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Characterizing Predictable Components in Excess Returns on Equity and Foreign Exchange Markets

GEERT BEKAERT and ROBERT J. HODRICK*

ABSTRACT

The paper first characterizes the predictable components in excess rates of returns on major equity and foreign exchange markets using lagged excess returns, dividend yields, and forward premiums as instruments. Vector autoregressions (VARs) demonstrate one-step-ahead predictability and facilitate calculations of implied long-horizon statistics, such as variance ratios. Estimation of latent variable models then subjects the VARs to constraints derived from dynamic asset pricing theories. Examination of volatility bounds on intertemporal marginal rates of substitution provides summary statistics that quantify the challenge facing dynamic asset pricing models.

THERE IS NOW CONSIDERABLE evidence that excess returns on a variety of assets are predictable. In equity markets around the world, predictable returns have been documented using dividend yields, short-term interest rates, default spreads, and yields in the term structure of interest rates as predictors.¹ In foreign exchange markets, predictable returns have been documented using the forward premium as a predictor.² What has yet to be established is whether this predictability is evidence of market inefficiency or

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¹For U.S. data, Fama and Schwert (1977) used nominal interest rates to predict stock returns. For recent uses of this instrument see Breen, Glosten and Jagannathan (1989) and Ferson (1989). Gultekin (1983) and Solnik (1983) extended the Fama and Schwert results to other countries. Dividend yields have been used as predictors of stock returns either alone or in conjunction with other instruments by Rozeff (1984), Shiller (1984), Keim and Stambaugh (1986), Fama and French (1988b), Campbell and Shiller (1988), Cochrane (1990), Campbell (1991), and Hodrick (1991), among others.

²Tryon (1979) and Bilson (1981) pioneered use of the foward premium in investigations of the efficiency of the foreign exchange market. See Hodrick (1987) for a survey of the empirical literature in this area. More recent contributions include Cumby (1988), Mark (1988), Kaminsky and Peruga (1990), Froot and Thaler (1990), and Bekaert and Hodrick (1991).

time-varying risk premiums in an efficient market. Although we do not resolve the issue here, our empirical analysis contributes to the debate in several ways. First, we characterize the predictability of returns in an integrated way. Second, we reject some simple models of market efficiency. Third, we provide a way of organizing the facts which demonstrates the challenge to the development of dynamic asset pricing models.

Our first purpose is to integrate the literature on the predictability of asset returns by characterizing the predictable components in excess returns in the equity markets of the U.S., Japan, the U.K., and Germany and in the foreign exchange markets of the dollar relative to the yen, the pound, and the Deutsche mark. We use dividend yields, forward premiums, and lagged excess returns as predictors.³ Our innovation is to investigate the equity and foreign exchange excess returns with vector autoregressive techniques. This facilitates calculations of various long-horizon summary statistics. The importance of long-horizon predictability of equity returns in the debate on market efficiency has been stressed by Fama and French (1988a, b), Campbell and Shiller (1988), Poterba and Summers (1988), and in an international context by Cutler, Poterba, and Summers (1989).

The first part of our empirical analysis answers questions like the following: "What is the variability of expected returns in equity and foreign exchange markets at various horizons?" "Are equity markets characterized by mean reversion in stock prices?" "Do dividend yields predict long-horizon equity returns?" "Do exchange rates exhibit mean reversion at long horizons?" "Does a forward premium on the foreign currency predict appreciation of the domestic currency at all horizons?" Answers to questions such as these provide a useful characterization of the data. But, since return predictability is only inconsistent with the simplest model of market efficiency that postulates a constant required return, we conduct additional analysis.

Once excess return predictability is established, one would like to know if the predictability is due to time varying risk premiums. Asset pricing models typically predict that expected returns on assets move proportionately (with different betas) in response to movements in underlying factors, such as the return on the market portfolio in the CAPM. Hence, if markets are efficient and internationally integrated, a low dimensional factor structure may characterize the co-movements in excess returns. The second part of our analysis therefore investigates several latent variable models in an effort to determine the empirical plausibility of this argument.⁴ Failure to reject such a

³Related papers include Giovannini and Jorion (1987, 1989), who examine models of risk premiums in several foreign exchange markets simultaneously with the risk premium in the U.S. stock market; Campbell and Hamao (1989) and Chan, Karolyi, and Stulz (1991), who examine excess equity returns in the U.S. and Japan; Cumby (1990), who examines real equity returns in the U.S., Germany, the U.K., and Japan; Solnik (1990), who examines out-of-sample predictability for eight countries' equity and bond returns; and Harvey (1991), who examines dollar denominated excess equity returns on seventeen countries.

⁴Hansen and Hodrick (1983) developed the latent variable model and applied it to the foreign exchange market. Gibbons and Ferson (1985) developed the model independently in an application to the stock market. In recent applications of the approach, Campbell and Hamao (1992) and Cumby (1990) examine integration of equity markets across countries.

model would be consistent with an efficient, integrated, world capital market in which the riskiness of an asset is determined by world market forces. Models with a single latent variable are strongly rejected, but the evidence against models with two latent variables is less strong.

Variation over time in expected returns poses a challenge for asset pricing theory because it requires an explicitly dynamic theory in contrast to the traditional static capital asset pricing model (CAPM). One way to quantify this challenge for a large class of models is to examine volatility bounds on the intertemporal marginal rate of substitution (IMRS) of investors. Hansen and Jagannathan (1991) derive such bounds nonparametrically by exploiting a duality between the mean-standard deviation frontier of returns and the mean-standard deviation frontier for the IMRS. Their most challenging volatility bounds arise when they use Treasury bill returns. We extend their analysis using dollar denominated excess returns on international investments and find even more restrictive bounds.

The paper is organized as follows. Section I contains a discussion of the data and some summary statistics. Section II provides the estimation of the vector autoregressions (VARs). In this section we also consider an alternative formulation of the VARs that uses the two nominal interest rates rather than the forward premium, which is the interest differential. The calculations of the long-horizon statistics are also reported here. Section III considers the latent variable models, and Section IV contains estimation of the Hansen-Jagannathan (1991) bounds. The last section contains concluding remarks.

I. Data and Summary Statistics

To facilitate the presentation and discussion of our empirical analysis, consider the following definitions. We subscript variables of the four countries with numbers: 1 for the U.S., 2 for Japan, 3 for the U.K. and 4 for Germany. Let the one-month nominal interest rate denominated in currency j that is set at time t for delivery at time t+1 be i_{jt} . Define r_{jt+1} to be the continuously compounded one-month rate of return denominated in currency j in the equity market of country j in excess of i_{jt} . In the VARs we include the U.S. excess rate of return, r_{1t+1} , and a second country's excess equity rate of return denominated in currency j, r_{jt+1} for j equal to either 2, 3, or 4. Let the rate of return in dollars on an uncovered investment in the currency j money market in excess of the U.S. nominal interest rate be rs_{jt+1} , for j=2,3,4. Including rs_{jt+1} in the VAR with r_{1t+1} and r_{jt+1} allows calculation of the excess rate of return on a country j equity investment from a dollar investor's perspective as $r_{jt+1} + rs_{jt+1}$. Similarly, the currency j rate of return on a U.S. equity investment in excess of i_{jt} is obtained as $r_{1t+1} - rs_{jt+1}$.

⁵A focus on excess rates of return arises naturally in theoretical frameworks if returns are lognormally distributed. Use of gross excess returns in our empirical work would not change inference about return predictability, but it would complicate many of our calculations since they would no longer be linear.

To understand these calculations, consider the following analysis. Let S_{jt} be the dollar price of currency j. Then, the continuously compounded rate of depreciation of the dollar relative to currency j is $s_{jt+1}-s_{jt}=\ln(S_{jt+1}/S_{jt})$. The uncovered dollar return on a continuously compounded currency j money market investment is $\exp(i_{jt})(S_{jt+1}/S_{jt})=\exp(i_{jt}+s_{jt+1}-s_{jt})$. Hence, the excess dollar rate of return on a currency j money market investment is:

$$rs_{it+1} = i_{it} + s_{it+1} - s_{it} - i_{1t}. (1)$$

Analogously, if the continuously compounded rate of return denominated in currency j in the country j equity market is R_{jt+1} , the dollar return in this equity market is $\exp(R_{jt+1} + s_{jt+1} - s_{jt})$. Hence, the excess rate of return from the U.S. perspective on a foreign equity market investment is $R_{jt+1} + s_{jt+1} - s_{jt} - i_{1t}$. Using equation (1) and the fact that $r_{jt+1} = R_{jt+1} - i_{jt}$, the excess rate of return from the U.S. perspective on a foreign equity market investment is $r_{jt+1} + rs_{jt+1}$.

market investment is $r_{jt+1} + rs_{jt+1}$. From interest rate parity, the dollar return on a foreign money market investment that is covered in the forward foreign exchange market to eliminate foreign exchange risk is the U.S. nominal return. Hence,

$$i_{1t} = i_{it} + f_{it} - s_{it} (2)$$

where $f_{jt} - s_{jt} = \ln(F_{jt}/S_{jt})$ is the continuously compounded forward premium on the foreign currency, which we denote fp_{jt} . Substituting from equation (2) into equation (1) notice that $rs_{jt+1} = s_{jt+1} - f_{jt}$. This is how we measure the excess money market rates of return, and we will refer to them as returns in the foreign exchange market.

Morgan Stanley Capital International (MSCI) constructs monthly equity returns, and we obtained our data from Ibbotson Associates, who report the total return, the capital appreciation, and an income return. While the capital appreciation is the actual percentage change in price, the reported income return is an estimate constructed from annualized dividends divided by the previous price. We use the MSCI total return denominated in the foreign currency in the construction of r_{jt+1} . Eurocurrency interest rates are subtracted from the equity returns to create excess returns. The Eurocurrency interest rate data are market-determined, end-of-month interest rates and are from Data Resources, Inc.⁶ We also use the MSCI series to calculate dividend yields as annualized dividends divided by current price for the U.S., Japan, and the U.K. Observations on dividend yields were compared to data from the Financial Times Actuaries, and two outlier observations for the U.K. and two for Japan were corrected. The German dividend yield series is taken from various issues of the Monthly Report of the Deutsche Bundesbank, Section VI., Table 6, from the column labelled "yields on shares including tax credit." We chose this series because beginning in January 1977,

⁶We thank Bob Korajczyk for the eurocurrency interest rate data which were obtained at INSEAD and are used in Korajczyk and Viallet (1990).

domestic investors in German equities receive a tax credit for the corporate tax paid on dividends, which eliminates the double taxation of dividends. The dividend yield in country j is denoted dy_{jt} .

Daily bid and ask exchange rate data were obtained from Citicorp Database Services. The data are captured from a Reuter's screen and represent quoted market prices. We ran several filter tests on the data to check for errors, and we corrected several with observations from the *International Monetary Market Yearbook* or the *Wall Street Journal*. Exchange rate data are sampled at the end of the month, and we construct true returns for the foreign exchange markets by incorporating the market rules governing delivery on foreign exchange contracts. We also incorporate transactions costs by buying a currency at the bank's ask price and selling a currency at the bank's bid price for foreign exchange.⁷

Some summary statistics on the data are reported in Table I. The monthly data are scaled by 1200 to express returns in percent per annum. The means of the excess equity rates of return estimate the unconditional equity risk premiums in the different countries. The estimates are 5% for the U.S., 9% for the U.K., 10% for Germany, and 15% for Japan. The estimates of the unconditional means of the excess foreign exchange returns are -1% for the dollar-yen, -4% for the dollar-pound, and -3% for the dollar-DM.

The excess rates of return are quite variable. The standard deviations of the annualized monthly data range from 57% for both the U.S. and Japan to 68% for the U.K. and 71% for Germany. The comparable statistics for the foreign exchange market excess returns indicate slightly less variability with standard deviations between 42% and 46%.

The estimated autocorrelations of the excess rates of return are all small, while the autocorrelations of the dividend yields are all quite large. The autocorrelations for the forward premiums and interest rates are also large. The standard deviations of the dividend yields, the forward premiums, and the interest rates are more than an order of magnitude smaller than those of the excess rates of return.

II. A Vector-Autoregressive Approach

One way to examine predictability of excess returns is to estimate VARs. We report two-country VARs for the United States and either Japan, the United Kingdom, or Germany. In each VAR we include the U.S. equity

⁷In Bekaert and Hodrick (1991), we explain the rules for delivery on forward contracts, and we compare proper and improper use of data, either through ignoring these rules or failing to account for bid-ask spreads, to determine whether previous inference about the predictive ability of the forward premium for foreign exchange returns is affected. We find essentially no differences in inference with monthly data.

⁸The reported standard deviations are not estimates of the standard deviation of the annual holding period return. If returns are i.i.d., the variance of the annual return is twelve times the variance of the one month return. To estimate the standard deviation of the annual holding period return, divide our reported numbers by $\sqrt{12}$ (= 3.464).

Table I Summary Statistics

The numerical subscripts denote countries: 1 for the U.S., 2 for Japan, 3 for the U.K. and 4 for Germany. The excess equity market rate of return in country j is r_{jt} , the excess dollar rate of return on a currency j money market investment is rs_{jt} , the dividend yield in country j is dy_{jt} , the forward premium on currency j in terms of U.S. dollars is fp_{jt} , and the interest rate on currency j is i_{jt} . The sample period is 1981:1 to 1989:12. The monthly data are scaled by 1200 to express returns in percent per annum. The standard error for the autocorrelations for the null hypothesis of no serial correlation is 0.096.

