SUPPLEMENT TO “FISCAL RULES AND DISCRETION UNDER PERSISTENT SHOCKS”  
(Econometrica, Vol. 82, No. 5, September 2014, 1557–1614)  

BY MARINA HALAC AND PIERRE YARED  

THIS SUPPLEMENTAL MATERIAL PROVIDES proofs for our results in the extension to a continuum of types in Appendix B and describes the algorithm that we use to compute the equilibria in Section 5.3 of the paper.  

S.A. PROOFS FOR APPENDIX B  

S.A.1. Proofs of Proposition 7 and Corollary 2  

To prove these results, consider the program isomorphic to (16)–(17) under a continuum of shocks. Define a function \( f(\theta_t) = \theta_t / \tilde{\theta}_t \) for \( \tilde{\theta}_t \) that depends on \( \theta_t \) as defined in (4). Let \( \omega_t = f(\theta_t) \in \Omega \equiv [\omega, \overline{\omega}] \), where it is clear that from Assumption 3, there is a one to one mapping from \( \theta_t \) to \( \omega_t \). Let \( h(\omega_t | \omega_{t-1}) \) correspond to the value of \( \tilde{\theta}_t p(\theta_t | \theta_{t-1}) (f'(\theta_t))^{-1} / \mathbb{E}[\tilde{\theta}_t p(\theta_t | \theta_{t-1}) | \theta_{t-1}] \), so that \( h(\omega_t | \omega_{t-1}) > 0 \),  

\[
h(\omega_t | \omega_{t-1}) d\omega_t = \tilde{\theta}_t p(\theta_t | \theta_{t-1}) / \mathbb{E}[\tilde{\theta}_t p(\theta_t | \theta_{t-1}) | \theta_{t-1}] d\theta_t,
\]

and \( \int_{\omega}^{\overline{\omega}} h(\omega_t | \omega_{t-1}) d\omega_t = 1 \), where we have used the fact that \( d\omega_t = f'(\theta_t) d\theta_t \). Therefore, \( h(\omega_t | \omega_{t-1}) \) is effectively a density function. Define \( H(\omega_t | \omega_{t-1}) \) as the associated cumulative distribution function (c.d.f.).  

Using this formulation, (16)–(17) can be rewritten as  

\[
\begin{align*}
\text{(S.A.1)} & \quad \max_{s_t(\omega_t), \omega_t \in \Omega} \int_{\omega}^{\overline{\omega}} h(\omega_t | \omega_{t-1}) \left( \omega_t U(1 - s_t(\omega_t)) + U(s_t(\omega_t)) \right) d\omega_t \\
\text{(S.A.2)} & \quad \omega_t U(1 - s_t(\omega_t)) + \beta U(s_t(\omega_t)) \\
& \geq \omega_t U(1 - s_t(\omega_t)) + \beta U(s_t(\omega_t)) \quad \forall \omega_t, \text{ and } \forall \omega_t \neq \omega_t.
\end{align*}
\]

Equations (S.A.1)–(S.A.2) are identical to (16)–(17), where we have used the one to one mapping from \( \theta_t \) to \( \omega_t \) to write the program as one of choosing a savings rate conditional on the report \( \omega_t \).  

Now consider a relaxed version of (S.A.1)–(S.A.2) that allows (1) to be an inequality:  

\[
\begin{align*}
\text{(S.A.3)} & \quad \max_{u_t(\omega_t), y_t(\omega_t)} \int_{\omega}^{\overline{\omega}} h(\omega_t | \omega_{t-1}) \left( \omega_t u_t(\omega_t) + y_t(\omega_t) \right) d\omega_t 
\end{align*}
\]

© 2014 The Econometric Society  

DOI: 10.3982/ECTA11207
subject to

(S.A.4) \[ U^{-1}(u_i(\omega_t)) + U^{-1}(y_i(\omega_t)) \leq 1 \quad \forall \omega_t, \]

