7] The cost of carrying cash could arise from an agency problem or from tax distortions. Cash retentions are tax disadvantaged because the associated tax rates generally exceed those on interest income (Graham, 2000). If $\lambda=0$, the firm has no incentives to pay out cash because keeping cash inside the firm does not have any disadvantages but still has the benefit of relaxing financial constraints. We could also let the cash-carrying cost vary with the state. For the sake of brevity, we do not pursue this generalization in this paper.

[8] See Riddick and Whited (2009) for an intertemporal model of a financially constrained firm with decreasing returns to scale.

large shock when the state of the economy changes. The small productivity shocks in any given state $s$ can be correlated with the aggregate market, and we denote the correlation coefficient by $\rho_s$. The large shock can affect the conditional moments of the firm's productivity or its external financing costs, or both.

How are these sources of systematic risk priced? Our model can allow for either risk-neutral or risk-averse investors. If investors are risk neutral, then the prices of risk are zero and the physical probability distribution coincides with the risk-neutral probability distribution. If investors are risk averse, we need to distinguish between physical and risk-neutral measures. We do so as follows.[9]

For the diffusion risk, we assume that there is a constant market price of risk $\eta_s$ in state $s$. The firm's risk-adjusted productivity shock (under the risk-neutral probability measure $\mathbb{Q}$) is then given by

$$dA_t = \widehat{\mu}_s\,dt + \sigma_s\,d\widehat{Z}_t^A, \tag{6}$$

where the mean of productivity shock accounts for the firm's exposure to the systematic diffusion risk:

$$\widehat{\mu}_s = \mu_s - \rho_s \sigma_s \eta_s, \tag{7}$$

and $\widehat{Z}_t^A$ is a standard Brownian motion under the risk-neutral probability measure $\mathbb{Q}$.

A risk-averse investor also requires a risk premium to compensate for the risk of the economy switching states. We capture this risk premium through the wedge between the transition intensity under the physical probability measure and under the risk-neutral probability measure $\mathbb{Q}$. Let $\widehat{\zeta}_G$ and $\widehat{\zeta}_B$ denote the risk-neutral transition intensities from state $G$ to state $B$ and from state $B$ to state $G$, respectively. Then,

$$\widehat{\zeta}_G = e^{\kappa_G}\zeta_G \quad \text{and} \quad \widehat{\zeta}_B = e^{\kappa_B}\zeta_B, \tag{8}$$

where the parameters $\kappa_G$ and $\kappa_B$ capture the risk adjustment for the change of state. In our calibrated model, state $G$ ($B$) is the state with good (bad) external financing conditions. We set $\kappa_G = -\kappa_B > 0$, which implies that the transition intensity out of state $G$ ($B$) is higher (lower) under the risk-neutral probability measure than under the physical measure. Intuitively, this reflects the idea that an investor's risk aversion towards a bad state is captured by making it more likely to transition into the bad state and less likely to leave it. In sum, it is as if a risk-averse investor were uniformly more 'pessimistic' than a risk-neutral investor: she thinks 'good times' are likely to be shorter and 'bad times' longer.

### 2.4. Firm optimality

The firm chooses its investment $I$, cumulative payout policy $U$, cumulative external financing $H$, and liquidation time $\tau$ to maximize firm value defined as follows (under the risk-neutral measure):

$$\mathbb{E}_0^{\mathbb{Q}}\left[\int_0^\tau e^{-\int_0^t r_u\,du}(dU_t - dH_t - dX_t) + e^{-\int_0^\tau r_u\,du}(L_\tau + W_\tau)\right], \tag{9}$$

where $r_u$ denotes the interest rate at time $u$. The first term is the discounted value of net payouts to shareholders, and the second term is the discounted value upon liquidation. Optimality could imply that the firm never liquidates. In that case we have $\tau = \infty$.

## 3. Model solution

Given that the firm faces external financing costs ($\phi_s > 0$, $\gamma_s \geq 0$), its value depends on both its capital stock $K$ and its cash holdings $W$. Thus, let $P(K,W,s)$ denote the value of the firm in state $s$. Since the firm incurs a carry cost $\lambda$ on its stock of cash, one would expect that it would choose to pay out some of its cash once its stock grows sufficiently large. Accordingly, let $\overline{W}_s$ denote the (upper) payout boundary. Similarly, if the firm's cash holdings are low, it could choose to issue equity. We therefore let $\underline{W}_s$ denote the (lower) issuance boundary.

*The interior regions*: $W \in (\underline{W}_s, \overline{W}_s)$ *for* $s = G,B$. When a firm's cash holdings $W$ are in the interior regions, $P(K,W,s)$ satisfies the following system of Hamilton-Jacobi-Bellman (HJB) equations under the risk-neutral measure:

$$\begin{aligned} r_s P(K,W,s) = \max_I\, & [(r_s-\lambda)W + \widehat{\mu}_s K - I - \Gamma(I,K,s)]P_W(K,W,s) \\ & + \frac{\sigma_s^2 K^2}{2} P_{WW}(K,W,s) + (I-\delta K)P_K(K,W,s) \\ & + \widehat{\zeta}_s(P(K,W,s^-) - P(K,W,s)). \end{aligned} \tag{10}$$

The first and second term on the right side of Eq. (10) represent the effects of the expected change in the firm's cash holdings $W$ and volatility of $W$ on firm value, respectively. Notice that the firm's cash grows at the net return $(r_s - \lambda)$ and is augmented by the firm's expected cash flow from operations (under the risk-neutral measure) $\widehat{\mu}_s K$ minus the firm's capital expenditure $I + \Gamma(I,K,s)$. Also, the firm's cash stock is volatile only to the extent that cash flows from operations are volatile, and the volatility of the firm's revenues is proportional to the firm's size as measured by its capital stock $K$. The third term represents the effect of capital stock changes on firm value, and the last term captures the expected change of firm value when the state changes from $s$ to $s^-$, where we use the notation $s^-$ to denote the state that is different from $s$.

Because firm value is homogeneous of degree one in $W$ and $K$ in each state, we can write $P(K,W,s) = p_s(w)K$, where $w \equiv W/K$ is the cash-capital ratio. Substituting this expression into Eq. (10) and simplifying, we obtain the following system of ordinary differential equations (ODEs) for $p_s(w)$:

$$\begin{aligned} r_s p_s(w) = \max_{i_s}\, & [(r_s-\lambda)w + \widehat{\mu}_s - i_s - g_s(i_s)]p_s'(w) + \frac{\sigma_s^2}{2} p_s''(w) \\ & + (i_s - \delta)(p_s(w) - w p_s'(w)) + \widehat{\zeta}_s(p_{s^-}(w) - p_s(w)). \end{aligned} \tag{11}$$

The first-order condition for the investment-capital ratio $i_s(w)$ is

$$i_s(w) = \frac{1}{\theta_s}\left(\frac{p_s(w)}{p_s'(w)} - w - 1\right) + \nu_s, \tag{12}$$

[9] In Appendix A, we provide a more detailed discussion of systematic risk premia. The key observation is that the adjustment from the physical to the risk-neutral probability measure reflects a representative risk-averse investor's stochastic discount factor in a dynamic asset pricing model.

where $p_s'(w)=P_W(K,W,s)$ is the marginal value of cash in state $s$.

*Payout boundary* $\overline{W}_s$ *and payout region* ($W \geq \overline{W}_s$): The firm starts paying out cash when the marginal value of cash held by the firm is less than the marginal value of cash held by shareholders. The payout boundary $\overline{w}_s=\overline{W}_s/K$ thus satisfies the following value matching condition:

$$p_s'(\overline{w}_s)=1. \tag{13}$$

That is, when the firm chooses to pay out, the marginal value of cash $p'(w)$ must be one. Otherwise, the firm can always do better by changing $\overline{w}_s$. Moreover, payout optimality implies that the following super contact condition (Dumas, 1991) holds:

$$p_s''(\overline{w}_s)=0. \tag{14}$$

We specify next the value function outside the payout boundary. If the marginal value of cash in state $s$ is such that $p_s'(w)<1$, the firm is better off reducing its cash holdings to $\overline{w}_s$ by making a lump-sum payout. Therefore, we have

$$p_s(w)=p_s(\overline{w}_s)+(w-\overline{w}_s), \quad w>\overline{w}_s. \tag{15}$$

This situation could arise when the firm starts off with too much cash or when the firm's cash holding in state $s^-$ is such that $\overline{w}_s<w<\overline{w}_{s^-}$ and the state of the economy suddenly changes from $s^-$ into $s$.

*The equity issuance boundary* $\underline{W}_s$ *and region* ($W \leq \underline{W}_s$): Similarly, we must specify the value function outside the issuance boundary. The firm could suddenly transition from the state $s^-$ with the financing boundary $\underline{w}_{s^-}$ into the other state $s$ with a higher financing boundary ($\underline{w}_s>\underline{w}_{s^-}$) and its cash holdings could lie between the two lower financing boundaries ($\underline{w}_{s^-}<w<\underline{w}_s$). In that case, if the firm is sufficiently valuable it then chooses to raise external funds through an equity issuance so as to bring its cash stock back into the interior region. But how much should the firm raise in this situation? Let $M_s$ denote the firm's cash level after equity issuance, which we refer to as the target level, and define $m_s=M_s/K$. Similarly, define $\underline{w}_s=\underline{W}_s/K$. Firm value per unit of capital in state $s$, $p_s(w)$, when $w \leq \underline{w}_s$ then satisfies

$$p_s(w)=p_s(m_s)-\phi_s-(1+\gamma_s)(m_s-w), \quad w \leq \underline{w}_s. \tag{16}$$

We thus have the following value matching and smooth pasting conditions for $\underline{w}_s$:

$$p_s(\underline{w}_s)=p_s(m_s)-\phi_s-(1+\gamma_s)(m_s-\underline{w}_s), \tag{17}$$

$$p_s'(m_s)=1+\gamma_s. \tag{18}$$

With fixed issuance costs ($\phi_s>0$), equity issuance is lumpy. The firm first pays the issuance cost $\phi_s$ per unit of capital and then incurs the marginal cost $\gamma_s$ for each unit raised. Eq. (17) states that firm value is continuous around the issuance boundary. In addition, the firm optimally selects the target $m_s$ so that the marginal benefit of issuance $p_s'(m_s)$ is equal to the marginal cost $1+\gamma_s$, which yields Eq. (18).

How does the firm determine its equity issuance boundary $\underline{w}_s$? We use the following two-step procedure. First, suppose that the issuance boundary $\underline{w}_s$ is interior ($\underline{w}_s>0$). Then, the standard optimality condition implies that

$$p_s'(\underline{w}_s)=1+\gamma_s. \tag{19}$$

Intuitively, if the firm chooses to issue equity before it runs out of cash, it must be the case that the marginal value of cash at the issuance boundary $\underline{w}_s>0$ is equal to the marginal issuance cost $1+\gamma_s$. If (19) fails to hold, the firm does not issue equity until it exhausts its cash holdings, i.e., $\underline{w}_s=0$.

We also need to determine whether equity issuance or liquidation is optimal, as the firm always has the option to liquidate. Under our assumptions, the firm's capital is productive and, thus, its going-concern value is higher than its liquidation value. Therefore, the firm never voluntarily liquidates itself before it runs out of cash.

However, when it runs out of cash, liquidation could be preferred if the alternative of accessing external financing is too costly. If the firm liquidates, then

$$p_s(0)=l_s. \tag{20}$$

The firm prefers equity issuance to liquidation as long as the equilibrium value $p_s(0)$ under external financing arrangement is greater than the liquidation value $l_s$.

For our later discussion it is helpful to introduce the following concepts. First, the enterprise value is often defined as firm value net of the value of short-term liquid assets. This measure is meant to capture the value created from productive illiquid capital. In our model, it equals $P(K,W,s)-W$. Second, we define average $q$ as the ratio between the enterprise value and the capital stock,

$$q_s(w)=\frac{P(K,W,s)-W}{K}=p_s(w)-w. \tag{21}$$

Third, the sensitivity of average $q$ to changes in the cash-capital ratio measures how much the enterprise value changes with an extra dollar of cash inside the firm. It is given by

$$q_s'(w)=p_s'(w)-1. \tag{22}$$

We also refer to $q_s'(w)$ as the net marginal value of cash. As $w$ approaches the optimal payout boundary $\overline{w}_s$, $q_s'(w)$ goes to 0.

## 4. Quantitative results

Having characterized the conditions that the solution to the firm's dynamic optimization problem must satisfy, we can now illustrate the numerical solutions for given parameter choices of the model. We begin by motivating our choice of parameters and then illustrate the model's solutions in the good and bad states of the world.

### 4.1. Parameter choice and calibration

In our choice of parameters, we select plausible numbers based on existing empirical evidence to the extent that it is available. For those parameters on which there is no empirical evidence we make an educated guess to reflect the situation we are seeking to capture in our model.

The capital liquidation value is set to $l_G = 1.0$ in state $G$, in line with the estimates provided by Hennessy and Whited (2007).[10] In the bad state, the capital liquidation value is set to $l_B = 0.3$ to reflect the severe costs of asset fire sales during a financial crisis, when few investors have sufficiently deep pockets or the risk appetite to acquire assets.[11] The model solution depends on these liquidation values only when the firm finds it optimal to liquidate instead of raising external funds.

