Financial Market Efficiency Tests

Tim Bollerslev and Robert J. Hodrick

1 Introduction

Testing for the efficiency of financial markets has generated enormous attention. In this chapter we provide a selective survey of the econometric tests and estimation procedures that have been employed in this literature. Throughout the chapter we illustrate the different ideas using monthly data on the New York Stock Exchange (NYSE) value-weighted price index and dividened series. Many of the results and techniques apply equally well to other financial markets, however.

As emphasized by Fama (1970; 1991), any test for market efficiency necessarily involves a joint hypothesis regarding the equilibrium expected rate of return and market rationality. The earliest tests for market efficiency were primarily concerned with short-horizon returns, where by "short horizon" we refer to holding periods within one year. These tests typically assumed that the expected rate of return was constant through time. It follows that if markets are efficient, the realized returns should be serially uncorrelated. Statistically significant own temporal dependencies at daily, weekly, and monthly frequencies have been documented for a wide variety of different asset categories, but the estimated autocorrelations are typically found to be numerically small. It has been argued that the autocorrelations are spurious or economically insignificant.

At the same time, however, most high frequency financial asset returns cannot be considered independently distributed over time since most returns are characterized by periods of relative tranquility followed by periods of turbulence. Since most modern asset pricing theories involve a direct mean–variance tradeoff, at least at the level of the market return, the explicit modeling of time variation in the conditional second-order moments of the returns and the underlying fundamentals process is potentially very important in tests for market efficiency. Of particular importance in these developments have been the autoregressive conditional heteroskedastic (ARCH) and generalized ARCH (GARCH) time series models. In the absence of any structural model explaining the time-varying second-order moments, simple ARCH models have provided a convenient statistical description of the conditional heteroskedasticity. Bollerslev, Chou, and Kroner (1992) provide a recent survey of this literature.

While the short-horizon tests generally suggest only minor violations of market efficiency, defined as constant expected returns, more recent evidence in Fama and French (1988a) and Poterba and Summers (1988), using multi-period regressions and variance ratio statistics, suggests that for longer return horizons a large proportion of the return variance is explainable from the history of past returns alone.[1] Of course, the finding of a large predictable component in long-horizon returns does not necessarily imply market inefficiency, as the variation in expected returns could be due to a time-varying risk premium. Indeed, consistent with the idea of a slowly moving equilibrium risk premium, Fama and French (1988b) find that the variation in dividend yields explains a large proportion of multi-year return predictability. Poterba and Summers (1988), though, argue that the magnitude and variability of the implied risk premium are too large to be explained by appeal to any rational asset pricing theory, and they suggest that asset prices are characterized by speculative fads in which market prices experience long systematic swings away from rational fundamentals prices. These highly serially correlated fads are difficult to distinguish from a martingale model on the basis of the earlier short-horizon tests for market efficiency, but their existence is more evident in long-horizon autocorrelations of returns.

Subsequent work has illustrated that the apparent predictability of the long-horizon returns should be interpreted very carefully. There are very few degrees of freedom, and the overlapping nature of the data in the multi-year return regressions gives rise to non-standard small sample distributions of the test statistics. Better approximations to the small sample distributions appear to be provided by the alternative asymptotic distribution discussed in Richardson and Stock (1989). The overlapping data problem may also be overcome by using the vector autoregressive techniques discussed in Baillie (1989) and Hodrick (1992).[2] Interestingly, while the own long-horizon return predictability may be spurious, the statistically more reliable return forecasting specifications employed in Campbell (1991), Hodrick (1992), and Bekaert and Hodrick (1992) suggest a statistically and economically significant long-horizon return predictability on the basis of fundamental variables including dividend yields and interest rates.

The variance bounds or volatility tests pioneered by Shiller (1979; 1981a) and LeRoy and Porter (1981) constitute another important class of tests for market efficiency. In the first generation of volatility tests the null hypothesis was taken to be the standard present value model with a constant discount rate. In the vast majority of these tests, the point estimates implied apparent clear rejections of market efficiency, with actual asset prices being excessively volatile compared to the implied price series calculated from the discounted value of the expected or actual future fundamentals. One possible explanation of excess volatility was the idea that asset prices may be characterized by self-fulfilling speculative bubbles that earn the fair rate of return, but cause prices to differ from their rational fundamentals. Flood and Hodrick (1990) provide a survey of this literature.

Although volatility tests appear different from traditional autocorrelation-based tests of market efficiency, they are equivalent to standard Euler-equation-based tests in the sense that each involves a joint hypothesis regarding the return generation process and the first-order condition for economic agents. Of course, as noted by Shiller (1981b), the power of the different approaches depends on the alternative hypothesis, and the volatility tests may have superior power against certain interesting alternatives.

Relaxing the null hypothesis of a constant discount rate results in much more mixed conclusions regarding excess volatility and market inefficiency. Additionally, many of the early volatility tests did not take seriously the non-stationarity of prices and fundamentals in calculating and interpreting the test statistics. At the same time, the non- stationarity gives rise to a robust testable cointegrating relationship based on the present value model that remains valid in the presence of stationary stochastic discount rates.

In summary, the current challenges for asset pricing theories can be expressed as the search for a model of expected return variability that is consistent with the empirical findings pertaining to the predictability of returns and that provides an explanation for the pronounced volatility clustering in returns. In this survey, we illustrate how a present value model for the NYSE price index that accounts for the time-varying uncertainty in dividend growth rates can actually explain most of the rejections of market efficiency on the basis of the different tests discussed above. In particular, the conditional mean and variance of monthly NYSE dividend growth rates both have a distinct seasonal pattern, whereas annual dividend growth rates show little serial correlation and appear homoskedastic. Using simulation methods, we demonstrate how incorporating this predictable monthly time variation into a model with stochastic discount rates provides a reconciliation of the actual empirical findings in tests for market efficiency with the present value relationship.[3] In addition to this new fundamental price process, we also report simulations for the conventional constant discount rate present value model, together with fads and bubble alternatives.

The plan for the rest of the chapter is as follows. The next section presents the different simulated models used throughout the chapter along with a discussion of the seasonal ARCH model for the fundamental dividend process. Section 3 reports a number of short-horizon summary statistics for the NYSE return series and illustrates how the volatility clustering in the returns may be conveniently modeled with a GARCH formulation for the conditional variance. Section 4 examines the long-horizon return tests. While the multi-period regression test statistics may be severely biased in small samples, we show how to develop tests based on an iterated version of the null hypothesis using Hansen's (1982) generalized method of moments (GMM). We find some improvement in the small sample performance of the test statistics using this method. In section 5, tests for market efficiency based on the ideas underlying cointegration are briefly analyzed. Section 6 considers a recent class of volatility tests derived by Mankiw, Romer, and Shapiro (1991) and examines the relation of excess volatility to expected return variability. Section 7 provides some concluding remarks.

2 Data Generation Mechanisms

In this section we describe the different data generation mechanisms used below in the Monte Carlo experiments. According to the standard present value relationship, the fundamental real price of an asset at time t is

$$P_t^{\mathrm{f}} \equiv E_t\left[\sum_{j=1}^{\infty}\left(\prod_{i=1}^{j}\rho_{t+i}\right)D_{t+i}\right], \tag{9.1}$$

where $E_t[\cdot]$ denotes the mathematical expectation conditional on all information available at time t, and D_t refers to the accumulated real dividend or other payoff on the asset from time $t-1$ through t.[4] Finally, the discount factor is

$$\rho_t \equiv \exp(-r_t), \tag{9.2}$$

where r_t is the continuously compounded required rate of return. Derivation of equation (9.1) imposes a transversality condition that the market fundamental price does not grow faster than the expected value of the product of the discount factors. If the present value model is true and markets are efficient, the observed price process should equal the fundamental price in equation (9.1) with the required rate of return being driven by a risk premium. This fundamental price relationship, coupled with an explicit formulation for the discount rate, r_t, forms the basis for three of our simulated models, considered below. Two alternative simulations involve explicit deviations of the actual price process, P_t, from the fundamentals price, P_t^{f}.

