Corporate Finance and the Monetary Transmission Mechanism

Patrick Bolton
Columbia University

Xavier Freixas
Universitat Pompeu Fabra

We analyze the transmission effects of monetary policy in a general equilibrium model of the financial sector, with bank lending and securities markets. Bank lending is constrained by capital adequacy requirements, and asymmetric information adds a cost to outside bank equity capital. In our model, monetary policy does not affect bank lending through changes in bank liquidity; rather, it operates through changes in the spread of bank loans over corporate bonds, which induce changes in the aggregate composition of financing by firms, and in banks' equity-capital base. The model produces multiple equilibria, one of which displays all the features of a "credit crunch."

This article is concerned with the monetary transmission mechanism through the financial sector, in particular the banking sector and securities markets. Specifically, it analyzes the effects of open-market operations on bank lending and securities issues in a real economy. By building on recent advances in the microeconomics of banking, it provides some underpinnings for the "credit view" of monetary policy, which, in its simplest form, relies on an exogenously assumed limited substitutability between bank loans and bonds.

The macroeconomics literature distinguishes between the "money view" and "credit view" of monetary policy transmission [Bernanke and Blinder (1988, 1992)]. The money view takes bonds and loans to be perfect substitutes and only allows for the effects of monetary policy on aggregate investment, consumption, and savings through changes in interest rates. The credit view allows for an additional effect on investment and economic activity operating through bank credit supply

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controlled through changes in bank reserve requirements. Two critical assumptions underlying the credit view are that firms cannot easily substitute bank loans for bonds and that banks cannot substitute reservable liabilities (deposits) and nonreservable liabilities (bank bond issues).

Our contribution in this article is to point to another channel operating through bank equity capital. This channel works not only through the equilibrium composition of funding between direct and intermediated finance but also through banks' incentives to raise equity capital. In our model, banks are capital constrained in equilibrium because equity capital is more costly than other sources of funding such as deposits or bonds. Banks are economizing on their cost of funding by holding no more than the required amount of equity capital. In addition, banks limit the size of their equity issues to economize their cost of capital.

The reason equity capital has a higher cost than other sources of funding in our model is due to asymmetric information and information dilution costs as in Myers and Majluf (1984). That is, when a bank decides to raise additional equity through a seasoned offer, the market tends to undervalue the issue for the better banks. But because it is the better banks that drive the decision whether to raise equity, the overall effect on all banks' equity issues (whether good or bad) is to reduce the amount of equity raised relative to the full information optimum. Thus, because of information asymmetries about the true value of bank assets, there is an endogenous cost of equity and, by extension, an endogenous cost of bank lending. Hence, banks' equity base (which includes retained earnings) is a key variable in determining the total amount of bank credit.

An important consequence of this endogenous cost of equity is that multiple equilibria may exist. In one equilibrium, the endogenous cost of capital (generated by self-fulfilling market beliefs) is high, whereas in the other it is low. The former has all the main features of a "credit crunch"[1] (i) bank lending is limited by a lower endogenous stock of bank capital; (ii) there is a correspondingly lower volume of bank credit; and, (iii) equilibrium bank spreads are high.[2] By contrast, the other equilibrium has a high stock of bank capital, a high volume of credit, and lower equilibrium bank spreads.

Another way of thinking about this multiplicity of equilibria is in terms of hysteresis in market beliefs about underlying bank values. Starting from a low level of equilibrium bank capital, a single bank's decision to issue equity is likely to be interpreted by the market as a bad signal about the issuing bank's value (resulting in a reduction in the market price of

[1] See, for example, Berger and Udell (1998, p. 655), for a review of the empirical literature on the credit crunch.

[2] "Bank spreads" here refer to the difference between the expected return on a bank loan and the yield on government bonds. These spreads are difficult to measure accurately, as banks do not systematically disclose the precise lending terms on the loans they extend.

bank equity), thus inhibiting new equity issues. Vice versa, in a situation where most banks are expanding their capital base, a failure to expand will be interpreted as a negative signal. This is the source of multiplicity of equilibria in our model.

This multiplicity of equilibria can give rise to potentially large monetary policy transmission effects if a change in monetary stance induces a switch from one equilibrium to another. One possible scenario, for example, is for a tightening in monetary policy to push the economy from an equilibrium with a high equity-capital base and high levels of bank lending into a credit-crunch equilibrium, with a low equity-capital base and low levels of bank lending. How can this happen? One effect of monetary tightening in our model is to reduce equilibrium bank spreads. Once those spreads hit a critically low level, it is no longer worth for banks to maintain a high equity-capital base. In other words, the high equity capital and high lending equilibrium are no longer sustainable, and only the credit-crunch equilibrium can arise for sufficiently low bank spreads. If there is hysteresis, as described earlier, then once the economy has settled in a credit-crunch equilibrium, a major change in interest rates may be required to pull it out of this low-lending equilibrium. That is, the economy may be stuck in the inefficient equilibrium as long as market beliefs are unchanged.

An important effect of monetary policy that our analysis highlights is related to the financial composition of the corporate sector between securities issues and bank credit. Recent empirical work suggests that one effect of monetary policy is to change firms' financing decisions, with corporations substituting bank lending for commercial paper issues. A common explanation for these changes is that when bank cash reserves are tight, firms turn to the securities market to raise funds [Gertler and Gilchrist (1994), Kashyap and Stein (1994)]. Our article, however, identifies a different and more-complex transmission mechanism, which operates through bank equity-capital constraints as opposed to bank reserves.

The corporate finance side of our model builds on the more-detailed analysis of our related article [Bolton and Freixas (2000)]. What distinguishes bank debt from corporate bond financing in our model is the flexibility of the two modes of financing: bank debt is easier to restructure, but because bank capital (and therefore bank loans) is in short supply, there is an endogenous cost of flexibility. In other words, what makes bank loans expensive is the existence of a capital requirement regulation together with a dilution cost for outside equity. This, along with the direct costs of running banks, is the source of the positive equilibrium spread between bank loans and bonds in our model.

Firms with higher default risk are willing to pay this intermediation cost because they have a greater benefit of flexibility. This is where monetary policy affects the composition of financing: by raising real interest rates and thus lowering the equilibrium bank spread, it induces some marginal firms to switch from bond financing to bank borrowing. This increase in the demand for bank loans is, however, offset by a decrease in bank lending, at the other end of the risk spectrum, to the riskiest firms, so that aggregate bank lending remains unaffected as long as bank equity capital remains unchanged.

In our model, the impact of monetary policy on aggregate investment is thus more complex than that in other models of the bank lending channel. A tightening of monetary policy not only results in the usual increase in interest rates but also gives rise to a decrease in spreads on bank loans. These two effects in turn induce a reduction in corporate securities issues and an improvement in the risk composition of bank loans. The latter effect is because the riskier firms are "priced out" of the bank credit market, as, with lower spreads, bank loans are cheaper relative to bonds than before. As a result, some safer firms switch away from bonds to bank loans, thus "crowding out" the riskier firms. The second major effect of a monetary tightening, as we show in Proposition 4, is on bank equity-capital issues (and thus on overall bank lending) that are reduced because of the lower profitability of banks.

Most of the predictions of our model are consistent with empirical findings in the banking literature. In particular, using a uniquely detailed data set from the Bank of Italy, Gambacorta and Mistrulli (2004) found that cross-sectional differences in bank lending responses to monetary policy relate to differences in equity-capital constraints, thus providing support for a bank capital channel. Also, Berger and Udell (1992) have shown that the spread of commercial bank loans over Treasury rates (either nominal or real) is a decreasing function of the Treasury rates, as our analysis predicts. Their study suggested many possible explanations for this finding. Interestingly, although this is not our objective here, our article suggests a new reason for the observed commercial loan rate stickiness.

Several related recent articles also deal with monetary policy transmission through a bank lending channel. The four most closely related ones are those of Gorton and Winton (1999), Van den Heuvel (1999), Schneider (1998), and Estrella (2001). The first two articles focused on banks' capital adequacy constraints and the macroeconomic effects of changes in bank lending induced by changes in banks' equity base. Bank capital is costly in Gorton and Winton's study because bank equity is risky and requires both a risk and a liquidity premium. Capital adequacy constraints impose a cost on banks whenever investors' optimal portfolio is less heavily weighted toward bank equity than is required by regulations. This is most likely to occur in recessions. Accordingly, the amplification effects of monetary policy are greatest at the onset of a recession, when higher interest rates affect aggregate investment both directly and indirectly through a reduction in bank lending capacity.

In Van den Heuvel's study, it is assumed that there is an imperfect market for banks' equity which implies that banks cannot readily raise new equity. Still, they can increase their capital stock through retained earnings. The amplification effects of monetary policy then work through their effects on retained earnings. Although Van den Heuvel's microeconomic model of banking is more rudimentary than that of Gorton and Winton, his dynamic macroeconomic analysis goes considerably further, exploring lagged effects of changes in interest rates. In the same vein, Schneider provided an extensive dynamic macroeconomic analysis, which relies on a combination of liquidity and bank capital effects. Finally, Estrella provided a similar dynamic analysis focusing on the cyclical effect of value-at-risk regulation.

Neither of these models, however, allows for other sources of corporate financing besides bank lending and therefore cannot explore composition effects of monetary policy.[3] Nor do these models allow for multiple equilibria and the possibility of what we describe as a credit-crunch equilibrium, where bank lending is constrained by investors' excessive pessimism about banks' underlying asset values.

Romer and Romer (1990) observed that if banks are able to obtain funds by tapping financial markets, monetary policy would affect banks only through changes in interest rates. There would be no specific bank lending channel. In response to Romer and Romer (1990), Lucas and McDonald (1992) and Stein (1998) have argued that nondeposit liabilities are imperfect substitutes for deposit liabilities (which are subject to reserve requirements) when banks have private information about their net worth. They show that when certificates of deposit (CDs) are risky, banks are unable to substitute perfectly CDs for deposits, so that bank lending may be partially controlled by monetary authorities through changes in reserve requirements. Our model emphasizes instead the imperfect substitutability of equity capital with other sources of funds and highlights that there is a bank lending channel operating through the bank equity-capital market even when banks have perfect access to the CD or bond market.

Because our model allows for the coexistence of bank lending and securities markets, it is also related to a third set of articles by Holmstrom and Tirole (1997), Repullo and Suarez (2000), and Bolton and Freixas (2000), which all characterize equilibria where bank lending and direct financing through securities issues are both present.

[3] From a conceptual point of view, a common weakness of models that allow for only bank lending without any other direct source of funding for firms is that banks in these models look essentially like nonfinancial firms, the only difference being that they are subject to capital adequacy requirements.

These three articles take intermediation costs to be exogenous and do not analyze the effects of monetary policy on bank equity capital. A major difference with these articles is thus that our model allows for an endogenous level of bank capital. This is obtained at the price of some simplifying assumptions. Still, our model can be straightforwardly extended to allow for different initial levels of bank capital across banks.

This article is organized as follows: Section 1 is devoted to the description of the model, whereas Section 2 deals with bank lending and asset–liability structure. Section 3 characterizes the general equilibrium when banks' equity is fixed, and Section 4 endogenizes the supply of bank equity and shows how a credit-crunch equilibrium may obtain. Section 5 considers comparative statics and the effects of monetary policy. Finally, Section 6 offers some concluding comments. The proofs of most results are given in Appendix A1.

1. The Model

We consider a real economy with a single consumption or production good, which can be thought of as wheat. Monetary policy in this economy operates through open-market operations (government bond issues G) that affect interest rates on government bills. For simplicity, the nominal side of this economy is not explicitly modeled.

1.1 Firms' investment projects and financial options

Each firm has one project requiring an investment outlay $I > 1$ at date $t = 0$. The project yields a return of $V > I$ when it succeeds. When it fails, the project can generate a value v, as long as the firm is restructured. If the firm is unable to restructure its debts following failure, the value of the project is zero.