		Standard		Autocor	relations	
Variable	Mean	Dev.	$\overline{ ho_1}$	$ ho_2$	$ ho_3$	$ ho_4$
r_{1t}	4.969	56.937	0.081	-0.048	-0.061	-0.048
r_{2t}	15.420	57.003	0.016	-0.057	-0.036	-0.010
r_{3t}	8.744	68.276	-0.094	-0.082	-0.041	0.040
r_{4t}	10.188	71.246	0.125	-0.001	0.041	-0.006
rs_{2t}	-0.894	42.251	0.128	0.051	0.172	-0.024
rs_{3t}	-3.620	45.884	0.021	0.165	0.041	0.070
rs_{4t}	-2.740	43.644	0.059	0.133	0.086	0.039
dy_{1t}	4.346	0.934	0.964	0.932	0.901	0.864
dy_{2t}	1.070	0.508	0.970	0.938	0.912	0.894
dy_{3t}	4.434	1.148	0.949	0.909	0.876	0.846
dy_{4t}	3.887	1.054	0.922	0.879	0.823	0.787
fp_{2t}	4.166	2.629	0.897	0.833	0.758	0.694
fp_{3t}	-1.438	2.877	0.877	0.780	0.668	0.559
fp_{4t}	3.630	1.622	0.620	0.456	0.353	0.226
i_{1t}	9.683	3.167	0.935	0.878	0.835	0.756
i_{2t}	5.785	1.195	0.900	0.829	0.794	0.731
i_{3t}^{-1}	11.320	2.023	0.893	0.791	0.688	0.597
i_{4t}	6.176	2.496	0.945	0.886	0.840	0.792

market excess return, the companion country equity market excess return, the relevant foreign exchange market excess return, the two dividend yields, and the forward premium. For example, the U.S.-Japan VAR contains $Y_t = [r_{1t}, r_{2t}, rs_{2t}, dy_{1t}, dy_{2t}, fp_{2t}]'$.

If Y_t follows a first-order VAR,

$$Y_{t+1} = \alpha_0 + AY_t + u_{t+1}, \tag{3}$$

where α_0 is a vector of constants, A is a (6 by 6) matrix, and u_{t+1} is the vector of innovations in Y_{t+1} relative to its past history. Higher order systems can be handled in exactly the same way by stacking the VAR into first-order companion form as in Campbell and Shiller (1988). In Table II we report the values of the Schwarz (1978) criteria for the choice of lag length in the VAR. In all cases the minimized value of the criterion is associated with the first-order system.

We estimate equations (3) with ordinary least squares and report heteroskedasticity consistent standard errors for the parameters. We test

Table II

Values of the Schwarz Criteria for Vector Autoregressions of Excess Stock Returns, the Excess Foreign Exchange Return, Dividend Yields, and the Forward Premium

The appropriate lag length for the VAR minimizes the Schwarz (1978) criterion. The sample period is 1981:1 to 1989:12. The monthly data are scaled by 1200 to express returns in percent per annum.

	Lag 1	Lag 2	Lag 3	Lag 4
U.SJapan	12.228	13.249	14.275	15.270
U.SU.K.	16.256	17.149	18.427	19.481
U.SGermany	17.574	18.482	19.376	20.306

one-step-ahead predictability of excess returns with a joint test of the six coefficients in the appropriate row of A. We also report the Cumby and Huizinga (1992) l-tests for serial correlation in the error processes. In contrast to more traditional tests for serial correlation, this test allows for the facts that the regressors are lagged dependent variables and that the error processes are conditionally heteroskedastic.

Estimation of the parameters of the VAR completely characterizes the unconditional mean, variance and covariances of the Y_t process since the series are assumed to be covariance stationary. In this case, the moving-average representation of Y_{t+1} is:

$$Y_{t+1} = \mu_0 + \sum_{j=0}^{\infty} A^j u_{t+1-j}.$$
 (4)

The unconditional mean of Y_t is $\mu_0 = (I - A)^{-1}\alpha_0$, where I is the six-dimensional identity matrix. If the innovation variance of u_t is V, the unconditional variance of the Y_t process can be derived from equation (4) to be $C(0) = \sum_{j=0}^{\infty} A^j V A^{j'}$, since u_t is serially uncorrelated. The jth order autocovariance of Y_t can similarly be derived to be $C(j) = A^j C(0)$.

A. Implied Long-Horizon Statistics

There is considerable interest in the characteristics of asset prices and returns at long horizons. For example, Fama and French (1988a) and Poterba and Summers (1988) examine variances and covariances of long-horizon stock returns to determine whether there are mean-reverting components in stock prices. Huizinga (1987) performs analogous computations for real currency depreciations. These authors note that when using short horizon or high frequency data it is often difficult to reject the hypothesis of no serial

⁹In actual calculations we truncate the infinite sum in C(0) at 255.

correlation in the logarithmic changes in asset prices, which are the primary part of an asset's return.¹⁰

One advantage of the VAR approach is that it uses additional variables that should be able to forecast returns under alternative hypotheses, which can improve the power of tests. It Furthermore, if there is long-horizon predictability in asset prices, there must be short-horizon predictability as well, since the long run is just a sequence of short runs. Characterizing long-run predictability can therefore be done with statistics that are functions of the autocovariances of the Y_t process. We consequently employ VAR methods to examine a number of implied long-horizon statistics.

The variance ratio for excess returns is defined to be the ratio of the variance of the sum of k one-period returns to k times the variance of the one-period return. The variance ratio equals one if returns are serially uncorrelated; it is greater than one if returns are positively autocorrelated; and it is less than one if the returns are negatively autocorrelated.

Rather than calculate variance ratios using sample variances of the returns over various horizons k, we calculate an implied variance ratio. To determine an implied variance ratio, first consider the sum of k consecutive Y_i 's. From equation (4) the variance of the sum of k Y_i 's can be derived to be

$$V_k = kC(0) + (k-1)[C(1) + C(1)'] + \cdots + [C(k-1) + C(k-1)']. \quad (5)$$

Define ei to be a six element vector of zeros except for the ith element which is one. Consequently, the total variance of the sum of k consecutive U.S. excess returns is $e1'V_ke1$. The variance ratio for the U.S. excess rate of return is therefore

$$VR(k) = \frac{e1' V_k e1}{ke1' C(0) e1}.$$
 (6)

The analogous variance ratios for the foreign country excess rate of return and the foreign exchange market excess return substitute *e*2 and *e*3, respectively, for *e*1 in equation (6).

We are also interested in variance ratios for dollar denominated rates of return to U.S. investors in the foreign equity markets and for foreign

¹⁰Poterba and Summers (1988) perform Monte Carlo experiments to examine the power of autocorrelation based tests. In the presence of highly serially correlated transitory components in prices, autocorrelation based tests have very low power. For example, such tests often incorrectly fail to reject the null hypothesis of no serial correlation in the changes in prices with probabilities of at least 0.8 when as much as 75% of the unconditional variance of the change in prices is due to transitory components.

¹¹Kandel and Stambaugh (1988) and Campbell (1991) employ VAR methods to examine long-horizon equity returns, and Cumby and Huizinga (1990) employ the technique to examine long-horizon forecasts of real exchange rates. Hodrick (1991) reports Monte Carlo analyses of the VAR technique and finds that the asymptotic distribution theory works very well given that the order of the VAR is correct.

¹²See Cochrane (1988), Lo and MacKinlay (1988) and Poterba and Summers (1988) for discussions of variance ratios.

currency denominated rates of return to foreign investors in the U.S. equity market. As noted above in Section I, these excess rates of returns are just linear combinations of the elements of Y_t , the first uses e7' = e2' + e3' and the second uses e8' = e1' - e3'. The final variance ratio we report is for depreciation of the dollar relative to the foreign currency, which is $e3' Y_{t+1} + e6' Y_t$.

Other long-horizon statistics can also be easily calculated. For example, Fama and French (1988b) regress long-horizon equity returns on the current dividend yield. The slope coefficient in such a regression is the covariance of the sum of returns from t+1 to t+k and the dividend yield at t divided by the variance of the dividend yield. Since a covariance involving a sum equals the sum of the covariances of the individual elements, an alternative estimator of this regression coefficient is

$$\beta_1(k) = \frac{e1'[C(1) + \cdots + C(k)]e4}{e4'C(0)e4}.$$
 (7)

Analogous coefficients for regressions of long-horizon foreign equity market returns on the foreign dividend yield are found by substituting e2 for e1 and e5 for e4 in equation (7). We also calculate the implied coefficient in the regression of the long-horizon foreign exchange market excess return on the forward premium. This is found by substituting e3 for e1 and e6 for e4 in equation (7).

Although the R^2 in regressions of one-step-ahead returns on current information is often quite small, the R^2 in long-horizon studies is often quite large, which reflects the negative serial correlation in long horizon returns. The explanatory power of the VAR at long horizons can also be assessed by examining the ratio of the explained variance of the sum of k returns to the total variance of the sum of k returns. These long-horizon R^2 coefficients can be calculated as one minus the ratio of the innovation variance in the sum of k returns to the total variance of the sum of k returns.

The innovation variance of the sum of k consecutive Y_t 's can be found from equation (4) to be

$$W_k = \sum_{j=1}^k (I - A)^{-1} (I - A^j) V (I - A^j)' (I - A)^{-1}'.$$
 (8)

Hence, the implied long-horizon \mathbb{R}^2 from the VAR for the U.S. equity return is

$$R^{2}(k) = 1 - \frac{e1'W_{k}e1}{e1'V_{k}e1}.$$
 (9)

Analogous long-horizon R^2 's can be produced for foreign excess equity returns and for the foreign exchange market by appropriate substitution for the indicator vector in equation (9).

B. Asymptotic Distributions for the Statistics

Each of the long-horizon statistics derived above is a function of the parameters of α_0 , A, and V. Let η_0 represent the vector of these distinct parameters, and let η_T be an estimate of η_0 from a sample of size T. Estimation of the parameters of the VAR can be thought of as an application of Hansen's (1982) Generalized Method of Moments (GMM) and can be done as a just-identified system. We use 63 orthogonality conditions in a GMM estimation to obtain the asymptotic distribution of η_T . This is a just-identified system because there are 42 coefficients in α_0 and A and 21 distinct parameters in V. The first 42 orthogonality conditions are the usual ordinary least squares conditions that the residuals are orthogonal to the right-hand-side instruments, $E(u_{t+1} \otimes Z_t) = 0$, where $Z_t' = (1, Y_t')$. The last 21 orthogonality conditions are given by stacking the distinct elements of $E(u_{t+1}u_{t+1}' - V) = 0$ into a vector.

In constructing the GMM weighting matrix, we allow a Newey-West (1987) lag of three (the 0.25 root of the sample size) for all of the orthogonality conditions since the deviations of the cross-products of the residuals from the elements of V can be arbitrarily serially correlated. The asymptotic distribution theory of GMM implies that $\sqrt{T}(\eta_T - \eta_0) \sim N(0, \Omega)$, where $\Omega = (D_0'S_0^{-1}D_0)^{-1}$, D_0 is the expectation of the gradient of the orthogonality conditions with respect to the parameters, and S_0 is the spectral density of the orthogonality conditions evaluated at frequency zero.

Let $H(\eta_0)$ represent the true value of one of the implied long-horizon statistics. The asymptotic distribution of the estimated function can be derived from a Taylor's series approximation to be

$$\sqrt{T} \left[H(\eta_T) - H(\eta_0) \right] \sim N(0, \nabla H \Omega \nabla H'). \tag{10}$$

Numerical derivatives can be used to calculate the gradient of H evaluated at η_T , which is denoted ∇H .

C. Interpretation of the Results of the VARs

The estimated VARs are reported in Panels A-C of Table III. The sample period is January 1981 to December 1989 for 108 observations. We use this sample because of the deregulation of international capital markets that took place at the end of the 1970s and the beginning of the 1980s, particularly in the U.K. and Japan. ¹³

We first analyze one-step-ahead predictability. A test that any of the excess returns is forecastable is a joint test that the six coefficients on the lagged variables are each zero. If we interpret the results of such tests as classical statisticians, we would reject the null hypothesis of no predictability if the value of the test statistic is greater than the prespecified critical value of a chi-square statistic with six degrees of freedom that is associated with a desired probability of a Type I error. Since we have no idea of the power of

¹³The sample corresponds to a sub-sample of Campbell and Hamao (1992) who describe the deregulation of Japanese financial markets.

these tests, and because Type II errors are also costly, we do not discuss the results in such terms. Instead, we report the confidence values of the test statistics which allows a quasi-Bayesian interpretation. We interpret large values of the test statistics as evidence against the hypothesis of no predictability.

Consider the results for the U.S.-Japan data in Panel A of Table III. For the U.S. equity market the test statistic is 24.245 with a confidence level of 0.999, for the Japanese equity market the test statistic is 13.955 with a confidence level of 0.970 and for the dollar-yen foreign exchange market the test statistic is 16.117 with a confidence level of 0.987. In each case there is some predictability of excess returns, but the returns are quite noisy, and the adjusted R^2 's are not large. The lagged variables explain 6.3% of the U.S. excess equity return, 5.2% of the Japanese excess equity return, and 10.9% of the dollar-yen foreign exchange market return.

The Cumby-Huizinga (1992) l-tests generally provide no strong evidence against the hypothesis that the residuals are serially uncorrelated. There is also no strong evidence against the hypothesis that the coefficients on the three lagged returns in each of the equations are zero. These results are in the row labelled Ret. Tests. Nevertheless, there are several coefficients on lagged returns in the return equations that are large relative to their standard errors. The point estimates indicate that expected excess returns in the U.S. and Japan respond positively to lagged U.S. returns and negatively to lagged Japanese returns. The forward premium enters all excess return equations with a negative sign, and the dividend yields enter the equity return equations with positive signs in the own-country equation and negative signs in the cross-country equation.

