

ECONOMETRICA

JOURNAL OF THE ECONOMETRIC SOCIETY

*An International Society for the Advancement of Economic
Theory in its Relation to Statistics and Mathematics*

<http://www.econometricsociety.org/>

Econometrica, Vol. 82, No. 5 (September, 2014), 1557–1614

FISCAL RULES AND DISCRETION UNDER PERSISTENT SHOCKS

MARINA HALAC

*Columbia Business School, New York, NY 10027, U.S.A. and University of
Warwick*

PIERRE YARED

Columbia Business School, New York, NY 10027, U.S.A. and NBER

The copyright to this Article is held by the Econometric Society. It may be downloaded, printed and reproduced only for educational or research purposes, including use in course packs. No downloading or copying may be done for any commercial purpose without the explicit permission of the Econometric Society. For such commercial purposes contact the Office of the Econometric Society (contact information may be found at the website <http://www.econometricsociety.org> or in the back cover of *Econometrica*). This statement must be included on all copies of this Article that are made available electronically or in any other format.

FISCAL RULES AND DISCRETION UNDER PERSISTENT SHOCKS

BY MARINA HALAC AND PIERRE YARED¹

This paper studies the optimal level of discretion in policymaking. We consider a fiscal policy model where the government has time-inconsistent preferences with a present bias toward public spending. The government chooses a fiscal rule to trade off its desire to commit to not overspend against its desire to have flexibility to react to privately observed shocks to the value of spending. We analyze the optimal fiscal rule when the shocks are persistent. Unlike under independent and identically distributed shocks, we show that the ex ante optimal rule is not sequentially optimal, as it provides dynamic incentives. The ex ante optimal rule exhibits history dependence, with high shocks leading to an erosion of future fiscal discipline compared to low shocks, which lead to the reinstatement of discipline. The implied policy distortions oscillate over time given a sequence of high shocks, and can force the government to accumulate maximal debt and become immiserated in the long run.

KEYWORDS: Asymmetric and private information, mechanism design, hyperbolic discounting, macroeconomic policy, political economy.

1. INTRODUCTION

GOVERNMENTS OFTEN IMPOSE RULES on themselves to constrain their behavior in the future. One of the most prevalent form of such rules is fiscal rules, typically adopted in response to rising public debts. In 2009, 80 countries had fiscal rules in place, a dramatic increase from 1990 when only 7 countries had them.²

Despite their prevalence, little is known about the optimal structure of fiscal rules. How much discipline does an optimal fiscal rule impose? How does this level of discipline evolve over time? And does an optimal rule restrict the growth of debt in the long run? Any theory of fiscal rules must take into account a fundamental trade-off between commitment and flexibility: on the one hand, rules provide valuable commitment as they can limit distorted incentives

¹We would like to thank Manuel Amador, Sylvain Chassang, Georgy Egorov, Navin Kartik, Ilan Kremer, Ciamac Moallemi, Emi Nakamura, Ali Shourideh, Paolo Siconolfi, Jon Steinsson, Bruno Strulovici, Steve Tadelis, Aleh Tsyvinski, Iván Werning, Steve Zeldes, seminar participants at Brown, Columbia, Duke, EIEF, Minneapolis Fed, New York Fed, Northwestern, Princeton, UC-Davis, UCSD, UPenn, USC, and Yeshiva, and participants at the NBER SI 2012 Political Economy Public Finance Workshop, the 2012 SED Conference, the 2013 Nemmers Prize Conference, the 2013 ESSET Conference, and the 2013 Minnesota Workshop in Macroeconomic Theory for comments. We are grateful to a co-editor and two referees for their valuable feedback. Sergey Kolbin provided excellent research assistance.

²See [International Monetary Fund \(2009\)](#). Of those 80 countries, 60 had a deficit limit, 60 had a debt limit, and 25 had a spending limit. These limits vary in their specification and the extent to which they adjust to levels of gross domestic product (GDP) and to the business cycle. Moreover, these limits also vary in the degree to which they are enforced. Some countries have formal escape clauses that allow for a temporary abandonment of fiscal rules; other countries—for example, France and Germany in 2003—have instead abandoned rules less formally.

in policymaking that result in a spending bias and excessive deficits; on the other hand, there is a cost of reduced flexibility as fiscal constitutions cannot spell out policy prescriptions for every single shock or contingency, and some discretion may be optimal.

This paper studies the trade-off between commitment and flexibility in a dynamic self-control setting. In our model, a present-biased government privately observes a shock to the economy in each period, and a fiscal rule is a mechanism that assigns a policy as a function of the government's reported shocks. We follow a similar approach to that used in [Amador, Werning, and Angeletos \(2006\)](#), but we depart from their work by considering an environment in which shocks are persistent over time.³ We are motivated by the fact that shocks underlying fiscal policy are likely to be autocorrelated, consistent with the observation that fiscal policy variables are persistent in the data.⁴ As is well known, persistence introduces new difficulties into the mechanism design problem; we consider a simple framework that allows for a full characterization of the optimal mechanism.⁵

Our environment is a small open economy in which the government makes repeated spending and borrowing decisions. In each period, a shock to the social value of deficit-financed government spending—either low or high—is realized, where this shock follows a first-order Markov process.⁶ The government in each period is benevolent *ex ante*, prior to the realization of the shock, but is present-biased *ex post*, when it is time to choose fiscal policy. This preference structure results naturally from the aggregation of heterogeneous, time-consistent citizens' preferences; see [Jackson and Yariv \(2012a, 2012b\)](#).⁷ We assume that the shocks to the value of spending are privately observed by the government, capturing the fact that not all contingencies in fiscal policy are contractible or observable. A fiscal rule is defined as a mechanism in which the government reports the shock in each period and is assigned a policy for

³See also [Amador and Bagwell \(2013\)](#), [Ambrus and Egorov \(2013b\)](#), and [Athey, Atkeson, and Kehoe \(2005\)](#). All these papers consider settings with independent and identically distributed (i.i.d.) shocks.

⁴For example, see [Barro \(1990\)](#) for evidence.

⁵Persistence complicates the mechanism design problem because single-crossing conditions generally used in the analysis may fail and a recursive representation is more difficult. Recent work addresses this issue in principal-agent and optimal taxation settings (see, e.g., [Battaglini and Lamba \(2012\)](#), [Farhi and Werning \(2013\)](#), [Golosov, Troshkin, and Tsyvinski \(2013\)](#), and [Pavan, Segal, and Toikka \(2014\)](#)), but this analysis does not apply to our self-control setting where cross-subsidization across types is not possible.

⁶We extend our analysis to a continuum of shocks in [Appendix B](#).

⁷In addition, in a fiscal policy framework, there are various microfoundations for how a deficit bias may emerge, including those in [Aguiar and Amador \(2011\)](#), [Alesina and Perotti \(1994\)](#), [Alesina and Tabellini \(1990\)](#), [Battaglini and Coate \(2008\)](#), [Caballero and Yared \(2010\)](#), [Lizzeri \(1999\)](#), [Persson and Svensson \(1989\)](#), and [Tornell and Lane \(1999\)](#). Our formulation of the government's preferences corresponds to the quasi-hyperbolic consumption model; see [Laibson \(1997\)](#).

every reported shock. Note that in the absence of private information, the first-best policy could be implemented with full commitment, that is, by giving the government no discretion and committing it to the efficient path of spending. Similarly, absent a present bias, the first best could be implemented with full flexibility, that is, by giving the government full discretion to choose spending in each period. In the presence of both private information and a present bias, however, a trade-off between commitment and flexibility arises, and the optimal rule is then not trivial.

We study the *ex ante* optimal fiscal rule and the sequentially optimal fiscal rule. The *ex ante* optimal rule is the optimal dynamic mechanism that the government chooses at the beginning of time. This is a sequence of spending and borrowing levels as a function of the history of the government's reports that maximizes *ex ante* social welfare subject to a sequence of dynamic incentive compatibility constraints. The sequentially optimal rule, on the other hand, corresponds to a static mechanism that is chosen by the government in every period t (prior to the realization of the shock) and that maximizes social welfare from t onward taking into account that future governments do the same.

Our motivation for studying sequentially optimal rules is twofold. First, in practice, fiscal rules often have a bite in the short term for the current fiscal year, but can be renegotiated and changed by the government in advance for the following year. Second, a central result from the previous literature is that the *ex ante* optimal and sequentially optimal rules coincide when the shocks are i.i.d.⁸ That is, under the *ex ante* optimal mechanism, at any given date (prior to the realization of the shock), the government would not want to change the continuation mechanism. The reason is that no dynamic incentives are provided, and the mechanism at any date depends only on the payoff-relevant states. Notably, this contrasts with the results of the principal-agent literature, where, even under i.i.d. shocks, dynamic incentives are provided to an agent with private information along the equilibrium path. The difference is due to the fact that there is a single player in a self-control setting, and dynamic incentives cannot be provided by increasing one player's welfare at the expense of another. Thus, under i.i.d. shocks, dynamic incentives would affect the government's welfare on the equilibrium path (when reporting the shock truthfully) and off the equilibrium path (when misreporting the shock) equally, and, therefore, result in no welfare gains.

We first show that, as in the case of i.i.d. shocks, the sequentially optimal fiscal rule under persistent shocks is history independent and simple, as, by definition, dynamic incentives are not provided to the government. This rule can be implemented with a debt limit that is a function of only the payoff-relevant states, namely the current level of debt and the previous period's shock, which forecasts the current shock.

⁸See Amador, Werning, and Angeletos (2006) and Athey, Atkeson, and Kehoe (2005).

Our main result, however, shows that unlike under i.i.d. shocks, the ex ante optimal rule does not coincide with the sequentially optimal rule when shocks are persistent. The ex ante optimal rule takes into account that the government in every period learns about its current and future spending needs, and now provides dynamic incentives for the government not to overspend and overborrow. Specifically, consider a government that is tempted to overspend today when its needs are low. Dynamic incentives can be provided by introducing excessively lax and ex post suboptimal rules tomorrow if the government chooses high levels of spending today. The expected cost to the government of such lax rules tomorrow is greater if spending needs are actually low today, as spending needs are then more likely to be low tomorrow. Thus, lax rules tomorrow affect welfare on and off the equilibrium path differently, and the threat of no discipline in the future ironically imposes discipline today.

We characterize the ex ante optimal rule and the dynamics that it induces in fiscal policy. We show that this rule is history dependent: the mechanism at a given date is a function of not only the payoff-relevant states, but also the entire history of shocks. History dependence follows from the fact that the mechanism provides dynamic incentives. Intuitively, because the shock at date $t - 2$ predicts the realization of the shock at $t - 1$ (since shocks follow a first-order Markov process), the shock at $t - 2$ affects the relative tightness of incentive compatibility constraints at $t - 1$, which in turn affect the policies that are chosen at t in providing dynamic incentives at $t - 1$. In particular, the shock at date $t - 2$ tells us how likely it is that the shock to the value of spending will be low at $t - 1$, and thus how beneficial it is to make rules excessively lax at t following high spending by the government at $t - 1$.

We explicitly characterize the dynamics implied by the ex ante optimal fiscal rule in an infinite horizon economy using a recursive technique similar to that developed by [Fernandes and Phelan \(2000\)](#) for a principal-agent setting. We show that high shocks to the value of spending lead to an erosion of future fiscal discipline compared to low shocks, which lead to the reinstatement of fiscal discipline. This is related to “no distortion at the top” in standard adverse selection problems, which in our setting results in a resetting of the mechanism whenever a low shock is realized. For a sequence of consecutive high shocks, we further show that fiscal policy exhibits oscillatory dynamics, with large (small) distortions being followed by relatively smaller (larger) distortions. The logic stems from the self-control nature of our problem: a reduction in fiscal discipline at date t to provide dynamic incentives at $t - 1$ relaxes incentive constraints at t and, hence, implies that a smaller reduction in fiscal discipline at date $t + 1$ is sufficient to provide dynamic incentives at t . Finally, we study the implied long-run debt dynamics when the market and social discount rates coincide. We show that while the self-insurance motive leads to the infinite accumulation of assets in the first-best and the sequentially optimal rule, periods of nondiscipline used to provide dynamic incentives in the ex ante optimal rule can force the government to accumulate maximal debt and become immiserated in the long run.

The paper is related to several literatures. First, as mentioned, the paper fits into the mechanism design literature on the trade-off between commitment and flexibility.⁹ Most papers here consider i.i.d. environments, whereas we analyze the optimal dynamic mechanism when shocks are persistent. One exception is [Bond and Sigurdsson \(2011\)](#), who examine when contracts can overcome commitment problems in a setting with negatively correlated shocks. They do not study infinite horizon dynamics, which is our focus. Second, our paper relates to the literature on the political economy of fiscal policy.¹⁰ Most closely related is [Azzimonti, Battaglini, and Coate \(2010\)](#), who consider the quantitative welfare implications of a balanced budget rule in an i.i.d. setting where the government is present-biased toward pork-barrel spending. Our main departure is that we study optimal fiscal rules in a private information economy and we use mechanism design tools to derive the optimal rule without restricting its structure. Third, our work is related to various papers studying principal–agent contracts under persistent private information, although, because our application is a self-control environment, their methods do not directly apply here.¹¹ Finally, more broadly, our paper contributes to the literature on hyperbolic discounting and the benefits of commitment devices.¹²

Section 2 describes the model. Section 3 defines the ex ante optimal and sequentially optimal fiscal rules. Section 4 illustrates the main insights from our model using a simple three-period example. Section 5 characterizes optimal fiscal rules in the infinite horizon economy and their implications for debt in the long run. Section 6 concludes and describes some additional applications of our model. Formal proofs and extensions can be found in Appendices A–E. The Supplemental Material ([Halac and Yared \(2014\)](#)) provides proofs for Appendix B and additional analysis.

2. THE MODEL

We study a simple model of fiscal policy in which a government makes repeated spending and borrowing decisions. Our environment is the same as that

⁹In addition to the work previously cited, see [Sleet \(2004\)](#) as well as [Bernheim, Ray, and Yeltekin \(2013\)](#) which considers the self-enforcement of commitment contracts. More generally, the paper relates to the literature on delegation in principal–agent settings, including [Alonso and Matouschek \(2008\)](#), [Ambrus and Egorov \(2013a\)](#), and [Holmström \(1977, 1984\)](#).

¹⁰In addition to the work cited in footnote 7, see [Acemoglu, Golosov, and Tsyvinski \(2008\)](#), [Azzimonti \(2011\)](#), [Krusell and Rios-Rull \(1999\)](#), [Song, Storesletten, and Zilibotti \(2012\)](#), and [Yared \(2010\)](#).

¹¹In addition to the work cited in footnote 5, see [Golosov and Tsyvinski \(2006\)](#), [Halac \(2012\)](#), [Kapicka \(2013\)](#), [Strulovici \(2011\)](#), and [Williams \(2011\)](#).

¹²See, for example, [Barro \(1999\)](#), [Bisin, Lizzeri, and Yariv \(2013\)](#), [Krusell, Kruscu, and Smith \(2010\)](#), [Krusell and Smith \(2003\)](#), [Laibson \(1997\)](#), [Lizzeri and Yariv \(2013\)](#), and [Phelps and Pollak \(1968\)](#).

analyzed in Amador, Werning, and Angeletos (2006), with the exception that we allow for persistent private information.¹³

Consider a small open economy. At the beginning of each period, $t \in \{0, 1, \dots\}$, the government observes a shock to the economy, which is the government's private information or *type*. The government's type can be low or high, $\theta_t \in \{\theta^L, \theta^H\} \equiv \Theta$, where $\theta^H > \theta^L > 0$.¹⁴ This type follows a first-order Markov process, with $p(\theta_{t+1}|\theta_t)$ corresponding to the probability of type θ_{t+1} at date $t+1$ conditional on type θ_t at date t . We consider $p(\theta^L|\theta^L) = p(\theta^H|\theta^H) \in [0.5, 1)$, and we compare the case where types are i.i.d., that is, $p(\theta^i|\theta^i) = 0.5$ for $\theta^i \in \{\theta^L, \theta^H\}$, to the case where types are persistent over time, that is, $p(\theta^i|\theta^i) > 0.5$ for $\theta^i \in \{\theta^L, \theta^H\}$.¹⁵

In each period t , following the realization of θ_t , the government chooses public spending $g_t \geq 0$ and debt b_{t+1} subject to a budget constraint

$$(1) \quad g_t = \tau + b_{t+1}/(1+r) - b_t,$$

where $\tau > 0$ is the exogenous fixed tax revenue collected by the government in each period, b_t is the level of debt with which the government enters the period, and r is the exogenous interest rate.¹⁶ b_0 is exogenous and $\lim_{t \rightarrow \infty} b_{t+1}/(1+r)^t = 0$, so that all debts must be repaid and all assets must be consumed. Note that this setting does not allow for cross-subsidization across types. Specifically, in contrast to other mechanism design models such as Atkeson and Lucas (1992) and Thomas and Worrall (1990), the net present value of public spending at any point cannot be different for the low type and the high type.¹⁷ We also note that constraint (1) can be rewritten as a weak inequality constraint to allow for money burning without affecting any of our results. Such a weak inequality constraint would take into account the possibility of introducing fines; we ignore this possibility here to simplify the exposition.¹⁸

The government's welfare at date t , *prior* to the realization of its type θ_t , is

$$(2) \quad \sum_{k=0}^{\infty} \delta^k \mathbb{E}[\theta_{t+k} U(g_{t+k}) | \theta_{t-1}],$$

¹³Amador, Werning, and Angeletos (2006) consider a two-period setting, but as shown in Amador, Werning, and Angeletos (2003), under i.i.d. shocks, the results apply directly to a multiple-period environment.

¹⁴Appendix B extends our analysis to an economy with a continuum of types.

¹⁵Our main results are robust to considering $p(\theta^i|\theta^i) < 0.5$, although the intuition is different. Given that fiscal policy variables are positively autocorrelated in the data, we focus our attention on $p(\theta^i|\theta^i) \geq 0.5$. Our results are also robust to considering $p(\theta^L|\theta^L) \neq p(\theta^H|\theta^H)$; we focus on the symmetric case to simplify the exposition.

¹⁶Our analysis also applies if instead of having an exogenous tax revenue, social welfare is an increasing function of the budget deficit.

¹⁷Other models that build on these papers and also feature cross-subsidization are Farhi and Werning (2007) and Sleet and Yeltekin (2006, 2008).

¹⁸In the proofs of our results in the Appendix, we allow (1) to be a weak inequality and show that this constraint must bind in equilibrium.

where $\theta_t U(g_t)$ is the social utility from public spending at date t and δ is the discount factor. The government's welfare *after* the realization of its type θ_t at date t , when choosing spending g_t , is

$$(3) \quad \theta_t U(g_t) + \beta \sum_{k=1}^{\infty} \delta^k \mathbb{E}[\theta_{t+k} U(g_{t+k}) | \theta_t],$$

where $\beta \in (0, 1)$.

There are two important features of this environment. First, the government's objective (3) following the realization of its type does not coincide with its objective (2) prior to this realization. In particular, the government's welfare after θ_t is realized overweighs the importance of current public spending compared to its welfare before θ_t is realized. As mentioned in the [Introduction](#), this structure arises naturally when the government's preferences aggregate citizens' preferences, even if the latter are time consistent. [Jackson and Yariv \(2012a\)](#) show that with any heterogeneity in preferences, every nondictatorial aggregation method that respects unanimity must be time inconsistent; moreover, any such method that is time separable must lead to a present bias.¹⁹ Our formulation can also be motivated by political turnover. For instance, preferences such as these may emerge in settings with political uncertainty where policymakers place a higher value on public spending when they hold power and can make spending decisions. A present bias given by $\beta \in (0, 1)$ for a current policymaker can be interpreted in this sense as the probability that the policymaker will be part of the government in the next and all future periods. In these settings with turnover, policymakers are then biased toward present public spending relative to future public spending and as a result incur excessively high debts; see, for example, [Aguilar and Amador \(2011\)](#).

The second feature of our environment is that the realization of θ_t —which affects the marginal social utility of public spending—is privately observed by the government. One possible interpretation is that θ_t is not verifiable ex post by a rule-making body; therefore, even if θ_t is observable, fiscal rules cannot explicitly depend on the value of θ_t .²⁰ An alternative interpretation is that the exact cost of public goods is only observable to the policymaker, who may be inclined to overspend on these goods. A third possibility is that citizens have heterogeneous preferences or heterogeneous information regarding the optimal level of public spending, and the government sees an aggregate that the citizens do not see (see [Sleet \(2004\)](#)).

¹⁹Specifically, any such time-separable aggregation method will be utilitarian and, by the results in [Jackson and Yariv \(2012b\)](#), it will thus be present-biased.

²⁰Because our focus is a fiscal constitution, we implicitly rule out other punishments that citizens can inflict on policymakers such as replacement. See [Ales, Maziero, and Yared \(2014\)](#) for a related model with private government information that allows for this possibility.

To facilitate an explicit characterization of optimal fiscal rules, we make the following assumption.²¹

ASSUMPTION 1: $U(g_t) = \log(g_t)$.

Assumption 1 implies that welfare is separable with respect to the level of debt. To see this, define

$$(4) \quad \tilde{\theta}^i = \sum_{k=1}^{\infty} \delta^k \mathbb{E}[\theta_{t+k} | \theta_t = \theta^i]$$

for $i = \{L, H\}$, so that at any date t , $\tilde{\theta}_t = \tilde{\theta}^i$ if $\theta_t = \theta^i$. Let the savings rate at t be $s_t \in [0, 1]$, corresponding to the fraction of lifetime resources that are not spent at t :

$$(5) \quad g_t = (1 - s_t)[(1 + r)\tau/r - b_t].$$

Using this notation and Assumption 1, welfare in (2), at date t prior to the realization of the type θ_t , can be rewritten as

$$(6) \quad \sum_{k=0}^{\infty} \delta^k \mathbb{E}[\theta_{t+k} U(1 - s_{t+k}) + \tilde{\theta}_{t+k} U(s_{t+k}) | \theta_{t-1}] + \chi(b_t)$$

for a constant $\chi(b_t)$ that depends on b_t .²² Analogously, welfare in (3), at date t following the realization of θ_t , can be rewritten as

$$(7) \quad \theta_t U(1 - s_t) + \beta \tilde{\theta}_t U(s_t) + \beta \sum_{k=1}^{\infty} \delta^k \mathbb{E}[\theta_{t+k} U(1 - s_{t+k}) + \tilde{\theta}_{t+k} U(s_{t+k}) | \theta_t] + \varphi(b_t)$$

for a constant $\varphi(b_t)$ that depends on b_t .²³ Given the representation in (6) and (7), hereafter we consider the problem of a government that chooses a savings rate s_t in every period t . In this environment, the first-best policy is defined by a stochastic sequence of savings rates that satisfy $s_t^{\text{fb}} = s^{\text{fb}}(\theta^i)$ if $\theta_t = \theta^i$ for $\theta^i \in \{\theta^L, \theta^H\}$, where

$$(8) \quad \theta^i U'(1 - s^{\text{fb}}(\theta^i)) = \tilde{\theta}^i U'(s^{\text{fb}}(\theta^i)).$$

²¹This assumption is made in previous work studying economies with hyperbolic discounting, such as Barro (1999). Our main result that dynamic incentives are provided under persistent shocks while not under i.i.d. shocks does not depend on this assumption.