We set the marginal cost of issuance in both states to be $\gamma = 6\%$ based on the estimates reported in Altinkilic and Hansen (2000). We keep this parameter constant across the two states for simplicity and focus only on changes in the fixed cost of equity issuance to capture changes in the firm's financing opportunities. The fixed cost of equity issuance in the good state is set at $\phi_G = 0.5\%$. In the benchmark model, this value implies that the average cost per unit of external equity raised in state $G$ is around 10%. This is in the ballpark with the estimates for seasoned offers in Eckbo, Masulis and Norli (2007).[12] As for the issuance costs in state $B$, we chose $\phi_B = 50\%$. No empirical study exists on which we can rely for the estimates of issuance costs in a financial crisis for the obvious reason that there are virtually no initial public offerings or secondary equity offerings in a crisis. Our choice for the parameter of $\phi_B$ reflects the fact that raising external financing becomes extremely costly in a financial crisis and only firms that are desperate for cash are forced to raise new funds. We later show that even with $\phi_B = 50\%$, firms that run out of cash in the crisis state still prefer raising equity to liquidation.

The transition intensity out of state $G$ is set at $\zeta_G = 0.1$, which implies an average duration of ten years for good times. The transition intensity out of state $B$ is $\zeta_B = 0.5$, with an implied average length of a financial crisis of two years. We choose the price of risk with respect to financing shocks in state $G$ to be $\kappa_G = \ln 3$, which implies that the risk-adjusted transition intensity out of state $G$ is $\widehat{\zeta}_G = e^{\kappa_G}\zeta_G = 0.3$. Due to symmetry, the risk-adjusted transition intensity out of state $B$ is then $\widehat{\zeta}_B = e^{-\kappa_G}\zeta_B = 0.167$. These risk adjustments are clearly significant. While we take these risk adjustments as exogenous in this paper, they can be generated in general equilibrium when the same financing shocks also affect aggregate output (see Chen, 2010).

The other parameters remain the same in the two states: the risk-free rate is $r = 5\%$, the volatility of the productivity shock is $\sigma = 12\%$, the rate of depreciation of capital is $\delta = 15\%$, and the adjustment cost parameter $v$ is set to equal the depreciation rate, so that $v = \delta = 15\%$. We rely on the technology parameters estimated by Eberly, Rebelo, and Vincent (2009) for these parameter choices. The cash-carrying cost is set to $\lambda = 1.5\%$. While we do not take a firm stand on the precise interpretation of the cash-carrying cost, it can be due to a tax disadvantage of cash or to agency frictions. Although in reality these parameter values can also change with the aggregate state, we keep them fixed in this model so as to isolate the effects of changes in external financing conditions.

We calibrate the expected productivity $\mu$ and the adjustment cost parameter $\theta$ to match the median cash-capital ratio and investment-capital ratio for US public firms during the period from 1981 to 2010. For the median firm, the average cash-capital ratio is 0.29, and the average investment-capital ratio is 0.17. The details of the data construction are given in Appendix B. We then obtain $\mu = 22.7\%$ and $\theta = 1.8$, both of which are within the range of empirical estimates found in previous studies (see for example Eberly, Rebelo, and Vincent, 2010; Whited, 1992). Finally, Table 1 summarizes all the parameter values.

### 4.2. Market timing in good times

When the firm is in state $G$, it can enter the crisis state $B$ with a 10% probability per year. Since the firm faces substantially higher external financing costs in state $B$, we show that the option to time the equity market in good times has significant value and generates rich implications for investment dynamics.

Fig. 1 plots average $q$ and investment-capital ratio for state $G$ as well as their sensitivities with respect to the cash-capital ratio $w$. Panel A shows as expected that the average $q$ increases with $w$ and is relatively stable in state $G$. The optimal external financing boundary is $\underline{w}_G = 0.027$. At this point, the firm still has sufficient cash to continue operating. Further deferring external financing would help the firm save on the time value of money for financing costs and also on subsequent cash-carrying costs. However, doing so would mean taking the risk that the favorable financing opportunities disappear and that the state of nature switches to the bad state when financing costs are much higher. The firm trades off these two margins and optimally exercises the equity issuance option by tapping securities markets when $w$ hits the lower barrier $\underline{w}_G$.

Should the firm start in state $G$ with $w \le \underline{w}_G$, or should its cash stock shrink to $\underline{w}_G$, it immediately raises external funds of an amount $(m_G - w)$ per unit of its capital stock. The lumpy size of the issue reflects the fact that it is efficient to economize on the fixed cost of issuance $\phi_G$. Similarly, should the firm start in state $G$ with $w \ge \overline{w}_G = 0.371$, or should its cash stock grow to $\overline{w}_G$, it responds by paying out the excess cash $(w - \overline{w}_G)$ because the net marginal value of cash (that is, the difference between the value of a dollar inside the firm and the value of a dollar in the hands of investors) would drop below zero if the firm holds onto the excessive cash reserve (see Panel B of Fig. 1).

When firms face external financing costs, it is optimal for them to hoard cash for precautionary reasons. This is why firm value is increasing and concave in financial slack in most models of financially constrained firms. In our model, while the precautionary motive for hoarding cash is still a key reason that firms save, stochastic financing

[10] They suggest an average value for $l$ of 0.9, so that the liquidation value in the good state should be somewhat higher.

[11] See Shleifer and Vishny (1992), Acharya and Viswanathan (2011), and Campello, Graham, and Harvey (2010).

[12] Eckbo, Masulis, and Norli (2007) report total costs of 6.09% for firm commitment offers, excluding the cost of the offer price discount and the value of Green Shoe options. They also report a negative average price reaction to a secondary equity offering announcement of $-2.22\%$.

**Table 1**
Summary of key variables and parameters.

This table summarizes the symbols for the key variables used in the model and the parameter values in the benchmark case. For each upper-case variable in the left column (except $K$, $A$, and $F$), we use its lower case to denote the ratio of this variable to capital. Whenever a variable or parameter depends on the state $s$, we denote the dependence with a subscript $s$. All the boundary variables are in terms of the cash-capital ratio $w_t$. All the parameter values are annualized when applicable.

| Variable | Symbol | Parameters | Symbol | State $G$ | State $B$ |
|---|---|---|---|---|---|
| *A. Baseline model* | | | | | |
| Capital stock | $K$ | Riskfree rate | $r$ | 5.0% | |
| Cash holding | $W$ | Rate of depreciation | $\delta$ | 15% | |
| Investment | $I$ | Mean productivity shock | $\mu$ | 22.7% | |
| Cumulative productivity shock | $A$ | Volatility of productivity shock | $\sigma$ | 12% | |
| Investment adjustment cost | $\Gamma$ | Adjustment cost parameter | $\theta$ | 1.8 | |
| Cumulative operating profit | $Y$ | Center of adjustment cost parameter | $\nu$ | 15% | |
| Cumulative external financing | $H$ | Proportional cash-carrying cost | $\lambda$ | 1.5% | |
| Cumulative external financing cost | $X$ | Proportional financing cost | $\gamma$ | 6% | |
| Cumulative payout | $U$ | Correlation between $Z_t^A$ and $Z_t^M$ | $\rho$ | 0.4 | |
| Firm value | $P$ | Price of risk for technology shocks | $\eta$ | 0.4 | |
| Average $q$ | $q$ | | | | |
| Net marginal value of cash | $q'$ | State transition intensity | $\zeta_s$ | 0.1 | 0.5 |
| Payout boundary | $\overline{w}$ | Capitalliquidation value | $l_s$ | 1.0 | 0.3 |
| Financing boundary | $\underline{w}$ | Fixed financing cost | $\phi_s$ | 0.5% | 50% |
| Target cash-capital ratio | $m$ | Price of risk for financing shocks | $\kappa_s$ | ln(3) | −ln(3) |
| Conditional risk premium | $\mu^R$ | | | | |
| *B. Hedging* | | | | | |
| Hedge ratio | $\psi$ | Market volatility | $\sigma_m$ | 20% | |
| Fraction of cash in margin account | $\alpha$ | | | | |
| Futures price | $F$ | Margin requirement | $\pi_s$ | 5 | 2 |
| Maximum-hedging boundary | $\hat{w}$ | | | | |
| Speculation boundary | $\tilde{w}$ | | | | |

opportunities introduce an additional motive for the firm to issue equity: timing equity markets in good times. This market timing option is more in the money near the equity issuance boundary, which causes firm value to become locally convex in $w$. In other words, the firm becomes endogenously risk-loving when $w$ is close to the lower barrier $\underline{w}_G$.

Panel B clearly shows that firm value is not globally concave in $w$. For sufficiently high $w$, $w \geq 0.061$, $q_G(w)$ is concave. This is because when the firm already has a considerable amount of cash, the benefits from timing the market are outweighed by the cash-carrying costs it would incur, so that the market timing option is out of the money. Corporate savings are then driven only by precautionary considerations, so that the firm behaves in a risk-averse manner. In contrast, for low values of $w$, $w \leq 0.061$, the firm is more concerned about the risk that financing costs could increase in the future when the state switches to $B$. A firm with low cash holding might want to issue equity while it still has access to relatively cheap financing opportunities, even before it runs out of cash.

Because the issuance boundary $\underline{w}_G$ and the target cash balance $m_G$ are optimally chosen, the marginal values of cash at these two points must be equal:

$$q'_G(\underline{w}_G) = q'_G(m_G) = \gamma. \tag{23}$$

The dash-dotted line in Panel B gives the (net) marginal cost of equity issuance: $\gamma = 0.06$. One immediate consequence of condition (23) is that $q_G(w)$ [or equivalently $p_G(w)$)] is not globally concave in $w$, which in turn has implications for investment, as can be seen from the expression for the cash-sensitivity of investment $i'_s(w)$ obtained by differentiating the optimal investment policy $i_s(w)$ in Eq. (12) with respect to $w$:

$$i'_s(w) = -\frac{1}{\theta}\frac{p_s(w)p''_s(w)}{p'_s(w)^2}. \tag{24}$$

As Eq. (24) highlights, investment increases with $w$ if and only if firm value is concave in $w$. For $0.061 \leq w \leq 0.371$, $q_G(w)$ is concave and corporate investment increases with $w$. In contrast, in the region where $w < 0.061$, $q_G(w)$ is convex in $w$, which implies that investment decreases with $w$, contrary to conventional wisdom. This surprising result is due to the interaction between stochastic external financing conditions and the fixed equity issuance costs.

Panels C and D of Fig. 1 highlight this non-monotonicity of investment in cash. Our model is thus able to account for the seemingly paradoxical behavior that the prospect of higher financing costs in the future can cause investment to respond negatively to an increase in cash today.

Another interesting observation from Panel C is that investment at the financing boundary $\underline{w}_G$ and the target $m_G$ are almost the same. That is, in a situation of equity issuance driven by market timing, the firm holds onto the cash raised and leaves its investment outlays more or less unchanged. By combining Eq. (12) and the boundary conditions one can show that we must have

$$i_G(m_G) - i_G(\underline{w}_G) = \frac{\phi_G}{\theta(1+\gamma)}, \tag{25}$$

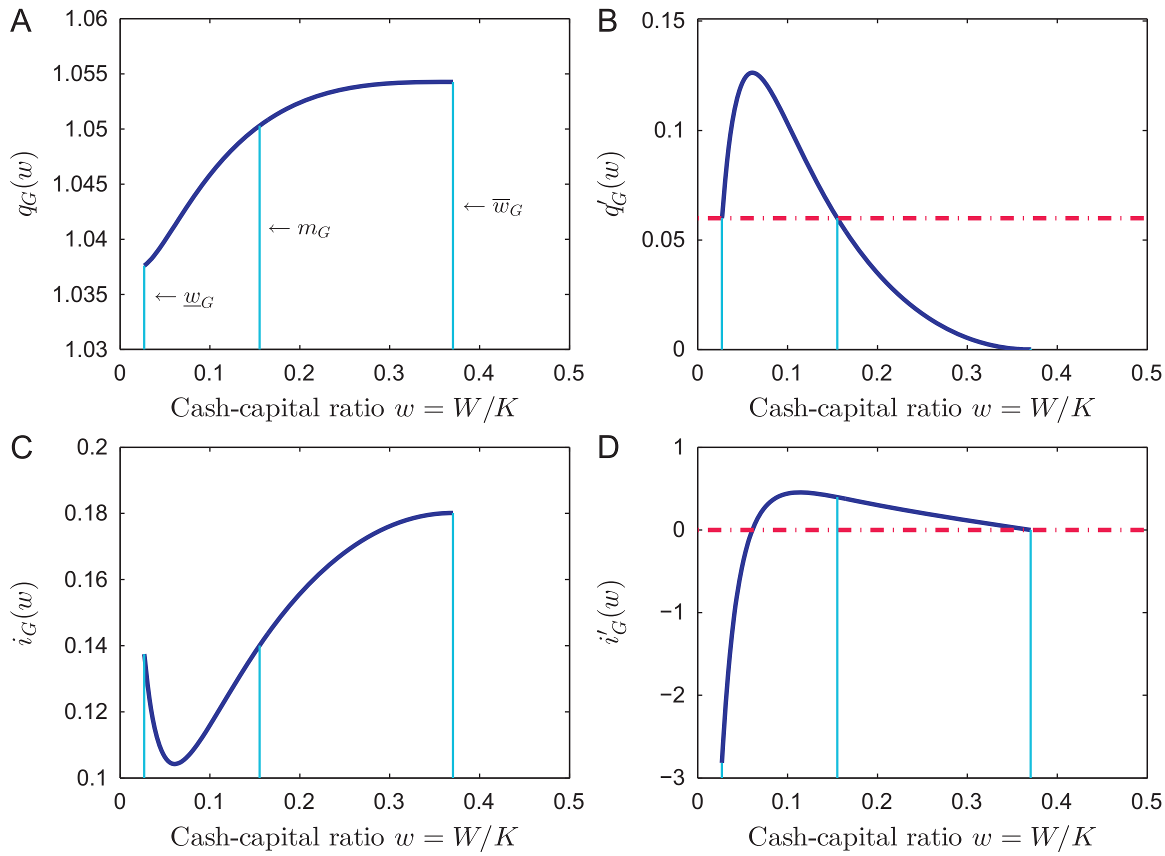


**Fig. 1.** Firm value and investment in the normal state, state $G$. The parameter values are given in Table 1. (A) Average $q$, $q_G(W)$, (B) Net marginal value of cash, $q'_G(W)$, (C) Investment-capital ratio $i_G(W)$, and (D) Investment-cash sensitivity $i'_G(W)$.

which is a small difference when the fixed cost of financing in the good state is low. This explains why most of the new cash raised in a market timing situation is hoarded.