The U.S.-U.K. data are investigated in Panel B of Table III, and the U.S.-German data are in Panel C. We view the results as qualitatively similar to those of the U.S.-Japan system. The confidence level of the test statistic that examines predictability of the excess return in the U.K. equity market is not as large as those of the U.S. and Japan, but the adjusted R^2 in this equation is comparable to the others, as are the coefficient estimates on the dividend yields and the forward premium. Similarly, the adjusted R^2 for the U.S. equity return in the U.S.-Germany VAR falls to zero, but the coefficient estimates on the dividend yields and the forward premium are very similar to the analogous coefficients in the other VARs, and the confidence level for the test of return predictability is 0.872. There is very strong evidence of predictability of the dollar-pound and dollar-mark excess returns. The confidence levels are never smaller than 0.999, and the adjusted R^2 's are 17.2% and 17.8%, respectively. 14

 $^{^{14}}$ Harvey (1991) reports a higher R^2 for the U.S. equity return, but he includes a term structure premium and a default premium. His R^2 's for other countries are approximately the same as ours even though his are denominated in dollars and ours are denominated in foreign currency. The R^2 's for dollar denominated returns on foreign equity investments using our data and our predictive variables are 12.6%, 13.0%, and 7.8% for Japan, the U.K., and Germany, respectively.

Table III

the Excess Foreign Exchange Return, Dividend Yields, and the Forward Premium First Order Vector Autoregressions of Excess Stock Returns,

returns in percent per annum. Heteroskedasticity consistent standard errors (SE) and associated confidence levels for the test that the coefficient is The numerical subscripts denote countries: 1 for the U.S., 2 for Japan, 3 for the U.K. and 4 for Germany. The excess equity market rate of return in premium on currency j in terms of U.S. dollars is fp_{jt} . The sample period is 1981:1 to 1989:12. The monthly data are scaled by 1200 to express zero are below the estimates. A superscript a indicates that the coefficient estimate and its SE have been multiplied by 100. The $\chi^2(6)$ statistic tests correlation of the residuals. It is robust to conditional heteroskedasticity and lagged dependent variables. We test five correlation coefficients. The country j is r_{ii} , the excess dollar rate of return on a currency j money market investment is r_{ii} , the dividend yield in country j is dy_{ii} , the forward the joint hypothesis that the six lagged variables have no predictive power. The $\chi^2(5)$ statistic is the Cumby-Huizinga (1992) 1-test for serial $\chi^2(3)$ statistic tests the joint hypothesis that the three lagged returns have zero coefficients in the forecasting equations.

			Coeffic	Coefficients on Regressors	ressors						
Dependent Variables	Contant ependent SE 'ariables Confidence Cc	r _{1t} SE Confidence	r_{2t} SE Confidence	rs_{2t} SE Confidence	$\frac{dy_{1t}}{\text{SE}}$ Confidence	$\frac{dy_{2t}}{\text{SE}}$ Confidence	fp_{2t} SE Confidence	R^2	$\chi^2(6)$ Confidence	$\chi^2(5)$ Confidence	$\chi^2(3)$ Confidence
			Par	Panel A: U.SJapan	ıpan						
	- 60.95	0.13	-0.15	-0.02	31.17	-35.62	-6.97		24.25	3.59	4.70
111+1	54.12	0.12	0.09	0.12	17.46	21.22	2.37	0.063	0.99	0.39	0.81
• •	0.74	0.72	0.92	0.14	0.93	0.91	0.99				
	92.58	0.19	-0.12	0.02	-28.60	49.02	-1.08		13.96	3.50	3.12
191+1	40.26	0.11	0.10	0.14	13.76	19.64	2.43	0.052	0.97	0.38	0.63
- - 1	0.98	0.92	0.78	0.30	96.0	0.99	0.34				
	60.26	-0.11	0.08	-0.04	-16.71	31.46	-5.48		16.12	7.13	2.16
1891+1	26.05	0.09	0.07	0.09	9.44	15.64	1.97	0.109	0.99	0.79	0.46
1	0.98	0.76	0.78	0.38	0.92	96.0	0.99				
	0.35	-0.03^{a}	0.06^a	0.02^a	0.85	0.15	0.03		2025	10.29	4.48
$dy_{1\ell+1}$	0.20	0.05	0.03	0.05	0.07	0.08	0.01	0.950	0.99	0.93	0.79
	0.92	0.46	0.93	0.36	0.99	0.95	0.99				

Table III—Continued

	$\chi^2(3)$ Confidence	4.52	2.56 0.54		2.89	4.84	7.98	2.88	3.53 0.68	2.09
	$\chi^2(5)$ Confidence	4.11	6.37		6.12	2.48	9.43 0.91	7.12 0.79	3.31 0.35	6.18 0.71
	$\chi^2(6)$ Confidence	11807	380.2 0.99		18.67	6.75	28.20	2137 0.99	2031 0.99	269 0.99
	R^2	0.989	0.853		0.041	0.064	0.172	0.949	0.943	0.814
	fp_{2t} SE Confidence	0.004 0.003 0.88	0.89 0.06 0.99		-7.22 2.24 0.99	-4.98 2.64 0.94	-8.15 2.48 0.99	0.03 0.01 0.99	0.03 0.01 0.98	0.84 0.07 0.99
	dy_{2t} SE Confidence	0.92 0.02 0.99	0.19 0.48 0.31		-10.18 12.96 0.57	6.62 13.27 0.38	-1.93 9.37 0.16	0.06 0.06 0.70	0.87 0.08 0.99	$-0.19 \\ 0.28 \\ 0.51$
essors	$\frac{dy_{1t}}{\text{SE}}$ Confidence	0.04 0.01 0.99	-0.08 0.33 0.18	K.	27.55 17.50 0.88	12.74 14.51 0.62	4.12 12.11 0.27	0.84 0.09 0.99	0.0 6 0.10 0.45	0.44 0.39 0.75
Coefficients on Regressors	$\frac{rs_{2t}}{\mathrm{SE}}$ Confidence	0.02 a 0.01 0.76	-0.001 0.003 0.38	Panel B: U.SU.K	-0.16 0.10 0.90	0.01 0.13 0.08	-0.15 0.11 0.84	0.06^a 0.04 0.87	0.02 a 0.05 0.38	0.20° 0.20 0.72
Coeffic	$\sum_{t=0}^{r_{2t}} SE$	0.01° 0.01 0.60	0.002 0.002 0.68	Par	-0.08 0.13 0.46	-0.32 0.17 0.95	-0.04 0.08 0.40	0.03^a 0.05 0.53	$0.11^a 0.06 0.92$	-0.05^a 0.30 0.13
	$\frac{r_{1t}}{\mathrm{SE}}$	-0.01^a 0.01 0.78	0.08 d 0.20 0.34		0.13 0.16 0.59	0.41 0.21 0.95	$-0.12 \\ 0.10 \\ 0.77$	-0.03^a 0.06 0.42	-0.13^a 0.08 0.90	-0.10^a 0.34 0.22
	Contant SE Confidence	- 0.10 0.03 0.99	0.47 0.85 0.42		-79.84 48.88 0.90	-82.24 49.91 0.90	-23.50 27.94 0.60	0.45 0.19 0.98	0.33 0.17 0.95	$-1.39 \\ 0.86 \\ 0.89$
	Dependent Variables	dy_{2t+1}	fp_{2t+1}		r_{1t+1}	r_{3t+1}	rs_{3t+1}	dy_{1t+1}	dy_{3t+1}	fp_{3t+1}

Table III—Continued

accorded to the contract of th	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	A	-5.04 9.93 5.45	$12.26 \qquad 7.29 \qquad 3.70 \qquad -0.003 \qquad 0.87 \qquad 0.64 \qquad 0.60 \\ 0.85 \qquad 0.89 \qquad 0.83 \qquad \qquad$	-2.14 -9.39 13.60 0.69		0.72 0.19 0.99	-11.93 -10.07 38.92 4.28 1		0.69 0.97 0.99	1488 7.56	$0.03 \qquad 0.02 \qquad 0.945 \qquad 0.99$	0.99 0.91 0.83	0.89 0.04	0.07 0.03 0.902 0.99 0.94	0.63 0.99 0.79	0.03 0.56 90.1	$0.12 \qquad 0.493 \qquad 0.999 \qquad 0.99$	
Coefficients on Regressors	r_{2t} r_{S_2t} dy_{1t} SE SE Confidence Confidence	Panel C: U.SGermany		0.08 0.11 0.69 0.14	-0.01 -0.30	0.12 0.17	0.08 0.92		4.90 0.10	0.02 0.83	7	0.03 0.04	0.77 0.36		0.05 0.07		0.16^a -0.14^a	0.14 0.32	•
	Contant r _{1t} SE SE Confidence Confidence C		0.12	$0.11 \\ 0.76$	0.26	0.16	06.0	-0.20	0.07	66.0	-0.03^{a}	0.04	0.58	-0.07^{a}	90.0	0.82	-0.13^a	0.19	•
	Contant SE Confidence		-7.70	34.36 0.18	-7.14	35.18	0.16	54.39	17.97	0.99	0.12	0.13	0.64	0.09	0.15	0.46	0.23	0.50	000
	Dependent Variables			r_{1t+1}		r_{3t+1}			r_{3t+1}			dy_{1t+1}			dy_{3t+1}			fp_{3t+1}	

Only for the dollar-DM forward premium does the Cumby-Huizinga (1992) l-test indicate strong evidence against the hypothesis that the residuals are serially uncorrelated. In contrast to the U.S.-Japan VAR, the tests for the significance of lagged returns as predictors indicate that past returns are useful for forecasting the dollar-pound and the dollar-DM foreign exchange market returns.

D. Sensitivity Analysis on the VARs

In the VARs reported above we employ the forward premium as a predictor. From equation (2) notice that the forward premium is the nominal interest rate differential between the U.S. and the other country. Fama and Schwert (1977) used the nominal interest rate to predict equity returns and found a negative coefficient. Here, we examine whether the VAR would be better specified if the two nominal interest rates are entered separately rather than being forced to enter with coefficients that are equal and opposite in sign. Several issues are worth noting.

First, if the true values of the coefficients are equal and opposite in sign, failure to impose a true constraint in a finite sample unnecessarily reduces degrees of freedom and lowers the power of tests. Even if the true values are different, the principle of parsimony (especially in a VAR) dictates imposition of a false constraint if the absolute values are not too different.

A second issue involves the persistence of the variables of the VAR. It is often argued that nominal interest rates are integrated processes (see King, Plosser, Stock, and Watson (1991)). From Table I it is clear that dividend yields are also highly persistent. If two interest rates are included with the two dividend yields, too many variables with near unit roots may be present in the VAR, which might negate the validity of the usual asymptotic distribution theory that we use to generate standard errors and test statistics.

A third issue involves cointegration of the interest rates. If the two nominal interest rates are each integrated processes but the forward premium is stationary, the two interest rates are cointegrated with a cointegrating vector of (1, -1). If the levels of the two variables are included as regressors in an equation in which the dependent variable is stationary, their influence on the dependent variable will enter through the cointegrating relation. That is, the coefficients on the two interest rates will be equal but opposite in sign.

We address these issues in Table IV. Panels A-C report three sets of tests for the three VARs. Since our primary focus is return predictability, we discuss only the evidence for these equations. The first columns report coefficient estimates on the nominal interest rates for each of the three excess returns. The coefficient estimates for the U.S. interest rate are always negative, and the coefficients on the other country interest rate are always positive. The $\chi^2(1)$ statistics test the constraints that the coefficients are equal and opposite in sign. In the U.S.-Japan system there is no evidence against this constraint. In the other two-country systems, only in the U.S.

Table IV

Sensitivity Analysis of the Basic VARs

The numerical subscripts denote countries: 1 for the U.S., 2 for Japan, 3 for the U.K. and 4 for Germany. The VAR of and the forward premium on currency j in terms of U.S. dollars, f_{ji} . The sample period is 1981:1 to 1989:12. The monthly data are scaled by 1200 to express returns in percent per annum. The second and third columns report the equal and opposite in sign. The fifth and sixth columns report the tests of return predictability, a $\chi^2(7)$, and the R^2 . The differential and a variable constructed by subtracting 0.9 times the lagged interest rate from the current interest rate. The Table III uses six variables: the excess equity market rates of return in the U.S., $r_{1\nu}$, and in country j, r_{ii} , the excess dollar rate of return on a currency j money market investment, rs_{ij} ; the dividend yield in the U.S., dy_{1i} , and in country j, dy_{ji} ; coefficients and standard errors for the levels of the interest rate on the dollar, i_1 , and on currency j, i_j , which replace the forward premium in the basic VAR. The fourth column tests the hypothesis that the coefficients on the interest rates are quasi-differenced specifications enter dividend yields relative to a twenty-four month moving average, the interest a columns use the quasi-differenced U.S. interest rate, and the b columns use the quasi-differenced foreign country interest rate. Only the statistics for the return equations are presented.

	VAI	VARs with Nominal Interest Rates	Interest Rates			Quasi-D	ifference	Quasi-Differenced Specifications	su
Dep. Var.	Coefficient i _{1t} (SE)	Coefficient i_{jt} (SE)	$\chi^2(1)$ Confidence	$\chi^2(7)$ Confidence	R^2	$\chi^2(7)^a$ Confidence	R^{2a}	$\chi^2(7)^b$ Confidence	R^{2b}
			Pane	Panel A: U.SJapan	п				
11+1	-8.298	12.439	0.502	29.247	0.079	36.914	0.074	35.752	0.075
794.1	-1.951	5.116	0.179	16.398	0.047	15.462	0.037	13.139	0.037
1+17.	(2.489)	(8.063)	0.317	0.978		0.961		0.931	
rs_{2t+1}	-4.837	9.657	0.979	14.056	0.092	13.933	0.084	12.800	0.083
	(2.164)	(5.202)	0.678	0.950		0.948		0.933	
			Pane	Panel B: U.SU.K					
r_{1t+1}	-10.904	5.083	4.366	21.278	0.072	17.611	0.032	18.412	0.027
! :	(2.873)	(2.613)	0.963	0.997		986.0		0.990	

able IV—Continued

	VAJ	VARs with Nominal Interest Rates	Interest Rates	100		Quasi-D	ifference	Quasi-Differenced Specifications	su
Dep. Var.	Coefficient i_{1t} (SE)	Coefficient i_{jt} (SE)	$\chi^2(1)$ Confidence	$\chi^2(7)$ Confidence	R^2	$\chi^2(7)^a$ Confidence	R^{2a}	$\chi^2(7)^b$ Confidence	R^{2b}
r_{3t+1}	-6.431 (3.441)	2.607 (3.301)	1.052 0.695	6.613 0.530	0.058	5.890 0.447	0.024	5.793 0.436	0.025
rs_{3t+1}	-8.003 (9.430)	9.430 (2.774)	0.321 0.429	30.771 0.999	0.171	35.167 0.999	0.196	34.382 0.999	0.189
			Panel	Panel C: U.SGermany	any				
r_{1t+1}	-10.732 (4.320)	4.237 (4.554)	5.648	21.386	0.040	14.333 0.954	0.030	13.139	0.017
r_{4t+1}	-8.493 (4.921)	3.274 (6.210)	1.483	13.074 0.930	0.035	10.016 0.812	0.038	9.910 0.806	0.042
rs_{4t+1}	-10.969 (2.813)	13.057 (2.951)	$0.877 \\ 0.651$	37.070 0.999	0.187	15.123 0.966	0.051	14.222 0.953	0.049

SE = standard error.

equity market equation is the test statistic sufficiently large to reject the restriction at the 5% level. The next columns report the $\chi^2(7)$ statistics testing overall predictability of returns and the adjusted R^2 statistics. The values of these statistics are not very different from their respective values in Table III. These statistics are all calculated under the assumption that interest rates are stationary.