²²This constant is equal to $\sum_{k=0}^{\infty} \delta^k \mathbb{E}[\theta_{t+k} U((1 + r)^k [\tau(1 + r)/r - b_t]) | \theta_{t-1}]$.

²³This constant is equal to $\theta_t U(\tau(1 + r)/r - b_t) + \beta \sum_{k=1}^{\infty} \delta^k \mathbb{E}[\theta_{t+k} U((1 + r)^k [\tau(1 + r)/r - b_t]) | \theta_t]$.

3. EQUILIBRIUM DEFINITION

We define a fiscal rule as a mechanism where the government reports the shock in every period and is assigned a policy as a function of the reports. We distinguish between the ex ante optimal rule and the sequentially optimal rule. The ex ante optimal rule is a dynamic mechanism chosen by the government at the beginning of time. In contrast, the sequentially optimal rule is a static mechanism chosen in every period by the current government, taking into account the future static mechanisms chosen by future governments.

We let $\theta^t = (\theta_0, \theta_1, \dots, \theta_t) \in \Theta^t$ denote the history of shocks through time t .

3.1. Ex ante Optimal Rule

Let $h_{t-1} = (\widehat{\theta}_0, \widehat{\theta}_1, \dots, \widehat{\theta}_{t-1}) \in \Theta^{t-1}$ be the history of reported types through time $t - 1$. A mechanism is a sequence of savings rates $s_t(h_{t-1}, \widehat{\theta}_t)$ for all $\{(h_{t-1}, \widehat{\theta}_t)\}_{t=0}^\infty$ that effectively specify levels of public spending $g_t(h_{t-1}, \widehat{\theta}_t)$ and debt $b_{t+1}(h_{t-1}, \widehat{\theta}_t)$ as a function of the history of past reports and the current report.

Given the mechanism, the government chooses a reporting strategy $m_t(h_{t-1}, \theta_t)$ for all $\{(h_{t-1}, \theta_t)\}_{t=0}^\infty$, where θ_t is the government’s type at date t and $m_t(h_{t-1}, \theta_t) \in \{\theta^L, \theta^H\}$ is the government’s report of its type at t . We restrict attention to public strategies, that is, strategies that depend only on the public history—reports and policies—and on the government’s current private information, but not on privately observed history. While common in the literature, this restriction is not without loss; see Section 4 for a discussion. It follows by standard arguments that if all future governments choose public strategies and if the mechanism is a function of the public history, then the current government’s best response is also a public strategy. From the revelation principle, we can restrict attention to truth-telling equilibria in which $m_t(h_{t-1}, \theta_t) = \theta_t$ for all h_{t-1} and θ_t .

A perfect Bayesian equilibrium of this revelation game is a mechanism and a reporting strategy such that the budget constraint (1) is satisfied in every period following every history, and the policy under the mechanism is incentive compatible, meaning that following every history and type realization, the government prefers to report $m_t(h_{t-1}, \theta_t) = \theta_t$ rather than $m_t(h_{t-1}, \theta_t) = \widehat{\theta}_t \neq \theta_t$. An ex ante optimal rule in this framework is one that selects a mechanism and a reporting strategy that maximize the ex ante welfare (6) in period 0.

We formulate the ex ante optimal rule as a solution to a sequence program. Given history θ^{t-1} , let $W_{t+1}(\theta^{t-1}, \theta_t)$ be the expected continuation value from $t + 1$ on, normalized by $b_t(\theta^{t-1})$, for a type θ_t who truthfully reports $\widehat{\theta}_t = \theta_t$:

$$(9) \quad W_{t+1}(\theta^{t-1}, \theta_t) = \sum_{\theta_{t+1} \in \{\theta^L, \theta^H\}} p(\theta_{t+1} | \theta_t) [\theta_{t+1} U(1 - s_{t+1}(\theta^{t-1}, \theta_t, \theta_{t+1})) + \widetilde{\theta}_{t+1} U(s_{t+1}(\theta^{t-1}, \theta_t, \theta_{t+1})) + \delta W_{t+2}(\theta^{t-1}, \theta_t, \theta_{t+1})].$$

In contrast, given history θ^{t-1} , let $V_{t+1}(\theta^{t-1}, \widehat{\theta}_t)$ be the expected continuation value from $t + 1$ on, normalized by $b_t(\theta^{t-1})$, for a type θ_t who lies and reports $\widehat{\theta}_t \neq \theta_t$.²⁴

$$(10) \quad V_{t+1}(\theta^{t-1}, \widehat{\theta}_t) = \sum_{\theta_{t+1} \in \{\theta^L, \theta^H\}} p(\theta_{t+1} | \theta_t) [\theta_{t+1} U(1 - s_{t+1}(\theta^{t-1}, \widehat{\theta}_t, \theta_{t+1})) + \widetilde{\theta}_{t+1} U(s_{t+1}(\theta^{t-1}, \widehat{\theta}_t, \theta_{t+1})) + \delta W_{t+2}(\theta^{t-1}, \widehat{\theta}_t, \theta_{t+1})].$$

Note that in the special case of i.i.d. shocks, expectations over future shocks do not depend on the current shock θ_t , implying that $W_{t+1}(\theta^{t-1}, \widehat{\theta}_t) = V_{t+1}(\theta^{t-1}, \widehat{\theta}_t)$.

Using (7), the incentive compatibility constraint for a government of type θ_t is

$$(11) \quad \begin{aligned} &\theta_t U(1 - s_t(\theta^{t-1}, \theta_t)) + \beta \widetilde{\theta}_t U(s_t(\theta^{t-1}, \theta_t)) + \beta \delta W_{t+1}(\theta^{t-1}, \theta_t) \\ &\geq \theta_t U(1 - s_t(\theta^{t-1}, \widehat{\theta}_t)) + \beta \widetilde{\theta}_t U(s_t(\theta^{t-1}, \widehat{\theta}_t)) \\ &\quad + \beta \delta V_{t+1}(\theta^{t-1}, \widehat{\theta}_t) \quad \text{for } \widehat{\theta}_t \neq \theta_t \text{ and all } \theta^{t-1}. \end{aligned}$$

Condition (11) says that the government prefers to report its true type θ_t , rather than to lie and report $\widehat{\theta}_t \neq \theta_t$. To understand this constraint, note that the government’s true type θ_t not only directly affects the government’s immediate payoff by determining the marginal cost and benefit of current savings, but if shocks are persistent, it can also affect the government’s continuation payoff by changing the expectations over the realizations of future types.²⁵

We emphasize again that the welfare functions above, W and V , are normalized by the level of debt, that is, they include constants that depend on the current debt level, which we are omitting as explained in our discussion of equations (6) and (7). These constants are implicitly included on each side of the incentive compatibility constraint (11), but they cancel each other out.

Let $\rho = \{\{s_t(\theta^t), W_{t+1}(\theta^t), V_{t+1}(\theta^t)\}_{\theta^t \in \Theta^t}\}_{t=0}^\infty$ be a stochastic sequence of savings rates and continuation values. The ex ante optimal rule solves the sequence problem

$$(12) \quad \begin{aligned} &\max_{\rho} \sum_{\theta_0 \in \{\theta^L, \theta^H\}} p(\theta_0 | \theta_{-1}) [\theta_0 U(1 - s_0(\theta^0)) + \widetilde{\theta}_0 U(s_0(\theta^0)) + \delta W_1(\theta^0)] \\ &\text{subject to } (9), (10), \text{ and } (11). \end{aligned}$$

It is clear that the solution to this program is invariant to the initial level of debt b_0 .

²⁴Because there are only two types, we only need to define one such expected continuation value.

²⁵Because θ_t follows a first-order Markov process, the single period deviation principle holds.

3.2. *Sequentially Optimal Rule*

The sequentially optimal fiscal rule is the one that results if, at every history, the government chooses a static mechanism that maximizes social welfare given that future governments will do the same. That is, given θ_{t-1} and b_t , the government chooses a mechanism $\{g_t(\widehat{\theta}_t), b_{t+1}(\widehat{\theta}_t)\}$, assigning a level of spending and debt conditional on the report $\widehat{\theta}_t$, which maximizes social welfare, taking the actions of future governments as given. The future government knows the true value of θ_t and, given θ_t and $b_{t+1}(\widehat{\theta}_t)$, it analogously chooses an optimal static mechanism, taking the actions of future governments as given.

The sequentially optimal rule thus solves the problem

$$(13) \quad J(\theta_{t-1}, b_t) = \max_{\{g_t(\theta_t), b_{t+1}(\theta_t)\}_{\theta_t \in \{\theta^L, \theta^H\}}} \sum_{\theta_t \in \{\theta^L, \theta^H\}} p(\theta_t | \theta_{t-1}) \times (\theta_t U(g_t(\theta_t)) + \delta J(\theta_t, b_{t+1}(\theta_t)))$$

subject to

$$(14) \quad g_t(\theta_t) = \tau + b_{t+1}(\theta_t)/(1+r) - b_t \quad \text{and}$$

$$(15) \quad \theta_t U(g_t(\theta_t)) + \beta \delta J(\theta_t, b_{t+1}(\theta_t)) \geq \theta_t U(g_t(\widehat{\theta}_t)) + \beta \delta J(\theta_t, b_{t+1}(\widehat{\theta}_t)) \quad \text{for } \widehat{\theta}_t \neq \theta_t.$$

$J(\theta_{t-1}, b_t)$ is the value at t under the payoff-relevant states θ_{t-1} and b_t if the current government chooses an optimal static mechanism given that future governments do the same. In a sense, $J(\theta_{t-1}, b_t)$ thus corresponds to the solution of a two-period mechanism design problem. In choosing its report $\widehat{\theta}_t$, the government's flow welfare is $\theta_t U(g_t(\widehat{\theta}_t))$ and its continuation welfare is $J(\theta_t, b_{t+1}(\widehat{\theta}_t))$. Condition (15) is the incentive compatibility constraint, where the government knows that if it lies and reports $\widehat{\theta}_t \neq \theta_t$, this affects its payoff from tomorrow onward only through the implied level of debt $b_{t+1}(\widehat{\theta}_t)$, since, given $b_{t+1}(\widehat{\theta}_t)$, the government, which knows the true value of θ_t , will choose an optimal static mechanism going forward. We interpret the sequentially optimal fiscal rule that emerges from this recursion as a rule that has a bite in the short term, but can be renegotiated for the future.²⁶

In the recursive program defined above, $J(\cdot)$ may have multiple solutions. We select a unique solution by considering the limit of a finite horizon economy with end date T as T approaches infinity. $J(\theta_{t-1}, b_t)$ is thus characterized recursively via backward induction. In Appendix A, we show that under log utility, the solution to (13)–(15) admits a savings rate $s_t(\theta_t)$ for each θ_t that

²⁶In our context in which we assume commitment to a rule within the period, this notion of sequential optimality is related to the notion of reconsideration-proofness in Kocherlakota (1996) and renegotiation-proofness in Farrell and Maskin (1989) and Strulovici (2011).

is invariant to the level of debt b_t and only depends on the previous shock θ_{t-1} , and, hence, using the welfare representation in (6) and (7) and given θ_{t-1} , (13)–(15) can be rewritten as

$$(16) \quad \max_{\{s_t(\theta_t)\}_{\theta_t \in \{\theta^L, \theta^H\}}} \sum_{\theta_t \in \{\theta^L, \theta^H\}} p(\theta_t | \theta_{t-1}) (\theta_t U(1 - s_t(\theta_t)) + \tilde{\theta}_t U(s_t(\theta_t)))$$

subject to

$$(17) \quad \theta_t U(1 - s_t(\theta_t)) + \beta \tilde{\theta}_t U(s_t(\theta_t)) \geq \theta_t U(1 - s_t(\hat{\theta}_t)) + \beta \tilde{\theta}_t U(s_t(\hat{\theta}_t)) \quad \text{for } \hat{\theta}_t \neq \theta_t.$$

Clearly, the sequentially optimal rule must satisfy the constraints of the problem defined by the ex ante optimal rule in (12). To see this, note that the incentive compatibility constraint (17) is more strict than the constraint (11), as, by definition, the solution to (16)–(17) must admit a sequence of savings rates satisfying $W_{t+1}(\theta^{t-1}, \theta_t) \geq V_{t+1}(\theta^{t-1}, \theta_t)$. Thus, naturally, the sequentially optimal rule provides weakly lower welfare than the ex ante optimal rule.

4. THREE-PERIOD EXAMPLE

To provide intuition for our main results, we start by considering a three-period economy with $t \in \{0, 1, 2\}$. The purpose is threefold. First, we show that in contrast to the case of i.i.d. shocks, when shocks are persistent, the ex ante optimal rule does not coincide with the sequentially optimal rule. Second, we show that, also unlike under i.i.d. shocks, the ex ante optimal rule under persistent shocks exhibits history dependence, that is, the mechanism at $t = 1$ depends on more than the payoff-relevant variables at $t = 1$. Finally, we describe properties of the solution that provide some insight into the dynamics of the infinite horizon economy discussed in Section 5.

Given log utility and the arguments of Section 3, one can think of the government as choosing a savings rate $s_0(\theta_{-1}, \theta_0)$ at date 0 and a savings rate $s_1(\theta_{-1}, \theta_0, \theta_1)$ at date 1.²⁷ It is immediate to show that these savings rates under the first-best policy are higher than the savings rates the government would choose were it allowed to flexibly choose borrowing and spending in each period.²⁸ Under full flexibility, the government would be left with excessively high debt in period 2: it is in this sense that fiscal rules can be beneficial by inducing more government saving.

²⁷Note that savings at date 2 are always equal to zero, since this is the final period.

²⁸In the first-best solution, the savings rates in periods 0 and 1 are $\delta \mathbb{E}[\theta_1 + \delta \theta_2 | \theta_0] / (\theta_0 + \delta \mathbb{E}[\theta_1 + \delta \theta_2 | \theta_0])$ and $\mathbb{E}[\theta_2 | \theta_1] / (\theta_1 + \delta \mathbb{E}[\theta_2 | \theta_1])$, respectively. In contrast, under full flexibility, these rates are $\beta \delta \mathbb{E}[\theta_1 + \delta \theta_2 | \theta_0] / (\theta_0 + \beta \delta \mathbb{E}[\theta_1 + \delta \theta_2 | \theta_0])$ and $\beta \mathbb{E}[\theta_2 | \theta_1] / (\theta_1 + \beta \delta \mathbb{E}[\theta_2 | \theta_1])$, respectively.

We begin by studying the sequentially optimal rule, which can be solved for by backward induction. At date 1, the government solves program (16)–(17), where $\tilde{\theta}_1$ is given by $\delta\mathbb{E}[\theta_2|\theta_1]$. To simplify the analysis, we assume that θ^L and θ^H are relatively close to each other:

$$(18) \quad \frac{\theta^H}{\theta^L} < \frac{1}{\beta}.$$

Condition (18) implies that the first-best savings rate at date 1, defined by (8), is not incentive compatible for the low type, who would want to pretend to be a high type so as to spend and borrow more. Moreover, this condition implies that the optimal static mechanism at date 1 features pooling: following the realization of θ_0 , the sequentially optimal rule assigns a fixed savings rate, independent of the type at date 1.²⁹ This pooled savings rate is chosen optimally given the probabilities of a high and low type at date 1, so it is given by

$$(19) \quad \mathbb{E}[\theta_1|\theta_0 = \theta^i]U'(1 - s_1(\theta_{-1}, \theta^i)) - \delta\mathbb{E}[\theta_2|\theta_0 = \theta^i]U'(s_1(\theta_{-1}, \theta^i)) = 0$$

for $\theta^i \in \{\theta^L, \theta^H\}$, where, with some abuse of notation, we have written $s_1(\cdot)$ as a function of θ_{-1} and θ_0 only given that savings at date 1 do not depend on θ_1 under pooling.

Consider next the ex ante optimal rule. Suppose by contradiction that the ex ante optimum and the sequential optimum coincide at date 1. We show that it is then possible to perturb the ex ante optimal mechanism in a way that reduces welfare from the perspective of date 1 but increases welfare from the perspective of date 0, thus increasing ex ante welfare and contradicting the assumption that the two rules coincide at date 1.

Assume that at date 0 the incentive compatibility constraint binds for the low type and is slack for the high type. This means that the low type is under-saving relative to first best, and he cannot be induced to save more as he would then want to pretend to be a high type to save less. Formally, the low type is indifferent between reporting the truth and receiving a payoff equal to

$$\begin{aligned} &\theta^L U(1 - s_0(\theta_{-1}, \theta^L)) + \beta\delta\mathbb{E}[\theta_1 + \delta\theta_2|\theta_0 = \theta^L]U(s_0(\theta_{-1}, \theta^L)) \\ &+ \beta\delta(\mathbb{E}[\theta_1|\theta_0 = \theta^L]U(1 - s_1(\theta_{-1}, \theta^L)) \\ &+ \delta\mathbb{E}[\theta_2|\theta_0 = \theta^L]U(s_1(\theta_{-1}, \theta^L))), \end{aligned}$$

²⁹Intuitively, separation when types are sufficiently close is suboptimal because it would require excessively low savings by the high type so as to satisfy the incentive compatibility constraint of the low type. A formal proof that pooling is optimal in this case follows closely the steps of the proof of Proposition 1 in Appendix C.

and lying and receiving a payoff equal to

$$\begin{aligned} & \theta^L U(1 - s_0(\theta_{-1}, \theta^H)) + \beta \delta \mathbb{E}[\theta_1 + \delta \theta_2 | \theta_0 = \theta^L] U(s_0(\theta_{-1}, \theta^H)) \\ & + \beta \delta (\mathbb{E}[\theta_1 | \theta_0 = \theta^L] U(1 - s_1(\theta_{-1}, \theta^H)) \\ & + \delta \mathbb{E}[\theta_2 | \theta_0 = \theta^L] U(s_1(\theta_{-1}, \theta^H))), \end{aligned}$$

where, by the contradiction assumption that the mechanism is sequentially optimal at date 1, $s_1(\theta_{-1}, \theta^L)$ is the pooled savings rate given by (19). Clearly, the low type could be induced to save more at date 0 if his payoff from pretending to be a high type could be reduced. Note that this payoff can be reduced by changing either the savings rate that is assigned at date 0 given a high reported type at date 0 or the savings rate that is assigned at date 1 given a high reported type at date 0. Moreover, the effect on welfare of changing the savings rate at date 1 depends on the probabilities of a low type and a high type at date 1, which under persistent shocks depend on the realized type at date 0.

With this observation in mind, consider a perturbation that reduces the savings rate at date 1 given a high type at date 0, $s_1(\theta_{-1}, \theta^H)$, by $\varepsilon > 0$ arbitrarily small. This perturbation clearly reduces *equilibrium* welfare from date 1 onward, that is, welfare at date 1 given a truthful report of $\hat{\theta}_0 = \theta^H$ at date 0, as it induces overspending at date 1. However, from the envelope condition in (19), this is a second-order loss, so this effect approaches 0 as ε approaches 0. In contrast, consider the effect of the perturbation on *off-equilibrium* welfare from date 1 onward, that is, welfare at date 1 given a nontruthful report of $\hat{\theta}_0 = \theta^H$ at date 0. As ε approaches 0, this effect takes the same sign as

$$(20) \quad \mathbb{E}[\theta_1 | \theta_0 = \theta^L] U'(1 - s_1(\theta_{-1}, \theta^H)) - \delta \mathbb{E}[\theta_2 | \theta_0 = \theta^L] U'(s_1(\theta_{-1}, \theta^H)).$$

If shocks are i.i.d., $\mathbb{E}[\theta_1 | \theta_0 = \theta^L] / \mathbb{E}[\theta_2 | \theta_0 = \theta^L] = \mathbb{E}[\theta_1 | \theta_0 = \theta^H] / \mathbb{E}[\theta_2 | \theta_0 = \theta^H]$, so given (19), (20) must equal zero. This means that the perturbation affects continuation welfare on and off the equilibrium path equally and, hence, cannot improve ex ante welfare. In contrast, when shocks are persistent, $\mathbb{E}[\theta_1 | \theta_0 = \theta^L] / \mathbb{E}[\theta_2 | \theta_0 = \theta^L] < \mathbb{E}[\theta_1 | \theta_0 = \theta^H] / \mathbb{E}[\theta_2 | \theta_0 = \theta^H]$, so (20) must be negative. Given (19), this means that the perturbation reduces continuation welfare off the equilibrium path without affecting continuation welfare on the path. Therefore, the low type's incentive compatibility constraint at date 0 can be relaxed at no social cost, and ex ante welfare can be increased.

The intuition behind the perturbation is simple. In every period, the government learns about its current spending needs. If shocks are persistent, the current shock also informs the government about its future spending needs. The government has a temptation to overspend today even if its needs are low, but it maximizes social welfare from tomorrow on. The ex ante optimal rule then has the feature that it becomes excessively lax in the future if spending is high today. The reason is that the expected cost of lax rules tomorrow is

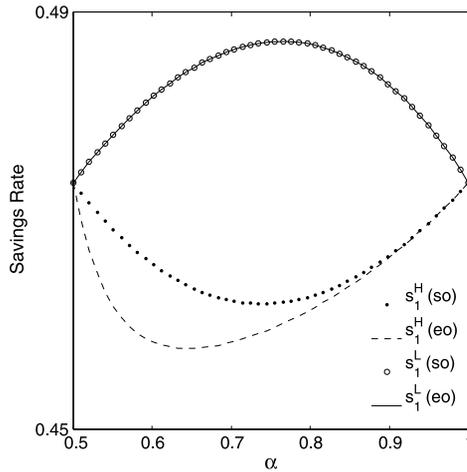


FIGURE 1.—Savings rate at $t = 1$ in the ex ante and sequentially optimal rules given $\theta_{-1} = \theta^H$.

greater if spending needs are low today, as spending needs are then likely to be low tomorrow. As a result, the threat of no discipline in the future ironically imposes discipline today.

