Market-Triggered Changes in Capital Structure: Equilibrium Price Dynamics

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Abstract

We analyze the internal consistency of using the market price of a firm's equity to trigger a contractual change in the firm's capital structure, given that the value of the equity itself depends on the firm's capital structure. Of particular interest is the case of contingent capital for banks, in the form of debt that converts to equity, when conversion is triggered by a decline in the bank's stock price. We analyze the problem of existence and uniqueness of equilibrium values for a firm's liabilities in this context, meaning values consistent with a market-price trigger. Discrete-time dynamics allow multiple equilibria. In contrast, we show that the possibility of multiple equilibria can largely be ruled out in continuous time, where the price of the triggering security adjusts in anticipation of breaching the trigger. Our main condition for existence of an equilibrium requires that the consequences of triggering a conversion be consistent with the direction in which the trigger is crossed. For the design of contingent capital with a stock price trigger, this condition may be interpreted to mean that conversion should be disadvantageous to shareholders, and it is satisfied by setting the trigger sufficiently high. Uniqueness follows provided the trigger is sufficiently accessible by all candidate equilibria. We illustrate precise formulations of these conditions with a variety of applications.

1 Introduction

This paper investigates the feasibility of using the market price of a firm's equity to trigger a contractual change in the firm's capital structure. Because the equity value itself depends on the firm's capital structure, such a mechanism creates a question of internal consistency in the design of the trigger.

Our analysis is motivated by proposals to enhance financial stability by having banks issue contingent capital in the form of debt that converts to equity if the bank's financial condition deteriorates.[1] This type of contingent convertible debt offers the potential of a private sector alternative to government intervention in a crisis. Should a bank's assets suffer a sharp decline in value, the bank would ordinarily need to raise new equity to remain solvent; as raising

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[1]The Basel Committee [2] and the Financial Stability Oversight Council [12] provide regulatory overviews.

equity near financial distress may be difficult, governments may feel compelled to inject public support if a bankruptcy is deemed sufficiently disruptive. Contingent capital would instead provide a built-in mechanism for increasing a bank's equity through conversion of a portion of its debt. This conversion could eliminate a perceived need for government support and should be relatively immune to regulatory forbearance.

One of the main challenges to the effective implementation of contingent capital is the design of the trigger for conversion from debt to equity. In the major issuances to date (most notably by several large European and British banks), conversion is triggered by a regulatory capital ratio falling below a threshold. Flannery [13, 14] introduced contingent capital with a market-based trigger (a bank's stock price), based on the view that the market value of equity provides a better indicator of a bank's capital adequacy. Regulatory capital relies on accounting-based measures that are slow to respond to new information and are subject to discretion in their implementation. In contrast, market signals in the form of stock prices, bond yields, and spreads on credit default swaps are credited with incorporating new information quickly and providing a forward-looking view of a firm's financial condition.[2]

Sundaresan and Wang [35] observe that using a stock price to trigger a conversion of debt to equity is potentially problematic because the stock price is itself affected by the possibility of conversion. Their discrete-time examples show that a conversion rule may admit multiple equilibria — that is, different stock prices consistent with the same mechanism — or no equilibrium.[3] They caution that indeterminacy or inconsistency in the stock price could create an opening for market manipulation to prompt or prevent hitting of the trigger. They relate the possibility of multiple equilibria to a value transfer between shareholders and contingent capital investors at the point of conversion; an equilibrium with greater transfer of value away from shareholders is associated with a lower stock price and an earlier conversion. They propose a modification to the terms of conversion to prevent a value transfer. Prescott [31] proposes contract modifications for similar reasons in a static formulation of the problem.

We analyze the existence and uniqueness of equilibrium for contingent capital with a stock

[2]McDonald [25] and Squam Lake Working Group [34] propose dual triggers measuring the condition of the market or banking sector generally as well as an individual institution, taking the view that contingent capital should convert only in the event of a broad threat and not simply to protect an individual institution. Most analyses to date (including Albul et al. [1], Chen et al. [9], Hilscher and Raviv [19], Himmelberg and Tsyplakov [20], and Koziol and Lawrenz [22]) have modeled conversion based on the value of a bank's assets or earnings; asset value is not directly observable but can be approximated using information about the value of bank liabilities (as in Calomiris and Herring [8] or Chen et al. [9]). Pennacchi [28] uses the combined value of a bank's equity and its contingent capital. Glasserman and Nouri [16] model an accounting-based capital ratio, allowing the book value and market value of assets to be imperfectly correlated. Bulow and Klemperer [7] use a stock price trigger. Hart and Zingales [18] consider a credit default swap trigger.

[3]These issues do not arise with a trigger based on asset value because asset value is not affected by conversion. The same holds for the trigger design in Pennacchi [28], which infers asset value from the market value of debt and equity.

price trigger and a broad class of related problems involving a change in capital structure determined by the market price of a firm's liability. We work in continuous time and show that this leads to starkly different conclusions than a discrete-time formulation. In continuous time, dynamic price adjustments allow the market price to reflect the anticipated effect of conversion. Within the general framework of problems we study, once we have at least one equilibrium, continuous updating of prices rules out the possibility of multiple equilibria under modest additional conditions; indeed, exceptions to these conditions and examples allowing multiple equilibria are then rather contrived. Existence of an equilibrium in our framework depends on consistency between the direction and consequences of the conversion trigger, as we explain shortly. In the basic case of contingent capital with a stock price trigger, we get one (and only one) equilibrium as long as the trigger is sufficiently high.

Our uniqueness results are easiest to describe when information arrives to the market continuously and prices evolve diffusively, although our analysis allows jumps as well. In this setting, consistency with rational expectations requires that an equilibrium stock price change continuously upon hitting the trigger, as only the sudden release of unanticipated information could cause the price to jump. Put differently, the ability of prices to adjust continuously is sufficient to preclude any transfer of value between holders of different claims when the trigger is reached; no contract modification is needed to enforce this property.

A continuous-time model is most relevant for a highly liquid market. Under this interpretation, our results suggest that the potential pitfalls in contingent capital with a market trigger lie primarily in the level of trading in the asset whose price triggers conversion rather than in the design of the contract itself.[4] In the case of the stock price of a major financial institution, the speed with which information is incorporated into the price makes continuous trading a sensible approximation. Corporate bonds are much less liquid, and indeed Bianchi, Hancock, and Kawano [3] find that trading frequency affects the yields on subordinated debt and impairs the value of these yields as measures of a bank's financial condition. Credit default swaps, with liquidity intermediate to that of stocks and corporate bonds, have been proposed as triggers by Hart and Zingales [18] and others. How introducing a trigger based on a security might affect the volume of trading in that security is an open question, though it seems plausible that trading frequency would increase near the trigger, where the distinction between intermittent and continuous price updating is most important.

For our existence results and indeed our entire framework, we introduce two hypothetical

[4]More precisely, this statement applies to concerns about multiple equilibria. There are other challenges to the feasibility of a market trigger, including questions about the information contained in stock prices as they approach a trigger and about investors' decisions at such times. For experimental evidence on the latter, see Davis, Korenok, and Prescott [10].

firms — a post-conversion firm for which the trigger has in effect been tripped prior to time zero, and a no-conversion firm not subject to a trigger. In the specific case of contingent capital, the first firm has already had debt converted to equity, and the second firm's debt is not convertible at all. More generally, we use "conversion" to refer to whatever change results from the trigger, which could also be a debt write-down or a forced deleveraging. Because neither the post-conversion firm nor the no-conversion firm is subject to an endogenous trigger, the prices of claims on these firms are automatically well-defined. The question of existence is whether one can define a price process (a stock price, for example) for the original firm that coincides with the post-conversion value after crossing the trigger and coincides with the no-conversion value at a terminal date if it does not reach the trigger prior to the terminal date.

Our results follow primarily from two types of conditions:

- For existence, we require that the no-conversion price be higher than the post-conversion price when either is above the trigger. Because we assume that conversion is triggered by a decline in a market price, this condition may be paraphrased as stating that the contract has been designed so that the consequences of conversion are consistent with the direction of the triggering event. In the case of contingent capital, one may interpret the condition to mean that conversion is disadvantageous to shareholders.[5]

- For uniqueness, we further require a trigger accessibility condition. At its core, this condition requires that the no-conversion price — and then any equilibrium stock price — have a sufficient chance of reaching the trigger. Indeed, if the no-conversion price cannot reach the trigger, it is often possible to construct one equilibrium in which conversion never occurs and another in which conversion is possible. Standard models satisfy our conditions, and we will argue that violations of our trigger accessibility condition in its various forms are unrealistic.

With this structure in place, we construct an equilibrium in which conversion occurs the first time the post-conversion price reaches the trigger; we then show that this is the only equilibrium. With price dynamics restricted to discrete time points, this becomes the *minimal* equilibrium, giving the earliest possible conversion date and the lowest stock price among all possible stock prices.

As a contrast to the dynamic models on which we mainly focus and to help introduce ideas, we begin with a static setting. Prices and the triggering event are determined simultaneously, and there is no opportunity for prices to anticipate the effect of conversion because there is no

[5] Pennacchi and Tchistyi [30] show that with perpetual debt a market-trigger equilibrium is sometimes possible under terms that favor shareholders. We return to this point in Section 6.3.

evolution of time. Here we may indeed have no equilibrium, multiple equilibria, or just one equilibrium, and we characterize each case.

This static setting shares some features with Bond, Goldstein, and Prescott [6], but there are also important differences in the problems considered. Bond et al. [6] consider a possible intervention in a firm based on the market price of the firm's equity and a private signal. The intervention affects the firm's fundamentals and thus the value of its equity, giving rise to questions about equilibrium. In our setting, the intervention has no effect on fundamentals — converting debt to equity has no effect on the value of the underlying assets. Bond et al. [6] have an unambiguous mapping from fundamentals to equity value; the possibility of multiple equilibria arises from uncertainty about fundamentals. In contrast, in our setting the source of the problem is that a market trigger can make contingent capital and equity *incomplete contracts*: even with perfect information about the value of a firm's assets, there are states in which the apportionment of firm value between contingent capital investors and equity holders may not be fully specified. This phenomenon of contract incompleteness has not been recognized in previous work on contingent capital.

For the dynamic setting on which we mainly focus, one can draw a partial analogy with the question of market completeness. A market that is dynamically complete with continuous trading typically becomes incomplete when trading is restricted to fixed dates. This incompleteness allows a range of values for the price of a contingent claim, each consistent with the absence of arbitrage (and thus with a market equilibrium). Discrete-time hedging of an option with the underlying assets leaves some residual risk at maturity, and the discrete-time version of our setting leaves some residual risk at conversion in the form of a possible value transfer between claimants. Under appropriate conditions, each of these price ranges shrinks to a point as trading frequency increases. But the analogy between the two settings should not be overstated. In the case of market incompleteness, the range of possible prices corresponds to a range of possible risk preferences; in our setting, specifying preferences for a representative agent does not pin down a single price. Instead, multiple equilibria arise in our setting because, as already noted, restricting the dates at which prices change impairs the ability of the market price to incorporate information and leaves ambiguity about the payouts to investors.

The rest of this paper is organized as follows. Section 2 introduces contingent capital, first detailing the static case and then expanding it to a dynamic capital structure model of the type introduced by Merton [26]. Section 3 abstracts from the specifics of contingent capital to consider more general changes in a firm triggered by the market value of a claim on the firm. The added generality clarifies the essential features leading to existence and uniqueness of equilibria. Section 4 extends these results to allow jumps in fundamentals. Section 5 works

through applications of these results, first revisiting contingent capital and then examining other capital structure changes including debt write-downs, forced deleveraging, capital access bonds, and automatic share repurchases. Section 6 extends our results to cover dividends and coupons, debt covenants and bankruptcy costs, and infinite-horizon problems. Most technical details are deferred to an appendix.

2 Contingent Capital

2.1 The Static Case

We begin by considering a static setting. This setting illustrates the possibility of having a unique equilibrium, multiple equilibria, or no equilibrium, depending on the level of the conversion trigger; it thus serves as a useful introductory case. It also describes the situation at maturity in a dynamic formulation we take up later.

We consider a firm with asset value A. The firm's liabilities consist of senior debt with face value B, contingent convertible debt with face value C, and equity. The value of the equity depends on whether the contingent capital converts, and conversion is triggered by the value of the equity, so equity value and the outcome of the conversion trigger are determined simultaneously. In the absence of conversion, the contingent capital remains debt and the value of the equity is

$$S = v(A) = (A - B - C)^+, \tag{1}$$

which is the residual value of the firm after the debt has been repaid, with $x^+ = \max\{x, 0\}$. If we adopt the normalization that there is a single share outstanding, then (1) is the price per share.

Upon conversion, the contingent capital is replaced with $m > 0$ newly issued shares of stock,[6] bringing the total number of shares to $1+m$. The total value of equity is now $(A-B)^+$, and the price per share is

$$S = u(A) = (A - B)^+/(1 + m). \tag{2}$$

If we posit that the contingent capital converts precisely if $S \leq L$, for some trigger level L, the question arises of whether these specifications are internally consistent and, if they are, whether they admit more than one solution. To make this precise, we introduce the following definition.

