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OUTSIDE AND INSIDE LIQUIDITY

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ABSTRACT

We consider a model of liquidity demand arising from a possible maturity mismatch between asset revenues and consumption. This liquidity demand can be met with either cash reserves (inside liquidity) or via asset sales for cash (outside liquidity). The question we address is, what determines the mix of inside and outside liquidity in equilibrium? An important source of inefficiency in our model is the presence of asymmetric information about asset values, which increases the longer a liquidity trade is delayed. We establish existence of an immediate-trading equilibrium, in which asset trading occurs in anticipation of a liquidity shock, and sometimes also of a delayed-trading equilibrium, in which assets are traded in response to a liquidity shock. We show that, when it exists, the delayed-trading equilibrium is Pareto superior to the immediate-trading equilibrium, despite the presence of adverse selection. However, the presence of adverse selection may inefficiently accelerate asset liquidation. We also show that the delayed-trading equilibrium features more outside liquidity than the immediate-trading equilibrium although it is supplied in the presence of adverse selection. Finally, long term contracts do not always dominate the market provision of liquidity.

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INTRODUCTION

The main goal of this paper is to propose a tractable model of maturity transformation by financial intermediaries and liquidity demand arising from the maturity mismatch between asset payoffs and desired redemptions. When financial intermediaries invest in long-term assets they may face redemptions before these assets mature. They therefore have a need for liquidity. Early redemptions can be met either with cash reserves – what we refer to as *inside liquidity* – or with the proceeds from asset sales to other investors with a longer horizon–what we refer to as *outside liquidity.* In reality financial intermediaries rely on both forms of liquidity and the purpose of our analysis is to determine the relative importance and efficiency of inside and outside liquidity in a competitive equilibrium of the financial sector.

Our model comprises two different groups of agents that differ in their investment horizons. One class of agents is short-run investors (SRs) who prefer early asset payoffs, and the second class is long-run investors (LRs) who are indifferent to the timing of asset payoffs. One can think of the long-run investors as wealthy individuals, endowments, hedge funds, pension funds or even sovereign wealth funds, and of the short-run investors as financial intermediaries, banks or mutual funds, catering to small investors with shorter investment horizons. Within this model the key question is, what determines the mix of inside and outside liquidity in equilibrium?

Our model describes a situation in which SRs invest in risky projects besides holding cash, and where LRs have sufficient knowledge about these projects to stand ready to buy them at a relatively good price.[1] An important potential source of inefficiency in reality and in our model is asymmetric information between SRs and LRs about project quality. That is, even when SRs turn to knowledgeable LRs to sell claims to their assets, the latter cannot always tell whether the sale is due to a sudden liquidity need or whether the SR investor is trying to pass on a *lemon.* This problem is familiar to market participants and has been widely studied in the literature in different contexts. The novel aspect our model focuses on is a *timing* dimension. SRs tend to learn more about their liquidity needs and underlying asset values over time. Therefore, when at the onset of a liquidity shock they choose to hold on to their positions – in the hope of riding out a temporary crisis – they run the risk of having to go to the market in a much worse position should the crisis be a prolonged one. The longer they wait the worse is the *lemons* problem and therefore the greater is the risk that they will have to sell assets at fire-sale prices. Yet, it makes sense for SRs not to rush to sell their projects, as

[1] Other less knowledgeable investors who are only ready to buy these assets at a much higher discount are not explicitly modeled.

these may mature and pay off soon enough so that SRs may ultimately not face any liquidity shortage. This timing decision by SRs as to *when* to sell their assets for cash creates the main tension in our model.

This timing of liquidity trades is the source of a common dynamic in liquidity crises, where the crisis deepens over time as asset prices decline. This aspect of liquidity crises has not been much analyzed nor previously modeled. We capture the essence of the unfolding of a liquidity crisis by establishing the existence of two types of rational expectations equilibria: an *immediate-trading equilibrium*, where SRs are rationally expected to trade at the onset of the liquidity shock, and a *delayed-trading equilibrium*, where they are instead expected to prefer attempting to ride out the crisis and to only trade as a last resort should the crisis be a prolonged one. We show that for some parameter values only the immediate-trading equilibrium exists, while for other values both equilibria coexist.

When two different rational expectations equilibria can coexist one naturally wonders how they compare in terms of efficiency. Which is better? Interestingly, the answer to this question depends critically on the ex-ante portfolio-composition decisions of both SR and LR investors. In a nutshell, under the expectation of immediate liquidity-trading, LRs expect to obtain the assets originated by SRs at close to fair value. In this case the returns of holding outside liquidity are low and thus there is little cash held by LRs. On the other side of the liquidity trade, SRs will then expect to be able to sell a relatively small fraction of assets at close to fair value, and therefore respond by relying more heavily on inside liquidity. In other words, in an immediate-trading equilibrium there is less *cash-in-the-market pricing* (to borrow a term from Allen and Gale, 1998) and therefore a lower supply of outside liquidity. The anticipated reduced supply of outside liquidity causes SRs to rely more on inside liquidity and, thus, bootstraps the relatively high equilibrium price for the assets held by SRs under immediate liquidity trading.

In contrast, under the expectation of delayed liquidity trading, SRs rely more on outside liquidity. Here the bootstrap works in the other direction, as LRs decide to hold more cash in anticipation of a larger future supply of the assets held by SRs. These assets will be traded at lower cash-in-the-market prices in the delayed-trading equilibrium, even taking into account the worse lemons problem under delayed trading. The reason is that in this equilibrium SRs originate more projects and therefore end up trading more assets following a liquidity shock. They originate more projects in this delayed trading equilibrium because the expected return for SRs to investing in a project is higher in the delayed-trading equilibrium, due to the lower overall probability of liquidating assets before they mature.

In sum, immediate trading equilibria are based on a greater reliance on inside liquidity than delayed-trading equilibria. And, to the extent that there is a greater reliance on outside liquidity in a delayed-trading equilibrium, one should expect – and we indeed establish – that equilibrium asset prices are lower in the delayed-trading than in the immediate-trading equilibrium. In other words, our model predicts the typical pattern of liquidity crises, where asset prices progressively deteriorate throughout the crisis. Importantly, this predictable pattern in asset prices is still consistent with no arbitrage, as short-run investors prefer to delay asset sales, despite the deterioration in asset prices, in the hope that they wont have to trade at all at these fire sale prices.[2]

Because of this deterioration in asset prices one would expect that welfare is also worse in the delayed-trading equilibrium. However, the Pareto superior equilibrium is in fact the delayed-trading equilibrium. What is the economic logic behind this somewhat surprising result? The answer is that the fundamental gains from trade in our model are between SRs who undervalue long term assets, and LRs who undervalue cash. Thus, the more SRs can be induced to originate projects and the more LRs can be induced to hold cash, the higher are the gains from trade and therefore the higher is welfare. In other words, the welfare efficient form of liquidity in our model is outside liquidity. Since the delayed-trading equilibrium relies more on outside liquidity it is more efficient.

In the presence of asymmetric information, however, outside liquidity involves a dilution of ownership cost so that SRs prefer to partially rely on inefficient inside liquidity. As the lemons' problem worsens – in particular, as SRs are less likely to trade for liquidity reasons when they engage in delayed-trading – the cost of outside liquidity rises. There is then a point when the cost is so high that SRs are better off postponing the redemption of their investments altogether, rather than realize a very low fire-sale price for their valuable projects. At that point the delayed-trading equilibrium collapses, as only lemons would get traded for early redemption.

In this paper we do not take an optimal mechanism design approach. We attempt instead to specify a model of trading opportunities that mimics the main characteristics of actual markets. The main advantage of our approach is that it facilitates interpretation and considerably simplifies aspects of the model that are not central to the questions we focus on. Still, we consider one contracting alternative to markets, in which SRs write a long-term contract for liquidity with LRs. Such a contract takes the form of an investment fund set up

[2]SRs' decision to delay trading has all the hallmarks of *gambling for resurrection*. But it is in fact unrelated to the idea of excess risk taking as SRs will choose to delay whether or not they are levered.

by LRs, in which the initial endowments of one SR and one LR are pooled, and where the fund promises state-contingent payments to its investors. Under complete information such a fund arrangement would always dominate any equilibrium allocation achieved through future spot trading of assets for cash. However, when the investor who manages the fund also has private information about the realized returns on the fund's investments then, as we show, the long-term contract cannot always achieve a more efficient outcome than the delayed-trading equilibrium. Indeed, the fund manager's private information then constraints the fund to make only incentive compatible state-contingent transfers to the SR investor, thus raising the cost of providing liquidity. We show that the fund allocation is dominated by the delayed-trading equilibrium in parameter regions for which there is a high level of origination and distribution of risky assets.

Given that neither financial markets nor long-term contracts for liquidity can achieve a fully efficient outcome, the question naturally arises whether some form of public intervention may provide an efficiency improvement. There are two market inefficiencies that public policy might mitigate. An ex-post inefficiency, which arises when the delayed-trading equilibrium fails to exist, and an ex-ante inefficiency in the form of an excess reliance on inside liquidity. It is worth noting that a common prescription against banking liquidity crises–to require that banks hold cash reserves or excess equity capital–would be counterproductive in our model. Such a requirement would only force SRs to rely more on inefficient inside liquidity and would undermine the supply of outside liquidity.

We discuss policy interventions in greater depth in Bolton, Santos and Scheinkman (2009), where we point out that the best form of public liquidity intervention relies on a complementarity between public and outside liquidity. Public liquidity in the form of a price support can restore existence of the delayed-trading equilibrium and thereby induce LRs to hold more outside liquidity. That is, such a policy would induce long-term investors to hold more cash in the knowledge that SRs rely less on inside liquidity, and thus help increase the availability of outside liquidity. Thus, far from being a substitute for privately provided liquidity, a commitment to providing price support in secondary asset markets in liquidity crises could be a complement and give rise to positive spillover effects on the provision of outside liquidity.

Related literature. Our paper is related to the literatures on banking and liquidity crises, and the limits of arbitrage. Our analysis differs from the main contributions in these literatures mainly in two respects: first, our focus on ex-ante efficiency and equilibrium portfolio composition, and second, the endogenous timing of liquidity trading. Still, our analysis shares several important themes and ideas with these literatures. We briefly discuss the most related contributions in each of these literatures.

Consider first the banking literature. Diamond and Dybvig (1983) and Bryant (1980) provide the first models of investor liquidity demand, maturity transformation, and inside liquidity. In their model a bank run may occur if there is insufficient inside liquidity to meet depositor withdrawals. In contrast to our model, investors are identical ex-ante, and are risk-averse with respect to future liquidity shocks. The role of financial intermediaries is to provide insurance against idiosyncratic investors' liquidity shocks.

Bhattacharya and Gale (1986) provide the first model of both inside and outside liquidity by extending the Diamond and Dybvig framework to allow for multiple banks, which may face different liquidity shocks. In their framework, an individual bank may meet depositor withdrawals with either inside liquidity or outside liquidity by selling claims to long-term assets to other banks who may have excess cash reserves. An important insight of their analysis is that individual banks may free-ride on other banks' liquidity supply and choose to hold too little liquidity in equilibrium.

More recently, Allen and Gale (2000) and Freixas, Parigi, and Rochet (2000) (see also Aghion, Bolton and Dewatripont, 2000) have analyzed models of liquidity provided through the interbank market, which can give rise to contagious liquidity crises. The main mechanism they highlight is the default on an interbank loan which depresses secondary-market prices and pushes other banks into a liquidity crisis. Subsequently, Acharya (2001) and Acharya and Yorulmazer (2005) have, in turn, introduced optimal bailout policies in a model with multiple banks and cash-in-the-market pricing of loans in the interbank market.

While Diamond and Dybvig considered idiosyncratic liquidity shocks and the risk of panic runs that may arise as a result of banks' attempts to insure depositors against these shocks, Allen and Gale (1998) consider aggregate business-cycle shocks and point to the need for equilibrium banking crises to achieve optimal risk-sharing between depositors. In their model aggregate shocks may trigger the need for asset sales, but their analysis does not allow for the provision of both inside and outside liquidity.

Another strand of the banking literature, following Holmstrom and Tirole (1998 and 2008) considers liquidity demand on the corporate borrowers' side rather than on depositors' side, and asks how efficiently this liquidity demand can be met through bank lines of credit. This literature emphasizes the need for public liquidity to supplement private liquidity in case of aggregate demand shocks.

Most closely related to our model is the framework considered in Fecht (2004), which itself builds on the related models of Diamond (1997) and Allen and Gale (2000). The models of Diamond (1997) and Fecht (2004) seek to address an important weakness of the Diamond and Dybvig theory, which cannot account for the observed coexistence of financial intermediaries and securities markets. Liquidity trading in secondary markets undermines liquidity provision by banks and obviates the need for any financial intermediation in the Diamond and Dybvig setting, as Jacklin (1987) has shown. To address this objection, Diamond (1997) introduces a model where banks coexist with securities markets due to the fact that households face costs in switching out of the banking sector and into securities markets. Fecht (2004) extends Diamond (1997) by introducing segmentation on the asset side between financial intermediaries' investments in firms and claims issued directly by firms to investors though securities markets. Also, in his model banks have local (informational) monopoly power on the asset side, and subsequently can trade their assets in securities markets for cash–a form of outside liquidity. Finally, Fecht (2004) also allows for a contagion mechanism similar to Allen and Gale (2000) and Diamond and Rajan (2005),[3] whereby a liquidity shock at one bank propagates itself through the financial system by depressing asset prices in securities markets.

Two other closely related models are Gorton and Huang (2004) and Parlour and Plantin (2007). Gorton and Huang also consider liquidity supplied in a general equilibrium model and also argue that publicly provided liquidity can be welfare enhancing if the private supply of liquidity involves a high opportunity cost. However, in contrast to our analysis they do not look at the optimal composition of inside and outside liquidity, nor do they consider the dynamics of liquidity trading. Parlour and Plantin (2007) consider a model where banks may securitize loans, and thus obtain access to outside liquidity. As in our setting, the efficiency of outside liquidity is affected by adverse selection. But in the equilibrium they characterize liquidity may be excessive for some banks–as it undermines their loan origination standards–and too low for other banks, who may be perceived as holding excessively risky assets.

The second literature our model is related to is the literature on liquidity and the dynamics of arbitrage by capital or margin-constrained speculators in the line of Dow and Gorton (1993) and Shleifer and Vishny (1997). The typical model in this literature (e.g. Kyle and

[3]Another feature in Diamond and Rajan (2005) in common with our setup is the idea that financial intermediaries possess superior information about their assets, which is another source of illiquidity.

Xiong, 2001 and Xiong, 2001) also allows for outside liquidity and generates episodes of fire-sale pricing–even destabilizing price dynamics–following negative shocks that tighten speculators' margin constraints. However, most models in this literature do not address the issue of deteriorating adverse selection and the timing of liquidity trading, nor do they explore the question of the optimal mix between inside and outside liquidity. The most closely related articles, besides Kyle and Xiong (2001) and Xiong (2001) are Gromb and Vayanos (2002), Brunnermeier and Pedersen (2007) and Kondor (2007). In particular, Brunnermeier and Pedersen (2007) also focus on the spillover effects of inside and outside liquidity, or what they refer to as funding and market liquidity.

II. THE MODEL

II.A Agents

There are two types of agents, short and long-run investors with preferences over periods $t = 1, 2, 3$. *Short run investors* (SRs), of which there is a unit mass, have preferences

$$u(C_1, C_2, C_3) = C_1 + C_2 + \delta C_3, \tag{1}$$

where $C_t \geq 0$ denotes consumption at dates $t = 1, 2, 3$ and $\delta \in (0, 1)$. These investors have one unit of endowment at date 0 and no endowments at subsequent dates. There is also a unit mass of *long run investors* (LRs), each with $\kappa > 0$ units of endowment at $t = 0$ and again no endowments at subsequent dates. Their utility function is simply given by

$$\hat{u}(C_1, C_2, C_3) = \sum_{t=1}^{3} C_t,$$

with $C_t \geq 0$.

II.B Assets

For simplicity and with almost no loss of generality we assume that the two types of investors have access to different investment opportunity sets. Both types can hold *cash* with a gross per-period rate of return of one. LR investors can also invest in a decreasing-returns-to-scale *long-maturity* asset that returns $\varphi(x)$ at date 3 for an initial investment at date 0 of $x = (\kappa - M)$, where $M \geq 0$ is the LRs' cash holding to which we refer as *outside liquidity.* Because LRs are risk neutral the assumption that the long run project is riskless is without loss of generality and nothing would change if we assumed that output from the long run project was random.

SR investors can invest up to one unit in a *risky* project (asset), which is a constant returns to scale technology, that returns $\widetilde{\rho}_t$ at dates $t = 1, 2, 3$ where $\widetilde{\rho}_t \in \{0, \rho\}$ and $\rho > 1$, per-unit invested. The return on the risky asset is the only source of uncertainty in the model and is shown in Figure 1. We assume that there is a first aggregate *maturity shock* that affects all risky assets. That is, agents learn first whether all risky assets mature at date 1, or at some later date. Subsequently, the realized value of a risky asset and whether it matures at date 2 or 3 is determined by an idiosyncratic shock.[4]

Formally, the SR chooses a size $\nu \leq 1$ for the risky project and the project either pays $\rho\nu$ at date 1 (in state $\omega_{1\rho}$), which occurs with probability λ, or it pays at a subsequent date, with probability $(1 - \lambda)$. In that case the asset yields either a return $\widetilde{\rho}_2 \in \{0, \rho\}$ at date 2, or a late return $\widetilde{\rho}_3 \in \{0, \rho\}$ at date 3 per unit invested. After date 1 shocks are idiosyncratic (i.e. independent across SRs) and are represented by two independent random variables: (i) an individual asset can either mature at date 2, with probability θ, or at date 3 with probability $(1 - \theta)$ (in idiosyncratic state ω_{2L}); (ii) when the asset matures at either dates $t = 2, 3$ it returns $\widetilde{\rho}_t = \rho$ with probability η (in idiosyncratic states $\omega_{2\rho}$ and $\omega_{3\rho}$, respectively) and $\widetilde{\rho}_t = 0$ with probability $(1 - \eta)$ for $t = 2, 3$ (in idiosyncratic states ω_{20} and ω_{30}.) The realization of idiosyncratic shocks is private information to the SR holding the risky asset.[5] We denote by m the amount of cash held by SRs and by $\nu = 1 - m$ the amount invested in the risky asset; m is thus our measure of *inside liquidity.*

Finally, in this model we ignore the presence of other agents for whom acquiring both the long run and the risky assets would only be attractive at much lower prices.[6]

II.C Assumptions

We introduce assumptions on payoffs that focus the analysis on the economically interesting outcomes and thus considerably shorten the discussion of the model. We begin with

[4]We assume that the shock in period 1 is aggregate to simplify the analysis and to focus on the informational failure induced by the idiosyncratic shock in period 2.