Of course, simulation of any of these price processes requires a characterization of the stochastic process governing dividends. All of the simulations are based on a model for an annualized dividend series, defined as the current value of the monthly dividends over the previous year,

$$D_t = \sum_{j=0}^{11} (ND_{t-j}/CPI_t) \prod_{h=1}^{j} (1 + i_{t-h+1}), \tag{9.3}$$

where the product from $h = 1$ to 0 is defined to be one. In equation (9.3) i_t denotes the monthly US Treasury bill rate at the beginning of month t, ND_t is the nominal value-weighted NYSE dividend series during month t, and CPI_t is the corresponding monthly US consumer price deflator. With monthly data from January 1926 through December 1987, there are 733 observations on D_t starting in December 1926. We chose to work with monthly observations on the annualized dividend series in equation (9.3), in contrast to modeling ND_t/CPI_t directly, because of the reduced seasonality and the consequent ease in computational burden. We note that most previous studies in this literature use annual data and are implicitly subject to similar problems of time aggregation. The same annualized dividend series is used in Hodrick (1992). Plots of the logarithmic dividend series, $d_t \equiv \ln(D_t)$, together with the logarithmic value-weighted, CPI-deflated, NYSE price index, $p_t \equiv \ln(P_t)$, are given in figure 9.1.

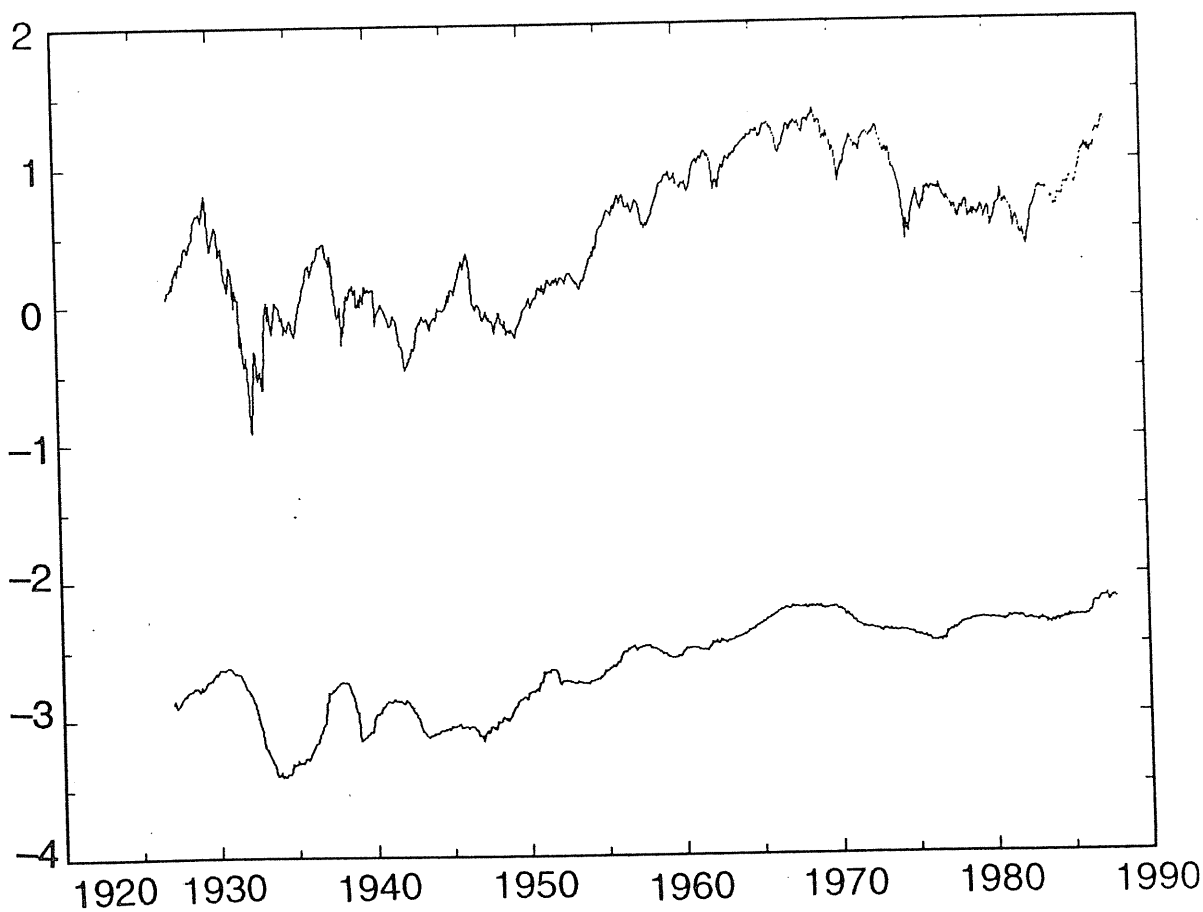

Figure 9.1 Log price and log dividend

It is apparent from figure 9.1 that both dividends and prices experienced growth in real terms during the sample period. In the subsequent analysis we therefore concentrate on modeling the growth rate in the dividend process, i.e. $\Delta d_t \equiv d_t - d_{t-1}$. Formal augmented Dickey and Fuller (1981) tests for a unit root in the autoregressive polynomial in the univariate time series representation for d_t are generally consistent with the idea that Δd_t is stationary. In particular, we consider the following regression:

$$\Delta d_t = \mu + \delta(t/T) + \gamma d_{t-1} + \phi_1 \Delta d_{t-1} + \ldots + \phi_{12} \Delta d_{t-12} + \varepsilon_t. \tag{9.4}$$

Note that this specification of the unit root test includes a linear deterministic trend under the alternative that $\gamma < 0$ in which case d_t is stationary around a linear time trend. Not including the time trend results in an inconsistent test against trend stationarity, as noted by West (1987a). We fail to reject the null hypothesis of no deterministic trend and a unit root in the autoregressive polynomial by the F-test of $\delta = 0$ and $\gamma = 0$ which equals 6.19. This value is slightly below the 5% critical value of 6.25. The t-statistic for $\gamma = 0$ equals -3.46. This value is also close to the 5% critical value of -3.41 and above the 1% critical value of -3.96 in the asymptotic unit root distribution given $\delta = 0$ tabulated in Dickey and Fuller (1981). We shall return to this and other tests for unit roots in both dividends and prices in our discussion of cointegration-based tests for market efficiency in section 5.

Examination of the correlation structure for Δd_t indicates a distinct seasonal pattern with highly significant autocorrelations at the seasonal frequencies and a clear cutoff in the partial autocorrelation function at lag 12.[5] This is consistent with the well-known observation that dividends are lumpy with payoffs concentrated at certain times of each quarter. To capture the seasonal dependence in the annual dividend series we estimate an unrestricted AR(12) model for Δd_t as in equation (9.4) with $\delta = 0$ and $\gamma = 0$. It is certainly possible that a seasonal ARMA model might provide a more parsimonious representation, but as a data generating process the unrestricted AR(12) model conveniently captures the own temporal dependencies in the conditional mean of the dividend series.