In order to address the issue of highly persistent variables in the VARs, the last four columns of Table IV report a $\chi^2(7)$ statistic and an adjusted R^2 for two VARs in which the four highly persistent variables enter in a quasi-differenced form. For dividend yields we subtract a moving average of the past twenty-four months from the current dividend yield variable in both specifications. For nominal interest rates we enter the interest differential in both specifications and one quasi-differenced interest rate obtained by subtracting 0.9 times the previous interest rate from the current interest rate. Unless the results on the predictability of returns, reported above, are spurious, the quasi-differenced variables should continue to explain returns although perhaps not with the same explanatory power.

The results are qualitatively quite similar to the specifications reported in levels for the U.S.-Japan and U.S.-U.K. systems. For the U.S.-German system, there is a decline in the statistical significance in all three equations and a substantial reduction in the \mathbb{R}^2 of the excess foreign exchange market return.

Given this evidence, we think that the original specification of the VAR is superior to the alternatives. Hence, the next section investigates long-horizon statistics calculated from the VARs of Table III.

E. Estimated Long-Horizon Statistics from the VARs

Tables V, VI, and VII report estimates of the implied long-horizon statistics derived in Section II.A with their associated asymptotic standard errors for the VARs of the U.S.-Japan, the U.S.-U.K., and the U.S.-Germany, respectively. Panel A of each table reports the implied unconditional means, standard deviations, and correlations of the series; Panel B reports several slope coefficients from implied OLS regressions; Panel C reports implied variance ratios; and Panel D reports implied R^2 's.

The results for Panel A are very similar across the three sets of countries. The point estimates of the unconditional mean excess returns implied by the VAR are similar in magnitude to the unconditional means calculated directly, but their standard errors are very large. The volatilities of the equity returns are larger than those of the foreign exchange market returns (between 50% and 70% for equities and between 40% and 50% for foreign exchange), and the correlations of the foreign exchange market returns with

¹⁵The large standard errors reflect imprecise estimation of the constant terms in the regressions and the near non-stationarity of the VAR caused by the inclusion of the highly serially correlated dividend yields and forward premiums.

Each Panel B of Tables V-VII reports implied slope coefficients, calculated analogously to equation (7), for the three sets of regressions. In the first two cases the own-country excess equity return compounded over various horizons is implicitly regressed on the own-country divided yield. In the third case, the compound excess foreign exchange return is implicitly regressed on the forward premium. Unfortunately, the large standard errors imply that the statistical significance of the estimates of the implied dividend yield coefficients is generally not as strong as that found in Hodrick (1991).¹⁷ The point estimates reported here are approximately the same size or slightly smaller than their standard errors for the U.S., the U.K., and Germany, and they are generally smaller than their standard errors for Japan.

Interpretation of the point estimates from these implied regressions is facilitated by dividing by the time horizon. The resulting coefficient is the increase in an annualized expected excess return for a 100 basis point increase in a dividend yield. For example, the estimates imply that a 1% increase in the own country dividend yield forecasts an increase in expected returns over the next 48 months of 2% per annum for the U.S., 3% per annum for Japan, 3.5% per annum for Germany, and 4% per annum for the U.K. These results are comparable to those of Fama and French (1988b) whose coefficient estimates imply that U.S. real returns increase by 4% per annum during 48 months when the U.S. dividend yield increases by 1%.

The last sets of implied coefficients in Panel B of Tables V-VII are from the implicit regressions of long-horizon excess foreign exchange returns on the own-market forward premiums. These coefficients are quite significantly different from zero. The coefficients at the one-, three-, six-, and twelve-month horizons are two to five times their standard errors. To interpret these coefficients, remember that the exchange rates are expressed as dollars per foreign currency and the excess rates of return are for uncovered investments in the foreign currency money markets in excess of the U.S. interest rate.

The coefficients at the one-month horizon imply that a one percentage point increase in the forward premium is associated with a 6% per annum decrease in the expected excess rate of return to investing in yen or pounds and an 8% per annum decrease to investing in Deutsche marks. At the twelve-month horizon, the coefficients imply that a one percentage point increase in the forward premium is associated with a 4% per annum decrease in the compound expected excess return from investing in the yen money market. Similar coefficients are found for the other currencies as well.

Each Panel C of Tables V-VII reports the implied long-horizon variance ratios. For the U.S. and the U.K. the point estimates indicate mean

¹⁷Hodrick (1991) uses the three variable VAR of Campbell (1991) composed of real returns, dividend yields and the short-term Treasury bill rate relative to its one year moving average. For a sample of monthly data from 1952 to 1987, the coefficients from the implied OLS regression of returns on dividend yields for comparable horizons to those of Tables III-V are often five to eight times their standard errors. Presumably, both the larger number of variables in the VAR (six vs. three) and the smaller sample size (108 vs. 431 observations) of this paper conspire to increase the standard errors here.

Table V Implied Long Horizon Statistics from the U.S.-Japan VAR

The implied long-horizon statistics are functions of the parameters of the vector autoregression. The asymptotic standard errors are calculated as in equation (10) and are in parenthesis below the estimates. The sample period is 1981:1 to 1989:12. The monthly data are scaled by 1200 to express returns in percent per annum. Panel A reports implied unconditional means and a matrix with the standard deviations on the diagonal and the correlation coefficients on the off-diagonal (see equation (4)). Panel B reports implied slope coefficients from the regression of a compound return for a given horizon onto a particular forecasting variable (see equation (7)). Panel C reports the implied ratio of the variance of returns compounded over a given horizon k to k times the variance of the one period return (see equation (6)). Panel D reports the implied R^2 from the VAR at horizon k which is one minus the ratio of the innovation variance to the total variance (see equation (9)). The numerical subscripts denote countries: 1 for the U.S., 2 for Japan, 3 for the U.K. and 4 for Germany. The excess equity market rate of return in country j is r_{jt} , the excess dollar rate of return on a currency j money market investment is rs_{jt} , the dividend yield in country j is dy_{jt} , the forward premium on currency j in terms of U.S. dollars is fp_{jt} .

Panel A: Implied Means Standard Deviations and Correlations

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(1.094) (0.541)
Panel B: Implied Slope Coefficients
-
Horizon: 1 3 6 12 24 36 48 60 ∞ U.S. Return and U.S. Dividend Yield
7.218 23.302 39.549 57.580 73.096 80.952 86.197 89.965 100.328
(7.325) (25.357) (44.156) (65.789) (89.815) (105.431) (116.315) (123.897) (140.084)
Japanese Return and Japanese Dividend Yield
0.161 2.313 9.271 29.685 71.450 104.494 129.104 147.235 197.458
(13.428) (38.832) (73.902) (135.131) (228.942) (294.165) (338.816) (368.947) (423.126)
Forward Bias and Forward Premium
-6.634 -18.807 -32.805 -49.211 -59.327 -60.572 -60.327 -59.626 -58.719
(1.843) (5.348) (10.272) (18.891) (31.339) (39.789) (46.048) (50.858) (64.827)

Table V—Continued

		Pane	l C: Implied	l Variance l	Ratios		
	Means		Standard	Deviations	and Corre	lation Matr	ix:
Horizon: 1	3	6	12	24	36	48	60
			U.S. I	Return			
1.000	1.023	0.919	0.768	0.625	0.559	0.521	0.496
(0.000)	(0.184)	(0.176)	(0.136)	(0.130)	(0.142)	(0.156)	(0.169)
			Japanes	e Return			
1.000	1.024	1.059	1.100	1.112	1.095	1.072	1.049
(0.000)	(0.102)	(0.175)	(0.288)	(0.425)	(0.504)	(0.557)	(0.597)
			Forwa	rd Bias			
1.000	1.246	1.594	2.114	2.679	2.936	3.068	3.145
(0.000)	(0.229)	(0.518)	(0.991)	(1.601)	(1.947)	(2.166)	(2.321)
		1	Oollar-Yen	Depreciatio	n		
1.000	1.186	1.460	1.872	2.317	2.513	2.608	2.660
(0.000)	(0.204)	(0.453)	(0.857)	(1.368)	(1.647)	(1.816)	(1.931)
		Dolla	r Return on	Japanese 1	Equity		, ,
1.000	1.234	1.493	1.846	2.182	2.305	2.351	2.367
(0.000)	(0.197)	(0.418)	(0.770)	(1.213)	(1.463)	(1.623)	(1.738)
		Ye	en Return o	n U.S. Equ	ity		, ,
1.000	1.046	0.980	0.925	0.933^{-}	0.954	0.969	0.978
(0.000)	(0.223)	(0.279)	(0.369)	(0.532)	(0.633)	(0.697)	(0.741)
			Panel D: Ir	nplied R^2 's	3		
Horizon: 1	3	6	12	24	36	48	60
				Return			
0.075	0.119	0.141	0.125	0.089	0.071	0.061	0.053
(0.044)	(0.079)	(0.100)	(0.101)	(0.100)	(0.100)	(0.098)	(0.094)
			Japanes	e Return			
0.103	0.128	0.151	0.142	0.093	0.063	0.048	0.039
(0.049)	(0.073)	(0.098)	(0.101)	(0.070)	(0.045)	(0.032)	(0.028)
			Forwa	rd Bias			
0.142	0.276	0.318	0.263	0.149	0.095	0.068	0.052
(0.100)	(0.165)	(0.172)	(0.149)	(0.105)	(0.077)	(0.061)	(0.050)

the equity returns are less than $\pm 14\%$ and are insignificantly different from zero. 16

The dollar forward premiums on the foreign currencies are always negatively correlated with all excess returns. Dividend yields are almost always negatively correlated with all excess returns, and, unsurprisingly, the statistical significance of the correlation of dividend yields with returns is concentrated primarily, but not exclusively, in the own-country equity market. Dividend yields are highly positively correlated across countries (at least 78% in all cases), and they are always positively correlated with the forward premiums.

¹⁶This latter observation forms the basis of recent interest in hedged foreign investment strategies in which the principal on a long-term foreign equity or bond investment is sold in the short-term forward market.

Table VI

Implied Long Horizon Statistics from the U.S.-U.K. VAR

The implied long-horizon statistics are functions of the parameters of the vector autoregression. The asymptotic standard errors are calculated as in equation (10) and are in parenthesis below the estimates. The sample period is 1981:1 to 1989:12. The monthly data are scaled by 1200 to express returns in percent per annum. Panel A reports implied unconditional means and a matrix with the standard deviations on the diagonal and the correlation coefficients on the off-diagonal (see equation (4)). Panel B reports implied slope coefficients from the regression of a compound return for a given horizon onto a particular forecasting variable (see equation (7)). Panel C reports the implied ratio of the variance of returns compounded over a given horizon k to k times the variance of the one period return (see equation (6)). Panel D reports the implied k^2 from the VAR at horizon k which is one minus the ratio of the innovation variance to the total variance (see equation (9)). The numerical subscripts denote countries: 1 for the U.S., 2 for Japan, 3 for the U.K. and 4 for Germany. The excess equity market rate of return in country k is k0. The excess dollar rate of return on a currency k1 money market investment is k2, the dividend yield in country k3 is k4, the forward premium on currency k3 in terms of U.S. dollars is k4.

