Internet Appendix for: Investor Information Choice with Macro and Micro Information

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1 Effect of f_M and f_S on attention equilibrium

Recall that for a fixed λ_U, the equilibrium λ_M^* (proportion of macro-informed) is determined by the condition $J_M/J_U = J_S/J_U$, in the case of an interior equilibrium. Because J_M/J_U does not depend on f_S, as f_S rises and J_S/J_U increases, J_M/J_U can increase only if λ_M increases (from Proposition 5.2). An interior equilibrium λ_M^* must therefore increase with f_S. As the benefit of being micro-informed falls due to more precise micro information, the fraction of informed investors that focus on macro information grows. The left panel of Figure 4 from the paper demonstrates this adjustment. For every λ_M, a higher f_S makes the micro-informed worse off, which pushes the equilibrium number of macro-informed higher.

Similarly, since J_S/J_U does not depend on f_M but decreases in λ_M (i.e., the micro-informed are better off as there are fewer micro-informed), if J_M/J_U decreases (increases) in f_M, then λ_M^* must increase (decrease) in f_M. The right panel of Figure 4 from the paper illustrates this phenomenon. In the figure, the equilibrium λ_M^* is sufficiently small so that macro precision makes the macro-informed better off. As macro precision f_M increases, for a range of sufficiently small λ_M, the macro-informed become better off, which increases λ_M^* (i.e., decreases the number of micro-informed, thus making the remaining micro-informed better off). Had the equilibrium λ_M been sufficiently high, the effect would have had the opposite sign.

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The preceding arguments establish the following result:

Proposition 1.1 (Effect of information precision on equilibrium)**.** *In the case of an interior equilibrium with $\lambda_M^* > 0$, the number of macro-informed increases as the micro signal becomes more precise:*

$$\frac{d\lambda_M^*}{df_S} > 0.$$

Condition (38) is necessary and sufficient for the number of macro-informed to increase as the macro signal becomes more precise:

$$\frac{d\lambda_M^*}{df_M} > 0 \ (< 0) \quad \text{if and only if} \quad \rho_F^2 < \frac{1}{1+f_M} \quad \left(> \frac{1}{1+f_M}\right).$$

2 Calibration

2.1 Supply shocks and turnover

In our model, equal weighted index turnover is given by $1/N \sum_{i=1}^{N} |X_F - \bar{X}_F + X_i|$. Using standard results, we get

$$\frac{1}{N}\sum_{i=1}^{N} |X_F - \bar{X}_F + X_i| = \sqrt{\frac{2}{\pi}} \times \sqrt{\sigma_{X_F}^2 + \sigma_X^2}.$$

Direct turnover for stock i is

$$\frac{1}{N}|X_F - \bar{X}_F + X_i|$$

and direct index turnover is

$$\frac{1}{N}\sum_{j=1}^{N} |X_F - \bar{X}_F + X_j|.$$

The conditions we want to satisfy are

$$E\left[\frac{1}{N}\sum_{j=1}^{N} |X_F - \bar{X}_F + X_j|\right] = 0.76$$

and

$$\rho^2(|X_F - \bar{X}_F + X_i|, \sum_{j=1}^{N} |X_F - \bar{X}_F + X_j|) = 0.47.$$

Writing $Y_j = X_F - \bar{X}_F + X_j$, we can express these conditions as

$$E|Y_j| = 0.76$$

and

$$\rho^2(|Y_i|, \sum_j |Y_j|) = 0.47. \tag{1}$$

As $\text{var}[Y_j] = \sigma^2_{X_F} + \sigma^2_X$, the first of these conditions gives

$$\sqrt{\frac{2}{\pi}}\sqrt{\sigma^2_{X_F} + \sigma^2_X} = 0.76 \Rightarrow \sigma^2_{X_F} + \sigma^2_X = .9073. \tag{2}$$

For the second equation, we have

$$\begin{aligned}
\rho(\sum_j |Y_j|, |Y_i|) &= \frac{\text{cov}[\sum_j |Y_j|, |Y_i|]}{\sigma(\sum_j |Y_j|)\sigma(|Y_i|)} \\
&= \frac{\text{cov}[\sum_{j\neq i} |Y_j|, |Y_i|] + \text{var}[|Y_i|]}{\sigma(\sum_j |Y_j|)\sigma(|Y_i|)} \\
&= \frac{(N-1)\rho(|Y_i|, |Y_j|)\sigma^2(|Y_i|) + \sigma^2(|Y_i|)}{\sigma(\sum_j |Y_j|)\sigma(|Y_i|)} \\
&= \frac{(N-1)\rho(|Y_i|, |Y_j|)\sigma^2(|Y_i|) + \sigma^2(|Y_i|)}{[N\sigma^2(|Y_i|) + N(N-1)\rho(|Y_i|, |Y_j|)\sigma^2(|Y_i|)]^{1/2}\sigma(|Y_i|)} \\
&= \frac{(N-1)\rho(|Y_i|, |Y_j|) + 1}{[N + N(N-1)\rho(|Y_i|, |Y_j|)]^{1/2}} \\
&\rightarrow \rho(|Y_i|, |Y_j|)^{1/2}, \quad \text{as } N \rightarrow \infty.
\end{aligned}$$