The residuals from this AR(12) model are uncorrelated, but they are clearly not independently distributed through time. Strong seasonality remains in the uncertainty associated with dividend growth, as manifest by highly significant autocorrelations of the squared residuals at the seasonal lags.[6] To account for this feature of the dividend process, we estimated a restricted ARCH(12) model for the conditional variance. ARCH models were first introduced by Engle (1982) and have subsequently found very wide use in the modeling of volatility clustering in high frequency financial data. This is discussed further in section 3. The maximum likelihood estimates for the AR(12)-ARCH(12) model for Δd_t obtained under the assumption of conditional normality for the sample period January 1928 through December 1987 are

$$\underset{(0.00038)}{\Delta d_t = 0.00033} - \underset{(0.048)}{0.035}\, \Delta d_{t-1} + \underset{(0.027)}{0.053}\, \Delta d_{t-2} + \underset{(0.062)}{0.337}\, \Delta d_{t-3} + \underset{(0.032)}{0.065}\, \Delta d_{t-4}$$

$$+ \underset{(0.023)}{0.018}\, \Delta d_{t-5} + \underset{(0.061)}{0.121}\, \Delta d_{t-6} + \underset{(0.026)}{0.010}\, \Delta d_{t-7} - \underset{(0.021)}{0.019}\, \Delta d_{t-8} + \underset{(0.036)}{0.182}\, \Delta d_{t-9}$$

$$- \underset{(0.026)}{0.009}\, \Delta d_{t-10} + \underset{(0.021)}{0.012}\, \Delta d_{t-11} - \underset{(0.058)}{0.238}\, \Delta d_{t-12} + \varepsilon_t \tag{9.5}$$

$$\varepsilon_t | I_{t-1} \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \underset{(0.000008)}{0.000042} + \underset{(0.078)}{0.0105}\, \varepsilon_{t-1}^2 + \underset{(0.075)}{0.289}\, \varepsilon_{t-3}^2 + \underset{(0.085)}{0.164}\, \varepsilon_{t-6}^2 + \underset{(0.113)}{0.287}\, \varepsilon_{t-12}^2$$

The notation I_{t-1} refers to the information set consisting of the past history of the dividend process. Robust asymptotic standard errors as in Bollerslev and Wooldridge (1992), which are discussed in section 3, are reported in parentheses. Notice that the seasonal AR and ARCH coefficients are all statistically important. Also, standard summary statistics, available upon request, indicate that this relatively simple time series model provides a good description of the own temporal dependencies in Δd_t.[7] We use this model throughout the chapter to forecast future dividends. Of course, agents may have a much richer information structure underlying their forecasts. This will generally lead to a more variable fundamental price series for a particular model of the return generating process as documented in LeRoy and Parke (1992).

The fundamental price series defined in equation (9.1) depends on forecasts of the levels of the future dividends, D_t, as opposed to the growth rates modeled in equation (9.5). Using a first-order Taylor series expansion, it follows that

$$E_t(D_{t+j}) \simeq \exp[E_t(d_{t+j}) + 0.5\mathrm{var}_t(d_{t+j})], \quad j = 1, 2, \ldots. \tag{9.6}$$

Equation (9.6) is satisfied exactly if the dividend growth rate is conditionally normally distributed. Closed-form expressions for $E_t(d_{t+j})$ and $\mathrm{var}_t(d_{t+j})$ are available by expressing the AR(12)-ARCH(12) model in first-order companion form, as in Baillie and Bollerslev (1992). These expressions are presented in the appendix.

In practice, the infinite sum in equation (9.1) is necessarily truncated at some value J.[8] Since the process for dividends is difference stationary, it follows that for large values of j,

$$\begin{aligned} E_t(d_{t+j+1}) &\simeq E_t(d_{t+j}) + \eta, \\ \mathrm{var}_t(d_{t+j+1}) &\simeq \mathrm{var}_t(d_{t+j}) + \xi, \end{aligned} \qquad j = J, J+1, \ldots, \tag{9.7}$$

where $\eta = E(\Delta d_t)$ denotes the unconditional expected real growth rate, and ξ denotes the unconditional increase in the prediction error uncertainty at long horizons. For the model estimates reported in equation (9.5), $\eta = 0.00066$ and $\xi = 0.00106$.[9] Also, suppose that the expected future discount rate associated with distant dividends is approximately constant,

$$E_t(\rho_{t+j+1}) \simeq \rho = \exp(-r), \quad j = J, J+1, \ldots, \tag{9.8}$$

In the implementation, we took $J = 120$, corresponding to a forecast horizon of ten years. Some informal sensitivity analysis revealed almost no change in the results with a longer truncation lag. Combining equations (9.1), (9.6), (9.7), and (9.8), the fundamental price may be approximated by

$$\begin{aligned}
P_t^{\mathrm{f}} &\simeq \sum_{j=1}^{J-1}\left(\prod_{i=1}^{j} E_t(\rho_{t+i})\right)\exp(E_t(d_{t+j}) + 0.5\mathrm{var}_t(d_{t+j})) \\
&\quad + \sum_{j=J}^{\infty}\left(\rho^{j-J}\prod_{i=1}^{J} E_t(\rho_{t+i})\right)\exp(E_t(d_{t+J}) + 0.5\mathrm{var}_t(d_{t+J}))\exp((\eta + 0.5\xi)(j-J)) \\
&= \sum_{j=1}^{J-1}\left(\prod_{i=1}^{j} E_t(\rho_{t+i})\right)\exp(E_t(d_{t+j}) + 0.5\mathrm{var}_t(d_{t+j})) \\
&\quad + \left(\prod_{i=1}^{J} E_t(\rho_{t+i})\right)\exp(E_t(d_{t+J}) + 0.5\mathrm{var}_t(d_{t+J}))\sum_{j=0}^{\infty}(\rho\exp(\eta + 0.5\xi))^j \\
&= \sum_{i=1}^{J-1}\left(\prod_{j=1}^{j} E_t(\rho_{t+i})\right)\exp(E_t(d_{t+j}) + 0.5\ \mathrm{var}_t(d_{t+j})) \\
&\quad + \left(\prod_{i=1}^{J} E_t(\rho_{t+i})\right)\exp(E_t(d_{t+J}) + 0.5\ \mathrm{var}_t(d_{t+J}))(1 - \rho\exp(\eta + 0.5\xi))^{-1}
\end{aligned} \tag{9.9}$$

where for the last equality to hold true it is assumed that $\rho^{-1} = \exp(r) > \exp(\eta + 0.5\xi)$. Note that a sufficient condition for the validity of the approximation in equation (9.9) is that the expected future discount rates and dividend growth rates are conditionally uncorrelated, which is trivially satisfied with a constant discount rate.

We simulate five different price series using the present value relationship in equation (9.9) and the estimated model for the dividend growth rate in equation (9.5). Each of the models is simulated 1,000 times. The Null1 prices are calculated under the assumption of a constant discount rate,

$$E_t(\rho_{t+j}) = \rho = \exp(-r), \quad j = 1, 2, \ldots, \tag{9.10}$$

and no ARCH effects in the dividend series, i.e. ε_t iid normally distributed with a variance equal to the implied unconditional variance from equation (9.5). The discount rate is set at $r = 0.00635$, corresponding to the 7.9% real annual return on the NYSE value-weighted index over the sample period underlying the estimation results for the dividend model in equation (9.5). The Null1 model is the standard present value relationship typically employed in volatility tests, most of which use annual data. The use of monthly data provides a richer representation of the data generation process and possibly avoids serious temporal aggregation bias.