(S.A.5) \[ \omega_t u_i(\omega_t) + \beta y_i(\omega_t) \geq \omega_t u_i(\omega_t) + \beta y_i(\omega_t) \quad \forall \omega_t \text{ and } \forall \omega_t \neq \omega_i. \]

Equations (S.A.3)–(S.A.5) are identical to (S.A.1)–(S.A.2) if the solution admits (S.A.4) holding with equality \( \forall \omega_t \). Equations (S.A.3)–(S.A.5) correspond to the problem analyzed in Section 3.2 of Amador, Werning, and Angeletos (2006) so that their analysis applies here as well.

The envelope condition that characterizes (S.A.5) implies that

(S.A.6) \[ \frac{\omega_t}{\beta} u(\omega_t) + y(\omega) = \int_{\omega}^{\omega_t} \frac{1}{\beta} u(\omega') d\omega' + \frac{\omega}{\beta} u(\omega) + y(\omega). \]

Standard arguments also require \( u(\omega_t) \) to be a nondecreasing function of \( \omega_t \). Thus, (S.A.6) and monotonicity are necessary for incentive compatibility. Substituting (S.A.6) into the objective function and the resource constraint, and integrating by parts allows us to rewrite the problem as

(S.A.7) \[ \max_{(u_i(\omega_t), y(\omega))} \left\{ \frac{\omega}{\beta} u(\omega) + y(\omega) + \frac{1}{\beta} \int_{\omega}^{\omega_t} (1 - G(\omega_t|\omega_{t-1}) u(\omega_t) d\omega_t \right\} \]

subject to

(S.A.8) \[ U(1 - U^{-1}(u_i(\omega_t))) + \frac{\omega_t}{\beta} u(\omega_t) = \frac{\omega}{\beta} u(\omega) - y(\omega) \]

\[ - \frac{1}{\beta} \int_{\omega}^{\omega_t} u(\omega') d\omega' \geq 0 \quad \text{and} \]

(S.A.9) \[ u_i(\omega_t) \text{ is nondecreasing} \]

for

\[ G(\omega_t|\omega_{t-1}) = H(\omega_t|\omega_{t-1}) + \omega_t(1 - \beta) h(\omega_t|\omega_{t-1}). \]

The above program can be solved using Lagrangian methods. Following Amador, Werning, and Angeletos (2006), define \( \omega_p(\omega_{t-1}) = \max\{\omega, \omega'\} \), where \( \omega' \) is the lowest \( \omega \in \Omega \) such that \( \forall \omega'' \geq \omega \),

(S.A.10) \[ \int_{\omega''}^{\omega'} (1 - G(\omega''|\omega_{t-1})) d\omega'' \leq 0. \]

Consider the following condition.

CONDITION 2: \( \forall \omega_{t-1} \text{ and } \forall \omega_t \leq \omega_p(\omega_{t-1}), G(\omega_t|\omega_{t-1}) \text{ is nondecreasing in } \omega_t. \)
Proposition 2 in Amador, Werning, and Angeletos (2006) states that the solution to (S.A.7)–(S.A.9) admits 

\[ s_t(\omega_t) = s_t(\omega_p(\omega_t - 1)) \]

if \( \omega_t \geq \omega_p(\omega_t - 1) \). Moreover, Proposition 3 in that paper states that if Condition 2 holds, then the solution to (S.A.7)–(S.A.9) admits (S.A.8) holding with equality, so that (S.A.4) also holds with equality. Furthermore, \( s_t(\omega_t) = s_t(\omega_t) \) if \( \omega_t < \omega_p(\omega_t - 1) \), where, with some abuse of notation, \( s_t(\omega_t) \) is defined as the flexible optimum given by

\[ \omega_t U'(1 - s_t(\omega_t)) = \beta U'(s_t(\omega_t)). \]