We illustrate the properties of the ex ante optimum and the sequential optimum with a numerical example.³⁰ It can be shown that under (18), the ex ante optimal rule also features pooling at date 1; thus, under the two rules, the savings rate at date 1 is independent of the realized type at date 1. Figure 1 shows this savings rate, $s_1(\theta_{-1}, \theta^i)$, in the ex ante optimal (eo) and sequentially optimal (so) rules for $\theta_0 = \theta^L$ (s_1^L) and $\theta_0 = \theta^H$ (s_1^H), given $\theta_{-1} = \theta^H$, as a function of the persistence of types, denoted $\alpha \equiv p(\theta^i | \theta^i)$ in the figure.

Three points are evident in Figure 1. First, consistent with the perturbation just described, the figure shows that the ex ante optimal savings rate at date 1 following $\theta_0 = \theta^H$ is below the sequentially optimal rate. As explained, inducing overspending ex post is efficient ex ante because it allows relaxing the low type’s incentive compatibility constraint and curbing his spending at date 0. Second, the figure shows that following $\theta_0 = \theta^L$, the ex ante optimal savings rate at date 1 coincides with the sequentially optimal rate. The logic for this “no distortion at the top” also stems from the low type’s incentive compatibility constraint at date 0: to relax this constraint, the ex ante optimal rule maximizes the continuation welfare for the low type given a truthful report at date 0, which corresponds to assigning the optimal static rate at date 1. Finally, the figure shows how the savings rate at date 1 depends on the persistence of types, and, in particular, that the ex ante optimal and sequentially optimal rates coincide if types are i.i.d. ($\alpha = 0.5$) or fully persistent ($\alpha = 1$). Intuitively,

³⁰We consider $\delta = 0.9$, $\beta = 0.6$, $\theta^L = 2$, and $\theta^H = 3$.

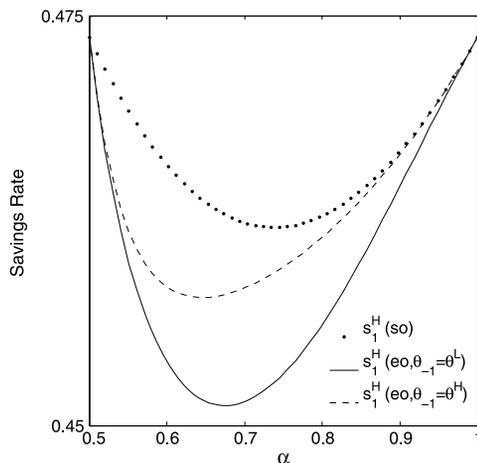


FIGURE 2.—Savings rate at $t = 1$ under different histories.

in these cases, $\mathbb{E}[\theta_1 | \theta_0 = \theta^i] = \mathbb{E}[\theta_2 | \theta_0 = \theta^i]$, so the marginal benefit of spending at dates 1 and 2 is the same. Given (19), this means that the sequentially optimal rate at date 1 is independent of θ_0 , and, thus, any perturbation would affect welfare on and off the equilibrium path equally.

Figure 2 explores whether the ex ante optimal mechanism exhibits history dependence. The figure shows that, indeed, the ex ante optimal savings rate at date 1 following $\theta_0 = \theta^H$ depends on the history, namely on θ_{-1} .³¹ Specifically, this rate is lower and farther away from the sequentially optimal rate if $\theta_{-1} = \theta^L$ than if $\theta_{-1} = \theta^H$, so fiscal rules become more excessively lax after a high shock at date 0 if $\theta_{-1} = \theta^L$. The intuition is that θ_{-1} affects the distribution of types at date 0, and, thus, the benefits and costs of perturbing the savings rate at date 1. Recall that the benefit of the perturbation is that it relaxes the low type's incentive compatibility constraint at date 0 and allows increasing savings given $\theta_0 = \theta^L$; the cost, on the other hand, is that it induces an ex post suboptimal savings rate at date 1 given $\theta_0 = \theta^H$. If $\theta_{-1} = \theta^L$, the probability of $\theta_0 = \theta^L$ is higher and, therefore, the relative benefits of the perturbation are larger.

Before turning to the infinite horizon economy, we use our three-period example to briefly discuss our restriction to public strategies. As previously noted, this restriction is not without loss. Suppose that the government's strategy can depend on its private history as follows: in addition to reporting its type at date 1, the date-1 government also reports the type that was realized at date 0. We show that this extension of the message space allows us to increase efficiency by relaxing the incentive compatibility constraints at date 0.

³¹The savings rate at date 1 following $\theta_0 = \theta^L$ is history independent, as it is sequentially optimal.

Specifically, consider a mechanism that is identical to the ex ante optimal mechanism (in public strategies), so it does not depend on the government’s report at date 1 of its type at date 0, except in one circumstance: when the date-0 government reported that its type was high and the date-1 government reports that its type was *low* at date 0 and is high at date 1. In this case, the mechanism assigns the date-1 government a savings rate such that the high type at date 1 is indifferent between this rate and the rate it would be assigned under the ex ante optimal mechanism. By construction, since the high type at date 1 wants to spend more than what is allowed in the ex ante optimum, the new savings rate is lower. Moreover, it can be verified that this rate satisfies incentive compatibility constraints; in particular, the low type at date 1 prefers its assigned savings rate at date 1 in the ex ante optimum to the new rate. Finally, because the date-1 government can now effectively “rat out” the date-0 government and cause savings to be even lower at date 1, it is clear that this modification makes it less attractive for the low type at date 0 to deviate and report to be a high type. Therefore, private strategies can in this way relax incentive compatibility constraints and enhance efficiency.³² While mechanisms using private strategies are interesting, in this paper we focus on public strategies as a first step to understanding optimal fiscal rules when shocks are persistent.

5. OPTIMAL FISCAL RULES

This section characterizes the optimal fiscal rules in an infinite horizon economy. Because the sequentially optimal rule is simple, we first study this rule in Section 5.1 and then use it as a benchmark in describing the ex ante optimal rule in Section 5.2. In Section 5.3, we consider the implications of these rules for the level of debt in the long run.

We develop our characterization under the assumption that types are relatively close to each other.³³

ASSUMPTION 2: *We have*

$$\frac{\theta^H}{\theta^L} - \frac{\tilde{\theta}^H}{\tilde{\theta}^L} < \frac{1 - p(\theta^i|\theta^i)}{p(\theta^i|\theta^i)} \left(\frac{1}{\beta} - 1 \right).$$

Assumption 2 is implied by condition (18) in the three-period example in Section 4 if the level of persistence is low enough, that is, if $p(\theta^i|\theta^i) \geq 1/2$ is

³²The proposed modification allows us to increase efficiency under private strategies when shocks are discrete. On the other hand, Amador, Werning, and Angeletos (2003) show that with a continuum of i.i.d. shocks, there is no loss in focusing on public strategies because there is zero mass on all types.

³³This assumption is consistent with the extension to a continuum of types in Appendix B.

sufficiently close to $1/2$.³⁴ We discuss the intuition and implications of Assumption 2 in the following subsections.

5.1. *Sequentially Optimal Rule*

Consider program (16)–(17) that defines the sequentially optimal fiscal rule. Given Assumption 2, it can be shown that this rule features pooling: the savings rate $s_t(\theta^t)$ is independent of the realization of θ_t and depends only on θ_{t-1} , which is used in predicting the value of θ_t . This is stated formally in the proposition below.

PROPOSITION 1—*Sequential Optimum: For all θ^t and θ^k , the sequential optimum features*

$$s_t(\theta^t) = s_k(\theta^k) \quad \text{if} \quad \theta_{t-1} = \theta_{k-1}.$$

Moreover, $s_t(\theta^t)$ satisfies

$$(21) \quad \mathbb{E}[\theta_t | \theta_{t-1}] U'(1 - s_t(\theta^t)) - \mathbb{E}[\tilde{\theta}_t | \theta_{t-1}] U'(s_t(\theta^t)) = 0.$$

For the proof, see Appendix C.

The intuition for this result is analogous to that in the three-period example of Section 4, where the optimal static mechanism features pooling when types are sufficiently close. Here the result follows from Assumption 2, which implies that the analog of condition (18) used for the three-period economy holds in the infinite horizon economy.³⁵

Given the budget constraint (1), Proposition 1 says that the sequentially optimal rule at date t prescribes a level of debt $b_{t+1}(\theta^t)$ as a function of θ_{t-1} and $b_t(\theta^{t-1})$ only. If $\theta_{t-1} = \theta^H$, the prescribed level of debt is higher than if $\theta_{t-1} = \theta^L$, as a high shock is then more likely and thus the sequentially optimal level of deficit-financed spending at date t is higher. Furthermore, if $b_t(\theta^{t-1})$ is relatively high, then $b_{t+1}(\theta^t)$ must also be relatively high to facilitate the servicing of the debt while simultaneously providing public goods. A useful implication is that the sequential optimum can be implemented with a renegotiated debt limit.

COROLLARY 1: *The sequentially optimal rule at any date t can be implemented with a history-independent debt limit, $\bar{b}(\theta_{t-1}, b_t(\theta^{t-1}))$.*

³⁴The strict inequality in Assumption 2 cannot be satisfied under full persistence ($p(\theta^i | \theta^j) = 1$). However, our problem in that case is rather trivial as the first-best savings rate is the same for both types. If types are close enough, the assumption is satisfied for any persistence level, including those arbitrarily close to 1.

³⁵Specifically, this condition is $\frac{\theta^H / \bar{\omega}^H}{\theta^L / \bar{\omega}^L} < \frac{1}{\beta}$.

It follows from Assumption 2 and equation (21) that under the sequentially optimal rule, both the low type and the high type would like to borrow and spend more than they are allowed. Thus, the sequentially optimal rule takes the form of a renegotiated debt limit where, given θ_{t-1} and $b_t(\theta^{t-1})$, both types choose the maximum allowable debt.³⁶

5.2. Ex ante Optimal Rule

We now consider the ex ante optimal fiscal rule defined in (12). We first develop a recursive representation of the problem, and then characterize the solution and discuss its implementation.

5.2.1. Recursive Representation

Our recursive representation is similar in spirit to that of Fernandes and Phelan (2000), who develop a recursive technique to study a principal-agent problem under persistent shocks. The idea of this representation is to solve program (12) by choosing $s_t(\theta^t)$ and $V_{t+1}(\theta^t)$ defined in (10) sequentially for each history θ^{t-1} , where associated with each choice of $V_{t+1}(\theta^t)$ is some continuation welfare $W_{t+1}(\theta^t)$ that is a function of the realized type θ_t and the chosen $V_{t+1}(\theta^t)$. To characterize this value of $W_{t+1}(\theta^t)$, let $W(\theta^i, V)$ for $\theta^i \in \{\theta^L, \theta^H\}$ correspond to the solution to (12) given $\theta_{-1} = \theta^i$ and subject to the additional constraint

$$(22) \quad V = \sum_{\theta_0 \in \{\theta^L, \theta^H\}} p(\theta_0 | \theta_{-1} = \theta^{-i}) \times [\theta_0 U(1 - s_0(\theta^0)) + \tilde{\theta}_0 U(s_0(\theta^0)) + \delta W_1(\theta^0)].$$

Constraint (22) is often referred to as a *threat-keeping constraint*. This constraint says that if θ_{-1} is equal to θ^{-i} as opposed to θ^i , the expected welfare under the savings rate sequence that solves the program must be equal to V . Using this formulation, we rewrite program (12) recursively as

$$(23) \quad W(\theta^i, V) = \max_{\{s^L, s^H, V^L, V^H\}} \{p(\theta^L | \theta^i)(\theta^L U(1 - s^L) + \tilde{\theta}^L U(s^L) + \delta W(\theta^L, V^L)) + p(\theta^H | \theta^i)(\theta^H U(1 - s^H) + \tilde{\theta}^H U(s^H) + \delta W(\theta^H, V^H))\}$$

subject to

$$(24) \quad V = \{p(\theta^L | \theta^{-i})(\theta^L U(1 - s^L) + \tilde{\theta}^L U(s^L) + \delta W(\theta^L, V^L)) + p(\theta^H | \theta^{-i})(\theta^H U(1 - s^H) + \tilde{\theta}^H U(s^H) + \delta W(\theta^H, V^H))\},$$

³⁶In Appendix B, we show that these results extend to an economy with a continuum of types, although in that case some types choose to borrow below the debt limit.

$$(25) \quad \theta^L U(1 - s^L) + \beta \tilde{\theta}^L U(s^L) + \beta \delta W(\theta^L, V^L) \\ \geq \theta^L U(1 - s^H) + \beta \tilde{\theta}^L U(s^H) + \beta \delta V^H,$$

$$(26) \quad \theta^H U(1 - s^H) + \beta \tilde{\theta}^H U(s^H) + \beta \delta W(\theta^H, V^H) \\ \geq \theta^H U(1 - s^L) + \beta \tilde{\theta}^H U(s^L) + \beta \delta V^L,$$

$$(27) \quad \underline{V}^L \leq V^L \leq \overline{V}^L, \quad \text{and} \quad \underline{V}^H \leq V^H \leq \overline{V}^H.$$

Equations (23)–(27) comprise a recursive representation of (12) starting from some history θ^{t-1} . The program selects savings rates s^i for $i \in \{L, H\}$ —which represent the values of $s_t(\theta^t)$ —and threats V^i for $i \in \{L, H\}$ —which represent the values of $V_{t+1}(\theta^t)$ —to maximize social welfare, taking into account that the continuation welfare conditional on $\theta_t = \theta^i$ is equal to $W(\theta^i, V^i)$ and subject to (24)–(27). Constraint (24) is a recursive representation of the threat-keeping constraint stating that if type θ^{-i} deviates and pretends to be θ^i in period $t - 1$, his continuation welfare at t is equal to V . As such, V^i , which is chosen in the current period t , is the continuation welfare at $t + 1$ to a type θ^{-i} who pretends to be θ^i at t . Constraints (25)–(26) are recursive representations of the incentive compatibility constraints in (11), and constraint (27) guarantees that the values of V^i are within a feasible range.³⁷

We next characterize $W(\theta^i, V)$. Clearly, if shocks are i.i.d., it must be that $W(\theta^i, V) = V$. If shocks are persistent, the following lemma holds.

LEMMA 1: *If shocks are persistent, $W(\theta^i, V)$ is strictly increasing, strictly concave, and continuously differentiable in V over the range $(\underline{V}^i, \overline{V}^i)$.*

See Appendix D for the proofs of all the lemmas and propositions in this subsection.

\underline{V}^i and \overline{V}^i are, respectively, the lowest and highest values of $V_{t+1}(\theta^t)$ that can be attained in the solution to (12). \overline{V}^i is the value of V that results from the sequence problem (12) given $\theta_{-1} = \theta^i$ and subject to the threat-keeping constraint (22) when this constraint does not bind, so that the solution effectively corresponds to the ex ante optimum. Values of $V_{t+1}(\theta^t)$ that exceed \overline{V}^i are never attained along the equilibrium path: such high values of $V_{t+1}(\theta^t)$ would tighten the incentive compatibility constraints (25) and (26) while simultaneously reducing the continuation welfare below $W(\theta^i, \overline{V}^i)$. Hence, only threats such that $V^i < \overline{V}^i$ are used, and the optimal level of threats depends on the

³⁷Note that our representation of the problem as one of choosing savings rates rather than spending and borrowing allows us to write the recursive problem without having an additional state variable for the level of debt.

benefits of relaxing incentive compatibility constraints relative to the costs of reducing continuation welfare $W(\theta^i, V^i)$.

To understand the role of Assumption 2 and the threat-keeping constraint (24), let λ be the Lagrange multiplier on this constraint. The envelope condition implies $W_V(\theta^i, V) = -\lambda$, so that given the strict concavity of $W(\cdot)$, lower values of V are associated with more negative values of λ . It follows that solving (23)–(27) is equivalent to solving the problem

$$(28) \quad \max_{\{s^L, s^H, V^L, V^H\}} \{ (p(\theta^L|\theta^i) + \lambda p(\theta^L|\theta^{-i})) \times (\theta^L U(1 - s^L) + \tilde{\theta}^L U(s^L) + \delta W(\theta^L, V^L)) + (p(\theta^H|\theta^i) + \lambda p(\theta^H|\theta^{-i})) \times (\theta^H U(1 - s^H) + \tilde{\theta}^H U(s^H) + \delta W(\theta^H, V^H)) \}$$

subject to (25)–(27).

Under persistent types, the role of the threat-keeping constraint is to effectively “twist” the probabilities assigned to each type. If, for example, $\theta^i = \theta^H$, the objective in (28) under-weighs welfare conditional on the low type relative to the high type, and this is done more severely the lower is V . The opposite is true if $\theta^i = \theta^L$. Hence, to satisfy the threat-keeping constraint, the program overweighs (underweighs) welfare at t conditional on the type that is less (more) likely to occur from the perspective of a deviating type θ^{-i} at $t - 1$.

Assumption 2 facilitates the characterization of $W(\theta^i, V)$ because it guarantees that along the equilibrium path, the value of V is never chosen to be so low that the objective in (28) would assign a negative weight to some type.³⁸ Intuitively, if V is very low, then types are sufficiently close that any benefits from providing better incentives today are outweighed by the costs of reduced welfare tomorrow. Note that these relative benefits and costs depend on the persistence of types; in particular, under a high level of persistence $p(\theta^i|\theta^i)$, the future welfare costs may be incurred by a type that is unlikely to occur. This explains why the condition in Assumption 2 depends on $p(\theta^i|\theta^i)$. (See Lemma 8 in Appendix D for details.)

Using the representation in (28), we also note that \underline{V}^i in (27) is defined so that whenever $V = \underline{V}^i$, one of the weights in (28) is equal to zero. That is, following type θ^H at time $t - 1$, if $V = \underline{V}^H$, then the program at time t assigns a zero weight to type θ^L , which is the type that is more likely to occur from the perspective of a deviating type at time $t - 1$.

³⁸In the case that the objective assigns a negative weight to a type, we are not able to prove that $W(\theta^i, V)$ is concave and differentiable, which prevents us from achieving a characterization of the equilibrium dynamics.

5.2.2. Characterization

We now characterize the ex ante optimal fiscal rule. Consider first the i.i.d. benchmark.

PROPOSITION 2—Ex ante Optimum Under i.i.d. Shocks: *If shocks are i.i.d., the ex ante optimum coincides with the sequential optimum.*

When shocks are i.i.d., the ex ante optimal rule prescribes the sequentially optimal savings rate in each period, which in this case is constant and which can be implemented with a renegotiated debt limit. This rule is thus history independent and does not provide dynamic incentives for truth-telling. The reason why dynamic incentives are inefficient is as discussed in the three-period example: under i.i.d. shocks, any perturbation in the ex post optimal rule affects continuation welfare on the equilibrium path (given a truthful report) and off the equilibrium path (given a nontruthful report) equally, and, therefore, cannot increase ex ante welfare. This result is analogous to that of Amador, Werning, and Angeletos (2006).

Consider next the case of persistent shocks. Let the solution to (23)–(27) be denoted by

$$(29) \quad \{s^{L*}(\theta^i, V), s^{H*}(\theta^i, V), V^{L*}(\theta^i, V), V^{H*}(\theta^i, V)\}.$$

LEMMA 2: *If shocks are persistent, the solution to (23)–(27) has the following properties for all $V \in (\underline{V}^i, \bar{V}^i]$:*

- (i) **Resetting.** $V^{L*}(\theta^i, V) = \bar{V}^L$ for $\theta^i \in \{\theta^L, \theta^H\}$.
- (ii) **Monotonicity of Threats.** $V^{H*}(\theta^H, V)$ is strictly decreasing in V and $V^{H*}(\theta^i, V)$ satisfies $\bar{V}^H > V^{H*}(\theta^H, V) > V^{H*}(\theta^L, \bar{V}^L)$.
- (iii) **Monotonicity of Savings Rates.** $s^{i*}(\theta^H, V)$ is strictly increasing in V and $s^{i*}(\theta^i, V)$ satisfies $s^{i*}(\theta^H, V) < s^{i*}(\theta^L, \bar{V}^L)$ for $i \in \{L, H\}$.

Lemma 2 describes the solution to (23)–(27) given θ^i and V . The first part of the lemma states that the equilibrium at $t + 1$ effectively “resets” if the low type is realized at t . This result is analogous to the result in the three-period example that the savings rate at date 1 is sequentially optimal if the low type is realized at date 0. As in that example, the intuition is that setting $V^{L*}(\theta^i, V) = \bar{V}^L$ maximizes the continuation payoff of the low type given a truthful report and, thus, maximally relaxes the incentive compatibility constraint of the low type (25) while maximizing social welfare.

The second part of Lemma 2 concerns the magnitude of $V^{H*}(\theta^i, V)$ across different values of V . Comparing $V^{H*}(\theta^H, V)$ and $V^{H*}(\theta^L, \bar{V}^L)$, the lemma states that the threat used in the ex ante optimum to induce the low type to report truthfully is more severe if $\theta^i = \theta^L$ than if $\theta^i = \theta^H$. This result is analogous

to the result in the three-period example that the savings rate at date 1 following $\theta_0 = \theta^H$ is lower if $\theta_{-1} = \theta^L$ than if $\theta_{-1} = \theta^H$. The intuition is also as in that example: when $\theta^i = \theta^L$, the low shock is more likely, so the benefit of using a more severe threat—namely, relaxing the low type’s incentive constraint and curbing his spending—is larger relative to the cost—namely, reducing the high type’s continuation welfare. The lemma also considers how $V^{H*}(\theta^H, V)$ varies with V . As discussed, the lower is V , the more twisted the effective objective function in (28) is in favor of welfare conditional on a high type, implying that threats are more costly and thus $V^{H*}(\theta^H, V)$ is higher.