Following Shiller (1981b; 1984), Summers (1986), and Poterba and Summers (1988) we also consider a fads model as a possible explanation for the actual empirical findings. According to the Fads alternative, the market price differs from the fundamental price by a highly serially correlated fad. More formally, let P_t^{f} denote the fundamental price under Null1 as described above. The Fads price series is then generated according to

$$P_t = \exp\left(\ln(P_t^{\mathrm{f}}) + \ln(F_t)\right), \tag{9.11}$$

where $\ln(F_t)$ follows the AR(1) process,

$$\ln(F_t) = \phi \ln(F_{t-1}) + u_t, \tag{9.12}$$

with u_t iid normally distributed. In the simulations, $\phi = 0.98$, and σ_u^2 equals 0.330 times the unconditional variance of $\Delta\ln(P_t^{\mathrm{f}})$ for the particular sample realization. With these parameter choices the process for the fad accounts for 25% of the unconditional variance in the change in logarithmic price.[10]

This particular parametric specification was chosen following Poterba and Summers (1988). The parameter choices differ from other studies such as West (1988) who uses annual S&P data from 1871 to 1985 to estimate the parameters of a fads model analogous to equations (9.11) and (9.12). West (1988) assumes that d_t follows a random walk with drift and that the fundamental rate of return is a constant. Thus, the fundamental price/dividend ratio would be constant. In order to match the sample means and variances of Δd_t and $p_t - d_t$ as well as the first-order autocorrelation of $p_t - d_t$ over this longer period of annual data, the fad must account for 57% of the variance of Δp_t.

In the Bubble alternative, the price differs from the Null1 present value relationship by a self-fulfilling speculative bubble. That is,

$$P_t = P_t^{\mathrm{f}} + B_t, \tag{9.13}$$

where B_t earns the required real rate of return, r,

$$E_{t-1}(B_t) = B_{t-1}\exp(r). \tag{9.14}$$

Notice that the existence of such bubbles violates the transversality condition underlying the fundamental pricing condition in equation (9.1). Stochastic collapsing bubbles were introduced by Blanchard and Watson (1982). The bubble continues with a certain conditional probability and collapses otherwise, where the probability-weighted average of these two events must satisfy equation (9.14). The bubble simulated here takes the form

$$\begin{aligned} B_t = {} & I_{z_t \leq \pi_{t-1}}(\pi_{t-1})^{-1}[\exp(r)B_{t-1} - (1-\pi_{t-1})(0.1P_{t-1}^{\mathrm{f}})]\exp(v_t - 0.5\sigma_v^2) \\ & + I_{z_t > \pi_{t-1}}(0.1P_{t-1}^{\mathrm{f}}). \end{aligned} \tag{9.15}$$

In equation (9.15) the probability of the bubble continuing is denoted π_{t-1}. If the bubble bursts, it does not collapse to zero but begins again at a value of 0.1 times the fundamental price, P_t^{f}.[11] To generate stochastic bubbles we drew an iid random variable z_t from the uniform distribution on the unit interval. The indicator variable $I_{z_t \leq \pi_{t-1}}$ signifies whether $z_t \leq \pi_{t-1}$. If the bubble continues, an innovation in the bubble is generated from v_t. By assumption, v_t is iid normally distributed with mean zero and variance σ_v^2, so that $E_{t-1}[\exp(v_t - 0.5\sigma_v^2)] = 1$. We allow the probability that the bubble will collapse to depend explicitly on the current deviation from the fundamental price,

$$1 - \pi_t = 2[1 - \Phi(P_t^{\mathrm{f}}/B_t)], \tag{9.16}$$

where $\Phi(\cdot)$ refers to the cumulative standard normal distribution function. The larger the bubble relative to the fundamental price, the greater the chance of a collapse. In the simulations we set $\sigma_v^2 = 0.0009$. This parameterization of the bubble process led to an average of eight collapsing bubbles, a minimum of two and a maximum of 16, across the 1,000 simulations of 720 months of data.[12]

The Null2 model incorporates the serial dependence in the conditional variances into the optimal forecasts for the levels of future dividends using the forecast formula for the conditional variances based on model (9.5). Details of these calculations are in the appendix. Given the strong conditional heteroskedasticity in the data, we think it is important in simulations to explicitly account for higher-order moment dependencies in the fundamentals in a pricing relationship as illustrated in equation (9.9). In accordance with the Null1, Fads and Bubble series, the Null2 alternative maintains the assumption of a constant discount rate, $r = 0.00635$.

The final model develops a crude time-varying risk premium or TVRP, price series. It extends the Null2 alternative by allowing for a variable discount rate. In order to keep things relatively simple and to avoid the use of additional data series, we do not work directly with any formal structural model of the variable discount rate. Instead, we simply postulate that the discount rate is a linearly increasing function of the change in the prediction error uncertainty associated with future values of the fundamental dividend process. Specifically, in equation (9.2) we set

$$r_{t+i} = \lambda[\mathrm{var}_t(d_{t+i}) - \mathrm{var}_t(d_{t+i-1})], \quad i = 1, 2, \ldots, \tag{9.17}$$

where $\lambda = 6.017$. This choice of λ ensures that the unconditional discount rate associated with long-horizon predictions converges to the sample real returns employed in the other simulations.

3 Short-Horizon Returns

In this section we review some of the time series techniques and test statistics used to examine the short-run temporal dependencies in asset returns and their relation to the market efficiency hypothesis. It is generally accepted that most high frequency returns are approximately linearly unpredictable, although this is not a requirement of an efficient market. It is also well recognized that returns are characterized by volatility clustering and leptokurtic unconditional distributions. The documentation of these facts dates back to at least Mandelbrot (1963) and Fama (1965).

The first column in table 9.1 reports a number of summary statistics for the real monthly value-weighted NYSE percentage rates of return for the sample period January 1928 through December 1987. These returns are denoted by R_t throughout the chapter. As noted in the previous section, the average real monthly return on the index over this period was 0.635%, or 7.9% on an annual basis. The realized return is very variable around this mean return, however, with a monthly variance of 33.5. Multiplying this monthly variance by 12, as if the returns are serially uncorrelated, produces an annualized

Table 9.1 Real monthly percentage returns, short-horizon summary statistics

	Data	*Asymp.*	*Null1*	*Fads*	*Bubble*	*Null2*	*TVRP*
μ	0.635	–	0.636	0.654	0.624	0.634	0.694
	(0.216)		[0.503]	[0.567]	[0.458]	[0.496]	[0.670]
σ^2	33.5	–	10.5	14.0	14.2	9.12	22.8
	(1.77)		[0.000]	[0.000]	[0.047]	[0.025]	[0.282]
b_3	0.389	0.000	0.097	0.112	−5.35	0.818	−1.32
	(0.091)	[0.000]	[0.000]	[0.000]	[0.003]	[0.893]	[0.126]
b_4	10.6	3.00	2.99	2.99	75.2	6.76	16.1
	(0.182)	[0.000]	[0.000]	[0.000]	[0.897]	[0.259]	[0.832]
ρ_1	0.116	0.000	−0.002	−0.004	−0.018	−0.003	−0.012
	(0.037)	[0.001]	[0.000]	[0.001]	[0.023]	[0.012]	[0.020]
Q_{12}	32.3	11.3	11.6	11.1	10.4	19.4	32.2
	(4.90)	[0.001]	[0.002]	[0.001]	[0.023]	[0.199]	[0.499]
$\rho_1^{(2)}$	0.289	0.000	−0.001	−0.003	0.005	0.077	0.045
	(0.037)	[0.000]	[0.000]	[0.000]	[0.007]	[0.023]	[0.013]
$Q_{12}^{(2)}$	398.4	11.3	11.3	11.1	0.896	172.1	190.2
	(4.90)	[0.000]	[0.000]	[0.000]	[0.002]	[0.064]	[0.139]