Let \( s_t(\omega_{t-1}) \) be defined by \( s_t(\omega_{t-1}) = s_t(\omega_p(\omega_t - 1)) \) if \( \omega_p(\omega_t - 1) > \omega \) and by

\[ \int_\omega^{\omega_p(\omega_t - 1)} h(\omega_t|\omega_{t-1})(\omega_t U'(1 - s_t(\omega_{t-1})) - U'(s_t(\omega_{t-1}))) d\omega = 0 \]

otherwise. \( \forall \omega' \), it follows then that the sequential optimum features

\[ s_t(\omega_t) = \max \{ s_t(\omega_t), s_t(\omega_{t-1}) \}. \]

This therefore means that the sequentially optimal rule at any date \( t \) can be implemented with a debt limit, which depends only on \( \omega_{t-1} \) and the current level of debt.

So as to complete the argument, we must verify that \( \theta_p(\theta_{t-1}) = f^{-1}(\omega_p(f(\theta_{t-1}))) \) and that Assumption 4 is identical to Condition 2. To show that \( \theta_p(\theta_{t-1}) = f^{-1}(\omega_p(f(\theta_{t-1}))) \), note that (S.A.10) can be rewritten as

\[ 1 - H(\omega''|\omega_{t-1}) \times \omega'' \left( \frac{1}{\beta} - \frac{\int_\omega^{\omega''} h(\omega''|\omega_{t-1}) d\omega''}{\int_\omega^{\omega''} h(\omega''|\omega_{t-1}) d\omega''} \right) \geq 0. \]

Letting \( \theta'' = f^{-1}(\omega'') \) and \( \theta''' = f^{-1}(\omega''') \) with associated values \( \tilde{\theta}'' \) and \( \tilde{\theta}''' \), the above condition becomes

\[ \left( \frac{1}{\beta} - \frac{\int_{\tilde{\theta}''}^{\tilde{\theta}'''} \theta'''' p(\theta''''|\theta_{t-1}) d\theta'''}{\tilde{\theta}''''} \right) \geq 0, \]

which becomes (B.1). This establishes that \( \theta_p(\theta_{t-1}) = f^{-1}(\omega_p(f(\theta_{t-1}))) \).

To show that Assumption 4 is identical to Condition 2, note that \( G(\omega_t|\omega_{t-1}) \) is continuously differentiable in \( \omega_t \) and first-order conditions imply that Condition 2 reduces to

(S.A.11) \[ \frac{d \log h(\omega_t|\omega_{t-1})}{d \log \omega_t} \geq -\frac{2 - \beta}{1 - \beta}. \]
We can show that (B.2) implies (S.A.11). Given the definition of $h(\cdot)$, note that the left hand side of (S.A.11) can be expanded so that (S.A.11) becomes

$$\frac{d \log \tilde{\theta}_t}{d \log \theta_t} + \frac{d \log p(\theta_t|\theta_{t-1})}{d \log \theta_t} - \frac{d \log \frac{d \tilde{\theta}_t}{d \theta_t}}{d \log \theta_t} \geq -\frac{2 - \beta}{1 - \beta},$$

which is equivalent to (B.2).

**S.A.2. Proof of Proposition 8**

Consider the case of i.i.d. shocks, and with some abuse of notation, let $W_t(\cdot)$ correspond to the continuous type analog of the continuation value defined in (9) divided by $\delta E[\theta_t]$, where we have taken into account that shocks are i.i.d. Let the range $[\underline{W}, \overline{W}]$ correspond to the feasible range of such continuation values. To write the period zero problem, we pursue an analogous strategy as in the proof of Proposition 7 by considering the relaxed problem that allows the resource constraint to hold as an inequality:

$$\begin{align*}
\max_{\omega_t} & \{u_t(\omega_t) + y_t(\omega_t) + W_t(\omega_t)\} \\
\text{subject to} & \\
&S.A.12) \quad U^{-1}(u_t(\omega_t)) + U^{-1}(y_t(\omega_t)) \leq 1 \quad \forall \omega_t, \\
&S.A.13) \quad \omega_t u_t(\omega_t) + \beta y_t(\omega_t) + \beta W_t(\omega_t) \\
&S.A.14) \quad \geq \omega_t u_t(\hat{\omega}_t) + \beta y_t(\hat{\omega}_t) + \beta W_t(\hat{\omega}_t) \\
&S.A.15) \quad \forall \omega_t, \text{ and } \forall \hat{\omega}_t \neq \omega_t, \quad \text{ and} \\
&S.A.16) \quad W \leq W_t(\omega_t) \leq \overline{W} \quad \forall \omega_t.
\end{align*}$$

The same envelope condition in (S.A.6) applies. Together with the monotonicity of $u_t(\omega_t)$, it implies incentive compatibility. Substituting (S.A.6) into (S.A.12), (S.A.13), and (S.A.15), the program can be rewritten as

$$\begin{align*}
\max_{\omega_t} & \left\{ \frac{\omega}{\beta} u(\omega) + y(\omega) \right\} \\
&S.A.16) \quad + W(\omega) + \frac{1}{\beta} \int_{\omega}^{\overline{W}} (1 - G(\omega_t|\omega_{t-1})) u(\omega_t) d\omega_t
\end{align*}$$
subject to
\[ (S.A.17) \quad U(1 - U^{-1}(u_i(\omega_i))) + \bar{W} + \frac{\omega_t}{\beta} u(\omega_t) - \frac{\omega}{\beta} u(\omega) - y(\omega) \]
\[ - W(\omega) - \frac{1}{\beta} \int_{\omega}^{\omega} u(\theta) d\theta' \geq 0 \quad \text{and} \]
\[ (S.A.18) \quad u_i(\omega_i) \text{ is nondecreasing.} \]

Equations (S.A.16)–(S.A.18) are identical to (S.A.7)–(S.A.9) since the term \( y(\omega) \) in (S.A.7)–(S.A.9) is replaced with \( y(\omega) + W(\omega) \) in (S.A.16)–(S.A.18) and the term \( U(1 - U^{-1}(u_i(\omega_i))) \) is replaced with \( U(1 - U^{-1}(u_i(\omega_i))) + \bar{W} \). Therefore, the same arguments as those of Amador, Werning, and Angeletos (2006) imply that the solution to (S.A.16)–(S.A.18) holding with equality, which then means that (S.A.13) holds with equality and \( W(\omega_t) = \bar{W} \). It thus follows that the solution admits a static mechanism and, therefore, the optimal mechanism is characterized as in Proposition 7.

S.A.3. Proof of Proposition 9

Part (i). Suppose by contradiction that the ex ante optimum coincides with the sequential optimum, so the solution is characterized by Proposition 7. With some abuse of notation, let \( V_{\theta_0}(\hat{\theta}_0) \) correspond to the expected date-1 welfare in the sequential optimum to a type \( \theta_0 \) who lies and claims to be a type \( \hat{\theta}_0 \). By lying, this type receives a mechanism associated with \( \hat{\theta}_0 \), evaluated using probabilities \( p(\theta_1|\theta_0) \). Given the description of the solution in Proposition 7, it is straightforward to show that \( V_{\theta_0}(\hat{\theta}_0) \) is continuously differentiable in \( \hat{\theta}_0 \) with \( V_{\theta_0}'(\hat{\theta}_0) = 0 \) for \( \hat{\theta}_0 = \theta_0 \).