[6] We limit ourselves to the case of fixed m. See Pennacchi [28], Pennacchi, Vermaelen, and Wolf [29], and Prescott [31] for discussions of alternative conversion rules.

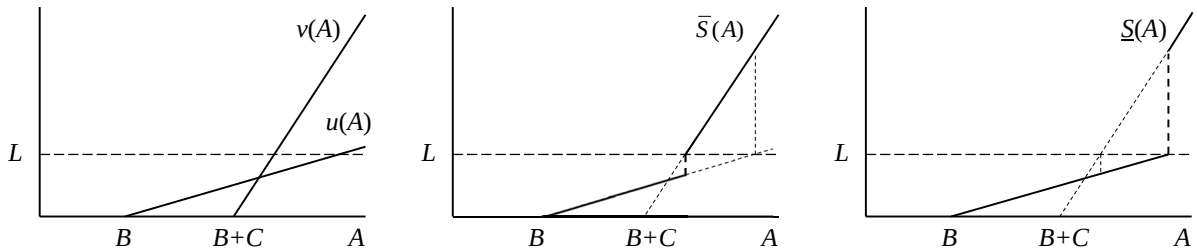

Figure 1: The equity values $u(A)$ and $v(A)$ cross at a height of C/m. With $L > C/m$, (3) admits an infinite number of solutions, all lying between $\underline{S}(A)$ and $\overline{S}(A)$.

Definition 2.1 *A mapping* $S : [0, \infty) \mapsto [0, \infty)$ *is an* equilibrium stock price *if it satisfies*

$$S(A) = \begin{cases} u(A), & \text{if } S(A) \leq L; \\ v(A), & \text{if } S(A) > L. \end{cases} \tag{3}$$

for all $A \in [0, \infty)$.

Equation (3) selects one of the two stock price formulas in (1) and (2), depending on whether the stock price is above the trigger L or not. Importantly, the stock price $S(A)$ appears on both sides of the equation.

The situation is illustrated in Figure 1. The left panel shows the functions $u(A)$ and $v(A)$. The lines cross at a height of C/m, so the figure has $L > C/m$. An equilibrium stock price is a mapping from A on the horizontal axis to $S(A)$ on the vertical axis consistent with (3). With $L > C/m$, there are multiple solutions. For example, we may take

$$\underline{S}(A) = \begin{cases} u(A), & \text{if } u(A) \leq L; \\ v(A), & \text{otherwise,} \end{cases} \tag{4}$$

as illustrated in the right panel of the figure. We may also take

$$\overline{S}(A) = \begin{cases} u(A), & \text{if } v(A) \leq L; \\ v(A), & \text{otherwise,} \end{cases} \tag{5}$$

as illustrated in the center panel. Both $\underline{S}$ and $\overline{S}$ satisfy the conditions in (3) to qualify as equilibrium stock prices. It is not hard to see (as shown more generally in the appendix) that any solution to (3) must lie between $\underline{S}$ and $\overline{S}$; conversely, any mapping S that lies between $\underline{S}$ and $\overline{S}$ and for which $S(A) \in \{u(A), v(A)\}$ for every A satisfies (3), even if it not monotone in the asset level A.

We can interpret the extremal solutions in (4) and (5) as follows. The equity price $\underline{S}(A)$ arises from conversion occurring at the highest asset level consistent with conversion (and all lower asset values); $\overline{S}$ arises from avoiding conversion at the lowest asset value consistent with non-conversion (and all higher asset values). If we imagine sliding A down from some large

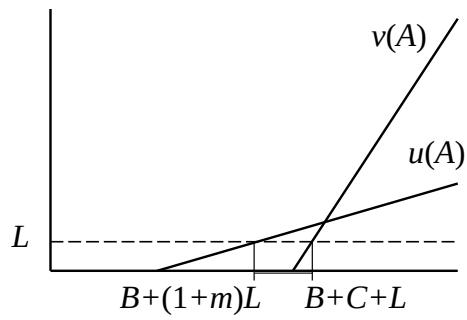

Figure 2: With $L < C/m$, there is no way to define $S(A)$ consistent with (3) for A between $B + (1 + m)L$ and $B + C + L$. Setting $S(A) = v(A) < L$ in this interval violates the first condition in (3), and setting $S(A) = u(A) > L$ violates the second condition.

value, moving from right to left in Figure 1, $\underline{S}$ results from converting at the first opportunity, and $\overline{S}$ results from converting at the last possible point.

When $C = mL$, the trigger L is exactly at the height at which $u(A)$ and $v(A)$ cross, $\overline{S} = \underline{S}$, and there is no other solution to (3), as any solution must lie between these two. If $C > mL$, there is no choice of mapping $S(A)$ consistent with (3). This case is illustrated in Figure 2. In particular, at any value of A for which $B + (1 + m)L < A < B + C + L$, we have

$$v(A) < L < u(A). \tag{6}$$

Thus, there is no way of defining $S(A)$ consistent with (3): if we set $S(A) = v(A) < L$ we violate the first condition in (3), and if we set $S(A) = u(A) > L$ we violate the second condition in (3). We summarize the situation as follows.

Proposition 2.2 *In the static problem (3), (i) if $C < mL$, the problem admits an infinite number of equilibrium stock prices, all lying between $\underline{S}$ and $\overline{S}$; (ii) if $C = mL$, the problem admits just one equilibrium stock price $S = \underline{S} = \overline{S}$; (iii) if $C > mL$, the problem admits no equilibrium stock price.*

In case (i) of the proposition, we get multiple equilibria because the seemingly simple rule defining a stock price in (3) — convert if the price is at or below the trigger — makes equity an incomplete contract. For asset levels between $B + C + L$ and $B + (1 + m)L$, marked by the vertical dashed lines in the center and right panels of Figure 1, the rule in (3) does not fully specify how asset value is apportioned between equity holders and contingent capital investors. An equity payout of either $u(A)$ or $v(A)$ is consistent with the conversion rule.

Generalizing beyond (1) and (2), the essential property of the functions v and u underpinning part (i) of the proposition is that

$$v(A) \geq u(A) \text{ whenever } u(A) \geq L. \tag{7}$$

This says that equity holders prefer non-conversion at high assets levels, and the trigger should be high enough so that this preference holds above the trigger. Condition (7) is equivalent to

$$v(A) \geq u(A) \text{ whenever } \max\{u(A), v(A)\} \geq L. \tag{8}$$

We do not require monotonicity of u or v. When (7) holds, (4) and (5) define equilibria, and they define lower and upper bounds on any other equilibria; if the sets where $v(A) \leq L$ and $u(A) \leq L$ coincide, there is just one equilibrium; and if (6) holds at some A, then no equilibrium is possible. See the appendix for details.

2.2 Dynamic Structural Model

There is a natural extension of this static case to a dynamic model of a firm on a time interval $[0, T]$, in the spirit of Merton [26]. In this formulation, A_t, $t \in [0, T]$, describes the evolution of the value of the assets held by the firm; to be concrete, take A_t to be geometric Brownian motion. The value of these assets — think of loans and securities in the case of a bank — does not depend on how the assets are financed. The firm has issued straight debt and contingent convertible debt, just as before, both maturing at T, with respective par values of B and C. We now seek a stock price *process* S_t, $t \in [0, T]$ satisfying

$$S_T = \begin{cases} u(A_T), & \text{if } S_t \leq L \text{ for some } t \in [0, T]; \\ v(A_T), & \text{otherwise.} \end{cases} \tag{9}$$

In this formulation, the trigger is in effect throughout $[0, T]$, and not just at the terminal date T. This fundamentally changes the problem because the stock price can adjust to anticipate the effect of the trigger — no such adjustment is possible in the static case.

The analysis of (9) depends on contractual terms like coupons on debt or covenants as in Black and Cox [4]. We return to these specific types of features after analyzing a setting that is both simpler and more general. As we develop the general setting, it will be useful to keep this example in mind.

3 Dynamic Setting

3.1 Formulation

In this section, we abstract from the details of contingent convertible debt and consider a more general problem of existence and uniqueness of an equilibrium with a market trigger. We postpone technical details as much as possible to the appendix. We do, however, need to lay out some assumptions. We assume that investors are risk-neutral and interest rates are zero.[7] The price of any asset is then the expected value of its future payouts, and the price

[7] These assumptions simplify notation; we relax them in Section A.3.

of an asset without dividends is a martingale. Information arrives to the economy diffusively and prices adjust continuously, though later we allow jumps. More precisely, we work on a probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t, t \in [0,T]\}, P)$ in which the information flow $\{\mathcal{F}_t, t \in [0,T]\}$ is generated by Brownian motion, possibly multidimensional. All processes we define are adapted to this information flow. The price of any asset that pays no dividends evolves continuously because any martingale adapted to a Brownian filtration is continuous.

We introduce two processes U_t and V_t which may be interpreted as, for example, stock price processes for two hypothetical firms investing in identical assets but with different liabilities and possibly different overall firm size. To connect this setting with contingent capital, think of U_t as the stock price for a firm in which the convertible debt has already been converted to equity, and think of V_t as the stock price for an otherwise identical firm in which the convertible debt is replaced with straight debt that can never convert.[8] By construction, these prices do not depend on an endogenous trigger and are well-defined. Our interest is in the existence and uniqueness of a stock price for a third firm with the same asset process but for which the contingent capital is genuinely convertible. More generally, we are interested in a process S_t satisfying

$$S_T = \begin{cases} U_T, & \text{if } S_t \leq L \text{ for some } t \in [0,T]; \\ V_T, & \text{otherwise,} \end{cases} \tag{10}$$

with U_T, V_T having finite expectations. Assuming this firm pays no dividends, we also require

$$S_t = E_t[S_T], \tag{11}$$

where E_t denotes conditional expectation given $\mathcal{F}_t$. Put abstractly, our problem is the existence and uniqueness of a process S_t satisfying (10) and (11), given U_T and V_T. We will call such a process an equilibrium stock price.

3.2 Existence

A natural counterpart of the condition (7) we used in the static setting is

$$U_T \leq V_T \text{ whenever } U_T \geq L. \tag{12}$$

This requires that the no-conversion price be higher than the post-conversion price above the trigger. It will often be easier to work with the simpler but stronger condition that the no-conversion price is always higher,

$$U_T \leq V_T. \tag{13}$$

[8] As adapted processes, U_t and V_t may be functions of the history of the underlying Brownian motion on $[0,t]$. In the specific case outlined in Sections 2.2 and to which we return several times for illustration, U_t and V_t are functions of a single state variable A_t, but we do not impose this restriction for our general results.

The two hypothetical firms pay no dividends (until Section 6), so

$$U_t = E_t[U_T] \text{ and } V_t = E_t[V_T], \tag{14}$$

and (13) implies

$$U_t \leq V_t, \quad \text{for all } t \in [0, T]. \tag{15}$$

This is already more than enough to ensure existence:

Theorem 3.1 *(i) If (13) holds, then there exists at least one equilibrium stock price. (ii) The same holds if (13) is relaxed to (12).*

Proof of part (i). We construct a solution to (10)–(11). Define

$$S_T^* = \begin{cases} U_T, & \text{if } U_t \leq L \text{ for some } t \in [0, T]; \\ V_T, & \text{otherwise,} \end{cases} \tag{16}$$

noting that the trigger event is here defined by U_t, so S_T^* is automatically well-defined. Further define $S_t^* = E_t[S_T^*]$, $t \in [0, T)$, so (11) holds. To show that S^* satisfies (10), we need to show that S_t^* reaches L if and only if U_t does, since then (10) follows from (16).

Because $S_T^* \in \{U_T, V_T\}$, we have $U_T \leq S_T^* \leq V_T$, and taking conditional expectations then yields $U_t \leq S_t^* \leq V_t$ for all $t \in [0, T]$. Consequently, if $S_t^* \leq L$, then $U_t \leq L$.

It remains to show that if $U_t \leq L$ for some $t \in [0, T]$, then the same is true of S^*. Define the stopping time

$$\tau_U = \inf\{t \in [0, T] : U_t \leq L\}, \tag{17}$$

the first time U_t is at or below the trigger, with the convention that $\tau_U = \infty$ if $U_t > L$ for all $t \in [0, T]$. On the event $\{\tau_U \leq T\}$, we have $S_T^* = U_T$ by definition, and then

$$S_{\tau_U}^* = E[S_T^* | \mathcal{F}_{\tau_U}] = E[U_T | \mathcal{F}_{\tau_U}] = U_{\tau_U} \leq L.$$

We have shown that S_t^* and U_t reach the trigger simultaneously or not at all, so we may replace U_t with S_t^* in the first condition in (16), and this yields (10). □

The construction is illustrated in Figure 3. The equilibrium stock price starts above the post-conversion value U_0 and coincides with the post-conversion value at the instant that both reach the trigger. Prior to conversion, there is some chance the trigger will not be reached and thus some chance the stock price will terminate at the higher value V_T; this possibility is reflected in the stock price being greater than U_t prior to the trigger being reached. We stress that the process U_t is merely used for the proof of existence and need not be observable in the market: the triggering event is ultimately determined solely by the equilibrium stock price process S_t^*.