We think of the number of stocks N as large, and passing to the limit simplifies the calculation. With this simplification, (1) becomes

$$\rho(|Y_i|, |Y_j|) = 0.47.$$

From formula (1,1,0) in Kamat (1958), p.26, we find that if (ξ_1, ξ_2) are bivariate normal with correlation ρ, then

$$\rho(|\xi_1|, |\xi_2|) = \frac{2}{\pi - 2}(\rho \arcsin(\rho) + \sqrt{1-\rho^2} - 1) \equiv \text{Kamat}(\rho).$$

Thus, we need

$$\rho(Y_i, Y_j) = \text{Kamat}^{-1}(0.47) = 0.714452.$$

But

$$\rho(Y_i, Y_j) = \frac{\sigma_{X_F}^2}{\sigma_{X_F}^2 + \sigma_X^2}. \tag{3}$$

Combining this with (2) we get

$$\sigma_{X_F}^2 = 0.714452 \times .9073 = .6482, \quad \sigma_X^2 = .9073 - .6482 = .2591,$$

and then

$$\sigma_{X_F} = .8051, \quad \sigma_X = .5090.$$

In this derivation, we have ignored the correlation $-1/(N-1)$ between idiosyncratic supply shocks X_i and X_j. Those correlations vanish if we take $N \to \infty$ (as we do above) and are negligible for large but finite N.

2.2 ETF and futures trading volume

Table 1 shows the annualized trading volume of the top ETFs and equity index futures on a representative trading day.

Name	Share Volume (daily)	Share Price	$ Traded per Year (trlns)
SPDR S&P 500 ETF TRUST	70,441,296	278.76	4.95
INVESCO QQQ TRUST SERIES 1	30,667,280	176.33	1.36
ISHARES RUSSELL 2000 ETF	17,498,722	166.99	0.74
ISHARES MSCI EMERGING MARKET	64,918,536	45.95	0.75
FINANCIAL SELECT SECTOR SPDR	49,738,832	28.07	0.35
ISHARES MSCI EAFE ETF	21,618,260	70.43	0.38
IPATH S&P 500 VIX S/T FU ETN	28,464,014	31.84	0.23
SPDR DJIA TRUST	3,857,796	253.06	0.25
ISHARES CHINA LARGE-CAP ETF	18,915,848	47.39	0.23
ISHARES CORE S&P 500 ETF	3,017,272	280.73	0.21
ISHARES IBOXX USD HIGH YIELD	13,330,054	85.91	0.29
TECHNOLOGY SELECT SECT SPDR	12,426,255	72.07	0.23
PROSHARES ULTRAPRO QQQ	12,553,372	62.10	0.20
ENERGY SELECT SECTOR SPDR	15,883,723	76.47	0.31
INDUSTRIAL SELECT SECT SPDR	10,715,862	76.04	0.21
		Total/Day (blns)	42.34
		Total/Yr (trlns)	10.67
		Russell 3000 Mkt Cap (trlns)	30.49
		% Turnover	35.0%

Name	Volume	Multiplier	Price	$ Annual Volume (trlns)
S&P 500 FUTURE Sep18	2,057	250	2,791.00	0.36
S&P 500 FUTURE Dec18	102	250	2,797.20	0.02
S&P500 EMINI FUT Sep18	189,059	50	2,791.50	6.65
S&P500 EMINI FUT Dec18	1,066	50	2,794.75	0.04
DJIA MINI e-CBOT Sep18	15,982	5	25,316.00	0.51
DJIA MINI e-CBOT Dec18	13	5	25,332.00	0.00
			Total/Day (blns)	30.07
			Total/Yr (trlns)	7.58
			Russell 3000 Mkt Cap (trlns)	30.49
			% Turnover	24.9%

Table 1: On June 13, 2018 we look at the top 15 most heavily traded ETFs (according to ETF.com), as well as the first two S&P500, S&P500 e-mini and Dow futures. The average daily trading volume over the prior 30 days for these 21 instruments represented an annualized dollar turnover of $18.25 trillion. Compared to the market capitalization of the Russell 3000 index of $30.49 trillion, this represents an annual turnover of 60%.

2.3 M as present value of dividends

Say monthly dividends follow

$$D_{t+1} = \mu_D + \rho D_t + \epsilon_{t+1},$$

with $\mu_D = 1 - \rho$ so the long run dividend level is 1 and $var(\epsilon) = \sigma^2$. Let us define the N period dividend at time $t+1$ as

$$M \equiv D_{t+12} + \beta D_{t+13} + \cdots + \beta^{N-1} D_{t+11+N}. \tag{4}$$

We think of this as the present value of the future N dividends starting at time $t+12$. We also assume that the first dividend follows the process $D_{12} = \mu_D + \rho D_0 + \epsilon_{12}$. If $D_0 = 1$, we can show that

$$E_t[M] = \frac{1-\beta^N}{1-\beta}.$$

The variance is given by

$$var_t(M) = \frac{\sigma^2}{(1-\beta\rho)^2}\left(\frac{1-\beta^{2N}}{1-\beta^2} - 2(\beta\rho)^N \frac{1-(\beta/\rho)^N}{1-\beta/\rho} + (\beta\rho)^{2N} \frac{1-(1/\rho)^{2N}}{1-(1/\rho)^2}\right).$$

Since we are normalizing the dividend level to 1, we are interested in the volatility per unit of expected dividend as N grows. Figure 1 shows the expectation, variance, volatility and normalized volatility

$$volatility\ of\ normalized\ dividend = \frac{\sqrt{var_t(M)}}{E_t[M]}$$

as a function of N.