[5]The assumption that adverse selection problems worsen during a liquidity crisis is consistent with the current episode. The risk profile of many financial intermediaries became difficult to ascertain as the residential real estate and mortgage markets' implosion unfolded in 2007 and 2008 (see Gorton 2007 and 2008). The freezing up of the interbank loan market was just one symptom of the difficulty in assessing the direct and indirect exposure of financial institutions to these toxic assets.

[6]We are currently exploring a model where the amount of capital available to absorb resales (κ in the current paper) is determined in equilibrium.

assumptions on the long run asset.

$$\varphi'(\kappa) > 1 \qquad \text{with} \qquad \varphi''(x) < 0 \quad \text{and} \quad \lim_{x \longrightarrow 0} \varphi'(x) = +\infty \tag{A1}$$

The assumption that $\varphi''(\cdot) < 0$ captures the fact that the opportunities that these long assets represent are scarce and cannot be exploited without limit. We also assume that LRs always want to invest some fraction of their endowment in this long asset, $\lim_{x\to 0} \varphi'(x) = +\infty$. The key assumption here though is that $\varphi'(\kappa) > 1$. This implies that if LRs carry cash it must be to acquire assets with expected returns at least as high as $\varphi'(\kappa)$. Given our assumption of risk neutrality this can only occur if asset purchases occur at *cash-in-the-market* prices. That is, assets must trade in equilibrium at prices that are below their expected payoff, for otherwise LRs would have no incentive to carry cash.

Our second assumption says that SRs would not invest in the risky asset in autarchy, even though investment in the risky asset may be more attractive than holding cash when the asset can be resold at a reasonable price:

$$\rho[\lambda + (1-\lambda)\eta] > 1 \qquad \text{and} \qquad \lambda\rho + (1-\lambda)[\theta + (1-\theta)\delta]\eta\rho < 1 \tag{A2}$$

Assumption A2 is needed to capture the economically interesting situation where liquidity of secondary markets at dates 1 and 2 affects asset allocation decisions at date 0. If instead we assumed that

$$\lambda\rho + (1-\lambda)[\theta + (1-\theta)\delta]\eta\rho \geq 1$$

then SRs would always choose to put all their funds in a risky asset irrespective of the liquidity of the secondary market at date 1.

Finally we assume that there are gains from trade at least at date 1. That is, $\varphi'(\kappa)$ is not as high as to rule out the possibility that LRs carry cash to trade at date 1. As will become clear below, assumption A3 implies that the agents' isoprofit lines cross in the right way:

$$\frac{\varphi'(\kappa) - \lambda}{(1-\lambda)\eta\rho} < \frac{1-\lambda}{1-\lambda\rho} \tag{A3}$$

III. EQUILIBRIUM

Given that all SRs are ex-ante identical, we shall restrict attention to competitive equilibria that treat all SRs symmetrically. We also restrict attention to *pooling equilibria*, in which observable actions cannot be used to distinguish among SRs.We will also assume that each LR gets exactly the same (expected) profit in equilibrium. Recall that trade between SRs and LRs can only take place in spot markets at dates 1 and 2, and that in period 1 there are strictly positive gains from trade only if aggregate state ω_{1L} obtains. We write P_1 for the price of one unit of risky project in period 1 if state ω_{1L} obtains, and P_2 for the price of one unit of the risky project in period 2. Given that SRs have private information about realized returns on their risky asset at date 2, they can condition their trading policy on their idiosyncratic state $\omega_{2\cdot}$. LRs, on the other hand, are unable to distinguish among potential SR sellers in any pooling equilibrium. We denote by q_1 the amount of the risky asset supplied by an SR at date 1 (in state ω_{1L}) and by q_2 the amount supplied at date 2, by an SR who is in the (idiosyncratic) state ω_{2L}. Notice that an SR who is in the (idiosyncratic) state ω_{20} would always sell all his risky assets at any price, since he is sure that the project will not yield any payoff, whereas a SR investor in state $\omega_{2\rho}$ might, as well, simply consume its output. Similarly, we denote by Q_1 and Q_2 the amount of the risky asset that an LR investor acquires at $t = 1$ and $t = 2$, respectively.[7] Finally each LR investor that chooses M units of cash has claims to $\varphi(\kappa - M)$ units of output in period 3 that, in principle, he can choose to sell to others in period 1 or 2. The risk neutrality of the LRs links the price of output at each point to the expected return on other assets held by the LRs.

Although it is not important, it will facilitate the interpretation of our results, as well as the discussion of the long term contract below, to assume that SRs have to sell their entire risky investment whenever they sell any. The interpretation of this assumption is that once a scale is chosen, a risky project is indivisible. This indivisibility is consistent with our assumption that each risky project has at most one SR owner, who is the only agent that observes the state of the risky project in period 2. To simplify the proof that follows, we will provisionally assume that LRs can share ownership of risky projects among themselves. Since, in equilibrium, LRs will hold to maturity any risky projects they eventually acquire and are risk-neutral, this possibility of sharing risky project has no informational impact. In addition, we will show below that making an analogous assumption concerning the LRs, namely that LRs can either buy a single risky project or none at all would not change much in our equilibrium analysis.

III.A The SR optimization problem

SRs must determine first how much to invest in cash and how much in a risky asset. Second, they must decide how much of the risky asset to trade at date 1 at price P_1 and at

[7] More formally, we could have written $P_1(\omega_{1L})$ and $P_2(\omega_{1L})$ to denote the prices of the risky asset at dates 1 and 2 and similarly $Q_1(\omega_{1L})$ and $Q_2(\omega_{1L})$ to denote the quantities acquired by LRs at different dates. Given that all trading occurs in the "lower branch" of the tree we adopt the simpler notation as there is no possible ambiguity.

date 2 at price P_2. Their objective function is then

$$\begin{aligned}\pi\left[m,q_1,q_2\right] &= m+\lambda\left(1-m\right)\rho \\ &+ \left(1-\lambda\right)q_1P_1 \\ &+ \left(1-\lambda\right)\theta\eta\left[\left(1-m\right)-q_1\right]\rho \\ &+ \left(1-\lambda\right)\theta\left(1-\eta\right)\left[1-m-q_1\right]P_2 \\ &+ \left(1-\lambda\right)\left(1-\theta\right)q_2P_2 \\ &+ \delta\left(1-\lambda\right)\left(1-\theta\right)\eta\left[\left(1-m\right)-q_1-q_2\right]\rho\end{aligned} \tag{2}$$

As already mentioned, notice that implicit in this objective function is the fact that an SR in (idiosyncratic) state ω_{20} liquidates his remaining position in the risky asset in its entirety since he is sure they are worthless. In addition, in states where the asset yields ρ we assume that the SRs consume the payoffs, since market prices can never exceed ρ. Finally we do not consider the possibility that an SR investor would acquire claims to output in period 3 from LRs, since SRs value such claims strictly less than the LRs.

The SR's optimal investment program is therefore given by:

$$\max_{m,q_1,q_2}\pi\left[m,q_1,q_2\right] \tag{$\mathcal{P}_{SR}$}$$

subject to

$$m\in[0,1]$$

and

$$q_1+q_2\leq 1-m \qquad \text{and} \qquad q_1,q_2\in\{0,1-m\}$$

The constraints simply state that the SR cannot invest more in the risky asset than the funds at its disposal and that it cannot sell more than what it holds. The last condition guarantees that when an SR sells his risky assets, he must sell everything he owns.

III.B The LR optimization problem

LR investors must first determine how much of their savings to hold in cash (outside liquidity), M, and how much in long term assets, $\kappa-M$. LRs must also decide at dates 1 and 2 how much of the risky assets to purchase at prices P_1 and P_2. Recall that, given assumption A1 cash is costly to carry for LRs and thus they never carry cash that they will never use. In other words, in some state of nature where trade occurs LR investors must completely exhaust their cash reserves to purchase the available supply of SR risky assets. With this observation in mind we can write the payoff an LR investor that purchases Q_1 at date 1 and Q_2 at date 2, as follows:

$$\begin{aligned} \Pi[M, Q_1, Q_2] &= M + \varphi(\kappa - M) \\ &+ (1-\lambda)[\eta\rho - P_1] Q_1 \\ &+ (1-\lambda) E[\tilde{\rho}_3 - P_2 | \mathcal{F}] Q_2 \end{aligned} \tag{3}$$

The first line in (3) is simply what the LR investor gets by holding an amount of cash M until date 3 without ever trading in secondary markets at dates 1 and 2. The third term is the net return from acquiring a position Q_1 in risky assets at unit price P_1 at date 1. Indeed, the expected gross return of a risky asset in state ω_{1L} is $\eta\rho$. The last term is the net return from trading in period 2. This net return depends on the payoff of the risky asset at date 3 and in particular on the quality of assets purchased at date 2. As we postulate rational expectations, the LR investor's information set, $\mathcal{F}$, will include the particular equilibrium that is being played. In computing conditional expectations the LRs assume that the mix of assets offered in period 2 corresponds to the one observed in equilibrium. An LR may also decide to acquire units of payoffs in period 3 from other LR's but risk neutrality of LR's guarantees that, in an equilibrium, such trades would be done at prices that do not produce any surplus.

We require a standard, and weak, rationality condition from LRs, that if they succeed in purchasing some SR projects in period 2 in an equilibrium that prescribes no sales in these states, and furthermore at a price for which SRs which are in state ω_{2L} strictly prefer to hold the asset until date 3 to selling it in period 2, then LR assumes that he is buying a worthless asset. In addition, LRs assume that SRs that weakly prefer to sell at price P_2 will sell their holdings. The LR investor's program is thus:

$$\max_{M, Q_1, Q_2} \Pi[M, Q_1, Q_2] \tag{$\mathcal{P}_{LR}$}$$

subject to

$$0 \leq M \leq \kappa \tag{4}$$

and

$$Q_1 P_1 + Q_2 P_2 \leq M \qquad \text{and} \qquad Q_1 \geq 0,\ Q_2 \geq 0 \tag{5}$$

The first constraint (4) is simply the LR investor's wealth constraint: LRs' cannot carry more cash than their initial capital κ and they cannot borrow. The second constraint (5) says that LRs cannot purchase more risky projects carried by the SRs than their money, M, can buy and that LRs cannot short risky projects.[8]

[8]Below when we comment in the case where the risky projects are nondivisible we will explore the case where

III.C Definition of equilibrium

A rational expectations competitive equilibrium is a vector of portfolio policies $[m^*, M^*]$, supply and demand choices $[q_1^*, q_2^*, Q_1^*, Q_2^*]$ and prices $[P_1^*, P_2^*]$ such that (i) at these prices $[m^*, q_1^*, q_2^*]$ solves $\mathcal{P}_{SR}$ and $[M^*, Q_1^*, Q_2^*]$ solves $\mathcal{P}_{LR}$ and (ii) markets clear in all states of nature. An equilibrium must also specify the price S_1^* that would obtain in event ω_{1L} for payoffs in period 3 and the price S_2^* for these payoffs that would prevail in period 2. However, the risk neutrality of the LR's tie these prices to the expected returns of risky projects and (or) cash.[9]

III.D Characterization of equilibria

An important property of our model is that it features multiple equilibria for a particular range of parameter values. Specifically, there are two (stable) equilibria; one where all the trading occurs at date 1 (in state ω_{1L}), and another where all the trading occurs at date 2. We refer to the first one as an *immediate-trading equilibrium* and the second as a *delayed-trading equilibrium.* We establish first existence of these equilibria and then proceed to characterize inside and outside liquidity across equilibria, as well as the comparative statics of equilibrium liquidity and prices with respect to θ. These comparative statics results are of central interest, as they determine both how desirable the risky asset is to SRs and the severity of the adverse selection at date 2. We conclude this section by studying the welfare properties of the different equilibria and in particular noting a novel form of inefficiency that arises in our model relative to other models that feature adverse selection.

III.D.1 Immediate and delayed-trading equilibria

The immediate-trading equilibrium. Under our stated assumptions we are able to establish first that there always exists an immediate-trading equilibrium.

some LRs may trade part of their long run projects in exchange for cash to acquire risky projects, in which case the budget constraint in (5) would be written differently.

[9] In particular, the price at which sure period 3 payoffs would trade in period 1 in event $\omega_{1\rho}$ is necessarily equal to 1 in any equilibrium in which LRs hold positive amounts of cash

Proposition 1. (*The immediate-trading equilibrium*) Assume A1-A3 hold then there always exists an immediate-trading equilibrium, where

$$M_i^* > 0 \qquad q_1^* = Q_1^* = 1 - m_i^* \qquad \text{and} \qquad q_2^* = Q_2^* = 0.$$

In this equilibrium cash-in-the-market pricing obtains and

$$P_{1i}^* = \frac{M_i^*}{1 - m_i^*} \geq \frac{1 - \lambda\rho}{1 - \lambda}. \tag{6}$$

The price in period 1 of claims to period 3 output, $S_{1i}^* = \frac{P_{1i}^*}{\eta\rho} < 1$, and the price in period 2 of claims to period 3 is $S_{2i}^* = 1$. Moreover the cash positions m_i^* and M_i^* are unique.

To gain some intuition on the construction of the immediate-trading equilibrium notice first that the first order conditions for m and M are:

$$P_{1i}^* \geq \frac{1 - \lambda\rho}{1 - \lambda} \qquad \text{and} \qquad \lambda + (1 - \lambda)\frac{\eta\rho}{P_{1i}^*} = \varphi'\left(\kappa - M_i^*\right), \tag{7}$$

when $m_i^* < 1$ and $M_i^* > 0$.[10] These expressions follow immediately from the maximization problem $\mathcal{P}_{SR}$ when we set $q_1^* = 1 - m_i^*$, and from problem $\mathcal{P}_{LR}$.

Next to determine the equilibrium price, let P_{1i} be the unique solution to the equation:

$$\lambda + (1 - \lambda)\frac{\eta\rho}{P_{1i}} = \varphi'\left(\kappa - P_{1i}\right), \tag{8}$$

which, given our assumptions, always exists. Assume first that the solution to (8) is such that

$$P_{1i} > \frac{1 - \lambda\rho}{1 - \lambda}.$$

In this case we can set $P_{1i}^* = P_{1i}$, $m_i^* = 0$, so that SRs are fully invested in the risky asset, and also $M_i^* = P_{1i}^*$ which by construction satisfies the LR's first order condition. Moreover, by assumption A1 it must also be the case that $M_i^* < \kappa$.

The key step in the construction of the immediate-trading equilibrium then, is that the price at date 2, P_{2i}^*, has to be such that both SRs and LRs have incentives to trade at date 1 and not at date 2. That is, it has to be the case that

$$P_{1i}^* \geq \theta\eta\rho + (1 - \theta\eta)\, P_{2i}^* \qquad \text{and} \qquad \frac{\eta\rho}{P_{1i}^*} \geq \frac{\mathbf{E}\left[\widetilde{\rho}_3 | \mathcal{F}\right]}{P_{2i}^*}. \tag{9}$$

[10] The proof of Proposition 1 establishes that assumption A3 rules out the possibility of a "no trade" immediate-trading equilibrium in which $M_i^* = 0$ and $m_i^* = 1$.

The first expression in (9) states that SRs prefer to sell assets at date 1 for a price P_{1i}^* rather than carrying it to date 2. Indeed if they do the latter, then with probability $\theta\eta$ the risky asset pays off ρ and with probability $(1-\theta\eta)$ they end up in either ω_{2L} or ω_{20} in which the SRs can sell the asset at price P_{2i}^*. If the price P_{2i}^* is low enough then SRs prefer to sell the asset at date 1.[11]

The expression on the right hand side of (9) states that for the LR the expected return of acquiring the asset in state ω_{1L} is higher than at date 2. To guarantee this outcome it is sufficient to set $P_{2i}^* < \delta\eta\rho$ for in this case SRs in state ω_{2L} would prefer to carry the asset to date 3 rather than selling it for that price. This then only leaves "lemons" in the market at date 2. LRs, anticipating this outcome, set their expectations accordingly, $\mathbf{E}\left[\widetilde{\rho}_3|\mathcal{F}\right] = 0$, and therefore for any strictly positive price $P_{2i}^* < \delta\eta\rho$ LRs prefer to acquire assets in state ω_{1L}.

Assume next that the solution to (8) is such that

$$P_{1i} \leq \frac{1-\lambda\rho}{1-\lambda}, \tag{10}$$

and set P_{1i}^* equal to the right hand side of (10). At this price, SRs are indifferent on the amount of cash carried. Then the solution to the LR's first order condition (see expression (7)) is such that:

$$M_i^* < P_{1i}^* = \frac{1-\lambda\rho}{1-\lambda}.$$

It is then sufficient to set $m_i^* \in [0,1)$ such that:

$$\frac{M_i^*}{1-m_i^*} = \frac{1-\lambda\rho}{1-\lambda}, \tag{11}$$

which is always possible.[12] Finally, the choice of P_{2i}^* can be taken to be the same as above.

The statements concerning S_{1i}^* and S_{2i}^* are immediate. Notice that in our framework, and by assumption A1, cash-in-the-market has to obtain and prices are lower than their discounted expected payoff, $P_{1i}^* < \eta\rho$, otherwise there would be no incentive for LRs to carry cash. Note also that this means that, by arbitrage, a unit of output from the long-run project at date 3 has to trade at a discount at date 1. Thus, in our setup cash-in-the-market pricing is necessarily transmitted in the form of *arbitrage contagion* across different markets even if no trading of the long-run asset occurs in equilibrium. This of course assumes, as we have done here, that no other capital would flow to absorb of firesales of neither the risky assets nor the long run projects.

[11] See expression (21) in the appendix for a precise upper bound on P_{2i}^* that has to hold to provide incentives for the SRs to sell at date 1 rather than at date 2.

[12] Notice that assumption A2 implies that $1-\lambda\rho > 0$.