The third part of Lemma 2 considers how the optimal savings rate depends on V . Comparing $s^{i*}(\theta^H, V)$ and $s^{i*}(\theta^L, \bar{V}^L)$, the lemma states that the savings rate conditional on the realization of either type is lower if $\theta^i = \theta^H$ than if $\theta^i = \theta^L$. The intuition is straightforward. Because of Assumption 2, the savings rate of the low (high) type is always below (above) the first-best rate for that type. If $\theta^i = \theta^H$, the objective function in (28) puts a higher weight on welfare conditional on a high type, so the optimal savings rate for this type must be lower and closer to first best. But then for the incentive constraint of the low type (25) to be satisfied, it must be that the savings rate of the low type is also lower, implying that $s^{i*}(\theta^H, V) < s^{i*}(\theta^L, \bar{V}^L)$ for $i \in \{L, H\}$. The lemma also compares $s^{i*}(\theta^H, V)$ across different values of V . The logic is the same as above: the lower is V , the higher is the weight in (28) assigned to the high type and, consequently, the lower is $s^{i*}(\theta^H, V)$.

We now use Lemma 2 to describe the ex ante optimal fiscal rule under persistent shocks. Define $\eta_t(\theta^{t-1})$ as the number of periods since the last time that θ^L was realized:

$$\eta_t(\theta^{t-1}) = k \quad \text{if} \\ \theta_{t-1-k} = \theta^L \quad \text{and} \quad \theta_{t-1-l} = \theta^H \quad \text{for all } l \in \{0, \dots, k-1\}.$$

PROPOSITION 3—Ex ante Optimum Under Persistent Shocks: *If shocks are persistent, the ex ante optimum has the following features:*

- (i) *For all θ^l and θ^k with $\theta_t = \theta_k$, $s_t(\theta^l) = s_k(\theta^k)$ if $\eta_t(\theta^{t-1}) = \eta_k(\theta^{k-1})$.*
- (ii) *There exist θ^l, θ^k with $\theta_t = \theta_k$ and $\theta_{t-1} = \theta_{k-1}$ for which $s_t(\theta^l) \neq s_k(\theta^k)$.*

Thus, the ex ante optimum does not coincide with the sequential optimum, and it exhibits history dependence.

Unlike the sequentially optimal rule or the optimal rule under i.i.d. shocks, the ex ante optimal rule under persistent shocks exhibits history dependence: the mechanism at history θ^t depends not only on the payoff-relevant variables, θ_{t-1} and $b_t(\theta^{t-1})$, but also on when the low shock last occurred and, thus, pos-

sibly on the entire history of shocks.³⁹ The logic stems from the fact that the mechanism provides dynamic incentives. Intuitively, because the shock at date $t - 2$ predicts the realization of the shock at $t - 1$, the shock at $t - 2$ affects the relative tightness of incentive compatibility constraints at $t - 1$, which in turn affect the policies that are chosen at t in providing dynamic incentives at $t - 1$. The result in Lemma 2 that the mechanism resets following a low shock implies, in fact, that the tightness of current incentive constraints depends on when this resetting began, which explains why prescribed policies depend on the time that passed since a low shock was realized.

The resetting property of the ex ante optimal rule is related to the “no distortion at the top” result of standard static adverse selection models as well as dynamic models such as Battaglini (2005). In our framework, the lack of cross-subsidization across types implies that maximizing the reward to the low type who tells the truth does not have any direct resource costs for the high type. Maximizing this reward by resetting the mechanism is then optimal as it allows us to maximally relax incentive constraints. To see this, consider an equilibrium starting from date 0. If $\theta_0 = \theta^L$, as explained, the equilibrium at date 1 optimally transitions to the ex ante optimum associated with $\theta_{-1} = \theta^L$. If instead $\theta_0 = \theta^H$, the fiscal rule at date 1 seeks to punish the low type at date 0 who would have lied, while not harming the truthful high type too much. There are two ways in which this can be done: first, spending at date 1 given a low type at date 1 can be made higher and further away from first best; second, the expected continuation welfare at date 1 from date 2 onward, given a low type at date 1, can be made lower. Because the low type at date 0 is more likely to be a low type at date 1, either of these changes hurts the deviating low type at date 0 more than the truthful high type at date 0. However, while the first option slackens the low type’s incentive constraint at date 1, the second option tightens this constraint. Consequently, it is cheaper from a date-0 perspective to provide incentives to the low type by increasing spending for a single period following a high shock, while resetting the equilibrium thereafter given a low shock.

Under the ex ante optimal rule, high shocks then lead to an erosion of future fiscal discipline compared to low shocks, which lead to the reinstatement of discipline. A natural question regards the equilibrium dynamics for a sequence of consecutive high shocks. These can be described using the second and third parts of Lemma 2, as shown in the following proposition.

PROPOSITION 4—Dynamics Under Persistent Shocks: *There exists a unique \widehat{V} such that for all θ^t , if $\theta_{t-1} = \theta_t = \theta^H$, then $V_{t+1}(\theta^t) > (<) \widehat{V}$ if $V_t(\theta^{t-1}) < (>) \widehat{V}$. Thus, the ex ante optimum features oscillations in savings rates.*

³⁹It is straightforward to show that for histories for which the low shock has never been realized (for which $\eta_t(\theta^{t-1})$ is not defined), policies are history dependent in the sense that they are a function of the date t .

Combined with Lemma 2, Proposition 4 states that under a sequence of consecutive high shocks, the equilibrium oscillates between periods of high spending and periods of low spending, around some fiscal rule associated with a threat \widehat{V} such that $V^{H*}(\theta^H, \widehat{V}) = \widehat{V}$.⁴⁰ In all of these periods, there is a lack of fiscal discipline in that the level of spending exceeds the level under $\theta^i = \theta^H$, $V = \overline{V}^H$.

To understand the oscillatory dynamics, consider again the equilibrium starting from date 0, given $\theta_{-1} = \theta^H$. As explained, if $\theta_0 = \theta^H$ is realized at date 0, the ex ante optimal rule induces high spending at date 1 given a low type at date 1, where this spending is further away from first best compared to spending at date 0 given a low type at date 0. It follows that at date 1, the low type's incentives to lie and pretend to be a high type are relatively lower, and, in turn, a smaller threat at date 2 following a high type at date 1 is sufficient to provide incentives at date 1. Yet, note that the equilibrium does not reset to the ex ante optimum at date 2; the reason is simply that if spending at date 2 corresponded to the ex ante optimal spending level given $\theta_{-1} = \theta^H$, incentive compatibility constraints at dates 0 and 1 could be relaxed by increasing such spending, implying a first-order gain and causing only a second-order loss at date 2.

The economics behind the oscillatory dynamics in Proposition 4 emerge from the self-control nature of the problem. The absence of discipline tomorrow is used to induce discipline today. In turn, the absence of discipline tomorrow allows increasing discipline the day after while preserving incentives tomorrow. Because the time-inconsistency problem lasts a single period in our setting, the oscillations under a sequence of consecutive high shocks are also each a single period long. Recall that under the political turnover interpretation of our preferences (see Section 2), our formulation corresponds to political cycles lasting one period; however, it is clear that the results are robust to different political turnover frequencies as long as these coincide with the frequency of economic shocks. Specifically, in a setting where turnover occurs every N periods and economic shocks also occur every N periods, our analysis would imply the presence of N -period oscillations. A related question concerns the form that oscillations would take if political turnover and economic shocks occurred at different frequencies; such an environment, however, would involve quite distinct considerations.⁴¹

⁴⁰This implies that during consecutive periods of high shocks, public spending under the ex ante optimal rule exhibits higher volatility than that implied by the shock process and, thus, higher volatility than under the sequentially optimal rule or the first-best policy. The oscillatory dynamics may imply convergence toward the fiscal rule associated with $V = \widehat{V}$, although this depends on the slope of the policy function. While we cannot characterize the slope of this policy function analytically, numerical simulations suggest that convergence toward $V = \widehat{V}$ indeed occurs in equilibrium.

⁴¹For the case where political turnover occurs every N periods while economic shocks occur at a higher frequency, we conjecture that the economy would experience oscillations in an N -period

5.2.3. Implementation

In our analysis of the sequentially optimal rule in Section 5.1, we showed that such a rule can be implemented with a history-independent debt limit. That is, in each period, the government is allowed to borrow any amount up to some threshold, where this threshold depends only on the accumulated level of debt and the previous period's shock.

A possible implementation of the ex ante optimal rule is also with the use of debt limits, although, importantly, these limits would now depend on the history. In particular, in the ex ante optimum, the level of debt that the government is allowed to incur in any period would depend not only on how much the government borrowed up to that period (i.e., the current level of debt), but also on how much the government chose to borrow relative to debt limits in previous periods.

More specifically, the ex ante optimal rule can be implemented as follows. In each period, the government is subject to two debt limits: a low debt limit that corresponds to the level of debt incurred under the low shock and a high debt limit that corresponds to the level of debt incurred under the high shock. The low debt limit can be seen as the limit that would prevail in "normal times," while the high debt limit applies if the government decides to utilize an "escape clause" to exceed the low limit up to some point.⁴² From Proposition 3, it is clear that these two limits depend not only on the payoff-relevant variables—the level of debt and the previous period's shock—but also on η_t , namely the number of periods that have passed since normal times last prevailed. Under this implementation, periods of nondiscipline are those that occur after escape clauses are triggered, while periods of discipline emerge once escape clauses stop being utilized.

It is worth noting here that we have considered throughout an environment where the government has the ability to commit within the period to a fiscal rule. In the implementation of the ex ante optimal rule just described, this means that even if an escape clause is utilized to exceed the low debt limit, the government is committed to not go beyond the high debt limit. A natural question is whether, in the absence of commitment power, such a debt limit—and, more generally, the dynamic fiscal rule—can be self-enforced. Using standard folk theorems, one can show that the ex ante optimal rule can indeed be self-enforced if society's discount factor (δ) is high enough. This follows from the fact that in each period, equilibrium social welfare in the ex ante optimum is strictly higher than welfare when the government is granted full flexibility to

contract every N periods, so that oscillations would again reflect the frequency of turnover. If political turnover occurs instead at a higher frequency than economic shocks, additional complications are introduced because the full persistence of shocks across periods of turnover implies that the full support assumption no longer holds.

⁴²Debt limits combined with escape clauses are a common form of fiscal rules around the world; see International Monetary Fund (2009).

choose fiscal policy; thus, self-enforcement can be achieved by using the threat of reversion to full flexibility upon deviations from the prescribed policies.⁴³ In terms of the implementation above, this means that any violation of the high debt limit—which never occurs along the equilibrium path—would result in the permanent abandonment of fiscal rules.

5.3. Implications for Debt in the Long Run

The analysis of the previous sections shows that the savings rates induced by the ex ante optimal and sequentially optimal rules differ when shocks are persistent. What are the implications for the path of debt? Do these rules imply different levels of debt in the long run?

To characterize the debt dynamics, we consider the case where the market discounts the future at the same rate as the government (prior to the realization of the shock):⁴⁴

$$(30) \quad \delta = 1/(1 + r).$$

For any sequence of savings rates s_t , $t \in \{0, 1, \dots\}$, that admits a long-run ergodic distribution, let \bar{s} be the mean of $\log(s_t/\delta)$ in this distribution. The following result will be useful to characterize debt dynamics under different policies.

LEMMA 3: *Assume (30) holds. If $\bar{s} > 0$, assets diverge to infinity: $b_t \rightarrow -\infty$ as $t \rightarrow \infty$. If $\bar{s} < 0$, debt becomes maximal: $b_t \rightarrow \tau/(1 - \delta)$ as $t \rightarrow \infty$.*

See Appendix E for the proofs of all lemmas and propositions in this subsection.

Consider now the path of debt under the first-best and full-flexibility policies. The first-best savings rates satisfy $s_t^{\text{fb}} = s^{\text{fb}}(\theta^i)$ if $\theta_t = \theta^i$, where $s^{\text{fb}}(\theta^i)$ is defined in (8). The full-flexibility savings rates satisfy $s_t^f = s^f(\theta^i)$ if $\theta_t = \theta^i$, where for $\theta^i \in \{\theta^L, \theta^H\}$,⁴⁵

$$(31) \quad \theta^i U'(1 - s^f(\theta^i)) = \beta \tilde{\theta}^i U'(s^f(\theta^i)).$$

⁴³By analogous reasoning, self-enforcement of the sequentially optimal fiscal rule is also possible. An interesting question for future research is how self-enforcement can be achieved when the discount factor is not sufficiently high. Bernheim, Ray, and Yeltekin (2013) characterize the worst “punishment” that can be imposed on a deviation to self-enforce commitment contracts in a related environment without private information.

⁴⁴We focus on this case for simplicity. An analogous analysis of the forces that push debt to increase or decrease in the long run can also be performed by considering the interest rate that guarantees a nondegenerate long-run level of debt.

⁴⁵Multiple equilibria can emerge when the government is given full flexibility in every period. Analogous to our selection of the sequential optimum, we select a unique solution by considering the limit of a finite horizon economy with end date T as T approaches infinity.

Using Lemma 3, we show that, as may be expected, the debt dynamics under these two policies are starkly different.

LEMMA 4: *Assume (30) holds.*

- (i) *Under first best, $\bar{s}^{\text{fb}} > 0$, so assets diverge to infinity.*
- (ii) *Under full flexibility, $\bar{s}^f < 0$, so debt becomes maximal.*

The intuition for the first part of Lemma 4 stems from the self-insurance motive of the government. As is common across a wide class of self-insurance models (e.g., Chamberlain and Wilson, 2000), when condition (30) holds, this motive dominates in the long run, and the first-best policy for the government is to perfectly insure itself by accumulating an infinite level of assets. The second part of Lemma 4, however, shows that if given the option to flexibly choose policy, a present-biased government would not engage in such an accumulation of infinite assets. To the contrary, the government's present bias would cause the economy to accumulate the maximal amount of debt in the long run, pushing public spending toward zero. This stark contrast between the first-best and full-flexibility economies highlights the importance of fiscal rules to alleviate the government's time-inconsistency problem.

Consider next the path of debt under the sequentially optimal rule as characterized in Proposition 1.

PROPOSITION 5—Long-Run Debt in Sequential Optimum: *Assume (30) holds. In the sequentially optimal rule, the following statements hold:*

- (i) *Under i.i.d. shocks, $\bar{s}^{\text{so}} = 0$ and the level of debt is constant.*
- (ii) *Under persistent shocks, $\bar{s}^{\text{so}} > 0$, so assets diverge to infinity.*

In the sequential optimum, the government commits at date $t - 1$ to a savings rate at date t . If shocks are i.i.d., the savings rate at t is independent of the government's information at $t - 1$ and, therefore, is constant over time. In fact, using condition (21), this rate is equal to the discount factor δ , and, hence, given (30), the level of debt is also constant over time. In contrast, when shocks are persistent, the savings rate at date t depends on the information that the government has at $t - 1$. In particular, if $\theta_{t-1} = \theta^L$ is realized, the government anticipates lower spending needs at t and thus commits to saving more at t to insure itself for the future. Analogous to the first-best case above, it is this self-insurance motive, combined with (30), that makes the government eventually accumulate an infinite amount of assets.

Finally, consider the path of debt under the ex ante optimal rule. The savings rates in the ex ante optimum are characterized by Proposition 2 when shocks are i.i.d. and by Proposition 3 when shocks are persistent. Note that from Proposition 3, the savings rate at date t under persistent shocks is a function only of the current shock and the number of periods since the last low

shock. Since the savings rate process is then a countable, irreducible, and recurrent Markov chain, it is ergodic.⁴⁶ Let $s^{co}(\theta^i, \eta)$ denote the savings rate when the current shock is θ^i and the number of periods since θ^L was last realized is $\eta \geq 0$. It follows that⁴⁷

$$(32) \quad \bar{s}^{co} = \frac{1}{2} [p(\theta^i|\theta^i) \log(s^{co}(\theta^L, 0)/\delta) + (1 - p(\theta^i|\theta^i)) \log(s^{co}(\theta^H, 0)/\delta)] \\ + \frac{1}{2} (1 - p(\theta^i|\theta^i)) \\ \times \sum_{k=1}^{\infty} p(\theta^i|\theta^i)^{k-1} [p(\theta^i|\theta^i) \log(s^{co}(\theta^H, k)/\delta) \\ + (1 - p(\theta^i|\theta^i)) \log(s^{co}(\theta^L, k)/\delta)].$$

Using Lemma 3, the following result obtains.

PROPOSITION 6—Long-Run Debt in ex ante Optimum: *Assume (30) holds. In the ex ante optimal rule, the following statements hold:*

- (i) *Under i.i.d. shocks, $\bar{s}^{co} = 0$ and the level of debt is constant.*
- (ii) *Under persistent shocks, there exists an open set of parameters $\{\theta^L, \theta^H, p(\theta^i|\theta^i), \delta, \beta\}$ satisfying Assumption 2 under which $\bar{s}^{co} > 0$ and thus assets diverge to infinity, and there exists another open set of parameters $\{\theta^L, \theta^H, p(\theta^i|\theta^i), \delta, \beta\}$ satisfying Assumption 2 under which $\bar{s}^{co} < 0$ and thus debt becomes maximal.*

If shocks are i.i.d., the ex ante optimal and sequentially optimal rules coincide, so it follows from Proposition 5 that debt is constant. If, instead, shocks are persistent, the long-run path of debt in the ex ante optimum may entail accumulating infinite assets or maximal debt, depending on parameters. The intuition stems from the interaction of two countervailing forces. On the one hand, there is an operational precautionary motive that pushes the government toward the accumulation of assets, just as in the first best and the sequential optimum. On the other hand, the ex ante optimum features the provision of dynamic incentives, whereby phases of nondiscipline along the equilibrium path are used to sustain discipline in earlier periods. These phases of nondiscipline push the government toward the accumulation of debt and can dominate the self-insurance motive if they involve sufficiently low savings rates. Therefore, while the ex ante optimal rule yields higher ex ante welfare than the sequentially optimal rule, it can induce the government to become immiserated in the long run, just as under full flexibility.⁴⁸

⁴⁶See Example 7.1.7 in Durrett (2004).

⁴⁷See Appendix E for details.

⁴⁸A similar force toward immiseration arises in the principal-agent models of Atkeson and Lucas (1992) and Thomas and Worrall (1990). As in our model, this force emerges as a conse-

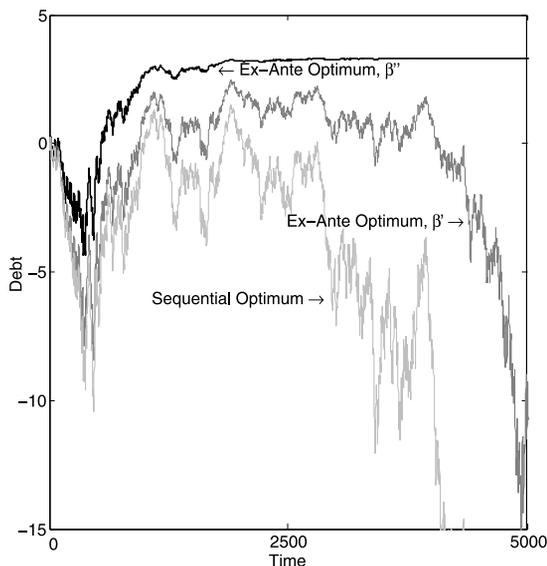


FIGURE 3.—Path of debt in the ex ante and sequentially optimal rules with $\delta = \frac{1}{1+r}$, $\beta'' > \beta'$.

To illustrate, we consider economies with different ex ante optimal debt dynamics. Figure 3 depicts two economies that differ in their degrees of time inconsistency.⁴⁹ The government's present bias affects the extent to which the ex ante optimal rule provides dynamic incentives: the larger this bias is, the less the government cares about its future welfare and, thus, the less effective is the threat of future nondiscipline to provide incentives in the present. Consequently, the figure shows that when the government's present bias is relatively large (that is, β is low), dynamic incentives are optimally weak and the

quence of dynamic incentive provision. However, note that those papers consider i.i.d. shocks, whereas in our self-control setting, dynamic incentives are provided only when shocks are persistent. Moreover, in our model, dynamic incentives need not always result in immiseration, as the countervailing self-insurance motive pushes toward asset accumulation.

⁴⁹We consider $\theta^L = 2$, $\theta^H = 3$, $p(\theta^i|\theta^i) = 0.7$, $\delta = 0.7$, $\beta' = 0.1$, $\beta'' = 0.3$, $b_0 = 0$, and $\tau = 1$. Recall that the sequential optimum is independent of β . For the ex ante optimum, the comparison across different values of β takes into account that, given other parameters, β must be low enough for Assumption 2 to be satisfied. To compute the equilibrium, we use the fact that there is a one-to-one mapping between λ in (28) and the threat V in (23), so that the value function depends on the last period's shock and λ . For a given λ , the value of the Lagrange multiplier on the incentive constraint implies savings rates from the first-order conditions. Moreover, first-order conditions also imply that the value of λ' for the next period depends only on λ and this Lagrange multiplier. We therefore guess an initial value function of λ and for each λ , we find a value of the Lagrange multiplier that makes the incentive constraint bind. We then feed this value and λ together with the implied savings rates and λ' back into the construction of the value function until the value function converges. The Supplemental Material describes the algorithm in more detail.