2.4 Calibration of f_M

Tables 2 and 3 show results of regressing

$$\frac{\overline{CF}[t, t+x] - CF[t]}{Book[t]}$$

on day t explanatory variables where $CF[t]$ is either last twelve month earnings or dividends of the S&P500 index on day t and $\overline{CF}[t, t+x]$ is the average $CF[t]$ over years $t+1, t+2, \cdots, t+x$. The regressions are run with overlapping daily data starting in

January 31, 1990. Data are obtained from Bloomberg.

2.5 Properties of equilibrium

Figures 2 and 4 shows equilibrium quantities for the model calibration in Section 7.1 of the paper for $\ell = 1, 2$ respectively. Figure 5 shows the number of macro and micro informed investors for the $\ell = 1, 2$ versions of the model calibration.

2.6 Trend in realized S&P500 volatility

Figure 6 shows the time trend in annual S&P500 volatility, using daily overlapping observations. Data are obtained from Bloomberg. The Newey-West t-statistic uses automatic lag selection.

3 Proof that attention in Kacperczyk et al. (2016) is weakly increasing in number of informed

We use the following notation, based on Kacperczyk et al. (2016):

$$\bar{\sigma}_i = \bar{\sigma}_i(\chi, k) = \left(\frac{1}{\sigma_i} + \chi k + \frac{\chi^2 k^2}{\rho^2 \sigma_x} \right)^{-1}$$

$$\lambda_i = \lambda_i(\chi, k) = \bar{\sigma}_i[1 + (\rho^2 \sigma_x + \chi k)\bar{\sigma}_i] + \rho^2 \bar{x}_i^2 \bar{\sigma}_i^2$$

The "true" λ functions depend on χ and k only through their product, but it will be useful to keep these arguments separate. We know from Kacperczyk et al. (2016) that λ_i is strictly decreasing in χ if $k > 0$ and strictly decreasing in k if $\chi > 0$.

Hold the total capacity K fixed and vary χ. Let $\{K_i(\chi)\}$ be equilibrium attention allocations at χ.

Proposition 3.1. *Each $\chi K_i(\chi)$ is increasing in χ. Consequently, each variance ratio is decreasing in χ.*

Proof. The second statement follows from the first using (A.15), which shows that variance ratios are decreasing in $\bar{K}_i$.

Write M_χ for the set of assets that attain the maximal λ: $i \in M_\chi$ iff $\lambda_i(\chi, K_i(\chi)) = \max_k \lambda_k(\chi, K_k(\chi))$. Let $0 < x < y$ be two values of χ. To argue by contradiction, suppose that for some asset i,

$$xK_i(x) > yK_i(y). \tag{5}$$

This requires $K_i(x) > 0$, which implies $i \in M_x$. By the strict monotonicity of λ_i, (5) implies $\lambda_i(x, K_i(x)) < \lambda_i(y, K_i(y))$. But then

$$\begin{aligned}
\lambda_j(x, K_j(x)) &\leq \max_k \lambda_k(x, K_k(x)) \\
&= \lambda_i(x, K_i(x)) \\
&< \lambda_i(y, K_i(y)) \\
&\leq \max_k \lambda_k(y, K_k(y)) = \lambda_j(y, K_j(y)), \quad \text{for all } j \in M_y.
\end{aligned}$$

By strict monotonicity of λ, this implies that

$$xK_j(x) > yK_j(y), \quad \text{for all } j \in M_y.$$

In equilibrium, the full capacity K is allocated, so

$$xK = x\sum_k K_k(x) \geq x \sum_{j \in M_y} K_j(x) > y \sum_{j \in M_y} K_j(y) = yK,$$

which contradicts the assumption that $x < y$. □

4 Graphing variance ratios in the model of Kacperczyk et al. (2016)

In order to graph variance ratios in the model of Kacperczyk et al. (2016), as we do in Figure 7, we need to solve the attention allocation problem in their paper, which we refer to as KVNV.

From KVNV (14) and p.605, item 4, we have, at $\bar{K}_i = k$,

$$\lambda_i(k) = \bar{\sigma}_i(k)[1 + (\rho^2\sigma_x + k)\bar{\sigma}_i(k)] + \rho^2\bar{x}_i^2\bar{\sigma}_i^2(k),$$

and

$$\bar{\sigma}_i(k) = \left(\sigma_i^{-1} + k + \frac{k^2}{\rho^2\sigma_x}\right)^{-1}.$$

We want to apply these expressions with $i = M$ or $i = S$, with

$$\sigma_i = \sigma_M^2, \sigma_S^2; \quad \rho = \gamma; \quad \sigma_x = \sigma_{X_F}^2, \sigma_{X_i}^2; \quad \bar{x}_i = \bar{X}_F, \bar{X}_S = 0.$$

Solution

If we assume a symmetric equilibrium, then in KVNV's (12)–(14) we can take $K = \bar{K}$ and $K_i = \bar{K}_i$ and drop the js. In other words, we are allocating the economy-wide capacity $\bar{K}$.

As we increase the capacity $\bar{K}$, we proceed as follows.