We now consider a perturbation that affects types \( \theta_0 < \theta_0(\theta_{-1}) \). To facilitate the construction of the perturbation, note first that one implementation of the sequentially optimal mechanism is as follows. The government can choose any savings rate above \( sf(\theta_0(\theta_{-1})) \). If the chosen savings rate is \( s_0 > sf(\theta_0(\theta_{-1})) \), then the mechanism at date 1 corresponds to the sequentially optimal mechanism for type \( sf^{-1}(s_0) \), where \( sf^{-1}(\cdot) \) is a function that uses (C.1) to derive the type from the chosen flexible savings rate. If, instead, the chosen savings rate is \( s_0 = sf(\theta_0(\theta_{-1})) \), then the government reports its type \( \hat{\theta}_0 \) and the mechanism at date 1 corresponds to the sequentially optimal mechanism for type \( \hat{\theta}_0 \).

Given this implementation, consider the following perturbation. If the chosen savings rate is \( s_0 = sf(\theta_0(\theta_{-1})) \), the mechanism is unchanged. If, instead, the chosen savings rate is \( s_0 > sf(\theta_0(\theta_{-1})) \), the mechanism at date 1 corresponds to the sequentially optimal mechanism for type \( sf^{-1}(s_0) + \varepsilon \mu(sf^{-1}(s_0)) \) for \( \varepsilon > 0 \) arbitrarily small and for some continuously differentiable function \( \mu(\cdot) \) satisfying \( \mu(\cdot) > 0, \mu'(\cdot) > -1 \), and \( \lim_{\theta_0 \to \theta_0(\theta_{-1})} \mu(\theta_0) = 0 \). Note that if
\( \varepsilon = 0 \), the original sequentially optimal mechanism is in place. If, instead, \( \varepsilon > 0 \) is arbitrarily small, then all types \( \theta_0 \geq \theta_p(\theta_{-1}) \) do not change their behavior, but the chosen savings rate \( s^*_0(\theta_{-1}, \theta_0) \) for types \( \theta_0 < \theta_p(\theta_{-1}) \) must satisfy the first-order condition

\[
(S.A.19) \quad -\theta_0U'(1 - s^*_0(\theta_{-1}, \theta_0)) + \beta\tilde{\theta}_0U'(s^*_0(\theta_{-1}, \theta_0)) = -\beta\delta V^{\theta_0}(s^{\varepsilon}(\theta_{-1}, \theta_0)) + \varepsilon \mu(s^{\varepsilon}(\theta_{-1}, \theta_0))) \\
\times \frac{d[s^{\varepsilon}(s^*_0(\theta_{-1}, \theta_0))]}{ds^*_0(\theta_{-1}, \theta_0)}.
\]

As \( \varepsilon \) approaches 0, \( s^*_0(\theta_{-1}, \theta_0) \) approaches \( s^f_0(\theta_0) \). We can show that it must approach it from above; that is, for sufficiently small \( \varepsilon > 0 \), it must be that \( s^*_0(\theta_{-1}, \theta_0) \geq s^f_0(\theta_0) \). If, instead, \( s^*_0(\theta_{-1}, \theta_0) < s^f_0(\theta_0) \), then type \( \theta_0 \) could make itself strictly better off by increasing its savings rate to \( s^*_0(\theta_0) \), as this maximizes its immediate welfare and raises its continuation utility.

Suppose that Condition 1 holds for \( \theta_{-1} \) and some \( \theta_0 \). Together with (S.A.19), this implies that \( s^*_0(\theta_{-1}, \theta_0) > s^f_0(\theta_0) \) for a positive measure of types \( \theta_0 \). Using (S.A.19), the change in welfare as \( \varepsilon \) approaches 0 for any such \( \theta_0 \) has the same sign as

\[
\left[ -\theta_0U'(1 - s^f_0(\theta_0)) + \tilde{\theta}_0U'(s^f_0(\theta_0)) \right] \frac{ds^*_0(\theta_{-1}, \theta_0)}{d\varepsilon} \bigg|_{\varepsilon=0} \geq 0,
\]

where we have used (C.1) and the fact that \( V^{\theta_0}(\theta_0) = 0 \). Therefore, the perturbation strictly increases welfare. Note that if Condition 1 does not hold for \( \theta_{-1} \), then it necessarily holds at some \( \theta_{-1} \), and the same perturbation can be performed at that date while continuing to satisfy all incentive compatibility constraints at \( t - k \) for \( k > 1 \).

