Value at Risk (D)

Multipliers

The attached table, taken from an article published in RISK magazine in 1996, displays risk measures disclosed by twenty major derivatives dealers in their 1995 annual reports. There are almost as many methods of measuring risk listed as there are dealers. How can these diverse measures be compared?

Value at risk, earnings at risk (EAR), daily earnings at risk (DEAR), and daily price volatility (DPV) have closely related interpretations. It is often possible to convert one of these measures into one of the others. But even if we focus on just the pure VaR measures, we cannot immediately compare risks across banks because of variations in the “confidence” levels applied (95%, 97.5%, and 99%), and differences in the VaR horizon used (1 day or 10 days). Is it possible to convert all VaR measures to a standard scale (e.g., 10 days, 99%) in order to be able to compare the risks reported by JPMorgan, Swiss Bank Corp, Bank of Tokyo, and Dresdner Bank?

Changing Confidence

For purposes of discussion, we will assume that each bank reports its VaR using the normal distribution as a model of its profit and loss distribution—i.e., the banks apply the RiskMetrics methodology. In this case, the VaR is simply a multiple of the standard deviation of the P&L. The appropriate multiplier is completely determined by the specified confidence level. The precise value required for each confidence level can be found from a table of the normal distribution.

Figure 1 from Risk Management: A Practical Guide, published by the RiskMetrics Group, illustrates the multipliers for three confidence levels: 90%, 95%, and 99%. Let’s use these values to convert HSBC’s reported risk from 95% confidence to 99% confidence.

At 95% confidence, the table shows a VaR of $26 million. From Figure 1, we see that the multiplier for 95% is 1.65. Thus,

\[
\text{HSBC 95% VaR} = 1.65 \sigma_{\text{HSBC}} = \$26 \text{ million},
\]

where \( \sigma_{\text{HSBC}} \) denotes the standard deviation of the bank’s one-day P&L. From this we see that

\[
\sigma_{\text{HSBC}} = \frac{\$26 \text{ million}}{1.65} = \$15.76 \text{ million}.
\]

From Figure 1 we also see that the multiplier for 99% is 2.33:

\[
\text{HSBC 99% VaR} = 2.33 \sigma_{\text{HSBC}}.
\]

Thus,

\[
\text{HSBC 99% VaR} = 2.33 \times \$15.76 \text{ million} = \$36.72 \text{ million}.
\]

We have converted a 95% VaR to a 99% VaR.

A bit more generally, the steps above show that

\[
99\% \text{ VaR} = \frac{2.33}{1.65} \times 95\% \text{ VaR}.
\]
Similarly, 

$$99\% \text{ VaR} = \frac{2.33}{1.96} \times 97.5\% \text{ VaR},$$

because the multiplier associated with 97.5% is 1.96.\(^1\)

**Changing Horizon**

It is reasonable to expect that a portfolio’s 10-day VaR should be larger than its 1-day VaR: a longer time horizon allows for a greater range of market movements and thus a greater probability of a large profit or loss. But just how much larger should the 10-day VaR be?

A natural first guess is that the 10-day VaR should be 10 times as large as the 1-day VaR. Further thought reveals, however, that this overstates the increase in risk over time—it ignores *time diversification*. Over a 10-day horizon, some days are likely to be good and others bad; to some extent, profits and losses will cancel each other out, resulting in a slower (i.e., slower than linear) increase in variability.

To make this idea more precise, let \(X_1, \ldots, X_{10}\) be the daily changes in value of a fixed portfolio over a 10-day period. The total change in the value of the portfolio over the 10 days is given by \(X_1 + \cdots + X_{10}\). Suppose that

- all \(X_i\) have the same standard deviation \(\sigma\);
- \(X_1, \ldots, X_{10}\) are independent of each other (i.e., daily price changes are independent).

At least over a short horizon, these assumptions represent a reasonable approximation of reality.

In light of the first assumption, the 1-day standard deviation is \(\sigma\); we can convert this to a 1-day VaR using an appropriate multiplier. What is the 10-day standard deviation? First observe that the assumed independence implies that

$$\text{Var}[X_1 + \cdots + X_{10}] = \text{Var}[X_1] + \cdots + \text{Var}[X_{10}].$$

Because all of the standard deviations are assumed equal, we have

$$\text{Var}[X_1] + \cdots + \text{Var}[X_{10}] = 10\sigma^2.$$

Taking square roots to get the standard deviation, we find that

$$\text{StdDev}[X_1 + \cdots + X_{10}] = \sqrt{10}\sigma.$$

In short, the 10-day standard deviation is \(\sqrt{10} (\approx 3.16)\) times larger than the 1-day standard deviation.

Exactly, the same argument applies to an arbitrary \(n\)-day horizon. The \(n\)-day standard deviation is \(\sqrt{n}\) times larger than the 1-day standard deviation.

\(^1\)This can be verified by looking up 0.975 in the body of the normal table. Alternatively, one can use NORMSINV(0.975) in Excel.
### 2. Risk disclosures for 20 major derivatives dealers based on fiscal year-end 1995 annual reports

<table>
<thead>
<tr>
<th>Date</th>
<th>Dealer</th>
<th>National Fair market value assets</th>
<th>Credit risk exposure</th>
<th>Market risk Method</th>
<th>Risk</th>
<th>Average Artificial VAR 1 day 1% 99%</th>
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</table>

**Notes:**
- **Credit Risk:** The table shows the credit risk exposure of major derivatives dealers as of fiscal year-end 1995. The exposure includes both principal and future exposure.
- **Market Risk:** The market risk is measured using various methods, including artificial VAR (Value at Risk) for different holding periods and confidence levels.
- **Exchange-Risk:** Exchange-risk derivatives are not included in this table.

**Exchange-Risk Derivatives:**
- Discussion on exchange-risk derivatives.

**Exchange-Risk:** Exchange-risk derivatives are calculated by examining the potential impact of exchange rate fluctuations on the dealers' positions.

**FAQ:**
- Questions related to exchange-risk derivatives and their calculation.

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**Figure 2:** Table of risk disclosures, from RISK magazine.