1. At zero capacity, we have $\bar{\sigma}_i(0) = \sigma_i$, and, at our calibration

$$\lambda_M(0) = \sigma_M^2[1 + \gamma^2\sigma_{X_F}^2\sigma_M^2] + \gamma^2\bar{X}_F^2\sigma_M^4 > \lambda_S(0) = \sigma_S^2[1 + \gamma^2\sigma_{X_i}^2\sigma_S^2].$$

   Thus, all capacity is initially allocated to M.

2. This continues until we reach the point k^* at which $\lambda_M(k^*) = \lambda_S(0)$: for $\bar{K} \in [0, k^*)$, the allocation is $\bar{K}_M = \bar{K}$ and $\bar{K}_{S_i} = 0$.

3. For $\bar{K} \geq k^*$, we will allocate capacity to M and to all N stocks in order to make all λs equal. Because all stocks have the same parameters, they will get the same allocation. So, for each $\bar{K} \geq k^*$, we need to find $k \leq \bar{K}$ such that

$$\lambda_M(\bar{K} - k) = \lambda_S(k/N),$$

   which allocates $\bar{K}_S = k/N$ to each stock and $\bar{K}_M = \bar{K} - k$ to M.

Once we have the optimal allocations $\bar{K}_M$ and $\bar{K}_S$, we can evaluate $\lambda_M(\bar{K}_M)$ and $\lambda_S(\bar{K}_S)$ and the posterior variances $\bar{\sigma}_i(\bar{K}_i)$, and from these we can calculate VR_M and VR_S.

References

Kacperczyk, M. T., S. Van Nieuwerburgh, and L. Veldkamp, 2016, "A rational theory of mutual funds' attention allocation," *Econometrica*, 84 (2), 571–626.

Kamat, A.R., 1958, "Incomplete and absolute moments of the multivariate normal distribution with some applications," *Biometrika*, 40 (1/2), 20–34.

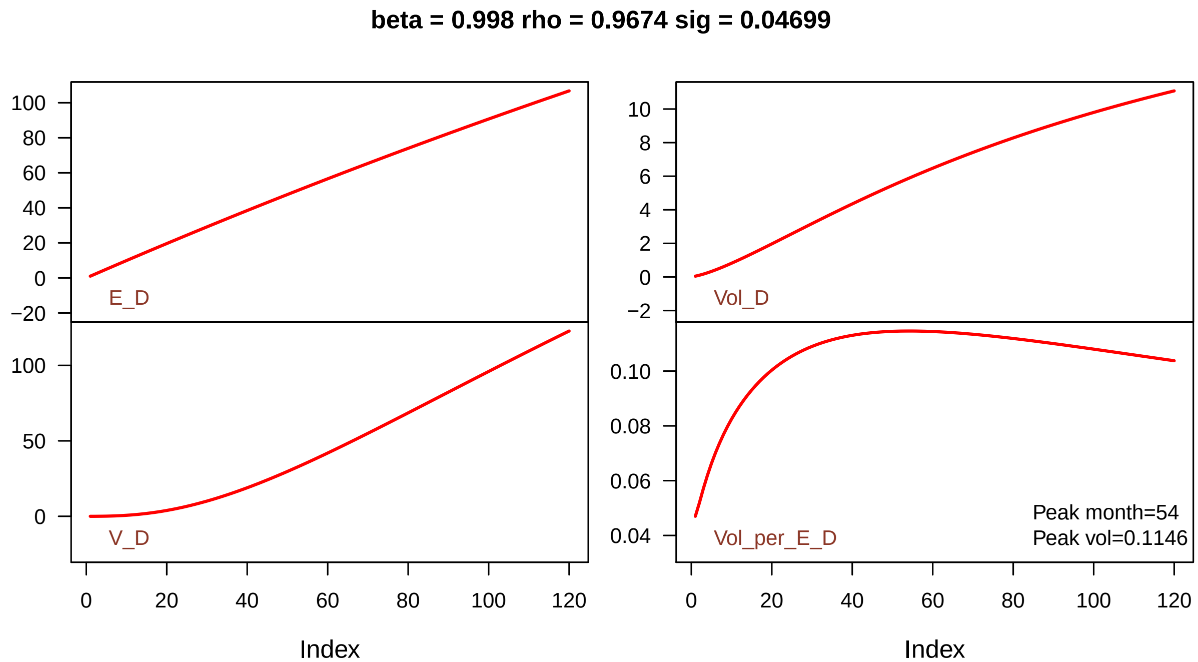

Figure 1: Expectation, variance, volatility and volatility per unit expectation of the N month discounted dividend process given in equation (4) with starting dividend $D_0 = 1$.