The following lemma, proved in the appendix, reduces part (ii) of Theorem 3.1 to part (i):

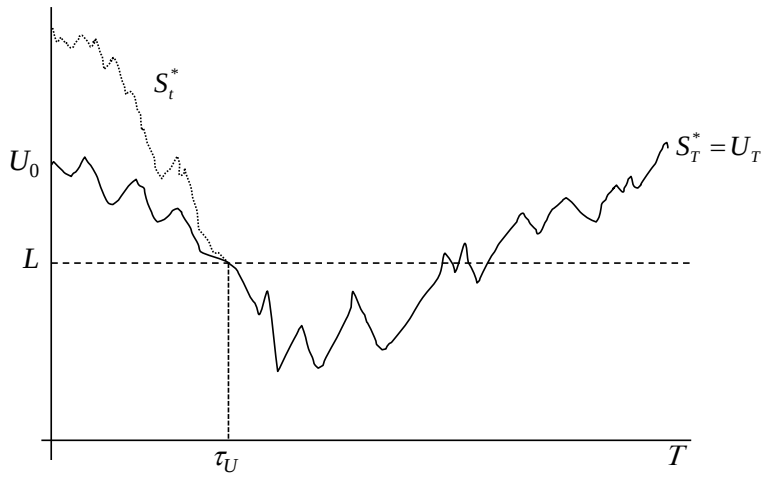

Figure 3: Illustration of the existence argument: By construction S_t^* reaches the trigger L together with U_t or not at all. In this equilibrium, conversion occurs the first time the post-conversion stock price reaches the trigger.

Lemma 3.2 *Suppose we relax (13) to (12). Then S_t is an equilibrium stock price with U_T and V_T in (10) if and only if it is an equilibrium stock price with $\tilde{U}_T = U_T$ and $\tilde{V}_T = \max\{U_T, V_T\}$.*

As already noted, condition (12) holds in the dynamic structural model of contingent capital in Section 2.2 if the trigger is high enough that $C \leq mL$. The simpler condition $U_T \leq V_T$ in (13) does not hold because the lines describing u and v in Figure 1 cross. But the value of the no-conversion payoff $v(A)$ below the trigger is irrelevant, so we are free to change $v(A)$ below the trigger to make it greater than or equal to $u(A)$. Lemma 3.2 formalizes this observation.

When the simple ordering condition (13) holds, the post-conversion price is always lower than the no-conversion price and existence of an equilibrium is automatic; this case is particularly convenient for analysis. Condition (12) has broader scope and is more directly applicable. In our examples, the interpretation of (12) will be that conversion is disadvantageous for the holders of S_t.

Theorem 3.1 provides sufficient conditions for existence of an equilibrium, so it is natural to ask to what extent conditions of this type are necessary. Dropping the ordering of U_t and V_t generally requires imposing other restrictions on the range of values they can take, as we illustrate through examples, starting with one that ensures existence without any condition on U_t by constraining V_t:

Proposition 3.3 *Suppose V_t can never reach the trigger L; that is, $P(V_t > L$ for all $t \in [0, T]) = 1$. Then $S_t = V_t$ is an equilibrium stock price for (10)–(11).*

This no-conversion equilibrium is of limited practical interest because it entails a strong restriction on the range of V_t, but it serves as a useful contrast as we consider what happens without restrictions on the range of U_t and V_t. The following is a partial converse to Theorem 3.1, precluding the possibility of an equilibrium if the no-conversion price can terminate below the trigger without the post-conversion price ever reaching the trigger:

Proposition 3.4 *There is no equilibrium stock price that reaches the trigger at a time when $U_t > L$. In particular, if*

$$P(V_T \leq L \text{ and } U_t > L \text{ for all } t \in [0, T]) > 0 \tag{18}$$

then there is no equilibrium.

Proof. Let S be an equilibrium stock price and define τ_S

$$\tau_S = \inf\{t \in [0, T] : S_t \leq L\}.$$

On the event $\{\tau_S \leq T\}$, $S_T = U_T$, so

$$S_{\tau_S} = E[S_T|\mathcal{F}_{\tau_S}] = E[U_T|\mathcal{F}_{\tau_S}] = U_{\tau_S}. \tag{19}$$

Thus, it is not possible to have $S_{\tau_S} \leq L < U_{\tau_S}$. For the second assertion in the proposition, if an equilibrium S_t exists and U_t does not reach L, then, by the first part of the proposition, S_t never reaches the trigger. But then $S_T = V_T \leq L$ yields a contradiction. □

Viewed from the perspective of this result, the ordering condition (12) that underpins Theorem 3.1 eliminates the problematic possibility in (18) that V_t ends up below the trigger without U_t ever having reached the trigger, which creates a logical inconsistency in trying to find a stock price that satisfies (10), just as (6) does in the static case. The practical implication of these results for contingent capital with a stock price trigger is that the trigger level L and the dilution factor m should be set so that the post-conversion stock price U_t is lower than the no-conversion stock price V_t when both are above the trigger, making conversion disadvantageous to shareholders. We discuss other types of triggers and conversion mechanisms in Section 5.

Equation (19), which we will invoke at several places, says that at conversion the stock price is precisely at the trigger L and exactly equal to the post-conversion price.[9] The fact that $U_{\tau_S} = L$ for any equilibrium stock price S_t reflects the absence of arbitrage: just after conversion, the original and post-conversion firms coincide, so their stock prices must be equal.

[9]This phenomenon is analogous to observations of Salant and Henderson [32] and Flood and Garber [15] in their models of speculative attacks. In particular, Flood and Garber [15, p.4] argue that the collapse of a fixed exchange rate regime occurs with the exchange rate equal to a shadow floating value to which it switches at the instant the fixed rate can no longer be sustained.

3.3 Uniqueness

We turn next to uniqueness. We describe the argument informally through reference to Figure 4, deferring technical details to the appendix. In the figure, S_t is a candidate equilibrium stock price. It is bounded above by the no-conversion stock price V_t, either because the ordering relation (13) holds or because of the weaker requirement (22) introduced below. In light of the bound, S_t must reach the trigger no later than V_t. However, we know from Proposition 3.4 (and (19) in particular) that S_t can reach the trigger only when the post-conversion stock price is at the trigger — that is, when $U_t = L$. To avoid a contradiction, we must therefore have S_t reach the trigger at τ_U, which then implies that S_t must coincide with the equilibrium S_t^* constructed in (16), establishing uniqueness. More precisely, we will argue that S_t must reach L before U_t reaches $L - \epsilon$, and we will then let ϵ shrink to zero.

Making this argument work relies on three types of conditions. First and most important is the type of ordering relation that pervades all of our results, requiring, in various ways, that the no-conversion stock price be higher than the post-conversion stock price when either is above the trigger. Second, we impose a *trigger accessibility* condition ensuring that that any candidate stock price can reach the trigger. We require, in effect, that the situation depicted in Figure 4 cannot be ruled out: after U_t reaches $L - \epsilon$, there is always a chance that V_t will reach the trigger before U_t does. In particular, the trigger accessibility condition will eliminate the possibility of an equilibrium stock price S_t that is somehow blocked from reaching the trigger for some time after U_t reaches the trigger. To complete the argument, we need a *right-continuity* condition stating that the first time U_t reaches $L - \epsilon$ approaches the first time it reaches L as ϵ decreases to zero. Models that violate these conditions are rather contrived, as we will see in two examples below.

Indeed, if it is impossible for V_t to reach the trigger, then, as in Proposition 3.3, $S_t \equiv V_t$ is automatically an equilibrium stock price, one in which conversion never occurs. However, the possibility of such an equilibrium suggests a deficiency in the design of the trigger; the main purpose of our trigger accessibility condition is to rule out this possibility.

To formulate these conditions precisely, let τ_U^ϵ denote the first time U_t is at or below $L - \epsilon$, as in (17) but with L replaced by $L - \epsilon$. The trigger accessibility condition requires that (i) $P(\tau_U^\epsilon < T) > 0$ for sufficiently small $\epsilon > 0$; and it further requires that (ii) conditional on $\tau_U^\epsilon < T$, we have, almost surely,

$$P(U_t < L \text{ for all } t \in [\tau_U^\epsilon, T] \text{ and } V_t \leq L \text{ for some } t \in [\tau_U^\epsilon, T] | \mathcal{F}_{\tau_U^\epsilon}) > 0. \tag{20}$$

In particular, U_t can reach $L - \epsilon$, and then V_t can reach the trigger L before U_t returns to L.

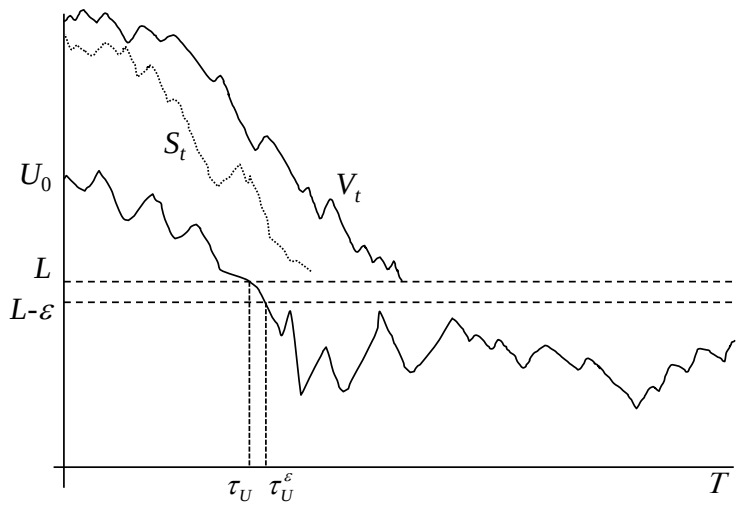

Figure 4: Illustration of the uniqueness argument. A candidate equilibrium S_t must reach the trigger no later than the no-conversion stock price V_t. However, it can reach the trigger only when the post-conversion stock price is at the trigger, $U_t = L$. To avoid a contradiction, S_t must reach the trigger the first time U_t does (or, more precisely, before U_t reaches $L - \epsilon$).

The right-continuity condition requires that, almost surely,

$$\tau_U^\epsilon \to \tau_U \quad \text{as} \quad \epsilon \downarrow 0, \tag{21}$$

Uniqueness means that any two equilibria agree with probability one at each time t.

Theorem 3.5 *Suppose the conditions of Theorem 3.1(i) hold, and suppose the trigger accessibility condition (20) and the right-continuity condition (21) hold. Then the equilibrium stock price is unique. The same holds under the conditions of Theorem 3.1(ii) with (12) replaced by*

$$U_t \geq L \Rightarrow U_t \leq V_t, \quad \textit{for all } t \in [0, T]. \tag{22}$$

In the dynamic structural model of Section 2.2, with the asset process A_t modeled by geometric Brownian motion, $U_t = E_t[u(A_T)]$ and $V_t = E_t[v(A_T)]$ are both given explicitly as functions of A_t by variants of the Black-Scholes formula. Our trigger accessibility condition is then satisfied because there is positive probability that A_t will drop sufficiently low to drive V_t below L before U_t returns to L. To further illustrate the argument, it is helpful to consider an example in which the trigger accessibility condition is violated and an example in which the right-continuity condition is violated:

Example 3.6 Consider a firm whose assets consist of 75 in cash and 25 in loans made to borrowers. The value of cash is constant; the value of the loans is stochastic, but it is always

between zero (default) and 25 (full repayment). The firm has borrowed 40 to finance its business and issued a single share of equity. The firm's debt converts to four additional shares if the stock price falls to the trigger level $L = 16$. Because the firm's asset value can never drop below 75, the no-conversion stock price V_t cannot drop below $75 - 40 = 35$, so V_t can never reach the trigger. The post-conversion stock price U_t is always between $75/(1+4) = 15$ and $100/(1+4) = 20$, so we are guaranteed to have $U_t \leq V_t$, for all t, as in (15). Applying the construction used in Theorem 3.1, we get one equilibrium S_t^* that converts the first time U_t reaches the trigger L (which is then also the first time S_t^* reaches L). However, we get a second equilibrium by setting $S_t = V_t$; this is a no-conversion equilibrium because V_t can never reach the trigger. The trigger accessibility condition is violated in this example because the large cash buffer creates a lower bound on V_t that is greater than the trigger. There is nothing to force the two equilibria to coincide.[10]

Example 3.7 Consider, now, a model in which the first time U_t reaches the trigger L it remains pegged at L for an exponentially distributed period of time ξ, violating (21). As before, we have an equilibrium S_t^* that reaches L at τ_U, the first time U_t reaches L. We can construct a second equilibrium $\tilde{S}_t$ that converts at $\tau_U + a$ if the exponential clock ξ runs for at least $a > 0$ time units and if $\tau_U + a \leq T$. More precisely, define $\tilde{S}$ the way we defined S^* in (16), but with the condition $\tau_U \leq T$ replaced by $\tau_U + a \leq \min\{\tau_U + \xi, T\}$. The argument in the proof of Theorem 3.1 showing that S_t^* is an equilibrium applies as well to $\tilde{S}_t$. Once S_t^* reaches the trigger, there is still a strictly positive probability that $\tilde{S}_t$ will never reach the trigger and thus that $\tilde{S}_T$ will terminate at V_T. This implies that $\tilde{S}_t$ is strictly greater than S_t^* during at least the interval in which only S_t^* has converted. Thus, these are two distinct equilibria. In fact, we get a separate equilibrium for each value of a, so this problem admits an infinite number of equilibrium stock prices.