The sample statistics are the mean, μ, the variance, σ^2, the skewness, b_3, the kurtosis, b_4, the first-order autocorrelations for returns, ρ_1, and for squared returns, $\rho_1^{(2)}$. The Ljung-Box portmanteau tests for up to 12th-order serial correlation in the levels of returns and the squared returns are denoted Q_{12} and $Q_{12}^{(2)}$, respectively. The "Data" column gives the sample statistics with asymptotic standard errors constructed under the null hypothesis of iid normally distributed constant expected returns in parentheses. The column labeled "Asymp." reports the medians and the p-values for this null hypothesis. The last five columns give the small sample or empirical medians of the sample statistics from the five Monte Carlo experiments with the corresponding p-values of the sample test statistic reported in square brackets.

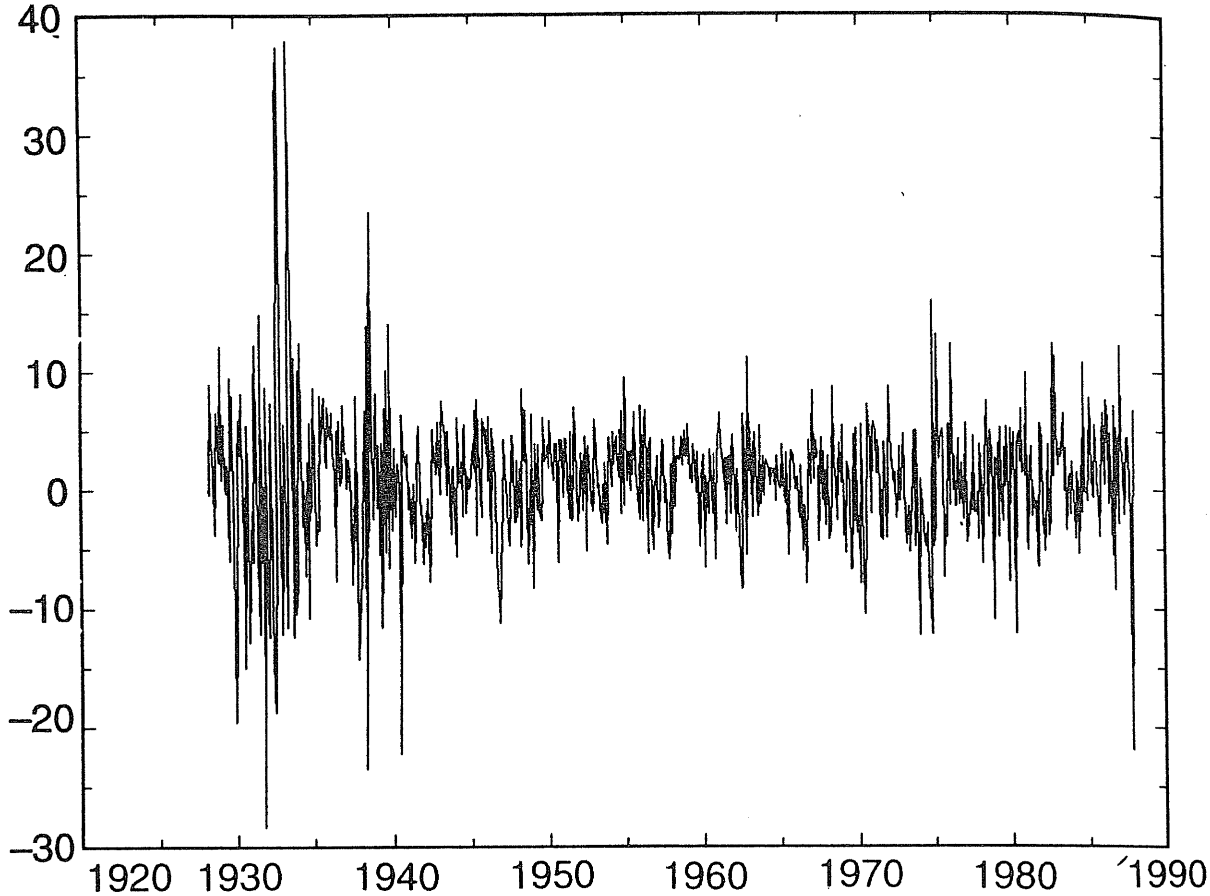

Figure 9.2 Value-weighted monthly real returns

standard deviation of 20.1%. This high degree of return variability is also obvious from the plot in figure 9.2.

The sample skewness and kurtosis coefficients are reported in the next two rows of table 9.1.[13] The column labeled "Asymp." provides the medians under the null hypothesis of iid normally distributed returns, i.e. $b_3 = 0$ and $b_4 = 3$. The p- values in the asymptotic distributions are reported in square brackets.[14] The results indicate positive skewness and very pronounced leptokurtosis in the sample unconditional distribution of returns.

The first-order sample autocorrelation coefficient is denoted by ρ_1. Although statistically significant, the estimated value implies that only 1.3% of the total variation in the return is explainable from last month's return alone. Furthermore, the predictability of this index return may, in part, be attributed to a non-synchronous trading phenomenon, as discussed in Lo and MacKinlay (1988) and Fama (1991). The next row in table 9.1 reports the Ljung and Box (1978) portmanteau test for up to 12th- order serial correlation, Q_{12}.[15] The second column indicates that this test statistic is highly significant at conventional levels in the asymptotic chi-square distribution under the null hypothesis of iid observations. As noted by Diebold (1986) and Cumby and

Huizinga (1992), however, the presence of conditional heteroskedasticity or excess kurtosis will bias the portmanteau test towards over-rejection of the less restrictive null hypothesis of uncorrelated returns. Also, excluding the first lag, the test for the joint significance of lags 2 through 12 equals only 22.6.

Recently, a number of authors have employed variance ratio statistics,

$$v(k) \equiv \mathrm{var}(R_{t+k-1} + R_{t+k-2} + \ldots + R_t)/[k\ \mathrm{var}(R_t)], \tag{9.18}$$

as an alternative way of summarizing own temporal dependencies, particularly at horizons longer than one year. If the returns at horizon k are dominated by positive autocorrelation, the variance ratio will be greater than one, whereas predominantly negative autocorrelation results in a variance ratio below one.[16] Consistent with the results reported in Lo and MacKinlay (1988), the variance ratio at the one-year horizon for the real value-weighted NYSE returns equals 1.151. This indicates only minor and statistically insignificant positive short-run own dependencies in returns. We defer discussion of longer-run dependencies to section 4.