The delayed-trading equilibrium. Proposition 2 establishes the existence of a delayed-trading equilibrium.[13]

Proposition 2 (*The delayed-trading equilibrium*) Assume A1-A3 hold and that δ is small enough[14] then there always exists an delayed-trading equilibrium, where $m_d^* \in [0,1)$, $M_d^* \in (0,\kappa)$,

$$q_1^* = Q_1^* = 0 \qquad \text{and} \qquad q_2^* = Q_2^* = (1-\theta\eta)\left(1-m_d^*\right).$$

In this equilibrium cash-in-the-market pricing obtains and

$$P_{2d}^* = \frac{M_d^*}{(1-\theta\eta)\left(1-m_d^*\right)} \geq \frac{1-\rho\left[\lambda+(1-\lambda)\,\theta\eta\right]}{(1-\lambda)\,(1-\theta\eta)}. \tag{12}$$

In addition, $S_{1d}^* = \frac{P_{1d}^*}{\eta\rho}$ and $S_{2d}^* = \frac{P_{2d}^*(1-\theta\eta)}{(1-\theta)\eta\rho}$. Moreover the cash positions m_d^* and M_d^* are unique.

The intuition of how we construct the delayed-trading equilibrium is broadly similar to the one for the immediate-trading equilibrium, with a few differences that we emphasize next. First, as stated in the proposition, δ needs to be small enough. Specifically, it has to be such that $\delta\eta\rho < P_{2d}^*$. Otherwise SRs in state ω_{2L} prefer to carry the asset to date 3 rather than selling it at date 2. This would destroy the delayed-trading equilibrium, as only lemons would then be traded at date 2. Second, a key difference with the immediate-trading equilibrium is that the supply of risky assets by SRs is reduced under delayed trading by an amount $\theta\eta$, which is the proportion of risky assets that pay off at that date.[15] As a result cash-in-the-market pricing under delayed trading is given by:

$$P_{2d}^* = \frac{M_d^*}{(1-\theta\eta)\left(1-m_d^*\right)}.$$

The mass of risky assets supplied in the market at $t=2$ is given by $(1-\theta\eta)\left(1-m_d^*\right)$. Thus delaying asset liquidation introduces both an adverse selection effect which depresses prices, and a lower supply of the risky asset, which, other things equal, increases prices.

[13] Recall that we are assuming that $q_1, q_2 \in \{0, 1-m\}$. If instead we had assumed that $0 \leq q_1, q_2 \leq 1-m$ there would also be a third equilibrium, which involves positive asset trading at both dates 1 and 2. We do not focus on this equilibrium as it is unstable.

[14] The proof of the proposition clarifies the upper bound on δ that guarantees existence, see expression (29) in the Appendix and the discussion therein.

[15] This is one of the key differences that arises when the shocks at date 2 are aggregate rather than idiosyncratic. In this case the supply of risky assets is always the same.

As under the immediate-trading equilibrium, to support a delayed-trading equilibrium requires that both SRs and LRs have incentives to trade at date 2 rather than at date 1, which means that

$$P_{1d}^{*} \leq \theta\eta\rho + (1-\theta\eta)\, P_{2d}^{*} \qquad \text{and} \qquad \frac{\eta\rho}{P_{1d}^{*}} \leq \frac{\mathbf{E}\left[\widetilde{\rho}_3|\mathcal{F}\right]}{P_{2d}^{*}}, \tag{13}$$

where now the expected payoff is given by

$$\mathbf{E}\left[\widetilde{\rho}_3|\mathcal{F}\right] = \frac{(1-\theta)\,\eta\rho}{(1-\theta\eta)}, \tag{14}$$

If (13) is to be met, the price in state ω_{1L} has to be in the interval

$$P_{1d}^{*} \in \left[\frac{1-\theta\eta}{1-\theta} P_{2d}^{*}, \theta\eta\rho + (1-\theta\eta) P_{2d}^{*}\right].$$

The key step of the proof of Proposition 2 is to show that this interval is non empty.

It is worth emphasizing that the delayed-trading equilibrium collapses to the immediate-trading equilibrium when $\theta = 0$. Indeed notice, for instance, that the lower bound in the price P_{2d}^{*} in (12) reduces to the lower bound in (6) for P_{1i}^{*}. The only difference between dates 1 and 2 is thus precisely the occurrence of an idiosyncratic shock that reveals to the SRs the true value of the risky asset. When $\theta = 0$ there is no informative idiosyncratic signal to be obtained as at date 1. This feature of our model plays an important role in what follows.

As before, notice that a unit of output from the long-run asset at date 3 trades at a discount both at dates 1 and 2. Thus, the liquidity event has effects in markets other than the one where distressed sales are taking place, and for as long as the crisis lasts. There may also be price changes in the long run asset even in the absence of any news about its underlying value, and even when there is no trading volume in the market for the long-run asset. As already mentioned, this is what we refer to as *arbitrage contagion*: Cash-in-the-market pricing transmits throughout financial markets inducing movements in prices of unrelated assets even when there is no news about these other assets and no need to liquidate them.

Before we close this section we introduce the following example to illustrate our results. As the parameter θ plays a critical role in our analysis it is the focus of our comparative statics, and we parameterize the set of economies that we consider throughout by the different values that θ takes. In light of assumption A2 it is then convenient to define $\overline{\theta}$ as the value for which

$$1 = \rho\left[\lambda + (1-\lambda)\eta\rho\left(\overline{\theta} + (1-\overline{\theta})\delta\right)\right], \tag{15}$$

for a given λ, δ, η, and ρ.

Example 1. In this example the parameter values are:

$$\lambda = .85 \qquad \eta = .4 \qquad \rho = 1.13 \qquad \kappa = .2 \qquad \delta = .1920 \qquad \varphi(x) = x^{\gamma} \quad \text{with} \quad \gamma = .4$$

Having fixed the value of δ, we need to restrict the values of our only free parameter θ to $\theta \leq \overline{\theta} = .4834$ to ensure that assumption A2 holds. It is immediate to check that in this example assumptions A1-A3 hold, as well as assumption A4 below. In particular, we have

$$\varphi'(\kappa) \approx 1.05 \qquad \text{and} \qquad \rho[\lambda + (1-\lambda)\eta] \approx 1.03$$

A summary of the main results is as follows:

- Both the immediate and delayed-trading equilibrium exist for $\theta \in [0, .4196)$; moreover in the delayed-trading equilibrium we have $m_d^* > 0$.
- For $\theta \in [.4196, .4628]$ both equilibria exist and the delayed-trading equilibrium is such that $m_d^* = 0$.
- For $\theta \in (.4628, .4834]$ the delayed-trading equilibrium does not exist. As we explain below, for this range of θ, the SR discount factor δ is not sufficiently small to induce SRs in idiosyncratic state ω_{2L} to trade at date 2; instead these SRs hold on to the risky assets until maturity at date 3. □

III.D.2 Inside and outside liquidity in the immediate and delayed-trading equilibria

How does the composition of inside and outside liquidity vary across equilibria? To build some intuition on this question it is useful to illustrate the immediate and delayed-trading equilibria that obtain in our example when $\theta = .35$. Figure 2 represents the immediate and delayed-trading equilibria in a diagram where the x axis measures the amount of cash carried by LRs, M, and the y axis the amount of cash carried by the SRs, m. The dashed lines are the *isoprofit curves* of the LRs and the straight (continuous) lines are the SR *isoprofit lines*.[16] To see the direction in which payoffs increase as one moves from one isoprofit curve to another, it is sufficient to observe that LRs prefer that SRs carry more risky projects for

[16] To generate these isoprofit lines note that we can construct an indirect expected profit function for SRs and LRs as a function of inside and outside liquidity, $\pi[M, m]$ and $\Pi[M, m]$ respectively. The lines plotted in Figures 2 and 3 simply give the combinations of m and M such that $\pi[m, M] = \overline{\pi}$ and $\Pi[m, M] = \overline{\Pi}$. Assumption A3 then simply says that the slope of the isoprofit lines at $M = 0$ at date 1 are such that there are gains from trade: the LR isoprofit curve is “flatter” than the SR isoprofit line.

a given level of outside liquidity, M. In other words, that m is lower. Along the other axis, LRs also prefer to carry less outside liquidity (lower M) for a given supply of risky projects by SRs. The converse is true for SRs. In the figure, we display the isoprofit lines for both the immediate and delayed-trading equilibrium. It is for this reason that isoprofit lines appear to cross in the plot: They simply correspond to different dates. Equilibria are located at the tangency points between the SR and LR isoprofit curves.

Consider first the immediate-trading equilibrium, located at the point marked $(M_i^*, m_i^*) = (.0169, .9358)$. There are two isoprofit curves going through that point; the straight line corresponds to the SR, and the dashed-dotted line corresponds to the LR isoprofit curve. In fact the straight line corresponds to the SR's reservation utility, $\overline{\pi} = 1$. Thus whatever gains from trade there are in the immediate-trading equilibrium they accrue entirely to the LRs. Turn next to the delayed-trading equilibrium, which is marked $(M_d^*, m_d^*) = (.0540, .4860)$ and features a mix of outside versus inside liquidity that is tilted towards the former relative to the latter when compared to the immediate-trading equilibrium. The SR's isoprofit line remains that associated with it's reservation value.

One way of understanding the portfolio choices in the immediate-trading equilibrium is that the risky asset is of high quality in state ω_{1L}, so that SRs must be compensated with a high price relative to the price that he would obtain if he were to delay the asset sale to $t = 2$, which also includes an *adverse selection discount*, to be willing to sell the asset at that point. This observation is reflected in the slope of the isoprofit lines in Figure 2: The SRs' isoprofit line in the immediate-trading equilibrium is *flatter* suggesting that SRs require a higher price per unit of risky asset sold at that date. But this higher price can only come at the expense of lower returns to holding cash for LRs. The latter are thus induced to cut back on their cash holdings. This, in turn, makes it less attractive for SRs to invest in the risky asset, and so on. The outcome is that in the immediate-trading equilibrium most of the liquidity is *inside liquidity* held by SRs, whereas the delayed-trading equilibrium features relatively more *outside liquidity* than inside liquidity.

The next proposition formalizes this discussion, specifically, it characterizes the mix of inside versus outside liquidity across the two types of equilibria. For this we make one additional assumption that allows for a particularly clean characterization of the aforementioned mix,

$$\frac{1-\lambda\rho}{1-\lambda} > \kappa \qquad (\mathbf{A4})$$

As the Result in the Appendix shows under assumption A4 the immediate-trading equilibrium is such that $m_i^* \in (0, 1)$, that is the SRs is carrying a strictly positive amount of cash.

Roughly, we need to guarantee that $m_i^* > 0$ in order to obtain non trivial cash allocation decisions for the SRs, which otherwise would be equal to 0 for both the immediate and the delayed-trading equilibria, as will become clear in Proposition 4. The present paper is concerned with the ex-ante efficiency costs associated with portfolio choices that result in the particular timing of the liquidation decisions and thus the most economically interesting case is the one where the economy is not "at a corner," that is $m_i^* = 0$, at the immediate-trading date. Armed with this new assumption we can prove the following

Proposition 3. (*Inside and outside liquidity across equilibria.*) Assume that A1-A4 hold and that δ is small enough so that a delayed-trading equilibrium exists for all $\theta \in (0, \overline{\theta}]$ then there exists a $\theta' \in \left(0, \overline{\theta}\right]$ such that $m_i^* > m_d^*$ and $M_i^* < M_d^*$ for all $\theta \in (0, \theta']$.

Thus for the range $\theta \in [0, \theta']$ the delayed-trading equilibrium features more outside liquidity and less inside liquidity than the immediate-trading equilibrium. In our example $\theta' = \overline{\theta}$ so that Proposition 3 holds for the entire range of admissible θs.[17]

We close this section by making two additional comments. First, note that all equilibria are *interim efficient*. That is, conditional on trade occurring in either dates there is no additional reallocation of the risky asset that would make both sides better off. As can be seen immediately in Figure 2, it is not possible to improve the ex-post efficiency of either equilibrium, as in each case the equilibrium allocation is located at the tangency point of the isoprofit curves. As we shall further explore below, in our model inefficiencies arise through distortions in the *ex-ante* portfolio decisions of SRs and LRs and through the particular timing of liquidity trades they give rise to. When agents anticipate trade in state ω_{1L}, SRs lower their investment in the risky asset and carry more *inside liquidity* m_i. In contrast LRs, carry less liquidity M_i as they anticipate fewer units of the risky asset to be supplied in state ω_{1L}.

A second observation is that A4, which implies that the immediate-trading equilibrium is such that $m_i^* > 0$, does not necessarily imply that $m_d^* > 0$. Indeed, Figure 3 shows the immediate and delayed-trading equilibrium when θ is increased from $\theta = .35$, as it was the case in Figure 2, to $\theta = .45$ and thus the adverse selection problem is relatively worse than in the previous case. The delayed-trading equilibrium is located in $(M_d^*, m_d^*) = (.0716, 0)$, the immediate-trading equilibrium being unaffected as it is independent of θ. Clearly the equilibrium is ex-post efficient, but now, unlike in the case considered in Figure 2, gains from trade do not solely accrue to the LRs but also to the SRs. In Figure 3 the isoprofit line marked

[17] In fact though we have been unable to prove it formally, we have not found an example of an economy that meets assumptions A1-A4 for which $\theta' < \overline{\theta}$.

IP_{SR} corresponds to the profit level $\overline{\pi} = 1$ for the SR, which is the same as under autarky. The isoprofit line through the delayed-trading equilibrium lies strictly to the right of IP_{SR}, which implies that SRs now command strictly positive profits. The reason is that at the corner when $m = 0$, SRs are at "full capacity" in supplying the risky asset at $t = 2$. In this case then they may earn *scarcity rents*, as LRs compete for the limited supply of the risky asset supplied by the SRs by increasing their bids for these assets.

III.D.3 Adverse selection and the delayed-trading equilibrium

We examine how changes in the adverse selection problem LRs face at date 2, as measured by changes in θ, affect equilibrium outcomes. In particular, we are interested in understanding how equilibrium cash holdings and equilibrium prices vary with θ.

Several important effects are at work as θ changes, some of which we have already mentioned. First, the incentives of both SRs and LRs to hold cash are affected by changes in θ. In addition, SRs' incentives to hold onto their asset position until date 2 (when the risky asset does not mature at date 1) are affected. As θ rises the risky asset is more likely to mature at date 2 and thus becomes more attractive to SRs. Other things equal, SRs are then both more likely to invest in the risky asset and to carry the asset from date 1 to date 2.

However, as θ rises the adverse selection problem at $t = 2$ is worsened and therefore equilibrium prices P_{2d}^* are likely to be lower. These lower prices that SRs face at $t = 2$ in turn reduce their incentives to invest in the risky asset and to carry it to date 2. An additional complication is that as θ increases the supply of the risky asset at date 2,

$$s_{2d}^* \equiv (1 - m_d^*(\theta))(1 - \theta\eta) \tag{16}$$

diminishes on account of the fact that a larger share of the available risky assets pay off and thus are not liquidated.

We are interested also in the expected return on acquiring the risky asset at date 2 in the delayed-trading equilibrium, which is defined as

$$R_{2d}^* \equiv \frac{(1 - \theta)\eta\rho}{(1 - \theta\eta)P_{2d}^*}$$

The next proposition establishes how these countervailing effects net out and how M_d^*, m_d^*, s_{2d}^*, and R_{2d}^* vary with θ. Throughout we assume, of course, that $\theta \leq \overline{\theta}$, defined in (15).

Proposition 4. (*Comparative statics.*) Assume that A1-A4 hold and that δ is small enough for all $\theta \in [0, \overline{\theta}]$ so that a delayed-trading equilibrium always exists, then there exists a unique $\widehat{\theta} \in [0, \overline{\theta}]$, possibly $\widehat{\theta} = \overline{\theta}$, such that:

I. The SR's cash position: (a) m_d^* is a (weakly) decreasing function of θ, (b) $m_d^* > 0$ for all $\theta \in [0, \widehat{\theta})$ $m_d^* = 0$ for all $\theta \in [\widehat{\theta}, \overline{\theta}]$ and (c) s_{2d}^* is a strictly increasing function of θ for $\theta \in [0, \widehat{\theta})$ and a strictly decreasing function of θ for $\theta \in [\widehat{\theta}, \overline{\theta}]$.

II. The LR's cash position: M_d^* is a strictly increasing function of θ for $\theta \in [0, \widehat{\theta})$ and a strictly decreasing function of θ for $\theta \in (\widehat{\theta}, \overline{\theta}]$.

III. Expected returns at date 2: R_d^* is an increasing function of θ for $\theta \in [0, \widehat{\theta})$ and a decreasing function of θ for $\theta \in (\widehat{\theta}, \overline{\theta}]$.

We illustrate the comparative statics described in Proposition 4 in our example, for which, given our parametric assumption, it can be shown that $\widehat{\theta} = .4196$. Figures 4 and 5 exhibit the comparative statics with respect to θ for the cash positions, m_d^* and M_d^*, and the expected return and the price of the risky asset at $t = 2$, R_{2d}^* and P_{2d}^* respectively.

Consider first Figure 4. As we would expect, based on our discussion above, the amount of cash carried by the SR is a decreasing function of θ, and $m_d^* = 0$ for $\theta \geq \widehat{\theta} = .4196$. It is less obvious how the amount of cash carried by LR investors varies with θ. Consider first the case where $\theta \leq \widehat{\theta}$. The amount of cash carried by LR investors is then an increasing function of θ. This is surprising: the more severe the adverse selection problem the more cash carried by LRs. What is the logic behind this result?

When $\theta \leq \widehat{\theta}$, an increase in θ does indeed worsen the adverse selection problem and would result in LRs reducing their supply of liquidity other things equal. But there is a countervailing effect, which is that an increase in θ also results in a higher supply of the risky asset by SRs at date 2. Indeed, as shown in Proposition 4-I-(c), s_d^* (which is defined in (16)) is an increasing function of θ in this range.[18] It is this higher supply of the risky asset that in turn increases the supply of outside liquidity. The latter effect dominates and thus results in an increasing M_d^* as a function of θ when $\theta \leq \widehat{\theta}$. Instead, when $\theta > \widehat{\theta}$ the supply effect gets reversed and

[18]There are two effects on s_{2d}^* when $\theta \leq \widehat{\theta}$. When θ increases, SRs carry more risky projects; that is m_d^* decreases as the risky project is more likely to pay off at date 2. On the other hand, the higher θ, the lower the fraction of risky projects carried by SRs that is supplied at date 2. Indeed, notice that the second term in (16), $1 - \theta\eta$, is a decreasing function of θ. Proposition 4-I shows that the first effect dominates the second over this range.

s_d^* is a decreasing function of θ. Both the supply side and the adverse selection effect then reduce the incentives of LRs to carry cash and it is for this reason that M_d^* is now a decreasing function of θ.