Although considerable debate exists in the literature on corporate investment about the sensitivity of investment to cash flow (see, e.g., Fazzari, Hubbard, and Petersen, 1988; Kaplan and Zingales, 1997), a general consensus is that investment is monotonically increasing with cash reserves or financial slack. In this context, our result that when firms face market-timing options, the monotonic relation between investment and cash holds only in the precautionary saving region is noteworthy, for it points to the fragility of seemingly plausible but misleading predictions derived from simple static models about how corporate investment is affected by financial constraints proxied by firms' cash holdings.

Another example of a potentially misleading proxy for financial constraints in our dynamic model relates to the cash flow sensitivity of cash of Almeida, Campello, and Weisbach (2004). They argue that constrained firms will tend to save more cash in periods of higher cash flows, but unconstrained firms will not. In a dynamic model, Riddick and Whited (2009) argue that an increase in the realized cash flow is correlated with a positive and persistent productivity shock and therefore leads to higher investment and lower cash savings. They find empirical support for the possibility that the corporate propensity to save out of cash flow could be negative. They suggest that the contrasting findings of their analysis and Almeida, Campello, and Weisbach (2004) could be due to differences in the measurement of Tobin's $q$. Our analysis above shows that, even when it faces constant investment opportunities, a firm with low cash holding might not necessarily save more out of cash flow if financing conditions are uncertain.

Finally, because productivity shocks are *i.i.d.* in our model, this implies that the firms that tend to time equity markets are those with low cash holdings, as opposed to those having better investment opportunities.[13] In reality there is likely to be a mix of firms with low cash or high investment opportunities, or both, timing the market.

We turn next to the investment and firm value in bad times (state $B$) and compare them with the results in good times (state $G$).

[13] Time-varying investment opportunities could also play a significant role on cash accumulation and external financing. Eisfeldt and Muir (2011) empirically document that liquidity accumulation and external financing are positively correlated, and they argue that a pure precautionary savings model can account for the empirical evidence.

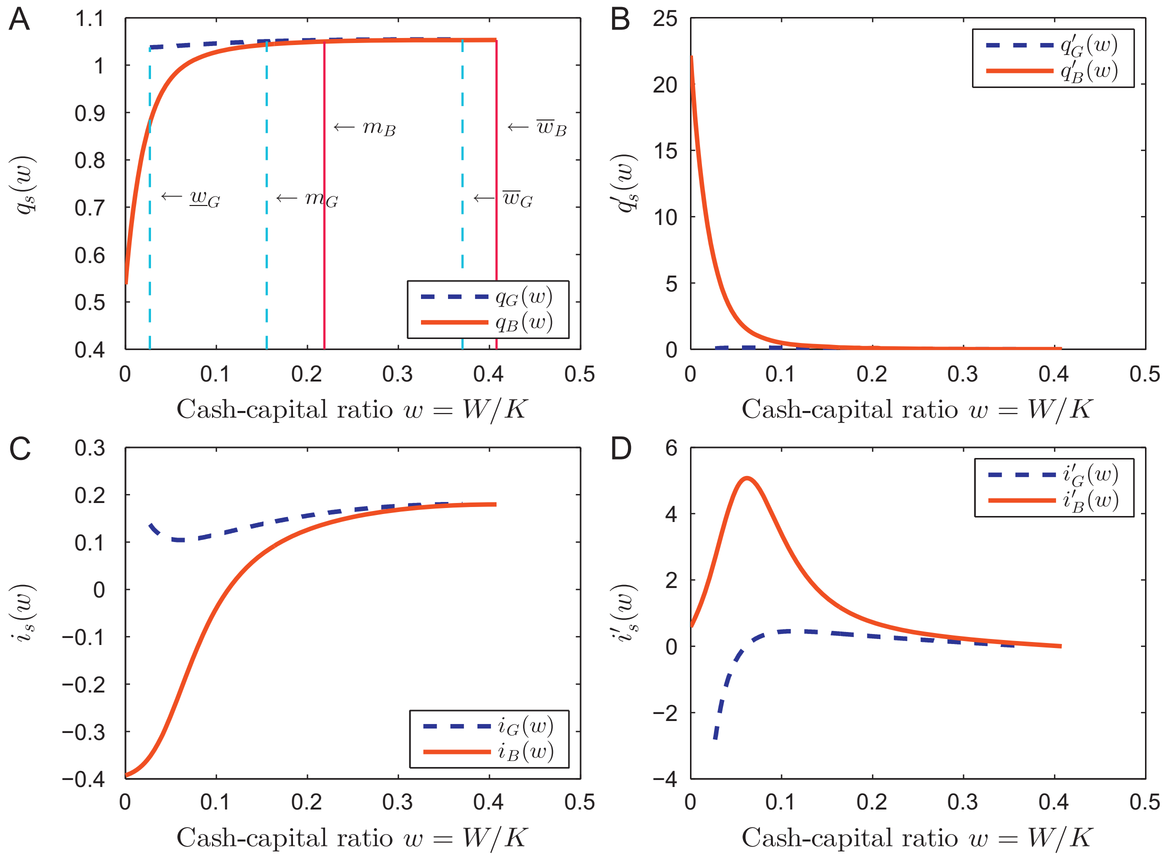


**Fig. 2.** Firm value and investment: Comparing states B with state G. The parameter values are given in Table 1. For both $s=G$ and $s=B$, we have panels (A) Average $q$, $q_s(w)$, (B) Net marginal value of cash $q_s'(w)$, (C) Investment-capital ratio $i_s(w)$ and (D) Investment-cash sensitivity $i_s'(W)$.

### *4.3. High financing costs in bad times*

Fig. 2 plots average $q$ and investment for both states and their sensitivities with respect to $w$. As expected, average $q$ in state $G$ is higher than in state $B$. More remarkable is the fact that the difference between $q_G$ and $q_B$ is very large for low levels of cash holdings $w$. Because productivity shocks in our calibration are the same in both states, this wedge in the average $q$ is solely due to the differences in financial constraints. An important implication of this observation for the empirical literature on corporate investment is that using average $q$ to control for investment opportunities and then testing for the presence of financing constraints by using variables such as cash-flow or cash is not generally valid. Panel C of Fig. 2 shows that investment in state $G$ is higher than in state $B$ for a given $w$, and again the difference is especially large when $w$ is low. Also, investment is much less variable with respect to $w$ in state $G$ than in state $B$.

In state $B$ there is no market timing and, hence, the firm issues equity only when it runs out of cash: $\underline{w}_B=0$. The amount of equity issuance is then $m_B=0.219$, which is much larger than $m_G-\underline{w}_G=0.128$, the amount of issuance in good times. The significant fixed issuance cost in bad times ($\phi_B=0.5$) causes the firm to be more aggressive should it decide to tap equity markets. The amount of issuance would be significantly decreased in bad times if we were to specify a proportional issuance cost $\gamma_B$ that is much higher than the cost $\gamma_G$ in good times. Also, because no market timing opportunity exists in state $B$, firm value is globally concave in bad times. The firm's precautionary motive is stronger in bad times, so that we should expect to see more cash hoarding by the firm. This is reflected in the lower levels of investment and the higher payout boundary $\overline{w}_B=0.408$, which is significantly larger than $\overline{w}_G=0.371$.

Panel B underscores the significant impact of financing constraints on the marginal value of cash in bad times, even though state $B$ is not permanent. In our model, when the firm runs out of cash ($w$ approaches zero) the net marginal value of cash $q_B'(w)$ reaches 23. Strikingly, the firm also engages in large asset sales and divestment to avoid incurring very costly external financing in bad times. Despite the fact that there is a steeply increasing marginal cost of asset sales, the firm chooses to sell its capital at up to a 40% annual rate near $w=0$ in bad times [$i_B(0)=-0.4$]. Finally, unlike in good times, investment is monotonic in $w$ because the firm behaves in a risk-averse manner and $q_B(w)$ is globally concave in $w$.

Conceptually, the firm's investment behavior and firm value are different in bad and good times. Quantitatively, the variation of the investment and firm value in bad times dwarfs the variation in good times. In particular, firm value at low levels of cash holdings is much more

**Table 2**
Conditional distributions of cash-capital ratio $w$, investment-capital ratio $i_s(w)$, and net marginal value of cash $q_s'(w)$.

The conditional distributions of the investment-capital ratio and the net marginal value of cash are computed based on the conditional distributions of the cash-capital ratio in state $G$ and $B$, respectively. All the parameter values are reported in Table 1.

| | Mean | 1% | 25% | 50% | 75% | 99% |
|---|---|---|---|---|---|---|
| Panel A. Cash-capital ratio: $w=W/K$ | | | | | | |
| $G$ | 0.283 | 0.088 | 0.240 | 0.300 | 0.341 | 0.370 |
| $B$ | 0.312 | 0.114 | 0.266 | 0.325 | 0.371 | 0.408 |
| Panel B. Investment-capital ratio: $i_s(w)$ | | | | | | |
| $G$ | 0.170 | 0.112 | 0.166 | 0.176 | 0.179 | 0.180 |
| $B$ | 0.161 | 0.003 | 0.159 | 0.173 | 0.178 | 0.180 |
| Panel C. Net marginal value of cash: $q_s'(w)$ | | | | | | |
| $G$ | 0.015 | 0.000 | 0.001 | 0.005 | 0.019 | 0.111 |
| $B$ | 0.031 | 0.000 | 0.001 | 0.008 | 0.028 | 0.351 |

volatile in state $B$ than in state $G$, as can be seen from Panel A. This could be an important reason why stock price volatility tends to rise sharply in downturns.

### 4.4. The stationary distribution

Table 2 reports the conditional stationary distributions for $w$, $i(w)$, and $q'(w)$ in both states $G$ and $B$. Panel A shows that the average cash holding in state $B$ (0.312) is higher than in state $G$ (0.283) by about 10%. Understandably, firms on average hold more cash for precautionary reasons under unfavorable financial market conditions. In addition, for a given percentile in the distribution, the cutoff cash reserve level is higher in state $B$ than in state $G$, meaning that the precautionary motive is unambiguously stronger in state $B$ than state $G$. Finally, it is striking that even at the bottom 1% of cash holdings the firm's cash-capital ratio is still reasonably high, 0.088 for state $G$ and 0.114 for state $B$, which reflects the firm's strong incentive to avoid running out of cash.

Panel B describes the conditional distribution of investment in states $G$ and $B$. The average investment-capital ratio $i(w)$ is lower in state $B$ (16.1%) than in state $G$ (17%), as cash is more valuable on average in state $B$ than in state $G$. Naturally, the underinvestment problem is more significant for firms with low cash holdings in state $B$ than state $G$. For example, the firm that ranks at the bottom 1% in state $B$ invests at the rate of only 0.3% of its capital stock, while the firm that ranks at the bottom 1% in state $G$ invests at the rate of 11.2% of its capital stock. Thus, firms substantially cut investment to decrease the likelihood of expensive external equity issuance in bad times. As soon as the firm piles up a moderate amount of cash, the underinvestment wedge between the two states disappears. In fact, the top half of the distributions of investments in the two states are almost identical. This result is in sharp contrast to the large gap between the investment-capital ratios $i_G(w)$ and $i_B(w)$ in Panel C of Fig. 2. It again illustrates the firm's ability to smooth out the impact of financing constraints on investment.

Panel C reports the net marginal value of cash $q'(w)$ in states $G$ and $B$. As one might expect, the marginal value of cash is higher in state $B$ than in $G$ on average. Quantitatively, the firm on average values a dollar of cash marginally at 1.015 in state $G$ and 1.03 in state $B$, implying a difference of 1.6 cents on the dollar between the two states. Remarkably, firms are able to optimally manage their cash reserves in anticipation of unfavorable market conditions and, therefore, end up spending very little time in regions of high marginal value of cash. For low cash holdings, the difference between the marginal value of cash in states $G$ and $B$ are larger. For example, at the 99th percentile, the firm values a dollar of cash marginally at 1.35 in state $B$ and 1.11 in state $G$, implying a difference of 22 cents on the dollar. This is a sizable difference, but it is still much less than the extreme cases illustrated in Panel B of Fig. 2.

### 4.5. Equity issuance waves and stock repurchase waves

One can see from Panel A in Fig. 2 that the payout boundary is higher in state $B$ than in state $G$: $\overline{w}_B = 0.408 > \overline{w}_G = 0.371$. This implies that any firm in state $B$ with cash holding $w \in (0.371, 0.408)$ will time the favorable market conditions by paying out a lump sum amount of $(w-\overline{w}_G)$ as the state switches from $B$ to $G$. To the extent that the payout is performed through a stock repurchase (as is common for non-recurrent corporate payouts), our model then provides a simple explanation for why stock repurchase waves tend to occur in favorable market conditions.

Similarly, the issuance boundary is lower in state $B$ than in state $G$: $\underline{w}_B = 0 < \underline{w}_G = 0.0277$, which implies that any firm in state $B$ with cash holding $w \in (0, 0.027)$ will time the favorable market conditions by issuing external equity of an amount $(m_G - w)$ per unit of capital as the state switches from $B$ to $G$. In other words, our model also generates equity issuance waves in good times.