In contrast to the weak evidence for autocorrelation in returns, the last two rows in table 9.1 highlight the importance of conditional heteroskedasticity. The first-order autocorrelation coefficient for the squared returns, $\rho_1^{(2)}$, and the portmanteau test for up to 12th-order serial correlation in the squared returns, $Q_{12}^{(2)}$, are highly significant at any reasonable level. The variance ratio statistic for the squared returns equals 3.027 at the one-year horizon, indicating very strong positive dependence. This pronounced dependence in the second-order moments is immediately evident from figure 9.2. It is worth noting that this finding of strong dependence in the even-ordered moments does not necessarily imply market inefficiency. The presence of high frequency volatility clustering is perfectly consistent with a martingale hypothesis for stock prices which is implied by the assumption of a constant short-run expected rate of return.[17] In addition, though, modern asset pricing theories rely crucially on time variation in the second-order moments of returns and market fundamentals as sources of rational time-varying risk premiums, e.g. Abel (1988), Hodrick (1989), and Bekaert (1992). We return to this issue in more detail below.

We next consider the simulation results for the various price processes discussed in section 2 as possible explanations of these empirical findings. The last five columns of table 9.1 report the medians for the sample statistics under the different data generating mechanisms along with the p-values of the statistics in the simulated distributions over the 1,000 replications. If the sample statistic does not fall within the empirical distribution generated by a particular model, we will judge the model as being inconsistent with the data along that dimension. We note that this interpretation of the test statistics does not incorporate parameter estimation uncertainty for the simulated models. It is possible that inclusion of this additional source of variation could overturn our assessment regarding the shortcomings of a model.

The Null1 constant discount rate model with no ARCH effects and the Fads alternative are both unable to explain the magnitude of the unconditional variance of returns. The medians of the Null2 and Bubble distributions are also considerably lower than the sample statistic of 33.5, although the distributions from these models are more dispersed.[18] Only for the TVRP alternative is the p-value relatively large, 0.282, and the median of the simulated distribution close to the sample unconditional one-month return variance. Similarly, neither the Null1 nor the Fads alternative with normally distributed errors is able to explain the non-normality of returns. In contrast, the Bubble specification results in a considerably higher median kurtosis coefficient than the sample analog. The simulated Null2 and TVRP alternatives are both consistent with the actual data regarding the unconditional sample kurtosis.

None of the five models produces any positive first-order autocorrelation in the medians of the test statistics. Both the Null2 and TVRP alternatives, however, lead to test statistics for the joint significance of the first 12 autocorrelations, Q_{12}, that are broadly consistent with the actual data.

As noted above, a striking feature of the monthly returns is volatility clustering. Although the portmanteau statistic for the 12 autocorrelations of the squared returns is inconsistent with the null hypothesis of iid observations in the asymptotic distribution and with the simulated Null1, Fads, and Bubbles models, the Null2 and TVRP price series yield test statistics that exceed the sample value of 398.4 in 6.4% and 13.9% of the replications, respectively.

In order to better understand the nature of this volatility clustering, we follow Bollerslev (1986; 1987) and estimate an MA(1)-GARCH(1,1) model for the monthly real percentage rates of return:

$$\begin{aligned} R_t &= \mu + \theta\varepsilon_{t-1} + \varepsilon_t \\ E_{t-1}(\varepsilon_t) &= 0, \\ E_{t-1}(\varepsilon_t^2) \equiv \sigma_t^2 &= \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2. \end{aligned} \tag{9.19}$$

This relatively simple non-linear time series model provides a useful characterization of the temporal variation in the second-order moments of returns for a wide variety of financial assets.

A number of alternative estimation schemes are available for the model in equation (9.19). The estimation results reported below are all obtained under the auxiliary distributional assumption of conditional normality; i.e. $\varepsilon_t | I_{t-1} \sim N(0, \sigma_t^2)$, where I_{t-1} refers to the information set at time $t-1$. In particular, if $\Xi' \equiv (\mu, \theta, \omega, \alpha, \beta)$ denotes the vector of unknown parameters, it follows by a standard prediction error decomposition argument that conditional on the initial observations, the quasi-log-likelihood function for the sample realizations $\{R_1, R_2, \ldots R_T\}$ takes the form

$$L_T(\Xi) = \sum_{t=1}^{T} l_t(\Xi) = \sum_{t=1}^{T} -0.5(\ln(\pi) + \ln(\sigma_t^2) + \varepsilon_t^2\sigma_t^{-2}). \tag{9.20}$$

While standard maximum likelihood theory requires the correct distributional assumptions, asymptotic standard errors that remain valid in the presence of conditional non-normality may be calculated from the matrix of the outer products of the gradients post-and pre-multiplied by an estimate of the hessian; see e.g. Domowitz and White (1982) and Weiss (1986). A simple expression for this estimator in the context of dynamic models with conditional heteroskedasticity that involves only first derivatives is given in Bollerslev and Wooldridge (1992).[19] The quasi-maximum-likelihood estimates obtained by maximizing equation (9.20) are given in the first column of table 9.2, with robust standard errors in parenthesis.

The significant MA(1) term in the conditional mean captures most of the autocorrelation in the returns. The Q_{12} portmanteau test for significant

Table 9.2 Real monthly returns, MA(1)-GARCH(1,1) quasi-maximum-likelihood estimates:

$$R_t = \mu + \theta\varepsilon_{t-1} + \varepsilon_t, \quad \sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \quad \varepsilon_t|I_{t-1} \sim N(0, \sigma_t^2)$$

	Data	Asymp.	Null2	TVRP
μ	0.778	–	0.643	0.431
	(0.184)		[0.092]	[0.002]
θ	0.101	–	–0.004	–0.002
	(0.044)		[0.006]	[0.030]
ω	0.668	–	0.233	0.563
	(0.328)		[0.008]	[0.383]
α	0.123	–	0.108	0.127
	(0.023)		[0.357]	[0.523]
β	0.861	–	0.865	0.853
	(0.026)		[0.554]	[0.435]
b_3	–0.547	0.000	0.555	–1.89
	(0.091)	[1.00]	[1.00]	[0.000]
b_4	4.39	3.00	4.39	10.5
	(0.182)	[0.000]	[0.499]	[1.00]
Q_{12}	14.2	11.3	11.0	12.6
	(4.90)	[0.288]	[0.263]	[0.396]
$Q_{12}^{(2)}$	11.8	11.3	64.9	22.5
	(4.90)	[0.462]	[0.984]	[0.804]

The "Data" column reports the quasi-maximum-likelihood estimates for the actual returns with robust asymptotic standard errors in parentheses. The summary statistics for the standardized residuals, $\hat{\varepsilon}_t\hat{\sigma}_t^{-1}$, are denoted as in table 9.1. The "Asymp." column reports the medians and the p-values in the asymptotic distribution under the null hypothesis of iid normally distributed standardized innovations. The Null2 and TVRP columns provide the medians and the p-values from the quasi-maximum-likelihood estimates for the data generated under the two hypotheses. The p-values are reported in square brackets.

autocorrelations within a year drops from 32.3 for the raw data to 14.2 for the standardized residuals, $\hat{\varepsilon}_t \hat{\sigma}_t^{-1}$, from the model in (9.19). The estimate for $\alpha + \beta$ indicates a very long memory in the conditional variance. The implied half-life of a shock to the conditional variance equals $\ln(1/2)/\ln(\hat{\alpha} + \hat{\beta}) = 43.0$ months. This high degree of persistence corresponds to the findings for a large number of other financial assets as noted by Bollerslev and Engle (1993). The parameterization for the conditional variance in equation (9.19) does a very good job of tracking the strong temporal dependence in the variance.[20] The $Q_{12}^{(2)}$ portmanteau test statistic for the squared standardized residuals, $\hat{\varepsilon}_t^2 \hat{\sigma}_t^{-2}$, equals 11.8 compared to 398.4 for the raw squared returns.