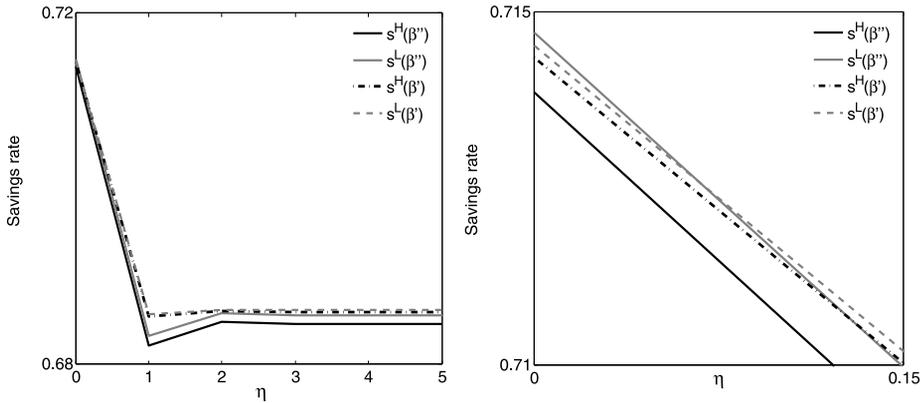


FIGURE 4.—Savings rates in the ex ante optimum with $\delta = \frac{1}{1+r}$, $\beta'' > \beta'$. The right graph zooms in around 0 to make the difference in savings rates more visible.

self-insurance motive dominates, so $\bar{s}^{co} > 0$ and the government accumulates infinite assets over time. Instead, when the government’s present bias is relatively small (that is, β is high), the ex ante optimal mechanism provides strong dynamic incentives, so periods of nondiscipline are severe enough that $\bar{s}^{co} < 0$ and the government becomes immiserated in the long run.⁵⁰

Figure 4 studies these dynamics further by considering the ex ante optimal savings rates. Recall that the savings rate at any date t in the ex ante optimum is a function only of the current shock θ_t and the number of periods since the last low shock η_t . Figure 4 depicts $s^{co}(\theta^i, \eta)$ for $\theta^i = \theta^H$ and $\theta^i = \theta^L$ as a function of η , for the two values of β considered in Figure 3. The left graph considers η from 0 to 5 and the right graph simply zooms in around 0 to make the comparison between savings rates more visible.⁵¹ When $\eta = 0$, the economy has reset into a period of discipline because θ^L was realized in the last period. The figure shows that if β is high, $s^{co}(\theta^L, 0)$ is higher; moreover, $s^{co}(\theta^H, 0)$ is lower, so there is more flexibility. Hence, if θ^L is realized and η remains at zero, the economy with a smaller present bias effectively experiences more discipline, and the level of debt grows at a slower pace than in the economy with a larger present bias. However, when $\eta \geq 1$, the economy enters a phase of nondiscipline, and this nondiscipline is more severe if β is high because dynamic incentives are then optimally stronger. Specifically, the figure shows that for $\eta \geq 1$, both $s^{co}(\theta^H, \eta)$ and $s^{co}(\theta^L, \eta)$ are lower if β is high, so the level of debt now grows at a faster pace than when β is low. Because high shocks

⁵⁰We find that the ex ante welfare benefit due to the additional flexibility in the ex ante optimum relative to the sequential optimum is quantitatively small. This is consistent with the more general observation by Lucas (1987) that the welfare benefit of smoothing out macroeconomic fluctuations is quantitatively small.

⁵¹Note that in both graphs, points are interpolated with a line for exposition.

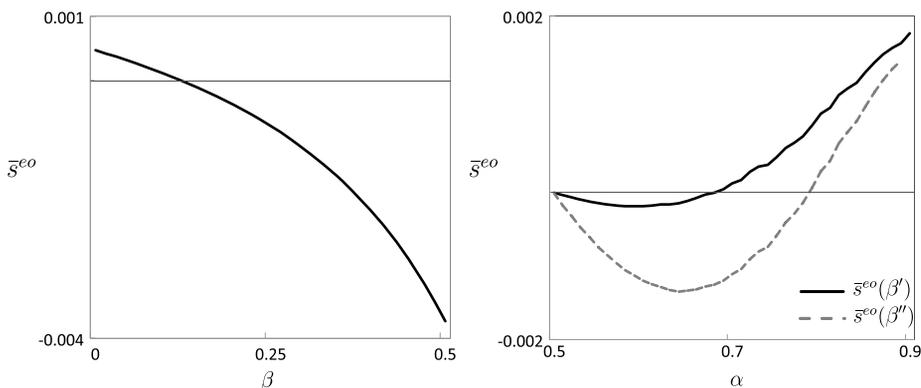


FIGURE 5.— \bar{s}^{eo} as a function of β (left graph) and α (right graph) with $\delta = \frac{1}{1+r}$, $\beta'' > \beta'$.

leading to phases of nondiscipline occur sufficiently often, the economy with a smaller present bias experiences a more rapid accumulation of debt over time.

Figure 5 builds on this example by performing a more in-depth numerical analysis of the determinants of \bar{s}^{eo} . The left graph shows that the results described above reflect a more general principle: higher levels of β are associated with lower levels of \bar{s}^{eo} and thus a greater tendency for long-run debt to become maximal. As explained, the intuition is that a smaller government present bias implies stronger dynamic incentives in the ex ante optimum, which in turn implies lower savings rates during phases of nondiscipline and a greater potential for immiseration. The right graph in Figure 5 studies the effects of changing the persistence of types, $\alpha \equiv p(\theta^i|\theta^i)$. The figure shows a U-shaped relationship between \bar{s}^{eo} and α , so that debt is more likely to become maximal for intermediate levels of persistence. The intuition mirrors that in the three-period economy of Section 4. If persistence is very low, dynamic incentives are less useful because the low type and the high type today place similar probabilities on realizations of shocks tomorrow. If persistence is very high, it is more difficult to separate the two types because their marginal rates of substitution are similar (i.e., $\theta/\tilde{\theta}$ varies less across types), so again dynamic incentives are less useful. It follows that in either of these extremes, optimal dynamic incentives are weak and the economy is more likely to accumulate infinite assets than for intermediate levels of persistence.^{52,53}

⁵²The left graph in Figure 5 computes the comparative static holding all parameters fixed as defined in footnote 49 while changing β within the range satisfying Assumption 2. The right graph is generated analogously changing α within the range satisfying Assumption 2.

⁵³One can also study the effects of changing the exogenous interest rate r . We find that in the cases where (30) does not hold, immiseration is more likely to occur if r is low.

6. CONCLUDING REMARKS

This paper has studied the role of persistence in determining the optimal structure of fiscal rules. We showed that when the shocks to the economy are i.i.d., the ex ante optimal fiscal rule coincides with the sequentially optimal rule, taking the simple form of a renegotiated debt limit. In contrast, when the shocks are persistent, the ex ante optimal rule is no longer sequentially optimal; this rule now provides dynamic incentives and exhibits history dependence. The ex ante optimal mechanism features rich dynamics, with high shocks leading to an erosion of future fiscal discipline, while low shocks reinstate discipline, and with policy distortions oscillating over time given a sequence of high shocks. Moreover, while the sequentially optimal rule leads to the accumulation of infinite assets over time, the ex ante optimal rule may induce the government to accumulate maximal debt and become immiserated in the long run.

We believe our paper leaves interesting questions for future research. One possible extension would be to consider more general time-inconsistent preferences with hyperbolic discounting. The nature of the problem would change compared to our quasi-hyperbolic setting, as the preferences of the current government regarding future policies would no longer coincide with those of society. As the government's bias extends to future periods, in fact, the problem becomes one of delegation. Also, our model can be enriched to consider the distortionary effects of taxation and the endogeneity of interest rates. A natural future direction would be to quantitatively assess the properties of optimal fiscal rules and how they depend on the economic structure.

While we have focused on fiscal policy, the insights of this paper can be applied to other settings as well. To give an example, consider recruiting in an academic department. There are a limited number of slots that the provost of the university can assign to a department over the years, and the provost wants that those slots be used for high-quality job candidates only. The department has the same preferences ex ante; however, when it comes to making decisions and there is a slot to fill in, the department is biased toward hiring now as it may lose the slot otherwise. The provost lacks the ability to evaluate candidates in each academic field and must thus rely on the department's assessments. Moreover, the quality of candidates is likely to be positively correlated over time: some academic fields are on the rise while others become less popular and productive. Our model can be directly applied to this setting and has clear implications for the optimal design of hiring rules. Other applications of our setup include individual self-control problems (such as those related to decisions on how much to consume or drink, or how often to go to the gym) and optimal monetary policy discretion.⁵⁴

⁵⁴Athey, Atkeson, and Kehoe (2005) consider the commitment versus flexibility trade-off in a monetary policy model with i.i.d. shocks.

APPENDIX A: DERIVING THE SEQUENTIALLY OPTIMAL RULE

In this appendix, we consider the problem defined by (13)–(15) for a finite horizon T -period economy and show that as $T \rightarrow \infty$, this program is equivalent to (16)–(17).

Define

$$(A.1) \quad J_T^T(\theta_{T-1}, b_T) = \sum_{\theta_T \in \{\theta^L, \theta^H\}} p(\theta_T | \theta_{T-1}) \theta_T U(\tau - b_T)$$

as the expected social welfare at date T conditional on θ_{T-1} and b_T , given the budget constraint (1) and given the end date T . Using $J_T^T(\theta_{T-1}, b_T)$, define the sequential optimum recursively as a solution to

$$(A.2) \quad J_t^T(\theta_{t-1}, b_t) = \max_{\{g_t(\theta_t), b_{t+1}(\theta_t)\}_{\theta_t \in \{\theta^L, \theta^H\}}} \sum_{\theta_t \in \{\theta^L, \theta^H\}} p(\theta_t | \theta_{t-1}) \times (\theta_t U(g_t(\theta_t)) + \delta J_{t+1}^T(\theta_t, b_{t+1}(\theta_t)))$$

subject to

$$(A.3) \quad g_t(\theta_t) = \tau + b_{t+1}(\theta_t)/(1+r) - b_t \quad \text{and}$$

$$(A.4) \quad \theta_t U(g_t(\theta_t)) + \beta \delta J_{t+1}^T(\theta_t, b_{t+1}(\theta_t)) \geq \theta_t U(g_t(\hat{\theta}_t)) + \beta \delta J_{t+1}^T(\theta_t, b_{t+1}(\hat{\theta}_t)) \quad \text{for } \hat{\theta}_t \neq \theta_t.$$

The only difference between (13)–(15) and (A.2)–(A.4) is that the latter takes into account the finite horizon, with $J_t^T(\theta_{t-1}, b_t)$ being welfare at t conditional on the end date T .

For $t < T$, define $\tilde{\theta}_t^T$ analogously to (4),

$$\tilde{\theta}_t^T = \sum_{k=1}^{T-t} \delta^k \mathbb{E}[\theta_{t+k} | \theta_t],$$

where it is clear that $\lim_{T \rightarrow \infty} \tilde{\theta}_t^T = \tilde{\theta}_t$ for $\tilde{\theta}_t$ defined in (4).

With some abuse of notation, define the finite horizon savings rate analogously to (5),

$$g_t(\theta_t) = (1 - s_t(\theta_t))(\tau((1+r)^{T-t+1} - 1)/(r(1+r)^{T-t} - b_t)),$$

where $s_T(\theta_T) = 0$. We now show that, given T , the program defined in (A.2)–(A.4) for any t is equivalent to

$$(A.5) \quad \max_{\{s_t(\theta_t)\}_{\theta_t \in \{\theta^L, \theta^H\}}} \sum_{\theta_t \in \{\theta^L, \theta^H\}} p(\theta_t | \theta_{t-1}) (\theta_t U(1 - s_t(\theta_t)) + \tilde{\theta}_t^T U(s_t(\theta_t)))$$

subject to

$$(A.6) \quad \theta_t U(1 - s_t(\theta_t)) + \beta \tilde{\theta}_t^T U(s_t(\theta_t)) \\ \geq \theta_t U(1 - s_t(\hat{\theta}_t)) + \beta \tilde{\theta}_t^T U(s_t(\hat{\theta}_t)) \quad \text{for } \hat{\theta}_t \neq \theta_t.$$

We show this by induction. First note that (A.2)–(A.4) is equivalent to (A.5)–(A.6) for $t = T - 1$. This follows from the fact that $J_T^T(\theta_{T-1}, b_T)$ satisfies

$$(A.7) \quad \delta J_T^T(\theta_{T-1}, b_T) = \tilde{\theta}_{T-1}^T U(s_{T-1}(\theta_{T-1})) + \chi_{T-1}^T(b_{T-1})$$

for some constant $\chi_{T-1}^T(b_{T-1})$ that depends only on b_{T-1} .⁵⁵ Substitution of (A.7) into the program (A.2)–(A.4) at $t = T - 1$ implies that (A.2)–(A.4) is equivalent to (A.5)–(A.6) at $t = T - 1$. Moreover, it also implies that

$$(A.8) \quad \delta J_{T-1}^T(\theta_{T-2}, b_{T-1}) = \tilde{\theta}_{T-2}^T U(s_{T-2}(\theta_{T-2})) + \chi_{T-2}^T(b_{T-2})$$

for some constant $\chi_{T-2}^T(b_{T-2})$ that depends only on b_{T-2} . Taking (A.8) into account, forward iteration of this reasoning implies that (A.2)–(A.4) is equivalent to (A.5)–(A.6) for all t . Taking $T \rightarrow \infty$, it is clear that (A.5)–(A.6) converges to (16)–(17), completing the argument.

APPENDIX B: EXTENSION TO CONTINUUM OF SHOCKS

In this appendix, we extend our analysis to a setting with a continuum of shocks. We consider this setting not only to explore the robustness of our main results, but also because economies with a continuum of shocks are the main focus of the mechanism design literature that studies the trade-off between commitment and flexibility.

The main complication that emerges in this extension is that under multiple shocks, one must ensure that not only local but also global incentive compatibility constraints are satisfied. This does not complicate the analysis of the sequential optimum, as under log utility that problem reduces to a two-period problem. However, this does complicate the analysis of the ex ante optimum, as the recursive method described in Section 5.2.1 no longer applies. Nonetheless, we show in this appendix that the main insights from the economy with two shocks continue to hold under a continuum of shocks.

B.1. Environment

Consider the benchmark environment described in Section 2 but with the government's type, $\theta_t > 0$, now being drawn from a continuous support

⁵⁵ Given this definition, note that the objective function in (A.5) is not exactly identical to (A.2) because we exclude the constant $\chi_t^T(b_t)$.

$\Theta \equiv [\underline{\theta}, \bar{\theta}]$. Let $p(\theta_t|\theta_{t-1})$ and $\tilde{\theta}_t$ be as previously defined, and assume that $p(\theta_t|\theta_{t-1})$ is strictly positive for all θ_t and θ_{t-1} and continuously differentiable with respect to θ_t and θ_{t-1} . Assumption 3 below, which holds in the two-shock economy, ensures that θ_t is mean reverting.

ASSUMPTION 3: $\theta_t/\tilde{\theta}_t$ is strictly increasing in θ_t .

We also make a technical assumption regarding the distribution of shocks. Define $\theta_p(\theta_{t-1}) = \max\{\underline{\theta}, \theta'\}$, where θ' is the lowest $\theta \in \Theta$ such that for all $\theta'' \geq \theta$,

$$(B.1) \quad \frac{\tilde{\theta}''}{\mathbb{E}[\tilde{\theta}_t|\theta_t \geq \theta'', \theta_{t-1}]} \frac{\mathbb{E}[\theta_t|\theta_t \geq \theta'', \theta_{t-1}]}{\theta''} \leq \frac{1}{\beta},$$

where $\tilde{\theta}''$ is the value of (4) associated with $\theta_t = \theta''$. Note that if shocks are i.i.d., $\theta_p(\theta_{t-1}) = \theta_p$, independent of θ_{t-1} . Given our assumptions, $\theta_p(\theta_{t-1})$ is a continuously differentiable function of θ_{t-1} . Using this definition, we make the following assumption.

ASSUMPTION 4: For all $\theta_t \leq \theta_p(\theta_{t-1})$,

$$(B.2) \quad \frac{d \log p(\theta_t|\theta_{t-1})}{d \log \theta_t} \geq -\frac{2 - \beta}{1 - \beta} + \frac{1}{1 - \beta} \frac{d \log \tilde{\theta}_t}{d \log \theta_t} + \frac{d \log(d\tilde{\theta}_t/d\theta_t)}{d \log \theta_t}.$$

Assumption 4 is isomorphic to Assumption A in Amador, Werning, and Angeletos’s (2006) study of an economy with i.i.d. shocks, where the main difference is that our condition incorporates the persistence of shocks through $d \log \tilde{\theta}_t/d \log \theta_t$ and $d \log(d\tilde{\theta}_t/d\theta_t)/d \log \theta_t$. This assumption is satisfied if the severity of the time-inconsistency problem is sufficiently low (i.e., β is sufficiently high), and the first and second derivatives of the density function $p(\theta_t|\theta_{t-1})$ with respect to both elements are bounded.⁵⁶

Definitions for the ex ante optimal and sequentially optimal fiscal rules analogous to those provided for the two-shock economy apply in this setting. We thus use the analysis of Section 3 to characterize the equilibrium.

B.2. *Sequentially Optimal Rule*

An analogous program to (16)–(17) defines the sequentially optimal fiscal rule. The incentive compatibility constraints (17) effectively imply that the problem is static and that global incentive constraints can be ignored. As such, analogous techniques to those used in the analysis of the two-period problem of Amador, Werning, and Angeletos (2006) apply here and can be used to

⁵⁶This follows from the fact that, given Assumption 3, $d \log \tilde{\theta}_t/d \log \theta_t < 1$.

characterize the sequential optimum. Let $s^f(\theta_t)$ be the flexible optimum of a government of type θ_t that is awarded full discretion as defined in (31).

PROPOSITION 7—Sequential Optimum Under Continuum of Shocks: *Let $\underline{s}(\theta_{t-1})$ be defined by $\underline{s}(\theta_{t-1}) = s^f(\theta_p(\theta_{t-1}))$ if $\theta_p(\theta_{t-1}) > \underline{\theta}$, and*

$$(B.3) \quad \mathbb{E}[\theta_t | \theta_{t-1}]U'(1 - \underline{s}(\theta_{t-1})) - \mathbb{E}[\tilde{\theta}_t | \theta_{t-1}]U'(\underline{s}(\theta_{t-1})) = 0$$

otherwise. For all θ^t , the sequential optimum features

$$s_t(\theta^t) = \max\{s^f(\theta_t), \underline{s}(\theta_{t-1})\}.$$

See the Supplemental Material for the proofs of Propositions 7–9.

COROLLARY 2: *The sequentially optimal rule at any date t can be implemented with a history-independent debt limit, $\bar{b}(\theta_{t-1}, b_t(\theta^{t-1}))$.*

This proposition and corollary state that if $\theta_p(\theta_{t-1}) > \underline{\theta}$, all types θ_t below $\theta_p(\theta_{t-1})$ are awarded full discretion, so they can choose their flexible optimal savings rate, and all types above $\theta_p(\theta_{t-1})$ are awarded no discretion, so they must choose the same savings rate as type $\theta_p(\theta_{t-1})$. If, instead, $\theta_p(\theta_{t-1}) = \underline{\theta}$, no type is given discretion and all types are assigned a savings rate $\underline{s}(\theta_{t-1})$ satisfying (B.3).

The dependence of the minimum savings rate $\underline{s}(\theta_{t-1})$ on θ_{t-1} captures the fact that the shock at date $t - 1$ provides information regarding the trade-off between commitment and flexibility at date t . Note that if shocks are i.i.d., $\underline{s}(\theta_{t-1})$ and the associated debt limit $\bar{b}(\theta_{t-1}, b_t(\theta^{t-1}))$ are independent of θ_{t-1} . Moreover, note that as β approaches 1, so that the time-inconsistency problem vanishes, $\theta_p(\theta_{t-1})$ approaches $\bar{\theta}$ and, thus, the sequentially optimal rule provides full discretion to all types.

As in the two-type case, the sequentially optimal rule does not provide dynamic incentives. It therefore takes the form of a set of allowable savings rates, from which the government chooses the one that is closest to its flexible optimum. To understand why very high types are given no discretion, note that because of the bounded distribution of shocks, allowing flexibility for these types has no ex ante welfare gain: such very high types would be overborrowing under *any* realized shock, so there is no trade-off between commitment and flexibility for them. $\theta_p(\theta_{t-1})$ can be interpreted as the type above which the value of flexibility is exceeded by the value of commitment.

To understand why all types below the cutoff $\theta_p(\theta_{t-1})$ are given full discretion, consider the alternative of having a mechanism that admits “holes,” namely, where some interior interval of savings rates is not allowed. Given such a hole, the types whose flexible optimum is inside the hole would choose savings rates at the boundaries of the hole. Specifically, those inside the hole

whose type is relatively high would reduce their savings by choosing the lower boundary—which is socially costly—and those inside the hole whose type is relatively low would increase their savings by choosing the upper boundary—which is socially beneficial. The resulting total change in welfare then depends on the slope of the density function; under Assumption 4, which effectively puts a lower bound on the elasticity of $p(\theta_t|\theta_{t-1})$ with respect to θ_t , the total welfare change of introducing a hole is always negative.

B.3. *Ex ante Optimal Rule*

We next consider the ex ante optimal fiscal rule. For this analysis, we assume that the mechanism at time t admits savings rates that are piecewise continuously differentiable with respect to the history θ^t .⁵⁷ As in the two-type case, we begin by considering the i.i.d. benchmark.

PROPOSITION 8—Ex ante Optimum Under Continuum of i.i.d. Shocks: *Suppose $p(\theta_t|\theta_{t-1})$ is independent of θ_{t-1} . Then the ex ante optimum coincides with the sequential optimum.*

The intuition for this result is analogous to that of the two-type case. Under i.i.d. shocks, any ex post suboptimality affects welfare on and off the equilibrium path equally, and, hence, cannot enhance efficiency. Consequently, dynamic incentives are not provided and the ex ante optimum coincides with the sequential optimum. It of course follows that when shocks are i.i.d., the ex ante optimum can be implemented with a renegotiated debt limit.

Consider now the case of persistent shocks, where $p(\theta_t|\theta_{t-1})$ depends on θ_{t-1} . We define persistence with the following condition.

CONDITION 1—Mechanism Relevance of Past Information: There is a positive measure of types θ_{t-1} and θ_t such that $\theta_p(\theta_{t-1}) > \underline{\theta}$, $s^f(\theta_t) > s^f(\theta_p(\theta_{t-1}))$, and $\underline{s}'(\theta_t) \neq 0$.

Condition 1 concerns the sequence of minimum savings policies implied by the sequential optimum described in Proposition 7. It states that in the sequentially optimal fiscal rule, there is a positive measure of types θ_{t-1} and θ_t with the property that, given θ_{t-1} , the government of type θ_t at date t has full discretion, so it spends above first-best level, and, moreover, such government has information that is locally relevant regarding the sequentially optimal mechanism at $t + 1$ (i.e., $\underline{s}'(\theta_t) \neq 0$). The analog of Condition 1 is trivially satisfied in a

⁵⁷This assumption is without loss in our setting in the case of i.i.d. shocks, and it is made in other settings in related work such as Athey, Atkeson, and Kehoe (2005). We use this assumption in proving the second part of Proposition 9.

two-type economy, as the low type always spends above first best in the sequential optimum, and this type also has information about the future sequentially optimal mechanism. In the setting with a continuum of types, Condition 1 is always satisfied if $\underline{s}'(\theta_t) \neq 0$ for some positive measure of types θ_t , so that current information is relevant to future mechanisms, and is always satisfied if β is sufficiently close to 1, so that such types θ_t with mechanism-relevant information have full discretion at t starting from some θ_{t-1} .