Firms' owner-managers invest $W < I$ in the firm and must raise $I - W = 1$ from outside. Firms differ in the observable probabilities p of success, where we assume that p is uniformly distributed on the interval [0,1].

Firms can choose to finance their project either by issuing bonds or by means of a bank loan. To keep the corporate financing side of the model to its bare essentials, we do not allow firms to issue equity or to combine bonds and bank debt.[4] The main distinguishing features of these two instruments are the following:

1. Bond financing: A bond issue specifies a time $t = 1$ repayment to bond holders of $R(p)$. If the firm is unable to meet this repayment,

[4] However, these options are examined in Bolton and Freixas (2000) for firms with a richer cash-flow structure.

the firm is declared bankrupt and is liquidated. Restructuring of debt is not possible because of the wide dispersion of ownership of corporate bonds [Bolton and Scharfstein (1996)].

2. Bank debt: A bank loan specifies a repayment $\widehat{R}(p)$. If the firm defaults following failure of the project, the bank is able to restructure the firm's debts and obtain a restructuring value of at most v.[5]

As we will show, in equilibrium, firms will be segmented by risk classes in their choice of funding, with all firms with $p \in (p^*,1)$ choosing bond financing and all firms with $p \in [0, p^*]$ preferring a bank loan.[6] Bond financing is preferred by low-risk firms (with a high p) because these firms are less likely to fail at date $t = 1$ and therefore have less of a need for the costly debt restructuring services provided by banks. These services are costly because banks themselves need to raise funds to be able to lend to firms.[7]

Having described the demand side for capital by firms, we now turn to a description of the supply side.

1.2 Households

There is a continuum of households in our economy represented by the unit interval [0,1], with utility function

$$U(C_1,C_2) = \log(1 + C_1) + \log(1 + C_2)$$

Each household is endowed with one unit of good. For a given fixed gross real interest rate, R_G aggregate investment and savings in this economy are determined by households' optimal savings decisions. That is, households determine their optimal savings $s = (1 - C_1)$ by maximizing their utility function $U(C_1,C_2)$ subject to the budget constraint

$$C_1 + \frac{C_2}{R_G} = 1.$$

It is straightforward to check that households' optimal savings function is then given by

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[5] We assume for convenience that the bank appropriates the entire restructuring value. In other words, the bank is an informational monopoly able to extract the entire continuation value as in Rajan (1992). Of course, banks' ability to extract this value will be anticipated by borrowers and priced into the exante loan terms.

[6] We restrict attention to parameter values such that the riskiest firms do not issue junk bonds in equilibrium. As we point out in Bolton and Freixas (2000), this is an option that may be attractive to the riskiest firms for some constellation of parameters.

[7] In Bolton and Freixas (2000), these fund-raising costs were specified exogenously. By contrast, here these costs are endogenized.

$$s^* = 1 - C_1 = 1 - \frac{1}{2R_G}.$$

Households can invest their savings in bank deposit accounts, bonds issued by firms, government bonds, or bank equity. We denote the supply of deposits by $D(R_D)$ [with $0 \leq D(R_D) \leq 1 - (1/2R_G)$], where R_D is the remuneration of deposits. We allow for perfect substitutability between deposit accounts and financial assets. For positive amounts of both deposits and bonds, this will lead to the no-arbitrage condition[8]

$$R_G = R_D.$$

Given this no-arbitrage condition, henceforth we refer to R_G as both the interest rate set on government bills and the remuneration on deposits. We will assume that bond returns and bank returns are independently distributed and that customers hold perfectly diversified portfolios. This simplifies the analysis by allowing us to model investments in an aggregate bond and bank equity portfolio as providing an essentially safe return. No arbitrage then also requires that the expected return on bonds $pR(p)$ is such that

$$pR(p) = R_G. \tag{1}$$

1.3 Banks

Banks, as firms, are run by self-interested managers, who have invested their personal wealth w in the bank. They can operate on a small scale by leveraging only their own capital w with (insured) deposits D, so as to fund a total amount of loans $w + D$. Their lending capacity will then be constrained by capital adequacy requirements:[9]

$$\frac{w}{w + D} \geq \kappa > 0.$$

Alternatively, banks can scale up their operations by raising outside equity capital E to be added to their own investment w. In that case, their lending capacity expands from w/κ to $(w + E)/\kappa$. However, when they raise outside equity, they may face informational dilution costs. Outside equity investors, having less information about the profitability

[8] Because in our model, banks do not default, this condition is the same even in the absence of deposit insurance. If banks could default with positive probability, we would have to introduce a demand for the payment services associated with deposits, so as to make bank deposits attractive while preserving their option to invest in Treasury bills as an investment vehicle that does not generate losses.

[9] The BIS capital adequacy rules in our highly simplified model are that $\kappa = 0.08$ for standard unsecured loans.

of bank loans will tend to misprice banks' equity issues. In particular, they will underprice equity issues of the most profitable banks. We assume that banks choose an amount of equity to issue within the interval $[0,\bar{E}]$, where $\bar{E} < \infty$.[10]

Because banks are perfectly diversified, they have a zero probability of default, a simplifying assumption that allows us to sidestep the complexities associated with banks' credit risk.

Banks face unit operating costs $c > 0$. These costs are best interpreted as operating costs banks must incur to attract depositors and potential borrowers and may also be thought of as screening costs the bank incurs on each loan to determine the probability of success p. Thus, these costs are incurred, whether the bank ends up extending loans to firms or not.

To model bank dilution costs of equity capital, we take as a basic premise that bank managers differ in their ability to profitably run their bank. Specifically, we assume that bank managers may be either *good* or *bad*. Good bank managers (or H-banks in our notation) are able to squeeze out a return v from restructuring a defaulting firm, whereas bad bank managers (or L-banks) can only obtain a return βv, $(1 > \beta > 0)$. There are obviously other perhaps more plausible ways of modeling the difference between good and bad banks, but the appeal of our formulation is its simplicity. Banks' outside investors do not know the bank's type; all they know is that any bank they face is an L-bank with probability μ and an H-bank with probability $1 - \mu$. This informational asymmetry about bank type gives rise to mispricing of each bank type's equity. It is the main source of costs of bank capital in our model.

A bank manager seeks to maximize his/her wealth and cares about both bank profits and the bank's share price. The reason a bank manager cares about share price is that he/she may need to sell his/her stake in the bank before the returns of the bank's loans are fully realized and known.

We model these objectives by assuming that bank managers may need to liquidate their stake in the bank at date $t = 1$ with probability $\lambda \in (0,1)$. Denoting by q the share price of the bank and by Π_2 the bank's accumulated profit up to period $t = 2$, the bank manager's objective is then to maximize[11]

$$\max[q, \lambda q + (1 - \lambda)\Pi_2].$$

[10] We justify the existence of an upper bound on E by the following potential incentive problem between bank mangers and bank shareholders: if the bank raises an amount superior to $\bar{E}$, bank managers may have an incentive to abscond with the money or use it to increase their private benefits. Indeed, the larger is E the greater the private benefits relative to the cost in terms of the loss of reputation.

[11] Note that this objective function is similar to that considered by Myers and Majluf (1984), but it is not vulnerable to the criticisms voiced against their specification [Dybvig and Zender (1991)].

If the manager is running an L-bank and he/she knows that $\Pi_2 < q$ (based on his/her private information), he/she will always sell his/her stake at date $t = 1$ and he/she will only care about the bank's share price. If, on the contrary, he/she is running an H-bank such that $\Pi_2 > q$, he/she will seek to maximize $\lambda q + (1 - \lambda)\Pi_2$.

Having determined the banks' objectives, their investment opportunities, and their sources of funds, we now can turn to an analysis of their optimal lending policy and asset–liability structure given fixed market terms.

Before doing so, we briefly summarize the sequence of moves and events and also recall the underlying information structure.

1.4 Timing

The following time line illustrates the order of decisions (Figure 1).

1. At date $t = 0$
   - The government sets G and announces an interest rate R_G, and firms issuing bonds quote their terms $R(p)$.
   - Banks quote their lending terms $\widehat{R}(p)$ to firms and choose the amount of new equity they want to issue, E.
   - Firms who prefer bank lending apply for a loan and those preferring bond financing tap financial markets.
   - Banks make their portfolio decision. In particular, they decide what proportion of their funds to invest in new loans and what proportion in government or corporate bonds. These decisions are unobservable to investors.

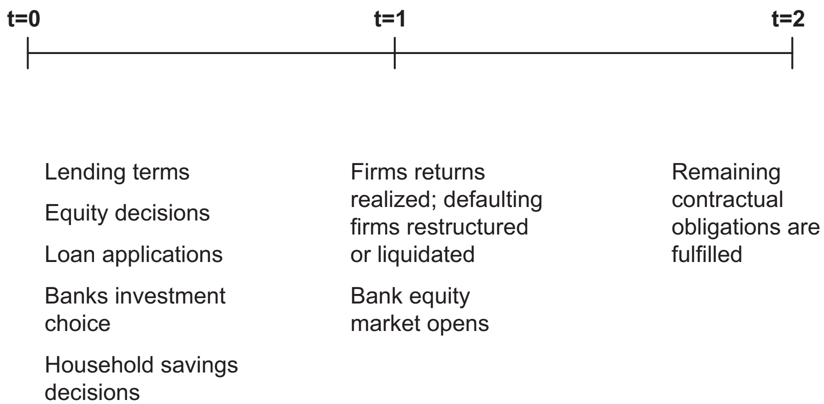

Figure 1
Timeline.

- Households determine the fraction of their endowment they want to save and the proportion of their savings they want to hold as deposits and in direct investments.

2. At date $t = 1$, firms' returns are realized. Those firms whose project has failed may be restructured if they have been financed with a bank loan.
   Bank managers have the option to sell their equity stake in the secondary market.
3. At the end of date $t = 2$, all remaining debts as well as dividends are paid and households consume their net income.

2. Bank Lending and Optimal Asset–Liability Structure

As we shall establish, in our model only pooling equilibria (where L-banks mimic H-banks) exist.[12] There may, of course, be many such equilibria. But, as we explain in Section 4, it is reasonable to focus on the pooling equilibria that are best for H-banks. Accordingly, we shall consider optimal lending and asset–liability management from the perspective of H-banks, who know that their observable actions are mimicked by L-banks (in a pooling equilibrium).

An H-bank contemplating an equity issue faces the following tradeoff. If it issues equity, it can increase lending and thus raise profits, but because its equity is undervalued in the financial market, the bank's manager does not appropriate the entire increase in profits. Depending on the profitability of loans and the extent of the undervaluation of equity, the H-bank may or may not decide to relax its lending constraint by issuing more equity. Thus, to determine an H-bank's choice, we need to specify the profitability of loans and the extent of dilution.

2.1 Optimal lending policy

In a pooling equilibrium, H-banks quote lending terms $\widehat{R}(p)$ to equalize the expected profit on every loan they make. We denote by ρ_H the expected net excess return per loan for H-banks over government bonds. The reason the spread on each loan for an H-bank must be the same at an optimum is that otherwise an H-bank could increase its profit by lending only to the firms with the highest spread. The spread ρ_H corresponds to the rents banks earn because of the specific role their funding plays in restructuring and because of the limited amount of capital they may have:

[12] More precisely, in Proposition 3, we establish that only pooling or semi-separating equilibria exist.

$$\rho_H = p\widehat{R}(p) + (1-p)v - R_G \tag{2}$$

Note that given these lending terms, H-banks get a higher return per loan than L-banks. Indeed, the expected return on a loan with success probability p for an L-bank is only

$$p\widehat{R}(p) + (1-p)\beta v - R_G. \tag{3}$$

Given the expected net excess return per loan over government bonds ρ_H and given rates on government bonds and bank deposits of R_G, an H-bank chooses its optimal mix of loans L and government bond holdings G_b to maximize expected profits subject to capital adequacy constraints:

$$\begin{cases} \max\limits_{(L,G_b)} \{L(R_G + \rho_H - c) + [G_b - D(R_G)]R_G\} \\ \text{subject to} \\ L + G_b = D(R_G) + w + E \quad (AL) \\ L \leq \frac{1}{\kappa}(w+E) \qquad\qquad (\kappa) \end{cases}$$

where (AL) is the asset–liability accounting identity and $D(R_G)$ satisfies $0 \leq D(R_G) \leq 1 - (1/2R_G)$.