Figure 5 illustrates how the price P_{2d}^* changes with θ. As can be seen, P_{2d}^* is a decreasing function of θ. But note that the decline is more pronounced when $\theta < \widehat{\theta}$. The reason has already been mentioned. As long as $\theta < \widehat{\theta}$ an increase in θ has a double effect. A higher θ worsens adverse selection concerns and thus the drop in prices. In addition, a higher θ increases investment in the risky asset, which gets liquidated whenever the SR is in idiosyncratic state ω_{20} or ω_{2L}. This supply effect produces a further decline in prices that is absent when $\theta \geq \widehat{\theta}$ for then $m_d^* = 0$ and there can be no further investment in the risky asset. Notice that for $\theta > \widehat{\theta}$ prices keep dropping but a lower rate for now the supply is decreasing and thus the competition for the risky asset amongst the LRs dampens the adverse selection effect on prices.

The pattern of returns is also revealing about the incentives of the LRs to carry outside liquidity to the delayed-trading equilibrium. For $\theta < \widehat{\theta}$, R_{2d}^* is an increasing function of θ: The expected payoff of the risky asset in the delayed-trading date is given by (14) which is a decreasing function of θ. But the price is dropping *faster* than returns on account both of the adverse selection effect and the supply effect. This produces the increasing pattern in returns. It is for this reason that the incentives of the LRs to carry outside liquidity are also increasing in θ. Instead when $\theta > \widehat{\theta}$, the expected payoff is still decreasing but the decrease in supply makes for a slow drop in prices as a function of θ, as we just saw, and thus the negative slope of R_{2d}^* as a function of θ in this range.

In conclusion then, for $\theta \in [0, \widehat{\theta}]$ the more severe the adverse selection problem, as measured by θ, the higher the amount of outside liquidity brought to the market and the lower the amount of inside liquidity carried by those holding the risky asset. This counterintuitive result is due to the drop in prices, which makes the risky asset more attractive to the LRs at $t = 2$. The larger the liquidity correction at date 2, the more attractive it is for LRs to carry cash and trade opportunistically. Armed with these insights we turn to the question of the Pareto ranking of the two equilibria.

III.D.4 Pareto ranking of the immediate and delayed-trading equilibria

Given these differences in *ex-ante* portfolio allocations an obvious question is whether there a clear ranking of the two equilibria in terms of Pareto efficiency when they coexist? Interestingly, the answer to this question is *yes* and, somewhat surprisingly, it is the delayed-trading equilibrium that *Pareto dominates* the immediate-trading equilibrium. This is surprising, as delayed trade is hampered by the information asymmetry that arises at $t = 2$, and therefore will take place at lower equilibrium prices.

Proposition 5. (*Pareto ranking of equilibria.*) Assume that A1-A4 hold and that δ is small enough for all $\theta \in [0, \overline{\theta}]$ so that a delayed-trading equilibrium always exists, then there exists a $\theta' \in (0, \overline{\theta}]$ such that $\pi_i^* \leq \pi_d^*$ and $\Pi_i^* < \Pi_d^*$ for all $\theta \in (0, \theta')$.

In our example $\theta' = \overline{\theta}$ and though we have not been able to prove a tighter characterization of Proposition 5, we have been unable to find an example that meeting assumptions A1-A4, features $\theta' < \overline{\theta}$. Thus in our example the delayed-trading equilibrium Pareto dominates the immediate-trading equilibrium for all $\theta \in (0, \overline{\theta}]$. This is illustrated in Figure 6, where the expected profits of both the SRs and LRs are plotted for a particular range of θs.[19]

Figure 6 shows the expected profits of SRs and LRs as a function of θ for the delayed-trading equilibrium. The top panel shows the SRs' expected profits. Notice that for $\theta \leq \widehat{\theta} = .4196$ the SRs are left at their reservation profits, which obtain if they were to be fully invested in cash. Indeed, the SRs' risky asset is a constant returns to scale technology and, a shown in Proposition 4, in this range they are not fully invested in the risky asset. Figure 2 offered a preview of this result. In that particular case $\theta = .35 < \widehat{\theta}$ and thus the delayed-trading equilibrium was located at the tangency point of the SR's isoprofit line which corresponds to its reservation value of $\overline{\pi} = 1$ and the LR's isoprofit line. The lower panel shows the LR's expected profit. The flat line corresponds to the LR's expected profit in the immediate-trading equilibrium, which is everywhere below the expected profit in the delayed-trading equilibrium. What may be at first surprising is that the LR's expected profits are, in this range, an increasing function of θ: The higher the adverse selection the higher the LR's expected profit. This again is due to the fact that as we increase θ the asset is more likely to pay at date 2, when SRs care the most for payoffs, and thus it becomes more attractive to them. This leads them to invest more in the risky asset and carry less inside liquidity, which translates into more goods for the LR in the event that the market opens at date 2. The liquidity premium associated with the adverse selection problem combined with the increased supply of assets translates, as we saw in Proposition 4, into an improvement of the investment opportunities available to the LRs at the interim stage, which can only make them better off.

For $\theta > \widehat{\theta}$ the SRs are fully invested in the risky asset and because of this fact they now acquire some rents. Indeed for this range $\pi_d^* > 1$ and increasing with θ, whereas for the LRs the expected profits are a decreasing function. Notice though that $\Pi_d^* > \Pi_i^*$ throughout. A

[19]The starting $\theta = .35$ is simply chosen to show the figures in a convenient scale.

particular example was depicted in Figure 3, where an example was shown where $\theta = .45 > \widehat{\theta}$. As could be seen there, the delayed-trading equilibrium was located strictly in the interior of the lens formed by the two reservation isoprofit lines.

III.D.5 Discussion: Welfare and outside versus inside liquidity

In our setup a higher social surplus can be achieved when the aggregate amount of cash held by investors is lowered and when investment in risky and long run projects is increased. But recall that under assumption A2, SRs only want to hold cash in autarchy and do not want to implement risky projects. They are only prepared to invest in risky projects if enough outside liquidity is provided by LRs to absorb the potential sales of risky projects following a liquidity shock at either dates 1 or 2. SRs are endowed with an investment opportunity they don't want to exploit in autarchy even though it is socially efficient to do so, unless they can *distribute* the investment to LRs in exchange for cash in some contingencies. The SR investment technology is a constant returns to scale technology. Therefore, from a social point of view efficiency requires minimization of inside liquidity. Thus the key trade-off is between the efficiency gain from lower inside liquidity and the efficiency loss from higher outside liquidity.

In the delayed-trading equilibrium, inside liquidity is lower and the amount of risky projects originated is larger than in the immediate-trading equilibrium. But, there is also more outside liquidity. The higher amount of risky projects originated is an efficiency gain, whereas the larger amount of outside liquidity is an efficiency loss.[20] However, in our model the efficiency gain more than offsets the efficiency loss. The reason is that the amount of outside liquidity that LRs hold in the delayed-trading equilibrium is not very large *relative* to the amount of cash they hold in the immediate-trading equilibrium. Also, in the delayed-trading equilibrium SRs retain the risky asset's upside (in idiosyncratic state $\omega_{2\rho}$), and they need to be compensated for giving up this upside option in the immediate-trading equilibrium. This compensation can only come in the form of more outside liquidity at date 1, as reflected in the higher price of the risky asset at that date compared with the price at date 2.

III.D.6 Trading of indivisible risky projects

In our equilibria SRs sell their entire risky investment conditional on selling any risky assets. The risky assets can be sold in pieces or whole and we have not imposed any restrictions

[20] Recall that we were able to prove this result for the range $\theta \in [0, \theta']$, where $\theta' \leq \overline{\theta}$, and not for the entire range (see Proposition 5 and the discussion therein.) To reiterate though, we have been unable to find an example for which $\theta' < \overline{\theta}$

on whether these assets are sold piecemeal or not. In this subsection we explore the consequences of restricting trade in risky assets to selling indivisible projects to LRs. The motivation for imposing this restriction is that assets may be physically indivisible and information about each project's quality is itself indivisible.

Recall that in the delayed trading equilibrium a fraction $\theta\eta$ of the risky projects return ρ and are not available for trading. If the risky projects are indivisible, a fraction $(1-\theta\eta)$ of the LRs must end up buying all the risky projects while the remaining LRs would only hold period 3 sure payoffs. We must therefore check whether LRs that end up with the risky projects have enough cash, and wealth,[21] to finance their purchases. The total value of the risky projects that are held in the delayed equilibrium by $\theta\eta$ LR's is $\theta\eta P^*_{2d}(1-m^*_d)$. In turn, the total value of period 3 sure payoffs held by the fraction $(1-\theta\eta)$ of LRs is:

$$(1-\theta\eta)\frac{P^*_{2d}(1-\theta\eta)}{(1-\theta)\eta\rho}\varphi(\kappa - M^*_d)$$

Hence our delayed trading equilibrium still obtains when discrete risky projects are traded provided that:

$$\frac{(1-\theta\eta)^2}{(1-\theta)\eta\rho}\varphi(\kappa - M^*_d) \geq \theta\eta(1-m^*_d) \tag{17}$$

Hence inequality (17) also holds for theta sufficiently small. In addition it can be verified that inequality (17) holds for every example of delayed equilibrium we have provided. Given this, it is immediate that LRs are indifferent between exchanging their cash for additional units of the long project or for the SRs' risky projects. Thus to be able to purchase "indivisible" risky projects at firesale prices, LRs may also have to sell some of their holdings on the long run project.

Alternatively, we could have assumed that the decreasing returns to long run projects are due to a pecuniary externality that depends on the average amount invested by all LRs. That is, the output produced in period 3 with x units invested at date 0 equals $x\phi(\bar{x})$, where $\bar{x}$ is the average LR investment and ϕ is a concave function. Under this interpretation, every LR is indifferent between holding cash or investing in the long run project in equilibrium. Besides capturing an important aggregate economic effect, this formulation also makes it easier to accommodate the discreteness of long-run projects.

[21] If LRs purchasing risky assets do not hold enough cash they can sell their claims to their period 3 consumption to other LRs who are not purchasing risky assets and thus obtain sufficient cash to absorb the firesales.

III.E Two forms of inefficiency

Adverse selection plays an important role in our framework and introduces two sources of inefficiency. The first one is standard and is discussed in section III.E.2. As in Akerloff (1970), when the adverse selection premium is large enough good risks (SRs in ω_{2L}) withdraw their supply of the risky asset leaving only "lemons" in the market, which leads to a market breakdown. The other, novel source of inefficiency (to the best of our knowledge), is that the presence of adverse selection at date 2 gives rise to an inefficient immediate-trading equilibrium at date 1, when the parties can trade under symmetric information. In other words, the presence of adverse selection may inefficiently accelerate asset liquidation. We begin by discussing this inefficiency in section III.E.1.

III.E.1 Existence of the immediate-trading equilibrium

We have shown that the delayed-trading equilibrium Pareto dominates the immediate-trading equilibrium. We now show that it is the presence of asymmetric information at date 2 that introduces the possibility of immediate trading in equilibrium at date 1.

Proposition 6 (*Unique full information equilibrium)* Assume that A1-A3 hold, that δ is small enough and that LRs observe whether SRs are in idiosyncratic state ω_{2L} or ω_{20}. Then the unique equilibrium is the delayed-trading equilibrium.

A formal derivation of the delayed-trading equilibrium is given in the appendix. Here we simply show that it is not possible to support an immediate-trading equilibrium when agents are symmetrically informed. Indeed, the conditions for a putative immediate-trading equilibrium are

$$P_{1i}^* \geq \theta\eta\rho + (1-\theta)\, P_{2i}^* \qquad \text{and} \qquad P_{2i}^* \geq P_{1i}^* \tag{18}$$

But these conditions imply that $P_{1i}^* \geq \eta\rho$, which, given (18), in turn imply that $P_{2i}^* \geq \eta\rho$. Given that the expected gross payoff of the asset for LRs is always $\eta\rho$ under symmetric information, it follows that the expected return of carrying cash for LRs cannot be greater than one. Thus $M_i^* = 0$, which implies $m_i^* = 0$. Notice that when LRs are not informed about the idiosyncratic state ω_{2L} or ω_{20}, it is possible to set $P_{2d}^* < \delta\eta\rho$ and to support an immediate-trading equilibrium as in the proof of Proposition 1. But this is no longer possible under symmetric information. Thus the presence of adverse selection can give rise to an *inefficient acceleration of liquidity trading* that would not occur if agents were symmetrically informed.

III.E.2 Non existence of the delayed-trading equilibrium

A maintained assumption throughout our analysis in section III.D is that δ is small enough that existence of the delayed-trading equilibrium is assured. In this subsection we explore the model solution for higher values of delta. An important feature of our model is that SRs in state ω_{2L} may prefer to carry the asset to date 3 rather than trading it for the (candidate) price P_{2d}^* at date 2.[22] When this happens the delayed-trading exchange cannot be supported as a competitive equilibrium for then only lemons would be traded in the market. SRs in state ω_{2L} would have an incentive to retain the asset and carry it to date 3 whenever the candidate delayed-trading equilibrium price is such that $P_{2d}^C < \delta\eta\rho$ where P_{2d}^C is the candidate price for the risky asset at date 2 constructed as in Proposition 2. In our example this occurs for the range of economies for which $\theta \in (.4628, .4834)$.

To illustrate graphically the welfare costs associated with this lack of existence, Figure 7 plots the expected profits for the SRs and the LRs as a function of θ where we have selected the Pareto superior equilibrium whenever both equilibria coexist. There are three regions in the plot. The first two correspond to the cases already discussed. In region A, $\theta \in (0, .4196)$ the delayed-trading equilibrium is Pareto superior and is such that $m_d^* > 0$. Region B, $\theta \in [.4196, .4628)$, also features the delayed-trading equilibrium as the Pareto superior one and in that range of θs, $m_d^* = 0$. Region C is one where, though assumptions A1-A4 are met, a delayed-trading equilibrium cannot be supported and thus the immediate-trading equilibrium gets selected.

The dashed line in both panels of Figure 7 shows the additional expected profits that would accrue to SRs and LRs if the former could commit ex-ante to liquidate their assets at the candidate price $\widetilde{P}_{2d}$ in state ω_{2L}. In this case, the LRs anticipating that the pool of assets supplied at date 2 would also include assets of higher quality would be willing to bring more outside liquidity than in the immediate-trading equilibrium. As shown above, this is always Pareto improving in our framework because it substitutes inside with outside liquidity.

III.F Monopolistic supply of liquidity and efficiency

So far we have assumed that outside liquidity is supplied competitively, so that LRs do not take into account the effect that their choices have on the equilibrium price P_{2d}^*. A monopoly LR, on the other hand, would internalize the effect of its supply of liquidity on the price. The obvious question then arises whether a monopoly LR might be more efficient in

[22]It is easy to check that A2 implies that it is never optimal to retain the asset in an immediate-trading equilibrium constructed as above.

situations where the competitive delayed-trading equilibrium fails to exist?

When $\theta < \widehat{\theta}$, where $\widehat{\theta}$ was defined in Proposition 4, SRs carry a strictly positive amount of inside liquidity $m_d^* > 0$ and make zero profits. All the surplus goes to the LRs, who cannot obtain higher profits by changing their holdings of liquidity. It follows then that in this range the competitive and monopoly solutions are identical. In contrast, when $\theta \geq \widehat{\theta}$, the level of inside liquidity in the competitive equilibrium is $m_d^* = 0$, LRs compete for a fixed supply of the risky asset, and SRs obtain some of the surplus from trade. In this situation, a monopoly LR would gain by restricting its supply of outside liquidity and thereby raising the price P_{2d}^*. This can be seen in Figure 8, where, in the top panel, the profits of the monopolist are plotted together with the competitive ones and in the bottom panel the prices in states $(\omega_{20}, \omega_{2L})$ are plotted against θ.

Notice first that in region **A**, which corresponds to the case $\theta < \widehat{\theta}$, the prices and profits in a monopoly are identical to those under perfect competition. In region **B**, SRs set the level of inside liquidity to $m_d^* = 0$ and the monopoly LR restricts the supply of outside liquidity so as to capture fully all the gains from trade. This explains why the price of the risky asset at $t = 2$ under a monopoly is below the competitive price.

But once θ exceeds a certain threshold the competitive delayed-trading equilibrium no longer exists. The monopoly LR in this case has to set the price for the risky asset to $\delta\eta\rho$ to guarantee a profitable trade at date 2. In this parameter region, region **C** in Figure 8, a monopoly LR will indeed improve efficiency. By carrying and supplying enough outside liquidity the monopolist elicits the supply of risky assets by SRs in state ω_{2L}, thus avoiding the break down of the delayed exchange. Notice that as shown in the top panel of Figure 8, the monopoly's profits are, in this region, above those that obtain in the immediate-trading equilibrium, which is the only one that exists with competitive LRs.[23]

IV. LONG-TERM CONTRACTS FOR LIQUIDITY

IV.A Long-term contracts

So far we have only allowed SRs and LRs to trade assets for cash at dates 1 and 2, after they have each made their portfolio investment decisions at date 0. We saw that when the lemon cost is severe a delayed-trading equilibrium may fail to exist and that there is value in SRs's commitment to sell the risky asset at $t = 2$ to avoid the market breakdown that occurs in

[23] It is worth emphasizing than in this region SR profits are such that $\pi > 1$. The reason is that the monopolist has to "leave some rents" to the SRs precisely to elicit the supply of the assets in state ω_{2L}.

its absence. A natural question arises whether some form of long term contract between an SR and LR at date 0 may improve on the outcome obtained in the immediate and delayed-trading equilibria. Allowing for a bilateral contract between the SR and the LR expands the set of states that can be contracted upon as, in principle, transfers can now be made contingent on $\omega_{2\rho}$, ω_{20}, and ω_{2L}. This was not possible when liquidity is provided via the market as in that case the price for transacting at time $t = 2$, P_2, cannot be made contingent on $\omega_{2\rho}$, ω_{20}, or ω_{2L}. A key point of this section is that, perhaps surprisingly, the additional contractibility that long term contracts allow for may not yield an efficiency improvement over the market provision of liquidity investigated in the previous section.