**Part (ii).** Suppose by contradiction that the mechanism does not exhibit history dependence. Let \( V^\theta(\tilde{\theta}) \) correspond to the continuation value of a type \( \theta \) who reports \( \tilde{\theta} \) under this history-independent mechanism, where, by assumption, \( V^\theta(\tilde{\theta}) \) is piecewise continuously differentiable. Given that the continuation mechanism is independent of the date and given that the ex ante optimal mechanism is chosen at date 0, it follows that \( \forall \theta_{-1} \), the continuation value at date 0 is \( V^\theta(\theta) \) if \( \theta = \theta_{-1} \) and, by optimality, \( V^\theta(\theta) = 0 \). Thus, the first-order conditions that guarantee truth-telling whenever the mechanism is differentiable imply that

\[
(S.A.20) \quad \left[ -\theta_tU'(1 - s_t(\theta_{t-1}, \theta_t)) + \beta\tilde{\theta}_tU'(s_t(\theta_{t-1}, \theta_t)) \right] \frac{dS_t(\theta_{t-1}, \theta_t)}{d\theta_t} = 0.
\]
This requires that either 

\[ s_i(\theta_{t-1}, \theta_t) = s^J(\theta_t) \] 

or 

\[ ds_i(\theta_{t-1}, \theta_t)/d\theta_t = 0. \]

Therefore, (S.A.20) effectively corresponds to the first-order condition to static incentive compatibility constraints. As such, the optimal mechanism is not dynamic. The solution to the program subject to a sequence of static incentive compatibility constraints coincides with the solution to the sequential optimum described in Proposition 7. However, part (i) shows that this mechanism is suboptimal.

**S.B. ALGORITHM**

This section describes the algorithm that we use to compute the equilibria in Section 5.3. We work from the first-order conditions to the program in (28). Let \( \phi \) be the Lagrange multiplier on the relevant incentive constraint (26). Define

\[
\Gamma = \frac{p(\theta^L|\theta^i) + \lambda p(\theta^L|\theta^{-i})}{1 + \lambda}
\]

and the functions

\[
s^L_\ast(\Gamma, \phi) = \left[ \frac{\theta^L}{\theta^L} \left( \frac{\Gamma + \phi}{\Gamma + \delta \phi} \right) + 1 \right]^{-1},
\]

\[
s^H_\ast(\Gamma, \phi) = \left[ \frac{\theta^H}{\theta^H} \left( \frac{1 - \Gamma - \phi \theta^L}{1 - \Gamma - \delta \phi \theta^L} \right) + 1 \right]^{-1},
\]

\[
\Gamma^{*\ast}(\Gamma, \phi) = \frac{(1 - \Gamma)(1 - \alpha) - \delta \phi \alpha}{(1 - \Gamma) - \delta \phi},
\]

where \( \alpha \equiv p(\theta^i|\theta^i) \). These equations correspond to the first-order conditions to (28), where \( \Gamma^{*\ast}(\Gamma, \phi) \) is the value of \( \Gamma \) in the next period given a particular \( \Gamma \) in the current period. We guess some initial functions \( V^L_0(\Gamma) \) and \( V^H_0(\Gamma) \) defined for \( \Gamma \in [0, \alpha] \). Then the next steps allow us to define \( V^L_t(\Gamma) \) and \( V^H_t(\Gamma) \) iteratively, where \( t \) denotes the number of the iteration.