Note first that because the cost of raising funds through bank bond issues or bank deposits is the same, we are only able to determine the net amount of bonds minus deposits (positive or negative) the bank holds: $G_b - D(R_G)$.

Second, it is easy to see from this program that the capital adequacy constraint is always binding if $\rho_H > c$. Indeed, if the excess return on bank lending is strictly positive, an H-bank can always make a profit by raising an extra dollar and investing it in a bank loan. If, on the contrary, $\rho_H < c$, it is best for the bank not to lend at all to firms and to invest only in the market.

We summarize these observations in the following lemma.

Lemma 1. *The optimal amount of lending for an H-bank is $L = (w+E)/\kappa$ if $\rho_H > c$ and $L = 0$ if $\rho_H < c$.*

This lemma highlights that banks' optimal lending policy and asset–liability structure is such that their equity-capital base is always a binding constraint on their lending capacity. Thus, in response to an increase in spreads, banks can only increase lending if they also increase their equity-capital base. Thus, it becomes essential to consider how banks' capital base is determined.

An important implication of this lemma is that monetary policy cannot affect bank lending by changing bank reserves (while keeping interest rates fixed). In other words, the classical bank lending channel of monetary policy is absent. As has already been noted by Romer and Romer (1990), when banks can perfectly substitute nonreservable liabilities for reservable ones, as in our model, monetary authorities can no longer control bank lending by controlling bank reserves. Bond issues may, of course, be imperfect substitutes for insured deposits if there is a risk of default, as Stein (1998) has noted. However, this imperfect substitutability of risky bonds for safe deposits is only a necessary condition for bank liquidity to affect bank lending. It is not sufficient if capital adequacy constraints remain binding.

Indeed, if we introduce reserve requirements into our model, by requiring banks to hold a fraction $\varphi \in (0,1)$ of their deposits as cash on hand (remunerated or not remunerated), and if we introduce an imperfect substitutability between bonds and insured deposits by, say, adding a small spread $\pi > 0$ on bank bond issues (so that a dollar raised in bonds costs the bank $R_G + \pi$), the effect would mainly be to raise the bank's overall cost of funds. But as long as bank spreads ρ_H remain sufficiently high, banks would continue to raise funds up to the point where the capital constraint binds. Thus, when the central bank changes reserve requirements on deposits, by changing φ, the only immediate effect is on bank profitability. It cannot affect bank lending if the capital constraint remains binding. Of course, profitability eventually or indirectly affects the availability of capital, so that, there may be an indirect or lagged effect on lending as in Van den Heuvel's study (1999). These points are made more formally in Section 5 dealing with comparative statics.

Our result that bank equity capital is always a binding constraint on bank lending appears to be counterfactual, as banks generally have a higher capital base than is required by BIS regulations. However, we show in Appendix A2 that the fact that equity capital is higher than is strictly required at any point in time does not necessarily mean that banks' capital constraint is not binding. The point is that if banks anticipate that their role as providers of flexible financing requires extending future lending to firms (as part of their loan commitments), they will hold capital reserves in anticipation of those future loan increases. This is why they may appear to be unconstrained, whereas in fact their equity base may actually constrain current lending. Introducing this idea formally into our model would have significantly increased its complexity. This is why we have chosen not to introduce it. Instead, we briefly describe the main changes to be made to the model to obtain a time 0 nonbinding capital constraint in Appendix A2.

3. General Equilibrium in the Credit Market

In this section, we take banks' equity capital as given and determine equilibrium rates R_G and $\widehat{R}(p)$ [or equivalently $R(p) = (R_G/p)$ and ρ_H] such that

1. the aggregate demand for bank credit is equal to aggregate supply;
2. the aggregate demand for bank equity, corporate, and government bonds is equal to the supply of funds to the securities markets;
3. bank demand for deposits equals deposit supply.

Because this last condition is met by setting $R_D = R_G$, for a given level of equity issues E, our equilibrium analysis boils down to solving a system of two equations in two unknowns, $R_G \geq 1$ and $\rho_H \geq 0$.[13]

The only difficulty in this analysis lies in constructing the aggregate demand and supply functions. Once these functions are determined, we can define two equilibrium schedules as functions of R_G and ρ_H—one for the bank credit market and the other for the securities markets. We then end up with a simple system of two equations as depicted in Figure 2, which can be solved straightforwardly.

Intuitively, to see why the credit market schedule is downward sloping, one should note that any increase in R_G that is not offset by a

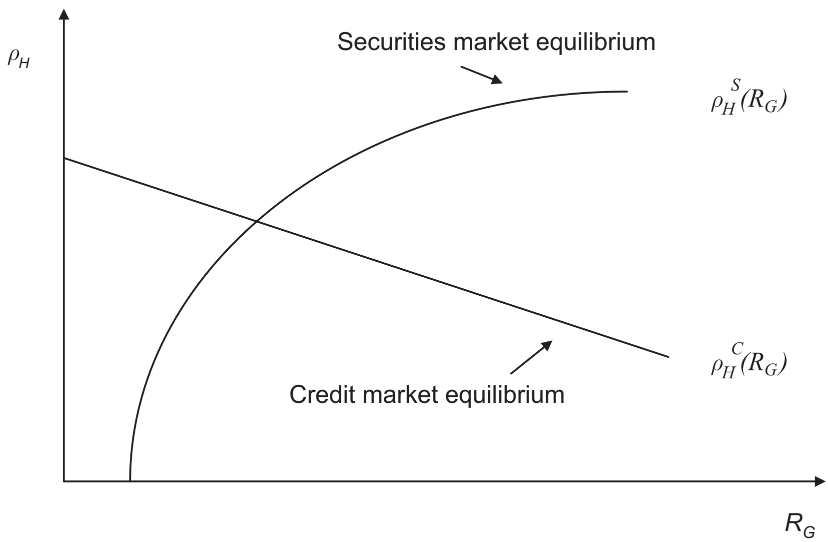

Figure 2
Bank lending equilibrium.

[13] We do not consider outcomes where $\rho_H < 0$, because banks always have the option to invest in securities, which provide a zero spread.

decrease in ρ_H results in an overall increase in the cost of bank credit. This increase results in a drop in aggregate effective demand for bank credit, as the marginal riskiest firms get priced out of the market. But, for a constant ρ_H, aggregate supply of bank lending remains unchanged. Therefore, to maintain equilibrium in the credit market, we require a fall in ρ_H. Similarly, the securities market schedule is upward sloping because any increase in R_G raises aggregate savings. When banks' equity capital E remains unchanged, banks' demand for deposits remains unchanged following an increase in R_G, so that household demand for securities has to increase. To meet this increase in demand, firms must raise their supply of bonds, which in turn requires an increase in ρ_H (Figure 2).

3.1 Equilibrium in the bank credit market

We focus our analysis on equilibria in the credit market such that (i) risky firms who cannot get a bank loan are also unable to get junk-bond financing and (ii) all bank types lend up to capacity.[14]

3.1.1 Firms funding choice. This equilibrium is such that all firms with a probability of default $(1-p) > (1-p^B)$ do not get any financing, all firms with a probability of default $(1-p^B) \geq (1-p) \geq (1-p^*)$ get bank financing, and all firms with a very low probability of default, $(1-p^*) > (1-p)$, get bond financing.

We now turn to a characterization of this equilibrium. Note first that, because firms do not appropriate any returns from restructuring, their demand for funds is simply driven by the cost of borrowing. A firm of risk characteristics p demands a bank loan if and only if

$$\widehat{R}(p) - R(p) \leq 0.$$

Using Equations (1) and (2), this is equivalent to

$$\frac{\rho_H - (1-p)v}{p} \leq 0.$$

Therefore, any firm with a probability of success lower than the threshold

[14] There are several possible equilibrium outcomes in our model. In some equilibria, there is a junk-bond financed segment of firms. These are highly risky firms that prefer bank financing but are not able to afford the intermediation cost. In other equilibria, L-banks do not extend any bank loans at all or do not lend to capacity, as they cannot find enough firms to lend to that provide a sufficiently high return [recall that L-banks are only able to generate a restructuring return βv (with $1>\beta$) where H-banks generate a return v]. Although these equilibria are of interest, we shall not analyze them, as they lead to a parallel, possibly more cumbersome analysis of the bank-capital monetary policy transmission channel.

$$p^* = \frac{v - \rho_H}{v}$$

prefers a bank loan to a bond issue, and any firm with a probability of success larger than p^* prefers to issue bonds.

This is quite intuitive. Banks obtain a rent from restructuring firms. Their comparative advantage is therefore higher when they face a riskier firm, which is more likely to go through a restructuring.[15]

Although all firms with $p < p^*$ apply for a bank loan, not all of these will be granted one. Indeed, some of these firms may be too risky and have too low a rating p to be worth investing in.[16] The threshold p^B below which firms do not obtain credit is given by

$$p^B V + (1 - p^B)v = R_G + \rho_H. \tag{4}$$

3.1.2 The demand for loans. Under our assumption that p is uniformly distributed on the unit interval, the mass of firms with $p \leq p^B$, which cannot get any funding at the cost of funds $(R_G + \rho_H)$ is given by[17]

$$p^B(R_G + \rho_H) = \frac{R_G + \rho_H - v}{V - v}. \tag{5}$$

And the aggregate demand for bank loans, comprising all firms with $p \in [p^B, p^*]$, is given by[18]

$$p^*(R_{G,}\rho_H) - p^B(R_{G,}\rho_H) = \left(\frac{v - \rho_H}{v}\right) - \left(\frac{R_G + \rho_H - v}{V - v}\right).$$

[15] This does not mean, necessarily, that riskier firms have to meet higher contractual repayments $\widehat{R}(p)$. But, it is easy to show that Assumption 1 implies that firms with higher risks will pay higher interest rates. The reason is simply that firms with higher risks also generate lower expected returns.

[16] The partition of firms into three classes—those that are credit rationed, those that are bank financed, and those that are financed through securities issues—has been obtained in earlier models but for different reasons. Holmstrom and Tirole (1997), for example, emphasized the role of collateral and had firms with more collateral issue securities. Berger and Udell (1998), on the contrary, assume that firms differ in the extent of their private information and obtain that those firms that have a higher level of asymmetric information are credit rationed, those with the least asymmetric information are funded by financial markets (arm-length finance), and those in between are funded by banks through monitored finance.

[17] Note that Assumption 1 implies that $p_1^B > 0$ for $\rho_H > 0$.

[18] As we have pointed out, firms with $p < p^B$ may be unable to get a bank loan but may possibly be able to get junk-bond financing. The minimum p for which bond financing is available is given by $pV = R_G$. If $(R_G/V) < p^B$, clearly the segment of risks between $\frac{R_G}{V}$ and p^B would be able to issue junk bonds. We do not consider such equilibria, as they are somewhat of a distraction.

3.1.3 The supply of loans. Equilibrium in the bank credit market requires that the aggregate supply of bank credit $L(R_G,\rho_H)$ equals this aggregate effective demand, $p^*(R_G,\rho_H) - p^B(R_G,\rho_H)$.

When $\rho_H > 0$, all H-banks supply as much credit as they can given their equity-capital stock. As we pointed out earlier, this is not necessarily true for L-banks, however. These banks only prefer to lend to the corporate sector if the return on their loans exceeds the return on bonds:

$$p\widehat{R}(p) + (1-p)\beta v \geq R_G$$

or

$$\rho_H - (1-p)(1-\beta)v \geq 0.$$

Notice that, in contrast to H-banks, the expected net excess return per loan over government bonds for L-banks is higher for loans with a lower risk of default. Therefore, L-banks concentrate on the safer segment of the bank loan market and cover a risk segment $[p^L,p^*]$, where $p^L > p^B$ is defined by the equation

$$p^* - p^L = \mu\left(\frac{w+E}{\kappa}\right), \tag{6}$$

when L-banks lend up to capacity. The RHS of this equation represents the maximum aggregate supply of loans by L-banks, and the LHS is the aggregate demand for bank loans by the safest segment of firms seeking bank financing.