The rather extreme situations described in Examples 3.6 and 3.7 are ruled out by the conditions we impose — the trigger accessibility condition and the right-continuity condition. We consider these two conditions very natural — indeed, examples that violate them seem rather contrived — so once we have the conditions of Theorem 3.1 in place for existence, uniqueness should be considered "typical."

[10] A referee notes that a firm can approximate the circumstances of this example by hedging the value of its assets. More generally, a firm's managers or other agents may take actions to manipulate the stock price. However, to yield more than one equilibrium, this example requires that it be completely impossible for the stock price V_t to fall sufficiently to reach the trigger. As long as there is some chance, no matter how small, that V_t will reach the trigger, our trigger accessibility condition applies, and the two equilibria will coincide. Without perfect and sustained price manipulation, the equilibrium is unique.

3.4 Contrasting Discrete and Continuous Time

The trigger accessibility condition provides additional insight into the difference between discrete-time dynamics, where multiple equilibria are possible, and the continuous-time setting, where the equilibrium is typically unique. Consider the situation illustrated in Figure 5, focusing initially on the discrete dates t_i. A discrete-time equilibrium stock price is defined exactly as in (10)–(11), but with t in (11) restricted to to discrete dates t_i. The circles and triangles show two possible discrete-time equilibrium stock price processes. At t_2, the circle path crosses the trigger and, consistent with having crossed, yields a lower price; the triangle path does not cross the trigger and, consistent with not having crossed, yields a higher price. At t_4, both have crossed, and so the two paths henceforth coincide.

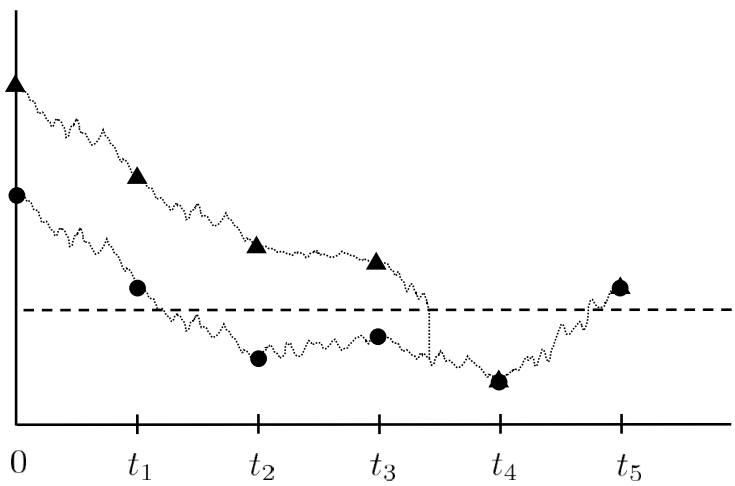

Figure 5: The circles and triangles show two discrete-time equilibrium stock price paths that cross the trigger at t_2 and t_4, respectively. But no interpolation of the triangle path can yield a valid path of a continuous-time equilibrium because at the moment it reaches the trigger, the path would have to jump to couple with the path that crossed earlier. This is illustrates why it may be possible to have multiple discrete-time equilibria yet only one continuous-time equilibrium under the trigger accessibility condition.

We now ask whether these two discrete-time equilibria are compatible with separate continuous-time equilibria. The continuous-time circle path reaches the trigger sometime between t_1 and t_2. The triangle path does not or the two paths would already have coupled by t_2. Under the trigger accessibility condition there is a chance the continuous-time triangle path will subsequently reach the trigger, as illustrated in the figure. When it does, it must jump to couple with the already converted circle path. But the jump is incompatible with being a continuous-time equilibrium stock price. The discrete-time triangle path may be a valid discrete-time equilibrium, but there is no interpolation of the triangles that yields a valid path for a continuous-time equi-

librium.[11] Any continuous-time equilibrium must have reached the trigger at the same time as the continuous-time circle path and must therefore coincide with that path at all earlier times.

In discrete time, the response of the stock price is determined simultaneously with the occurrence or non-occurrence of conversion. As we saw in Section 2.1, this can leave ambiguity and allow either outcome, as happens at t_2 in Figure 5. In continuous time, the consequences of the trigger are fully anticipated at the moment the trigger is reached; there is no stock price response left to determine and, in particular, conversion cannot produce a jump in the price. This is the key difference between the discrete-time and continuous-time settings.

We can take these observations a step further to compare the stock price levels in the two settings. The construction of S_t^* in the existence proof of Theorem 3.1 works in discrete time as well. Indeed, by the ordering argument used there, no equilibrium stock price can reach or cross the trigger before the post-conversion price U_t does. If we have $U_T \leq V_T$ (which we may assume in light of Lemma 3.2), then converting later translates to a higher stock price because it entails a higher likelihood of terminating at V_T rather than U_T; and a higher stock price confirms that conversion occurs later. Thus, the discrete-time counterpart of the unique continuous-time equilibrium gives the *lowest* stock price of all possible discrete-time equilibria. If another discrete-time equilibrium exists, it may be interpreted as a case of self-fulfilling speculation (lifting the stock price) that conversion can be delayed. Moreover, because the continuous-time equilibrium reaches the trigger no later than any discrete-time equilibrium (the continuous-time process can reach the trigger between ticks of the discrete clock), it yields a lower stock price. Thus, for any discrete time-step, the full swath of discrete-time equilibrium stock prices lies above the continuous-time equilibrium.

We illustrate this phenomenon with a numerical example. We consider contingent capital in the dynamic structural model of Section 3, so $U_T = u(A_T)$, $V_T = v(A_T)$, A_t is geometric Brownian motion, and v and u are given by (1) and (2). We will see in Section 5.1 that Theorem 3.5 implies the existence of a unique continuous-time equilibrium if $C \leq mL$, but we have multiple discrete-time equilibria when $C < mL$, so we choose this parameters in this range.

Figure 6 illustrates the convergence from discrete to continuous time. The figure plots the smallest and largest possible equity prices and contingent capital prices against the number of discrete time steps, calculated in a binomial lattice. Consistent with the explanation just given, both the upper and lower bounds for equity prices appear to decrease toward a common

[11] The argument extends to processes with jumps as long as there is a positive probability of reaching the trigger without a jump. Briefly, a jump can cause a trigger crossing but reaching the trigger cannot by itself cause a jump. See Section 4.

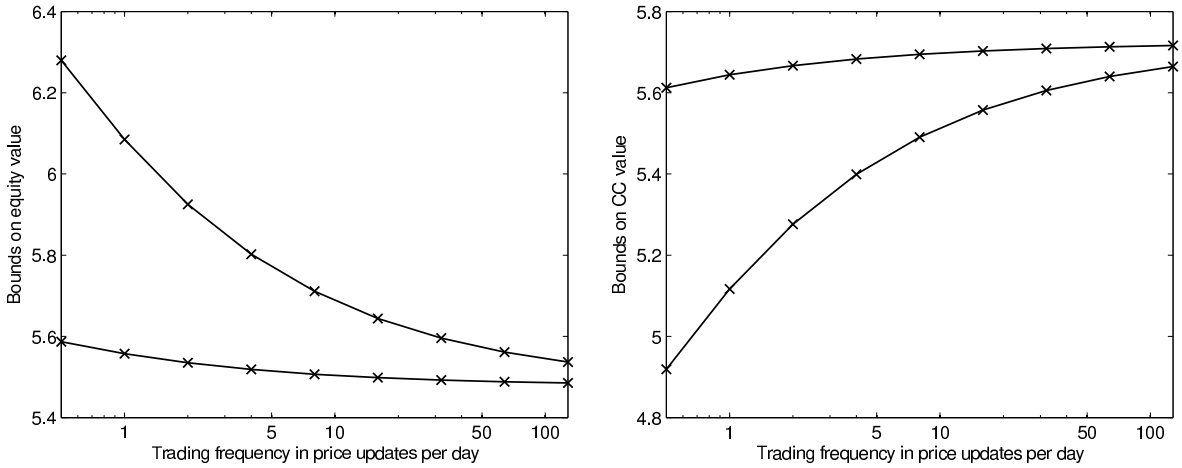

Figure 6: Convergence of discrete-time prices to continuous-time prices as the trading frequency increases in a binomial lattice. The left panel shows the range of stock prices and the right panel shows the range of prices for contingent capital. The example has $A_0 = B = 100$, $C = 6$, trigger level $L = 1$, maturity $T = 5$, an asset volatility of 6%, and a risk-free interest rate of 2%.

continuous-time limit.[12] This example uses parameters $A_0 = B = 100$, $C = 6$, $L = 1$, $T = 5$, an asset volatility of 6%, and a risk-free interest rate of 2%.

4 Jumps

Our analysis thus far has relied on the assumption introduced in Section 3 that the filtration or information flow to which all processes are adapted is generated by an underlying (possibly multidimensional) Brownian motion. As a consequence of this assumption, all of our price processes have been continuous because all Brownian martingales are continuous. We now introduce jumps using a Poisson process N_t and arbitrary fixed dates $0 < t_1 < t_2 < \cdots < t_k < T$ at which jumps may occur. The t_i could represent the dates of scheduled announcements, for example. Let

$$J_t = \sum_{n=1}^{N_t} Y_n + \sum_{n:t_n \le t} \tilde{Y}_n,$$

where $\{Y_1, Y_2, \dots\}$ and $\{\tilde{Y}_1, \tilde{Y}_2, \dots\}$ are i.i.d. sequences of random vectors, independent of each other and of the Poisson process N_t and of the underlying Brownian motion W_t. For example, this framework would allow us to extend the dynamic structural model of Section 2.2 to the

[12] Numerical results suggest that the range shrinks in proportion to the square root of the time step. The binomial calculation entails a discretization of price levels as well as time, and this may contribute to the magnitude of the range of prices without necessarily changing the convergence rate.

case of an asset value process given by a Merton [27] jump-diffusion model

$$\frac{dA_t}{A_{t-}} = \mu\, dt + \sigma\, dW_t + dJ_t^1, \tag{23}$$

where J_t^1 denotes the first coordinate of J_t, with jump sizes greater than -1, and μ and σ are constants. Such a model with Poisson jumps is also used in Chen et al. [9] and Pennacchi [28]. We return to this specific application after formulating a general result. We take all price processes to be continuous from the right and have limits from the left. The size of a jump in a process X at t is $X_t - X_{t-}$, with the left limit X_{t-} giving the value just before the jump.

The information set $\mathcal{F}_t$ is now the one generated by the joint history of the underlying Brownian motion and the jump process J in $[0, t]$. We require that all processes be adapted to this information. Given martingales U_t and V_t we seek an equilibrium stock price process S_t as in (10)–(11). In previous sections, all martingales (all prices in the absence of dividends) were necessarily continuous. In this section, the martingale representation theorem (as in Lemma 4.24 of Jacod and Shiryaev [21]) implies that a martingale can jump only when the process J_t jumps. In other words, the arrival of new information, such as a jump in fundamentals or an earnings announcement, can cause a price to jump, but a conversion from debt to equity cannot by itself cause a price to jump. A jump in a stock price may cause a trigger to be breached, but reaching the trigger cannot itself generate a jump in the stock price. This is the key property that allows us to extend our earlier results.

The construction of the equilibrium stock price process S_t^* in Theorem 3.1 is based on a monotonicity argument but does not rely on continuity of paths. In contrast, the uniqueness argument in Theorem 3.5 does use continuity and therefore requires modification. As a first step, it is important to note that if S_t is an equilibrium stock price process, then in the presence of jumps we can no longer assume that $S_{\tau_S} = L$ because it is possible to have $S_{\tau_S} < L$; however, (19) still holds, so we still have $S_{\tau_S} = U_{\tau_S}$ whenever $\{\tau_S \leq T\}$. This simply states that the stock price just after conversion must coincide with the post-conversion stock price, and it is essentially a no-arbitrage property. As an immediate consequence, we see that it is not possible for two equilibrium stock price processes to trigger conversion at the same instant by jumping to two different prices at or below L. If they both trigger conversion at some τ, then they must both agree with U_τ at the moment of conversion.

To establish uniqueness, we need to make two modifications to the trigger accessibility condition (20). Jumps make it possible for V_t to jump below U_t, so for the upper bound in Figure 4 we introduce

$$\tilde{V}_t = E_t[\max\{U_T, V_T\}].$$

Any equilibrium stock price S_t satisfies $S_T \leq \max\{U_T, V_T\}$ and therefore $S_t \leq \tilde{V}_t$ for all t. Let

T_ϵ denote the first time after τ_U^ϵ of a jump in J_t, taking $T_\epsilon = \infty$ if either $\tau_U^\epsilon = \infty$ or if there is no jump in $(\tau_U^\epsilon, T]$. More explicitly, set

$$T_\epsilon = \inf\{t > \tau_U^\epsilon : N_t = N_{t-} + 1 \text{ or } t \in \{t_1, \ldots, t_K\}\}.$$

We replace the endpoint T in (20) with T_ϵ and require

$$P(U_t < L \text{ for all } t \in [\tau_U^\epsilon, T_\epsilon) \text{ and } \tilde{V}_t \leq L \text{ for some } t \in [\tau_U^\epsilon, T_\epsilon) | \mathcal{F}_{\tau_U^\epsilon}) > 0, \tag{24}$$

almost surely. In other words, after U_t is at or below $L - \epsilon$, it remains possible for $\tilde{V}_t$ to reach the trigger even without a jump of the J_t process. But then in Figure 3.3 there is positive probability that S_t will jump from L to $U_{\tau_S} < L - \epsilon$ without a jump in J_t, which is incompatible with the martingale property and violates rational expectations. In the case of (23), condition (24) necessitates $\sigma > 0$. Condition (24) also rules out the discrete-time case in which U_t and V_t change values only at the fixed dates $t_1, \ldots, t_K$ and remain constant in between.