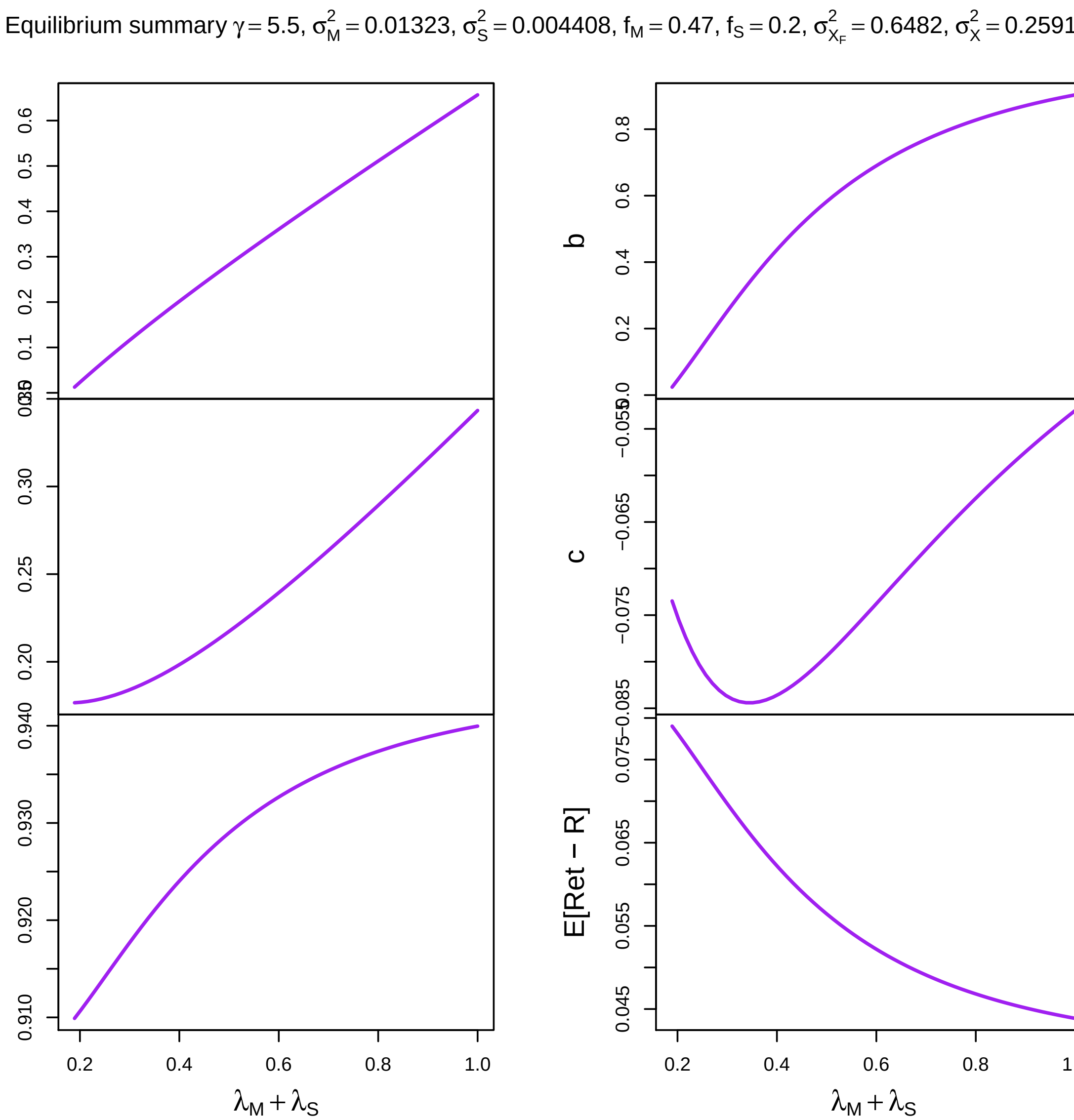

Figure 2: Properties of equlibrium with $\ell = 1$.