3.1.4 Equilibrium in the credit market. The aggregate supply of bank credit in the equilibrium where all banks lend to capacity is given by $L = (w+E)/\kappa$. In this equilibrium, we have a schedule for the bank credit market given by the following lemma:

Lemma 2. *The equilibrium bank credit schedule relating ρ_H to R_G is given by*

$$\rho_H^C(R_G) = v\mathcal{A}(E) - \frac{vR_G}{V}, \tag{7}$$

where

$$\mathcal{A}(E) = \left[1 - (1 - \frac{v}{V})\frac{w+E}{\kappa}\right].$$

Proof. Equating supply and demand for bank loans, we have

$$\frac{w+E}{\kappa} = \frac{v-\rho_H}{v} - \frac{R_G+\rho_H-v}{V-v} \tag{8}$$

or

$$\rho_H \frac{V}{v} = \left(1 - \frac{w+E}{\kappa}\right)(V-v) + v - R_G.$$

Rearranging, we obtain the desired expression.

Notice that the equilibrium schedule (7) is independent of G. The reason is that banks do not compete directly with the government bond market in the equilibria we focus on. They only compete with the corporate bond market. Notice also that the schedule (7) defines a decreasing linear function in R_G. That is, a higher R_G is associated with a lower equilibrium spread ρ_H. The reason is that, with a fixed supply of bank loans (constrained by bank equity capital E), demand for bank loans can stay equal to supply only if an increase in R_G is partially offset by a decrease in ρ_H.

An equilibrium where L-banks lend to capacity and where aggregate loan supply is $L(\rho_H) = (w+E)/\kappa$ can be obtained when the equilibrium spread ρ_H^* is sufficiently high. In the following section, we provide a sufficient condition under which such an equilibrium exists.

3.2 Securities market equilibrium

Securities markets clear when aggregate bond issues by the highest-rated firms with total mass $[1-p^*(R_G,\rho_H)]$ together with government bond issues G and equity issues by banks E are equal to the aggregate supply of household savings (net of deposits) invested in financial markets $[1-1/(2R_G)-D(R_G)]$ plus aggregate investments by the banking sector in government bonds G_b. Equating aggregate demand and supply of securities, we therefore obtain the following lemma:

Lemma 3. *The equilibrium securities market schedule relating ρ_H to R_G is given by*

$$\rho_H^S(R_G) = v\mathcal{B}(E) - \frac{v}{2R_G}, \tag{9}$$

where

$$\mathcal{B}(E) = 1 - \left[\frac{E}{\kappa} + w(\frac{1}{\kappa} - 1) + G\right].$$

Proof. The securities market equilibrium condition is given by

$$1 - \frac{1}{2R_G} - D(R_G) = 1 - p^*(R_G,\rho_H) + E + G - G_b. \tag{10}$$

Replacing $p^*(R_G,\rho_H)$ by its value and replacing $(D - G_b)$ by $(w + E)[(1/\kappa) - 1]$, from the accounting identity (AL), this condition becomes

$$\frac{v - \rho_H}{v} = \frac{1}{2R_G} + \frac{E}{\kappa} + w(\frac{1}{\kappa} - 1) + G.$$

Rearranging, we obtain the desired expression.

Thus, the securities market equilibrium condition defines an increasing concave schedule $\rho_H^S(R_G)$ parameterized by G. The reason the schedule is increasing in R_G is that any increase in R_G raises household savings. To be able to invest these increased savings in corporate bonds, there has to be an equivalent increase in corporate bond issues. These issues, in turn, can only come from firms that otherwise would have taken out a bank loan. Thus, to get these firms to switch away from bank loans to bond issues, there has to be an increase in the relative cost of bank loans—that is, an increase in ρ_H. Similarly, the reason the schedule is concave in R_G is that the household savings function is concave in R_G.

Finally, notice that an increase in G induces a downward shift in the equilibrium schedule (9). The reason is that the corporate bond market also competes for household savings against the government bond market. Therefore, any increase in government bond issues must be met in part by an increase in savings (requiring in turn an increase in interest rates R_G) and by a contraction in corporate bond issues (requiring a reduction in ρ_H).

A general equilibrium in the securities and bank loan markets (ρ_H^*,R_G^*) is obtained when the two functions $\rho_H^C(R_G)$ and $\rho_H^S(R_G)$ intersect (as shown in Figure 2). In the next subsection, we give two sufficient conditions that guarantee the existence of a unique general equilibrium with maximal bank lending.

3.3 Existence

We now establish that a unique general equilibrium (ρ_H^*,R_G^*) with maximum bank lending exists if the following two conditions hold.

Assumption 1. $\frac{1}{2} - \frac{1}{V} + G > w - \frac{v}{V}\frac{w}{\kappa}$

Assumption 2. $w - \left[1 + \mu\left(\frac{1-\beta}{\beta}\right)\right]\left(\frac{w+E}{\kappa}\right) > G - \frac{1}{2}$

The first condition guarantees that the two functions $\rho_H^C(R_G)$ and $\rho_H^S(R_G)$ intersect for some $R_G \geq 1$. The second condition guarantees that the equilibrium spread is sufficiently high that both H- and L-banks want to lend to capacity. As is easily seen, these two conditions are satisfied within a range of G. If G is too low, the two functions $\rho_H^C(R_G)$ and $\rho_H^S(R_G)$ may intersect only for $R_G < 1$. But we must have $R_G \geq 1$ for households to invest any savings in firms. Similarly, if G is too large, the second condition is violated. In that case, government borrowing is so large and equilibrium interest rates R_G are so high that ρ_H is too low to make it profitable for L-banks to lend to the corporate sector.

The two conditions also hold for a range of bank equity capital, w or $(w+E)$. If bank equity capital is too large, bank loan supply to the corporate sector is so large that the loan market cannot clear with a spread ρ_H that is high enough to induce L-banks to engage in maximal lending to the corporate sector. Note, finally, that Assumption 2 is more likely to hold for a lower μ and a higher β. This is again easy to understand intuitively. A lower μ means that the mass of L-banks is smaller. Other things equal, therefore, any L-bank is able to lend to a better risk pool (p_L is higher), which raises the return on corporate lending. Similarly, a higher β raises the return on lending for L-banks.

Proposition 1. *Under Assumptions 1 and 2, a unique maximum bank lending equilibrium* (R_G^*, ρ_H^*) *exists.*

Proof. Notice first that for R_G sufficiently large we always have $\rho_H^C(R_G) < \rho_H^S(R_G)$ (this is obvious from Figure 2). Next we will prove that under Assumption 1 $\rho_H^C(1) \geq \rho_H^S(1)$, and when this inequality holds, the two equilibrium schedules can only intersect at some $R_G \geq 1$. To show $\rho_H^C(1) \geq \rho_H^S(1)$, subtract Equation (9) from (7) to obtain

$$\mathcal{A} - \mathcal{B} - \frac{1}{V} + \frac{1}{2} \geq 0$$

and, substituting for the values of $\mathcal{A}$ and $\mathcal{B}$,

$$\frac{1}{2} - \frac{1}{V} + G \geq w - \frac{v}{V}\left(\frac{w+E}{\kappa}\right),$$

an inequality implied by Assumption 1.

Second, when Assumption 2 holds, we have $\rho_H^C(1) \geq (1 - p_L)(1 - \beta)v$ so that even at a spread $\rho_H^C(1)$, L-banks prefer to lend up to capacity to the corporate sector. A fortiori, then they are lending to capacity at the equilibrium spread $\rho_H^*\left(R_G^*\right) > \rho_H^C(1)$.[19]

Having established the existence of a unique general equilibrium for a fixed equity-capital base for banks $(w + E)$ that satisfy Assumptions 1 and 2, we now turn to the endogenous determination of banks' equity capital. To guarantee the existence of an equilibrium with endogenous equity issues, we shall take it that Assumptions 1 and 2 hold for all $E \in [0,\bar{E}]$.

4. Endogenous Bank Equity

In this section, we take into account banks' incentives to issue equity and allow for the *endogenous* determination of bank equity capital. Banks' incentives to issue equity depend on the equilibrium beliefs of investors. We therefore face the standard equilibrium problem of the joint determination of equilibrium strategies and beliefs.

When banks must pay a premium for equity capital, they will expand their equity base only if the rate of return on bank loans exceeds the cost of equity capital. This is why equilibrium bank spreads $[p\widehat{R}(p) + (1 - p)v - R_G]$ will be strictly greater than banks' average operating costs, c. Also, given that bank spreads are strictly positive in equilibrium, banks have an incentive to lend up to the point where the capital constraint binds.[20]

As is well known, signaling games generally have multiple equilibria. We argue in this section that this observation may have important implications for equilibrium bank lending and the monetary transmission mechanism. Indeed, we show that a high-lending equilibrium, with low rationally expected dilution costs and low equilibrium bank spreads, may exist along with a low-lending *credit-crunch equilibrium*, with high spreads and high rationally expected dilution costs. In this context, even a small change in interest rates R_G induced by monetary policy can have large effects on aggregate bank lending, if it induces a switch from the high-lending equilibrium to the low-lending equilibrium, or vice versa.

Ultimately, the relevant equilibrium is tied down by market beliefs and, as Spence (1974) has compellingly argued, a complete theory of how

[19] To see this, one should note that if $x > (1 - p_L(x))(1 - \beta)v$, then $y > x$ implies that $y > (1 - p_L(y))(1 - \beta)v$. Indeed, replacing $p_L(x)$ by its value yields $x > [(x/v) + \mu[(w + E/\kappa)]](1 - \beta)v$, which is equivalent to $\beta x > \mu[(w + E)/\kappa](1 - \beta)v$. Therefore, if this inequality holds for x, it must also hold for $y > x$.

[20] This is always true in our model for H-banks, but only true for L-banks under Assumption 2. We also explain in Appendix A1 that dynamic considerations may induce banks to keep a small equity-capital "cushion," for inventory-management reasons. This is an additional reason why banks in reality hold equity capital in excess of 8%.

market beliefs are formed involves historical, psychological, and cultural considerations, which go beyond the scope of our analysis. We can, however, narrow down somewhat the equilibrium set by appealing to intuitive refinement ideas along the lines of Cho and Kreps (1987). Nevertheless, as we show in this section, a fundamental multiplicity will remain even after the elimination of equilibria that are supported by market beliefs that do not satisfy Cho–Kreps' *intuitive criterion*. We believe that this is a strength and not a weakness of the theory, as it provides the underpinnings for the notion of a credit-crunch equilibrium.

We begin the section by considering a single bank's incentive to issue new equity given equilibrium rates of return on lending ρ_H and ρ_L. We then derive the optimal amount of equity banks choose to issue, the aggregate supply of bank credit, and credit spreads in a general equilibrium of the capital and bank credit market.

In our equilibria, H-banks issue equity only if the bank spread ρ_H is high enough and bank loans are sufficiently lucrative to compensate for the dilution cost on new equity issues. As for L-banks, they prefer to mimic H-banks to benefit from better prices on the sale of their shares.

4.1 Bank owner-manager payoffs and optimal equity issues

To be able to determine a bank's incentive to issue equity, we need first to derive the return on equity and the bank manager's payoff function for each type of bank.

Characterizing the return on equity for each bank type $J = H,L$ turns out to be straightforward for an H-bank, but somewhat more involved for an L-bank. To simplify our notation, we shall denote the return on equity capital for a J-bank by Γ_J. Both types of bank must incur a cost per asset unit of $c > 0$, and H-banks earn a unit spread of ρ_H over R_G on each loan they make, whereas L-banks earn only $p\hat{R}(p)+(1-p)\beta v - R_G$ on a loan to a firm with a probability of success p. Therefore, we obtain the following characterization for the return on equity for each bank type.