	Panel	l A: Implie	ed Means,	Standard I	Deviations a	and Correla	itions	
	Means		Stan	dard Devia	tions and C	orrelation	Matrix:	
r_{1t}	4.386 (6.112)	r_{1t} 56.297 (5.918)	$r_{3t} = 0.700 = (0.066)$	$rs_{3t} - 0.005 \ (0.088)$	$ \begin{array}{c} dy_{1t} \\ -0.191 \\ (0.052) \end{array} $	$ \begin{array}{c} dy_{3t} \\ -0.122 \\ (0.065) \end{array} $	$fp_{3t} - 0.187 \ (0.063)$	
r_{3t}	3.173 (5.630)		68.083 (6.029)	0.024 (0.093)	-0.079 (0.089)	-0.106 (0.087)	-0.028 (0.071)	
rs_{3t}	6.592 (7.277)			45.303 (3.779)	-0.228 (0.147)	-0.219 (0.150)	-0.342 (0.128)	
dy_{1t}	3.646 (0.661)				0.875 (0.356)	0.925 (0.064)	0.685 (0.162)	
dy_{3t}	3.646 (0.745)					1.006 (0.348)	0.567 (0.225)	
fp_{3t}	-2.893 (1.447)						2.722 (0.777)	
		F	anel B: In	nplied Slop	e Coefficier	ıts		
Horizon: 1	3	6	12	24	36	48	60	∞
		U.	S. Return	and U.S. I	Dividend Y	ield		
2.127	8.715	19.736	41.226	75.000	96.536	109.722	117.728	130.034
(5.672)	(20.072)	(41.450)	(77.962)	(124.343)	(145.204)	(152.639)	(153.944)	(143.896)
				and U.K.	dividend yi	eld		
8.594	23.262	45.482	84.735	140.433	174.187	194.608	206.972	225.966
(6.719)	(19.184)	((64.600)	(- , - ,	(105.533)	` ,	(116.245)	(132.688)
6 000	10.500				vard Premi		00.404	100 100
-6.333		-29.315			-81.491	-88.971	-93.491	-100.432
(1.160)	(3.363)	(7.188)	(16.215)	(35.035)	(51.556)	(64.736)	(74.673)	(97.602)

Table VI-Continued

		Pane	l C: Implied	Variance l	Ratios		
	Means		Standard	Deviations	and Corre	lation Matr	ix:
Horizon: 1	3	6	12	24	36	48	60
			U.S. I	Return			
1.000	1.111	1.161	1.166	1.087	0.997	0.919	0.857
(0.000)	(0.187)	(0.242)	(0.263)	(0.308)	(0.377)	(0.435)	(0.477)
			U.K. 1	Return			
1.000	0.880	0.847	0.792	0.700	0.630	0.577	0.537
(0.000)	(0.111)	(0.142)	(0.157)	(0.167)	(0.180)	(0.189)	(0.194)
			Forwa	rd Bias			
1.000	1.103	1.362	1.781	2.366	2.766	3.057	3.276
(0.000)	(0.191)	(0.429)	(0.861)	(1.587)	(2.175)	(2.657)	(3.054)
		D	ollar-Pound	Depreciati	ion		
1.000	1.034	1.214	1.511	1.929	2.215	2.425	2.582
(0.000)	(0.159)	(0.344)	(0.680)	(1.236)	(1.681)	(2.042)	(2.338)
		Dol	lar Return	on U.K. Eq	luity		
1.000	0.872	0.927	1.014	1.084	1.108	1.119	1.125
(0.000)	(0.119)	(0.199)	(0.318)	(0.489)	(0.618)	(0.719)	(0.799)
			ınd Return	on U.S. Eq	uity		
1.000	1.178	1.217	1.221	1.211	1.202	1.196	1.190
(0.000)	(0.185)	(0.268)	(0.368)	(0.545)	(0.688)	(0.798)	(0.883)
			Panel D: Ir	nplied R^2 's	3		
Horizon: 1	3	6	12	24	36	48	60
				Return			
0.074	0.093	0.096	0.082	0.074	0.075	0.074	0.071
(0.065)	(0.077)	(0.088)	(0.105)	(0.146)	(0.168)	(0.172)	(0.168)
				Return			
0.111	0.118	0.151	0.196	0.267	0.304	0.314	0.307
(0.102)	(0.112)	(0.151)	(0.215)	(0.305)	(0.341)	(0.346)	(0.339)
				rd Bias			
0.198	0.319	0.391	0.397	0.331	0.268	0.219	0.181
(0.096)	(0.174)	(0.215)	(0.250)	(0.273)	(0.266)	(0.245)	(0.219)

reversion in stock prices at long horizons, with the U.S. evidence being the strongest in the U.S.-Japan VAR. The 48- and 60-month variance ratios fall to 0.50 or 0.60, which is consistent with the results of Poterba and Summers (1988) and Hodrick (1991). There is no evidence of mean reversion in Japanese or German excess returns. There is slight evidence that German excess equity returns are positively correlated at short horizons since the variance ratios rise to 1.2 at six months. For the excess returns in the foreign exchange market the point estimates indicate that returns are highly positively serially correlated. The variance ratios increase monotonically to above 2.9 for all currencies.

Each Panel D of Tables V-VII reports the implied long-horizon R^2 for the three excess returns. The U.S., Japanese and German excess returns show some predictability at long horizons, but the ratio of explained variance to total variance never rises above 15.1% for these countries. In contrast, the

Table VII

Implied Long Horizon Statistics from the U.S.-Germany VAR

The implied long-horizon statistics are functions of the parameters of the vector autoregression. The asymptotic standard errors are calculated as in equation (10) and are in parenthesis below the estimates. The sample period is 1981:1 to 1989:12. The monthly data are scaled by 1200 to express returns in percent per annum. Panel A reports implied unconditional means and a matrix with the standard deviations on the diagonal and the correlation coefficients on the off-diagonal (see equation (4)). Panel B reports implied slope coefficients from the regression of a compound return for a given horizon onto a particular forecasting variable (see equation (7)). Panel C reports the implied ratio of the variance of returns compounded over a given horizon k to k times the variance of the one period return (see equation (6)). Panel D reports the implied k^2 from the VAR at horizon k which is one minus the ratio of the innovation variance to the total variance (see equation (9)). The numerical subscripts denote countries: 1 for the U.S., 2 for Japan, 3 for the U.K. and 4 for Germany. The excess equity market rate of return in country k is k0. The excess dollar rate of return on a currency k1 money market in investment is k2, the dividend yield in country k3 is k4, the forward premium on currency k5 in terms of U.S. dollars is k4.

Panel A: Implied Means, Standard Deviations and Correlations

(6.472) (22.990) (46.300) (85.844) (138.777) (166.844) (180.436) (186.183) (180.468) German Return and German Dividend Yield 3.202 13.467 29.098 59.700 109.612 143.626 166.007 180.626 208.014 (5.568) (18.210) (37.702) (74.659) (130.985) (164.804) (184.399) (196.070) (217.750) Forward Bias and Forward Premium		Means		Stan	dard Devia	tions and C	Correlation	Matrix	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	r_{1t}	4.584	56.497	0.461	-0.046	-0.186	-0.126	-0.188	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		(6.415)	(6.475)	(0.118)	(0.081)	(0.052)	(0.061)	(0.072)	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	r_{4t}	6.125		70.856	-0.101	-0.061	-0.177	-0.105	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		(9.367)		(5.971)	(0.076)	(0.114)	(0.077)	(0.098)	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	rs_{4t}	6.826			42.602	-0.169	-0.222	-0.355	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		(9.079)			(2.845)	(0.122)	(0.121)	(0.098)	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	dy_{1t}	3.660				0.877	0.776	0.510	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		(0.807)				(0.386)	(0.161)	(0.205)	
$fp_{4t} = \begin{array}{ccccccccccccccccccccccccccccccccccc$	dy_{4t}	3.239					0.915	0.368	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		(0.749)					(0.242)	(0.250)	
Panel B: Implied Slope Coefficients Horizon: 1 3 6 12 24 36 48 60 ∞ U.S. Return and U.S. Dividend Yield 2.538 9.881 20.191 39.010 67.890 87.105 99.686 107.895 123.269 (6.472) (22.990) (46.300) (85.844) (138.777) (166.844) (180.436) (186.183) (180.468) German Return and German Dividend Yield 3.202 13.467 29.098 59.700 109.612 143.626 166.007 180.626 208.014 (5.568) (18.210) (37.702) (74.659) (130.985) (164.804) (184.399) (196.070) (217.750) Forward Bias and Forward Premium - 8.096 -18.702 -29.336 -45.237 -67.905 -82.638 -92.236 -98.492 -110.206	fp_{4t}	2.988						1.537	
Horizon: 1 3 6 12 24 36 48 60 ∞ U.S. Return and U.S. Dividend Yield 2.538 9.881 20.191 39.010 67.890 87.105 99.686 107.895 123.269 (6.472) (22.990) (46.300) (85.844) (138.777) (166.844) (180.436) (186.183) (180.468) German Return and German Dividend Yield 3.202 13.467 29.098 59.700 109.612 143.626 166.007 180.626 208.014 (5.568) (18.210) (37.702) (74.659) (130.985) (164.804) (184.399) (196.070) (217.750) Forward Bias and Forward Premium -8.096 -18.702 -29.336 -45.237 -67.905 -82.638 -92.236 -98.492 -110.206		(0.740)						(0.391)	
U.S. Return and U.S. Dividend Yield 2.538 9.881 20.191 39.010 67.890 87.105 99.686 107.895 123.269 (6.472) (22.990) (46.300) (85.844) (138.777) (166.844) (180.436) (186.183) (180.468) German Return and German Dividend Yield 3.202 13.467 29.098 59.700 109.612 143.626 166.007 180.626 208.014 (5.568) (18.210) (37.702) (74.659) (130.985) (164.804) (184.399) (196.070) (217.750) Forward Bias and Forward Premium -8.096 -18.702 -29.336 -45.237 -67.905 -82.638 -92.236 -98.492 -110.206			I	Panel B: Ir	nplied Slop	e Coefficier	nts		
2.538 9.881 20.191 39.010 67.890 87.105 99.686 107.895 123.269 (6.472) (22.990) (46.300) (85.844) (138.777) (166.844) (180.436) (186.183) (180.468 German Return and German Dividend Yield 3.202 13.467 29.098 59.700 109.612 143.626 166.007 180.626 208.014 (5.568) (18.210) (37.702) (74.659) (130.985) (164.804) (184.399) (196.070) (217.750 Forward Bias and Forward Premium -8.096 -18.702 -29.336 -45.237 -67.905 -82.638 -92.236 -98.492 -110.206	Horizon: 1	3	6	12	24	36	48	60	∞
(6.472) (22.990) (46.300) (85.844) (138.777) (166.844) (180.436) (186.183) (180.468) German Return and German Dividend Yield 3.202 13.467 29.098 59.700 109.612 143.626 166.007 180.626 208.014 (5.568) (18.210) (37.702) (74.659) (130.985) (164.804) (184.399) (196.070) (217.750) Forward Bias and Forward Premium -8.096 -18.702 -29.336 -45.237 -67.905 -82.638 -92.236 -98.492 -110.206			U	.S. Return	and U.S.	Dividend Y	ield		
German Return and German Dividend Yield 3.202	2.538	9.881	20.191	39.010	67.890	87.105	99.686	107.895	123.269
3.202 13.467 29.098 59.700 109.612 143.626 166.007 180.626 208.014 (5.568) (18.210) (37.702) (74.659) (130.985) (164.804) (184.399) (196.070) (217.750 Forward Bias and Forward Premium -8.096 -18.702 -29.336 -45.237 -67.905 -82.638 -92.236 -98.492 -110.206	(6.472)	(22.990)	(46.300)	(85.844)	(138.777)	(166.844)	(180.436)	(186.183)	(180.468)
(5.568) (18.210) (37.702) (74.659) (130.985) (164.804) (184.399) (196.070) (217.750) Forward Bias and Forward Premium -8.096 -18.702 -29.336 -45.237 -67.905 -82.638 -92.236 -98.492 -110.206			Germ	an Return	and Germ	an Dividen	d Yield		
Forward Bias and Forward Premium -8.096 -18.702 -29.336 -45.237 -67.905 -82.638 -92.236 -98.492 -110.200	3.202	13.467	29.098	59.700	109.612	143.626	166.007	180.626	208.014
-8.096 -18.702 -29.336 -45.237 -67.905 -82.638 -92.236 -98.492 -110.206	(5.568)	(18.210)	` ,	,	,	. ,	,	(196.070)	(217.750)
			\mathbf{F}	orward Bi	as and For	ward Premi	um		
$(1.918) \qquad (6.183) (12.772) (25.865) (50.230) (71.238) (102.146) (102.146) (140.820) (12.772)$	-8.096	-18.702	-29.336	-45.237	-67.905	-82.638	-92.236	-98.492	-110.206
	(1.918)	(6.183)	(12.772)	(25.865)	(50.230)	(71.238)	(102.146)	(102.146)	(140.820)

implied R^2 at the 60-month horizon for the U.K. is 31%. The excess returns in the foreign exchange market are more predictable. At the twelve-month horizon the implied R^2 's are 26% for the yen, 40% for the pound, and 30% for the mark.

Table VII-Continued

		Dono	C. Immlied	Variance I	Dation		
		Pane					
	\mathbf{Means}			Deviations			
Horizon: 1	3	6	12	24	36	48	60
				Return			
1.000	1.089	1.089	1.062	0.982	0.907	0.845	0.794
(0.000)	(0.190)	(0.242)	(0.259)	(0.292)	(0.355)	(0.417)	(0.466)
				Return			
1.000	1.162	1.192	1.194	1.150	1.099	1.054	1.017
(0.000)	(0.161)	(0.232)	(0.276)	(0.283)	(0.268)	(0.254)	(0.242)
			Forwa	rd Bias			
1.000	1.118	1.327	1.643	2.104	2.443	2.703	2.907
(0.000)	(0.173)	(0.346)	(0.643)	(1.151)	(1.586)	(1.960)	(2.282)
]	Dollar-DM 1	Depreciatio	n		
1.000	1.083	1.258	1.529	1.927	2.220	2.444	2.620
(0.000)	(0.161)	(0.311)	(0.571)	(1.014)	(1.390)	(1.713)	(1.990)
		Dolla	r Return on	Germany 1	Equity		
1.000	1.081	1.190	1.308	1.421	1.479	1.515	1.540
(0.000)	(0.131)	(0.218)	(0.322)	(0.486)	(0.637)	(0.770)	(0.884)
		\mathbf{D}	M Return o	n U.S. Equ	ity		
1.000	1.136	1.143	1.135	1.110	1.089	1.072	1.058
(0.000)	(0.166)	(0.222)	(0.267)	(0.344)	(0.419)	(0.483)	(0.536)
			Panel D: Ir	nplied R^2 's	3		
Horizon: 1	3	6	12	24	36	48	60
			U.S. 1	Return			
0.039	0.055	0.065	0.068	0.067	0.068	0.067	0.065
(0.044)	(0.067)	(0.091)	(0.126)	(0.173)	(0.197)	(0.205)	(0.202)
()	(=	()	` '	n Return	(= == -)	,,	(=)
0.096	0.059	0.059	0.078	0.109	0.122	0.124	0.120
(0.081)	(0.075)	(0.104)	(0.164)	(0.232)	(0.252)	(0.250)	(0.238)
()	()	()	` '	rd Bias	()	()	()
0.185	0.214	0.256	0.298	0.298	0.265	0.229	0.196
(0.075)	(0.128)	(0.173)	(0.220)	(0.257)	(0.265)	(0.257)	(0.241)

III. Latent Variable Models

This section examines several latent variable models that are constrained counterparts of the excess return equations of the VAR. As in Hansen and Hodrick (1983), we note that these models are not precise tests of a particular equilibrium theory of international asset pricing. Rather, they are best interpreted either as tests motivated by asset pricing theories with additional restrictions or simply as empirical investigations of parsimonious characterizations of the expected excess returns. ¹⁸ This analysis is also motivated by Campbell and Hamao (1992) who report latent variable models for the U.S.