While an explicit characterization of the ex ante optimal rule becomes more complicated under a continuum of persistent shocks, we show that under Condition 1, this rule does not coincide with the sequentially optimal rule, and it exhibits history dependence.

PROPOSITION 9—Ex ante Optimum Under Continuum of Persistent Shocks: *Suppose Condition 1 is satisfied. The ex ante optimum has the following features:*

- (i) *It does not coincide with the sequential optimum.*
- (ii) *It exhibits history dependence: there exist θ^t and θ^k with $\theta_t = \theta_k$ and $\theta_{t-1} = \theta_{k-1}$ for which $s_t(\theta^t) \neq s_k(\theta^k)$.*

The intuition for Proposition 9 is familiar from the two-type economy. For the first part of the proposition, suppose by contradiction that the ex ante optimum coincided with the sequential optimum. We show that ex ante welfare can then be improved with a perturbation that induces ex post suboptimality. For concreteness, suppose that $\theta_p(\theta_{-1}) > \underline{\theta}$ for all θ_{-1} and $\theta'_p(\theta_{-1}) > 0$, so the sequentially optimal rule at date 0 becomes more relaxed the higher is the shock θ_{-1} . Consider perturbing the mechanism by assigning at date 1 the fiscal rule associated with a cutoff $\theta_p(\theta_0 + \mu(\theta_0))$ for some $\mu(\theta_0) > 0$ arbitrarily small. This relaxes incentive compatibility constraints at date 0, allowing one to increase the savings rates assigned to all types $\theta_0 < \theta_p(\theta_{-1})$, from $s^f(\theta_0)$ to $s^f(\theta_0) + \varepsilon(\theta_0)$, for some $\varepsilon(\theta_0) > 0$ that satisfies incentive compatibility. By envelope arguments analogous to those of Section 4, we can show that ex ante welfare increases as a result: the first-order gain of bringing savings closer to first best at date 0 outweighs the second-order loss of assigning suboptimal rules at date 1. Therefore, as in the two-type case, the ex ante optimal mechanism uses a threat of lack of fiscal discipline in the future to induce discipline today.

The logic for the second part of Proposition 9 is also analogous to that used for the two-type economy. History dependence arises because the mechanism provides dynamic incentives. That is, because the shock at date $t - 2$ predicts the realization of the shock at $t - 1$, the shock at $t - 2$ affects the relative tightness of incentive compatibility constraints at $t - 1$ and, therefore, it affects the policies that are chosen at t to provide dynamic incentives to the government at $t - 1$.

APPENDIX C: PROOFS FOR SECTION 5.1

C.1. *Proof of Proposition 1 and Corollary 1*

Consider the problem defined by (16)–(17). We first show that Assumption 2 implies that the solution to this problem admits pooling. We use the definition of the first-best benchmark given in (8) and we define the full flexibility benchmark as a sequence of savings rates satisfying $s_t^f = s^f(\theta^i)$ if $\theta_t = \theta^i$, where for $\theta^i \in \{\theta^L, \theta^H\}$,

$$(C.1) \quad \theta^i U'(1 - s^f(\theta^i)) = \beta \tilde{\theta}^i U'(s^f(\theta^i)).$$

Assumption 2 together with the concavity of $U(\cdot)$ implies

$$(C.2) \quad s^f(\theta^H) < s^f(\theta^L) < s^{\text{fb}}(\theta^H) < s^{\text{fb}}(\theta^L).$$

Hence, (17) must bind for some type in the solution to (16)–(17); otherwise, the solution would satisfy (8), but given (C.2) and the strict concavity of $U(\cdot)$, (17) would then be violated.

Note furthermore that the solution must admit $s_t(\theta^H) \leq s_t(\theta^L)$. This follows since pooling both types to the same savings rate must weakly lower welfare. Specifically, a perturbation that assigns the low type's savings rate to the high type must weakly lower welfare:

$$(C.3) \quad \frac{\theta^H}{\tilde{\theta}^H} (U(1 - s_t(\theta^H)) - U(1 - s_t(\theta^L))) + (U(s_t(\theta^H)) - U(s_t(\theta^L))) \geq 0.$$

Analogously, a perturbation that assigns the high type's savings rate to the low type must weakly lower welfare:

$$(C.4) \quad \frac{\theta^L}{\tilde{\theta}^L} (U(1 - s_t(\theta^L)) - U(1 - s_t(\theta^H))) + (U(s_t(\theta^L)) - U(s_t(\theta^H))) \geq 0.$$

Adding (C.3) and (C.4) gives

$$\left(\frac{\theta^H}{\tilde{\theta}^H} - \frac{\theta^L}{\tilde{\theta}^L} \right) (U(1 - s_t(\theta^H)) - U(1 - s_t(\theta^L))) \geq 0,$$

which requires that $s_t(\theta^H) \leq s_t(\theta^L)$.

Finally, note that it must be that (17) is an equality for $\theta_t = \theta^L$. Suppose instead that it were a strict inequality. Then given the above argument, (17) must be an equality for $\theta_t = \theta^H$. Substitution of (17) into the left hand side of (C.3) then implies

$$(C.5) \quad (1 - \beta) [U(s_t(\theta^H)) - U(s_t(\theta^L))] \geq 0,$$

which implies $s_t(\theta^H) \geq s_t(\theta^L)$. Given that $s_t(\theta^H) \leq s_t(\theta^L)$, this means that $s_t(\theta^H) = s_t(\theta^L)$, and thus (17) is an equality for $\theta_t = \theta^L$.

To establish that there is pooling, suppose that there is separation so that (17) is ignored for the high type and consider the first-order conditions to (16)–(17). Let ϕ correspond to the Lagrange multiplier on (17) for the low type. First-order conditions yield

$$(C.6) \quad \frac{1 + \frac{\phi}{p(\theta^L|\theta_{t-1})}}{1 + \beta \frac{\phi}{p(\theta^L|\theta_{t-1})}} = \frac{\tilde{\theta}^L U'(s_t(\theta^L))}{\theta^L U'(1 - s_t(\theta^L))},$$

$$(C.7) \quad \frac{1 - \frac{\theta^L}{\theta^H} \frac{\phi}{p(\theta^H|\theta_{t-1})}}{1 - \beta \frac{\tilde{\theta}^L}{\tilde{\theta}^H} \frac{\phi}{p(\theta^H|\theta_{t-1})}} = \frac{\tilde{\theta}^H U'(s_t(\theta^H))}{\theta^H U'(1 - s_t(\theta^H))}.$$

Equation (C.6) implies $s_t(\theta^L) < s^{fb}(\theta^L)$. Moreover, note that Assumption 2 implies

$$(C.8) \quad \frac{\theta^L}{\tilde{\theta}^L} > \beta \frac{\theta^H}{\tilde{\theta}^H},$$

and thus (C.7) implies $s_t(\theta^H) > s^{fb}(\theta^H)$. Therefore, from (C.2), this means that $s^f(\theta^L) < s_t(\theta^H) < s_t(\theta^L)$, but this violates (17) since the low type can make itself strictly better off by claiming to be a high type. Therefore, (17) must hold with equality for both types and the mechanism features pooling.

Since the equilibrium features pooling, it follows that the optimal pooling level necessarily satisfies first-order condition (21) where the pooling level depends only on θ_{t-1} . Since the solution to (21) admits a savings rate between $s^{fb}(\theta^H)$ and $s^{fb}(\theta^L)$, and since this savings rate is then above both $s^f(\theta^H)$ and $s^f(\theta^L)$, it follows that the equilibrium can be implemented with a required minimum savings rate that solves (21), which both types would choose. From (1), it is clear that requiring such a minimum savings rate is equivalent to imposing a maximum debt limit, which proves the corollary.

APPENDIX D: PROOFS FOR SECTION 5.2

D.1. Proof of Lemma 1

To prove this result, we begin by considering a relaxed program that effectively lets (1) be a weak inequality so as to allow for money burning:

$$(D.1) \quad Q(\theta^i, Z) = \max_{\{u^L, y^L, u^H, y^H, Z^L, Z^H\}} \{p(\theta^L|\theta^i)(\theta^L u^L + \tilde{\theta}^L y^L + \delta Q(\theta^L, Z^L)) + p(\theta^H|\theta^i)(\theta^H u^H + \tilde{\theta}^H y^H + \delta Q(\theta^H, Z^H))\}$$

subject to

$$(D.2) \quad U^{-1}(u^L) + U^{-1}(y^L) \leq 1,$$

$$(D.3) \quad U^{-1}(u^H) + U^{-1}(y^H) \leq 1,$$

$$(D.4) \quad Z = \{p(\theta^L|\theta^{-i})(\theta^L u^L + \tilde{\theta}^L y^L + \delta Q(\theta^L, Z^L)) \\ + p(\theta^H|\theta^{-i})(\theta^H u^H + \tilde{\theta}^H y^H + \delta Q(\theta^H, Z^H))\},$$

$$(D.5) \quad \theta^L u^L + \beta \tilde{\theta}^L y^L + \beta \delta Q(\theta^L, Z^L) \geq \theta^L u^H + \beta \tilde{\theta}^L y^H + \beta \delta Z^H,$$

$$(D.6) \quad \theta^H u^H + \beta \tilde{\theta}^H y^H + \beta \delta Q(\theta^H, Z^H) \geq \theta^H u^L + \beta \tilde{\theta}^H y^L + \beta \delta Z^L,$$

$$(D.7) \quad \underline{Z}^L \leq Z^L \leq \overline{Z}^L, \quad \text{and} \quad \underline{Z}^H \leq Z^H \leq \overline{Z}^H.$$

Equations (D.1)–(D.7) are identical to (23)–(27) if (D.2) and (D.3) hold with equality, in which case $u^i = U(1 - s^i)$ and $y^i = U(s^i)$. We will eventually establish that in the solution, (D.2) and (D.3) hold with equality, and thus (D.1)–(D.7) is identical to (23)–(27). Note that in this program, \underline{Z}^i and \overline{Z}^i are, respectively, the lowest and highest values of Z given θ^i for which a solution to the program exists. We establish some preliminary lemmas regarding the solution to the relaxed program.

LEMMA 5: $Q(\theta^i, Z)$ is weakly concave in $Z \in [\underline{Z}^i, \overline{Z}^i]$.

PROOF: Consider the sequence problem defined by (D.1)–(D.7), and let

$$\gamma^*(\theta^i, Z) = \left\{ \left\{ u(\theta^t), y(\theta^t) \right\}_{\theta^t \in \theta^i} \right\}_{t=0}^{\infty}$$

correspond to the stochastic sequence of u 's and y 's defined by forward iteration that solves (D.1)–(D.7) for some given $\theta_{-1} = \theta^i$ and $Z_0 = Z$. Suppose that $Z' \in [\underline{Z}^i, \overline{Z}^i]$ and $Z'' \in [\underline{Z}^i, \overline{Z}^i]$ with $Z'' > Z'$, and consider the solution to the sequence problem for $Z = \kappa Z' + (1 - \kappa)Z'' \equiv Z^\kappa$. Define a potential solution

$$\gamma^\kappa = \kappa \gamma^*(\theta^i, Z') + (1 - \kappa) \gamma^*(\theta^i, Z'').$$

γ^κ is a convex combination of the stochastic u and y sequences under Z' and Z'' . Because the set of u and y sequences that satisfy the sequence analog of the constraint set in (D.2)–(D.7) is convex, it follows that γ^κ satisfies the constraints of the problem for $Z = Z^\kappa$. Therefore, the value of $Q(\theta^i, Z^\kappa)$ must be weakly greater than the welfare under γ^κ ; that is,

$$(D.8) \quad Q(\theta^i, Z^\kappa) \geq \kappa Q(\theta^i, Z') + (1 - \kappa) Q(\theta^i, Z'').$$

It follows from (D.8) that $Q(\theta^i, Z)$ is weakly concave.

Q.E.D.

Define $Z_{\max}^i = \arg \max_{Z \in [\underline{Z}^i, \bar{Z}^i]} Q(\theta^i, Z)$. We now use Lemma 5 to characterize Z_{\max}^i and $Q(\theta^i, Z_{\max}^i)$.

LEMMA 6: *The solution to (D.1)–(D.7) for $Z = Z_{\max}^i$ has the following properties:*

- (i) *Equation (D.5) holds with equality and (D.6) holds as a strict inequality.*
- (ii) *Equations (D.2) and (D.3) both hold with equality.*

PROOF: *Part (i).* We establish this result in three steps. We take into account that the solution to (D.1)–(D.7) for $Z^i = Z_{\max}^i$ is equivalent to the solution to (D.1)–(D.7) ignoring (D.5).

Step 1. Either (D.5) or (D.6) holds with equality. Suppose by contradiction that this is not the case. Then (D.5) and (D.6) can be ignored, and the solution admits the first-best allocation defined by (8) and $Z^i = Z_{\max}^i$ for $i = \{L, H\}$, as this maximizes welfare. This implies that the infinite repetition of the first-best allocation is incentive compatible, which means that, in fact, $Q(\theta^i, Z^i) = Z^{-i}$ for $i = \{L, H\}$. But the same arguments as in the proof of Proposition 1 then imply that (D.5) is violated, leading to a contradiction.

Step 2. Suppose that in the solution, (D.6) holds with equality. Then there is bunching, with $u^L = u^H$ and $y^H = y^L$, and (D.2) and (D.3) both hold with equality. To see why, note first that the solution admits $u^H \geq u^L$ and $y^L \geq y^H$. This follows from the fact that a perturbation that changes the high type’s allocation to the low type’s (so that there is bunching) and sets $Z^i = Z_{\max}^i$ for $i = \{L, H\}$ must weakly lower welfare:

$$(D.9) \quad \theta^H (u^H - u^L) + \tilde{\theta}^H (y^H - y^L) \geq 0.$$

It can be verified that this perturbation satisfies (D.2)–(D.3) and (D.5)–(D.7). Analogously, a perturbation that changes the low type’s allocation to the high type’s must weakly lower welfare:

$$(D.10) \quad \theta^L (u^L - u^H) + \tilde{\theta}^L (y^L - y^H) \geq 0.$$

Adding (D.9) and (D.10) gives

$$\left(\frac{\theta^H}{\tilde{\theta}^H} - \frac{\theta^L}{\tilde{\theta}^L} \right) (u^H - u^L) \geq 0 \quad \text{and} \quad \left(\frac{\tilde{\theta}^L}{\theta^L} - \frac{\tilde{\theta}^H}{\theta^H} \right) (y^L - y^H) \geq 0,$$

which requires $u^H \geq u^L$ and $y^L \geq y^H$.

Next, note that a perturbation that changes the high type’s allocation and continuation allocation to be the same as the low type’s must also weakly lower welfare:

$$\theta^H (u^H - u^L) + \tilde{\theta}^H (y^H - y^L) + \delta(Q(\theta^H, Z^H) - Z^L) \geq 0.$$

Substitution of (D.6) holding with equality into the above condition gives

$$\left(1 - \frac{1}{\beta}\right)\theta^H(u^H - u^L) \geq 0,$$

which means that $u^H \leq u^L$. Combined with the result above, this implies $u^H = u^L$.

Finally, note that conditional on $u^H = u^L$, setting $y^H = y^L$, so that (D.2) and (D.3) hold with equality and $Z^i = Z_{\max}^i$ for $i = \{L, H\}$, yields the highest feasible welfare. Moreover, such a solution satisfies (D.5) and (D.6), so it is incentive compatible. Therefore, there is bunching and the resource constraints hold with equality.

Step 3. A solution where (D.6) holds with equality is suboptimal. From Step 2, such a solution admits bunching, implying that $Q(\theta^H, Z_{\max}^H) = Z_{\max}^L$. By construction, it must also be that $Q(\theta^L, Z_{\max}^L) = Z_{\max}^H$. Thus, the solution admits the same repeated level of bunching that is independent of the previous period's shock. However, this level of bunching yields a welfare strictly below that achieved in the sequential optimum described in Proposition 1, where the level of bunching depends on the previous period's shock. Therefore, (D.6) holds as a strict inequality in the solution and by Step 1, (D.5) holds with equality.

Part (ii). By the previous steps, the solution is such that (D.5) holds as an equality and (D.6) holds as an inequality. We now show that (D.2) and (D.3) hold with equality. Suppose first that (D.2) is a strict inequality. Consider a perturbation that decreases u^L and increases y^L while holding $\theta^L u^L + \beta \tilde{\theta}^L y^L$ constant. This perturbation clearly increases welfare. Moreover, this perturbation leaves (D.5) unaffected and for a small enough perturbation, (D.6) continues to hold. Therefore, this perturbation is incentive compatible and strictly increases welfare. Suppose next that (D.3) is an inequality. Consider a perturbation that decreases u^H and increases y^H while holding $\theta^L u^H + \beta \tilde{\theta}^L y^H$ constant. It follows from Assumption 2, which implies (C.8), that this perturbation increases welfare. Moreover, this perturbation leaves (D.5) unaffected and for a small enough perturbation, (D.6) continues to hold. Therefore, this perturbation is incentive compatible and strictly increases welfare.

Finally, we show that in the solution, $u^L < u^H$ and $y^L > y^H$. From Step 2 in the proof of part (i), $u^L \leq u^H$ and $y^L \geq y^H$. Since (D.2) and (D.3) hold with equality, if it is the case that $u^L = u^H$, then it must be that $y^L = y^H$ and vice versa, so that there would be bunching. However, by Step 3 of the proof of part (i), a solution that admits bunching is suboptimal. It thus follows that $u^L < u^H$ and $y^L > y^H$. Q.E.D.

We now use the fact that Z_{\max}^i characterizes the global optimum of $Q(\theta^i, Z)$ to characterize $Q(\theta^i, Z)$.

LEMMA 7: $Q(\theta^i, Z)$ has the following properties:

- (i) $\exists Z_{\min}^i \in [Z^i, Z_{\max}^i)$ subject to $Q(\theta^i, Z)$ is continuously differentiable in Z if $Z \in (Z_{\min}^i, Z_{\max}^i]$.
- (ii) If $Z \in (Z_{\min}^i, Z_{\max}^i)$, $Q(\theta^i, Z)$ is strictly concave in Z .

PROOF: We establish these results in four steps.

Step 1. We first establish that $Q(\theta^i, Z)$ is continuously differentiable in Z at $Z = Z_{\max}^i$ with a derivative equal to zero. Consider the solution to (D.1)–(D.7) for $Z = Z_{\max}^i$. Let s^L and s^H correspond to the associated savings rates in the solution, where by the proof of part (ii) of Lemma 6, $s^H < s^L$. Define $s_\varepsilon^L = s^L + \varepsilon$ and s_ε^H as the solution to

$$(D.11) \quad \theta^L(U(1 - s^L) - U(1 - s^H)) + \beta\tilde{\theta}^L(U(s^L) - U(s^H)) \\ = \theta^L(U(1 - s_\varepsilon^L) - U(1 - s_\varepsilon^H)) + \beta\tilde{\theta}^L(U(s_\varepsilon^L) - U(s_\varepsilon^H)).$$

Let

$$(D.12) \quad Z_\varepsilon = Z_{\max}^i + \{p(\theta^L|\theta^{-i}) \\ \times (\theta^L(U(1 - s_\varepsilon^L) - U(1 - s^L)) + \tilde{\theta}^L(U(s_\varepsilon^L) - U(s^L))) \\ + p(\theta^H|\theta^{-i}) \\ \times (\theta^H(U(1 - s_\varepsilon^H) - U(1 - s^H)) + \tilde{\theta}^H(U(s_\varepsilon^H) - U(s^H)))\}$$

and

$$(D.13) \quad Q_\varepsilon = Q(\theta^i, Z_{\max}^i) + \{p(\theta^L|\theta^i) \\ \times (\theta^L(U(1 - s_\varepsilon^L) - U(1 - s^L)) + \tilde{\theta}^L(U(s_\varepsilon^L) - U(s^L))) \\ + p(\theta^H|\theta^i) \\ \times (\theta^H(U(1 - s_\varepsilon^H) - U(1 - s^H)) + \tilde{\theta}^H(U(s_\varepsilon^H) - U(s^H)))\}.$$

From the definition of Z_{\max}^i , it must be that the derivative of Q_ε with respect to ε at $\varepsilon = 0$ is zero, as the objective attains a local maximum, where we have taken into account that the perturbation continues to satisfy (D.2)–(D.3) and (D.5)–(D.7). Note that it cannot be that the derivative of Z_ε with respect to ε at $\varepsilon = 0$ is also zero. Given that $p(\theta^L|\theta^{-i}) \neq p(\theta^L|\theta^i)$ and $p(\theta^H|\theta^{-i}) \neq p(\theta^H|\theta^i)$, this could be true if s^L and s^H are chosen at the first-best level, but this is also ruled out in Step 1 in the proof of part (i) of Lemma 6. Therefore, it cannot also be that Z_ε is at its local maximum. It thus follows that one can use this perturbation to apply Lemma 1 of Benveniste and Scheinkman (1979) to show that $Q(\theta^i, Z)$ is continuously differentiable in Z at $Z = Z_{\max}^i$ and has a derivative of zero.