Lemma 4. *The return on equity capital for respectively H-banks and L-banks is given by*

$$\Gamma_H = R_G + \frac{\rho_H - c}{\kappa} \text{ and } \Gamma_L = R_G + \frac{\rho_L - c}{\kappa},$$

where

$$\rho_L(\rho_H) = \rho_H\beta - Z \text{ and } Z = (1-\beta)v\frac{\mu}{2}\left(\frac{w+E}{\kappa}\right).$$

Proof. See Appendix A1. ■

The returns Γ_H and Γ_L are the actual returns for each type of bank. Investors, however, do not observe banks' types and can only draw inferences about a bank's likely type given the bank's observed actions, in particular the bank's observed equity issue E. We shall denote by Γ_E market beliefs about a bank's type, conditional on the bank issuing equity E.

We now turn to the characterization of each bank type's optimal equity issue decision given market beliefs Γ_E. A bank raising an amount of equity E from new shareholders must relinquish equity ownership α equal to

$$\alpha\Gamma_E(w+E) = E \cdot R_G. \tag{11}$$

Recall that bank managers face liquidity shocks that force them to unwind their equity holdings with probability λ at date $t=1$. Therefore, as highlighted in Section 1.3, an H-bank owner-manager's expected payoff following a new equity issue E is given by

$$(1-\alpha)[\lambda\Gamma_E + (1-\lambda)\Gamma_H](w+E).$$

Under the same market expectations, an L-bank manager's expected payoff following a new equity issue E is given by

$$(1-\alpha)\Gamma_E(w+E).$$

The expressions for the payoffs of the two types differ because a manager of an L-bank is always better off selling his/her equity stake at date $t=1$ than holding on to it, because the market always (weakly) overvalues the shares of an L-bank.

Now, if we denote by Γ_0 the market's expected return on equity capital when a bank issues no equity ($E=0$) and by Γ_E the market expected return on equity capital when a bank raises an amount of equity $E>0$, each type of bank's optimal equity issue decision is given by the following lemma:

Lemma 5. *An H-bank manager is better off issuing equity $E>0$ than issuing no equity if and only if*

$$\frac{\Gamma_E(w+E) - ER_G}{\Gamma_E}[\lambda\Gamma_E + (1-\lambda)\Gamma_H] \geq [\lambda\Gamma_0 + (1-\lambda)\Gamma_H]w. \tag{12}$$

An L-bank manager is better off issuing equity $E>0$ than issuing no equity if and only if

$$\Gamma_E(w+E) - ER_G \geq \Gamma_0 w. \tag{13}$$

Proof. See Appendix A1. ■

As will become clear in the next section, conditions (12) and (13) will be central for the determination of the full equilibrium with endogenous bank equity capital.

For future reference, it is helpful to put more structure on market conditional beliefs Γ_E. To that end, recall the timing of moves in our model: banks begin by quoting lending terms $\hat{R}(p)$ to firms and announcing new equity issues, E.[21] Households then respond by deciding how they want to allocate their savings among the different financial instruments, all yielding an expected return of R_G in equilibrium. When evaluating the return on bank equity, households will be able to update their beliefs about a bank's type given the observed policy of the bank, $\hat{R}(p)$ and E. Thus, if we denote by $\delta_H(\hat{R}(p),E)$ and $\delta_L(\hat{R}(p),E)$ investors' updated beliefs about the bank's type we have the following lemma:

Lemma 6. *The return on equity capital* Γ_E *expected by investors in equilibrium is given by*

$$\Gamma_E = R_G + \frac{\delta_H(\hat{R}(p),E)\rho_H + \delta_L(\hat{R}(p),E)\rho_L - c}{\kappa}. \tag{14}$$

Proof. See Appendix A1. ■

4.2 Pooling equilibria in the banking sector

We now turn to the derivation of the aggregate equilibrium supply of bank loans, by characterizing the Bayes-Nash equilibrium of the equity-capital issue game banks play.

Definition. A Bayes-Nash equilibrium in the banking sector given R_G and $\hat{R}(p)$ is characterized by

- banks' best equity issue strategies, $E^i \in [0,\bar{E}]$, $i = H, L$ given equity-market conditional beliefs, $\delta_H(\hat{R}(p), E)$ and $\delta_L(\hat{R}(p), E)$, and
- equity-market conditional beliefs about the bank's type that are consistent with the banks' best responses; that is, equity-market conditional beliefs that are consistent with Bayesian updating.

As one might expect, there may be many Bayes-Nash equilibria in our game. We shall show, however, that separating equilibria such that $E^H \neq E^L$ do not exist. The intuition for this result is as follows. Two

[21] In general, one might expect that H-banks would make different equity issue decisions E^H than the decisions E^L by L-banks. However, as the next section makes clear, in our model only pooling equilibria such that $E^H = E^L = E$ can exist.

different types of separating equilibria are conceivable. One type is where the H-bank manager chooses not to issue equity ($E^H = 0$) while the L-bank manager issues new equity ($E^L > 0$). The other type of separating equilibrium has the L-bank and H-bank managers switch roles.[22]

It is easy to see that the latter type of separating equilibrium cannot exist. The reason is simply that the L-bank manager would have a profitable deviation by mimicking the H-bank' s strategy. This would not only raise the L-bank' s stock price but also allow the bank to sell overvalued equity. Symmetrically in the former type of separating equilibrium, for an H-bank to decide not to issue equity in a separating equilibrium in which it incurs no dilution costs à la Myers–Majluf, it must be the case that loans have a very low return (namely, $\rho_H < R_G$). But then it is even less profitable for L-banks to lend. Therefore, neither H- nor L-banks have an incentive to issue equity.

Proposition 3. *There exist no separating Bayes-Nash equilibria in the banking sector, such that* $E^H \neq E^L$, $\delta_H(\hat{R}(p),E^H) = 1$, *and* $\delta_L(\hat{R}(p),E^L) = 1$.

Proof. See Appendix A1. ■

Pooling equilibria can be supported by out-of-equilibrium beliefs such that $\delta_L(\hat{R}(p),\hat{E}) = 1$ for all $\hat{E} \neq E$.[23] A pooling equilibrium with bank equity-capital issues E is characterized by equilibrium spreads $\rho_H(E)$, $\rho_L(E)$, and interest rates $R_G(E)$ given by the unique solution to the equilibrium Equations (9) and (7) such that $R_G(E) \geq 1$.[24]

As is to be expected, a continuum of pooling equilibria characterized by the size of the equilibrium equity issue $E \leq \bar{E}$ may exist, because out-of-equilibrium beliefs can be set arbitrarily. We shall, however, restrict attention to the pooling equilibria that are best from the point of view of an H-bank manager, partly on the grounds that the Cho–Kreps intuitive criterion would select these equilibria over all other pooling equilibria in our game [Cho and Kreps (1987)].[25]

The optimal equity issue for an H-bank, in a pooling equilibrium with lending terms $\hat{R}(p)$ and R_G, is generically either 0 or $\bar{E}$, as is established in the following lemma.

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[22] Or, to issue less new equity.

[23] Semi-separating equilibria, where one or both of the types randomize over two levels of equity issues $\hat{E} = 0$ and $\hat{E} > 0$, may also exist. To avoid a lengthy digression, we shall not consider semi-separating equilibria here.

[24] This system of equations admits two possible solutions for $R_G(E)$ and $\rho_H(E)$, but only one root $R_G(E)$ is greater than one under Assumptions 1 and 2.

[25] Note that our qualitative results and comparative statics analysis do not depend on this refinement in any way. What is important for our analysis is essentially that pooling equilibria do indeed exist and that bank lending may vary with equity-market beliefs.

Lemma 7. *Given lending terms $\hat{R}(p)$ such that $p\hat{R}(p) + (1-p)v - R_G = \rho_H$, the optimal equity issue for an H-bank under pooling is*

- $\bar{E}$, if $\delta_H(\hat{R}(p), E)\rho_H + \delta_L(\hat{R}(p), E)\rho_L > c$ or
- 0, if $\delta_H(\hat{R}(p), E)\rho_H + \delta_L(\hat{R}(p), E)\rho_L < c$ or
- undetermined, if $\delta_H(\hat{R}(p), E)\rho_H + \delta_L(\hat{R}(p), E)\rho_L = c$.

Proof. We compute the sign of the derivative with respect to E of the manager's payoff. The manager's payoff is $V_H(E) = [(\Gamma_E(w+E) - ER_G)/\Gamma_E][\lambda\Gamma_E + (1-\lambda)\Gamma_H]$, and its derivative's sign is therefore given by the sign of $\Gamma_E - R_G$, which value we are able to compute as

$$\Gamma_E - R_G = \frac{\delta_H(\hat{R}(p),E)\rho_H + \delta_L(\hat{R}(p),E)\rho_L - c}{\kappa}$$

This establishes the lemma ■

Consider first the pooling equilibrium with $E^H = E^L = 0$. This equilibrium admits spreads $\rho_H(0)$, $\rho_L(0)$, and interest rates $R_G(0) \geq 1$, given by the unique solution to the equations

$$\rho_H = v\mathcal{A}(0) - \frac{vR_G}{V} \tag{15}$$

and

$$\rho_H = v\mathcal{B}(0) - \frac{v}{2R_G}. \tag{16}$$

Similarly, the pooling equilibrium with $E^H = E^L = \bar{E}$ admits spreads $\rho_H(\bar{E})$, $\rho_L(\bar{E})$, and interest rates $R_G(\bar{E}) \geq 1$, given by the unique solution to the equations

$$\rho_H = v\mathcal{A}(\bar{E}) - \frac{vR_G}{V} \tag{17}$$

and

$$\rho_H = v\mathcal{B}(\bar{E}) - \frac{v}{2R_G}. \tag{18}$$

When does either of these equilibria obtain? And, can both equilibria coexist? The next proposition provides sufficient conditions under which a pooling equilibrium always exists and under which both may coexist.

Proposition 4. *Under Assumptions 1, 2, and*

Assumption 3. $$\beta < \frac{(1-\mu)\rho_H(E)}{\rho_H(0) - \mu\rho_H(E)},$$

a pooling equilibrium always exists. When, in addition,

Assumption 4. $$\rho_H(0)\beta < \min\{c + (1-\beta)v\frac{\mu w}{2\kappa}; (1-\mu+\mu\beta)\rho_H(E)\} \quad (19)$$

holds, both pooling equilibria may coexist.

Proof. See Appendix A1. ■

We have imposed three conditions to guarantee the existence of at least one and possibly multiple pooling equilibria with maximal bank lending. The reader may wonder at this point whether the set of parameters that satisfy Assumptions **1**, **2**, and **3** is not an empty set. Careful inspection of these three conditions reveals that the first two conditions impose a restriction on G, whereas the third one restricts the values of the parameter β. In Appendix A1:1, we provide a numerical example in which both pooling equilibria with maximal bank lending do coexist. Note also that the range of parameter values for which pooling equilibria with maximal bank lending exist is larger than the set of parameters that satisfy Assumptions 1, 2, and 3, as these are sufficient conditions.

Nevertheless, the reader may be concerned about the robustness of our equilibria. The reason we require Assumptions **1, 2,** and **3** is that we want to focus on a specific type of equilibrium where L-banks lend to full capacity.

Clearly, Assumption **2** guaranteeing that L-banks want to engage in maximal lending to firms is unnecessarily restrictive. We only focus on this equilibrium because it is the simplest to characterize. But an equally plausible equilibrium could be one where L-banks do not lend up to capacity. An interesting and empirically relevant comparative statics feature of this equilibrium is that it produces an aggregate bank credit supply function (by L-banks) that is downward sloping in R_G. Indeed, the higher R_G the lower is ρ_L; therefore, the lower is the credit supply of L-banks. Assumption 1, on the contrary, is quite natural as we do not expect to obtain an equilibrium if the amount of government bond issues is not sustainable. Finally, Assumption 3 simply allows us to disregard semi-separating equilibria that are more cumbersome and do not bring any additional insight (Figure 3).