¹⁸See Campbell (1987, pp. 394-396) for an extensive discussion relating theoretical intertemporal asset pricing models with additional auxiliary assumptions to empirical latent variable models. See Wheatley (1989) for a critique of the latent variable approach to testing asset pricing models.

and Japanese equity markets with returns denominated in dollars and yen. We include the dollar-yen money market as well.

Some intuition about latent variable models is the following. It is possible that there are K risks in the world economy that are priced and that the expected rates of returns on all assets depend linearly on these risk factors with constant betas. In this case each rate of return in the world economy would have the following representation:

$$E_{t}(r_{t+1}^{i} - r_{t+1}^{f}) = \sum_{k=1}^{K} \beta_{ik} E_{t}(r_{t+1}^{k} - r_{t+1}^{f}).$$
 (11)

In equation (11) the r_{t+1}^k are rates of return on portfolios that are perfectly correlated with the sources of risks and r_{t+1}^f is the risk free rate. If Θ is the (N by M) matrix of reduced form coefficients from regressions of N excess returns on M explanatory variables, the K-dimensional latent variable model is the restriction that the rank of Θ is K. The restriction arises because the explanatory power of a regressor must come through its ability to explain one of the K fundamental sources of risk.

Table VIII reports models with a single latent variable for each of the three country pairs, U.S.-Japan in Panel A, U.S.-U.K. in Panel B, and U.S.-Germany in Panel C. In each case, the three excess returns are the U.S. equity return, the foreign country equity return, and the relevant foreign exchange market return. In the VAR there are seven forecasting variables including a constant in each equation. Hence, there are twenty-one free coefficients in the three excess return equations. The single latent variable model constrains the explanatory power of the seven variables to be proportional across the three excess returns.

For example, with $Z'_t = (1, Y'_t)$, the U.S.-Japan system is

$$r_{1t+1} = \alpha' Z_t + \varepsilon_{1t+1} \tag{12}$$

$$r_{2t+1} = \beta_1 \alpha' Z_t + \varepsilon_{2t+1} \tag{13}$$

$$rs_{2t+1} = \beta_2 \alpha' Z_t + \varepsilon_{3t+1} \tag{14}$$

which results in nine free parameters or twelve constraints on the VAR coefficients. The nonlinear system of three equations is estimated with GMM using the 21 orthogonality conditions $E_t(\varepsilon_{it+1}Z_t) = 0$, for i = 1-3. Table VIII reports the estimated β as well as the constrained reduced form coefficients.

Models with two latent variables are reported in Table IX. These may be written as

$$r_{1t+1} = \alpha_1' Z_1 + \varepsilon_{1t+1} \tag{15}$$

$$r_{2t+1} = \alpha_2' Z_t + \varepsilon_{2t+1} \tag{16}$$

$$rs_{2t+1} = (\beta_1 \alpha_1' + \beta_2 \alpha_2') Z_t + \varepsilon_{3t+1}$$

$$\tag{17}$$

which allows sixteen free parameters with twenty-one orthogonality conditions. We report several chi-square statistics that examine the adequacy of the models. If the models are good representations of the data, the chi-square statistics that test the overidentifying restrictions should be small. On the

other hand, since there is evidence that each of the excess returns is forecastable in the unconstrained systems, the chi-square statistics for a particular equation that test the explanatory power of the constrained variables ought to be large.

For the U.S.-Japan system, a confidence level of 0.941 for the test of the overidentifying restrictions indicates evidence against the single latent variable model that is about as strong as the evidence in Campbell and Hamao (1992), who examine just the two excess equity returns. Hence, adding the foreign exchange market with its strong predictability did not strengthen the evidence against the model. Examination of the reduced form coefficients in Table III suggests one reason why the model is inconsistent with the data. In the unconstrained VAR, the own dividend yield enters the own country equity return equation with a positive coefficient and the foreign country excess return equation with a negative coefficient. Since the single latent variable model constrains all of the coefficients of a particular forecasting variable to be the same sign across equations, it clearly cannot fit the data.

In the models with two latent variables in Table IX, the evidence against the constrained U.S.-Japan system is essentially the same as found above, even though there are now only five constraints. The confidence level of the overall test is 0.92. The constrained reduced form coefficients now fit the pattern of the unconstrained system described above, but the explanatory power of the variables in the foreign exchange market equation is not as statistically significant as in the unconstrained system.

For the U.S.-U.K. single latent variable system, the dollar-pound foreign exchange market excess return is not well explained. In the constrained model, the beta for the foreign exchange market is essentially zero. The substantive evidence against the model from the confidence level of the overall test statistic of 0.988 appears to be driven by feedback effects from lagged returns to current returns present in the unconstrained model that cannot be captured in the constrained case.

The model with two latent variables for the U.S.-U.K. system works very well. The value of the chi-square statistic that tests the overidentifying restrictions is less than its mean. Notice that if equations (15) and (16) were estimated as unconstrained equations, β_1 and β_2 in equation (17) would measure the influence of predictable components of the U.S. and U.K. equity returns on the predictable part of the foreign exchange return. Because estimation of the system is done in a constrained way, this interpretation is not literally valid, but the positive β_1 and negative β_2 do suggest the following interpretation. Market forces that increase the U.S. equity risk premium also increase the risk premium on uncovered pound money market investments, and market forces that increase the U.K. equity risk premium also increase the risk premium on uncovered dollar money market investments made with pounds. The statistical significance of the betas suggests that the former effect is more important than the latter.

For the U.S.-German data, the model with two latent variables also works better than the single latent variable model. In the unconstrained VAR there

Table VIII

Models With One Latent Variable

the dividend yield in country j is dy_{ji} , the forward premium on currency j in terms of U.S. dollars is fp_{ji} . The sample period is 1981:1 to 1989:12. The monthly data are scaled by 1200 to express returns in percent per annum. By definition, The numerical subscripts denote countries: 1 for U.S., 2 for Japan, 3 for the U.K. and 4 for Germany. The excess equity market rate of return in country j is r_{jt} , the excess dollar rate of return on a currency j money market investment is $r_{S_{jt}}$ $Z_i' = [1, r_1\iota, r_j\iota, r_s]\iota\iota, dy_1\iota\iota, dy_j\iota, fp_{j\iota}]$. The single latent variable model imposes twelve cross equation constraints on the three excess return equations of the VAR as in the following:

$$r_{1t+1} = \alpha' Z_t + \varepsilon_{1t+1}$$

 $r_{jt+1} = \beta_1 \alpha' Z_t + \varepsilon_{2t+1}$
 $r_{jt+1} = \beta_2 \alpha' Z_t + \varepsilon_{3t+1}$

The GMM estimation uses 21 orthogonality conditions; each of the 3 error terms should be orthogonal to the 7 elements of Z_t . The overall test of the model is therefore a $\chi^2(12)$ statistic. The reported parameter estimates are the quasi-reduced-form coefficients, which are β 's multiplied by α 's. The test of return predictability is the $\chi^2(6)$ statistic.

Dependence Variable	Consistent (SE) Confidence	$^{r_{1t}}_{ m (SE)}$ Confidence	$\sum_{j,t}^{r_{j,t}}$ (SE)	$rs_{ji} $ (SE)	dy_{1t} (SE) e Confidence	$\frac{dy_{jt}}{(\mathrm{SE})}$ ee Confidence	fp_{jt} (SE)	$\chi^2(6)$ Confidence
Panel A: U.SJapan		betas (SE)	$\beta_1 = 1$ (C)	$\beta_1 = 1.794$ $\beta_2 = (0.571)$	0.628 (0.358)	$\chi^2(12) =$ Confidence =	= 20.474 e = 0.941	
111+1	54.812 (26.641) 0.960	0.047 (0.046) 0.699	-0.036 (0.040) 0.622	0.002 (0.059) 0.022	-13.947 (8.032) 0.918	23.357 (11.423) 0.959	-2.872 (1.242) 0.979	10.245
121+1	98.323 (34.857) 0.995	0.085 (0.083) 0.691	-0.064 (0.069) 0.645	0.003 (0.105) 0.022	$-25.019 \\ (11.415) \\ 0.972$	$41.899 \\ (15.925) \\ 0.991$	-5.151 (1.972) 0.991	25.355 0.999
rs_{2t+1}	34.439 (17.113) 0.956	0.030 (0.032) 0.644	-0.022 (0.025) 0.624	0.001 (0.037) 0.022	-8.763 (4.987) 0.921	14.676 (7.743) 0.942	-1.804 (1.041) 0.917	5.167 0.360

Table VIII—Continued

			Table	anie vill—continued	nea			
Dependence Variable	Consistent (SE) Confidence	$^{r_{1t}}_{(\mathrm{SE})}$ Confidence	r_{jt} (SE)	$^{rs_{jt}}_{(\mathrm{SE})}$ Confidence	$\frac{dy_{1t}}{(\mathrm{SE})}$ Confidence	$\frac{dy_{jt}}{(\mathrm{SE})}$ Confidence	$\begin{array}{c} fp_{jt} \\ \text{(SE)} \\ \text{Confidence} \end{array}$	$\chi^2(6)$ Confidence
Panel B: U.SU.K.		betas (SE)	$\beta_1 = 1.319$ (0.231)	1.319 $\beta_2 = (0.231)$	- 0.003 (0.246)	$\chi^2(12) = 25.554$ Confidence = 0.988	$\chi^2(12) = 25.554$ offidence = 0.988	
1111	-92.228 (36.611) 0.988	0.179 (0.140) 0.799	-0.230 (0.116) 0.952	-0.065 (0.085) 0.557	28.764 (10.465) 0.994	-7.980 (9.607) 0.594	-5.790 (1.771) 0.999	18.833
r _{3t+1}	$-121.615 \\ (46.227) \\ 0.991$	0.236 (0.190) 0.786	-0.303 (0.152) 0.954	-0.086 (0.110) 0.563	37.929 (13.284) 0.996	$-10.522 \\ (12.685) \\ 0.593$	-7.635 (2.320) 0.999	20.043 0.997
rs_{3t+1}	0.300 (22.732) 0.011	-0.001 (0.044) 0.011	0.001 (0.057) 0.011	0.0001 (0.016) 0.010	-0.094 (7.085) 0.011	0.026 (1.963) 0.011	0.019 (1.428) 0.011	0.001
Panel C: U.SGermany		betas (SE)	$\beta_1 = 2$	$2.856 \beta_2 = (1.178)$	- 1.369 (0.826)	$\chi^2(12) = 16.183$ Confidence = 0.817	$\chi^2(12) = 16.183$ offidence = 0.817	
r_{1t+1}	-10.608 (9.745) 0.724	0.119 (0.067) 0.925	-0.005 (0.026) 0.138	-0.070 (0.053) 0.814	4.937 (3.857) 0.799	-1.052 (2.479) 0.329	-1.130 (1.100) 0.696	4.409
r_{4t+1}	-30.733 (25.839) 0.759	0.341 (0.118) 0.996	-0.013 (0.076) 0.137	-0.200 (0.125) 0.891	14.103 (8.958) 0.885	-3.006 (6.783) 0.342	-3.227 (2.795) 0.752	21.361 0.997
<i>r</i> 84 <i>t</i> +1	14.519 (11.726) 0.784	-0.163 (0.057) 0.996	0.006 (0.036) 0.136	0.096 (0.065) 0.863	-6.758 (4.662) 0.853	1.440 (3.327) 0.335	$1.546 \\ (1.526) \\ 0.689$	11.385 0.877

SE = standard error.

Models With Two Latent Variables

is rs_{i} , the dividend yield in country j is dy_{j} , the forward premium on currency j in terms of U.S. dollars is fp_{j} . The sample period is 1981:1 to 1989:12. The monthly data are scaled by 1200 to express returns in percent per annum. By definition, $Z_t' = [1, r_{1t}, r_{jt}, r_{jt}, r_{jt}, dy_{1t}, dy_{jt}, fp_{jt}]$. The model with two latent variables imposes five cross equation con-The numerical subscripts denote countries: 1 for the U.S., 2 for Japan, 3 for the U.K. and 4 for Germany. The excess equity market rate of return in country j is r_{ji} , the excess dollar rate of return on a currency j money market investment straints on the three excess return equations of the VAR as in the following:

$$r_{1t+1} = \alpha_1' Z_t + \varepsilon_{1t+1}$$

$$r_{jt+1} = \alpha_2' Z_t + \varepsilon_{2t+1}$$

$$r_{jt+1} = \left(\beta_1 \alpha_1' + \beta_2 \alpha_2'\right) Z_t + \varepsilon_{3t+1}$$

The GMM estimation uses 21 orthogonality conditions; each of the 3 error terms should be orthogonal to the 7 elements of Z_{ℓ} . The overall test of the model is the $\chi^2(5)$ statistic. The reported parameter estimates are the quasi-reduced-form coefficients, which are β 's multiplied by α 's. The test of return predictability is the $\chi^2(6)$ statistic.