Specifically we allow for long term contract between one SR and one LR by which the SR transfers to the LR both his investment opportunity as well as his unit of endowment.[24] The fund run by the LR allocates the total endowment $(1 + \kappa)$ of the two investors in a portfolio that may comprise the long-run asset, the risky asset, and cash. Since there is only one unit of risky projects available, we assume that each fund cannot invest more than one unit of endowment in the risky asset. The LR manager in turn promises payments that are contingent on the announced state of nature,

$$C_t(\omega) \quad \text{for} \quad t = 1, 2, 3 \quad \text{and} \quad \omega \in \{\omega_{1\rho}, \omega_{1L}, \omega_{2\rho}, \omega_{20}, \omega_{2L}, \omega_{30}, \omega_{3\rho}\}, \tag{19}$$

where $C_t(\omega)$ is the transfer from the from the LR to the SR at date t when the announcement is that the state of the risky project is given by ω. The key assumption is that now, the LR in the long term contract with the SR is *managing* the funds and investments and thus information regarding the realization of the risky project at $t = 2$ and $t = 3$ accrues only to him: If the LR chooses to invest part of the endowments in the risky asset then it is the LR who privately observes the realization of the idiosyncratic shocks affecting the risky asset at dates 2 and 3. Intuitively then an LR investing in the risky asset faces incentive compatibility constraints, which limit the efficiency of the long-term contract.[25]

Finally we assume throughout that $\delta\varphi(\kappa) < 1$, so that the LR investor would not find it optimal to simply invest the whole endowment $(1 + \kappa)$ in the long asset and repay the SR at date 3.

[24]Obviously, a long term contract in which the entire LR industry contracts with the entire SR industry could achieve a better outcome by virtue of effectively eliminating the asymmetric information between parties.

[25]Note that if the SR investor can also observe the realization of idiosyncratic shocks then the asymmetric information problem in the delayed-trading equilibrium would not be present, so that the long-term contract at date 0 would clearly yields a superior outcome. The more consistent and interesting case, however, is when the observation of idiosyncratic shocks is private information to the manager of the risky asset.

IV.B Feasibility, Participation and incentive compatibility constraints

As mentioned, information about the risky asset accrues now to the LR and thus incentives have to be provided for a truthful revelation of the state of nature. We start with date 3; having announced at date 2 that the state of nature is ω_{2L} incentive compatibility requires that at date 3

$$C_3\left(\omega_{3\rho}\right) = C_3\left(\omega_{30}\right),$$

for otherwise the LR simply announces the state which involves the lower payment to SR. Given this, date 2 incentive compatibility requires that,

$$C_2\left(\omega_{2L}\right) + C_3\left(\omega_{30}\right) = C_2\left(\omega_{20}\right) + C_3\left(\omega_{20}\right) = C_2\left(\omega_{2\rho}\right) + C_3\left(\omega_{2\rho}\right)$$

Otherwise, again, LR would always announce the state which involves the lowest total payout.

As for the feasibility constraints, given that the SR is indifferent between consumption at date 1 and 2 then, without loss of generality, we can restrict contracts to those that set $C_1\left(\omega_{1\rho}\right) = C_1\left(\omega_{1L}\right) = 0$. Then, for instance, the feasibility constraints associated with $\omega_{1\rho}$ are

$$C_{2\rho}\left(\omega_{1\rho}\right) \leq \alpha_x \rho + M_x \quad \text{and} \quad C_2\left(\omega_{1\rho}\right) + C_3\left(\omega_{1\rho}\right) \leq \alpha_x \rho + M_x + \varphi\left(y_x\right),$$

where $\alpha_x \leq 1$ is the amount that the LR invests in the risky project under the long term contract, M_x is the cash position and

$$y_x = \kappa + 1 - \alpha_x - M_x$$

is the amount invested in the long project. The rest of the feasibility constraints follow along similar lines and in the interest of space they are relegated to the Appendix.

Finally, because our purpose in this section is to assess to what extent the long run contract can do better than the market provision of liquidity, we simply leave the SR with the expected profits he would obtain in the delayed-trading equilibrium and thus assign all the surplus associated with the long term contract to the LR. The question then is simply whether $\Pi_x^* \geq \Pi_d^*$, where Π_x^* is the LR's expected profit under the ex-ante contract.

IV.C Long term contracts and the market provision of liquidity

As is easy to see, when SR investors expect the immediate-trading equilibrium, then any pair of LR and SR investors are weakly better off writing a long-term contract at date 0. Indeed, at worst the contract simply replicates the allocation under immediate trading. But, the contract can also implement other allocations that are not feasible under the immediate-trading equilibrium, in particular by investing part of the SR endowment in the LR project and by specifying payments to SR that (weakly) dominate the allocation under immediate trading:

$$C_1(\omega_{1\rho}) \geq m_i^* + \lambda(1 - m_i^*)\rho$$

$$C_1(\omega_{1L}) \geq m_i^* + (1 - m_i^*)P_{1i}^*.$$

The following proposition, which is given without proof, then obtains,

Proposition 7. (*Ranking of long-term contract and immediate-trading equilibrium*) The optimal long-term contract weakly (and sometimes strictly) dominates the equilibrium allocation under immediate trading.

In contrast, when SR investors expect the delayed-trading equilibrium, then the long-term contract cannot always replicate the allocation under delayed trading. The reason is that under delayed trading, SRs face different incentive constraints at date 2 from those faced by LRs under the long-term contract.

Under delayed-trading, the SR investor seeking to trade assets for cash at date 2 must trade assets at the same price in states ω_{20} and ω_{2L} to be induced to truthfully reveal these two states to LRs. However, in state $\omega_{2\rho}$ there is no trade between SR and LR, and therefore also no need to reveal that state truthfully to LR. In other words, no incentive constraint applies in this state of nature.

Under the long-term contract, however, LR promises transfers to SR as in (19). As we have argued above, transfer must satisfy incentive compatibility. Therefore, LR simply cannot replicate the allocation under the delayed-trading equilibrium with a suitable long-term contract. Given that the delayed-trading equilibrium allocation is not in the feasible set for the long-term contract it is not obvious a priori which allocation is superior. To be able to answer this question we must first characterize the optimal long-term contract (when SR requires a payoff at least as high as under the delayed-trading equilibrium).

Solving the long-term contracting problem is a somewhat tedious constrained optimization problem, as it involves two investment variables (α, M) and, effectively, seven state-contingent transfers to SR. This problem can be simplified to some extent, as the next proposition establishes, since the combination of all the incentive and feasibility constraints reduce the long-term contracting problem to the determination of optimal values for only: i) the amount $\alpha \in [0, 1]$ invested in the risky SR project, ii) the amount M of cash held by the fund, and iii) payments to SR in states $\omega_{1\rho}$, $\omega_{2\rho}$ and ω_{30}.

Proposition 8. *(Characterization of the long term contract)*

I. Without loss of generality, any feasible, incentive-compatible long-term contract between LR and SR takes the form:

	$C_2(\omega)$	$C_3(\omega)$
$\omega_{1\rho}$	$M+\alpha\rho$	$C_3\left(\omega_{1\rho}\right)$
$\omega_{2\rho}$	$C_2(\omega_{2\rho})$	$C_3(\omega_{2\rho})$
ω_{20}	M	$C_3\left(\omega_{30}\right)$
ω_{2L},ω_{30}	M	$C_3\left(\omega_{30}\right)$
$\omega_{2L},\omega_{3\rho}$	M	$C_3\left(\omega_{3\rho}\right)$

II. Suppose that δ is close to zero and that

$$\eta(1-\lambda)\rho+\varphi(0)\leq\varphi(\kappa), \tag{\textbf{A5}}$$

then the optimal long-term contract is such that $C_3(\omega_{1\rho})=C_3(\omega_{2\rho})=0$.

Given that SR discounts date 3 consumption by δ it seems inefficient to offer any date 3 consumption to SR. Still, we cannot rule out that $C_3\left(\omega\right)>0$ for either $\omega\in\{\omega_{1\rho},\omega_{2\rho},\omega_{30},\omega_{3\rho}\}$ since a date 3 transfer in one state may be required for LR to satisfy all incentive constraints he faces. That is, to be able to credibly disclose that the realized state is ω_{20}, for example, LR may have to promise a high transfer $C_3\left(\omega_{30}\right)$ at date 3. Nevertheless, intuition suggests that if δ is very small, λ sufficiently large, and the opportunity cost of holding cash for LR is bounded, then the optimal contract ought to specify $C_3(\omega_{1\rho})=C_3(\omega_{2\rho})=0$. This is indeed what Proposition 8-II establishes.

With this characterization of the optimal long-term contract we are able to numerically solve for the optimal contract and to compare LR payoffs under the contract to LR equilibrium payoffs under the delayed-trading equilibrium. We then show that for some parameter values, the long-term contract is dominated by the delayed-trading equilibrium outcome for high values of θ.

The economic logic behind this result is that when θ is high then the SR risky asset already matures most of the time at dates 1 or 2. The added value of additional liquidity offered by LR through a long-term contract is then not that high. In addition, when θ is high LR also faces high costs of meeting incentive constraints under the long-term contract. To be able to credibly claim that the risky asset did not yield a return ρ at either dates 1 or 2, LR must commit to wasteful date 3 payments $C_3(\omega_{3\rho})=C_3(\omega_{30})>0$ which SR does not value much. The deadweight cost of these distortions exceeds the benefit of extra liquidity insurance, which is why the delayed-trading equilibrium outcome is superior.

Example 2. In our example we keep θ as a free parameter and fix the other parameters to the following values:

$$\lambda = .7 \qquad \eta = .4 \qquad \rho = 1.25 \qquad k = .12 \qquad \delta = .1 \quad \text{and} \quad \varphi(x) = x^{\gamma} \quad \text{with} \quad \gamma = .19.$$

Note that all our assumptions are then met as long as $\theta \leq .8148$, which is the first value of θ for which the assumption $[\lambda + (1-\lambda)\,\eta\,[\theta + (1-\theta)\delta]] \leq 1$ is violated. Accordingly our plots below are restricted to the interval $\theta \in [0, .8148]$.

The payoffs of SR and LR under the long-term contract are given by respectively:

$$\begin{aligned}
\pi_x^* &= \lambda[M + \alpha\rho + \delta C_3(\omega_{1\rho})] \\
&+ (1-\lambda)[\theta\eta(C_2(\omega_{2\rho}) + \delta C_3(\omega_{2\rho})) + (1-\theta\eta)(M + \delta C_3(\omega_{30}))] \\
\Pi_x^* &= M + \varphi[\kappa + (1-\alpha) - M] + \lambda[\alpha\rho - (C_2(\omega_{1\rho}) + C_3(\omega_{1\rho}))] \\
&+ (1-\lambda)[\eta\alpha\rho - (M + C_3(\omega_{30}))].
\end{aligned}$$

As mentioned we set $\pi_x^* = \pi_d^*$ the SR payoff in the delayed-trading equilibrium. Numerical computations show that for the chosen parameter values the optimal long-term contract is such that $C_3(\omega_{1\rho}) = C_3(\omega_{2\rho}) = C_3(\omega_{30}) = 0$, and therefore that $C_2(\omega_{2\rho}) = M$.

Note that unlike in the previous example, a delayed-trading equilibrium always exists here. In the top panel of Figure 9 we graph the expected utility of the SR in the delayed-trading equilibrium as a function of θ whereas the bottom panel shows the expected utility of the LR in the delayed-trading equilibrium, Π_d^*, as well as the LR's expected payoff under the long-term contract, Π_x^*. For $\theta > \tilde{\theta}$ this payoff is less than what LR gets in the delayed-trading equilibrium. The bottom panel of Figure 10 shows that when θ increases, the amount of cash carried by the LR to fulfill his commitments under the long-term contract increases, making the contract less efficient, in sharp contrast with the total amount of cash $m_d^* + M_d^*$ carried by both the LRs and SRs in the delayed-trading equilibrium, shown in the top panel of the same figure. This increase in cash follows because the expected payoff of SR in the delayed-trading equilibrium increases with θ. Incentive constraints limit the difference in payments in states $\omega_{2\rho}$ and ω_{20}, and since payments at date 3 are very inefficient, the contract specifies higher payments at date 2, which requires carrying more cash. □

V. CONCLUSIONS

This paper is concerned with two questions. First, what determines the mix of inside and outside liquidity in equilibrium? Second, does the market provide an efficient mix of inside and outside liquidity? In addition we asked whether the provision of market liquidity can be Pareto improved upon by long term contracts between those with potential liquidity needs and those who are likely to supply it. A novel dimension of our model is the cross sectional supply of liquidity, which seems to be a core feature of modern financial markets, where different actors outside the regulated financial intermediary sectors that stand ready to absorb asset sales by distressed financial intermediaries.[26] The incentives of the different parties to carry liquidity in our model are driven by their different opportunity costs and different investment horizons. An important question we address is whether a competitive price mechanism would elicit the optimal cross-sectional cash reserve decisions by all the different actors.

A second element in the model that departs from the existing literature is the endogenous timing of asset sales and the deterioration of adverse selection problems over time. Financial intermediaries face the choice of raising liquidity early, or in anticipation of a crisis, before adverse selection problems set in, or in the midst of a crisis at more depressed prices. The benefit of delaying asset sales and attempting to ride through the crisis is that the intermediary may be able to entirely avoid any sale of assets at distressed prices should the crisis be short and mild. We show that when the adverse selection problem is not too severe there are multiple equilibria, an immediate-trading and a delayed-trading equilibrium. In the first equilibrium, intermediaries liquidate their positions in exchange for cash early in the liquidity crisis. In the second equilibrium, liquidation takes place late in the liquidity event and in the presence of adverse selection problems.

We show that surprisingly the latter equilibrium Pareto-dominates the former because it saves on cash reserves, which are costly to carry. However, the delayed-trading equilibrium does not exist when the adverse selection problem is severe enough. The reason is that in this case prices are so depressed as to make it profitable for the agents holding good assets to carry them to maturity even when it is very costly to do so. We show that if they were able

[26]For instance, a recent article in the Wall Street Journal of Friday May 9th 2008 by Lingling Wei and Jennifer S. Forsyht emphasized that the discounts on commercial real estate debt are less pronounced than in the previous real-estate collapse of the early 1990s. As the authors point out "[t]oday there are at least 55 active or planned commercial real-estate debt funds seeking to raise $33.8 billion, according to Real Estate Alert, a trade publication. And many have begun to do deals." In the recent period of distress that started in the summer of 2007, the role of sovereign funds has been notorious and major source of recapitalization for institutions such as Citi.

to do so, intermediaries would be better off committing ex-ante to liquidating their assets at these depressed prices in the distressed states. We also show, perhaps more surprisingly, that a monopoly supplier of liquidity may be able to improve welfare.

We argued in Bolton, Santos and Scheinkman (2009) that the role of the public sector as a provider of liquidity has to be understood in the context of a competitive provision of liquidity by the private sector very much like the one we propose in this paper. In particular the public provision of liquidity can act as a complement for private liquidity in situations where lemon's problems are so severe that the market would break down without any public price support. Clearly for the intervention to be effective the public liquidity provider needs to know whether the crisis is in date 1 or 2. An important remaining task is then to analyze the benefits of public policy in our model under the assumption that the public agency may be ignorant about the true state of nature in which it is intervening.

Another central theme in our analysis is the particular timing of the liquidity crisis that we propose. Liquidity crisis have, in our opinion, always real origins, small as they may be. In our framework the onset of the liquidity event starts with a real deterioration of the quality of the risky asset held by financial intermediaries. The assumption that adverse selection problems worsen during the liquidity crisis is perhaps the most novel of our modeling assumptions and seems reasonable in some crisis like the present one. Here our framework captures the fact that intermediaries were holding securities which had a degree of complexity that made for a costly assessment of the actual risk they were exposed to (see Gorton (2008) for an elaboration of this important point.) Once problems in the mortgage market were widely reported in early 2007 banks turned to an assessment of the actual risks buried in their books. As emphasized by Holmstrom (2008), the opacity of these securities was initially the source of liquidity. Once the crisis started though banks and intermediaries started the costly process of risk discovery in their books, which immediately led to an adverse selection problem. Financial institutions here faced a choice of whether to liquidate early or ride out the crisis in the hope that the asset may ultimately pay off. This trade-off is unrelated to the incentives that may force institutions to liquidate at particular times due to accounting and credit quality restrictions in the assets they can hold that have featured more prominently in the literature. Understanding the effect that these restrictions have on the portfolio decisions of the different intermediaries remains an important question to explore in future research.

Finally, in our model LRs are those with sufficient knowledge to be able to value and absorb the risky assets for sale by financial intermediaries. Only their capital and liquid reserves matter for equilibrium pricing to the extent that they are the only participants with the knowledge to perform an adequate valuation. Other, less knowledge intensive, capital will only step in at steeper discounts for which there may be no market. Our current work focuses precisely on understanding how different knowledge-capital gets "earmarked" to specific markets. What arises is a theory of market segmentation and contagion that, we believe may shed light on the behavior of financial markets in states of crisis.

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APPENDIX

Proof of Proposition 1. We proceed by constructing an immediate-trading equilibrium with prices P_{1i}^* and P_{2i}^*. We show that under those prices SRs prefer to sell the risky asset at date 1, rather than selling at date 2 or alternatively carrying the asset to date 2, taking the chance that the asset may payoff in $\omega_{2\rho}$, or to date 3 if in ω_{2L}, or swapping the risky asset for units of the long asset (trading at S_{1i}^*).

The first order condition of the LR is

$$\lambda + (1-\lambda)\frac{\eta\rho}{P_{1i}} \leq \varphi'(\kappa - M) . \tag{20}$$

First we establish that it is not possible to support an equilibrium with $M_i^* = 0$ and $m_i^* = 1$. Indeed if $m_i^* = 1$ it has to be the case that the price in state ω_{1L} is such that

$$P_{1i}^* \leq \frac{1-\lambda\rho}{1-\lambda}$$

but by assumption A3 this implies

$$\lambda + (1-\lambda)\frac{\eta\rho}{P_{1i}^*} > \varphi'(\kappa),$$

and thus $M_i^* > 0$ a contradiction.

Having ruled the no trade immediate-trading equilibrium we proceed next as follows. Start by solving the following equation in P_{1i}

$$\lambda + (1-\lambda)\frac{\eta\rho}{P_{1i}} = \varphi'(\kappa - P_{1i}),$$

and define

$$\underline{P} = \frac{1-\lambda\rho}{1-\lambda},$$

a positive number by assumption A2.