The coexistence of two pooling equilibria is of interest for the study of the effects of monetary policy. Figure 3 illustrates this case. When the economy is initially in the high-lending equilibrium with $E^H = E^L = \bar{E}$, a

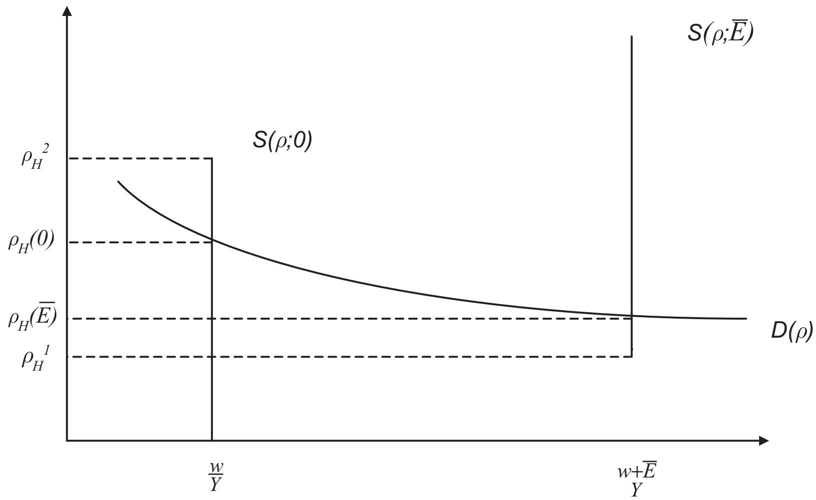

Figure 3
Multiple pooling equilibria.

contractionary monetary policy resulting in an increase in R_G and a reduction in bank spreads ρ_H may induce a switch from the high-lending equilibrium to the low-lending one characterized by $E^H = E^L = 0$ and thus lead to a magnified contractionary effect. In other words, a contractionary monetary policy may drive the economy into a region of the parameter space, where only the pooling equilibrium such that $E^H = E^L = 0$ exists. This switch is inevitable if spreads $\rho_H(\bar{E})$ are reduced to the point where an H-bank finds it optimal to deviate from the high-lending equilibrium policy ($E^H = E^L = \bar{E}$) by setting $E^H = 0$.

To state this point more formally, if initially the economy is characterized by a sufficiently low $R_G(\bar{E})$ and a sufficiently high $\rho_H(\bar{E})$ to satisfy

$$\frac{\Gamma_{\bar{E}}(w+\bar{E}) - \bar{E}\cdot R_G(\bar{E})}{\Gamma_{\bar{E}}}\left[\lambda\Gamma_{\bar{E}} + (1-\lambda)\Gamma_H^{\bar{E}}\right] \geq \left[\lambda\Gamma_L^{\bar{E}} + (1-\lambda)\Gamma_H^{\bar{E}}\right]w, \tag{20}$$

where

$$\Gamma_{\bar{E}} = \bar{\Gamma} = \mu\Gamma_L^{\bar{E}} + (1-\mu)\Gamma_H^{\bar{E}},$$

$$\Gamma_H^{\bar{E}} = R_G(\bar{E}) + \frac{\rho_H(\bar{E}) - c}{\kappa},$$

and

$$\Gamma_L^{\bar{E}} = R_G(\bar{E}) + \frac{\rho_H(\bar{E})\beta - (1-\beta)v\frac{\mu}{2}\left(\frac{w+E}{\kappa}\right) - c}{\kappa},$$

then a severe monetary contraction may increase $R_G(\bar{E})$ and lower $\rho_H(\bar{E})$ to the point where

$$\frac{\Gamma_{\bar{E}}(w+\bar{E}) - \bar{E}\cdot R_G(\bar{E})}{\Gamma_{\bar{E}}}\left[\lambda\Gamma_{\bar{E}} + (1-\lambda)\Gamma_H^{\bar{E}}\right] < \left[\lambda\Gamma_L^{\bar{E}} + (1-\lambda)\Gamma_H^{\bar{E}}\right]w, \quad (21)$$

in which case it is optimal for banks to deviate from the high-lending equilibrium policy by setting $E^H = E^L = 0$.

We interpret the switch from the equilibrium with high bank equity capital to the one with low equity capital as a form of *credit crunch* induced by a *bank equity crunch* similar to the credit crunch in some northeastern states in 1990 described by Bernanke and Lown (1991).

To summarize, a key prediction of our analysis in this section is that a monetary tightening not only brings about the expected increase in the cost of borrowing R_G and $R_G + \rho_H$, but as it induces a decrease in bank spreads ρ_H, it also reduces banks' incentives to increase their equity-capital base. Capital-adequacy regulations thus may induce a decrease in the supply of loans through a contraction of the equity-capital base and thus magnify the contractionary effect of monetary policy.

5. Comparative Statics and the Effects of Monetary Policy

Having highlighted how changes in interest rates may produce a switch from a high-lending equilibrium to a credit-crunch equilibrium, we now turn to a discussion of "local" effects of monetary policy by exploring the comparative statics of changes in G. We shall illustrate these comparative statics effects in a diagram and confine the formal analysis to Appendix A1. What we have in mind here is a central bank holding a stock of Treasury bills and conducting monetary policy by buying and selling these bills against cash. An expansionary monetary policy is then implemented by buying Treasury bills (decreasing G) and a contractionary one by increasing G.

The effect of an increase in G is shown in Figure 4. When the supply of bonds increases from G to G', it does not affect the credit market equilibrium schedule, as Equation (7) is independent of G. However, it shifts the security market equilibrium schedule (9) outward, as government borrowing crowds out corporate and bank bond issuers. One expected effect of increased government borrowing is an increase in interest rates, R_G. The other effect in our model is a decrease in bank

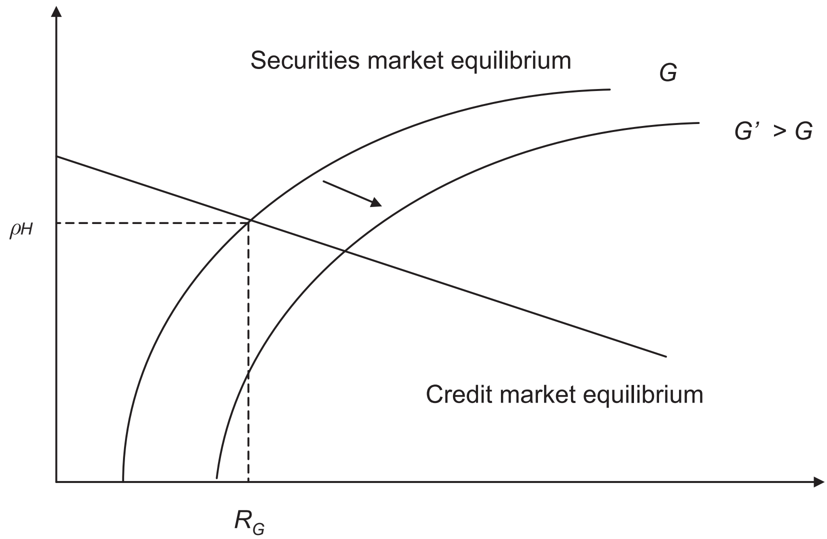

Figure 4
Crowding-out effect.

spreads ρ_H. The reason bank spreads must decrease is that otherwise total demand for bank loans would decrease to a point where banks would not be able to lend to capacity. But then it would be profitable for banks to lower ρ_H slightly to be able to lend more (Figure 4).

These effects on ρ_H and R_G in turn affect the equilibrium level of bank lending and bond issues by shifting overall financing to safer firms. That is, both $p(R_G + \rho_H)$ and $p_1^*(R_G + \rho_H)$ shift to the right by an amount proportional to the increase in G (see Appendix A1), so that total bank lending remains unchanged, but the increased supply of Treasury bills partially crowds out corporate bond issues. It does not fully crowd out corporate bond issues because the increase in R_G also induces an increase in savings.[26]

Thus, the effect of a monetary tightening on individual firms is to cut off the riskiest firms from bank lending and to induce the substitution of bond financing for relatively cheaper bank lending at the firms with the lowest bond ratings. Overall, the total share of bank lending to corporate bond issues increases in response to a monetary tightening. We summarize these findings in the following proposition.

[26] An interesting observation emerging from this analysis is that it is possible to observe at the same time a reduction in bank spreads, accompanied by a contraction in aggregate private investment, and an increase in the remuneration of deposits.

Proposition 5. *Effects of open-market operations: A contractionary monetary policy (increase in G) has the effect of (i) increasing Treasury bill rates R_G, (ii) decreasing bank spreads ρ_H, (iii) increasing the overall cost of bank loans $R_G + \rho_H$, (iv) decreasing corporate bond issues by an amount that is smaller than the additional amount of government borrowing, and (v) leaving the aggregate amount of bank lending unchanged. However, bank lending is now directed to safer firms, so that on balance it is the marginal firms with the highest risks that are forced out of investment by the tightening in monetary policy.*

Proof. See Appendix A1. ■

Thus, for the pooling equilibria with maximal bank lending we have focused on, the only local effect of a monetary tightening and reduction in bank spreads is to change the composition of financing. Aggregate bank lending remains unchanged, but there is a shift away from bond financing toward borrowing through bank loans. However, when equilibrium bank spreads decrease to the point that Assumption 2 no longer holds, then a further increase in R_G also induces L-banks to switch away from direct lending to the corporate sector and to investing in securities. This switch will then result in a contraction of aggregate bank lending.

How do these results relate to empirical findings? The recent empirical literature on the monetary transmission mechanism has uncovered one broad finding on the composition effects of a contractionary policy. Kashyap, Stein, and Wilcox (1993) have found that an important response to a monetary tightening is a surge in commercial paper issuance. They interpret this finding as a change in the composition of financing by firms in response to a monetary tightening: firms substituting bank debt for commercial paper. However, Gertler and Gilchrist (1994) and Oliner and Rudebush (1996) found that the main factor behind this surge in commercial paper issuance is inventory build-up by large firms financed by commercial paper issuance. Small firms do not rely on commercial paper issues at all. Moreover, these firms bear the brunt of the monetary tightening. Thus, the story that seems to emerge from these studies is that

> ... the main effect of a monetary contraction is to shift financing of all types from small firms to large firms. This shift produces a decline in the aggregate bank-loan share because large firms rely less heavily on bank debt than do small firms. [Oliner and Rudebush (1996, p. 301)]

Our results are consistent with these findings to the extent that they explain the shift in overall financing from small (or riskier) firms to large (or safer) firms that results from an increase in interest rates R_G. They seem, however to be in contradiction to the empirical evidence to the extent that in our model the aggregate bank loan share can increase as a result of the monetary tightening. But note that our results concern the aggregate share of bank lending to long-term bond issues and not the share of bank lending to commercial paper or other short-term debt, which is the focus of most of the empirical literature. The only study that investigates the ratio of bank loans to long-term bonds is that of Gertler and Gilchrist (1993). They show that this ratio only declines slightly following a tightening in monetary policy.[27]

Note also that our result that ρ_H decreases in response to a monetary tightening is consistent with the stylized fact that the yield on bank loans is sticky relative to the Treasury rate. Our model thus provides an alternative explanation for the observed stickiness of bank loan rates.

Other comparative statics effects could be explored, such as the effects of a tightening in capital regulations (an increase in κ), a change in the quality composition of banks (a change in μ, or a change in β), and a change in the profitability of investment projects (a change in V or v). For the lack of space, we omit the discussion of these effects. But, it should be clear that the analysis of the comparative statics with respect to G can be easily adapted to explore these effects.

6. Conclusion

This article proposes a model of the interface between corporate financing decisions and monetary policy in a general equilibrium model (of the capital market), which traces the effects of monetary policy on firms' investment decisions.

The model developed here, which abstracts from many other relevant considerations, generates several qualitative predictions about the joint equilibrium in the credit and securities markets and the effects of open-market operations on the real sector, which are broadly consistent with stylized facts on the effects of monetary policy on investment and firm financing uncovered by recent empirical studies.