Theorem 4.1 *(i) If $U_T \leq V_T$ whenever $U_T \geq L$ then there exists at least one equilibrium stock price. (ii) If in addition the trigger accessibility condition (24) and the right-continuity condition (21) hold, then the equilibrium is unique.*

As an illustration, we can consider contingent capital in the dynamic structural model of Section 2.2 with asset value A_t described by (23). In this case, the processes U_t and V_t reduce to $U_t = u_t(A_t)$ and $V_t = v_t(A_t)$, with u_t and v_t given explicitly by a version of Merton's [27] formula for the price of an option in a jump-diffusion model.[13] The terminal value of the upper bound $\tilde{V}_T = \max\{U_T, V_T\}$ can be replicated with a portfolio of options, so $\tilde{V}_t = \tilde{v}_t(A_t)$ is given by a linear combination of variants of the same formula. It is easy to see that once U_t reaches $L - \epsilon$, the asset process A_t can decline to any positive level in an arbitrarily small time interval without the occurrence of a jump, ensuring that $\tilde{V}_t$ will reach or cross L as required by (24). In short, the only restriction imposed by the theorem on this example is in part (i), the same condition we had in the continuous case. We will see in the next section that the key requirement for existence and uniqueness is then simply $L \geq C/m$.

We can revisit Figure 5 to further illustrate the uniqueness result in Theorem 4.1. The trigger accessibility condition (24) requires that the trigger be reachable with positive probability without a jump in J_t — for example, through diffusion. In Figure 5, this means that there is positive probability that the continuous-time triangle path will reach the trigger without a jump

[13] The standing adaptedness assumption of this section allows U_t and V_t to be functions of the paths of the underlying Brownian motion and compound jump process J on $[0, t]$. This example specializes this general framework to the case in which U_t and V_t are functions of a single state variable A_t.

in J_t while the continuous-time circle path is below the trigger. As illustrated in the figure, this means that the continuous-time triangle path will jump upon hitting the trigger with no jump in J_t, which is impossible. A valid continuous-time stock price must have reached or crossed the trigger the first time the post-conversion price reached or crossed it, and this then defines the only continuous-time equilibrium.

5 Applications and Extensions

This section applies the theoretical results of the previous section to various examples. We start by revisiting contingent capital in the form of debt that converts to equity and then examine other types of changes in capital structure and other types of market triggers. Throughout this section, we use the dynamic structural model of Section 2.2. An equilibrium stock price is now a solution to (9) for some $v(A_T)$ and $u(A_T)$. The asset value process A_t is geometric Brownian motion (or the jump-diffusion (23)), and $U_t = E_t[u(A_T)]$ and $V_t = E_t[v(A_T)]$ are functions of the current asset value A_t.

5.1 Contingent Capital Revisited

In the case of contingent capital, v and u are given by (1) and (2), $0 < m < \infty$.

Corollary 5.1 *If $C \leq mL$, there is a unique equilibrium stock price. If $C > mL$ and $U_0 > L > 0$, there is no equilibrium.*

This result reflects a sharp contrast with its static counterpart, Proposition 2.2: *in this continuous-time setting there is no possibility of multiple equilibria.* Moreover, the possibility of having no equilibrium is easily ruled out by setting the trigger L (or the dilution factor m) to be sufficiently high. In light of the construction of S^* in Theorem 3.1, the unique equilibrium is easy to describe: conversion occurs the first time the post-conversion stock price reaches or crosses the trigger.[14]

Multiple equilibria will exist in the case $C < mL$ if the market evolves in discrete time steps, a case considered in Sundaresan and Wang [35] and illustrated schematically in Figure 5. In discrete time, it is possible for the post-conversion price U_t to be below the trigger when the trigger is breached. In the terminology of Sundaresan and Wang [35], this would result in a value transfer between contingent capital investors and shareholders. Sundaresan and Wang [35] propose modifying the terms of the contingent capital contract to eliminate the possibility of a value transfer. The purpose of their contract modification would be (in our notation) to

[14] In the trivial case $U_0 \leq L$ excluded from the statement of the corollary, we always get a unique equilibrium by setting $S_t = U_t$, and if $L = 0$ then the only equilibrium is $S_t = V_t$.

have U_t equal the trigger level at the moment of conversion. With continuous-time dynamics and no jumps, we have $U_{\tau_S} \equiv L$ *automatically*; no contract modification is needed to ensure this property holds.[15]

Once we have jumps, it is possible to have $U_{\tau_S} < L$, even in continuous time. This prompts a closer examination of how the notion of a value transfer relates to our results. As we have already noted, the conditions (7) and (12) that we use for existence of an equilibrium may be interpreted as stating that conversion is disadvantageous to shareholders. In other words, they may be interpreted as precluding a transfer of value *to* shareholders; but a transfer *from* shareholders is consistent with the existence of an equilibrium. Intuitively, a situation in which a drop in the stock price below the trigger caused the stock price to jump above the trigger would create a logical inconsistency that precludes existence of an equilibrium. There is no inconsistency if conversion is associated with a decline in the stock price.

These informal statements need to be interpreted with care because, as we have already stressed, the consequences of conversion (including the possibility $U_{\tau_S} < L$ in the case of jumps) are anticipated in an equilibrium stock price. A more precise statement of the transfer of value away from shareholders is that we have $S_t \geq U_t$ prior to conversion and $S_t = U_t$ after conversion, as illustrated in Figure 3.

We end this section with a final observation on the difference between the dynamic and static settings. It may seem paradoxical that multiple equilibria are possible in the static formulation of Proposition 2.2, which coincides with $T = 0$, but not for any $T > 0$. This apparent paradox is easily resolved. In the static problem with $C < mL$, the bounds in Proposition 2.2 imply that multiple equilibria can differ only at values of A where $u(A) \leq L < v(A)$; every equilibrium stock price has $S(A) = v(A)$ at larger values of A and $S(A) = u(A)$ at lower values of A. In the dynamic setting, we would therefore seem to face the possibility of multiple equilibria if $U_T \leq L < V_T$. However, if $U_T < L$, then $U_t < L$ for some $t < T$ (by continuity, or, in the case of jumps, because the probability of a jump right at T is zero) and any equilibrium must have converted before T and satisfies $S_T = U_T$. If $U_T = L$, then we have two potential candidates for an equilibrium stock price, $S_T = U_T$ and $S_T = V_T$, but $P(U_T = L) = P(A_T = B + (1 + m)L) = 0$. In short, the possibility of A_t reaching the problematic interval where $u(A_T) \leq L < v(A_T)$ (where the static problem admits multiple equilibria) without conversion

[15] Theorem 1 in [35] asserts a necessary condition for existence of an equilibrium which rules out the case $C < mL$ and is at odds with our Corollary 5.1. The proof in [35] has an error just after (A10), where an equality that holds at the instant of conversion is assumed to hold at all times t. In more detail, let C_t denote the value of the contingent capital at time t, and suppose for simplicity that there are no jumps. We automatically have $C_{\tau_S} = mS_{\tau_S} = mL$; the argument in [35] incorrectly concludes that then $C_t = mL$ for all t. A similar comment applies to Theorem 2 in [35] through its use of (A18). These points are also made independently in Pennacchi and Tchistyi [30].

having already occurred has probability zero.

5.2 Debt Triggers and Principal Write-Downs

An automatic debt write-down is often categorized as another form of contingent capital; see, for example, FSOC [12] and Sidley Austin [33]. Rabobank issued debt of this type in March 2010, designed to have the principal reduced to 25% of its original value if a regulatory capital ratio fell below a 7% trigger. Here we apply our results to the case of a market trigger.

To this point, we have taken the market trigger to be a stock price and formulated our equilibrium conditions in terms of stock prices. This interpretation however is not essential to our results, and the example of this section is convenient for illustrating the alternative of a debt trigger.

We adapt the basic dynamic structural model of Section 2.2 to the case of a principal write down. As before, let B denote the face value of the firm's debt, and now let αB, $0 < \alpha < 1$, be the level to which principal is reduced at the occurrence of a triggering event. We drop the contingent convertible debt from the capital structure (taking $C = 0$) to focus on the write-down. Consider the possibility of using the market value of the firm's debt as the trigger. Credit default swap spreads have been proposed as triggers for contingent capital, and, in the setting of our simple model a CDS trigger is equivalent to a bond price trigger. Writing Δ_t for the market price of the debt, the condition we need is

$$\Delta_T = \begin{cases} \alpha B, & \text{if } \Delta_t \leq L \text{ for some } t \in [0,T]; \\ B, & \text{otherwise,} \end{cases} \tag{25}$$

and then $\Delta_t = E_t[\Delta_T]$ for $t \in [0,T)$. Here we have implicitly assumed that the debt matures at T and pays no coupons; we consider coupons in Section 6.

If we now set

$$U_t = E_t[\min(A_T, \alpha B)] \quad \text{and} \quad V_t = E_t[\min(A_T, B)],$$

then these are the post-write-down and no-write-down bond prices in the basic dynamic structural model. It is immediate that $U_t \leq V_t$ for all t, so Theorem 3.1 ensures existence of an equilibrium — one that triggers a write-down the first time the post-write-down bond value is at or below the trigger. With A_t as in (23), the trigger accessibility and right-continuity conditions are also satisfied and we have uniqueness.

A stock price trigger for a principal write-down leads to a very different outcome. The post-write-down and no-write-down stock prices are given by

$$U_t = E_t[(A_T - \alpha B)^+] \quad \text{and} \quad V_t = E_t[(A_T - B)^+].$$

Thus, we have $U_t \geq V_t$, which is the opposite of what we need. In fact, with A_t geometric Brownian motion and $U_0 > L$, (18) holds *and there is no equilibrium stock price.* By replacing A_t with a suitably constrained process, we could ensure that V_t is always above L, and then taking $S_t = V_t$ would yield an equilibrium in which the trigger is never reached and the write-down never occurs, but this exception is rather contrived. The overall conclusion of this example is that an equity trigger is incompatible with a principal write-down.[16]

Taken together, this example and the contingent capital bond of Section 5.1 provide valuable insight into our results. The stock price trigger works for the contingent capital bond because the stock price decreases to the trigger and the post-conversion stock price is lower than the no-conversion price. Similarly, in (25), the bond price decreases to the trigger and the post-write-down bond price is lower than the no-write-down bond price. When we try to use a stock price trigger with a principal write-down, we create a situation in which as the stock price decreases toward the trigger the anticipated *increase* in the stock price resulting from the principal write-down pushes the stock price up, creating an inconsistency that precludes any equilibrium that can actually reach the trigger. The value transfer goes the wrong way.

This difficulty could be circumvented by setting L above U_0 and having the stock price reach the trigger from below. This is the mirror image of the setting of Section 3, and the results given there hold here with the directions of the inequalities reversed; see the appendix. However, the result is an unpalatable outcome in which a rising stock price triggers a principal write-down.[17]

One case remains to complete this discussion: a senior bond price trigger for contingent capital in the form of debt that converts to equity, as in Section 5.1. This case is easiest of all because the price of the senior debt is $\Delta_t = E_t[\min(A_T, B)]$ regardless of whether the contingent capital converts. There is no ambiguity or potential inconsistency in defining the conversion trigger to be the first time the value of the senior debt falls to some threshold because the debt value is unaffected by the conversion.[18]

[16] However, Pennacchi and Tchistyi [30] show that an automatic write-down is sometimes consistent with a stock price trigger in the case of perpetual debt; see the discussion in Section 6.3.

[17] A better example qualitatively consistent with a trigger in this direction is provided by performance-sensitive debt for which the coupon decreases as the stock price increases. Manso et al. [24] value contracts of this type, but with the coupon tied to a firm's asset value rather than its stock price, in which case equilibrium is not a concern.

[18] The trigger in Pennacchi [28] and Pennacchi, Vermaelen, and Wolff [29] has a similar property. It uses the sum of the market values of equity and contingent capital, which is unaffected by the conversion of one to the other.

5.3 Forced Deleveraging

Consider, next, a firm that issues equity and senior debt with a covenant that requires deleveraging in the event of a sufficiently large decline in the firm's stock price. More specifically, the covenant specifies that if the stock price reaches L, then a fraction $1-\beta$ of the firm's assets are transferred to the bond holders to retire a fraction $1-\alpha$ of the firm's debt, $0 < \alpha, \beta < 1$. Equivalently, we may think of a fraction $1-\alpha$ of the debt as being secured by a fraction $1-\beta$ of the assets and the bond holders seizing this collateral when the trigger is reached.