**Step 1.** Define \( F_i(\Gamma, \phi) \) for a given \( V^L_t(\Gamma) \) and \( V^H_t(\Gamma) \):

\[
F_i(\Gamma, \phi) = \theta^L U(1 - s^L_\ast(\Gamma, \phi)) + \delta \tilde{\theta}^L U(s^L_\ast(\Gamma, \phi)) + \delta \beta V^L_t(\alpha)
- \theta^L U(1 - s^H_\ast(\Gamma, \phi)) - \delta \tilde{\theta}^L U(s^H_\ast(\Gamma, \phi))
- \delta \beta V^L_t(\Gamma^{*\ast}(\Gamma, \phi)).
\]

Note that \( F_i(\Gamma, \phi) \) captures the incentive constraint in (26).
Step 2. For each $\Gamma$, choose the value of $\phi$ that minimizes the absolute value of $F_t(\Gamma, \phi)$, effectively letting (26) hold with equality. Denote this value by $\phi^*_t(\Gamma)$.

Step 3. Given $\phi^*_t(\Gamma)$, define the functions

$$V_{i+1}^L(\Gamma) = \left\{ \alpha \left[ \theta^L U \left( 1 - s^{L*}(\Gamma, \phi^*_t(\Gamma)) \right) + \tilde{\theta}^L U \left( s^{L*}(\Gamma, \phi^*_t(\Gamma)) \right) \right] 
+ (1 - \alpha) \left[ \theta^H U \left( 1 - s^{H*}(\Gamma, \phi^*_t(\Gamma)) \right) + \tilde{\theta}^H U \left( s^{H*}(\Gamma, \phi^*_t(\Gamma)) \right) \right] 
+ \alpha \beta V_{i}^L(\alpha) + (1 - \alpha) \beta V_{i}^H(\Gamma^\alpha(\Gamma, \phi^*_t(\Gamma))) \right\},$$

$$V_{i+1}^H(\Gamma) = \left\{ (1 - \alpha) \left[ \theta^L U \left( 1 - s^{L*}(\Gamma, \phi^*_t(\Gamma)) \right) + \tilde{\theta}^L U \left( s^{L*}(\Gamma, \phi^*_t(\Gamma)) \right) \right] 
+ \alpha \left[ \theta^H U \left( 1 - s^{H*}(\Gamma, \phi^*_t(\Gamma)) \right) + \tilde{\theta}^H U \left( s^{H*}(\Gamma, \phi^*_t(\Gamma)) \right) \right] 
+ (1 - \alpha) \beta V_{i}^L(\alpha) + \alpha \beta V_{i}^H(\Gamma^\alpha(\Gamma, \phi^*_t(\Gamma))) \right\}.$$

$V_{i+1}^i(\Gamma)$ is the continuation welfare given last period’s shock $\theta$ and given the weight $\Gamma$ assigned to the low type. Our construction takes into account the resetting property, so $V_{i}^L(\alpha)$ is the continuation value with weight $\alpha$ if the low shock was realized in the previous period.1

Step 4. Given $V_{i+1}^L(\Gamma)$ and $V_{i+1}^H(\Gamma)$, repeat Steps 1–3 until $V_{i+1}^L(\Gamma)$ and $V_{i+1}^H(\Gamma)$ converge.

Using the values of $V^L(\Gamma)$ and $V^H(\Gamma)$ that emerge from this iteration, we can define the policy functions $s^{L*}(\Gamma, \phi^*(\Gamma))$, $s^{H*}(\Gamma, \phi^*(\Gamma))$, and $\Gamma^\alpha(\Gamma, \phi^*(\Gamma))$. These policy functions can then be used to simulate a path of shocks and calculate the value of $\bar{s}$.

**REFERENCE**


Columbia Business School, 3022 Broadway, Uris Hall 616, New York, NY 10027, U.S.A. and Dept. of Economics, University of Warwick; *mhalac@columbia.edu*

and

Columbia Business School, 3022 Broadway, Uris Hall 823, New York, NY 10027, U.S.A. and NBER; *pyared@columbia.edu.*

*Manuscript received November, 2012; final revision received April, 2014.*

1When the equilibrium resets, $\lambda = 0$ because the threat-keeping constraint is slack and, thus, $\Gamma = \alpha$. 