- Case 1: Assume first that $P_{1i} \geq \underline{P}$, then set $P_{1i}^* = M_i^* = P_{1i}$ and $m_i^* = 0$, which meets the first order condition of the SRs as can be checked by inspection of expression (2).
- Case 2: Assume next that $P_{1i} < \underline{P}$, then set $P_{1i}^* = \underline{P}$ and M_i^* to be the solution to

  $$\lambda + (1-\lambda)\frac{\eta\rho}{\underline{P}} \leq \varphi'(\kappa - M_i^*),$$

  which by assumption A3 is such that $M_i^* > 0$ and clearly it has to be such that $M_i^* < \underline{P}$. Because, given these prices, the SRs are indifferent on the level of cash carried set m_i^* so that

  $$P_{1i}^* = \frac{1-\lambda\rho}{1-\lambda} = \frac{M_i^*}{1-m_i^*}$$

As for prices at date 2 they have to be such that both the SRs and the LRs prefer to trade at ω_{1L}. For this set

$$P_{2i}^* < \delta\eta\rho. \tag{21}$$

Given this price the LR investors expect only lemons (assets with zero payoff) in the market at $t = 2$ and thus the demand is equal to zero $Q_2^* = 0$. As for the SRs notice that if they wait to liquidate at $t = 2$ they obtain

$$\theta\eta\rho + (1-\theta\eta)\,P_{2i}^* < \theta\eta\rho + \frac{1-\rho\,[\lambda + (1-\lambda)\theta\eta]}{1-\lambda} = P_{1i}^*,$$

and thus SRs set $q_2^* = 0$ and $q_1^* = 1 - m_i^* = Q_1^*$.

Notice as well that under these prices SRs prefer to liquidate rather than carry the asset to date 2 or 3. Indeed, given that we have established that SRs do not want to sell at $t = 2$, if instead they were to carry the asset to dates $t = 2$ (where the asset pays with probability $\theta\eta$) or take its chances at date $t = 3$ (in which case the asset is worth $\delta\eta\rho$ in ω_{2L}) it must be because:

$$P_{1i}^* < \theta\eta\rho + (1-\theta)\delta\eta\rho \tag{22}$$

Recall that

$$P_{1i}^* \geq \frac{1-\lambda\rho}{1-\lambda}. \tag{23}$$

Then substitution yields

$$\frac{1-\lambda\rho}{1-\lambda} < \theta\eta\rho + (1-\theta)\delta\eta\rho \tag{24}$$

which, once rearranged, yields

$$1 < \lambda\rho + (1-\lambda)\left[\theta + (1-\theta)\delta\right]\eta\rho, \tag{25}$$

a contradiction with A2. Finally, it is obvious that the SRs do not want to trade into the long asset. Indeed, assume they do. In this case the number of units of the long asset that they can acquire are $\eta\rho$, which are only worth $\delta\eta\rho$ to them which is clearly below P_{1i}^*.

As for the prices of the long asset at dates 1 and 2, S_{1i}^* and S_{2i}^* they follow immediately from arbitrage. □

Proof of Proposition 2. We first construct a *candidate* delayed-trading equilibrium and then establish the conditions on δ under which the candidate delayed-trading equilibrium is indeed an equilibrium.

First notice that since $\varphi'(\kappa) > 1$ in any delayed-trading equilibrium there must be cash-in-the-market pricing thus

$$M_d^* = P_{2d}^*(1-\theta\eta)(1-m_d^*)$$

Define P_{2d} to be the solution to

$$\lambda + (1-\lambda)\frac{(1-\theta)\eta\rho}{(1-\theta\eta)P_{2d}} = \varphi'(\kappa - (1-\theta\eta)P_{2d})$$

This equation always a unique solution which in addition satisfies

$$P_{2d} \in \left(0, \frac{\kappa}{1-\theta\eta}\right).$$

There are two cases to consider:

- Case 1: P_{2d} is such that

$$P_{2d} < \frac{1-\rho\left[\lambda + (1-\lambda)\theta\eta\right]}{(1-\lambda)(1-\theta\eta)} = \underline{P}. \tag{26}$$

  In this case set

$$P_{2d}^* = \underline{P},$$

  and set M_d^* to be the solution of

$$\lambda + (1-\lambda)\frac{(1-\theta)\eta\rho}{(1-\theta\eta)P_{2d}^*} = \varphi'(\kappa - M_d^*), \tag{27}$$

  which from the strict concavity of $\varphi(\cdot)$ is

$$M_d^* < (1-\theta\eta)P_{2d}^*.$$

By A3 $M_d^* > 0$. Indeed, define

$$\psi\left(\theta\right) = \lambda + \left(1-\lambda\right)^2 \left(\frac{(1-\theta)\eta\rho}{1-\rho\left[\lambda+(1-\lambda)\theta\eta\right]}\right),$$

which is the left hand side of the LR's first order condition as shown in (27). Notice that A3 can be simply written as $\psi(0) > \varphi'\left(\kappa\right)$. Straightforward algebra shows that

$$\psi_\theta \propto \rho\left[\lambda + (1-\lambda)\eta\right] - 1 > 0,$$

by A2.

Then choose m_d^* such that

$$P_{2d}^* = \frac{M_d^*}{\left(1-\theta\eta\right)\left(1-m_d^*\right)}$$

Notice that because $P_{2d}^* = \underline{P}$ the SRs are indifferent in the level of cash held. Both types of traders would prefer to wait to trade at date 2 provided that P_{1d}^* is in the interval

$$\left[\frac{\left(1-\theta\eta\right)P_{2d}^*}{1-\theta}, \theta\eta\rho + \left(1-\theta\eta\right)P_{2d}^*\right],$$

which is non empty if and only if

$$P_{2d}^* \leq \frac{\left(1-\theta\right)\eta\rho}{1-\theta\eta} = \overline{P}. \tag{28}$$

Clearly, given assumption A1, specifically the fact that $\varphi'\left(\kappa\right) > 1$, and equation (27), equation (28) is trivially met. Clearly, given assumption A1, specifically the fact that $\varphi'\left(\kappa\right) > 1$, and equation (27), equation (28) is trivially met.

Notice that P_{2d}^* is independent of δ and for $\delta \leq \overline{\delta}$, where

$$\overline{\delta} = \frac{P_{2d}^*}{\eta\rho}, \tag{29}$$

the SR (weakly) prefers to trade at date 2 for a price P_{2d}^* than carrying the asset to date 3.

- Case 2: $P_{2d} \geq \underline{P}$, then choose

$$P_{2d}^* = P_{2d} \qquad M_d^* = P_{2d}^*\left(1-\theta\eta\right) > 0 \qquad \text{and} \qquad m_d^* = 0.$$

Except for establishing inequality (28), the remainder of the proof follows as in the previous case. To establish that P_{2d}^* meets (28) it is enough to substitute $\underline{P}$ in (28) and appeal to assumption A2. □

Before proving Propositions 3, 4, and 5, it is useful to establish the following

Result. Assume A1-A4 hold. Then the immediate-trading equilibrium is such that $m_i^* \in (0,1)$.

Proof. By the SR's first order condition if the price at date 1 is given by

$$P_{1i}^* = \frac{1-\lambda\rho}{1-\lambda}$$

then the SR investor is indifferent about the cash position carried. Let M_i^* be the solution to

$$\lambda + \left(1-\lambda\right)^2 \frac{\eta\rho}{1-\lambda\rho} = \varphi'\left(\kappa - M_d^*\right),$$

which by assumption A3 exists and is unique. By assumption A4,

$$\frac{1-\lambda\rho}{1-\lambda} > \kappa > M_i^*.$$

Then set $m_i^* \in (0,1)$ so that

$$\frac{1-\lambda\rho}{1-\lambda} = \frac{M_i^*}{1-m_i^*}.$$

The construction now of the immediate-trading equilibrium follows as in the proof of Proposition 1. □

We prove Proposition 4 first. The proof of Proposition 3 following trivially after that.

Proof of Proposition 4. First notice that by the result above, the immediate-trading equilibrium is such that $m_i^* > 0$ (and, obviously, $M_i^* > 0$). Thus because the delayed-trading equilibrium specializes to the immediate-trading equilibrium when $\theta = 0$, it follows that there exists a neighborhood $(0, \tilde{\theta})$ such that $m_d^* > 0$. Then from the LR's and SR's first order conditions, combined with cash in the market pricing, M_d^* and m_d^* are fully determined by

$$\psi^{(M)} = \lambda + (1-\lambda) R_d^*(\theta) - \varphi'(\kappa - M_d^*) = 0 \tag{30}$$

$$\psi^{(m)} = (1-m_d^*)(1-\rho(\lambda+(1-)\theta\eta)) - (1-\lambda) M_d^* = 0 \tag{31}$$

Expression (30) is the LR's first order condition. Expression (31) is the SR's first order condition combined with the cash-in-the-market pricing equation. These two equations determine M_d^* and m_d^*. In the above expression

$$R_d^* = \frac{(1-\theta)\eta\rho}{(1-\theta\eta) P_{2d}^*},$$

where P_{2d}^* is given by $\underline{P}$ (see expression (26). Then basic algebra shows that

$$R_{d,\theta}^* = \frac{\partial R_d^*}{\partial \theta} \propto \rho[\lambda + (1-\lambda)\eta] - 1 > 0,$$

by assumption (A1).

$$\partial_x \psi = \begin{pmatrix} \psi_M^{(M)} & \psi_m^{(M)} \\ \psi_M^{(m)} & \psi_m^{(m)} \end{pmatrix} \quad \text{and} \quad \partial_\theta \psi = \begin{pmatrix} \psi_\theta^{(M)} \\ \psi_\theta^{(m)} \end{pmatrix}, \tag{32}$$

where

$$\begin{aligned} \psi_M^{(M)} &= \varphi''(\kappa - M_d^*) < 0 \\ \psi_m^{(M)} &= 0 \\ \psi_m^{(m)} &= -[1-\rho(\lambda+(1-\lambda)\theta\eta)] < 0 \\ \psi_M^{(m)} &= -(1-\lambda) \\ \psi_\theta^{(M)} &= (1-\lambda) R_{d,\theta}^* > 0 \\ \psi_\theta^{(m)} &= -(1-m_d^*)(1-\lambda)\eta\rho < 0, \end{aligned}$$

First,

$$|\partial_x \psi| = -[1-\rho(\lambda+(1-\lambda)\theta\eta)]\varphi''(\kappa - M_d^*) > 0$$

Second, by an application of the implicit function theorem

$$M_{d,\theta}^* = \frac{\partial M_d^*}{\partial \theta} = -\begin{bmatrix} 1 & 0 \end{bmatrix} (\partial_x \psi)^{-1} \partial_\theta \psi \quad \text{and} \quad m_{d,\theta}^* = \frac{\partial m_d^*}{\partial \theta} = -\begin{bmatrix} 0 & 1 \end{bmatrix} (\partial_x \psi)^{-1} \partial_\theta \psi.$$

After some algebra:

$$
\begin{aligned}
m^*_{d,\theta} &= -|\partial_x \psi|^{-1}\left[-\psi^{(m)}_M (1-\lambda) R^*_{d,\theta} - \psi^{(M)}_M (1-m)(1-\lambda)\eta\rho\right] \\
&= -|\partial_x \psi|^{-1}\left[(1-\lambda)^2 R^*_{d,\theta} - \varphi''(\kappa - M^*_d)(1-m^*_d)(1-\lambda)\eta\rho\right] \\
&< 0
\end{aligned}
\tag{33}
$$

and

$$
\begin{aligned}
M^*_{d,\theta} &= -|\partial_x \psi|\left[\psi^{(m)}_m \psi^{(M)}_\theta - \psi^{(M)}_m \psi^{(m)}_\theta\right] \\
&= -|\partial_x \psi|\psi^{(m)}_m \psi^{(M)}_\theta \\
&> 0
\end{aligned}
$$

Because m^*_d is strictly decreasing in θ if $m^*_d = 0$ for some $\widehat{\theta}$, then $m^*_d = 0$ for all $\theta \geq \widehat{\theta}$. For $\theta \geq \widehat{\theta}$ the LR's first order condition is given by

$$
\lambda + (1-\lambda)\frac{(1-\theta)\eta\rho}{M^*_d} = \varphi'(\kappa - M^*_d),
$$

where we have made use of the fact that cash-in-the-market pricing obtains and $m^*_d = 0$. Then a basic application of the implicit function theorem shows that $M^*_{d,\theta} < 0$ for $\theta > \widehat{\theta}$. As for the behavior of expected returns when $\theta > \widehat{\theta}$, notice that the LR's first order condition is written as

$$
\lambda + (1-\lambda) R^*_d = \varphi'(\kappa - M^*_d),
$$

and thus given that $M^*_{d,\theta} < 0$ for $\theta > \widehat{\theta}$, it follows that $R^*_{d,\theta} < 0$ for that range.

We turn now to the properties of the aggregate supply of the risky asset at date 2 in the delayed-trading equilibrium s^*_d. Using (33),

$$
\begin{aligned}
s^*_{d,\theta} &= |\partial_x \psi|^{-1}(1-\lambda)^2 R^*_{d,\theta}(1-\theta\eta) \\
&- |\partial_x \psi|^{-1}\varphi''(\kappa - m^*_d)(1-m^*_d)(1-\lambda)\eta\rho(1-\theta\eta) - \eta(1-m^*_d)
\end{aligned}
\tag{34}
$$

Tedious algebra shows that (34) is equal to

$$
(1-m^*_d)\eta\left[\frac{\rho - 1}{1-\rho(\lambda + (1-\lambda)\theta\eta)}\right],
$$

which is positive by assumption A2. This completes the proof of Proposition 3. □

Proof of Proposition 3. That $m^*_i > m^*_d$ follows immediately from the fact that $m^*_i = m^*_d(\theta = 0)$ and Proposition 3. Clearly for $\theta \leq \widehat{\theta}$ where $\widehat{\theta}$ was defined in the proof of Proposition 3, $M^*_i < M^*_d$. For $\theta > \widehat{\theta}$ M^*_d is a decreasing function of θ and thus, by continuity there exists a (unique) θ', possibly higher than $\overline{\theta}$, for which $M^*_d(\theta') = M^*_i$; for any $\theta < \theta'$, $M^*_i < M^*_d$. □

Proof of Proposition 5. Under assumption A4, $m^*_i > 0$ and thus $\pi^*_i = 1 \leq \pi^*_d$. As for the expected profits of the LR investors, firs notice that

$$
\frac{\partial \Pi^*_d}{\partial \theta} = \Pi^*_{d,\theta} = (1-\lambda) R^*_{d,\theta} M^*_d.
$$

Given that $\Pi_i^* = \Pi_d^* (\theta = 0)$ and the characterization of expected returns in Proposition 3 the result follows immediately. □

Proof of Proposition 6. The proof follows along similar lines as that of Proposition 2. First define P_2 to be the solution to[27]

$$\lambda + (1-\lambda)\frac{\eta\rho}{P_2} = \varphi'\left(\kappa - (1-\theta)P_2\right),$$

which always exists, is unique, and immediately implies that $P_2 < \eta\rho$ by assumption A1. Define next

$$\underline{P}^{fi} = \frac{1-\rho\left[\lambda + (1-\lambda)\theta\eta\right]}{(1-\theta)(1-\lambda)},$$

where fi stands for full information. We have, as before two cases.

- Case 1: $P_2 < \underline{P}^{fi}$; then set $P_2^* = \underline{P}^{fi}$ and set M^* to be the unique solution to

$$\lambda + (1-\lambda)\frac{\eta\rho}{P_2^*} = \varphi'\left(\kappa - M^*\right), \tag{35}$$

  the LR's first order condition, which now takes into account the fact that the acquired assets have expected payoff $\eta\rho$ as there is no asymmetric information (compare this with the conditional expected payoff under asymmetric information, expression (14)). Clearly $M^* < (1-\theta)P_2^*$. Then, set $m^* \in (0,1)$ to solve

$$P_2^* = \frac{M^*}{(1-\theta)(1-m^*)},$$

  which, obviously exists, is unique, and satisfies the SR's first order condition. SRs and LRs postpone trading to date 2 as long as

$$P_1^* \in \left[P_2^*, \theta\eta\rho + (1-\theta)P_2^*\right], \tag{36}$$

  which is non-empty by assumption A2. Finally we show that $M^* > 0$. Notice that the LR's first order condition (35) can be written as

$$\psi(\theta) = \varphi'\left(\kappa - M^*\right) \qquad \text{where} \qquad \psi(\theta) = \lambda + (1-\lambda)^2 \frac{(1-\theta)\,\eta\rho}{1-\rho\left[\lambda + (1-\lambda)\,\theta\eta\right]}$$

  and notice that again, assumption A3 can be written as $\psi(0) > \varphi'(\kappa)$. Differentiating, rearranging and by assumption A2 we obtain that $\psi_\theta > 0$, which proves that $M^* > 0$.

- Case 2: $P_2 \geq \underline{P}^{fi}$; then set $P_2^* = P_2$, $M^* = (1-\theta)P_2^*$ and $m^* = 0$, which by construction satisfy the LR's and SR's first order condition, respectively. Notice that given that $P_2^* \leq \eta\rho$, it immediately follows that the interval in (36) is non-empty. Finally, to support the equilibrium at date 2 it has to be the case that $\delta \leq \overline{\delta}$ where $\overline{\delta} = P_2^*/\eta\rho$, which concludes the proof. □

Proof of Proposition 8.

I. Given a choice M of cash carried by LR and αinvested in the SR risky project $0 \leq \alpha \leq 1$, the feasibility constraints on transfers to SR are given by:

$$\begin{aligned} C_1(\omega_{1\rho}) &\leq \alpha\rho + M, \\ C_1(\omega_{1\rho}) + C_3(\omega_{1\rho}) &\leq \alpha\rho + M + \varphi\left[\kappa + (1-\alpha) - M\right], \\ C_1(\omega_{1L}) + C_2(\omega_{20}) &\leq M, \end{aligned}$$

[27]Throughout we drop the subscript d to emphasize that now the only equilibrium is a delayed one.

$$
\begin{aligned}
C_1(\omega_{1L}) + C_2(\omega_{2L}) &\leq M, \\
C_1(\omega_{1L}) + C_2(\omega_{2\rho}) &\leq \alpha\rho + M, \\
C_1(\omega_{1L}) + C_2(\omega_{2\rho}) + C_3(\omega_{2\rho}) &\leq \alpha\rho + M + \varphi[\kappa + (1-\alpha) - M], \\
C_1(\omega_{1L}) + C_2(\omega_{20}) + C_3(\omega_{20}) &\leq M + \varphi[\kappa + (1-\alpha) - M], \\
C_1(\omega_{1L}) + C_2(\omega_{2L}) + C_3(\omega_{30}) &\leq M + \varphi[\kappa + (1-\alpha) - M].
\end{aligned}
$$

Consider next the following observations concerning equilibrium contracts:

(A) State $\omega_{1\rho}$ is observable and since there is no discounting between periods 1 and 2 we may assume without any loss of generality that $C_1(\omega_{1\rho}) = C_1(\omega_{1L}) = 0$.