The post-deleveraging and no-deleveraging stock prices are given by

$$U_t = E_t[(\beta A_T - \alpha B)^+] \quad \text{and} \quad V_t = E_t[(A_T - B)^+].$$

Here we need to distinguish two cases. If $\alpha \geq \beta$, then $U_t \leq V_t$ and we get existence of an equilibrium stock price, with uniqueness under the trigger accessibility and right-continuity conditions of Section 3.3. If $\alpha < \beta$, we do not necessarily have $U_t \leq V_t$, but this ordering does hold at all sufficiently high asset levels. In particular, if we set the trigger so that $L \geq (\beta-\alpha)B/(1-\beta)$, then we can apply Lemma 3.2, modify V_T below the trigger (where its value is irrelevant), and get $U_t \leq V_t$ with this modification. (The appendix on linear contracts provides the details.) Indeed, from the perspective of equity holders this contract is nearly equivalent to the contingent capital setting, and Corollary 5.1 applies with only minor modification: If $L \geq (\beta-\alpha)B/(1-\beta)$, there is a unique equilibrium stock price; if L is below this level and $U_0 > L > 0$, no equilibrium exists.

5.4 Convertible Bonds

The holder of a conventional convertible bond has the option to convert the bond to equity and will do so when the share price rises. This contrasts with contingent capital that converts when the share price falls. To sharpen the parallel between the two cases, we consider convertible bonds that convert automatically the first time the stock price is greater than or equal to L. The no-conversion and post-conversion terminal equity claims are $(A_T - B - C)^+$ and $(A_T - B)^+/(1+m)$, just as in the case of contingent capital, with C now denoting the face value of the convertible bonds and m denoting the number of shares to which the bonds convert. The only difference is that the trigger is now approached from below rather than from above. In this case, if $C \geq mL$ there is a unique equilibrium and if $C < mL$ and $U_0 < L$ there is no equilibrium. The symmetry of this example is rather particular to this case — the roles of U_t and V_t are not symmetric so we cannot automatically reverse the direction from which the trigger is approached and expect the same conclusions to apply. See Proposition A.2 in the appendix for details.

Bolton and Samama [5] propose a *capital access bond* with an embedded option for the issuing bank to repay the bond with new equity. In their proposal, the conversion option is exercised by the bank when the stock price is sufficiently low. If the conversion threshold were a fixed level of the stock price, this security would function like our contingent capital example, with a unique equilibrium with a sufficiently high trigger. With the bank holding the option to set the trigger, it has an incentive to lower the trigger. At maturity the optimal level is clearly $L = C/m$, as Bolton and Samama [5] note. In other words, in the static case, the optionality moves the trigger to the only level that results in a unique equilibrium stock price.

6 Dividends, Coupons, and Infinite-Horizon Settings

To this point, we have excluded payments of any dividends or coupons, and this has allowed us to express all prices as conditional expectations of terminal payoffs. We now extend the analysis to allow intermediate cash flows. This also allows us to consider infinite-horizon problems in which there are no terminal payouts. In order to focus on a single extension at a time, in this section we first omit jumps and, as in Section 3, assume that the underlying filtration is generated by Brownian motion. We combine jumps and discrete dividends in Section 6.4.

6.1 Dividends

We denote by D_t^U and D_t^V the cumulative dividends *per share* paid in $[0, t]$ by the post-conversion and no-conversion firms. We assume that these are continuous processes and defer discrete dividends to Section 6.4. The corresponding stock prices are now given by

$$U_t = E_t[U_T + D_T^U - D_t^U] \quad \text{and} \quad V_t = E_t[V_T + D_T^V - D_t^V]; \tag{26}$$

in other words, the current price U_t is the conditional expectation of the sum of the terminal payment U_T and the future dividends $D_T^U - D_t^U$, and similarly for V_t.

We get a simple extension of previous results if we impose the ordering relation (13) and add

$$D_T^U - D_t^U \leq D_T^V - D_t^V, \quad \text{for all } 0 \leq t < T. \tag{27}$$

In light of (26), these conditions imply $U_t \leq V_t$, a property we have used before. When the dividends are given by *per share* payout rates δ_t^U, δ_t^V in the sense that

$$D_t^U = \int_0^t \delta_s^U \, ds \quad \text{and} \quad D_t^V = \int_0^t \delta_s^V \, ds, \tag{28}$$

then (27) is implied by $\delta_t^U \leq \delta_t^V$. These conditions on dividends are broadly consistent with the original ordering relationship (13): the condition $U_T \leq V_T$ indicates that the post-conversion

equity claim is always less than the no-conversion equity claim, suggesting that the conversion is punitive to shareholders. Reducing dividend payouts after conversion is then consistent with this interpretation. As we did in Theorem 3.1, we will use somewhat weaker and more broadly applicable conditions that impose the ordering when the stock prices are sufficiently high.

We need to modify our definition of an equilibrium stock price to incorporate dividends. The requirement on S_T in (10) continues to hold, but we also need to define an equilibrium dividend process associated with an equilibrium stock price. Writing τ_S for the first time $S_t \leq L$, we require

$$D_t^S = \begin{cases} D_{\tau_S}^V + D_t^U - D_{\tau_S}^U, & \text{if } \tau_S \leq t; \\ D_t^V, & \text{otherwise.} \end{cases} \tag{29}$$

In other words, the cumulative dividends paid by an equilibrium stock price coincide with those of the no-conversion process V_t before the stock price reaches the trigger; once the trigger is reached, the accumulation of subsequent dividend payments coincides with that of the post-conversion process U_t. To complete the definition of an equilibrium pair (S_t, D_t^S) of a price process and dividend process, we need to replace (11) with

$$S_t = E_t[S_T + D_T^S - D_t^S]. \tag{30}$$

We can now extend our previous results on existence and uniqueness.

Theorem 6.1 *(i) If (13) and (27) hold, there is an equilibrium stock price and dividend process. (ii) The same holds if*

$$U_{\tau_U \wedge T} \leq V_{\tau_U \wedge T} \quad \text{and} \quad E_t[D_{\tau_U \wedge T}^U - D_t^U] \leq E_t[D_{\tau_U \wedge T}^V - D_t^V], \tag{31}$$

for all $t \leq \tau_U$. (iii) In (i), if the right-continuity and trigger accessibility conditions hold, the equilibrium is unique. (iv) The same holds in (ii) with the addition of condition (22).

The first condition in (31) is implied by (22). We get a simpler formulation of the second condition in (31) when the cumulative dividends are given by dividend rates in the sense of (28). In the following corollary, we require that the post-conversion price and the post-conversion dividend rate should both be lower than their no-conversion counterparts when the post-conversion price is above the trigger.[19] Put differently, we need the trigger to be sufficiently high to imply these orderings.

[19] This dividend ordering is consistent with the example of performance-sensitive debt mentioned in Section 5.2 if a drop in the stock price triggers an increase in the debt coupon. With the total payout rate held fixed, increasing the debt coupon decreases the dividends to shareholders.

Corollary 6.2 *If $U_t \leq V_t$ whenever $U_t \geq L$ and also*

$$\delta_t^U \leq \delta_t^V \text{ whenever } U_t \geq L \tag{32}$$

then an equilibrium stock price and dividend process exist. If the right-continuity and trigger accessibility condition hold, the equilibrium is unique.

6.2 Dynamic Structural Model and Debt Covenants

To ground the discussion, we revisit the basic dynamic structural model of Section 2.2. The firm's assets A_t are modeled as geometric Brownian motion, and we posit, as in for example Leland [23], that the assets have a constant[20] payout rate δ and thus generate cash at rate δA_t at time t. The firm's senior debt pays a continuous coupon at rate b_t on a principal of B, and the contingent convertible debt pays a continuous coupon at rate c_t on a principal of C. The coupon rates could depend on the current asset level. The difference $\delta A_t - b_t B - c_t C$ is the rate at which dividends are paid to equity. Here, as in Leland [23] and much of the dynamic capital structure literature, this rate may become negative, in which case it should be interpreted as the issuance of small amounts of additional equity to existing shareholders.

After conversion of the contingent capital, the net dividend per share is $(\delta A_t - b_t B)/(1+m)$. Condition (27) is satisfied if the no-conversion dividend rate is higher than the post-conversion dividend rate,

$$\delta A_t - b_t B - c_t C \geq (\delta A_t - b_t B)/(1+m), \tag{33}$$

or, equivalently, if $m(\delta A_t - b_t B)/(1+m) \geq c_t C$.

We can relax (33) to the weaker condition in (32), which requires only that (33) hold when $U_t \geq L$. Suppose the coupon rates $b_t \equiv b$ and $c_t \equiv c$ are constant. By the Markov property of geometric Brownian motion, V_t is given by a deterministic function $v_t(A_t)$ of time and the underlying asset level, and similarly for U_t. Assuming a drift of $-\delta$ for A_t (consistent with the assumed payout rate of δ), we get

$$u_t(A_t) = E_t[(A_T - B)^+/(1+m)|A_t] + \frac{A_t}{1+m}(1 - e^{-\delta(T-t)}) - \frac{bB(T-t)}{1+m}$$

and

$$v_t(A_t) = E_t[(A_T - B - C)^+|A_t] + A_t(1 - e^{-\delta(T-t)}) - bB(T-t) - cC(T-t)$$

by evaluating the conditional expectations of the terminal payoffs and the expected dividends. Both $u_t(\cdot)$ and $v_t(\cdot)$ are unbounded and strictly increasing in $A_t > 0$ for every $t < T$. It is

[20] One could also consider the possibility that the payout rate changes at conversion but for simplicity we consider only constant δ.

easy to see that (33) holds for all sufficiently large A_t, and thus for all sufficiently large U_t, as required by (32). In particular, (32) holds if

$$L \geq \frac{cC}{m\delta} + \frac{(b-\delta)B}{(1+m)\delta}.$$

Some algebra shows that $u_t(A_t) \leq v_t(A_t)$ whenever

$$A_t \geq a_t = B(1 + b(T-t)) + (1+m)C(1 + c(T-t))/m,$$

so we can ensure that (22) holds if the trigger is sufficiently high — high enough that $u_0(a_0) \leq L$. Moreover, with geometric Brownian motion and constant coupon rates, the right-continuity and trigger accessibility conditions hold; thus, we get existence and uniqueness of an equilibrium stock price by setting the trigger sufficiently high.

A similar analysis applies with a debt covenant of the type used in Black and Cox [4]. In their model, a debt covenant requires that the firm's assets be liquidated to pay the firm's debt if the asset level falls below a specified boundary. If the boundary is lower than the face value of the debt, then the liquidation results in a loss to bond holders, which could be interpreted as a bankruptcy cost.

Consider, then, a liquidation boundary at an asset level of kB, for some k. With $k < 1$, bond holders suffer a loss at liquidation and there is no residual payment to equity holders. Let τ_A denote the first time A_t reaches the level kB. With constant coupon rates, the no-conversion stock price is given by

$$V_t = E_t\left[(A_T - B - C)^+ \mathbf{1}\{\tau_A > T\} + \int_t^{\tau_A \wedge T} (\delta A_s - bB - cC)\, ds\right],$$

and the post-conversion stock price by

$$U_t = E_t\left[\frac{1}{1+m}(A_T - B)^+ \mathbf{1}\{\tau_A > T\} + \frac{1}{1+m}\int_t^{\tau_A \wedge T} (\delta A_s - bB)\, ds\right].$$

If in addition to $k < 1$ we have

$$k \geq \frac{b}{\delta} + \frac{(1+m)}{m}\frac{c}{\delta}\frac{C}{B},$$

then the no-conversion dividend rate is always at least as large as the post-conversion dividend rate, because this lower bound on k implies that (33) holds whenever $A_t \geq kB$. If $L \geq C/m$, then (22) holds. The values of U_t and V_t are now given by barrier option formulas, rather than the Black-Scholes formula. The unique equilibrium stock price with $C \leq mL$ triggers conversion the first time $U_t = L$; and because $U_t = 0$ at τ_A, the contingent capital is necessarily converted to equity before a liquidation is triggered by the debt covenant.

6.3 Infinite-Horizon Formulation

A small step takes us from a model with dividends to an infinite-horizon formulation. Without a terminal date T, equity value is given by the present value of all future dividends. The appendix examines this case in detail. Here we present a simple example.

Consider the basic dynamic structural model of Section 2.2 with payout rate δ and constant coupon rates b and c on perpetual senior debt with face value B and contingent convertible debt with face value C. To get finite stock prices, we introduce a constant discount rate $r > 0$. The no-conversion equity value is

$$V_t = E_t \left[\int_t^\infty e^{-r(s-t)} (\delta A_s - bB - cC)\, ds | A_t \right] = A_t - \frac{bB + cC}{r},$$

and the post-conversion equity value is

$$U_t = E_t \left[\int_t^\infty e^{-r(s-t)} (\delta A_s - bB)/(1+m)\, ds | A_t \right] = \frac{A_t}{1+m} - \frac{bB}{r(1+m)}.$$

We have implicitly assumed that the drift of A_t is $(r - \delta)$ so that A_t is itself the present value of its future payouts δA_s, $s \geq t$, and zero is an absorbing state for U_t and V_t corresponding to bankruptcy. Algebraic simplification now shows that we can ensure that $U_t \leq V_t$ whenever $U_t \geq L$ by setting $cC/r \leq mL$. This is essentially the same condition as in the finite-horizon setting of Corollary 5.1, except that the principal value C has been replaced with the capitalized value of the coupon payments cC/r.