The model considered in this article is already somewhat complex, and we have chosen to leave some interesting extensions for future research. An obvious immediate extension is to differentiate firms according to both the underlying risk of their cash flows and their size. If, in addition, one then introduces a fixed issuing cost for bonds (representing legal and administrative costs), we would expect to obtain an equilibrium segmentation where only the largest and safest firms issue bonds. Such a model

[27] The model we have considered here is an oversimplification of reality to the extent that we have ruled out equity financing by firms. In Bolton and Freixas (2000), we allow firms to raise funds in any form they like, including equity. The analysis in that model suggests that another possible effect of a monetary tightening is to induce substitution away from equity financing toward bank lending and bond financing.

could also be used to investigate how the size distribution of firms as well as the relative costs of securitization affect the aggregate composition of financing in the economy.

Another obvious but more ambitious extension is to introduce a final goods market and sticky prices. Extending the model to a multiperiod setting to explore the dynamics of the monetary transmission mechanism is perhaps the most interesting and difficult challenge.

Appendix A1: Mathematical Appendix

Proof of Proposition 3. The two functions ψ_H and ψ_L below represent the net payoff of issuing equity worth $\mathrm{E} = \bar{\mathrm{E}}$ instead of $\mathrm{E} = 0$ for an H- and an L-bank, respectively:

$$\psi_H(\Gamma_E,\Gamma_0) = (\Gamma_E - R_G)E\left[\lambda + (1-\lambda)\frac{\Gamma_H}{\Gamma_E}\right] - \lambda(\Gamma_0 - \Gamma_E)w \tag{A1}$$

and

$$\psi_L(\Gamma_E,\Gamma_0) = (\Gamma_E - R_G)E - (\Gamma_0 - \Gamma_E)w \tag{A2}$$

Combining Expressions (A1) and (A2) yields the following convenient expression:

$$\psi_H(\Gamma_E,\Gamma_0) = \lambda\psi_L(\Gamma_E,\Gamma_0) + (\Gamma_E - R_G)E(1-\lambda)\frac{\Gamma_H}{\Gamma_E} \tag{A3}$$

The different types of equilibria are characterized by the signs of the functions ψ_H and ψ_L as follows:

1. The pooling equilibrium with $E = 0$ is such that $\psi_H(\Gamma_E,\Gamma_0) \leq 0$ and $\psi_L(\Gamma_E,\Gamma_0) \leq 0$.
2. The pooling equilibrium with $E = \bar{E}$ is such that $\psi_H(\Gamma_E,\Gamma_0) \geq 0$ and $\psi_L(\Gamma_E,\Gamma_0) \geq 0$.
3. Separating equilibria are such that $\psi_H(\Gamma_E,\Gamma_0) > 0$ and $\psi_L(\Gamma_E,\Gamma_0) \leq 0$ or $\psi_H(\Gamma_E,\Gamma_0) \leq 0$ and $\psi_L(\Gamma_E,\Gamma_0) > 0$.
4. Semi-separating equilibria are such that either

$$\psi_L(\Gamma_E,\Gamma_0) = 0 \quad \text{or} \quad \psi_H(\Gamma_E,\Gamma_0) = 0,$$

so that either or both types are indifferent between issuing and not issuing new equity.

To prove Proposition 3, we first establish a preliminary lemma:

Lemma 8.

(i) *If* $\Gamma_E > R_G$, *then* $\psi_L(\Gamma_E,\Gamma_0) \geq 0$ *implies* $\psi_H(\Gamma_E,\Gamma_0) > 0$
(ii) *If* $\Gamma_E < R_G$, *then* $\psi_L(\Gamma_E,\Gamma_0) \leq 0$ *implies* $\psi_H(\Gamma_E,\Gamma_0) < 0$
(iii) $\psi_L(\Gamma_E,\Gamma_0) = 0$ *implies* $\psi_H(\Gamma_E,\Gamma_0) = (\Gamma_E - R_G)E\left[(1-\lambda)\frac{\Gamma_H}{\Gamma_E}\right]$, *so that if* $\Gamma_E = R_G$, $\psi_H(\Gamma_E,\Gamma_0) = 0$.

Proof of Lemma 8. This is straightforward using (A3).

We now proceed to prove Proposition 3:
If $\psi_H > 0$, and $\psi_L < 0$, we have $\Gamma_E - R_G > 0$, as otherwise (ii) would yield a contradiction. But then the equilibrium beliefs are such that $\Gamma_0 = \Gamma_L$, and Expression (A2) tells us that ψ_L is the sum of two positive terms, a contradiction. If instead $\psi_H < 0$, and $\psi_L > 0$, then, symmetrically, this implies $\Gamma_E - R_G < 0$, because of (i) and equilibrium beliefs are such that $\Gamma_E = \Gamma_L$, so that Expression (A2) is the sum of two negative terms, again contradicting $\psi_L = 0$. This establishes the proposition.

Lemma 4. *The return on equity capital for respectively H-banks and L-banks is given by*

$$\Gamma_H = R_G + \frac{\rho_H - c}{\kappa} \quad \textit{and} \quad \Gamma_L = R_G + \frac{\rho_L - c}{\kappa},$$

where

$$\rho_L(\rho_H) = \rho_H\beta - Z, \quad \textit{and} \quad Z = (1-\beta)v\frac{\mu}{2}\left(\frac{w+E}{\kappa}\right).$$

Proof of Lemma 4. The expression for the return on equity capital for H-banks is obvious. As for L-banks, they generate an average return of

$$\Gamma_L = \frac{\int_{p_L}^{p^*} \Gamma_L(p)dp}{p^* - p_L},$$

where

$$\Gamma_L(p) = R_G + \frac{\rho_H - (1-p)(1-\beta)v - c}{\kappa}.$$

So that

$$\begin{aligned}\Gamma_L &= R_G + \frac{\rho_H - c - \frac{(1-\beta)v}{p^*-p_L}\int_{p_L}^{p^*}(1-p)dp}{\kappa} \\ &= R_G + \frac{\rho_H - c - (1-\beta)v(1-\frac{p^*+p_L}{2})}{\kappa}\end{aligned}$$

or, after substituting for p_L and p^*,

$$\Gamma_L = R_G + \frac{\rho_H\beta - c - (1-\beta)v\frac{\mu}{2}\left(\frac{w+E}{\kappa}\right)}{\kappa}.$$

Writing $Z = (1-\beta)v(\mu/2)[(w+E)/\kappa]$ and $\rho_L(\rho_H) = \rho_H\beta - Z$ and rearranging, we obtain the desired expression for the the return on equity capital for L-banks.

Lemma 5. *An H-bank manager is better off issuing equity $E > 0$ than issuing no equity if and only if*

$$\frac{\Gamma_E(w+E) - ER_G}{\Gamma_E}[\lambda\Gamma_E + (1-\lambda)\Gamma_H] \geq [\lambda\Gamma_0 + (1-\lambda)\Gamma_H]w. \tag{A4}$$

An L-bank manager is better off issuing equity $E > 0$ than issuing no equity if and only if

$$\Gamma_E(w + E) - ER_G \geq \Gamma_0 w. \tag{A5}$$

Proof of Lemma 5. An H-bank owner-manager making a new equity issue E and holding on to his/her shares until date $t = 2$ gets an expected return $(1 - \alpha)\Gamma_H(w + E)$. The same manager making a new equity issue E and selling his/her shares at date $t = 1$, before loan returns are realized and the bank's type is revealed, gets $(1 - \alpha)\Gamma_E(w + E)$. Substituting for the value of α in (11) and rearranging, the manager's payoff can be rewritten as

$$V_H(E) = \frac{\Gamma_E(w + E) - ER_G}{\Gamma_E}[\lambda\Gamma_E + (1 - \lambda)\Gamma_H] \tag{A6}$$

Similarly, the manager's expected payoff from issuing no equity is given by $\lambda\Gamma_0 + (1 - \lambda)\Gamma_H w$. Comparing these payoffs, we obtain condition (12).

Under the same market expectations, an L-bank manager issuing equity $E > 0$ gets an expected return $\Gamma_E(w + E) - ER_G$, and a payoff $\Gamma_0 w$ when issuing no equity. Comparing these payoffs, we obtain condition (13).

Lemma 6. *The return on equity capital Γ_E expected by investors in equilibrium is given by*

$$\Gamma_E = R_G + \frac{\delta_H(\hat{R}(p),E)\rho_H + \delta_L(\hat{R}(p),E)\rho_L - c}{\kappa} \tag{A7}$$

Proof of Lemma 6. Given the observed bank actions $\hat{R}(p)$ and E, investors' updated beliefs about the bank's type are

$$\Gamma_E = \delta_H(\hat{R}(p),E)\Gamma_H + \delta_L(\hat{R}(p),E)\Gamma_L$$

Substituting for Γ_H and Γ_L and rearranging, we obtain the desired expression.

Proof of Proposition 4. For the pooling equilibrium with $E = 0$ to exist, we must have

$$\frac{\Gamma_L^0(w + \bar{E}) - \bar{E} \cdot R_G(0)}{\Gamma_L^0}\left[\lambda\Gamma_L^0 + (1 - \lambda)\Gamma_H^0\right] \leq \left[\lambda\Gamma_0 + (1 - \lambda)\Gamma_H^0\right]w \tag{A8}$$

and

$$\Gamma_L^0(w + \bar{E}) - \bar{E}R_G(0) \leq \Gamma_0 w, \tag{A9}$$

where

$$\Gamma_0 = \bar{\Gamma} = \mu\Gamma_L^0 + (1 - \mu)\Gamma_H^0,$$

$$\Gamma_H^0 = R_G(0) + \frac{\rho_H(0) - c}{\kappa},$$

and

$$\Gamma_L^0 = R_G(0) + \frac{\rho_L(0) - c}{\kappa}.$$

It is easy to see that inequalities (A8) and (A9) both hold when $\Gamma_L^0 \leq R_G(0)$. Thus, a sufficient condition for this pooling equilibrium to exist is that

$$c + Z_0 \geq \rho_H(0)\beta, \tag{A10}$$

where

$$Z_0 = (1-\beta)v\frac{\mu w}{2\kappa}.$$

Consider next the pooling equilibrium with $E^H = E^L = \bar{E}$. Again, for this equilibrium to exist, we must have

$$\frac{\Gamma_{\bar{E}}(w+\bar{E}) - \bar{E} \cdot R_G(\bar{E})}{\Gamma_{\bar{E}}}\left[\lambda\Gamma_{\bar{E}} + (1-\lambda)\Gamma_H^{\bar{E}}\right] \geq \left[\lambda\Gamma_L^{\bar{E}} + (1-\lambda)\Gamma_H^{\bar{E}}\right]w \tag{A11}$$

and

$$\Gamma_{\bar{E}}(w+\bar{E}) - \bar{E}R_G(\bar{E}) \geq \Gamma_L^{\bar{E}}w, \tag{A12}$$

where

$$\Gamma_{\bar{E}} = \bar{\Gamma} = \mu\Gamma_L^{\bar{E}} + (1-\mu)\Gamma_H^{\bar{E}},$$

$$\Gamma_H^{\bar{E}} = R_G(\bar{E}) + \frac{\rho_H(\bar{E}) - c}{\kappa},$$

and

$$\Gamma_L^{\bar{E}} = R_G(\bar{E}) + \frac{\rho_L(\bar{E}) - c}{\kappa}.$$

Note that inequalities (A11) and (A12) both hold when $\Gamma_{\bar{E}} \geq R_G(\bar{E})$. The latter inequality in turn is equivalent to

$$c \leq (1-\mu)\rho_H(\bar{E}) + \mu\rho_L(\bar{E})$$

or

$$c + Z_{\bar{E}} \leq (1-\mu+\mu\beta)\rho_H(\bar{E}),$$

where

$$Z_{\bar{E}} = (1-\beta)v\frac{\mu}{2}\left(\frac{w+\bar{E}}{\kappa}\right).$$

Thus, a sufficient condition for the pooling equilibrium with $E^H = E^L = \bar{E}$ to exist is that

$$c + Z_{\bar{E}} \leq (1-\mu+\mu\beta)\rho_H(\bar{E}). \tag{A13}$$

In sum, if (A10) holds, the pooling equilibrium with $E = 0$ exists. If, however, (A10) does not hold, but

$$\beta < \widehat{\beta} \equiv \frac{(1-\mu+\mu\beta)\rho_H(\bar{E})}{\rho_H(0)}$$

or,

Assumption 3. $\beta < \dfrac{(1-\mu)\rho_H(\bar{E})}{\rho_H(0) - \mu\rho_H(\bar{E})}$

then (A13) holds and the pooling equilibrium with $E = \bar{E}$ exists. Indeed, we then have

$$c + Z_{\bar{E}} \leq \rho_H(0)\beta \leq (1-\mu+\mu\beta)\rho_H(\bar{E}).$$

Thus, when Assumption 3 holds a pooling equilibrium always exists. What is more, both pooling equilibria (with respectively $E = 0$ and $E = \bar{E}$) coexist when