(B) If $C_3(\omega_{1\rho}) > 0$ then $C_2(\omega_{1\rho}) = \alpha\rho + M$. For if $C_2(\omega_{1\rho}) < \alpha\rho + M$, both agents can be made better off by increasing $C_2(\omega_{1\rho})$ and decreasing $C_3(\omega_{1\rho})$.

(C) Incentive compatibility requires that $C_3(w_{30}) = C_3(w_{3\rho})$. Hence any feasible and incentive compatible payment in histories that follow from ω_{2L} is also feasible in histories that follow ω_{20}. Incentive compatibility also requires that

$$
C_2(\omega_{2L}) + C_3(\omega_{30}) = C_2(\omega_{20}) + C_3(\omega_{20}).
$$

Therefore any payment prescribed for the histories starting at ω_{2L} must also be prescribed for histories starting at ω_{20}:

$$
C_2(\omega_{2L}) = C_2(\omega_{20}),
$$

and

$$
C_3(\omega_{30}) = C_3(\omega_{20}).
$$

(D) If $C_3(\omega_{30}) > 0$ then $C_2(\omega_{2L}) = M$. For if $C_2(\omega_{2L}) < M$ SR can be made better off, while keeping LR indifferent, by increasing the payment at date 2 and decreasing by the same amount the payments in states ω_{30} and $\omega_{3\rho}$ at date 3. The same reason, together with observation 3, implies that if $C_3(\omega_{20}) > 0$ then $C_2(\omega_{20}) = M$. We can also use the same reasoning to show that if $C_3(\omega_{2\rho}) > 0$ then $C_2(\omega_{2\rho}) = M + \alpha\rho$.

(E) Since $(\lambda + (1-\lambda))\,\rho > 1$ and $\varphi'(\kappa) > 1$, if cash is carried by the LR it must be distributed in some state (at either dates 1 or 2). Hence either $C_2(\omega_{1\rho}) = M + \alpha\rho$, or $C_2(\omega_{2\rho}) = M + \alpha\rho$ or $C_2(\omega_{20}) = C_2(\omega_{2L}) = M$. Note furthermore from observation 4 and incentive compatibility that we must have $C_2(\omega_{2\rho}) > 0$ and $C_2(\omega_{20}) > 0$ unless SR consumption is zero in all histories starting at ω_{1L}. However, in the latter case, because of discounting and $\delta\phi'(k) < 1$ the ex-ante contract is dominated by autarky. Hence we may assume that $C_2(\omega_{2\rho}) > 0$ and $C_2(\omega_{20}) > 0$. In an analogous fashion we can establish that $C_2(\omega_{1\rho}) > 0$.

(F) Suppose, that $C_2(\omega_{1\rho}) \leq M + \alpha\rho - \mu$ for some $\mu > 0$, and let $\gamma > 0$ be small enough that $\gamma < \frac{\mu}{2}$ and $\gamma\frac{\lambda}{1-\lambda} < \min\{C_2(\omega_{2\rho}); C_2(\omega_{20})\}$. Consider the payment

$$
\hat{C}_2(\omega_{1\rho}) = C_2(\omega_{1\rho}) + \gamma
$$

and lower date 2 payments for all realizations following ω_{1L} by $\gamma\frac{\lambda}{1-\lambda}$. This new contract, leaves SR indifferent and economizes in cash. This cash can be invested in the LR project, which has a marginal product above one, and yield extra utility for LR at date 3. Hence the initial contract cannot be optimal.

(G) Suppose that $C_2(\omega_{2L}) < M$, then from observation 4, $C_3(\omega_{30}) = 0$. Hence $C_2(\omega_{20}) < M$ and $C_2(\omega_{2\rho}) < M + \alpha\rho$. Using the same logic as in observation 6 we may then show that this contract is not optimal.

(H) Incentive compatibility requires that

$$C_2(\omega_{2\rho}) + C_3(\omega_{2\rho}) = M + C_3(\omega_{30}).$$

Since $C_2(\omega_{2\rho}) = M$ satisfies the LR budget constraint, it follows that

$$C_3(\omega_{2\rho}) \leq C_3(\omega_{30}).$$

II. Under assumption (**A5**) LR's opportunity cost of holding cash, $\varphi'(\kappa + (1-\alpha) - M)$, is bounded. To see this, note first from lemma 7 that LR must pay SR at least M following the realization of state ω_{1L}. LR's date 0 expected payoff therefore cannot exceed:

$$\eta(1-\lambda)\alpha\rho + \varphi(\kappa + (1-\alpha) - M).$$

Since participation by LR requires that

$$\eta(1-\lambda)\alpha\rho + \varphi(\kappa + (1-\alpha) - M) \geq \varphi(\kappa),$$

we must have $\kappa + (1-\alpha) - M > 0$, by assumption **A5**. It follows that

$$\varphi'(\kappa + (1-\alpha) - M) < B, \quad \text{for some } B > 0.$$

Now, suppose by contradiction that $C_3(\omega_{1\rho}) \geq \epsilon > 0$. Then lowering $C_3(\omega_{1\rho})$ by ϵ and increasing M by $\delta\lambda\epsilon$ keeps SR indifferent, but makes LR strictly better off if $B\delta < 1$. Similarly, if $\min\{C_3(\omega_{2\rho}); C_3(\omega_{30})\} = C_3(\omega_{2\rho}) \geq \epsilon$, a decrease of $C_3(\omega_{30})$ and $C_3(\omega_{2\rho})$ by ϵ and an increase of M by $(1-\lambda)\delta\epsilon$, again keeps SR indifferent but makes LR better off (provided that $B\delta < 1$). □

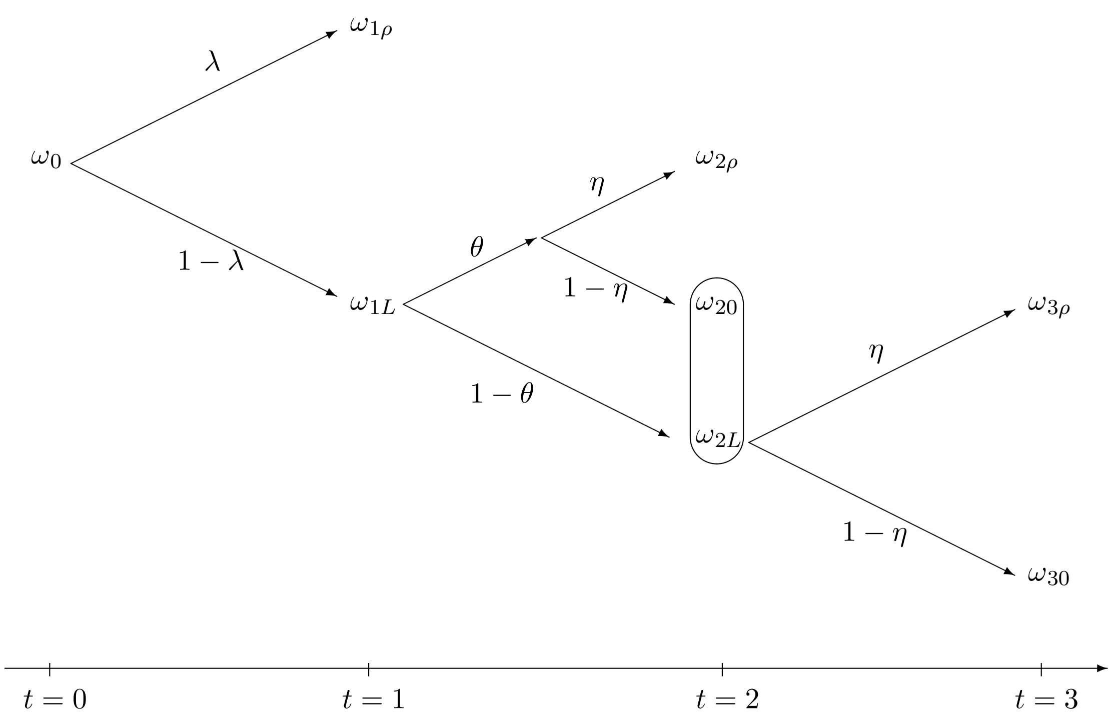

Figure 1. The risky asset. There are four dates. Investment in the risky asset occurs at date0. At date 1 there is an *aggregate* shock. Specifically there are two possible aggregate states, $\omega_{1\rho}$, which occurs with probability λ, and ω_{1L}, which occurs with probability $1-\lambda$. In $\omega_{1\rho}$ the risky asset matures at date 1 and yields a cash dividend ρ. ω_{1L} is the state when the long duration asset matures later than date 1 (either at dates 2 or 3). At date 2, there are three *idiosyncratic* states of nature, $\omega_{2\rho}$, which occurs with probability $\theta\eta$, ω_{20}, which occurs with probability $\theta\,(1-\eta)$,and ω_{2L}, which occurs with probability $1-\theta$. $\omega_{2\rho}$ is the state when the asset matures at date 2 and yields dividend ρ. Thus the probabilities also denote the mass of SRs which are in the corresponding states of nature. ω_{20} is the state when the asset matures at date 2 but yields no dividends. In ω_{2L} the risky asset is known to mature at date 3. Finally at date 3 there are again two states, $\omega_{3\rho}$, which occurs with probability η, and ω_{30}, which occurs with probability $1-\eta$. In state $\omega_{3\rho}$ the asset matures at date 3 and yields dividend ρ and in state ω_{30} the asset matures at date 3 and yields zero dividends. The information set of the LRs at date 2 is denoted by the oval encompassing ω_{20} and ω_{2L}, which generates the adverse selection problem that is key in the analysis. At date 1, agents are symmetrically informed.

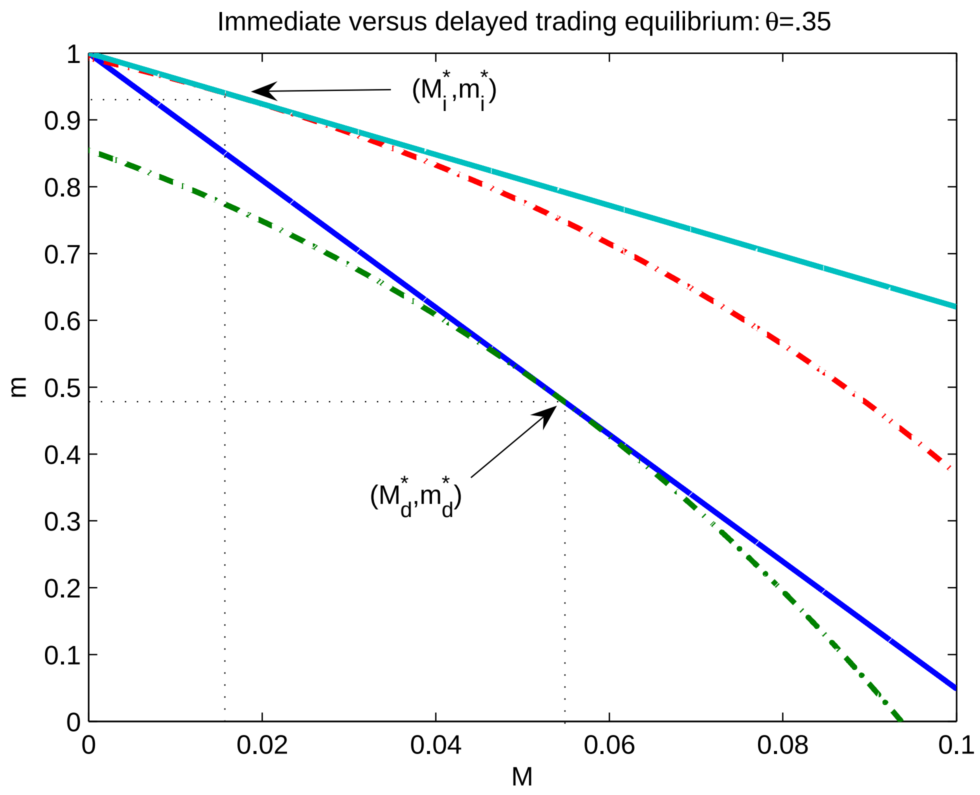

Figure 2. Immediate and delayed-trading equilibria in Example 1 for the case $\theta = .35$. The graph represents cash holdings, with the cash holdings of the LRs, M, in the x-axis and the cash holdings of the SRs, m in the y-axis. The dashed curves represent isoprofit lines for the LR and the straight continuous lines represent the SR's isoprofit lines, for both when the exchange occurs in state ω_{1L} and in date 2. The isoprofit lines for the SR correspond to its reservation profits $\pi_i^* = \pi_d^* = 1$. The immediate and delayed-trading equilibrium cash holdings are marked (M_i^*, m_i^*) and (M_d^*, m_d^*), respectively.

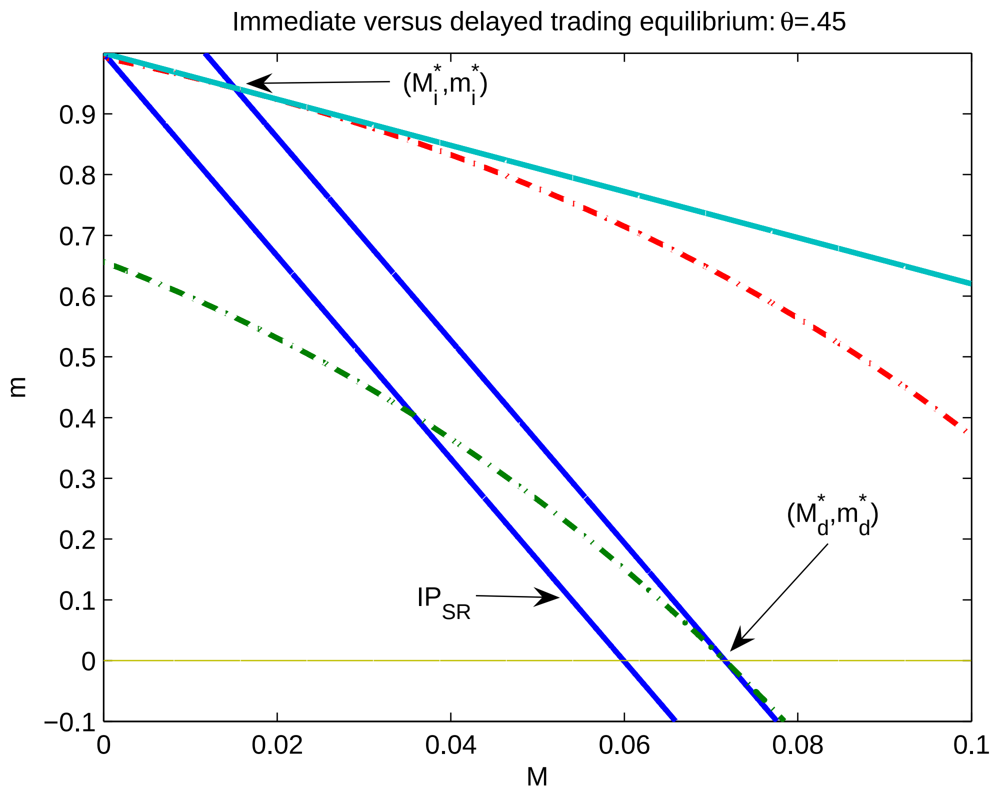

Figure 3. Immediate and delayed-trading equilibria in Example 1 when $\theta = .45$. The graph represents cash holdings, with the cash holdings of the LRs, M, in the x-axis and the cash holdings of the SRs, m in the y-axis. The dashed curves represent isoprofit lines for the LR and the straight continuous lines represent the SR's isoprofit lines, for both when the exchange occurs in state ω_{1L} and in date 2. As opposed to the case in Figure 2 now the delayed-treading equilibrium, marked (M_d^*, m_d^*), has the SRs commanding strictly positive profits, $\pi_d^* > 1$. The line marked IP_{SR} denotes the SR's reservation isoprofit line in states $(\omega_{20}, \omega_{2L})$.

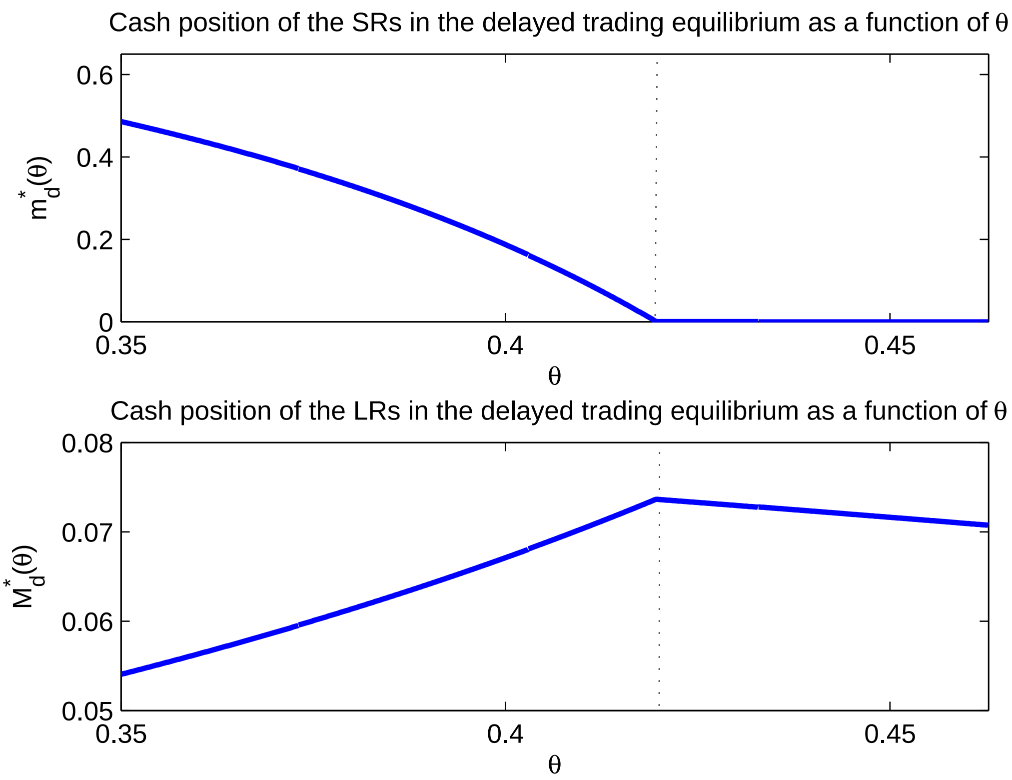

Figure 4. Cash holdings as a function of θ for Example 1. Panel A represents the SR's cash holdings in the delayed-trading equilibrium, m_d^* as a function of θ and Panel B does the same for the LR, M_d^*. The dashed vertical line, which sits at $\widehat{\theta} = .4196$ delimits the set of θs for which $m_d^* > 0$ and the one for which $m_d^* = 0$.