Step 2. Define Z_{\min}^i as follows: If

$$\lim_{\varepsilon>0, \varepsilon \rightarrow 0} \frac{Q(\theta^i, \underline{Z}^i + \varepsilon) - Q(\theta^i, \underline{Z}^i)}{\varepsilon} < \frac{1 - p(\theta^i|\theta^i)}{p(\theta^i|\theta^i)},$$

then $Z_{\min}^i = \underline{Z}^i$; otherwise, Z_{\min}^i is the highest point in $[\underline{Z}^i, \bar{Z}^i]$ such that

$$\begin{aligned} \text{(D.14)} \quad & \lim_{\varepsilon>0, \varepsilon \rightarrow 0} \frac{Q(\theta^i, Z_{\min}^i + \varepsilon) - Q(\theta^i, Z_{\min}^i)}{\varepsilon} \\ & \leq \frac{1 - p(\theta^i|\theta^i)}{p(\theta^i|\theta^i)} \leq \lim_{\varepsilon>0, \varepsilon \rightarrow 0} \frac{Q(\theta^i, Z_{\min}^i) - Q(\theta^i, Z_{\min}^i - \varepsilon)}{\varepsilon}, \end{aligned}$$

where such a point necessarily exists given Step 1 and the concavity of $Q(\cdot)$. Note further that (D.1)–(D.7) can be represented by

$$\begin{aligned} \text{(D.15)} \quad & \max_{\{u^L, y^L, u^H, y^H, Z^L, Z^H\}} \{ (p(\theta^L|\theta^i) + \lambda p(\theta^L|\theta^{-i})) \\ & \times (\theta^L u^L + \tilde{\theta}^L y^L + \delta Q(\theta^L, Z^L)) \\ & + (p(\theta^H|\theta^i) + \lambda p(\theta^H|\theta^{-i})) (\theta^H u^H + \tilde{\theta}^H y^H + \delta Q(\theta^H, Z^H)) \} \\ & \text{subject to (D.2), (D.3), (D.5), (D.6), and (D.7).} \end{aligned}$$

Program (D.15) corresponds to (D.1)–(D.7), where λ is the Lagrange multiplier on the threat-keeping constraint (D.5). By the envelope condition,

$$\begin{aligned} & \lim_{\varepsilon>0, \varepsilon \rightarrow 0} \frac{Q(\theta^i, Z + \varepsilon) - Q(\theta^i, Z)}{\varepsilon} \\ & \leq -\lambda \leq \lim_{\varepsilon>0, \varepsilon \rightarrow 0} \frac{Q(\theta^i, Z) - Q(\theta^i, Z - \varepsilon)}{\varepsilon}. \end{aligned}$$

It thus follows from concavity and the definition of Z_{\min}^i above that if $Z \in (Z_{\min}^i, Z_{\max}^i]$, then

$$\text{(D.16)} \quad p(\theta^L|\theta^i) + \lambda p(\theta^L|\theta^{-i}) > 0 \quad \text{and} \quad p(\theta^H|\theta^i) + \lambda p(\theta^H|\theta^{-i}) > 0,$$

so that the objective in (D.15) puts positive weight on the realization of both states.

Step 3. Analogous arguments to those used in the proof of Lemma 6 imply that given (D.16), the solution to (D.15) has the following properties: (D.5) holds with equality and (D.6) as a strict inequality; (D.2) and (D.3) hold with equality; the solution admits $s^L > s^H$. Given these observations, a perturbation as the one used in Step 1 can be used here, and the same arguments from Step 1 imply that $Q(\theta^i, Z)$ is continuously differentiable in Z .

Step 4. We now show that $Q(\theta^i, Z)$ is strictly concave in this range. Consider the argument of concavity in the proof of Lemma 5. The potential solution yields welfare $\kappa Q(\theta^i, Z') + (1 - \kappa)Q(\theta^i, Z'')$, while leaving the resource constraints (D.2) and (D.3) as strict inequalities in the initial date. Given the arguments in Step 3, welfare can be strictly increased by letting (D.2) and (D.3) hold as equalities, implying that (D.8) must be a strict inequality. Thus, $Q(\theta^i, Z)$ is strictly concave. *Q.E.D.*

We now characterize the solution to the program for $Z \in (Z_{\min}^i, Z_{\max}^i)$, where we denote by $Z^{L*}(\theta^i, Z)$ and $Z^{H*}(\theta^i, Z)$ the optimal values of Z^L and Z^H , respectively, given θ^i and Z .

LEMMA 8: *If $Z \in (Z_{\min}^i, Z_{\max}^i]$, the solution to (D.1)–(D.7) has the following properties:*

- (i) *Equations (D.2) and (D.3) hold with equality.*
- (ii) $Z^{L*}(\theta^i, Z) = Z_{\max}^L$ for $\theta^i \in \{\theta^L, \theta^H\}$.
- (iii) $Z^{H*}(\theta^i, Z) < Z_{\max}^H$ for $\theta^i \in \{\theta^L, \theta^H\}$.
- (iv) $Z^{H*}(\theta^H, Z)$ is strictly decreasing in Z .
- (v) $Z^{H*}(\theta^H, Z) > Z^{H*}(\theta^L, Z_{\max}^L) > Z_{\min}^H$.

PROOF: *Part (i).* This follows from Step 3 in the proof of Lemma 7.

Part (ii). Consider the problem as represented in (D.15). Suppose it were the case that $Z^{L*}(\theta^i, Z) < Z_{\max}^L$. Then a perturbation that moves Z^L in the direction of Z_{\max}^L strictly increases welfare by increasing $Q(\theta^L, Z^L)$. Furthermore, it relaxes (D.5), and since (D.6) is a strict inequality in the solution to the program (from Step 3 in the proof of Lemma 7), (D.6) continues to hold.

Part (iii). Consider the solution to (D.15) given that (D.2) and (D.3) hold with equality. Let s^L and s^H correspond to the savings rates in the low and high shocks, respectively. Let ϕ be the Lagrange multiplier on (D.5). First-order conditions with respect to s^L and s^H yield

$$(D.17) \quad \frac{U'(1 - s^L)}{U'(s^L)} = \left(\frac{\tilde{\theta}^L \frac{1 + \beta \frac{\phi}{p(\theta^L|\theta^i) + \lambda p(\theta^L|\theta^{-i})}}{\theta^L}}{1 + \frac{\phi}{p(\theta^L|\theta^i) + \lambda p(\theta^L|\theta^{-i})}} \right),$$

$$(D.18) \quad \frac{U'(1 - s^H)}{U'(s^H)} = \left(\frac{\tilde{\theta}^H - \beta \tilde{\theta}^L \frac{\phi}{p(\theta^H|\theta^i) + \lambda p(\theta^H|\theta^{-i})}}{\theta^H - \theta^L \frac{\phi}{p(\theta^H|\theta^i) + \lambda p(\theta^H|\theta^{-i})}} \right).$$

Note that $\phi > 0$; otherwise, the same arguments as those used in Step 1 in the proof of part (i) of Lemma 6 would imply that the repeated first-best allocation is incentive compatible, leading to a contradiction. Finally, first-order

conditions with respect to Z^H yield

$$\begin{aligned}
 \text{(D.19)} \quad & \lim_{\varepsilon > 0, \varepsilon \rightarrow 0} \frac{Q(\theta^H, Z^H) - Q(\theta^H, Z^H - \varepsilon)}{\varepsilon} \\
 & \geq \beta \frac{\phi}{p(\theta^H | \theta^i) + \lambda p(\theta^H | \theta^{-i})} \\
 & \geq \lim_{\varepsilon > 0, \varepsilon \rightarrow 0} \frac{Q(\theta^H, Z^H + \varepsilon) - Q(\theta^H, Z^H)}{\varepsilon},
 \end{aligned}$$

where we have taken into account that Z^H may be below Z_{\min}^H , in which case $Q(\theta^H, Z^H)$ may not necessarily be differentiable. Given the definition of Z_{\max}^H and the fact that $Q_Z(\theta^H, Z_{\max}^H) = 0$, it follows from (D.19) that $Z^{H*}(\theta^i, Z) < Z_{\max}^H$, since $\phi > 0$.

Part (iv). Suppose that $\theta^i = \theta^H$, and consider $Z' \in (Z_{\min}^H, Z_{\max}^H]$ and $Z'' \in (Z_{\min}^H, Z_{\max}^H]$ with $Z'' > Z'$. Given part (ii) of Lemma 7, it follows that Z' is associated with multiplier λ' in (D.15) and Z'' is associated with multiplier λ'' in (D.15), where $\lambda' < \lambda'' < 0$. We will establish that $Z^{H*}(\theta^H, Z'') < Z^{H*}(\theta^H, Z')$. Suppose instead that $Z^{H*}(\theta^H, Z'') \geq Z^{H*}(\theta^H, Z')$. Let ϕ' and ϕ'' correspond to the Lagrange multipliers on (D.5) for the program for $Z = Z'$ and $Z = Z''$, respectively. From (D.19) together with the concavity of $Q(\cdot)$, it must be that

$$\text{(D.20)} \quad \frac{\phi'}{p(\theta^H | \theta^H) + \lambda' p(\theta^H | \theta^L)} \geq \frac{\phi''}{p(\theta^H | \theta^H) + \lambda'' p(\theta^H | \theta^L)}.$$

Substituting into (D.18), taking into account (C.8), this implies that the solution $s^{H*}(\cdot)$ must satisfy

$$\text{(D.21)} \quad s^{\text{fb}}(\theta^H) < s^{H*}(\theta^H, Z'') \leq s^{H*}(\theta^H, Z'),$$

where $s^{\text{fb}}(\theta^H)$ is defined in (8). Moreover, since $p(\theta^H | \theta^H) / p(\theta^H | \theta^L) > p(\theta^L | \theta^H) / p(\theta^L | \theta^L)$, it follows that

$$\text{(D.22)} \quad \frac{\phi'}{p(\theta^L | \theta^H) + \lambda' p(\theta^L | \theta^L)} > \frac{\phi''}{p(\theta^L | \theta^H) + \lambda'' p(\theta^L | \theta^L)}.$$

Substituting into (D.17) implies that the solution $s^{L*}(\cdot)$ must satisfy

$$\text{(D.23)} \quad s^{L*}(\theta^H, Z') < s^{L*}(\theta^H, Z'') < s^{\text{fb}}(\theta^L),$$

where $s^{\text{fb}}(\theta^L)$ is defined in (8). From Step 3 of the proof of Lemma 7,

$$\text{(D.24)} \quad s^{H*}(\theta^H, Z') < s^{L*}(\theta^H, Z').$$

Note that $\theta^L U(1 - s) + \beta \tilde{\theta}^L U(s)$ is strictly decreasing in s for $s \geq s^{fb}(\theta^H) > s^i(\theta^L)$, where we have appealed to (C.2). Therefore, conditions (D.21), (D.23), and (D.24) combined together imply that

$$\begin{aligned} & \theta^L (U(1 - s^{L*}(\theta^H, Z'')) - U(1 - s^{H*}(\theta^H, Z''))) \\ & \quad + \beta \tilde{\theta}^L (U(s^{L*}(\theta^H, Z'')) - U(s^{H*}(\theta^H, Z''))) \\ & < \theta^L (U(1 - s^{L*}(\theta^H, Z')) - U(1 - s^{H*}(\theta^H, Z'))) \\ & \quad + \beta \tilde{\theta}^L (U(s^{L*}(\theta^H, Z')) - U(s^{H*}(\theta^H, Z'))). \end{aligned}$$

Given that (D.5) holds with equality and $Z^{L*}(\theta^i, Z) = Z_{max}^L$ from part (ii), this implies

$$\beta \delta(Z^{H*}(\theta^H, Z'') - Q(\theta^L, Z_{max}^L)) < \beta \delta(Z^{H*}(\theta^H, Z') - Q(\theta^L, Z_{max}^L)),$$

which contradicts the fact that $Z^{H*}(\theta^H, Z'') \geq Z^{H*}(\theta^H, Z')$.

Part (v). First note that we can show $Z^{H*}(\theta^H, Z_{max}^H) > Z^{H*}(\theta^L, Z_{max}^L)$ using analogous arguments to those used in the proof of part (iv), taking into account that the associated value of λ is 0 for $Z = Z_{max}^i$, $i = \{L, H\}$. Then using the fact that $Z^{H*}(\theta^H, Z)$ is decreasing in Z from part (iv), it follows that $Z^{H*}(\theta^H, Z) > Z^{H*}(\theta^L, Z_{max}^L)$ for any $Z \in [Z_{min}^H, Z_{max}^H]$. We are then left to show that $Z^{H*}(\theta^L, Z_{max}^L) > Z_{min}^H$. Suppose instead that $Z^{H*}(\theta^L, Z_{max}^L) \leq Z_{min}^H$. By the definition of Z_{min}^H in Lemma 7, using (D.19) and the fact that $\lambda = 0$ for $Z = Z_{max}^i$, it follows that

$$(D.25) \quad \phi \geq \frac{1}{\beta} \frac{(1 - p(\theta^i|\theta^i))^2}{p(\theta^i|\theta^i)}.$$

Note that by the arguments in the proof of part (iv), $s^{H*}(\theta^L, Z_{max}^L) < s^{fb}(\theta^L)$. From (D.18), taking into account that $\lambda = 0$, this means that

$$(D.26) \quad \frac{\theta^H}{\theta^L} - \frac{\tilde{\theta}^H}{\tilde{\theta}^L} > \phi \frac{(1 - \beta)}{1 - p(\theta^i|\theta^i)}.$$

Combining (D.25) with (D.26) implies that

$$(D.27) \quad \frac{\theta^H}{\theta^L} - \frac{\tilde{\theta}^H}{\tilde{\theta}^L} > \frac{1 - p(\theta^i|\theta^i)}{p(\theta^i|\theta^i)} \left(\frac{1}{\beta} - 1 \right).$$

But this contradicts Assumption 2, completing the proof. Q.E.D.

By Lemma 8, the relaxed program in (D.1)–(D.7) is equivalent to the original program in (23)–(27) if we define $\underline{V}^i = Z_{min}^i$ and $\bar{V}^i = Z_{max}^i$. This is because (D.2) and (D.3) both hold with equality, and for $Z \in (Z_{min}^i, Z_{max}^i]$, $Z^{L*}(\theta^i, Z) \in$

$(Z_{\min}^L, Z_{\max}^L]$ and $Z^{H*}(\theta^i, Z) \in (Z_{\min}^H, Z_{\max}^H]$, so that \underline{V}^i and \bar{V}^i effectively correspond to the minimum and maximum values of V that would ever be reached given that the equilibrium begins with the ex ante optimum with $Z_0 = Z_{\max}^i$. It follows then from Lemma 7 that $W(\theta^i, V)$ is strictly increasing in V , strictly concave, and continuously differentiable in V over the range $(\underline{V}^i, \bar{V}^i)$.

D.2. Proof of Proposition 2

If shocks are i.i.d., $p(\theta^L|\theta^i) = p(\theta^H|\theta^i) = 0.5$ and $W(\theta^i, V) = V$. Moreover, $\bar{V}^L = \bar{V}^H = \bar{V}$ and $\underline{V}^L = \underline{V}^H = \underline{V}$ since the value and solution to (23)–(27) is independent of θ^i . Now consider the program starting from $V = \bar{V}$, which is the solution to the ex ante optimum. Let us denote the solution as in (29).

Analogous arguments to those used in the proof of Lemma 6 imply that in the solution, either (25) and (26) both hold as equalities with $s^{L*}(\theta^i, \bar{V}) = s^{H*}(\theta^i, \bar{V})$, so that there is bunching, or, alternatively, (25) holds as an equality and (26) holds as a strict inequality. We consider the latter case first and rule it out. Note that the same arguments as those used in the proof of Proposition 1 imply that if there is separation, the solution admits

$$(D.28) \quad s^{fb}(\theta^H) < s^{H*}(\theta^i, \bar{V}) < s^{L*}(\theta^i, \bar{V}) < s^{fb}(\theta^L).$$

This means that if the solution admits $V^{L*}(\theta^i, \bar{V}) = V^{H*}(\theta^i, \bar{V}) = \bar{V}$, then from (C.2), (25) would be violated. Therefore, for (25) to hold as an equality and (26) to hold as a strict inequality, it is necessary that $V^{L*}(\theta^i, \bar{V}) < \bar{V}$ or $V^{H*}(\theta^i, \bar{V}) < \bar{V}$. It is clear that setting $V^{L*}(\theta^i, \bar{V}) = \bar{V}$ maximizes social welfare while fully relaxing (25), so the only possibility to consider is $V^{H*}(\theta^i, \bar{V}) < \bar{V}$. Consider an increase in V^H and an increase in s^H that leaves the following term unchanged:

$$\theta^L U(1 - s^H) + \beta \tilde{\theta}^L U(s^H) + \beta \delta V^H.$$

Such a perturbation leaves (25) unchanged, and since (26) was satisfied with strict inequality, (26) continues to hold for a small enough perturbation. One can show that given (C.8) and (D.28), the change in welfare from an arbitrarily small perturbation takes the same sign as $\theta^L/\beta - \theta^H$, which is positive, so that the perturbation strictly increases welfare.

This implies that the equilibrium admits bunching with $s^{L*}(\theta^i, \bar{V}) = s^{H*}(\theta^i, \bar{V})$. Conditional on bunching, the optimal mechanism assigns $V^{L*}(\theta^i, \bar{V}) = \bar{V}$ and $V^{H*}(\theta^i, \bar{V}) = \bar{V}$ since (25) and (26) are trivially satisfied without the use of dynamic incentives. Therefore, the ex ante optimum corresponds to the sequential optimum and Proposition 1 applies.

D.3. Proof of Lemma 2

Parts (i) and (ii) follow directly from Lemma 8 in the solution to the relaxed problem. We are left then to prove part (iii). Let ϕ correspond to the Lagrange multiplier on (25). First-order conditions with respect to s^L , s^H , and V^H , respectively, yield

$$(D.29) \quad \frac{U'(1 - s^L)}{U'(s^L)} = \left(\frac{\tilde{\theta}^L \left(1 + \beta \frac{\phi}{p(\theta^L|\theta^i) + \lambda p(\theta^L|\theta^{-i})} \right)}{\theta^L \left(1 + \frac{\phi}{p(\theta^L|\theta^i) + \lambda p(\theta^L|\theta^{-i})} \right)} \right),$$

$$(D.30) \quad \frac{U'(1 - s^H)}{U'(s^H)} = \left(\frac{\tilde{\theta}^H - \beta \tilde{\theta}^L \frac{\phi}{p(\theta^H|\theta^i) + \lambda p(\theta^H|\theta^{-i})}}{\theta^H - \theta^L \frac{\phi}{p(\theta^H|\theta^i) + \lambda p(\theta^H|\theta^{-i})}} \right),$$

$$(D.31) \quad W_V(\theta^i, V^H) = \beta \frac{\phi}{p(\theta^H|\theta^i) + \lambda p(\theta^H|\theta^{-i})}.$$

Consider first the case with $\theta^i = \theta^H$. Consider $V'' > V'$, so that from the envelope condition and the strict concavity of $W(\theta^i, V)$, the associated multipliers satisfy $\lambda' < \lambda'' < 0$. Since $V^{H*}(\theta^H, V)$ is strictly decreasing in V , this means from (D.31) that

$$\frac{\phi''}{p(\theta^H|\theta^i) + \lambda'' p(\theta^H|\theta^{-i})} > \frac{\phi'}{p(\theta^H|\theta^i) + \lambda' p(\theta^H|\theta^{-i})}.$$

Combining this inequality with (D.30) implies that $s^{H*}(\theta^i, V'') > s^{H*}(\theta^i, V')$. From parts (i) and (ii), we know that $V^{L*}(\theta^i, V'') = V^{L*}(\theta^i, V')$ and $V^{H*}(\theta^i, V'') < V^{H*}(\theta^i, V')$. Since (25) binds, this implies that

$$\begin{aligned} & \theta^L (U(1 - s^{L*}(\theta^i, V'')) - U(1 - s^{L*}(\theta^i, V'))) \\ & \quad + \beta \tilde{\theta}^L (U(s^{L*}(\theta^i, V'')) - U(s^{L*}(\theta^i, V'))) \\ & = \theta^L (U(1 - s^{H*}(\theta^i, V'')) - U(1 - s^{H*}(\theta^i, V'))) \\ & \quad + \beta \tilde{\theta}^L (U(s^{H*}(\theta^i, V'')) - U(s^{H*}(\theta^i, V'))) \\ & \quad + \beta (V^{H*}(\theta^i, V'') - V^{H*}(\theta^i, V')) \\ & < 0. \end{aligned}$$

For this inequality to hold, it must be that $s^{L*}(\theta^i, V'') > s^{L*}(\theta^i, V')$, where we have used the arguments in the proof of part (iv) of Lemma 8 that require

$$s^{fb}(\theta^L) > s^{L*}(\theta^i, V) > s^{H*}(\theta^i, V) > s^{fb}(\theta^H) > s^f(\theta^L).$$

Finally, note that we can show $s^{i*}(\theta^H, \overline{V}^H) < s^{i*}(\theta^L, \overline{V}^L)$ using analogous arguments to those above, taking into account that the associated value of λ is 0 for $V = \overline{V}^i$ for $i = \{L, H\}$. Then using the fact that $s^{i*}(\theta^H, V)$ is increasing in V , we obtain $s^{i*}(\theta^H, V) < s^{i*}(\theta^L, \overline{V}^L)$ for all $V \in [\underline{V}^H, \overline{V}^H]$.

D.4. Proof of Proposition 3

Part (i). From part (i) of Lemma 2, if $\eta_t(\theta^{t-1}) = 0$, then $s_t(\theta^{t-1}, \theta^i) = s^{i*}(\theta^L, \overline{V}^L)$ for $i = \{L, H\}$. Moreover, if $\eta_t(\theta^{t-1}) = 1$, then $s_t(\theta^{t-1}, \theta^i) = s^{i*}(\theta^H, V^{H*}(\theta^L, \overline{V}^L))$ for $i = \{L, H\}$. Forward iteration implies that if $\eta_t(\theta^{t-1}) = k$ for $k > 1$, then

$$s_t(\theta^{t-1}, \theta^i) = s^{i*}(\theta^H, V^{H*^{k-1}}(\theta^H, V^{H*}(\theta^L, \overline{V}^L))),$$

where, with some abuse of notation, $V^{H*^{k-1}}(\cdot)$ corresponds to $k - 1$ iterations of the operator $V^{H*}(\theta^H, \cdot)$. Therefore, $\eta_t(\theta^{t-1})$ determines $s_t(\theta^i)$ conditional on θ_t .

Part (ii). Suppose that $\theta_{t-1} = \theta_{k-1} = \theta^H$ and that $\eta_t(\theta^{t-1}) = 1$ while $\eta_k(\theta^{k-1}) = 2$. From part (ii) of Lemma 2, it must be that $V_t(\theta^{t-1}) < V_k(\theta^{k-1})$. From part (iii) of Lemma 2, this implies that $s_t(\theta^i) < s_k(\theta^k)$ if $\theta_t = \theta_k$.