The condition $cC/r \leq mL$ may be interpreted to mean that the terms of conversion are disadvantageous to shareholders. Pennacchi and Tchistyi [30] show that when A_t follows geometric Brownian motion, it is sometimes possible to construct an equilibrium under the reverse inequality $cC/r > mL$, where conversion favors shareholders. They show that this construction (and a similar result for an automatic debt write-down) requires that the asset volatility be sufficiently large. This condition makes their result qualitatively different from most of ours: in our main existence results, we have emphasized structural features that are not sensitive to specific stochastic assumptions.

As observed by Pennacchi and Tchistyi [30], their results also highlight differences between finite-maturity and perpetual debt: a lump-sum payment at a maturity date T can preclude existence of an equilibrium under conditions that would support an equilibrium with debt paying a continuous coupon stream but no principal.

6.4 Discrete Dividends and Jumps

We now introduce discrete dividends and reintroduce jumps in the form we used in Section 4. We assume that dividend payments are restricted to fixed dates $0 < t_1 < \cdots < t_n < T$. To

separate these dividend dates from other types of jumps, we omit the second term in J_t. To keep the setting simple, we will suppose that the post-conversion firm pays no dividends, and we will further suppose that the probability of the post-conversion firm reaching or crossing the trigger at a dividend date is zero, $P(\tau_U \in \{t_1, \ldots, t_n\}) = 0$.

We define a cumulative dividend *per share* D_t^V for V_t by writing

$$D_t^V = \sum_{i:t_i \leq t} \delta_i^V,$$

for some payments δ_i^V, which, for simplicity we take to be positive. We augment $\mathcal{F}_t$ to include payments made in $[0, t]$. We define an equilibrium stock price and dividend process as a pair (S_t, D_t^S) with terminal value S_T as in (10) and satisfying the pricing relation $S_t = E_t[S_T + D_T^S - D_t^S]$, $t \in [0, T]$, with

$$D_t^S = \sum_{i:t_i \leq t,\, t_i < \tau_S} \delta_i^V; \tag{34}$$

in other words, S_t pays the same dividends as V_t prior to conversion at τ_S.

To establish uniqueness, we modify the trigger accessibility condition (24). We need a process $\tilde{V}_t$ that upper bounds any possible equilibrium stock price. For example, we can take

$$\tilde{V}_t = E_t[\max\{U_T, V_T\} + D_T^V - D_t^V].$$

As in (24), we take T_ϵ to be the minimum of the next dividend date and the time of the next Poisson jump after τ_U^ϵ. Thus, there are no jumps between τ_U^ϵ and T_ϵ either from dividends or from the Poisson process.

Theorem 6.3 *Under the foregoing conditions, (i) if $U_t \leq V_t$ whenever $U_t \geq L$ then there exists at least one equilibrium stock price and dividend process. (ii) If in addition the trigger accessibility condition and the right-continuity condition hold, then the equilibrium is unique.*

7 Concluding Remarks

We have analyzed the problem of internal consistency that arises when a firm's stock price (or the market price of any other liability of the firm) is used to trigger a change in the firm's capital structure. Within a general framework, we have shown that continuous-time dynamics largely rule out the possibility of multiple equilibria by allowing the market price to adjust in anticipation of reaching the trigger. A practical interpretation of this phenomenon is that trading in the triggering security should be sufficiently active to make the continuous-time paradigm a reasonable reflection of reality. Existence of an equilibrium follows from conditions that ensure consistency between the direction of a market trigger and its consequences. In the

case of contingent capital, we need the conversion trigger or the dilution ratio at conversion to be sufficiently high so that conversion is disadvantageous to shareholders.

The cases we have analyzed involve a contractual change in capital structure, as proposed in the design of contingent capital for banks. In practice, most changes in capital structure involve managerial discretion. Investors seek to anticipate a firm's decisions, and these decisions are in part informed by market prices, so similar questions about equilibria arise. These are settings in which the trigger itself is not well-defined; the approach developed here may nevertheless be relevant to these settings as well.

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A Appendix

A.1 The Static Case

We abstract from the specific payoff functions in (1) and (2) and let u and v be arbitrary functions from $\mathbb{R}$ to $\mathbb{R}$. We continue to use the definitions in (3)–(5).

Proposition A.1 *Suppose the functions u and v satisfy (7). (i) Then $\underline{S}$ and $\overline{S}$ are equilibrium stock prices. (ii) Every equilibrium stock price satisfies*

$$S(A) \in \{u(A), v(A)\} \quad \textit{and} \quad \underline{S}(A) \leq S(A) \leq \overline{S}(A),$$

for all A, and, conversely, any function satisfying this property is an equilibrium stock price. (iii) If the sets $\{A : u(A) \geq L\}$ and $\{A : v(A) \geq L\}$ coincide there is just one equilibrium stock price, $S = \underline{S} = \overline{S}$. (iv) If (7) does not hold and if $\overline{S}(A) < \underline{S}(A)$ at some A, then no equilibrium exists.

Proof. (i) If $\underline{S}(A) \leq L$ and $\underline{S}(A) = v(A)$ then $v(A) \leq L$, but then (7) implies $u(A) \leq L$ and then $\underline{S}(A) = u(A)$ in light of (4). If $\underline{S}(A) > L$ and $\underline{S}(A) = u(A)$ then $u(A) > L$ so $\underline{S}(A) = v(A)$. Thus, $\underline{S}$ satisfies (3). If $\overline{S}(A) \leq L$ and $\overline{S}(A) = v(A)$ then $v(A) \leq L$ and $\overline{S}(A) = u(A)$. If $\overline{S}(A) > L$ and $\overline{S}(A) = u(A)$ then $u(A) > L$ so $v(A) > L$ and then $\overline{S}(A) = v(A)$. Thus, $\overline{S}$ satisfies (3).

(ii) Suppose S is an equilibrium stock price. If $S(A) \leq L$ then $S(A) = u(A) = \underline{S}(A)$; also, either $\overline{S}(A) = u(A)$ or $\overline{S}(A) = v(A) > L$. If $S(A) > L$ then $S(A) = v(A) \geq u(A)$, so $S(A) \geq \underline{S}(A)$; also, $\overline{S}(A) = v(A)$. Thus $\underline{S} \leq S \leq \overline{S}$. Conversely, let S be any function with the property in (ii). If $S(A) \leq L$ and $S(A) = v(A)$ then $\overline{S}(A) = u(A)$ and $\underline{S}(A) \leq L$

so $\underline{S}(A) = u(A)$; thus $S(A) = u(A)$. If $S(A) > L$ and $S(A) = u(A)$ then $\underline{S}(A) = v(A)$ and $v(A) \geq u(A)$ so $S(A) = v(A)$. Thus, S is an equilibrium stock price.

(iii) If the sets coincide then $\underline{S} = \overline{S}$ so this is the only equilibrium. (iv) Of the four possible combinations that can be constructed by pairing one of the outcomes $u(A) \leq L$ and $u(A) > L$ with either $v(A) \leq L$ or $v(A) > L$, the only one consistent with $\overline{S}(A) < \underline{S}(A)$ is $u(A) > L$ and $v(A) \leq L$. As in the discussion of (6), this rules out the possibility of an equilibrium. □

A.2 Existence and Uniqueness

To complete the proof of Theorem 3.1(ii), we just need to prove Lemma 3.2.

Proof of Lemma 3.2. Suppose S_t is an equilibrium stock price in the sense of (10)–(11). If $S_t > L$ for all $t \in [0, T]$, then $S_T = V_T$ so $V_T > L$, in which case (12) implies $V_T \geq U_T$. Thus, (10) continues to hold if we replace V_T with $\tilde{V}_T = \max\{U_T, V_T\}$. Conversely, suppose (10) holds with V_T replaced by $\tilde{V}_T$. If $V_T \geq U_T$, then $\tilde{V}_T = V_T$, so suppose $U_T > V_T$. In light of (12), this requires $U_T \leq L$ and thus $V_T \leq L$. But then $S_T \leq L$ and $S_T = U_T$. In other words, if $U_T > V_T$ then the value of V_T is irrelevant, so if (10) holds with $\tilde{V}_T$ it holds with V_T. □

Proof of Theorem 3.5. We prove the second case in the theorem. In the first case, any equilibrium must satisfy $U_t \leq S_t \leq V_t$ so a simpler version of the same argument applies.

Let τ_V denote the first time V_t reaches L; condition (22) implies $\tau_U \leq \tau_V$. Let S_t be an equilibrium stock price and let τ_S be the first time it reaches L. If $t \leq \tau_S$ then

$$\begin{aligned} S_t &= E_t[S_{\tau_S \wedge T}] \\ &= E_t[S_{\tau_S}\mathbf{1}\{\tau_S \leq T\} + S_T\mathbf{1}\{\tau_S > T\}] \\ &= E_t[U_{\tau_S}\mathbf{1}\{\tau_S \leq T\} + V_T\mathbf{1}\{\tau_S > T\}], \end{aligned}$$

and with (22), $U_T > V_T$ would require $S_T \leq \max\{U_T, V_T\} < L$, so

$$U_t = E_t[U_{\tau_S}\mathbf{1}\{\tau_S \leq T\} + U_T\mathbf{1}\{\tau_S > T\}] \leq S_t \leq E_t[V_{\tau_S}\mathbf{1}\{\tau_S \leq T\} + V_T\mathbf{1}\{\tau_S > T\}] = V_t. \quad (35)$$

This in turn implies that $\tau_S \leq \tau_V$.

Write the event in the trigger accessibility condition (20) as

$$\mathcal{A}_\epsilon = \{U_t < L \text{ for all } t \in [\tau_U^\epsilon, T] \text{ and } V_t \leq L \text{ for some } t \in [\tau_U^\epsilon, T]\}.$$

On the event $\mathcal{A}_\epsilon$, we have $\tau_V \leq T$ and therefore $\tau_S \leq T$. However, we know from Proposition 3.4 (and (19) in particular) that $U_{\tau_S} = L$, whereas $U_t < L$ for all $t \in [\tau_U^\epsilon, T]$ on $\mathcal{A}_\epsilon$. Thus, almost surely,

$$P(\mathcal{A}_\epsilon \cap \{\tau_S > \tau_U^\epsilon\} | \mathcal{F}_{\tau_U^\epsilon}, \tau_U^\epsilon < T) = 0. \quad (36)$$

Because the event $\{\tau_S > \tau_U^\epsilon\}$ is contained in $\mathcal{F}_{\tau_U^\epsilon}$, its conditional probability is 0 or 1, so in view of the trigger accessibility condition, (36) implies

$$P(\tau_S > \tau_U^\epsilon | \mathcal{F}_{\tau_U^\epsilon}, \tau_U^\epsilon < T) = 0,$$

almost surely. This implies $P(\tau_S \leq \tau_U^\epsilon | \tau_U^\epsilon < T) = 1$. The right-continuity condition implies $P(\tau_S \leq \tau_U^\epsilon < T) \to P(\tau_S \leq \tau_U < T)$ and $P(\tau_U^\epsilon < T) \to P(\tau_U < T)$. The right-continuity condition also implies $P(\tau_U = T) = 0$; otherwise, there would be positive probability that $\tau_U^\epsilon = \infty$ for all $\epsilon > 0$ while $\tau_U = T$. Thus, we have $P(\tau_S \leq \tau_U | \tau_U \leq T) = 1$. From the proof of Theorem 3.1, we know that $\tau_S \geq \tau_U$. We conclude that $P(\tau_S = \tau_U) = 1$, and thus $S = S^*$, the equilibrium constructed in the proof of Theorem 3.1. □

It is evident from the proof that in the trigger accessibility condition we could replace T in the interval $[\tau_U^\epsilon, T]$ with $[\tau_U^\epsilon, T \wedge \tau_U^\epsilon + h]$, for any $h > 0$, or with the interval $[\tau_U^\epsilon, \tau_V]$.

Proof of Theorem 4.1. (i) In the proofs of Lemma 3.2 and Theorem 3.1, we need to replace "reaches the trigger" or "reaches L" with "reaches $(-\infty, L]$," because conversion may now be triggered by a jump that crosses L. With this modification, the proofs hold exactly as before because the arguments there do not use continuity.

(ii) Even with jumps, we have $S_{\tau_S} = U_{\tau_S}$, by the definition (10)–(11) of an equilibrium S and the fact $S_{\tau_S} = S_{\tau_S+}$ and $U_{\tau_S} = U_{\tau_S+}$ record values after any jump at τ_S. The argument in the proof of Theorem 3.5 showing in (35) that $U_t \leq S_t$ for all $t \leq \tau_S$ therefore applies. We do not necessarily have $U_{\tau_S} \leq V_{\tau_S}$ because a jump at τ_S could result in $V_{\tau_S} < U_{\tau_S} < L$, so the upper bound in (35) may no longer hold, but we clearly have $S_t \leq \tilde{V}_t$ for all $t \in [0, T]$.