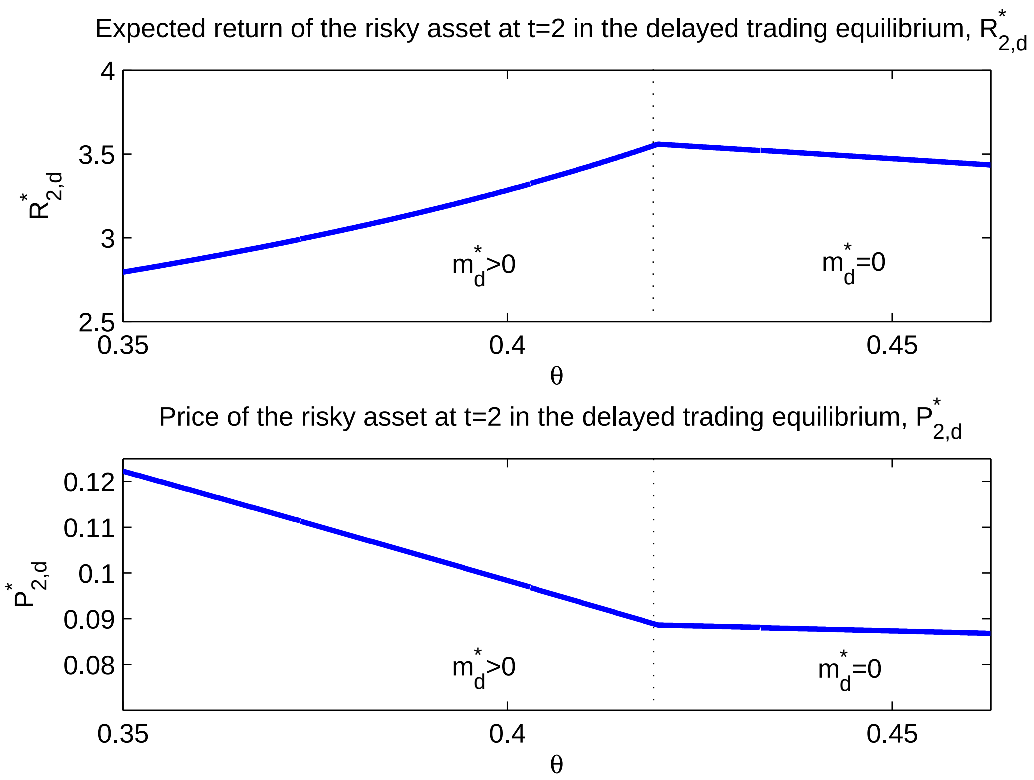

Figure 5. The top panel shows the expected return of the risky asset, R^*_{2d}, as a function of θ at date 2 in the delayed-trading equilibrium. The bottom panel shows the price of the risky asset at $t = 2$, P^*_{2d}, as a function of θ at date 2 in the delayed-trading equilibrium. The dashed vertical line corresponds to $\widehat{\theta} = .4196$. Both panels correspond to the case considered in Example 1.

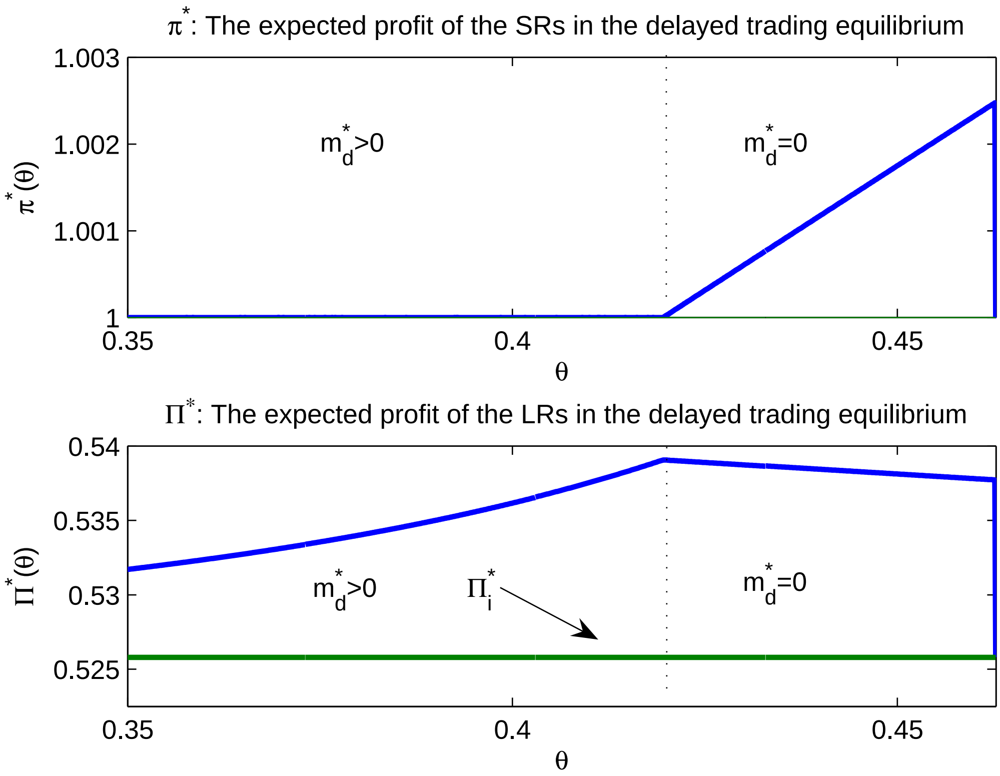

Figure 6. Expected profits for the SR, π^*, (top panel) and the LR (bottom panel), Π^*, as a function of θ in the delayed-trading equilibrium for the case considered in Example 1. The dashed vertical line corresponds to $\widehat{\theta} = .4196$.

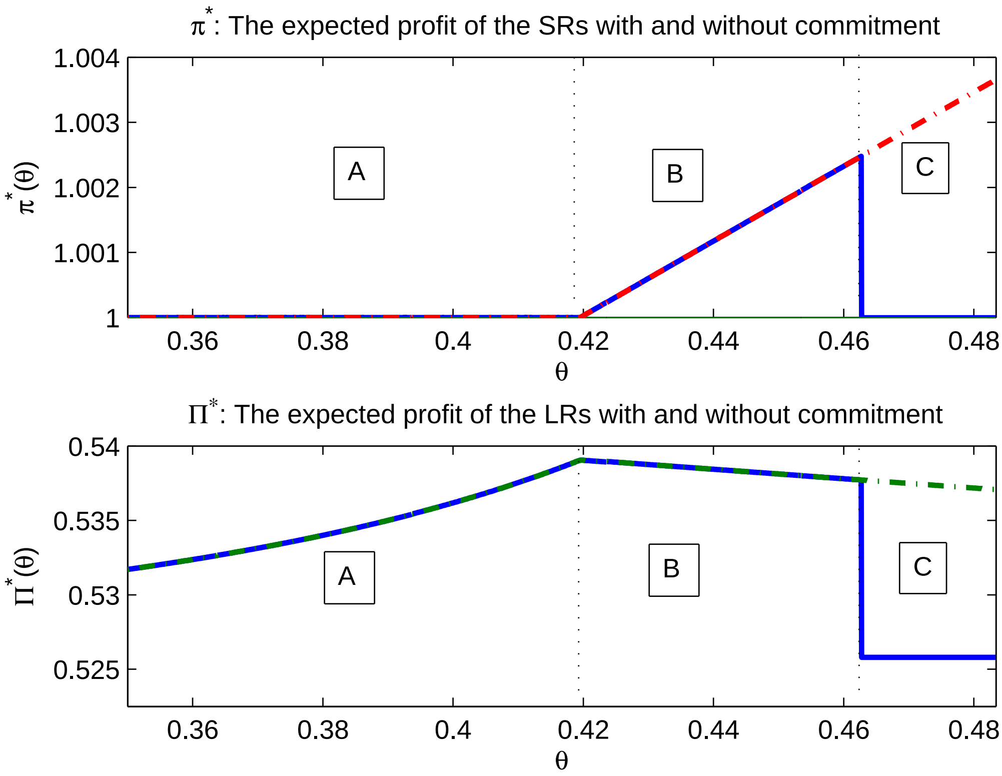

Figure 7. Expected profits for the SR, π^*, (top panel) and the LR (bottom panel), Π^*, as a function of θ for the case considered in Example 1. The first dashed vertical line corresponds to $\widehat{\theta} = .4196$. The continuous line plots the expected profits when the Pareto superior equilibrium is chosen. In regions A and B, the delayed-trading equilibrium exists and it is the Pareto superior equilibrium. In region C, which corresponds to $\theta \in (.4628, .4834]$, the delayed-trading equilibrium no longer exists as $P_d^* < \delta\eta\rho$ and the sole equilibrium is the immediate-trading equilibrium. The dashed line corresponds to the expected profits when the SRs can commit to liquidate assets in state ω_{2L}.

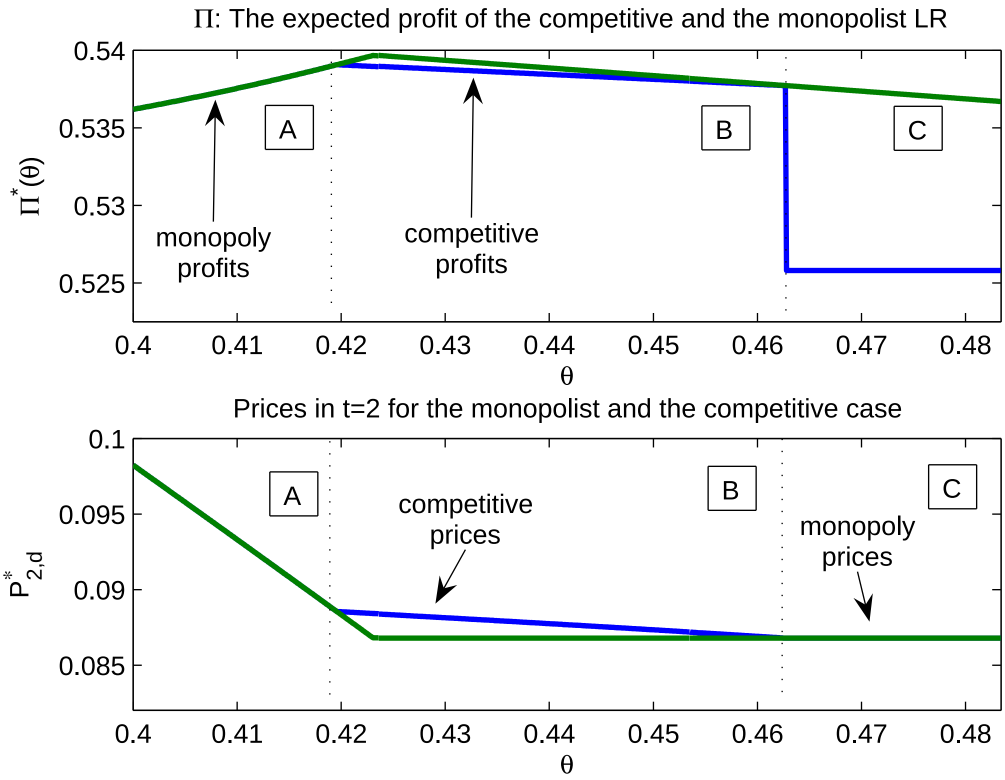

Figure 8. Top panel: Expected profits of the monopolist (the thick line) and the competitive LR (the thin line) as a function of θ. Bottom panel: Prices at date 2, P^*_{2d}, in the monopolist (the thick line) and the competitive (the thin line) LR case. Both panels correspond to the case considered in Example 1.

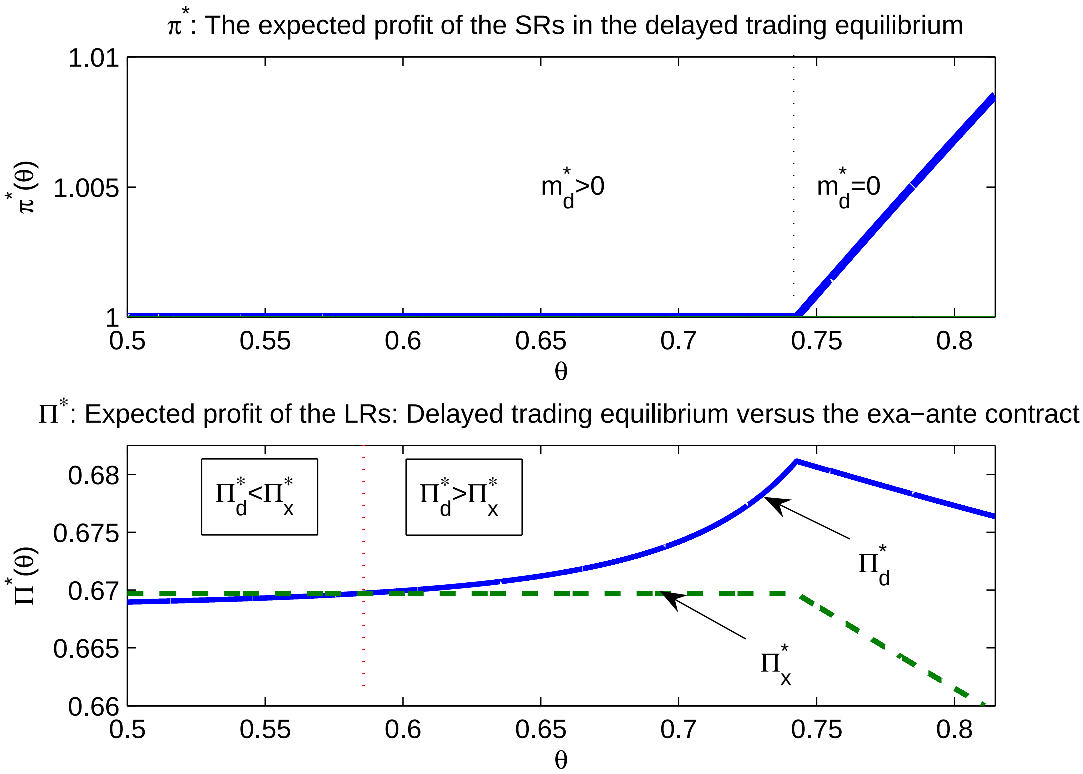

Figure 9. Top panel: Expected profits for the SR in the ex-ante contract when the outside value is the expected profit associated with the delayed-trading equilibrium. Bottom panel: Expected profits of the LR in the ex-ante contract, Π_x^*, when the outside value of the SR is the expected profit associated with the delayed-trading equilibrium (π_d^*). Also included are the expected profit of the LR in the delayed-trading equilibrium, Π_d^*, and in the immediate-trading equilibrium, Π_i^*. Both panels correspond to the case considered in Example 2.

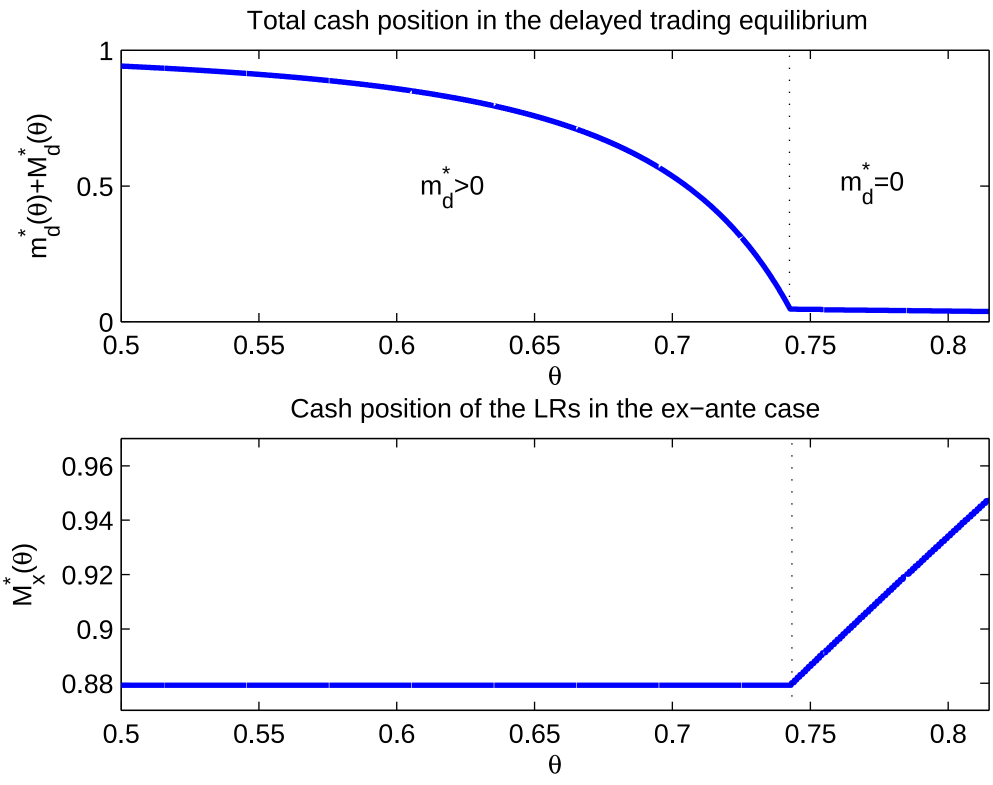

Figure 10. Top panel: Total cash position, $m_d^* + M_d^*$ in the delayed-trading equilibrium as a function of θ. Bottom panel: Cash position of the LR in the ex-ante contract case as a function of θ when the outside opportunity of the SR is the expected profit in the delayed-trading equilibrium. Both panels correspond to the case considered in Example 2.