Taken together, these two results provide a plausible explanation for the Dittmar and Dittmar (2008) finding that equity issuance waves coincide with stock repurchase waves. As financing conditions improve, firms' precautionary demand for cash is reduced, which translates into stock repurchases by cash-rich firms. Note that our model does not predict repurchases when firms are undervalued, the standard explanation for repurchases in the literature. As Dittmar and Dittmar (2008) point out, this theory of stock repurchases is inconsistent with the evidence on repurchase waves. They further suggest that the market timing explanations by Loughran and Ritter (1995) and Baker and Wurgler (2000) are rejected by their evidence on repurchase waves. However, as our model shows, this is not the case. It is possible to have at the same time market timing through equity issues by cash-poor firms and market timing through repurchases by cash-rich firms. These two very different market timing behaviors can be driven by the same change in external financing conditions. The difference in behavior is driven only by differences in internal financing conditions, the amount of cash held by firms.

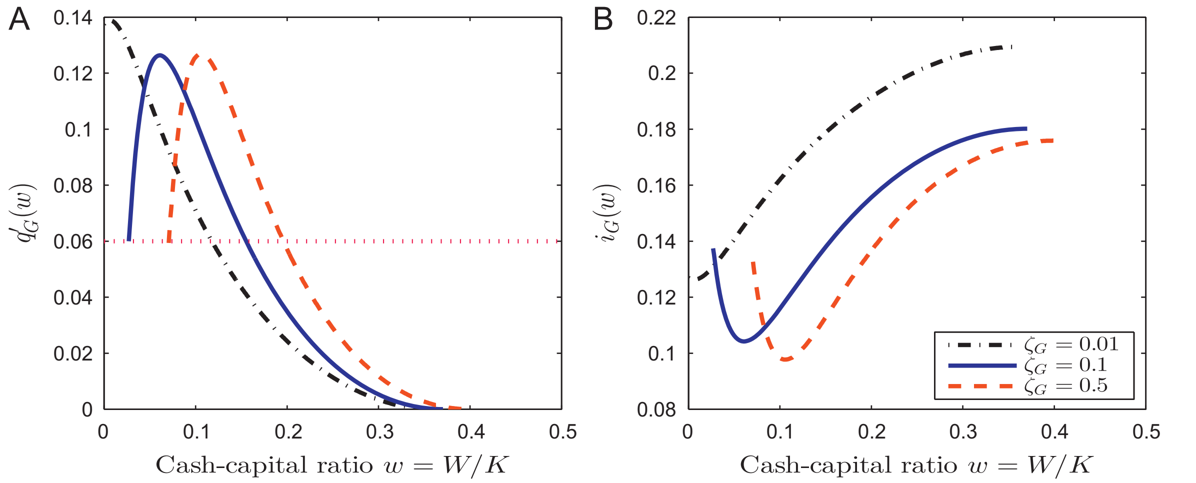


**Fig. 3.** The effect of duration in state $G$ on $q'_G(w)$ and $i_G(w)$. This figure plots (A) the net marginal value of cash $q'_G(w)$ and (B) investment-capital ratio $i_G(w)$ for three values of transition intensity, $\zeta_G = 0.01, 0.1, 0.5$. All other parameter values are given in Table 1.

### 4.6. Comparative analysis

Next, we study the effect of changes in the duration of state $G$ and then analyze the effect of changes in the issuance cost $\phi_G$.

#### 4.6.1. Effect of changes in the duration of state G

How does the transition intensity $\zeta_G$ out of state $G$ (or, equivalently, the duration $1/\zeta_G$ of state $G$) affect firms' market timing behavior? Consider the case in which state $G$ has a very high average duration of one hundred years ($\zeta_G = 0.01$). Not surprisingly, in this case the firm taps equity markets only when it runs out of cash ($\underline{w}_G = 0$). Firm value $q_G(w)$ is then globally concave in $w$ and $i_G(w)$ increases with $w$ everywhere. Essentially, the expected duration of favorable market conditions is so long that the market timing option has no value for the firm.

With a sufficiently high transition intensity $\zeta_G$ (say $\zeta_G = 0.1$), however, the firm could time the market by selecting an interior equity issuance boundary $\underline{w}_G > 0$. The firm then equates the net marginal value of cash at $\underline{w}_G$ with the proportional financing cost $\gamma : q'_G(\underline{w}_G) = \gamma = 6\%$, as can be seen from Panel A of Fig. 3. Because the net marginal value of cash at the target cash-capital ratio, $m_G$, also satisfies $q'_G(m_G) = 6\%$, it follows that, for $w \in [\underline{w}_G, m_G]$, the net marginal value of cash $q'_G(w)$ first increases with $w$ and then decreases, as Panel A again illustrates.

When $\zeta_G$ increases from 0.1 to 0.5, the firm taps the equity market even earlier ($\underline{w}_G$ increases from 0.027 to 0.071) and holds onto cash longer (the payout boundary $\overline{w}_G$ increases from 0.370 to 0.400) for fear that favorable financial market conditions could be disappearing faster. For sufficiently high $w$, the firm facing a shorter duration of favorable market conditions (higher $\zeta_G$) values cash more at the margin [higher $q_G(w)$] and invests less [lower $i_G(w)$]. However, for sufficiently low $w$, the opposite holds because the firm with a shorter lived market timing option taps equity markets sooner so that the net marginal value of cash is lower. Consequently and somewhat surprisingly, the underinvestment problem can be smaller for a firm facing a narrower window of good financing conditions and its investment is higher, as Panel B illustrates.

Both investment and the net marginal value of cash are highly nonlinear and non-monotonic in cash despite the fact that the real side of our model is time invariant. Our model thus suggests that the typical empirical practice of detecting financial constraints is conceptually flawed. Using average $q$ to control for investment opportunities and then testing for the presence of financing constraints by using variables such as cash flow or cash (which is often done in the empirical literature) would be misleading and miss the rich dynamic adjustment involved to balance the firm's market timing and precautionary saving motives.

**Table 3**
Fixed cost of equity issuance.
This table reports the amount of equity issuance relative to capital, $m_G - \underline{w}_G$, the average issuance cost, the equity issuance boundary, $\underline{w}_G$, and the payout boundary, $\overline{w}_G$, for different fixed cost of equity issuance $\phi_G$ in state $G$. All the parameter values are reported in Table 1.

| $\phi_G$ | $m_G - \underline{w}_G$ | Average cost | $\underline{w}_G$ | $\overline{w}_G$ |
|---|---|---|---|---|
| 0 | 0.000 | 0.060 | 0.092 | 0.357 |
| 0.5% | 0.128 | 0.099 | 0.027 | 0.370 |
| 1.0% | 0.153 | 0.126 | 0.013 | 0.375 |
| 2.0% | 0.176 | 0.174 | 0.000 | 0.380 |
| 5.0% | 0.189 | 0.324 | 0.000 | 0.388 |
| 10.0% | 0.199 | 0.563 | 0.000 | 0.394 |

#### 4.6.2. The effect of changes in issuance cost $\phi_G$

Table 3 reports the effects of changes in the issuance cost parameter $\phi_G$ on the issuance boundary $\underline{w}_G$, the issuance amount $(m_G - \underline{w}_G)$, the average issuance cost, and the payout boundary $\overline{w}_G$.

Consistent with basic economic intuition, when the fixed issuance cost $\phi_G$ increases, the firm becomes less willing to issue equity, holds onto its cash longer, and issues more equity whenever it taps the equity market. These responses are reflected in a lower issuance boundary $\underline{w}_G$, a higher payout boundary $\overline{w}_G$, and a larger amount of issuance $(m_G - \underline{w}_G)$. While a firm with a larger fixed cost $\phi_G$ issues more, the average issuance cost is still higher. Without any fixed cost $(\phi_G = 0)$,[14] the firm issues just enough equity to stay away from its optimally chosen financing boundary $\underline{w}_G = 0.092$, and the net marginal value of cash at issuance equals $q'_G(w) = \gamma = 6\%$, so that the average issuance cost is precisely 6%. In this extreme case, the marginal value of cash $q'_G(w)$ is monotonically decreasing in $w$, and firm value is again globally concave in $w$ even under market timing.

When the fixed cost of issuing equity is positive but not very high (consider $\phi_G = 1\%$), the firm times equity markets at the optimally chosen issuance boundary of $\underline{w}_G = 0.013$ and issues the amount $(m_G - \underline{w}_G) = 0.153$. Neither the marginal value of cash nor investment is then monotonic in $w$ in the region where $w \in [\underline{w}_G, m_G]$. Moreover, higher fixed costs lead firms to choose larger issuance sizes $(m_G - \underline{w}_G)$. Also, $\underline{w}_G = 0$ when $\phi_G = 2\%$. This result shows that market timing does not necessarily lead to a violation of the pecking order between internal cash and external equity financing and that $\underline{w}_G > 0$ is not necessary for the convexity of the value function. Finally, when the fixed cost of issuing equity is very high, the market timing effect is so weak that the precautionary motive dominates again, so that the net marginal value of cash is monotonically decreasing in $w$.

Having determined why the value function could be locally convex, we next explore the implications of convexity for investment. Recall from Eq. (24) that the sign of the investment-cash sensitivity $i'_s(w)$ depends on $p''_s(w)$. Thus, in the region where $p_G(w)$ is convex, investment is decreasing in cash holdings $w$.

There are other ways to generate a negative correlation between changes in investment and cash holdings. First, when the firm moves from state $G$ to $B$, this not only results in a drop in investment, especially when $w$ is low (see Panel C in Fig. 2), but also in an increase in the payout boundary, which could explain why firms during the recent financial crisis have increased their cash reserves and cut back on capital expenditures, as Acharya, Almeida, and Campello (forthcoming) show. Second, in a model with persistent productivity shocks (as in Riddick and Whited, 2009), when expected future productivity falls, the firm cuts investment and the cash saved could also result in a rise in its cash holding.[15]

Is it possible to distinguish empirically between these two mechanisms? In the case of a negative productivity shock, the firm has no incentive to significantly raise its cash holding. In fact, reduced investment might lead the firm to hold less cash. This prediction is opposite to the prediction related to a negative financing shock. Thus, following a negative technology shock firms that already have high cash holdings could pay out cash but would hold even more cash after a negative financing shock.

Another empirical prediction that differentiates our model from other market timing models concerns the link between equity issuance and corporate investment. Our model predicts that underinvestment is substantially mitigated when the firm is close to the equity financing boundary. Moreover, the positive correlation between investment and equity issuance in our model is not driven by better investment opportunities (as the real side of the economy is held constant across the two states). It is driven solely by the market timing and precautionary demand for cash.

## 5. Real effects of financing shocks

Several empirical studies have attempted to measure the impact of financing shocks on corporate investment by exploiting the recent financial crisis as a natural experiment.[16] A central challenge for any such study is to determine the degree to which the financial crisis has been anticipated by corporations. As long as corporations put a nonzero probability on a financial crisis occurring, the real effects of the financing shock would already be present before the realization of the shock. Any real response following the shock would merely be a residual response. Our model is naturally suited to contrast the impact of more versus less anticipated financing shocks on investment and firm value.

The fact that shocks are anticipated does not necessarily mean that the firm knows exactly when a financial crisis will occur. It simply means that the firm (and everyone else in the economy) attaches a certain probability to the crisis. In our benchmark model the firm solves the value maximization problem in the good state assuming that $\zeta_G = 0.1$. This is a scenario in which a negative financing shock is thought to be likely, at least compared with a scenario in which the firm assumes that $\zeta_G = 0.01$. What are the real effects of an increase in $\zeta_G$ from 0.01 to 0.1 both before and after the economy switches from state $G$ to state $B$? We explore this question below while keeping the transition intensity in the bad state fixed at $\zeta_B = 0.5$.

A higher probability of a crisis leads firms to respond by holding more cash, adopting more conservative investment policies, and raising external financing sooner, etc. As a result, the ex post impact of the financing shock on investment and other real decisions can appear to be small due to the fact that the shock has already been partially smoothed out through precautionary savings.

Fig. 4 illustrates this idea. A comparison of the two scenarios with different probabilities of a negative financing shock demonstrates that the firm smoothes out financing shocks in two ways. First, a heightened concern about the incidence of a financial crisis pushes firms to invest more conservatively in state $G$ most of the time. Second, a firm anticipating a higher probability of crisis

[14] The case with low (close to zero) financing costs is empirically relevant. Baker and Wurgler (2000) claim that financing costs can be precipitously close to zero in market conditions that can be identified (in sample) by econometricians.

[15] This mechanism is captured in our model with the two states corresponding to two different values for the return on capital $\mu_s$.

[16] See Campello, Graham, and Harvey (2010), Campello, Giambona, Graham, and Harvey (2011), Duchin, Ozbas, and Sensoy (2010), and Almeida, Campello, Laranjeira, and Weisbenner (2012), among others.

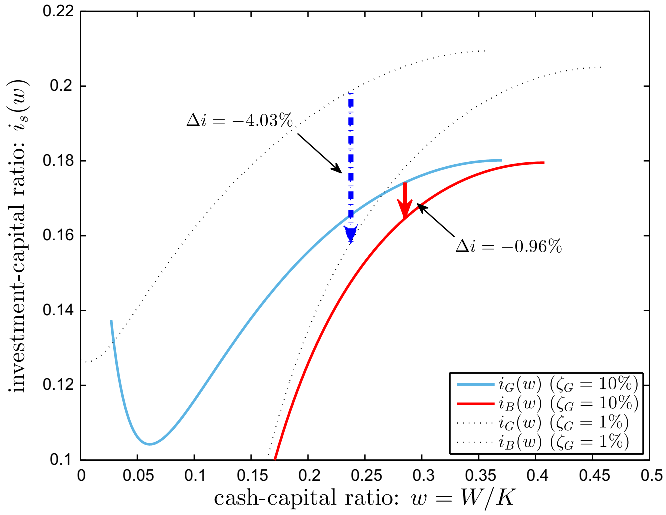


**Fig. 4.** Impact of financing shocks on investment. This figure illustrates the response in investment when the financing shock occurs. The two solid lines plot the investment-capital ratio in the case of more anticipated shocks, with $\zeta_G = 0.1$. The two dotted lines are for the case of less anticipated shocks, where $\zeta_G = 0.01$.