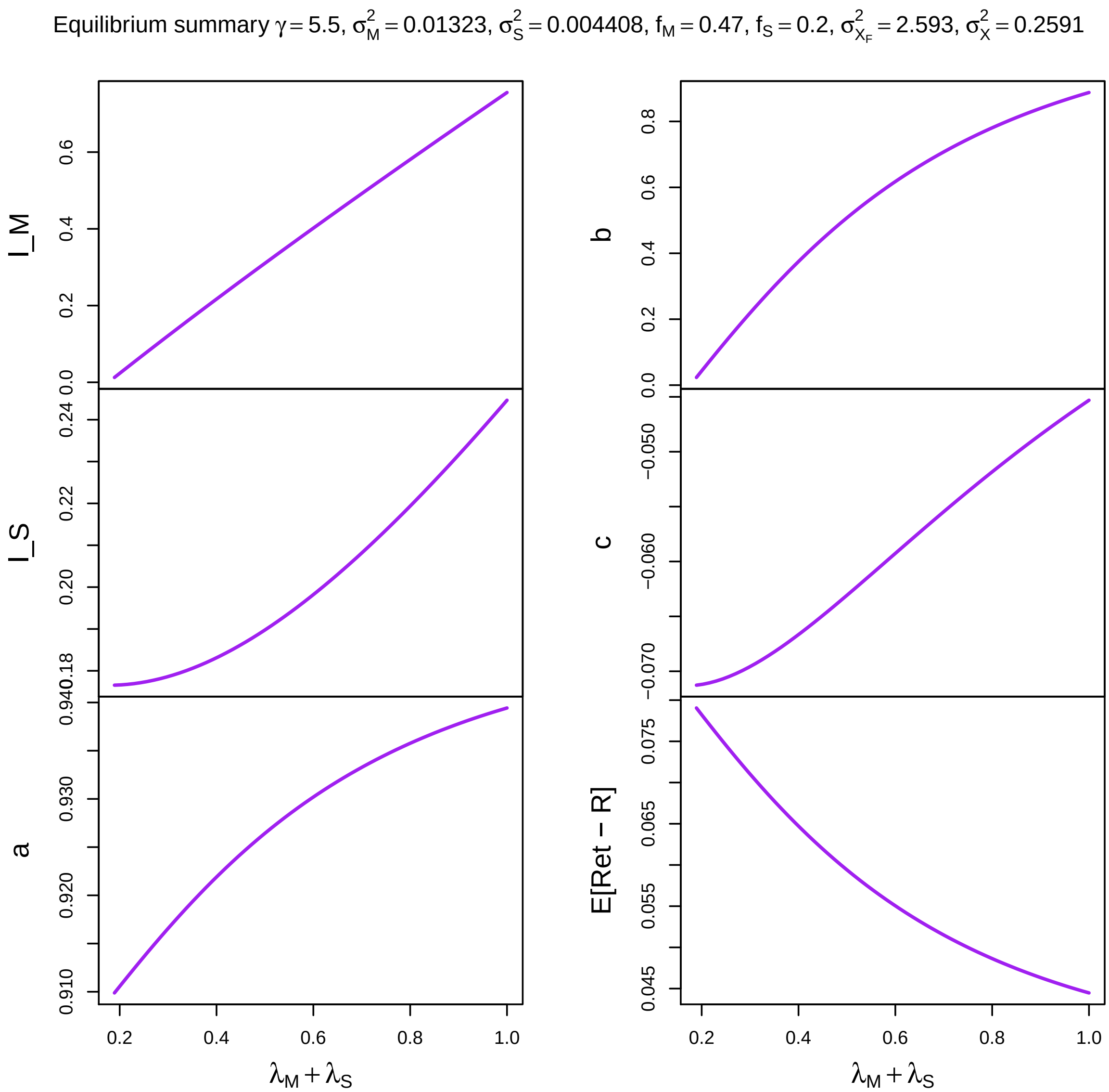

Figure 3: Properties of equlibrium with $\ell = 2$.

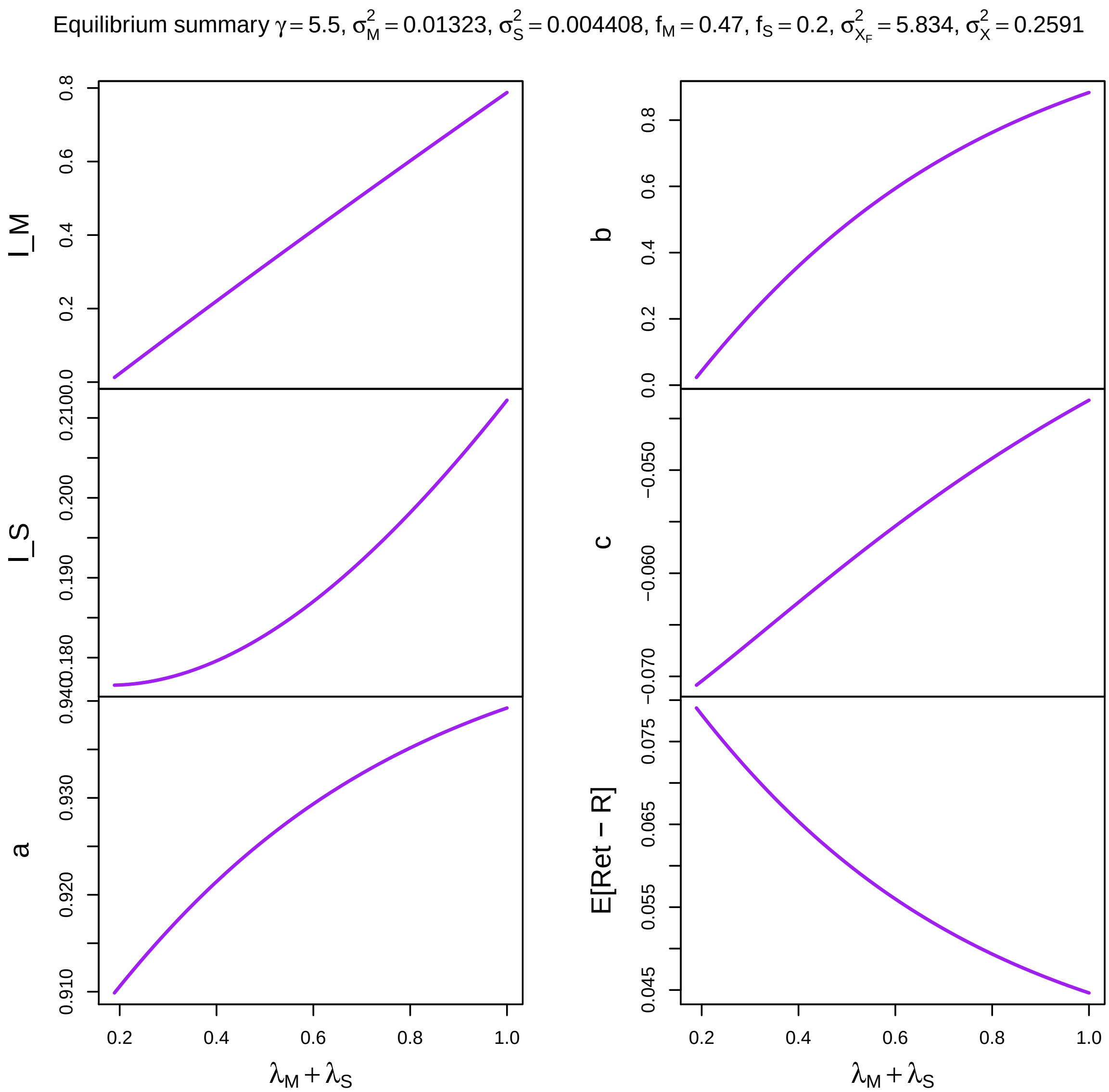

Figure 4: Properties of equlibrium with $\ell = 3$.