Let $\mathcal{A}_\epsilon$ denote the event in the modified trigger accessibility condition (24). On this event, we have $\tau_S \in [\tau_U, T_\epsilon)$ because the upper bound $\tilde{V}_t$ reaches or crosses the trigger before T_ϵ and the lower bound U_t does so at τ_U. However, the path of S_t is continuous throughout the interval $[\tau_U^\epsilon, T_\epsilon)$ because J_t has no jumps in this interval. Thus, for S_t to reach the trigger in this interval we would need to have $S_{\tau_S} = L$ and then $U_{\tau_S} = S_{\tau_S} = L$, which is impossible on $\mathcal{A}_\epsilon$. Thus, (36) holds and the rest of the proof goes through exactly as before. □

Proof of Corollary 5.1. If $C \leq mL$, then the terminal payments (1) and (2) satisfy (7), so (12) holds and we get existence from Theorem 3.1(ii). By the Markov property, $U_t = u_t(A_t)$ for some deterministic function of time and asset level; $u_t(A_t)$ is essentially the Black-Scholes formula (or Merton's [27] extension with jumps) for a call option (monotone and continuous in t and A), and it inherits the right-continuity property (21) from A_t. Let $\tilde{V}_T = \max\{U_T, V_T\}$ and $\tilde{V}_t = E_t[\tilde{V}_T] = E_t[\max\{u(A_T), v(A_T)\}]$. By Lemma 3.2, it suffices to show that the trigger accessibility condition holds with $\tilde{V}_t$ in place of V_t. But $\tilde{V}_t \leq u_t(A_t) + v_t(A_t)$, where $v_t(A_t) =$

$E_t[v(A_T)]$ is again given by a variant of the Black-Scholes formula. It is clear that after A_t has fallen far enough that $u_t(A_t) = L - \epsilon$, there is positive probability that it will subsequently fall to the point where $u_t(A_t) + v_t(A_t) \leq L$ before returning to a level at which $u_t(A_t) = L$ (and before the next jump in J_t).

If $C > mL$, then there is strictly positive probability of having A_T end up in the interval $(u_T^{-1}(L), v_T^{-1}(L))$, which is $(B + (1+m)L, B + C + L)$, while having $u_t(A_t) > L$ for all t. In other words, (18) holds and there is no equilibrium. □

A.3 Stochastic Discounting

We have assumed that investors are risk-neutral and interest rates are zero largely for notational convenience: under these assumptions, the prices U_t, V_t, and S_t are simply conditional expectations of future payouts. Here we briefly show that our results hold without these assumptions.

We assume the existence of a stochastic discount factor $Z_t > 0$ such that all price processes satisfy relations of the form $U_t = E_t[Z_T U_T]/Z_t$, $V_t = E_t[Z_T V_T]/Z_t$. In a general equilibrium asset pricing model, the stochastic discount factor encodes the time and risk preferences of a representative agent, with Z_t measuring the marginal utility of optimal consumption at time t. Even without such a framework, the absence of arbitrage is essentially equivalent to the existence of a stochastic discount factor.

In modifying our results for this setting, we leave the definition of an equilibrium in (10) unchanged but replace (11) with the pricing relation $S_t = E_t[Z_T S_T]/Z_t$. Observe that $U_T \leq V_T$ continues to imply $U_t \leq V_t$, as in (15), with stochastic discounting, and Lemma 3.2 continues to hold as well. Theorems 3.1 and 3.5 hold in this setting with the price processes U_t and V_t now defined through the stochastic discounting relation.

A.4 Linear Contracts

In this section, we provide a result that covers several of the examples in Section 5 and others, all formulated within the basic dynamic structural model of Section 2.2. The no-conversion and post-conversion stock prices are given by

$$U_t = E_t[(aA_T - bB_t)^+] \quad \text{and} \quad V_t = E_t[(A_T - B)^+], \tag{37}$$

for some positive constants $a \neq 1$ and b. (For the case $a = 1$, see Section 5.2.) The existence and uniqueness of an equilibrium stock price with a conversion trigger at L depend on these coefficients. We will refer to (10) as a *lower-trigger* equilibrium because the trigger condition is $S_t \leq L$; we will also consider the case of an *upper-trigger* equilibrium in which the condition is $S_t \geq L$. To rule out uninteresting cases, we assume $U_0 > L$ in the lower-trigger case and

$U_0 < L$ in the upper-trigger case. Set $L^* = (a-b)B/(1-a)$; this is the level at which the lines $A \mapsto aA - bB$ and $A \mapsto A - B$ intersect.

Proposition A.2 *In the setting of (14) and (37) with A_t geometric Brownian motion or the jump-diffusion (23), we have the following:*

(i) *If $a > 1$ and $b \leq a$, there is no lower-trigger equilibrium and there is a unique upper-trigger equilibrium for any L.*

(ii) *If $b < a < 1$, there is a unique lower-trigger equilibrium for any $L \geq L^*$ and no lower-trigger equilibrium otherwise. There is a unique upper-trigger equilibrium for any $L \leq L^*$ and no upper-trigger equilibrium otherwise.*

(iii) *If $a < 1$ and $a \leq b$, there is a unique lower-trigger equilibrium and no upper-trigger equilibrium for any L.*

(iv) *If $1 < a < b$, there is no lower-trigger equilibrium if $L > L^*$ and no upper-trigger equilibrium if $L < L^*$.*

Proof. In case (i), $U_t \geq V_t$ for all $t \in [0,T]$, so existence and uniqueness in the upper-trigger case follows from the arguments used in Theorems 3.1 and 3.5. In the lower-trigger case, (18) holds. The lower-trigger case of (ii) becomes equivalent to the setting of Corollary 5.1 if we replace B with $\tilde{B} = B + C$ and set $a = 1/(1+m)$ and $b = aB/(B+C)$. A symmetric argument works for the upper-trigger case. In (iii), $V_t \geq U_t$ for all $t \in [0,T]$, so existence and uniqueness in the lower-trigger case follows from Theorems 3.1 and 3.5. For the upper-trigger case, (18) holds with the inequalities reversed, ruling out an equilibrium.

For case (iv), observe that the Markov property of A_t allows us to write $U_t = u_t(A_t)$ and $V_t = v_t(A_t)$, where u_t and v_t are deterministic functions of t and A_t (both given by variants of the Black-Scholes or Merton formulas). Let $u_t^{-1}(L)$ be the asset level that makes $U_t = L$; i.e., that solves $u_t(u_t^{-1}(L)) = L$, and define $v_t^{-1}(L)$ accordingly. If $L > L^*$, then $u_T^{-1}(L) < v_T^{-1}(L)$, and then by continuity of these inverses $u_t^{-1}(L) < v_t^{-1}(L)$ for all $t \in [T-\epsilon, T]$ for some $\epsilon > 0$. Now there is strictly positive probability that A_t will be above $u_t^{-1}(L)$ for all $0 \leq t \leq T$ (recall that $U_0 > L$ so $A_0 > u_0^{-1}(L)$) and then $A_t \in (u_T^{-1}(L), v_T^{-1}(L))$ for all $t \in [T-\epsilon, T]$. But then $V_T = v_T(A_T) < L$ whereas $U_t = u_t(A_t) > L$ for all $t \in [0,T]$; in other words, (18) holds. The second assertion of (iv) follows by a symmetric argument. □

There are two possibilities not covered by Proposition A.2, both arising in case (iv). If $L \leq L^*$, we cannot rule out the possibility of a lower-trigger equilibrium; however, we cannot ensure that $S_t^* \geq U_t$, so the argument in Theorem 3.1 does not apply. Similar comments apply for an upper-trigger equilibrium with $L \geq L^*$.

A.5 Dividends

Proof of Theorem 6.1. Part (i) follows from (ii). For (ii), define S_T^* as in (16) and define a dividend process D^* as in (29) but with τ_S replaced by τ_U. For $t < T$, define

$$S_t^* = E_t[S_T^* + D_T^* - D_t^*].$$

The process D_t^* is continuous because D_t^V and D_t^U are continuous; the process $S_t^* + D_t^*$ is continuous because it is a Brownian martingale; it follows that S_t^* is continuous. For $t \leq \tau_U$,

$$\begin{aligned} S_t^* &= E_t[S_{\tau_U \wedge T}^*] + E_t[D_{T\wedge\tau_U}^V - D_t^V] \\ &= E_t[U_{\tau_U}\mathbf{1}\{\tau_U \leq T\} + V_T\mathbf{1}\{\tau_U > T\}] + E_t[D_{T\wedge\tau_U}^V - D_t^V] \\ &\geq E_t[U_{\tau_U}\mathbf{1}\{\tau_U \leq T\} + U_T\mathbf{1}\{\tau_U > T\}] + E_t[D_{T\wedge\tau_U}^U - D_t^U] \\ &= U_t. \end{aligned} \tag{38}$$

It follows that S_t^* first reaches L at τ_U, and thus that S_t^* is an equilibrium, as in the proof of Theorem 3.1.

For uniqueness, first observe that any equilibrium stock price S_t is continuous by the argument used for S_t^*, so $S_{\tau_S} = U_{\tau_S} = L$. For (iii), once we have $U_t \leq V_t$ for all $t \in [0, T]$, uniqueness follows as in Theorem 3.5. For (iv), if S_t is an equilibrium stock price process then

$$\begin{aligned} S_{\tau_S\wedge T} &= S_{\tau_S}\mathbf{1}\{\tau_S \leq T\} + S_T\mathbf{1}\{\tau_S > T\} \\ &= U_{\tau_S}\mathbf{1}\{\tau_S \leq T\} + V_T\mathbf{1}\{\tau_S > T\} \leq V_{\tau_S\wedge T}, \end{aligned}$$

using $U_{\tau_S} = L$ and (22). It follows that if $t \leq \tau_S \wedge T$, then

$$S_t = E_t[D_{\tau_S\wedge T}^V - D_t^V + S_{\tau_S\wedge T}] \leq V_t;$$

and if $\tau_S < t \leq T$, then $S_t = U_t$, so $S_t \leq V_t$ if $S_t \geq L$. Combining these two cases, we conclude that $S_t \leq V_t$ for all $t \leq \tau_V$. The proof of Theorem 3.5 now applies. □

A.6 Infinite Horizon

For the infinite-horizon formulation, we set $U_t = E_t[D_\infty^U - D_t^U]$ and $V_t = E_t[D_\infty^V - D_t^V]$, where D_t^U and D_t^V are continuous dividend processes, their limits D_∞^U and D_∞^V are almost surely finite, and U_t and V_t uniformly integrable. An equilibrium pair (S_t, D_t^S) satisfies $S_t = E_t[D_\infty^S - D_t^S]$, where

$$D_t^S = \begin{cases} D_{\tau_S}^V + D_t^U - D_{\tau_S}^U, & t \geq \tau_S; \\ D_t^V, & t < \tau_S. \end{cases} \tag{39}$$

We modify the trigger accessibility condition by fixing a $T > 0$ and requiring $P(\tau_U^\epsilon < \infty) > 0$ and, conditional on $\tau_U^\epsilon < \infty$, we have

$$P(U_t < L \text{ for all } t \in [\tau_U^\epsilon, \tau_U^\epsilon + T] \text{ and } V_t \leq L \text{ for some } t \in [\tau_U^\epsilon, \tau_U^\epsilon + T] | \mathcal{F}_{\tau_U^\epsilon}) > 0, \tag{40}$$

almost surely.

Proposition A.3 *(i) If $E_t[D^V_{\tau_U} - D^V_t] \geq E_t[D^U_{\tau_U} - D^U_t]$ for all $t < \tau_U$, then there exists at least one equilibrium stock price and corresponding dividend process. (ii) If $E_t[D^V_{\tau_V} - D^V_t] \geq E_t[D^U_{\tau_V} - D^U_t]$ for all $t < \tau_V$, and the trigger accessibility condition (40) and the right-continuity condition (21) hold, then the equilibrium is unique.*

Proof. Define D^*_t as in (39) but with τ_U in place of τ_S, and then set $S^*_t = E_t[D^*_\infty - D^*_t]$. The condition in (i) ensures that $S^*_t \geq U_t$ and thus that S^*_t is an equilibrium by the argument used previously. The condition in (ii) ensures that any equilibrium satisfies $S_t \leq V_t$, and then uniqueness follows by the argument used previously. □

A.7 Jumps and Discrete Dividends

Proof of Theorem 6.3. If $t < \tau_U$, then throughout the interval $[t, \tau_U \wedge T)$ we have $U_t > L$ and then $U_t \leq V_t$. Define S^* as in the proof of Theorem 6.1, and define D^* as in (34) but with τ_S replaced by τ_U. On the event $\tau_U < T$, we have $S^*_{\tau_U} = U_{\tau_U}$ by construction, and then $S^*_t \geq U_t$ for all $t \leq \tau_U$, as in (38). We conclude that (S^*_t, D^*_t) is an equilibrium as in Theorems 3.1 and 6.1. Uniqueness follows from the trigger accessibility and right-continuity conditions, together with the bound $S_t \leq \tilde{V}_t$, for all $t \in [0, T]$, which holds by construction. □

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