**Table 4**
Distribution of investment responses.

This table reports the distributions of instantaneous investment responses when firms in state $G$ (good state) experience a negative financing shock. The distributions of investment responses are computed based on the stationary distributions of the cash-capital ratio $W$ conditional on being in state $G$. For example, the column 25% gives the investment response by a firm whose cash holding is at 25th-percentile of the stationary distribution in state $G$. The mean responses are integrated over the conditional stationary distribution in state $G$.

| | Mean | 1% | 5% | 25% | 50% | 75% |
|---|---|---|---|---|---|---|
| Panel A. Financing shock | | | | | | |
| $\zeta_G = 1\%$ | −6.59 | −43.17 | −23.66 | −7.06 | −3.66 | −2.23 |
| $\zeta_G = 10\%$ | −1.78 | −18.11 | −6.49 | −1.67 | −0.76 | −0.39 |
| Panel B. Shock to expected productivity | | | | | | |
| $\zeta_G = 1\%$ | −6.59 | −6.84 | −6.84 | −6.81 | −6.67 | −6.40 |
| $\zeta_G = 10\%$ | −3.15 | −3.17 | −3.17 | −3.17 | −3.16 | −3.15 |

also holds more cash on average, which further reduces the impact of financial shocks on investment.

We specify in Fig. 4 the size of the investment response to a financing shock at the average cash holdings in state $G$. With a lower probability of a financing shock ($\zeta_G = 0.01$), the average cash holding in state $G$ is 0.224, at which point investment drops by 4.03% following the shock. In contrast, with a higher probability ($\zeta_G = 0.1$), the average cash holding in state $G$ rises to 0.283, and the drop in investment reduces to 0.96% at this level of cash holding.

This analysis reveals that a small observed investment response to a financing shock does not imply that financing shocks are unimportant for the real economy. As Fig. 4 shows, with a higher risk of a crisis, the firm responds by taking actions ahead of the realization of the shock. Thus, the firm substantially scales back its investment in state $G$ when the transition intensity $\zeta_G$ rises from 1% to 10%, which is a main contributor to the overall reduction in the firm's investment response. The ex ante responses of the firm in state $G$ such as lower levels of investment, higher (costly) cash holdings, and earlier use of costly external financing are all reflections of the impending threat of a negative financing shock and all of them contribute to the real costs of financing shocks.

Panel A of Table 4 provides information about the entire distribution of investment responses to financing shocks for $\zeta_G = 1\%$ and 10%. The average investment reduction following a negative financing shock is 1.78% when $\zeta_G = 10\%$, compared with 6.59% when the shock was perceived to be less likely ($\zeta_G = 1\%$). The median investment decline is 0.76% for $\zeta_G = 10\%$, which is again lower than 3.66%, the median investment drop when $\zeta_G = 1\%$. Moreover, in the scenario in which the financing shock was seen to be less likely, the distribution of investment responses also has significantly fatter left tails. For example, at the 5th percentile, the investment decline is 23.66% when $\zeta_G = 1\%$, which is much larger than the drop of 6.49% when $\zeta_G = 10\%$. In other words, when a financial crisis strikes, only those firms that happen to have low cash holdings will be forced to cut investment dramatically.

This result is consistent with the survey evidence of Campello, Graham, and Harvey (2010). They report that during the financial crisis in 2008 the chief financial officers they surveyed planned to cut capital expenditures by 9.1% on average when their firm was financially constrained, while the unconstrained firms planned to keep capital expenditures essentially unchanged (on average, CFOs of these firms reported a cutback of investment of only 0.6%). Our model further demonstrates that the fraction of firms that have to significantly cut investment (e.g., by over 5%) following a severe financing shock decreases significantly as firms assign higher probabilities to a financial crisis shock.

Furthermore, the average investment response in our scenario of more anticipated shocks quantitatively matches the empirical findings from the recent financial crisis. Duchin, Ozbass, and Sensoy (2010) find that corporate investment on average declined by 6.4% from its unconditional mean level before the crisis. Based on the average investment-capital ratio of 17%, this number translates into a 1% decline in the investment-capital ratio, which is between the mean and median investment response for $\zeta_G = 10\%$ in Panel A of Table 4.

While firms can effectively shield investment from financing shocks by hoarding more cash, changes in cash reserves have almost no effect on firms' investment responses when they are hit by a shock to expected productivity. To compare the effects of shocks to expected productivity against the effects of financing shocks, we carry out the following experiment. Holding the financing cost constant ($\phi_G = \phi_B = 0.5\%$ and $\gamma = 6\%$), we instead assume that the conditional mean return on capital ($\mu_s$) is higher in state $G$ than $B$. Specifically, we hold $\mu_G$ at 22.7% as in the benchmark model but calibrate $\mu_B = 19.25\%$ such that the average drop in investment following a productivity shock is 6.59% when $\zeta_G = 1\%$, the same as in the scenario with financing shocks (see Panel A of Table 4). Again, we consider the two scenarios with $\zeta_G = 1\%$ and 10% respectively, while holding $\zeta_B = 0.5$. The results for the distribution of investment responses immediately following a productivity shock are reported in Panel B of Table 4.

A higher transition intensity $\zeta_G$ means that the high-productivity state is expected to end sooner on average, and the firm invests less aggressively in state $G$ as a result. This is why the average decline in investment following a productivity shock is smaller when $\zeta_G = 10\%$ than when $\zeta_G = 1\%$. More interestingly, unlike the effects of financing shocks for which there is significant heterogeneity in investment responses across firms with different levels of cash-capital ratios, the investment responses following a productivity shock are essentially the same across all firms. The contrast of investment responses to financing shocks and shocks to expected productivity in our calibrated model suggests that financing and productivity shocks can have significantly different implications for investment responses among firms with different amount of financial slack.[17]

In summary, our model provides two new insights about the real effects of financing shocks. First, it will be wrong to conclude that financing shocks have small effect on the real economy just because the ex post investment responses to financing shocks are small. The effects of financing shocks on a firm's investment policy crucially depend on two variables: (1) the probability that the firm attaches to the financing shock, and (2) the firm's cash holding. A relatively small rise in the probability of a financing shock can already cause firms to underinvest significantly and hold onto excess cash in good times, which in turn leads to small average investment responses to financing shocks ex post.

Second, the heterogeneity in investment responses across firms with different cash holdings can help us distinguish between financing shocks and productivity shocks empirically. There will be significant variation in the investment responses across firms with different cash holdings following a financing shock, but little such variation following a shock to expected productivity.

## 6. Financial constraints and the risk premium

In this section, we explore how aggregate financing shocks affect the risk premium for a financially constrained firm.[18] Without external financing constraints, the firm in our model has a constant risk premium. When the firm's financing conditions remain the same over time, a conditional CAPM (capital asset pricing model) holds in our model, in which the conditional beta is monotonically decreasing in the firm's cash-capital ratio. In the presence of aggregate financing shocks, however, the conditional risk premium is determined by a two-factor model, which prices both the aggregate shocks to profitability and the shocks to financing conditions.[19]

A heuristic derivation of the firm's (risk-adjusted) expected return involves a comparison of the HJB equations under the physical and risk-neutral measures $\mathbb{P}$ and $\mathbb{Q}$. Let the firm's conditional risk premium in state $s$ be $\mu_s^R(w)$. We can then write the HJB equation under the physical measure as

$$\begin{aligned}(r_s+\mu_s^R(w))p_s(w) = \max_{i_s}[(r_s-\lambda)w+\mu_s-i_s-g_s(i_s)]p_s'(w) \\ +\frac{\sigma_s^2}{2}p_s''(w)+(i_s-\delta)(p_s(w)-wp_s'(w)) \\ +\zeta_s(p_{s^-}(w)-p_s(w)),\end{aligned} \tag{26}$$

where $\mu_s$ and $\zeta_s$, respectively, denote the expected return on capital and the transition intensity from state $s$ to $s^-$ under the physical probability measure. By matching terms in the HJB equations (11) and (26), and using the risk adjustments specified in (7) and (8), we then obtain the following expression for the conditional risk premium:

[17] Fixed capital adjustment costs can also generate heterogenous investment responses to shocks to expected productivity. However, the response will depend more on how close the firm is to the adjustment boundary rather than on the firm's cash holding.

[18] Livdan, Sapriza, and Zhang (2009) also study the effect of financing constraints on stock returns. Their model, however, does not allow for stochastic financing conditions or cash accumulation.

[19] One interpretation of the pricing model in this section is that all investors are rational risk-averse investors who anticipate shocks to a firm's financing opportunities, which could be driven by (unmodeled) shocks to financial intermediation costs or changes in the opaqueness of the firm's balance sheets. An alternative interpretation is that the firm's external financing costs are driven by (unmodeled) changes in market sentiment. This behavioral interpretation is still consistent with the view that investors require compensation for the risk with respect to changes in the firm's financing opportunities if one takes the approach based on differences of opinion *à* la Scheinkman and Xiong (2003). The point is that, in the differences of opinion model, investors are aware that at any moment there could be other more optimistic or pessimistic investors. Each investor is not always the marginal investor, and to the extent that each investor is aware of this (as is assumed in the differences of opinion model) he faces risk with respect to other investors' optimism (which here takes the form of risk with respect to the firm's financing opportunities) for which he requires compensation.

$$\mu_s^R(w) = \eta_s \rho_s \sigma_s \frac{p_s'(w)}{p_s(w)} - (e^{\kappa_s}-1)\zeta_s \frac{(p_{s^-}(w) - p_s(w))}{p_s(w)}. \tag{27}$$

The first term in Eq. (27) is the productivity risk premium, which is the product of the firm's exposure to aggregate (Brownian) productivity shocks $\rho_s \sigma_s p_s'(w)/p_s(w)$ and the price of Brownian risk $\eta_s$ (where $\rho_s$ is the conditional correlation between the firm's productivity shock $dA_t$ and the stochastic discount factor in state $s$). This term is positive for firms whose productivity shocks are positively correlated with the aggregate market.

The second term is the financing risk premium, which compensates risk-averse investors for the firm's exposure to aggregate financing shocks. Financing shocks are priced when their arrival corresponds to changes in the stochastic discount factor. As seems empirically plausible, we suppose that the stochastic discount factor jumps up when aggregate financing conditions deteriorate, that is, $\kappa_G = -\kappa_B > 0$ in our two state model. In other words, investors demand an extra premium for investing in firms whose values drop during times when external financing conditions worsen [$p_G(w) > p_B(w)$].

In the first-best setting where a firm has free access to external financing, its risk premium is constant and can be recovered from Eq. (27) by setting $\eta_s$, $\rho_s$, and $\sigma_s$ to constants and dropping the second term. We then obtain the standard CAPM formula:

$$\mu^{FB} = \eta \rho \sigma \frac{1}{q^{FB}}. \tag{28}$$

The comparison between $\mu_s^R(w)$ and $\mu^{FB}$ highlights the impact of external financing frictions on the firm's cost of capital.

When financing opportunities are constant over time, financial constraints affect only the cost of capital by amplifying (or dampening) a firm's exposure to productivity shocks. This effect is captured by the productivity (diffusion) risk premium in Eq. (27). As the cash-capital ratio $w$ increases, the firm tends to become less risky for two reasons. First, if a greater fraction of its assets is in cash, the firm beta is automatically lower due to a simple portfolio composition effect. As a financially constrained firm hoards more cash to reduce its dependence on costly external financing, the firm beta becomes a weighted average of its asset beta and the beta of cash, which is equal to zero. In particular, with a large enough buffer stock of cash relative to its assets, this firm could be even safer than a firm facing no external financing costs and therefore holding no cash. Second, an increase in $w$ relaxes the firm's financing constraint and therefore reduces the sensitivity of firm value to cash flow, which also tends to reduce the risk of the firm.

Time-varying external financing costs affect the cost of capital for a financially constrained firm in two ways. First, the firm's exposure to productivity shocks changes as financing conditions change, because the marginal value of cash $p_s'(w)$ and firm value $p_s(w)$ both depend on the state $s$. Second, when external financing shocks are priced, investors demand an extra premium for investing in firms that do poorly when financing conditions worsen. This effect is captured by the second term in Eq. (27). Note that $(p_{s^-}(w) - p_s(w))/p_s(w)$ gives the percentage change of firm value if financing conditions change. Intuitively, the financing risk premium is larger the bigger the relative change in firm value due to a change in external financing conditions.

Fig. 5, Panel A, plots the productivity risk premium [the first term in Eq. (27)] in state $G$ as a function of the cash-capital ratio $w$. This premium is generally decreasing in the cash-capital ratio, except near the financing boundary. In the benchmark case ($\zeta_G = 0.1$), the risk with respect to higher future financing costs generates market timing behavior and non-monotonicity in the marginal value of cash (Fig. 1, Panel B), which in turn can cause the productivity risk premium to be locally increasing in $w$ for low levels of $w$. As the non-monotonicity in the marginal value of cash is partially offset by the asset composition effect, the non-monotonicity in the productivity risk premium is relatively weak. Similarly, holding $w$ fixed at a low level, market timing can lower $p_G'(w)$ as the transition intensity $\zeta_G$ increases. This explains why the productivity risk premium may be decreasing in the transition intensity for low $w$. When the transition intensity is sufficiently low (e.g., $\zeta_G = 0.01$), the non-monotonicity in the productivity risk premium disappears.