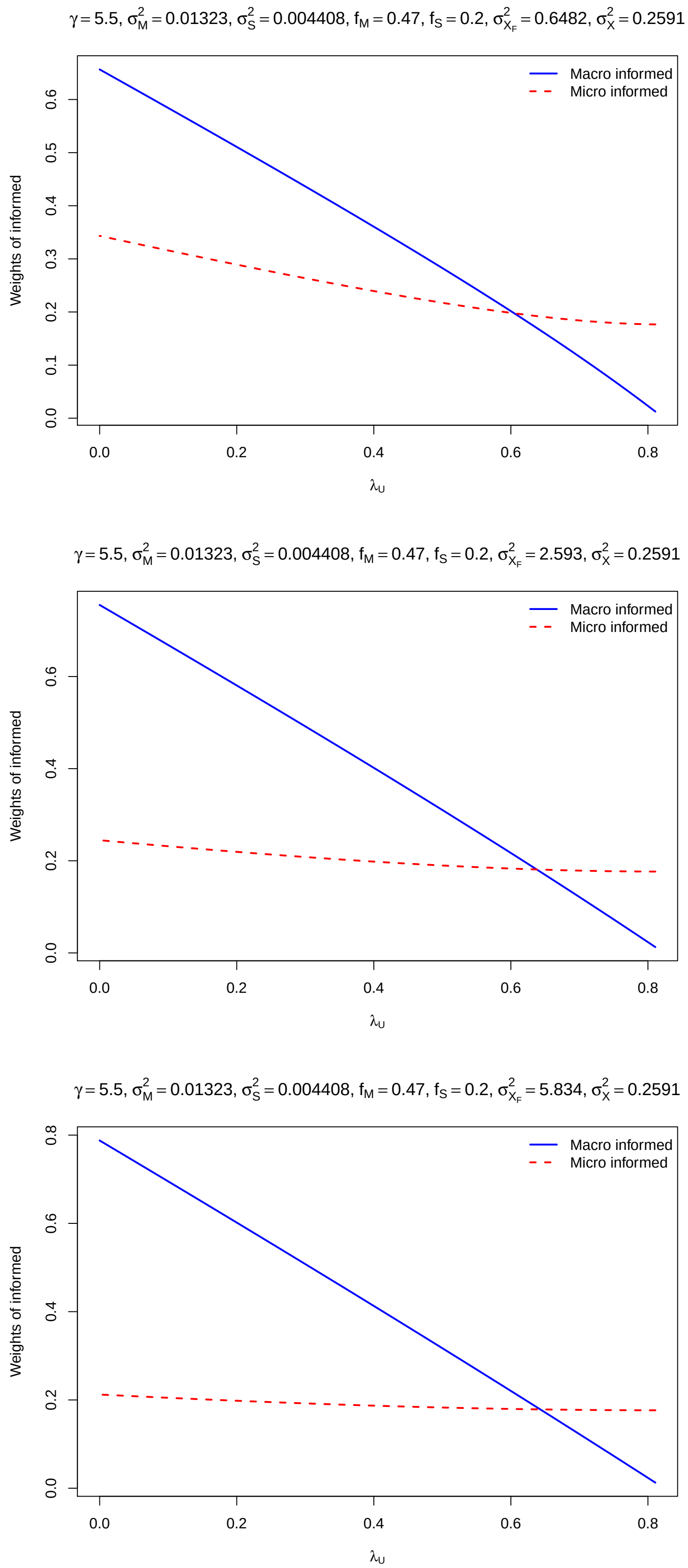

Figure 5: Number of informed in equlibrium.