Dependence Variable	Consistent (SE)	r_{1t} (SE) Confidence	$r_{jt} \ m (SE)$ Confidence	rs_{jt} (SE)	dy_{1t} (SE) Confidence	$dy_{jt} \ (\mathrm{SE})$ Confidence	$\begin{array}{c} fp_{jt} \\ (\mathrm{SE}) \end{array}$ Confidence	$\chi^2(6)$ Confidence
Panel A: U.SJapan		betas (SE)	$\beta_1 = 0$	$\beta_1 = 0.052$ $\beta_2 = (0.236)$	0.366 (0.190)	$\chi^2(5$ Confidence	$\chi^2(5) = 9.840$ Confidence = 0.920	
1,1+1	-63.364 (47.969) 0.813	0.113 (0.113) 0.685	-0.137 (0.079) 0.918	-0.0001 (0.114) 0.001	32.113 (15.514) 0.962	-31.786 (18.805) 0.909	-8.533 (2.174) 0.999	31.786
r_{2t+1}	93.192 (39.039) 0.983	0.118 (0.103) 0.745	-0.037 (0.074) 0.379	-0.029 (0.116) 0.197	-28.146 (13.618) 0.961	54.880 (19.516) 0.995	-3.833 (2.322) 0.901	20.278 0.998
rs_{2t+1}	30.788 (23.536) 0.809	0.049 (0.047) 0.700	-0.021 (0.038) 0.415	-0.011 (0.044) 0.189	-8.620 (9.033) 0.660	18.420 (12.536) 0.858	-1.849 (1.710) 0.720	5.360 0.384

Table IX-Continued

			ranie	able IA—Continued	nai			
Dependence Variable	Consistent (SE) Confidence	$^{r_{1t}}_{({ m SE})}$ Confidence	$\sum_{j,t}^{r_{j,t}} (\mathrm{SE})$	$\overset{rs_{jt}}{(\mathrm{SE})}$ Confidence	$dy_{1t} \ (\mathrm{SE})$	$dy_{jt} \ (\mathrm{SE})$	$f_{P_{jt}} \ (\mathrm{SE})$ Confidence	$\chi^2(6)$ Confidence
Panel B: U.SU.K.		betas (SE)	$\beta_1 = 1.$ (0.	$ \begin{array}{ccc} 1.973 & \beta_2 = - \\ (0.758) & \end{array} $	- 0.669 (0.505)	$\chi^2(5$ Confidenc	$\chi^2(5) = 3.347$ Confidence = 0.353	
711+1	- 43.575 (23.223) 0.939	0.091 (0.089) 0.695	-0.154 (0.085) 0.930	-0.094 (0.062) 0.873	10.142 (7.542) 0.821	-1.472 (6.016) 0.193	-6.242 (1.660) 0.999	17.988
r_{3t+1}	$-70.814 \\ (37.333) \\ 0.942$	0.373 (0.171) 0.971	-0.392 (0.140) 0.995	0.045 (0.114) 0.305	$11.154 \\ (12.644) \\ 0.622$	6.062 (10.980) 0.419	-4.497 (2.354) 0.944	10.447 0.893
rs_{3t+1}	- 38.571 (26.695) 0.852	-0.070 (0.083) 0.601	-0.041 (0.072) 0.430	-0.216 (0.093) 0.980	$12.543 \\ (11.644) \\ 0.741$	-6.960 (8.023) 0.614	-9.304 (2.463) 0.999	27.180
Panel C: U.SGermany		betas (SE)	$\beta_1 = 4.308$ (4.402)	$4.308 \beta_2 = -4.402$	- 1.593 (1.743)	$\chi^2(5)$ Confidence	$\chi^2(5) = 5.300$ Confidence = 0.620	
111+1	$13.855 \\ (15.547) \\ 0.627$	0.075 (0.073) 0.699	-0.027 (0.044) 0.454	-0.062 (0.063) 0.679	5.508 (4.905) 0.739	-3.886 (4.298) 0.634	-4.946 (2.697) 0.933	6.770 0.657
744+1	8.448 (32.230) 0.207	0.310 (0.148) 0.964	-0.059 (0.120) 0.378	-0.121 (0.150) 0.578	10.138 (10.895) 0.648	-3.015 (8.469) 0.278	-7.442 (3.509) 0.966	15.881 0.986
rs_{4t+1}	46.229 (17.275) 0.993	-0.169 (0.066) 0.989	$-0.021 \\ (0.048) \\ 0.339$	-0.077 (0.094) 0.589	7.579 (5.836) 0.806	-11.937 (5.370) 0.974	-9.453 (2.159) 0.999	37.349 0.999

SE = standard error.

is strong positive feedback from U.S. equity returns to German equity returns but negative feedback from U.S. equity returns to the excess return in the foreign exchange market. This forces the betas in the single latent variable model to have opposite signs and causes the coefficient on the forward premium, which is negative in the unconstrained foreign exchange market equation to be positive in the constrained case. The model with two latent variables works quite well. The test statistics of the overidentifying restrictions has a confidence level of 0.62, and the joint statistical significance of the constrained reduced form coefficients is almost as large as in the unconstrained systems. The estimates of β_1 and β_2 are positive and negative, respectively, although neither is precisely estimated.

A. A Three-Country System

The results of two three-country latent variable models are reported in Table X (one latent variable in Panel A and two latent variables in Panel B). We include the three excess equity returns of the U.S., Japan, and the U.K., and the two foreign exchange market returns for a five equation system. We use a constant, the three dividend yields and the two forward premiums as the instruments in Z_t . The single latent variable system is:

$$r_{1t+1} = \alpha' Z_t + \varepsilon_{1t+1} \tag{18}$$

$$r_{2t+1} = \beta_1 \alpha' Z_t + \varepsilon_{2t+1} \tag{19}$$

$$r_{3t+1} = \beta_2 \alpha' Z_t + \varepsilon_{3t+1} \tag{20}$$

$$rs_{2t+1} = \beta_3 \alpha' Z_t + \varepsilon_{4t+1} \tag{21}$$

$$rs_{3t+1} = \beta_4 \alpha' Z_t + \varepsilon_{5t+1} \tag{22}$$

Hence, there are thirty orthogonality conditions with ten free parameters in the single latent variable model.

The two latent variable model may be written as:

$$r_{1t+1} = \alpha_1' Z_t + \varepsilon_{1t+1} \tag{23}$$

$$r_{2t+1} = \alpha_2' Z_t + \varepsilon_{2t+1} \tag{24}$$

$$r_{3t+1} = (\beta_1 \alpha_1' + \beta_2 \alpha_2') Z_t + \varepsilon_{3t+1}$$

$$(25)$$

$$rs_{2t+1} = (\beta_3 \alpha_1' + \beta_4 \alpha_2') Z_t + \varepsilon_{4t+1}$$
(26)

$$rs_{3t+1} = (\beta_5 \alpha_1' + \beta_6 \alpha_2') Z_t + \varepsilon_{5t+1}$$

$$(27)$$

which allows eighteen free parameters with thirty orthogonality conditions. There is evidence against the two models, since the confidence levels for the overall test statistics are 0.980 and 0.893. Nevertheless, in the two latent variable model there is also strong evidence of statistically significant

¹⁹We did not use the two German returns because we considered estimation of a model with seven returns using all dividend yields and all forward premiums as instruments in all equations to be inappropriate given our sample size.

forecasting power for all excess returns but the U.K. equity market. Examination of the significance of the individual coefficients in the constrained reduced form in Panel B reveals an interesting pattern, which should also be interpreted with care given the high correlation of the instruments. Most of the coefficients on the forward premiums are negative, and these variables are important in forecasting the U.S. equity return and the two foreign exchange returns. The U.S. dividend yield has an important negative influence on the Japanese and the U.K. equity returns, but it is insignificant in the U.S. equity equation. The Japanese dividend yield enters all equations positively, and it is most important in the Japanese and U.K. equity equations.

IV. Hansen-Jagannathan (1991) Bounds

The linear predictability of equity and foreign exchange returns across countries documented above is not necessarily inconsistent with equilibrium asset pricing models, although there is currently no dynamic equilibrium model that has been shown to be consistent with it. One way to summarize the implications of this predictability for a rich class of dynamic models is to investigate volatility bounds on investors' IMRS as pioneered by Hansen and Jagannathan (1991).²⁰ The analysis builds on the observation that if time varying risk premiums are the source of the predictability, there must be volatility in an investor's IMRS.

To understand the derivation of these statistics, recognize that in models of rational maximizing behavior, investment decisions are dictated by intertemporal Euler equations that relate the loss in marginal utility from sacrificing a dollar at time t in purchasing an asset to the expected gain in marginal utility from holding the asset and selling it at time t+1. Let Q_{t+1} be the intertemporal marginal rate of substitution of a dollar between period t and t+1, and let R_{t+1} be a return at t+1 on a dollar invested at t. The typical Euler equation is:

$$E_t(Q_{t+1}R_{t+1}) = 1. (28)$$

Equation (28) is the foundation of many theoretical and empirical investigations of asset pricing. In the most basic representative agent model, e.g., Lucas (1982), the IMRS is

$$Q_{t+1} = \beta U'(C_{t+1}) \pi_{t+1} / U'(C_t) \pi_t, \tag{29}$$

which is the agent's discount factor times the ratio of the marginal utility of consumption at time t+1 multiplied by the purchasing power of a dollar at time t+1 to the product of these variables at time t.

²⁰Snow (1991) extends the methodology of Hansen and Jagannathan (1991) to extract information about the IMRS from additional moments of the distribution of asset returns. He obtains bounds on moments of the IMRS other than its mean and variance, and he examines the information in returns on portfolios sorted by firm size.

Table X U.S., Japan, U.K. Latent Variable Models

U.K. The excess equity market rate of return in country j is r_{ji} , the excess dollar rate of return on a currency j money market investment is r_{Sji} , the dividend yield in country j is dy_{ji} , the forward premium on currency j in terms of U.S. variables for each equation are a constant, the three dividend yields and the two forward premiums. The GMM estimation uses thirty orthogonality conditions; each of the five error terms should be orthogonal to the six instruments. The reported parameter estimates are the quasi-reduced-form coefficients, which are β 's multiplied by α 's. The test of return predictability is the $\chi^2(5)$ statistic. The numerical subscripts denote countries: 1 for the U.S., 2 for Japan, and 3 for the The five dependent variables are the three equity and two foreign money market excess returns. The instrumental dollars is p_{μ} . The sample period is 1981:1 to 1989:12. The monthly data are scaled by 1200 to express returns in percent

Panel A: One Latent Variable

The model with one latent variable imposes twenty cross-equation constraints:

$$r_{1t+1} = \alpha' Z_t + \varepsilon_{1t+1}$$

$$r_{2t+1} = \beta_1 \alpha' Z_t + \varepsilon_{2t+1}$$

$$r_{3t+1} = \beta_2 \alpha' Z_t + \varepsilon_{3t+1}$$

$$r_{2t+1} = \beta_3 \alpha' Z_t + \varepsilon_{4t+1}$$

$$r_{3t+1} = \beta_4 \alpha' Z_t + \varepsilon_{5t+1}$$

The overall test of the model is a $\chi^2(20)$ statistic.

Dependent Variables	Consistent (SE) Confidence	$dy_{1t} \ (\mathrm{SE})$ Confidence	$dy_{2t} \ ({ m SE})$ Confidence	dy_{3t} (SE) Confidence	fp_{2t} (SE)	fp_{3t} (SE)	$\chi^2(5)$ Confidence
	betas (SE)	$\beta_1 = 1.216$ (0.486)	$\beta_2 = 1.312$ (0.412)	$\beta_3 = 0.416$ (0.363)	$\beta_4 = -0.489$ (0.360)	$\chi^2(20) =$ Confidence =	35.019 0.980
111+1	-26.848 (21.052) 0.798	8.331 (6.165) 0.823	-0.888 (16.004) 0.044	4.122 (7.147) 0.436	. –4.725 (2.076) 0.977	0.221 (1.675) 0.105	9.050
r_{2t+1}	-32.650 (23.551) 0.834	10.132 (6.840) 0.861	-1.080 (19.449) 0.044	5.013 (8.666) 0.437	-5.749 (2.319) 0.987	0.268 (2.045) 0.104	15.048 0.990

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	Table X—(

	$\chi^2(5)$ Confidence	35.019 0.980		16.129	0.994		1.784	0.122		3.083	0.313	
	fp_{3t} (SE)	$\chi^2(20) =$ Confidence =		0.289	(2.201)	0.105	0.092	(669.0)	0.104	-0.108	(0.801)	0.107
	fp_{2t} (SE) Confidence	$\beta_4 = -0.489$ (0.360)	le	-6.201	(2.473)	0.988	-1.965	(1.584)	0.785	2.312	(1.454)	0.888
Table X—Continued	$\begin{array}{c} dy_{3t} \\ (\text{SE}) \\ \text{Confidence} \end{array}$	$\beta_3 = 0.416$ (0.363)	Panel A: One Latent Variable	5.410	(9.385)	0.436	1.715	(3.007)	0.432	-2.017	(3.442)	0.442
Table X	$\frac{dy_{2t}}{(\mathrm{SE})}$ Confidence ($\beta_2 = 1.312$ (0.412)	Panel A: One	-1.166	(21.021)	0.044	-0.370	(8.608)	0.045	0.435	(7.749)	0.045
	dy_{1t} (SE) Confidence	$\beta_1 = 1.216$ (0.486)		10.935	(7.650)	0.847	3.466	(3.541)	0.672	-4.076	(3.303)	0.783
	Consistent (SE) Confidence	betas (SE)		-35.238	(26.296)	0.820	-11.169	(11.337)	0.675	13.136	(11.317)	0.754
	Dependent Variables				r_{3t+1}			rs_{2t+1}			rs_{3t+1}	

The model with two latent variables imposes twelve cross-equation constraints:

Panel B: Two Latent Variables

$$\begin{split} r_{1t+1} &= \alpha_1' Z_t + \varepsilon_{1t+1} \\ r_{2t+1} &= \alpha_2' Z_t + \varepsilon_{2t+1} \\ r_{3t+1} &= (\beta_1 \alpha_1' + \beta_2 \alpha_2') Z_t + \varepsilon_{3t+1} \\ rs_{2t+1} &= (\beta_3 \alpha_1' + \beta_4 \alpha_2') Z_t + \varepsilon_{4t+1} \\ rs_{2t+1} &= (\beta_3 \alpha_1' + \beta_4 \alpha_2') Z_t + \varepsilon_{4t+1} \\ rs_{3t+1} &= (\beta_5 \alpha_1' + \beta_6 \alpha_2') Z_t + \varepsilon_{5t+1} \end{split}$$

The overall test of the model is a $\chi^2(12)$ statistic.