Second, Panel B plots the financing risk premium. The size of this premium depends on the relative change in firm value when external financing conditions change. It is increasing in the transition intensity $\zeta_G$, but decreasing in $w$. Intuitively, when the cash holding is low, a sudden worsening in external financing conditions is particularly costly to the firm as it leads deep cuts in investment, asset fire sales, and costly equity issuance. However, when the cash holding is high, the firm can still maintain relatively high levels of investment using the internal funds.

In Panels C and D, both the productivity risk premium and financing risk premium in state $B$ are monotonically and rapidly decreasing in the firm's cash holding. When $w$ is close to zero, the annualized conditional productivity risk premium can exceed 80%. The high premium and sharp decline with $w$ mirror the rapid decline in the marginal value of cash (see Fig. 2, Panel B). High marginal value of cash in the low $w$ region can dramatically amplify the firm's sensitivity to productivity shocks. The productivity risk premium eventually falls below 2% when the firm is near the payout boundary. Similarly, the conditional financing premium can exceed 30% when $w$ is close to zero, this is due to the large jump in firm value when the financing state changes (see Fig. 2, Panel A).

Quantitatively, the level and variation of the conditional risk premium generated by financing constraint should be interpreted in conjunction with the stationary distributions of cash holdings in Section 4.4. Because the firm's cash holdings rarely drop to very low levels, its risk premium is small and smooth most of the time in our model.

Our model has several implications for the expected returns of financially constrained firms. Controlling for productivity and financing costs, the model predicts an inverse relation between the expected returns and corporate cash holdings, which has been shown by Dittmar and Mahrt-Smith (2007) and others. Our analysis points out that this negative relation might not be due to agency problems, as they emphasize, but could be driven by

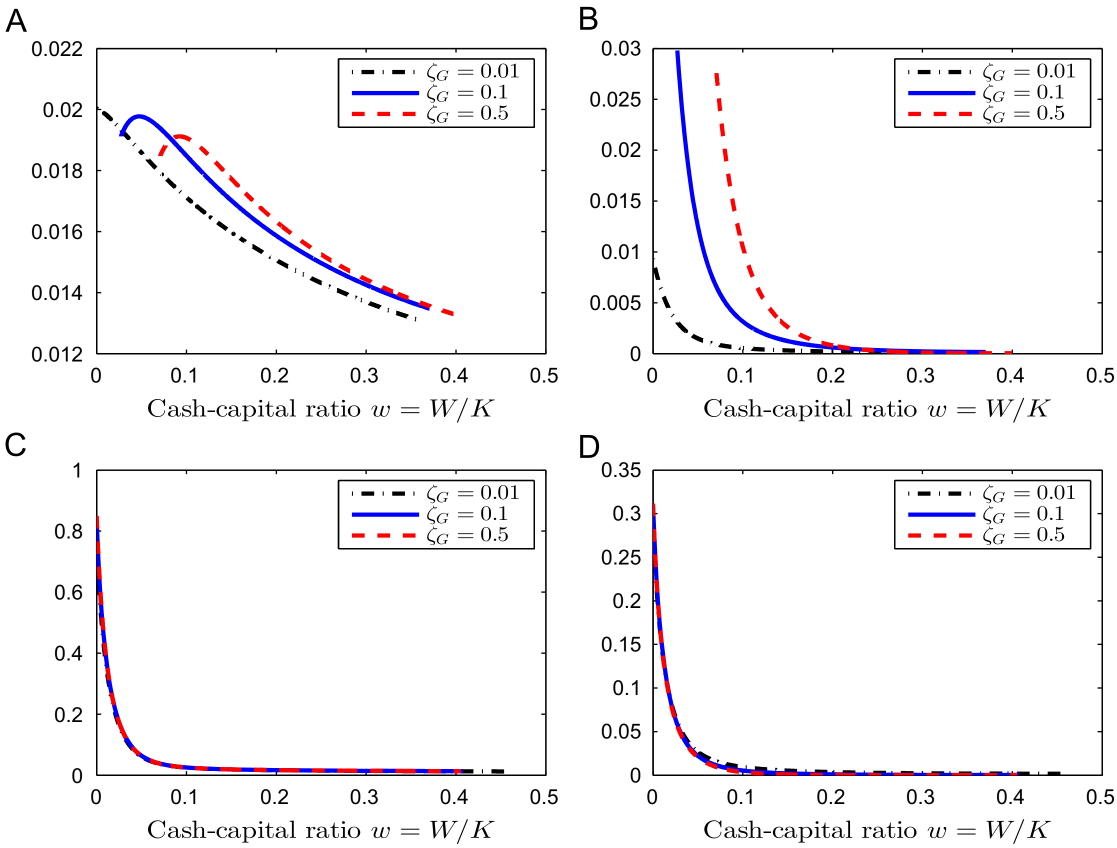


**Fig. 5.** The effects of stochastic financing conditions on the cost of capital. This figure plots firm risk premium in state $G$ and $B$. We consider three levels of transition intensity from $G$ (good state) to $B$ (bad state): $\zeta_G = 0.01, 0.1, 0.5$. All other parameter values are given in Table 1. (A) Productivity risk premium in state $G$, (B) Financing risk premium in state $G$, (C) Productivity risk premium in state $B$ and (D) Financing risk premium in state $B$.

relaxed financing constraints and a changing asset composition of the firm. When heterogeneity in productivity and financing costs is difficult to measure, it is important to take into account the endogeneity of cash holdings when comparing firms with different cash holdings empirically. A firm with higher external financing costs tends to hold more cash. However its risk premium could still be higher than for a firm with lower financing costs and consequently lower cash holdings. Thus, a positive relation between returns and corporate cash holdings across firms could still be consistent with our model [see Palazzo, 2012 for a related model and supporting empirical evidence].

With time-varying financing conditions, our model can be seen as a conditional two-factor model to explain the cross section of returns (we provide details of the derivation in Appendix C). A firm's risk premium is determined by its productivity beta and its financing beta. Other things equal, a firm whose financing costs move closely with aggregate financing conditions has a larger financing beta and earns higher returns than one with financing costs independent of aggregate conditions. Empirically, this two-factor model can be implemented using the standard market beta plus a beta with respect to a portfolio that is sensitive to financing shocks (e.g., a banking portfolio). This model, in particular, shows how a firm's conditional beta depends on the firm's cash holdings.

## 7. Market timing and dynamic hedging

We have thus far restricted the firm's financing choices to internal funds and external equity financing. In this section, we extend the model to allow the firm to engage in dynamic hedging via derivatives such as market-index futures. How does market timing behavior interact with dynamic hedging? This is the question we address in this section. We denote by $F$ the index futures price for a market portfolio that is already completely hedged against financing shocks. Under the risk-neutral probability measure, the futures price $F$ then evolves according to

$$dF_t = \sigma_m F_t \, d\widehat{Z}_t^M, \tag{29}$$

where $\sigma_m$ is the volatility of the market index portfolio and $\{\widehat{Z}_t^M : t \geq 0\}$ is a standard Brownian motion under $\mathbb{Q}$ that is correlated with the firm's productivity shock $\{Z_t^A : t \geq 0\}$ with a constant correlation coefficient $\rho$.

Futures contracts require that investors hold cash in a margin account. We denote the notional amount of the futures contract by $\psi_t W_t$, and denote the fraction of the firm's total cash $W_t$ held in the margin account by $\alpha_t \in [0,1]$. Cash held in this margin account incurs a flow unit cost $\epsilon \geq 0$. Futures market regulations typically require that an investor's futures position (in absolute value) cannot exceed a multiple $\pi$ of the amount of cash $\alpha_t W_t$ held in the margin account. We let this multiple be state dependent and denote it by $\pi(s_t)$. The margin requirement in state $s$ then imposes the following limit on the firm's futures position: $|\psi_t| \leq \pi(s_t)\alpha_t$. As the firm can costlessly reallocate cash between the margin account and its regular interest-bearing account, it optimally holds the minimum amount of cash necessary in the margin account when $\epsilon > 0$. For simplicity, we shall ignore this haircut on the margin account and assume that $\epsilon = 0$. Under this assumption, we do not need to keep track of cash allocations in the margin account and outside the account. We can then simply set $\alpha_t = 1$.

The firm's cash holdings evolve as

$$dW_t = K_t[\mu(s_t)\,dt + \sigma(s_t)\,dZ_t] - (I_t + \Gamma_t)\,dt + dH_t - dU_t + [r(s_t) - \lambda]W_t\,dt + \psi_t W_t \sigma_m\,dZ_t^M, \tag{30}$$

where $|\psi_t| \leq \pi(s_t)$. To avoid unnecessary repetition, we consider only the case with positive correlation, i.e., $\rho > 0$. Next, we examine the crisis state.

### 7.1. In state B

Given that firm value is always concave in cash in state $B$ ($P_{WW}(K,W,G) < 0$), the firm in state $B$ faces the same decision problem as the firm in BCW. BCW show that the optimal hedge ratio (with time-invariant opportunities) is given by

$$\psi_B^*(w) = \max\left\{\frac{-\rho\sigma_B}{w\sigma_m}, -\pi_B\right\}. \tag{31}$$

Intuitively, the firm chooses the hedge ratio $\psi$ so that the firm faces only idiosyncratic volatility after hedging. The hedge ratio that achieves this objective is $-\rho\sigma_B\sigma_m^{-1}/w$. However, this hedge ratio might not be attainable due to the margin requirement. In that case, the firm chooses the maximally admissible hedge ratio $\psi_B^*(w) = -\pi_B$. Eq. (31) captures the effect of margin constraints on hedging. Because there is no haircut (i.e., $\epsilon = 0$), the hedge ratio $\psi$ given in (31) is independent of firm value and depends only on $w$. We next turn to the focus of this section: hedging in state $G$.

### 7.2. In state G

Before entering the crisis state, the firm has external financing opportunities. Moreover, the margin requirement could be different (i.e., $\pi_G > \pi_B$). In state $G$, the firm chooses its investment policy $I$ and its index futures position $\psi W$ to maximize firm value $P(K,W,G)$ by solving the following HJB equation:

$$\begin{aligned} r_G P(K,W,G) = \max_{I,\psi} [&(r_G - \lambda)W + \mu_G K - I - \Gamma(I,K,G)]P_W \\ &+ (I - \delta K)P_K + \frac{1}{2}(\sigma_G^2 K^2 + \psi^2 \sigma_m^2 W^2 \\ &+ 2\rho\sigma_m\sigma_G \psi W K)P_{WW} \\ &+ \zeta[P(K,W,G) - P(K,W,B)], \end{aligned} \tag{32}$$

subject to $|\psi| \leq \pi_G$.

When firm value is concave in cash, we have the same solution as in state $B$, but with margin $\pi_G$. That is,

$$\psi_G^*(w) = \max\left\{\frac{-\rho\sigma_G}{w\sigma_m}, -\pi_G\right\}. \tag{33}$$

However, market timing opportunities combined with fixed costs of equity issuance imply that firm value could be convex in cash, i.e., $P_{WW}(K,W,G) > 0$ for certain regions of $w = W/K$. With convexity, the firm naturally speculates in derivatives markets. Given the margin requirement, the firm takes the maximally allowed futures position, i.e., the corner solution $\psi_G(w) = \pi_G$. The firm is long in futures despite positive correlation between its

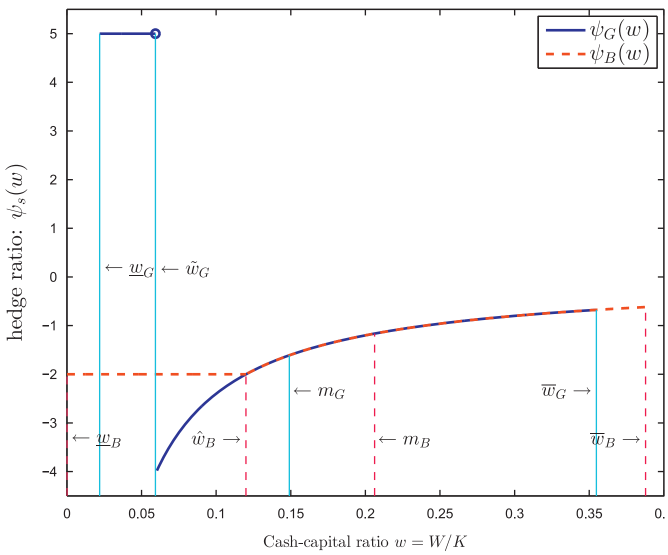


**Fig. 6.** Optimal hedge ratios $\psi^*(w)$ in states $G$ (good states) and $B$ (bad states) when state $B$ is absorbing. The parameter values are: market volatility $\sigma_m = 20\%$, correlation coefficient $\rho = 0.4$, and margin requirements $\pi_G = 5$ and $\pi_B = 2$. All other parameter values are given in Table 1.

productivity shock and the index futures. Let $\widehat{w}_G$ denote the endogenously chosen point at which $P_{WW}(K,W,G)=0$, or $p''_G(\widehat{w}_G)=0$. We now summarize the firm's futures position in state $G$ as

$$\psi^*_G(w)=\begin{cases}\max\left\{\dfrac{-\rho\sigma_G}{w\sigma_m},-\pi_G\right\} & \text{for } w\geq\widehat{w}_G,\\ \pi_G & \text{for } \underline{w}_G\leq w\leq\widehat{w}_G.\end{cases} \tag{34}$$

Note the discontinuity of the hedge ratio $\psi^*_G(w)$ in $w$. The firm switches from a hedger to a speculator when its cash-capital ratio $w$ falls below $\widehat{w}_G$.

For numerical illustration, we choose the correlation between index futures and the firm's productivity shock to be $\rho=0.4$ and a market return volatility of $\sigma_m=20\%$. The margin requirements in states $G$ and $B$ are set at $\pi_G=5$ and $\pi_B=2$, respectively. All other parameter values are the same as in the previous sections.