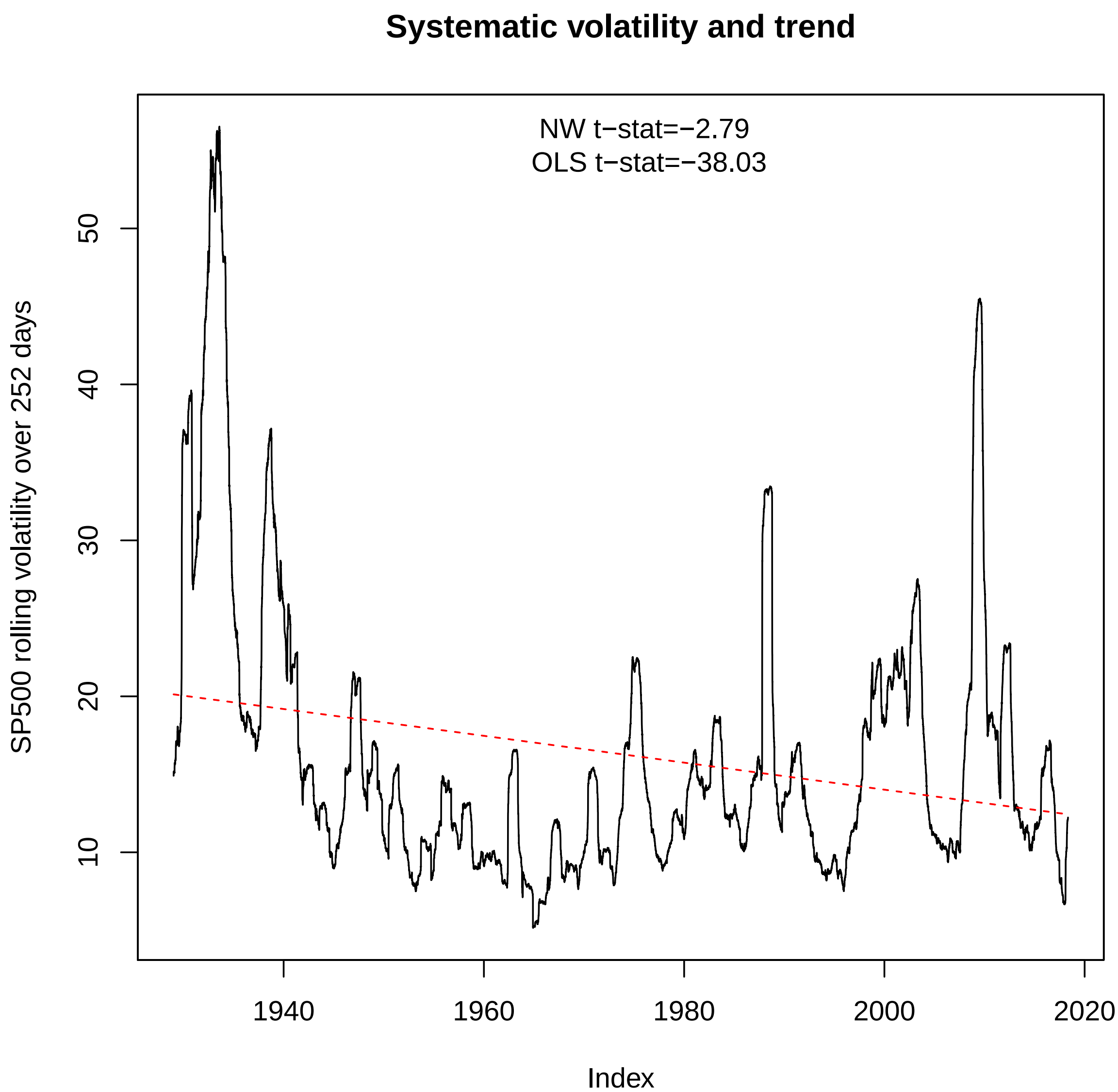

Figure 6: S&P500 volatility, daily observations of lagged annual volatility. The time trend coefficient is -3.4 basis points per day. The Newey-West t-statistic for the time trend uses auto lag selection. Volatility observations range from January 3, 1929 to July 16, 2018.

Table 2: Earnings regression.

	Dependent variable:				
	Forward average EPS				
	(1 yr)	(2 yrs)	(3 yrs)	(4 yrs)	(5 yrs)
E_dEPS	1.837^{**}	1.688	0.976	0.870	0.968
	(0.901)	(1.053)	(1.060)	(1.154)	(1.230)
E1_dEPS	−1.067	−0.911	−0.131	0.007	−0.122
	(0.709)	(0.799)	(0.852)	(0.965)	(1.056)
dEPS_l12m	0.317	0.374^{**}	0.380^{**}	0.374^{**}	0.352
	(0.205)	(0.157)	(0.156)	(0.180)	(0.236)
Price_Book	−0.001	−0.004	-0.007^{*}	-0.008^{**}	-0.006^{**}
	(0.004)	(0.004)	(0.004)	(0.003)	(0.003)
EBIT_Margin	−0.004	−0.007	−0.008	-0.010^{*}	-0.013^{**}
	(0.006)	(0.007)	(0.007)	(0.006)	(0.005)
Constant	0.060	0.113	0.116	0.146^{*}	0.184^{***}
	(0.092)	(0.098)	(0.093)	(0.076)	(0.068)
Observations	6,577	6,326	6,075	5,824	5,573
R^2	0.303	0.373	0.472	0.583	0.689
Adjusted R^2	0.302	0.372	0.471	0.582	0.688
Note:			$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$		

Table 3: Dividend regressions.

	Dependent variable:				
	Forward average dividend per share				
	(1 yr)	(2 yrs)	(3 yrs)	(4 yrs)	(5 yrs)
E_dEPS	0.225*	0.317*	0.340*	0.330	0.302
	(0.128)	(0.164)	(0.176)	(0.208)	(0.257)
E1_dEPS	−0.089	−0.178	−0.199	−0.201	−0.185
	(0.081)	(0.169)	(0.205)	(0.244)	(0.280)
dEPS_l12m	0.050**	0.074	0.086	0.084	0.072
	(0.022)	(0.050)	(0.066)	(0.083)	(0.106)
Price_Book	−0.002***	−0.003***	−0.004***	−0.004***	−0.004**
	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)
EBIT_Margin	0.002***	0.001*	0.001	0.001	0.001
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)
Constant	−0.012	−0.003	0.004	0.011	0.016
	(0.009)	(0.011)	(0.015)	(0.019)	(0.023)
Observations	6,577	6,326	6,075	5,824	5,573
R^2	0.639	0.572	0.476	0.380	0.296
Adjusted R^2	0.639	0.571	0.475	0.380	0.296
Note:					*p<0.1; **p<0.05; ***p<0.01