Assumption 4. $\rho_H(0)\beta < \min\{c + (1-\beta)v\dfrac{\mu w}{2\kappa}; (1-\mu+\mu\beta)\rho_H(\bar{E})\}$ (A14)

Proof of Proposition 5. We use Cramer's rule to prove the different comparative statics results in the equilibrium where $R_G > 1$. Define first the functions $\Phi(R_G,\rho_H)$ and $\Omega(R_G,\rho_H)$ from Equations (7) and (9) so that both have zero on the RHS:

$$\Phi(R_G,\rho_H) \equiv R_G + \rho_H\frac{V}{v} - \left(1 - \frac{w+E}{\kappa}\right)(V-v) - v = 0 \tag{A15}$$

$$\Omega(R_G,\rho_H) \equiv \frac{1}{2R_G} + \rho_H\frac{1}{v} + \frac{E}{\kappa} + w\left(\frac{1}{\kappa} - 1\right) + G - 1 = 0$$

Then

$$\begin{bmatrix} 1 & \frac{V}{v} \\ -\frac{1}{2R_G^2} & \frac{1}{v} \end{bmatrix}\begin{bmatrix} dR_G \\ d\rho_H \end{bmatrix} = \begin{bmatrix} -\frac{\partial\Phi}{\partial G} \\ -\frac{\partial\Omega}{\partial G} \end{bmatrix} dG$$

and

$$\begin{bmatrix} -\frac{\partial \Phi}{\partial G} \\ -\frac{\partial \Omega}{\partial G} \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

Straightforward computations then lead to

$$\frac{dR_G}{dG} = \frac{2R_G^2 V}{2R_G^2 + V} > 0,$$

$$\frac{d\rho_H}{dG} = -v\frac{2R_G^2 G}{2R_G^2 + V} < 0,$$

and

$$\frac{d(R_G + \rho_H)}{dG} = \frac{2R_G^2}{2R_G^2 + V}(V - v) > 0.$$

The effect on p^* and p^B of a change in G is thus

$$\frac{dp^*}{dG} = \frac{2R_G^2}{2R_G^2 + V} > 0$$

and

$$\frac{dp^B}{dG} = \frac{2R_G^2}{2R_G^2 + V} > 0.$$

In words, there is a shift of bank lending toward safer firms with a constant aggregate amount of bank lending lending. Thus, the effect of an increase in G is only to reduce the size of the corporate bond market. ■

A1.1 An example

We have imposed several conditions to guarantee the existence of at least one and possibly multiple pooling equilibria (with maximal bank lending and no junk-bond financing). The reader may wonder whether the set of parameters that satisfy Assumptions 1, 2, and 3 is an empty set and whether there are indeed parameter values for which multiple pooling equilibria may exist. For this reason, we provide a numerical example below in which multiple pooling equilibria, with maximal bank lending and no junk-bond financing, exist.

Let the value of corporate projects be $V = 2$ and $v = \frac{1}{2}$. Next, let bank characteristics be $\kappa = 0.08$, $w = 0.01$, $\bar{E} = 0.01$, $\mu = 0.1$, $c = 0.1$, $\lambda = 0.5$, and $\beta = 0.5$. Set government borrowing at $G = 0.1$.

Then, straightforward calculations yield

- $\frac{\bar{E}+w}{\kappa} = 0.25$,
- $A(0) = 0.90625$,
- $B(0) = 0.785$,
- $A(0) - B(0) = 0.12125$,
- $A(\bar{E}) = 0.8125$,

- $B(\bar{E}) = 0.66$, and
- $A(\bar{E}) - B(\bar{E}) = 0.1525$.

Combining (7) and (9), we obtain a quadratic equation:

$$-R_G^2 + [\mathcal{A}(E) - \mathcal{B}(E)]VR_G + \frac{V}{2} = 0$$

and the solution:

- $R_G(0) = 1.128573961$ for $E = 0$ and
- $R_G(\bar{E}) = 1.164061293$ for $E = \bar{E}$.

Replacing for R_G in either (7) or (9) then yields

- $\rho_H(0) = 0.17098151$ and therefore $\rho_L(0) = 0.054240755$ and
- $\rho_H(\bar{E}) = 0.115234677$ and therefore $\rho_L(\bar{E}) = 0.026367338$.

For these values of bank spreads, L-banks prefer to lend to firms rather than to invest in the financial market. This can be seen by observing that the spread $\rho_L(E,p_L(E))$ on the riskiest firm $p_L(E)$ is positive for these numbers:

$$\rho_H(E) - [1 - p_L(E)](1 - \beta)v \geq 0 \text{ for } E = 0,\bar{E}.$$

To verify these two inequalities, we begin by computing $p_L(0)$ and $p_L(\bar{E})$ from the formula

$$p_L(E) = 1 - \frac{\rho_H(E)}{v} - \mu\left(\frac{w + E}{\kappa}\right)$$

and get $p_L(0) = 0.87901849$ and $p_L(\bar{E}) = 0.922265323$. From these values, we then check that the spread $\rho_L(E,p_L(E))$ on the riskiest firm is positive for $E = 0,\bar{E}$. For the parameter values of our example, we get $\rho_L(0,\, p_L(0)) = 0.023995377$ and $\rho_L(0,\, p_L(0)) = 0.006933669$, which are both positive.

Using these numbers, it is also easy to check that Assumptions 1 and 2 are satisfied and that

$$p_B(0) < \frac{R_G(0)}{V} \text{ and } p_B(\bar{E}) < \frac{R_G(\bar{E})}{V}.$$

Assumption 3 also holds because $\beta\rho_H(0) = 0.085490755$ while $(1 - \mu + \mu\beta)\rho_H(\bar{E}) = 0.109472943$.

Finally to check the existence of the two pooling equilibria, we compute the returns on bank equity:

- $\Gamma_H^0 = 2.015842833$,
- $\Gamma_L^0 = 0.556583397$,
- $\Gamma_0 = 1.869916889$,
- $\Gamma_H^{\bar{E}} = 1.354494752$,
- $\Gamma_L^{\bar{E}} = 0.243653023$, and
- $\Gamma_{\bar{E}} = 1.243410579$.

For these returns, the pooling equilibrium with $E = 0$ exists, if

$$\frac{\Gamma_L^0(w+\bar{E}) - \bar{E}\cdot R_G}{\Gamma_L^0}\left[\lambda\Gamma_L^0 + (1-\lambda)\Gamma_H^0\right] \le \left[\lambda\Gamma_0 + (1-\lambda)\Gamma_H^0\right]w \tag{A16}$$

and

$$\Gamma_L^0(w+\bar{E}) - \bar{E}R_G(0) \le \Gamma_0 w \tag{A17}$$

The first condition is satisfied, as

$$\frac{\Gamma_L^0(w+\bar{E}) - \bar{E}\cdot R_G}{\Gamma_L^0}\left[\lambda\Gamma_L^0 + (1-\lambda)\Gamma_H^0\right] - \left[\lambda\Gamma_0 + (1-\lambda)\Gamma_H^0\right]w = -0.019784844.$$

The second condition is also satisfied, because $\Gamma_L^0 - R_G(0) < 0$.

Finally, the pooling equilibrium with $E^H = E^L = \bar{E}$ also exists if

$$\frac{\Gamma_{\bar{E}}(w+\bar{E}) - \bar{E}\cdot R_G(\bar{E})}{\Gamma_{\bar{E}}}\left[\lambda\Gamma_{\bar{E}} + (1-\lambda)\Gamma_H^{\bar{E}}\right] \ge \left[\lambda\Gamma_L^{\bar{E}} + (1-\lambda)\Gamma_H^{\bar{E}}\right]w \tag{A18}$$

and

$$\Gamma_{\bar{E}}(w+\bar{E}) - \bar{E}R_G(\bar{E}) \ge \Gamma_L^{\bar{E}}w. \tag{A19}$$

The first inequality is satisfied, as

$$\frac{\Gamma_{\bar{E}}(w+\bar{E}) - \bar{E}\cdot R_G(\bar{E})}{\Gamma_{\bar{E}}}\left[\lambda\Gamma_{\bar{E}} + (1-\lambda)\Gamma_H^{\bar{E}}\right] - \left[\lambda\Gamma_L^{\bar{E}} + (1-\lambda)\Gamma_H^{\bar{E}}\right]w = 0.005827725$$

And the second condition also holds as $\Gamma_{\bar{E}} > R_G$.

Appendix A2: Non-Binding Capital Adequacy Constraints

In practice, most banks have an equity capital base in excess of the BIS equity capital requirement. If equity capital involves a higher cost of capital than other sources of external funding, the obvious question arises as to why banks hold equity capital in excess of regulatory requirements.

The answer seems to be that banks want to maintain a lending capacity to be able to meet unexpected new lending opportunities or to be able to carry out future loan commitments. We can model this idea by assuming that when a firm debt is to be renegotiated, it involves a complete restructuring with an additional cash injection that we assume equal to l_1. Obviously, if the firm is successful at time 1, no additional external funds are required. Assumptions *A*0 and 1 have to be adapted then:

Assumption A0 stating that the average project has a positive net present value has to be modified as follows:

Assumption A0. $\pi_H + \nu\pi_H - (1 - p_1)l_1 > 1 + \omega$.

For simplicity, we assume that banks can only issue equity at time 0. (In practice, it is difficult to tap the equity market too often.) Then, the bank is able to compute its time 1 capital constraint

$$\frac{E_1}{L_1} \geq \kappa \geq 0,$$

where L_1 is the amount of outstanding loans at time 1 [which depends on the expected amount of additional loans, $\int_{p_1^B}^{p_1^*}(1 - p_1)l_1 dp_1$, the nominal rate on these loans, the expected repayment, $\pi_H \int p_1 dp_1$, as well as the expected loan losses $(L_0 - A)(1 - \nu)\int p_1 dp_1)$] and E_1 is the equity capital base at time $t = 1$, which includes time 0 net profits if we assume a zero dividend at time $t = 0$.

Depending on the expected amount of profit, additional loans, and loan losses, the binding capital constraint will be the one at time $t = 1$ while it will not be binding at time $t = 0$. Thus, we will observe equity capital slack because of the profitability of making additional loans to good firms in distress. Notice therefore that the slack will depend on the business cycle because the proportion of good firms in distress, the proportion of repayments, and the banks profits themselves depend on the business cycle.

There is an additional condition that is required for banks to perform their role in the loan renegotiation process——that is, a loan to a firm in distress is more profitable than a new loan to the average firm:

$$\frac{\pi_H}{l_1} \geq \rho_H.$$

This condition which is absolutely natural when we think in terms of the incentives and of the credibility of banks to renegotiate their loans is also interesting, as it shows banks benefiting from their captive unlucky borrowers. This condition will always hold for high cash flows π_H and low cost overruns l_1 and low future expected profitability on new loans ρ_H.

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