### 7.3. *Optimal hedge ratios* $\psi^*_s(w)$

Fig. 6 plots the optimal hedge ratios in the two states: $\psi^*_G(w)$ and $\psi^*_B(w)$. First, for sufficiently high $w$, the firm hedges the same way in both states. Hedging is then unconstrained by the firm's cash holding. The firm chooses its hedge ratio to be equal to $-\rho\sigma\sigma_m^{-1}/w$ so as to eliminate its exposure to systematic volatility of the productivity shock. This explains the concave and overlapping parts of the hedging policies in Fig. 6.

Second, for low $w$, hedging strategies differ in the two states. In state $B$ the hedge ratio hits the constraint $\psi^*_B(w)=-\pi_B=-2$ for $w\leq 0.12$. In state $G$, the firm issues equity at $\underline{w}_G=0.0219$ and firm value is convex in $w$ (due to market timing) for $w\leq\tilde{w}_G=0.0593$ [where $p''(\tilde{w}_G)=0$]. In other words, for $w\in(\underline{w}_G,\tilde{w}_G)$ firm value is convex in $w$ and the firm engages in maximally allowed speculation by setting $\psi^*_G(w)=\pi_G=5$ for $w\in(0.0219,0.0593)$.

Hedging lowers the firm's precautionary holding of cash and, hence, lowers its payout boundary from 0.371 (no hedging benchmark) to 0.355 in state $G$ and from 0.408 to 0.385 in state $B$. Intuitively, because both cash hoarding and risk management mitigate financial constraints, they act as substitutes for each other. Leland (1998) studies the effect of agency costs on leverage and risk management and finds that risk management allows the firm to choose a higher leverage. We find that risk management allows the firm to lower its cash holding, although the mechanism is different from Leland (1998).

For sufficiently low cash holdings, the ability to speculate lowers the firm's issuance boundary $\underline{w}_G$ because the marginal value of cash for a firm with speculation or hedging opportunity is higher. The ability to increase the volatility of the cash accumulation process makes the equity issuance option more valuable and hence causes the issuance boundary $\underline{w}_G$ to be lowered from 0.0268 (no hedging or speculation benchmark) to 0.0219.

Froot, Scharfstein, and Stein (1993) argue that hedging increases firm value by mitigating its underinvestment problem. We show that this result does not hold generally in a dynamic setting. For sufficiently high cash holdings, hedging mitigates the firm's underinvestment problem by reducing exposure to systemic volatility. However, when the firm's cash holdings are sufficiently low, the firm optimally engages in speculation to take advantage of its market timing option.

Rampini and Viswanathan (2010) argue that more financially constrained firms hedge less because the firm's financing needs for investment override hedging concerns. An important link then exists between financing and risk management, both of which involve promises to pay by the firm that are limited by collateral. Like Rampini and Viswanathan (2010), our model for corporate risk management is also consistent with the evidence that smaller firms hedge less. These firms are likely to be more financially constrained and would have limited hedging capacity due to the difficulty in meeting the margin requirements.

## 8. Conclusion

Mounting evidence show large market-wide swings in the financing conditions. What is more, in rare episodes of financial crises, primary markets essentially shut down. Firms have also become increasingly aware of the risks and opportunities they face with respect to these external financing costs, and they appear to time equity markets as suggested by Baker and Wurgler (2002). However, despite the rapid growth in empirical research on the effects of shocks to the supply of capital on firms' corporate policies (see Baker and Wurgler, 2011 for a survey of the literature), very few theoretical analyses are available on the implications of stochastic external financing costs for the dynamics of corporate investment and financing. This paper aims to close this gap by taking the perspective of a rational firm manager maximizing shareholder value by timing favorable equity market conditions and shielding the firm against crisis episodes through precautionary cash holdings.

We show that firms optimally hoard cash and issue equity in favorable market conditions even when they do not have immediate funding needs. As simple as this market timing behavior by the firm appears to be, we show that it has subtle implications for the dynamics of corporate investment, risk management, and stock returns. The key driver of these surprising implications is the finite duration of favorable financing conditions combined with the fixed issuance costs firms incur when they tap equity markets. Finally, we highlight how much a firm that optimally times equity markets and holds optimal precautionary cash buffers is able to shield itself against large shocks to external financing conditions. A firm entering a crisis state with an optimally replenished cash buffer in good times is able to maintain its investment policy almost unaltered and, thus, substantially smooth out adverse external financing shocks.

One natural question for future research is how would time-varying financing conditions interact with time-varying investment opportunities to affect firms' financing constraints. Our analysis in Appendix D shows that a firm with positively correlated financing and investment opportunities can sometimes be more financially constrained than when the two are negatively correlated. Another interesting question is how time-varying uncertainty about the financing conditions could affect corporate decisions. Bloom (2009) studies uncertainty shocks about productivity for financially unconstrained firms. The recent financial crisis suggests that uncertainty shocks about external financing conditions can potentially have first-order effects on investment and output as well.

## Appendix A. General model setup

Our analysis in the paper focuses on the special case of two states of the world. It is straightforward to generalize our model to a setting with more than two states, denoted by $s_t = 1, \ldots, n$. The transition matrix in the $n$-state Markov chain is then given by $\boldsymbol{\zeta} = [\zeta_{ij}]$. The $n$-state Markov chain can capture aggregate and firm-specific shocks, as well as productivity and financing shocks. In sum, the firm's expected return on capital, volatility, and financing costs could all change over time under the general formulation.

### A.1. Risk adjustments

To determine the adjustments for systematic risk in the model, we assume that the economy is characterized by a stochastic discount factor (SDF) $\Lambda_t$, which evolves as

$$\frac{d\Lambda_t}{\Lambda_{t^-}} = -r(s_{t^-})\,dt - \eta(s_{t^-})\,dZ_t^M + \sum_{s_t \neq s_{t^-}} (e^{\kappa(s_{t^-}, s_t)} - 1)\,dM_t^{(s_{t^-}, s_t)}, \tag{35}$$

where $r(s)$ is the risk-free rate in state $s$, $\eta(s)$ is the price of risk for systematic Brownian shocks $Z_t^M$, $\kappa(i,j)$ is the relative jump size of the discount factor when the Markov chain switches from state $i$ to state $j$, and $M_t^{(i,j)}$ is a compensated Poisson process with intensity $\zeta_{ij}$,

$$dM_t^{(i,j)} = dN_t^{(i,j)} - \zeta_{ij}\,dt, \quad i \neq j. \tag{36}$$

In Eq. (35), we have made use of the result that an $n$-state continuous-time Markov chain with generator $[\zeta_{ij}]$ can be equivalently expressed as a sum of independent Poisson processes $N_t^{(i,j)}$ $(i \neq j)$ with intensity parameters $\zeta_{ij}$ (see e.g., Chen, 2010).[20] The above SDF captures two different types of risk in the market: small systematic shocks generated by the Brownian motion, and large systematic shocks from the Markov chain. We assume that $dZ_t^M$ is partially correlated with the firm's productivity shock $dZ_t^A$, with instantaneous correlation $\rho\,dt$. Chen (2010) shows that the SDF in Eq. (35) can be generated from a consumption-based asset pricing model.

The SDF defines a risk-neutral probability measure $\mathbb{Q}$, under which the process for the firm's productivity shocks becomes Eq. (6). In addition, if a change of state in the Markov chain corresponds to a jump in the SDF, then the corresponding large shock carries a risk premium, which leads to an adjustment of the transition intensity under $\mathbb{Q}$ as follows:

$$\widehat{\zeta}_{ij} = e^{\kappa(i,j)}\zeta_{ij}, \quad i \neq j. \tag{37}$$

[20] More specifically, the process $s$ solves the following stochastic differential equation: $ds_t = \sum_{k \neq s_{t^-}} \delta_k(s_{t^-})\,dN_t^{(s_{t^-},k)}$, where $\delta_k(j) = j - i$.

### A.2. Solution of the n-state model

Under the first best, the HJB equation for the $n$-state model is

$$r_s q_s^{FB} = \widehat{\mu}_s - i_s^{FB} - \frac{1}{2}\theta_s (i_s^{FB} - \nu_s)^2 + q_s^{FB}(i_s^{FB} - \delta) + \sum_{s' \neq s} \widehat{\zeta}_{ss'}(q_{s'}^{FB} - q_s^{FB}), \tag{38}$$

where for each state $s = 1, \ldots, n$ the average $q$ is given by

$$q_s^{FB} = 1 + \theta_s (i_s^{FB} - \nu_s). \tag{39}$$

While there are no closed form solutions for $n > 2$, it is straightforward to solve the system of nonlinear equations numerically.

With financial frictions, the HJB equation is generalized from Eq. (11) as follows:

$$\begin{aligned} r_s P(K,W,s) = \max_I [(r_s - \lambda)W + \widehat{\mu}_s K - I - \Gamma(I,K,s)] P_W(K,W,s) \\ + \frac{\sigma_s^2 K^2}{2} P_{WW}(K,W,s) + (I - \delta K) P_K(K,W,s) \\ + \sum_{s' \neq s} \widehat{\zeta}_{ss'}(P(K,W,s') - P(K,W,s)), \end{aligned} \tag{40}$$

for each state $s = 1, \ldots, n$, and $\underline{W}_s \le W \le \overline{W}_s$. As before, firm value is homogeneous of degree one in $W$ and $K$ in each state, so that

$$P(K,W,s) = p_s(w)K, \tag{41}$$

where $p_s(w)$ solves the following system of ODE:

$$\begin{aligned} r_s p_s(w) = \max_{i_s} [(r_s - \lambda)w + \widehat{\mu}_s - i_s - g_s(i_s)] p_s'(w) + \frac{\sigma_s^2}{2} p_s''(w) \\ + (i_s - \delta)(p_s(w) - w p_s'(w)) + \sum_{s' \neq s} \widehat{\zeta}_{ss'}(p_{s'}(w) - p_s(w)). \end{aligned} \tag{42}$$

The boundary conditions in each state $s$ are then defined in similar ways as in Eqs. (13)–(16).

## Appendix B. Calibration

We use annual data from COMPUSTAT to calculate the moments of the investment-capital ratio and cash-capital ratio for our model calibration. The sample is from 1981 to 2010 and excludes utilities (Standard Industrial Classification (SIC) codes 4900-4999) and financial firms (SIC codes 6000-6999). We require firms to be incorporated in the United States and have positive assets and positive net PPE (property, plant, and equipment). In addition, because our model does not allow for lumpy investment, mergers and acquisitions, or dramatic changes in profitability, we eliminate firm-years for which total assets or sales grew by more than 100% or investments exceeded 50% of capital stock from the previous year.

Capital investment is measured using capital expenditure ($CAPX_t$). Because our calibrated model does not allow for short-term debt, we measure cash holdings as the difference between cash and short-term investments ($CHE_t$) and average short-term borrowing ($BAST_t$). Capital stock is the total net PPE ($PPENT_t$). Then, the cash-capital ratio for year $t$ is defined as $(CHE_t - BAST_t)/PPENT_t$, and the investment-capital ratio for year $t$ is $CAPX_t/PPENT_{t-1}$. We first compute moments for the cash-capital ratio and the investment-capital ratio at the firm level and then calibrate the model parameters to match the moments of the median across firms.

## Appendix C. Beta representation

Because this model has two sources of aggregate risk, the CAPM does not hold. Instead, expected returns reflect aggregate risk driven by a two-factor model. We, thus, assume that there are two diversified portfolios $T$ and $F$, each subject only to one type of aggregate shock, the technology shock or financing shock. Suppose their return dynamics are given as

$$dR_t^T = (r_s + \mu_s^T)\,dt + \sigma_s^T\,dZ_t^M \tag{43}$$

and

$$dR_t^F = (r_s + \mu_s^F)\,dt + (e^{\kappa_1^F} - 1)\,dM_t^1 + (e^{\kappa_2^F} - 1)\,dM_t^2. \tag{44}$$

Then, the stochastic discount factor in Eq. (35) implies that

$$\mu_s^T = \sigma_s^T \eta_s \tag{45}$$

and

$$\mu_s^F = \zeta_s (e^{\kappa_s^F} - 1)(e^{\kappa_s} - 1). \tag{46}$$

We can now rewrite the risk premium in Eqs. (43) and (44) using betas as follows:

$$\mu_s^R(w) = \beta_s^T(w)\mu_s^T + \beta_s^F(w)\mu_s^F, \tag{47}$$

where

$$\beta_s^T(w) = \frac{\rho_s \sigma_s}{\sigma_s^T} \frac{p_s'(w)}{p_s(w)} \tag{48}$$

and

$$\beta_s^F(w) = \frac{p_{s^-}(w) - p_s(w)}{p_s(w)(e^{\kappa_s^F} - 1)} \tag{49}$$

are the technology beta (beta with respect to portfolio $T$) and financing beta (beta with respect to portfolio $F$) for the firm in state $s$. The technology beta is large when the marginal value of cash relative to firm value is high. The financing beta is large when the probability that financing conditions will change is high or when the change in financing conditions has a large impact on firm value.

## Appendix D. Correlated investment and financing opportunities

To focus on the effects of stochastic financing shocks, we set the productivity shocks to be *i.i.d* in the paper. In this appendix, we relax this restriction and explore the implications of correlation between investment and financing opportunities.

We conduct the following thought experiment. Suppose two firms face identical financing conditions as in the benchmark model (determined by the states $G$ and $B$). To make the two states symmetric, we assume the risk-neutral transition intensities are the same in the two states. Firm 1's investment opportunities are positively correlated with financing opportunities. That is, its expected return on capital (i.e., expected productivity shock $\mu_s$) is higher when the financing cost ($\phi_s$) is lower. The opposite is true for Firm 2, in that $\mu_s$ is higher when the financing cost ($\phi_s$) is higher.