The negative skewness coefficient for $\hat{\varepsilon}_t \hat{\sigma}_t^{-1}$ in table 9.2 contrasts sharply with the positive value for b_3 reported in table 9.1. Negative skewness is consistent with the so-called leverage effect in which volatility increases with bad news but decreases with good news, as analyzed by Black (1976) and Christie (1982). This observation also provides one of the primary motivations behind the exponential GARCH model in Nelson (1991) that allows both the sign and the magnitude of past shocks to influence the conditional variance.[21]

Even though the GARCH(1,1) model does a very good job of capturing the dependence in the second-order moments, the temporal variation in the conditional variance does not explain all of the leptokurtosis in the data, necessitating the use of robust inference procedures. Again, this is not unique to the present return series. As an alternative to the robust quasi-maximum-likelihood procedures, the conditional distribution for $\varepsilon_t \sigma_t^{-1}$ could be parameterized directly or estimated by non-parametric methods, as discussed by Bollerslev (1987), Gallant and Tauchen (1989), and Engle and Gonzalez-Rivera (1991).

The last two columns of table 9.2 report the results of estimating the same MA(1)-GARCH(1,1) model in (9.19) for each of the 1,000 realizations of the 720 monthly simulated Null2 and TVRP returns. These are the only two models that exhibit significant heteroskedasticity as evidenced by the $Q_{12}^{(2)}$ statistic in table 9.1. The similarity between the estimated GARCH coefficients for the artificially generated returns and the actual data is striking. The median values of the quasi-maximum-likelihood estimates for α over the 1,000 replications for the Null2 and TVRP alternatives are 0.108 and 0.127, respectively, compared to 0.123 for the real data. Similarly, the median values for the estimates of β are 0.865 and 0.853, respectively, compared to 0.861 for the actual returns. Note also that although the Null2 alternative is unable to explain the negative skewness in the standardized returns, the TVRP hypothesis results in excess negative skewness and excess leptokurtosis compared to the real data.

In summary, the results in table 9.2 illustrate how explicitly allowing for time-varying uncertainty in the fundamental real dividend growth process within the context of a simple present value relationship may endogenously account for the observed ARCH effects in the data. Reconciliation of a fundamental model and the volatility of short-horizon real returns appears to require time variation in the discount factor.

4 Long-Horizon Tests

The previous section documents that the evidence in autocorrelations of short-horizon returns against the hypothesis of a constant conditional mean return is not very strong. At the same time, the results indicate that returns appear to be too volatile relative to the models with a constant discount factor. Shiller (1981b; 1984) and Summers (1986) argued that fads would make returns more volatile. However, it would be difficult to detect a fads alternative hypothesis, as developed above, with short-horizon autocorrelation tests because of their low power when transitory components are very highly serially correlated. As noted by Poterba and Summers (1988) and Fama and French (1988a; 1988b), the negative serial correlation in returns implied by such a model would manifest itself more transparently at longer horizons. Consequently, Poterba and Summers (1988) investigated long-horizon variance ratios as in equation (9.18), while Fama and French (1988a) analyzed regressions of long-horizon returns on lagged long-horizon returns. Of course, if the researcher has a specific alternative hypothesis in mind, such as the parametric Fads model in equations (9.11) and (9.12), the asymptotically most powerful procedure would be to estimate the resulting parametric model for one-period returns by maximum likelihood using the full data set. Nothing can be gained by lengthening the return horizon.

The statistical properties of the Fama and French (1988a) analysis have generated much controversy in the literature. For example, Jegadeesh (1990), Kim, Nelson, and Startz (1991), Mankiw, Romer, and Shapiro (1991), Richardson (1993), and Richardson and Stock (1989) all argue that the case for predictability of long-horizon stock returns is weak when one corrects for the small sample biases in the test statistics. Our simulations demonstrate these biases. We then present two additional ways that hypotheses regarding the long-horizon predictability of returns can be investigated which are not as severely biased. These techniques apply generally in other long-horizon forecasting situations.

To begin, let $\ln(R_{t+k,k}) = \ln(R_{t+1}) + \ldots + \ln(R_{t+k})$ denote the continuously compounded k-period rate of return. Then, a typical OLS specification of Fama and French (1988b) is the following:

$$\ln(R_{t+k,k}) = \alpha_{k,k} + \beta_{k,k}\ln(R_{t,k}) + u_{t+k,k}. \tag{9.21}$$

Note that the k-period error term $u_{t+k,k}$ is not realized until time $t+k$. Therefore, if the data are sampled more finely than the compound return interval, $u_{t+k,k}$ is serially correlated, even under the null hypothesis of constant expected returns. If all of the data are employed, $u_{t+k,k}$ is correlated with

$k-1$ previous error terms as discussed in Hansen and Hodrick (1980).[22] Under alternative hypotheses in which returns have a variable conditional mean, however, $u_{t+k,k}$ can be arbitrarily serially correlated if lagged returns do not capture all of the variation in the conditional mean.

Since lagged returns are predetermined but not strictly exogenous, asymptotic distribution theory must be used to determine the properties of an estimator for $\kappa_{k,k} = (\alpha_{k,k}, \beta_{k,k})'$. Ordinary least squares provides consistent estimates, but traditional OLS standard errors are not appropriate asymptotically since the error term is serially correlated when forecasting more than one period ahead. Furthermore, the variability of the conditional variance of returns, documented in section 3, makes it inappropriate to assume homoskedasticity which underlies the derivation of the conventional OLS standard errors.

Nevertheless, the OLS coefficient estimator from equation (9.21), $\hat{\kappa}_{k,k}$, is readily interpreted as a generalized method of moments (GMM) estimator based on the instruments $x_t = (1, R_{t,k})'$. Hence, it is straightforward to derive appropriate asymptotic standard errors. Following Hansen (1982), it can be demonstrated that $(\sqrt{T})(\hat{\kappa}_{k,k} - \kappa_{k,k}) \sim N(0, \Omega)$, where $\Omega = Z_0^{-1} S_0 Z_0^{-1}$, $Z_0 = E(x_t x_t')$, and S_0 denotes the spectral density evaluated at frequency zero of $w_{t+k,k} = u_{t+k,k} x_t$. Under the null hypothesis that the returns are not predictable,

$$S_0 = \sum_{j=-k+1}^{k-1} E(w'_{t+k,k} w_{t+k-j,k}). \tag{9.22}$$

This matrix is consistently estimated by

$$S_T^a = C_T(0) + \sum_{j=1}^{k-1} [C_T(j) + C_T(j)'], \tag{9.23}$$

where $C_T(j) = (1/T)\Sigma_{t=j+1}^{T}(\hat{w}_{t+k,k}\hat{w}'_{t+k-j,k})$, and $\hat{w}_{t,k}$ denotes $w_{t,k}$ evaluated at the estimated residuals. It is important to note that the estimator in equation (9.23) is not guaranteed to be positive definite in finite samples. Various weighting schemes may be imposed on the terms in equation (9.23) to guarantee that the estimator is positive definite; see Newey and West (1987; 1994) and Andrews (1991). In all of the simulations reported below, we employed the unweighted estimator, and in no case was the estimated covariance matrix negative semi-definite. A consistent estimator for Z_0 is given by $Z_T = (1/T)\Sigma_{t=1}^{T} x_t x_t'$.