D.5. Proof of Proposition 4

From part (ii) of Lemma 2, $V^{H*}(\theta^H, V)$ is strictly decreasing in V . We can establish that there exists a unique \widehat{V} with the property that $V^{H*}(\theta^H, \widehat{V}) = \widehat{V}$. Note that $\overline{V}^H > V^{H*}(\theta^H, \overline{V}^H)$ and it can be shown that $\underline{V}^H < V^{H*}(\theta^H, \underline{V}^H)$. To see why, consider the solution to the program given $V = \underline{V}^H$, taking into account that this corresponds to the solution to (28) as λ approaches $-(1 - p(\theta^i|\theta^i))/p(\theta^i|\theta^i)$. The value of the objective for $\lambda = -(1 - p(\theta^i|\theta^i))/p(\theta^i|\theta^i)$ is weakly exceeded by

$$\theta^H U(1 - s^{fb}(\theta^H)) + \tilde{\theta}^H U(s^{fb}(\theta^H)) + \delta p(\theta^H|\theta^H)W(\theta^H, \overline{V}^H),$$

which is the unconstrained value of the objective function uniquely attained under $s^{H*}(\theta^H, \underline{V}^H) = s^{fb}(\theta^H)$ and $V^{H*}(\theta^H, \underline{V}^H) = \overline{V}^H$. Note that a solution that satisfies the constraints of the problem and achieves this unconstrained maximum exists. For instance, a potential candidate solution assigns $s^{H*}(\theta^H, \underline{V}^H) = s^{L*}(\theta^H, \underline{V}^H) = s^{fb}(\theta^H)$ and $V^{H*}(\theta^H, \underline{V}^H) = V^{L*}(\theta^H, \underline{V}^H) = \overline{V}^H$, and satisfies all the constraints of the problem. Therefore, $V^{H*}(\theta^H, \underline{V}^H) = \overline{V}^H > \underline{V}^H$. Given

that $V^{H^*}(\theta^H, V)$ monotonically declines in V , there thus exists a unique \widehat{V} with the property that $V^{H^*}(\theta^L, \widehat{V}) = \widehat{V}$.

Given that $V^{H^*}(\theta^H, V)$ is monotonically declining in V , it follows that if $V < (>) \widehat{V}$, then $V^{H^*}(\theta^H, V) > (<) \widehat{V}$. Therefore, for $\theta_{t-1} = \theta_t = \theta^H$, if $V_t(\theta^{t-1}) < (>) \widehat{V}$, then $V_{t+1}(\theta^t) = V^{H^*}(\theta^H, V_t(\theta^{t-1})) > (<) \widehat{V}$.

APPENDIX E: PROOFS FOR SECTION 5.3

E.1. Proof of Lemma 3

From (1) and (5), given (30), we have

$$(E.1) \quad (\tau/(1 - \delta) - b_{t+1}) = (\tau/(1 - \delta) - b_t)(s_t/\delta).$$

Taking the log of both sides of (E.1), taking the sum over t between 0 and T , and dividing by T yields

$$\frac{\log(\tau/(1 - \delta) - b_{T+1})}{T} = \frac{\log(\tau/(1 - \delta) - b_0)}{T} + \frac{1}{T} \sum_{t=0}^T \log(s_t/\delta),$$

so that

$$(E.2) \quad \lim_{T \rightarrow \infty} \frac{\log(\tau/(1 - \delta) - b_{T+1})}{T} = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^T \log(s_t/\delta).$$

If the long-run sequence of savings rates is ergodic, the Birkhoff theorem implies that the right hand side of (8) equals \bar{s} .⁵⁸ It therefore follows that if $\bar{s} > 0$, we obtain from (E.2) that $\lim_{T \rightarrow \infty} \log(\tau/(1 - \delta) - b_{T+1}) = \infty$, which implies $\lim_{T \rightarrow \infty} b_{T+1} = -\infty$. In contrast, if $\bar{s} < 0$, then analogous reasoning implies that $\lim_{T \rightarrow \infty} \log(\tau/(1 - \delta) - b_{T+1}) = -\infty$ and, thus, $\lim_{T \rightarrow \infty} b_{T+1} = \tau/(1 - \delta)$.

E.2. Proof of Lemma 4

Part (i). Consider the value of \bar{s}^{fb} , taking into account that the symmetry of $p(\theta^i|\theta^j)$ implies that θ^L and θ^H must occur with equal probability in the long run:

$$(E.3) \quad \bar{s}^{fb} = \left[\frac{1}{2} \log\left(\frac{1}{\delta} \frac{\tilde{\theta}^H}{\theta^H + \tilde{\theta}^H}\right) + \frac{1}{2} \log\left(\frac{1}{\delta} \frac{\tilde{\theta}^L}{\theta^L + \tilde{\theta}^L}\right) \right].$$

⁵⁸See Durrett (2004, p. 337).

Note that

$$\begin{aligned}
 \text{(E.4)} \quad & \frac{1}{2} \log\left(\frac{1}{\delta} \frac{\tilde{\theta}^H}{\theta^H + \tilde{\theta}^H}\right) + \frac{1}{2} \log\left(\frac{1}{\delta} \frac{\tilde{\theta}^L}{\theta^L + \tilde{\theta}^L}\right) \\
 &= -\left[\frac{1}{2} \log\left(\delta \frac{\theta^H + \tilde{\theta}^H}{\tilde{\theta}^H}\right) + \frac{1}{2} \log\left(\delta \frac{\theta^L + \tilde{\theta}^L}{\tilde{\theta}^L}\right)\right] \\
 &> -\log\left(\frac{1}{2} \delta \frac{\theta^H + \tilde{\theta}^H}{\tilde{\theta}^H} + \frac{1}{2} \delta \frac{\theta^L + \tilde{\theta}^L}{\tilde{\theta}^L}\right) \\
 &\geq -\log\left(\delta \frac{\theta^H + \tilde{\theta}^H + \theta^L + \tilde{\theta}^L}{\tilde{\theta}^H + \tilde{\theta}^L}\right) = 0,
 \end{aligned}$$

where the second inequality follows from Jensen’s inequality and the third inequality follows from algebraic manipulation, taking into account that $\theta^H > \theta^L$. Combined with Lemma 3, $\bar{s}^{\text{fb}} > 0$ implies that assets diverge to infinity in the long run.

Part (ii). Consider the value of \bar{s}^f , taking into account that the symmetry of $p(\theta^i|\theta^i)$ implies that θ^L and θ^H must occur with equal probability in the long run:

$$\text{(E.5)} \quad \bar{s}^f = \left[\frac{1}{2} \log\left(\frac{1}{\delta} \frac{\beta \tilde{\theta}^H}{\theta^H + \beta \tilde{\theta}^H}\right) + \frac{1}{2} \log\left(\frac{1}{\delta} \frac{\beta \tilde{\theta}^L}{\theta^L + \beta \tilde{\theta}^L}\right)\right].$$

Note that

$$\text{(E.6)} \quad \frac{1}{\delta} \frac{\beta \tilde{\theta}^H}{\theta^H + \beta \tilde{\theta}^H} \leq \frac{1}{\delta} \frac{\beta \tilde{\theta}^L}{\theta^L + \beta \tilde{\theta}^L} < \frac{1}{\delta} \frac{\tilde{\theta}^H}{\theta^H + \tilde{\theta}^H} < 1,$$

where we have used (C.8) and the fact that $\theta^H > \theta^L$. Combined with Lemma 3, $\bar{s}^f < 0$ implies that debt becomes maximal in the long run.

E.3. Proof of Proposition 5

Part (i). If shocks are i.i.d., then (21) implies that $s_t = \delta$ for all t under the sequential optimum. This implies that $\bar{s}^{\text{so}} = 0$. Moreover, from (E.1), $s_t = \delta$ for all t implies that the level of debt is constant for all t .

Part (ii). Consider the value of \bar{s}^{so} under persistent shocks, taking into account that the symmetry of $p(\theta^i|\theta^i)$ implies that θ^L and θ^H must occur with equal probability in the long run, and taking into account that the savings rates satisfy (21):

$$\begin{aligned}
 \text{(E.7)} \quad \bar{s}^{\text{so}} = & \left[\frac{1}{2} \log\left(\frac{1}{\delta} \frac{p(\theta^i|\theta^i)\tilde{\theta}^H + (1 - p(\theta^i|\theta^i))\tilde{\theta}^L}{p(\theta^i|\theta^i)(\theta^H + \tilde{\theta}^H) + (1 - p(\theta^i|\theta^i))(\theta^L + \tilde{\theta}^L)}\right)\right. \\
 & \left. + \frac{1}{2} \log\left(\frac{1}{\delta} \frac{(1 - p(\theta^i|\theta^i))\tilde{\theta}^H + p(\theta^i|\theta^i)\tilde{\theta}^L}{(1 - p(\theta^i|\theta^i))(\theta^H + \tilde{\theta}^H) + p(\theta^i|\theta^i)(\theta^L + \tilde{\theta}^L)}\right)\right].
 \end{aligned}$$

Note that by Jensen’s inequality, the right hand side of (E.7) is strictly larger than

$$\begin{aligned}
 & -\log\left(\frac{1}{2}\left(\frac{1}{\delta}\frac{p(\theta^i|\theta^i)(\theta^H + \tilde{\theta}^H) + (1 - p(\theta^i|\theta^i))(\theta^L + \tilde{\theta}^L)}{p(\theta^i|\theta^i)\tilde{\theta}^H + (1 - p(\theta^i|\theta^i))\tilde{\theta}^L}\right)\right. \\
 & \left. + \frac{1}{2}\left(\frac{1}{\delta}\frac{(1 - p(\theta^i|\theta^i))(\theta^H + \tilde{\theta}^H) + p(\theta^i|\theta^i)(\theta^L + \tilde{\theta}^L)}{(1 - p(\theta^i|\theta^i))\tilde{\theta}^H + p(\theta^i|\theta^i)\tilde{\theta}^L}\right)\right),
 \end{aligned}$$

which by algebraic manipulation, taking into account that $\theta^H > \theta^L$, can be shown to be weakly larger than

$$-\log\left(\delta\frac{\theta^H + \tilde{\theta}^H + \theta^L + \tilde{\theta}^L}{\tilde{\theta}^H + \tilde{\theta}^L}\right) = 0.$$

Combined with Lemma 3, $\bar{s}^{\text{co}} > 0$ implies that assets diverge to infinity in the long run.

E.4. Proof of Proposition 6

Part (i). This follows directly from Proposition 2 and Proposition 5.

Part (ii). Consider \bar{s}^{co} under persistent shocks in the ex ante optimum. Recall that the savings rate at any date t is a function only of the current shock θ_t and the number of periods since the last low shock, η_t . Moreover, note that given Lemma 2 and Proposition 3, in the long-run ergodic distribution,

$$\begin{aligned}
 \Pr[\eta_t = 0] &= \Pr[\eta_{t-1} = 0]p(\theta^i|\theta^i) + \sum_{k=1}^{\infty} \Pr[\eta_{t-1} = k](1 - p(\theta^i|\theta^i)), \\
 \Pr[\eta_t = 1] &= \Pr[\eta_{t-1} = 0](1 - p(\theta^i|\theta^i)), \\
 \Pr[\eta_t = k] &= \Pr[\eta_{t-1} = k - 1]p(\theta^i|\theta^i) \quad \text{for } k > 1.
 \end{aligned}$$

Since $\Pr[\eta_t = k] = \Pr[\eta_{t-1} = k]$ for all $k \geq 0$ and since $\sum_{k=0}^{\infty} \Pr[\eta_t = k] = 1$, it follows by substituting above that $\Pr[\eta_t = 0] = 1/2$ and $\Pr[\eta_{t-1} = k] = (1 - p(\theta^i|\theta^i))p(\theta^i|\theta^i)^{k-1}/2$ for $k > 0$. Moreover, conditional on $\eta_t = 0$, the probability of $\theta_t = \theta^L$ is $p(\theta^i|\theta^i)$, and conditional on $\eta_t > 0$, the probability of $\theta_t = \theta^H$ is $p(\theta^i|\theta^i)$. This establishes that \bar{s}^{co} in (32) corresponds to the mean of $\log(s_t/\delta)$ in the invariant distribution of savings rates in the ex ante optimum.

Finally, to show that both $\bar{s}^{\text{co}} > 0$ and $\bar{s}^{\text{co}} < 0$ may arise for different parameter values, consider the values for $\{\theta^L, \theta^H, p(\theta^i|\theta^i), \delta, \beta\}$ used in Figure 3 (see footnote 49). It is easy to verify using computational methods that if $\beta = 0.01$, then $\bar{s}^{\text{co}} > 0$; by continuity this is also true for a neighborhood of parameter vectors. In this case, Lemma 3 implies that assets diverge to infinity in the long

run. Similarly, one can verify that if $\beta = 0.4$, then $\bar{s}^{\text{co}} < 0$, and again by continuity, this is true for a neighborhood of parameters. Here Lemma 3 implies that debt becomes maximal in the long run. Last, note that in all these cases, parameter values satisfy Assumption 2.

REFERENCES

- ACEMOGLU, D., M. GOLOSOV, AND A. TSYVINSKI (2008): "Political Economy of Mechanisms," *Econometrica*, 76, 619–641. [1561]
- AGUIAR, M., AND M. AMADOR (2011): "Growth Under the Shadow of Expropriation," *Quarterly Journal of Economics*, 126, 651–697. [1558,1563]
- ALES, L., P. MAZIERO, AND P. YARED (2014): "A Theory of Political and Economic Cycles," *Journal of Economic Theory*, 153, 224–251. [1563]
- ALESINA, A., AND R. PEROTTI (1994): "The Political Economy of Budget Deficits," Working Paper 4637, NBER. [1558]
- ALESINA, A., AND G. TABELLINI (1990): "A Positive Theory of Fiscal Deficits and Government Debt," *Review of Economic Studies*, 57, 403–414. [1558]
- ALONSO, R., AND N. MATOUSCHEK (2008): "Optimal Delegation," *Review of Economic Studies*, 75, 259–293. [1561]
- AMADOR, M., AND K. BAGWELL (2013): "The Theory of Delegation With an Application to Tariff Caps," *Econometrica*, 81 (4), 1541–1599. [1558]
- AMADOR, M., I. WERNING, AND G.-M. ANGELETOS (2003): "Commitment vs. Flexibility," Working Paper, NBER. [1562,1573]
- (2006): "Commitment vs. Flexibility," *Econometrica*, 74, 365–396. [1558,1559,1562,1578,1592]
- AMBRUS, A., AND G. EGOROV (2013a): "Delegation and Nonmonetary Incentives," Working Paper. [1561]
- (2013b): "A Comment on Commitment and Flexibility," *Econometrica*, 81, 2113–2124. [1558]
- ATHEY, S., A. ATKESON, AND P. J. KEHOE (2005): "The Optimal Degree of Discretion in Monetary Policy," *Econometrica*, 73, 1431–1475. [1558,1559,1589,1594]
- ATKESON, A., AND R. LUCAS (1992): "On Efficient Distribution With Private Information," *Review of Economic Studies*, 59 (3), 427–453. [1562,1585]
- AZZIMONTI, M. (2011): "Barriers to Investment in Polarized Societies," *American Economic Review*, 101, 2182–2204. [1561]
- AZZIMONTI, M., M. BATTAGLINI, AND S. COATE (2010): "Analyzing the Case for a Balanced Budget Amendment to the U.S. Constitution," Working Paper. [1561]
- BARRO, R. (1990): "On the Predictability of Tax-Rate Changes," in *Macroeconomic Policy*. Cambridge: Harvard University Press, 268–297. [1558]
- (1999): "Ramsey Meets Laibson in the Neoclassical Growth Model," *Quarterly Journal of Economics*, 114, 1125–1152. [1561,1564]
- BATTAGLINI, M. (2005): "Long-Term Contracting With Markovian Consumers," *American Economic Review*, 95 (3), 637–658. [1580]
- BATTAGLINI, M., AND S. COATE (2008): "A Dynamic Theory of Public Spending, Taxation, and Debt," *American Economic Review*, 98, 201–236. [1558]
- BATTAGLINI, M., AND R. LAMBA (2012): "Optimal Dynamic Contracting," Working Paper. [1558]
- BENVENISTE, L. M., AND J. A. SCHEINKMAN (1979): "On the Differentiability of the Value Function in Dynamic Models of Economics," *Econometrica*, 47, 727–732. [1601]
- BERNHEIM, B. D., D. RAY, AND S. YELTEKIN (2013): "Poverty and Self-Control," Working Paper. [1561,1583]
- BISIN, A., A. LIZZERI, AND L. YARIV (2013): "Government Policy With Time Inconsistent Voters," Working Paper. [1561]

- BOND, P., AND G. SIGURDSSON (2011): "Commitment Contracts," Working Paper. [1561]
- CABALLERO, R., AND P. YARED (2010): "Future Rent-Seeking and Current Public Savings," *Journal of International Economics*, 82, 124–136. [1558]
- CHAMBERLAIN, G., AND C. A. WILSON (2000): "Optimal Intertemporal Consumption Under Uncertainty," *Review of Economic Dynamics*, 3 (3), 365–395. [1584]
- DURRETT, R. (2004): *Probability: Theory and Examples* (Third Ed.). Andover: Cengage Learning. [1585,1609]
- FARHI, E., AND I. WERNING (2007): "Inequality and Social Discounting," *Journal of Political Economy*, 115 (3), 365–402. [1562]
- (2013): "Insurance and Taxation Over the Life Cycle," *Review of Economic Studies*, 80, 596–635. [1558]
- FARRELL, J., AND E. MASKIN (1989): "Renegotiation in Repeated Games," *Games and Economic Behavior*, 1, 327–360. [1567]
- FERNANDES, A., AND C. PHELAN (2000): "A Recursive Formulation for Repeated Agency With History Dependence," *Journal of Economic Theory*, 91, 223–247. [1560,1575]
- GOLOSOV, M., AND A. TSYVINSKI (2006): "Designing Optimal Disability Insurance: A Case for Asset Testing," *Journal of Political Economy*, 114, 257–279. [1561]
- GOLOSOV, M., M. TROSHKIN, AND A. TSYVINSKI (2013): "Redistribution and Social Insurance," Working Paper. [1558]
- HALAC, M. (2012): "Relational Contracts and the Value of Relationships," *American Economic Review*, 102, 750–779. [1561]
- HALAC, M., AND P. YARED (2014): "Supplement to 'Fiscal Rules and Discretion Under Persistent Shocks'," *Econometrica Supplemental Material*, 82, http://www.econometricsociety.org/ecta/supmat/11207_proofs.pdf. [1561]
- HOLMSTRÖM, B. (1977): "On Incentives and Control in Organizations," Ph.D. Thesis, Stanford University. [1561]
- (1984): "On the Theory of Delegation," in *Bayesian Models in Economic Theory*, ed. by M. Boyer and R. Kihlstrom. New York: North-Holland. [1561]
- INTERNATIONAL MONETARY FUND (2009): "Fiscal Rules—Anchoring Expectations for Sustainable Public Finances," Working Paper, IMF. [1557,1582]
- JACKSON, M. O., AND L. YARIV (2012a): "Collective Dynamic Choice: The Necessity of Time Inconsistency," Working Paper. [1558,1563]
- (2012b): "Present Bias and Collective Dynamic Choice in the Lab," *American Economic Review* (forthcoming). [1558,1563]
- KAPICKA, M. (2013): "Efficient Allocations in Dynamic Private Information Economies With Persistent Shocks: A First Order Approach," *Review of Economic Studies*, 80, 1027–1054. [1561]
- KOCHERLAKOTA, N. (1996): "Reconsideration-Proofness: A Refinement for Infinite Horizon Time Inconsistency," *Games and Economic Behavior*, 15, 33–54. [1567]
- KRUSELL, P., AND J.-V. RIOS-RULL (1999): "On the Size of U.S. Government: Political Economy in the Neoclassical Growth Model," *American Economic Review*, 89, 1156–1181. [1561]
- KRUSELL, P. AND A. SMITH, JR. (2003): "Consumption-Savings Decisions With Quasi-Geometric Discounting," *Econometrica*, 71, 365–375. [1561]
- KRUSELL, P., B. KRUSCU, AND A. SMITH, JR. (2010): "Temptation and Taxation," *Econometrica*, 78, 2063–2084. [1561]
- LAIBSON, D. (1997): "Golden Eggs and Hyperbolic Discounting," *Quarterly Journal of Economics*, 112, 443–478. [1558,1561]
- LIZZERI, A. (1999): "Budget Deficits and Redistributive Politics," *American Economic Review*, 66, 909–928. [1558]
- LIZZERI, A., AND L. YARIV (2013): "Collective Self-Control," Working Paper. [1561]
- LUCAS, R. (1987): *Models of Business Cycles*. New York: Basil Blackwell. [1587]
- PAVAN, A., I. SEGAL, AND J. TOIKKA (2014): "Dynamic Mechanism Design: A Myersonian Approach," *Econometrica*, 82, 601–653. [1558]

- PERSSON, T., AND L. SVENSSON (1989): "Why a Stubborn Conservative Would Run a Deficit: Policy With Time-Inconsistent Preferences," *Quarterly Journal of Economics*, 104, 325–345. [1558]
- PHELPS, E., AND R. POLLAK (1968): "On Second Best National Savings and Game-Equilibrium Growth," *Review of Economic Studies*, 35, 185–199. [1561]
- SLEET, C. (2004): "Optimal Taxation With Private Government Information," *Review of Economic Studies*, 71, 1217–1239. [1561,1563]
- SLEET, C., AND S. YELTEKIN (2006): "Credibility and Endogenous Societal Discounting," *Review of Economic Dynamics*, 9 (3), 410–437. [1562]
- (2008): "Politically Credible Social Insurance," *Journal of Monetary Economics*, 55 (1), 129–151. [1562]
- SONG, Z., K. STORESLETTEN, AND F. ZILIBOTTI (2012): "Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt," *Econometrica*, 80 (6), 2785–2803. [1561]
- STRULOVICI, B. (2011): "Contracts, Information Persistence, and Renegotiation," Working Paper. [1561,1567]
- THOMAS, J., AND T. WORRALL (1990): "Income Fluctuation and Asymmetric Information: An Example of a Repeated Principal–Agent Problem," *Journal of Economic Theory*, 51 (2), 367–390. [1562,1585]
- TORNELL, A., AND P. LANE (1999): "The Voracity Effect," *American Economic Review*, 89, 22–46. [1558]
- WILLIAMS, N. (2011): "Persistent Private Information," *Econometrica*, 79, 1233–1275. [1561]
- YARED, P. (2010): "Politicians, Taxes and Debt," *Review of Economic Studies*, 77, 806–840. [1561]

Columbia Business School, 3022 Broadway, Uris Hall 616, New York, NY 10027, U.S.A. and Dept. of Economics, University of Warwick; mhalac@columbia.edu

and

Columbia Business School, 3022 Broadway, Uris Hall 823, New York, NY 10027, U.S.A. and NBER; pyared@columbia.edu.

Manuscript received November, 2012; final revision received April, 2014.