Fig. D1 plots the investment-capital ratio for the two firms in state $G$ (with low financing costs) and $B$ (high financing costs). Not surprisingly, Firm 1 has higher return on capital in state $G$ and, thus, invests more in this state, while the opposite is true in state $B$, especially when cash holdings are high. Firm 1 on average holds onto more cash in state $G$ than Firm 2, but less in state $B$ (as indicated by the payout boundaries on the right ends of the curves). These results indicate that Firm 1 can sometimes have a stronger precautionary motive than Firm 2 due to the fact that better investment opportunities call for more cash holding to reduce underinvestment.

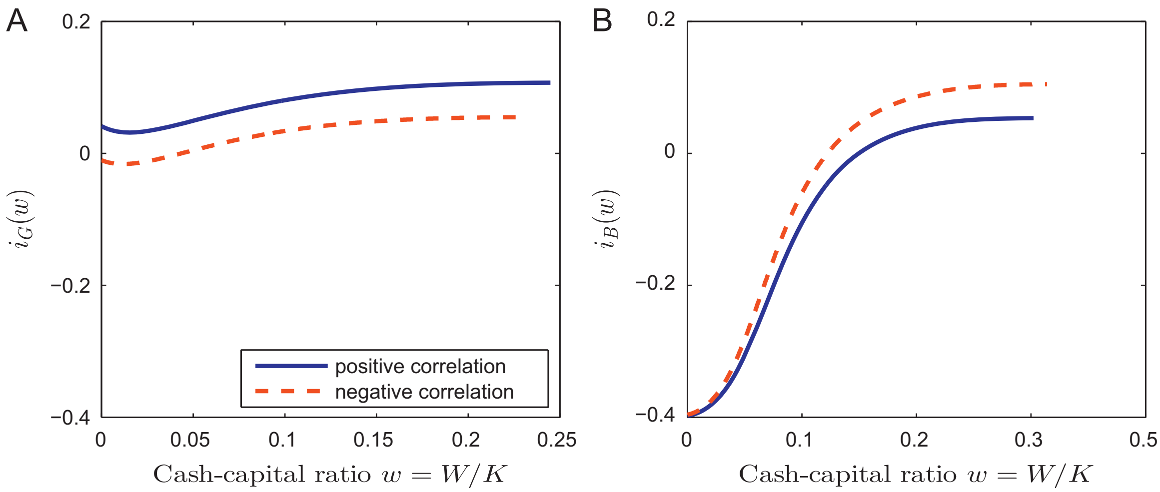


**Fig. D1.** Investment-capital ratio for firms with different correlation between investment and financing opportunities. The financing costs are the same as in our benchmark model (see Table 1 in the paper). We assume Firm 1 has $\mu_G = 22.7\%$ and $\mu_B = 19.7\%$. Firm 2 has $\mu_G = 19.7\%$ and $\mu_B = 22.7\%$. Finally, we set the risk-neutral transition intensities $\widehat{\zeta}_G = \widehat{\zeta}_B = 0.1$. (A) State $G$ and (B) State $B$.

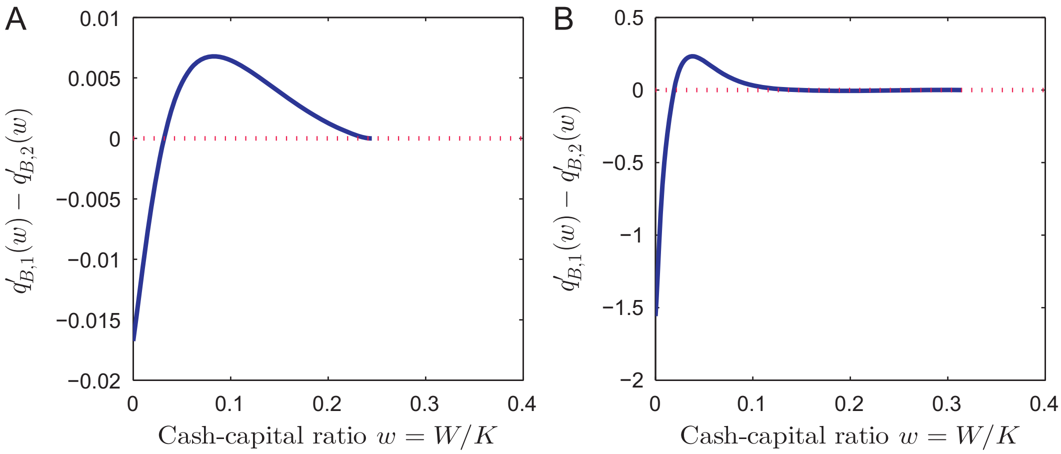


**Fig. D2.** Differences in the marginal value of cash for two firms with different correlation between investment and financing opportunities. Panels A and B plot the differences in the marginal value of cash in states (A) *G* and (B) *B*, respectively. See Fig. 1 caption for details of the setup.

More interestingly, we find that Firm 1 can be more financially constrained as measured by a higher marginal value of cash than Firm 2 in both states of the world. Fig. D2 plots the differences in the marginal value of cash for the two firms. When the cash holding is low, the marginal value of cash for Firm 1 (with positively correlated investment and financing opportunities) is lower in both state *G* and *B*, suggesting that the positive correlation makes the firm less constrained. However, as the cash holding rises, this order gets reversed for both states. (In fact, in state *B*, the marginal values of cash for the two firms cross each other twice.)

Firm 1 can have a higher marginal value of cash in state *G* because of its higher productivity in this state. In state *B*, the reason is more subtle. A positive correlation can make the good state even more valuable, making survival of bad states of the world all the more important. As Fig. D2 shows, this effect can more than overcome the effect of low productivity in the bad state.

## References


Acharya, V.V., Almeida, H., Campello, M. Aggregate risk and the choice between cash and lines of credit. Journal of Finance, forthcoming.

Acharya, V.V., Viswanathan, S., 2011. Leverage, moral hazard, and liquidity. Journal of Finance 66, 99–138.

Almeida, H., Campello, M., Laranjeira, B., Weisbenner, S., 2012. Corporate debt maturity and the real effects of the 2007 credit crisis. Critical Finance Review 1, 3–58.

Almeida, H., Campello, M., Weisbach, M.S., 2004. The cash flow sensitivity of cash. Journal of Finance 59, 1777–1804.

Altinkilic, O., Hansen, R.S., 2000. Are there economies of scale in underwriting fees? Evidence of rising external financing costs. Review of Financial Studies 13, 191–218.

Baker, M., 2010. Capital market-driven corporate finance. Annual Review of Financial Economics 1, 181–205.

Baker, M., Stein, J.C., Wurgler, J., 2003. When does the market matter? Stock prices and the investment of equity-dependent firms. Quarterly Journal of Economics 118, 969–1005.

Baker, M., Wurgler, J., 2000. The equity share in new issues and aggregate stock returns. Journal of Finance 55, 2219–2257.

Baker, M., Wurgler, J., 2002. Market timing and capital structure. Journal of Finance 57, 1–32.

Baker, M., Wurgler, J., 2011. Behavioral corporate finance: an updated survey. Unpublished working paper. Harvard University and New York University.

Bates, T.W., Kahle, K.M., Stulz, R.M., 2009. Why do U.S. firms hold so much more cash than they used to? Journal of Finance 64, 1985–2021.

Birru, J., 2012. Inefficient markets, efficient investments? Unpublished working paper. New York University.

Bloom, N., 2009. The impact of uncertainty shocks. Econometrica 77, 623–685.

Bolton, P., Chen, H., Wang, N., 2011. A unified theory of Tobin's *q*, corporate investment, financing, and risk management. Journal of Finance 66, 1545–1578.

Bolton, P., Scharfstein, D.S., 1990. A theory of predation based on agency problems in financial contracting. American Economic Review 80, 93–106.

Campello, M., Giambona, E., Graham, J.R., Harvey, C.R., 2011. Liquidity management and corporate investment during a financial crisis. Review of Financial Studies 24, 1944–1979.

Campello, M., Graham, J.R., Harvey, C.R., 2010. The real effects of financial constraints: evidence from a financial crisis. Journal of Financial Economics 97, 470–487.

Chen, H., 2010. Macroeconomic conditions and the puzzles of credit spreads and capital structure. Journal of Finance 65, 2171–2212.

DeAngelo, H., DeAngelo, L., Stulz, R., 2010. Seasoned equity offerings, market timing, and the corporate lifecycle. Journal of Financial Economic 95, 275–295.

Decamps, J., Mariotti, T., Rochet, J., Villeneuve, S., 2011. Free cash flow, issuance costs, and stock prices. Journal of Finance 66, 1501–1544.

DeMarzo, P.M., Fishman, M.J., 2007a. Agency and optimal investment dynamics. Review of Financial Studies 20, 151–188.

DeMarzo, P.M., Fishman, M.J., 2007b. Optimal long-term financial contracting. Review of Financial Studies 20, 2079–2128.

DeMarzo, P., Fishman, M., He, Z., Wang, N., 2012. Dynamic agency and the *q* theory of investment. Journal of Finance 67 (6), 2295–2340.

DeMarzo, P.M., Sannikov, Y., 2006. A continuous-time agency model of optimal contracting and capital structure. Journal of Finance 61, 2681–2724.

Dittmar, A.K., Dittmar, R.F., 2008. The timing of financing decisions: an examination of the correlation in financing waves. Journal of Financial Economics 90, 59–83.

Dittmar, A., Mahrt-Smith, J., 2007. Corporate governance and the value of cash holdings. Journal of Financial Economics 83, 599–634.

Duchin, R., Ozbas, O., Sensoy, B.A., 2010. Costly external finance, corporate investment, and the subprime mortgage credit crisis. Journal of Financial Economics 97, 418–435.

Dumas, B., 1991. Super contact and related optimality conditions. Journal of Economic Dynamics and Control 4, 675–685.

Eberly, J., Rebelo, S., Vincent, N., 2009. Investment and value: a neoclassical benchmark. Unpublished working paper. Northwestern University.

Eckbo, B.E., Masulis, R.W., Norli, O., 2007. Security offerings. In: Eckbo, B.E. (Ed.), Handbook of Corporate Finance: Empirical Corporate Finance, vol. 1. , Elsevier, North-Holland, pp. 233–373.

Eisfeldt, A., Muir, T., 2011. The joint dynamics of internal and external finance. Unpublished working paper. University of California at Los Angeles.

Fama, E.F., French, K.R., 2005. Financing decisions: Who issues stock?. Journal of Financial Economics 76, 549–582.

Fazzari, S.M., Hubbard, R.G., Petersen, B.C., 1988. Financing constraints and corporate investment. Brookings Papers on Economic Activity 1988, 141–206.

Froot, K., Scharfstein, D., Stein, J., 1993. Risk management, coordinating corporate investment, and financing policies. Journal of Finance 48, 1629–1658.

Gomes, J.F., 2001. Financing investment. American Economic Review 91, 1263–1285.

Graham, J.R., 2000. How big are the tax benefits of debt? Journal of Finance 55, 1901–1941.

Hayashi, F., 1982. Tobin's marginal $q$ and average $q$: a neoclassical interpretation. Econometrica 50, 215–224.

Hennessy, C.A., Whited, T.M., 2005. Debt dynamics. Journal of Finance 60, 1129–1165.

Hennessy, C.A., Whited, T.M., 2007. How costly is external financing? Evidence from a structural estimation. Journal of Finance 62, 1705–1745.

Huang, R., Ritter, J.R., 2009. Testing theories of capital structure and estimating the speed of adjustment. Journal of Financial and Quantitative Analysis 44, 237–271.

Hugonnier, J., Malamud, S., Morellec, E., 2012. Capital supply uncertainty, cash holdings, and investment. Working paper, Swiss Finance Institute.

Ivashina, V., Scharfstein, D., 2010. Bank lending during the financial crisis of 2008. Journal of Financial Economics 97, 319–338.

Kaplan, S.N., Zingales, L., 1997. Do investment-cash flow sensitivities provide useful measures of financing constraints? Quarterly Journal of Economics 112, 169–215.

Leland, H., 1998. Agency costs, risk management, and capital structure. Journal of Finance 53, 1213–1243.

Livdan, D., Sapriza, H., Zhang, L., 2009. Financially constrained stock returns. Journal of Finance 64, 1827–1862.

Loughran, T., Ritter, J., 1995. The new issues puzzle. Journal of Finance 50, 23–52.

Palazzo, D., 2012. Cash holdings, risk, and expected returns. Journal of Financial Economics 104, 162–185.

Rampini, A.A., Viswanathan, S., 2010. Collateral, risk management, and the distribution of debt capacity. Journal of Finance 65, 2293–2322.

Rampini, A.A., Viswanathan, S., 2011. Collateral and capital structure. Unpublished working paper. Duke University.

Riddick, L.A., Whited, T.M., 2009. The corporate propensity to save. Journal of Finance 64, 1729–1766.

Scheinkman, J.A., Xiong, W., 2003. Overconfidence and speculative bubbles. Journal of Political Economy 111, 1183–1219.

Shleifer, A., Vishny, R.W., 1992. Liquidation values and debt capacity: a market equilibrium approach. Journal of Finance 47, 1343–1366.

Stein, J., 1996. Rational capital budgeting in an irrational world. Journal of Business 69, 429–455.

Wang, C., Wang, N., Yang, J., 2012. A unified model of entrepreneurship dynamics. Journal of Financial Economics 106, 1–23.

Whited, T.M., 1992. Debt, liquidity constraints, and corporate investment: evidence from panel data. Journal of Finance 47, 1425–1460.