Table X-Continued

$x^2(5)$ e Confidence	9 $\chi^2(12) = 18.29$ (9) Confidence = 0.893		1	30) 0.993	. 96	53 17.617	33) 0.997	61	30 6.204	17) 0.713	14	36 12.650	35) 0.973	32	93 26.623	666.0 (99	66
fp _{3t} (SE) Confidence	$\beta_6 = -0.519$ (0.519)		-5.033	(1.730)	966.0	-3.653	(3.183)	0.749	-3.460	(2.617)	0.814	-4.136	(1.565)	0.992	-8.493	(2.566)	666.0
fp_{2t} (SE) Confidence	$\beta_5 = 2.064$ (0.883)	riables	-0.171	(1.357)	0.100	-1.020	(3.371)	0.238	-0.533	(1.921)	0.219	-0.042	(0.961)	0.035	0.177	(1.922)	0.073
dy_{3t} (SE) Confidence	$\beta_4 = -0.109$ (0.277)	Panel B: Two Latent Variables	-9.533	(6.694)	0.846	-5.839	(16.918)	0.270	-6.051	(9.895)	0.459	-7.951	(5.049)	0.885	-16.648	(8.750)	0.943
dy_{2t} (SE) Confidence	$\beta_3 = 0.901$ (0.489)	Panel B:	22.905	(16.297)	0.840	76.680	(33.145)	0.979	43.616	(24.585)	0.924	12.256	(12.608)	0.669	7.464	(19.312)	0.301
$\frac{dy_{1t}}{(\mathrm{SE})}$ Confidence	$\beta_2 = 0.464$ (0.209)		2.370	(8.455)	0.221	-32.490	(12.415)	0.991	-14.248	(10.117)	0.841	5.678	(6.907)	0.590	21.765	(11.263)	0.947
Consistent (SE) Confidence	$\beta_1 = 0.351 $ (0.401)		5.363	(25.718)	0.165	101.477	(40.318)	0.988	48.975	(33.766)	0.853	-6.260	(19.077)	0.257	-41.663	(27.349)	0.872
Dependent Variables	betas (SE)			r_{1t+1}			r_{2t+1}			r_{3t+1}			rs_{2t+1}			rs_{3t+1}	

SE = standard error.

Hansen and Jagannathan (1991) use data on returns to compute bounds on the variability of an agent's real IMRS that any model implying an Euler equation like (28) must satisfy. Whereas Hansen and Jagannathan (1991) investigate real returns using only U.S. dollar assets, we consider the nominal IMRS and use dollar returns on domestic and international investments to see if this makes the bounds more restrictive. Below, we discuss the variability of the IMRS that is implied by parameterizing and simulating an international extension of equation (29) after we discuss the estimation of the bounds.

Bounds on the variability of Q_{t+1} using excess returns are derived as follows. Let x_{t+1} denote a vector of n excess returns. One can think of these excess returns as dollar payoffs on assets that have zero prices. From equation (28), by the law of iterated expectations, we know that

$$E(Q_{t+1}x_{t+1}) = 0. (30)$$

Let P denote the space spanned by x_{t+1} , and let P^a be the space P augmented with a unit payoff. If Q_{t+1} were observable, we could run a regression of Q_{t+1} on a constant and x_{t+1} to recover the linear projection of Q_{t+1} onto P^a . The predicted part of Q_{t+1} would be $\alpha + \beta' x_{t+1}$. Because there will typically be a projection error, the variance of the nominal IMRS, which is the dependent variable in the regression, must be greater than $\beta' \Sigma \beta$, which is the variance of the explained part of Q_{t+1} , where Σ is the unconditional covariance matrix of x_{t+1} . From the algebra of least squares, we know that the true projection coefficient is:

$$\beta = \Sigma^{-1} \left[E(Q_{t+1} x_{t+1}) - E(Q_{t+1}) E(x_{t+1}) \right]$$

= $-\Sigma^{-1} E(Q_{t+1}) E(x_{t+1}).$ (31)

By substituting from equation (31) into $\beta' \Sigma \beta$, it is straightforward to derive a bound on the variance of Q_{t+1} :

$$\sigma^{2}(Q_{t+1}) > \left[E(Q_{t+1})\right]^{2} E(x_{t+1})' \Sigma^{-1} E(x_{t+1}). \tag{32}$$

Since $E(Q_{t+1})$ is unobservable, we obtain a bound on the coefficient of variation of the nominal IMRS implied by the mean and the variance of excess dollar returns:

$$\frac{\sigma(Q_{t+1})}{E(Q_{t+1})} > \left(E(x_{t+1})' \Sigma^{-1} E(x_{t+1})\right)^{1/2}. \tag{33}$$

Notice that if only one excess return is used, the bound is immediately given by rewriting equation (30) as $cov(Q_{t+1}, x_{t+1}) + E(Q_{t+1})E(x_{t+1}) = 0$ and using the Cauchy-Schwarz inequality. The bound then restricts the coefficient of variation of the nominal IMRS to be greater than or equal to the Sharpe ratio of the excess return, i.e., $|E(x_{t+1})|/\sigma(x_{t+1})$. The right-hand-side of equation (33) is similarly the Sharpe ratio of the return on a portfolio formed

Table XI Hansen-Jagannathan (1991) Bounds on the Coefficient of Variation of the Nominal Dollar Intertemporal Marginal Rate of Substitution

All returns are dollar denominated. The sample period is 1981:1 to 1989:12. The bound is the right-hand-side of equation (33). The unscaled bounds use excess returns. The scaled bounds use excess returns and pseudo excess returns generated by scaling an equity return with the lagged own dividend yield and a foreign exchange return with the lagged own forward premium. The cross-scaled bounds use both the excess returns and the scaled returns with additional pseudo returns generated by scaling an equity return with the lagged dollar-yen forward premium and a foreign exchange return with the lagged U.S. dividend yield. Standard errors are in parenthesis and are calculated using a Taylor's series approximation and three Newey-West (1987) lags.

Excess Returns Included in the Tests	Bound	Bound	Bound
	(unscaled)	(scaled)	(cross-scaled)
	(SE)	(SE)	(SE)
U.S. Equity	0.112	0.116	0.337
	(0.153)	(0.100)	(0.086)
Japanese Equity	0.225	0.239	0.410
	(0.210)	(0.095)	(0.069)
U.K. Equity	0.100	0.103	0.271
	(0.107)	(0.093)	(0.064)
German Equity	0.128	0.183	0.381
	(0.148)	(0.101)	(0.074)
Japanese Foreign Exchange	0.004	0.320	0.320
	(0.111)	(0.082)	(0.082)
U.K. Foreign Exchange	0.060	0.394	0.405
	(0.133)	(0.068)	(0.075)
German Foreign Exchange	0.045	0.319	0.337
	(0.127)	(0.066)	(0.073)
U.S. Equity, Japanese	0.305	0.474	0.598
Equity and Foreign Exchange	(0.098)	(0.097)	(0.093)
U.S. Equity, U.K. Equity and Foreign Exchange	0.181	0.474	0.579
	(0.111)	(0.068)	(0.089)
U.S. Equity, German	0.181	0.384	0.519
Foreign Exchange	(0.124)	(0.093)	(0.097)
Japanese, U.K. and German	0.077	0.477	0.479
Foreign Exchange	(0.104)	(0.075)	(0.077)
U.S., Japanese, U.K. and	0.237	0.301	0.585
German Equity	(0.105)	(0.089)	(0.080)
U.S. Equity, Japanese, U.K., and German Equity and Foreign Exchange	0.331 (0.111)	0.641 (0.088)	0.776 (0.083)

with the excess returns x_{t+1} , where the portfolio weights are given by the optimal portfolio in a mean-variance framework.

Table XI provides estimates of a variety of volatility bounds for the dollar IMRS calculated from our dollar denominated domestic and foreign excess returns. The column labelled (unscaled) contains bounds derived using only the raw excess returns listed in the first column. The bounds estimated only with foreign exchange investments are not very demanding (they are never larger than 0.07), nor are they precisely estimated. The volatility bound implied by all the equity market investments is 0.237. Using all of the foreign exchange returns with all of the equity market returns increases the bound to 0.331, which is not much larger than the bound implied by considering the Japanese foreign exchange and equity returns with the U.S. excess equity return.

Hansen and Jagannathan (1991) note that the payoff space can be increased by considering returns that are scaled by elements in the agents' information set. Essentially, scaling a return based on the realization of a random variable amounts to changing the investment in an asset as in a trading rule. The return from a trading rule is different from the return on the underlying asset. The empirical results from this paper suggest that incorporating conditioning information should be important because the returns are forecastable. ²¹

The column of Table XI labeled (scaled) reports bounds generated from using the original unscaled excess returns and the scaled excess returns. The scaling factors are the own dividend yields for equity returns and the own forward premiums for the foreign exchange returns. The column labeled (cross-scaled) adds additional pseudo returns constructed by scaling the equity returns with the dollar-yen forward premium and the foreign exchange returns with the U.S. dividend yield.

For the scaled bounds, except for Germany, the use of dividend yields tends not to increase the volatility bounds, while the effect of using the forward premiums with the foreign exchange returns is dramatic. Whenever a foreign exchange return that is scaled by its forward premium is included in the analysis, the bound invariably exceeds 0.30 with a standard error less than 0.10. The volatility bound implied by all of our excess returns including the scaled ones is 0.641 with a standard error of 0.088.

Cross-scaling the equity returns with the dollar-yen forward premium tends to increase the volatility bounds quite substantially, but the effect of scaling the foreign exchange returns with the U.S. dividend yield is minimal. The volatility bound implied by all assets rises to 0.776 with a standard error of 0.083.

These bounds can be compared to some benchmarks provided by Bekaert (1991), who simulates a two country, general equilibrium, Lucas (1982) model using an estimated VAR of two money growth rates and two

²¹Gallant, Hansen, and Tauchen (1990) discuss efficient use of conditioning information using seminonparametric methods.

consumption growth rates to provide realistic exogenous processes. ²² Intertemporal preferences are separable across periods, and the period utility function is parameterized with constant relative risk aversion (CRRA) preferences. Equation (29) applies with consumption measured as a geometric average of foreign and home goods with equal weights on the two goods. For a risk aversion coefficient of 2, the coefficient of variation of Q_{t+1} is of the order 0.010. To obtain bounds on the coefficient of variation of Q_{t+1} of around 0.2, the CRRA coefficient must be increased to over 40. Obtaining a bound of 0.78 requires a CRRA coefficient over 140.

Hansen and Jagannathan (1991) report bounds that are less restrictive than the ones we report, except when they examine returns from the U.S. Treasury bill market. They argue that such restrictive bounds may be incorrectly estimated since Treasury bills may provide liquidity services to investors who hold them to maturity as cash substitutes. While this argument may apply to money market investments, we find bounds that are equally restrictive using only equity returns. The bound from the four equity returns, including the scaled and cross-scaled pseudo returns, is 0.585 with a standard error of 0.080.

V. Conclusions

In this paper we characterize the linear predictability of excess returns in major equity and foreign exchange markets. Variables such as dividend yields, that were known to predict excess equity returns, are demonstrated to have predictive power for excess returns in the foreign exchange market. Similarly, variables such as forward premiums, that were known to predict excess returns in the foreign exchange market, are demonstrated to have predictive power for excess equity returns. We establish these results in VAR that allow calculation of a variety of long-horizon statistics.

We find evidence of long-horizon mean reversion in stock prices in the U.S. and the U.K., but not in Japan or Germany. The excess returns in the foreign exchange market have strong positive persistence. This implies, for example, that a U.S. investor faces mean reversion in the U.S. equity market, but not in the dollar-denominated Japanese equity market, and from the Japanese perspective, there is no evidence of mean reversion in the Japanese equity market nor in the yen-denominated return on the U.S. equity market.

We investigate the implications of a change in the dividend yield for long-horizon equity returns finding that a 1% increase in dividend yields implies between a 2-4% per annum increase in expected returns over the next forty-eight months. Increases in the forward premium on foreign currencies imply large decreases in excess returns in the foreign exchange market

 $^{^{22}}$ Bekaert (1991) uses data from the U.S. and Japan. The consumption data are quarterly non-durables and services obtained from the OECD, and the money stocks are measures of M2 from International Financial Statistics.

that are quite significant at shorter horizons. The forecasting power of the forward premium (that appears so puzzling to some researchers in the foreign exchange market) is also present in the equity excess returns. Increases in the forward premium (dollars/foreign currency) forecast lower expected excess equity returns in all countries. Latent variable models, which are constrained counterparts to the VAR analysis, require at least two latent variables to capture the covariance structure of excess returns, but even these models are not successful.

Our final results demonstrate that bounds on the nominal dollar IMRS derived from considering U.S. investments jointly with foreign money market and stock market investments with appropriate conditioning information are considerably higher than those obtained when attention is restricted only to the U.S. excess equity return. Whether the predictability of returns and the derived volatility bounds represent evidence of highly variable risk premiums, regime switching, peso problems, learning about policy changes, or market inefficiencies remains an open question.

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²³See Froot (1990) for a recent investigation of short-term nominal interest rates as predictors of returns on a variety of assets. Froot argues that risk premiums cannot be the source of the predictive power because the nominal interest rates have similar predictive power for the forecast errors from surveys of expected returns.

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