Solutions to Midterm
1.a. The question asks for the probability that an A-rated bond will default. From the transition matrix we find the answer is 0.03.
1.b. An A-rated bond has probability 0.03 of defaulting and a B-rated bond has probability 0.10. Since 60% of the bonds are A-rated and 40% are B-rated, the probability that a randomly chosen bond defaults is $0.60 \cdot 0.03 + 0.40 \cdot 0.10 = 0.058$.
1.c. Expected value for an A-rated bond is $100 \cdot 0.92 + 75 \cdot 0.05 + 50 \cdot 0.03 = 97.25$.
For a B-rated bond, expected value is $100 \cdot 0.10 + 75 \cdot 0.80 + 50 \cdot 0.10 = 75$.
Thus, $E[\text{Portfolio}] = 60 \cdot 97.25 + 40 \cdot 75 = 8835$.
1.d. There are four possible outcomes, depending on who defaults:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Payoff</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Big only</td>
<td>20</td>
<td>0.1 \cdot 0.9</td>
</tr>
<tr>
<td>Bob only</td>
<td>20</td>
<td>0.9 \cdot 0.1</td>
</tr>
<tr>
<td>Both</td>
<td>20</td>
<td>0.1 \cdot 0.1</td>
</tr>
<tr>
<td>Neither</td>
<td>0</td>
<td>0.9 \cdot 0.9</td>
</tr>
</tbody>
</table>

Multiplying across and adding, we get 3.8.
2.a. Leisure bids are $N(150, (20)^2)$. Need to find $x$ so that $P(X > x) = 0.75$. From the table we find that the $z$ value for 0.75 is 0.67 so the minimum bid should be 0.67 standard deviations below the mean: $x = 150 - 0.67 \cdot 20 = 136.60$.
2.b. Need to find $P(X > 136.60)$ where now $X \sim N(220, (45)^2)$. Standardizing, we find that this is the same as $P(Z > (220 - 136.6)/45) = P(Z > -1.85)$, $Z \sim N(0, 1)$. From the table we find this is 0.9678.
3.b. $Var[Y] = \sigma^2_G + \sigma^2_M + 2\sigma_G\sigma_M\rho$ with $\sigma_G = 25$, $\sigma_M = 22$, and $\rho = 0.38$. Taking the square root, we get a standard deviation of 39.08.
3.c. Need to find $P(X_G > X_M)$ which is $P(X_G - X_M > 0)$. So, now let $Y = X_G - X_M$. 

$E[Y] = 228 - 312 = -74$ and 

$\sigma_Y = \sqrt{\sigma^2_G + \sigma^2_M - 2\sigma_G\sigma_M\rho} = 26.29$.

Standardizing, we get $P(Y > 0) = P(Z > 74/26.29) = P(Z > 2.81) = 1 - 0.9975 = 0.0025$.
3.d. (i) Increasing the correlation decreases $\sigma_Y$ (see 3c) and therefore decreases the probability in 3c. Intuitively, if good years in Guatemala coincide with good years in Mexico, it becomes harder for Guatemala to outproduce Mexico.