We present the results for this estimation of equation (9.21) for five horizons in table 9.3a. The first column indicates the horizon k equal to 1, 12, 24, 36, and 48 months. As in the previous tables, the second column labeled "Data" provides the sample statistics, which in this case are the OLS estimates of the slope coefficients, $\beta_{k,k}$, with asymptotic standard errors in parentheses. The medians of the t-tests for the hypothesis $\beta_{k,k} = 0$ from the asymptotic distribution and the five simulated economies are presented in the next six columns. The p-values of the test statistics are in square brackets.

Notice that at the 24-month horizon, the asymptotic p-value is effectively zero, while the one-month and 36-month values both equal 0.162. Richardson (1993) notes that considerable care must be exercised when examining the individual test statistics since they are highly correlated. Hence, we also report the $\chi^2(5)$ statistic that examines the joint test that all five slope coefficients are zero. It has a value of 15.8, which corresponds to an asymptotic p-value of 0.007. Clearly, if the asymptotic distributions are validated by the simulations, this would be strong evidence of long-horizon return predictability.

Examination of the columns for Null1 and Null2 indicates that the asymptotic distribution is a poor approximation to the distribution of the test

Table 9.3 Real monthly returns, multi-period regressions:

$$\ln(R_{t+k,k}) = \alpha_{k,k} + \beta_{k,k}\ln(R_{t,k}) + u_{t+k,k}$$

(a) Traditional GMM standard errors

k	*Data*	*Asymp.*	*Null1*	*Fads*	*Bubble*	*Null2*	*TVRP*
Slope coefficients							
1	0.095	0.000	−0.064	−0.127	−0.359	−0.098	−0.251
	(0.068)	[0.162]	[0.145]	[0.158]	[0.108]	[0.175]	[0.217]
12	−0.068	0.000	−0.244	−0.511	−0.801	−0.324	−0.713
	(0.140)	[0.626]	[0.649]	[0.690]	[0.741]	[0.680]	[0.730]
24	−0.200	0.000	−0.363	−0.637	−0.768	−0.507	−1.14
	(0.044)	[0.000]	[0.006]	[0.014]	[0.008]	[0.012]	[0.018]
36	−0.205	0.000	−0.448	−0.738	−0.785	−0.553	−1.31
	(0.146)	[0.162]	[0.327]	[0.368]	[0.371]	[0.362]	[0.516]
48	−0.187	0.000	−0.611	−0.704	−0.884	−0.664	−1.40
	(0.227)	[0.408]	[0.577]	[0.610]	[0.612]	[0.590]	[0.737]
$\chi^2(5)$ *statistics*							
	15.8	4.35	8.61	9.77	10.6	11.1	14.3
		[0.007]	[0.237]	[0.278]	[0.311]	[0.324]	[0.460]
Coefficients of multiple correlation, R^2							
1	0.009		0.001	0.001	0.001	0.001	0.002
			[0.014]	[0.017]	[0.049]	[0.061]	[0.102]
12	0.005		0.006	0.006	0.011	0.008	0.014
			[0.535]	[0.554]	[0.661]	[0.606]	[0.712]
24	0.045		0.012	0.013	0.014	0.015	0.031
			[0.169]	[0.204]	[0.217]	[0.226]	[0.378]
36	0.058		0.018	0.021	0.020	0.021	0.042
			[0.228]	[0.270]	[0.235]	[0.245]	[0.410]
48	0.046		0.026	0.032	0.028	0.024	0.048
			[0.353]	[0.422]	[0.385]	[0.368]	[0.512]

Table 9.3 (*Cont.*)

(b) GMM standard errors calculated under the null hypothesis

k	*Data*	*Asymp.*	*Null1*	*Fads*	*Bubble*	*Null2*	*TVRP*
Slope coefficients							
1	0.095	0.000	−0.064	−0.127	−0.360	−0.098	−0.250
	(0.069)	[0.171]	[0.152]	[0.166]	[0.110]	[0.172]	[0.208]
12	−0.068	0.000	−0.232	−0.446	−0.481	−0.253	−0.587
	(0.170)	[0.688]	[0.691]	[0.717]	[0.675]	[0.678]	[0.748]
24	−0.200	0.000	−0.309	−0.472	−0.433	−0.345	−0.834
	(0.244)	[0.411]	[0.410]	[0.443]	[0.372]	[0.414]	[0.578]
36	−0.205	0.000	−0.331	−0.538	−0.487	−0.372	−0.836
	(0.223)	[0.357]	[0.380]	[0.397]	[0.344]	[0.368]	[0.504]
48	−0.187	0.000	−0.423	−0.600	−0.518	−0.449	−0.857
	(0.172)	[0.276]	[0.291]	[0.335]	[0.255]	[0.289]	[0.414]
$\chi^2(5)$ *Statistics*							
	3.37	4.35	1.48	1.68	1.50	1.30	1.84
		[0.644]	[0.373]	[0.380]	[0.382]	[0.341]	[0.396]

The "Data" column provides the sample statistics with asymptotic standard errors in parentheses. The "Asymp." column reports the medians and the p-values in the asymptotic distributions for testing the null hypothesis $\beta_{k,k} = 0$. The remaining five columns provide the medians from the empirical distributions for the sample statistics together with the p-values in square brackets.

statistics in small samples. The median values of the sample $\chi^2(5)$ statistics for these two simulated economies are 8.61 and 11.1, respectively, compared to the median of a true $\chi^2(5)$ of 4.35. The corresponding p-values from the two distributions are 0.237 and 0.324. Hence, there is actually little evidence against the null hypothesis. The deterioration of the t-tests is apparent as the horizon is increased, with the median values becoming increasingly negative.[23]

Notice also that the median and the p-value of the $\chi^2(5)$ statistic for the Bubble model are close to those of the Null1 and Null2 simulations. This is not surprising, as the Bubble model maintains a constant expected return. More surprisingly, perhaps, are the medians and p-values for the Fads alternative which are only slightly larger than those under the Null1 hypothesis. This latter finding illustrates the low power of these tests when type I error rates are fixed at the traditional levels. Given the large overlap in the distributions of the test statistics under the null and the alternative hypotheses, the probability of failing to reject the null hypothesis when it is actually false is quite high. Finally, notice that the p-value for the $\chi^2(5)$ statistic for the TVRP model is 0.460 which is the largest of any of the models.

The bottom part of table 9.3a reports the coefficients of multiple correlation, R^2 statistics, for the five horizons. Although the sample values increase from 0.009 at the one-month horizon to 0.058 at the 36-month horizon, the smallest p-values of the simulated economies are actually at the one-month horizon. The serial correlation of the residuals induced by using overlapping forecasting intervals causes a spurious regression phenomenon as in Granger and Newbold (1974).

We next develop an alternative estimator of S_0 that is valid only under the null hypothesis. This estimator utilizes the fact that the values of unconditional expectations of covariance stationary time series depend only on the time intervals between the observations.[24] In particular, notice that under the null hypothesis, $u_{t+k,k} = (e_{t+1} + \ldots + e_{t+k})$, where e_{t+1} is the serially uncorrelated one-step-ahead forecast error of returns. Estimates of e_{t+1} can be obtained from the residuals of a regression of $\ln(R_{t+1})$ on a constant because under the null hypothesis $e_{t+1} = u_{t+1,1}$. To derive the alternative estimator, examine a typical term, $E(w_{t+k,k}w'_{t+k-j,k})$, where $k > j > 0$. Substituting $(e_{t+1} + \ldots + e_{t+k})$ for $u_{t+k,k}$,

$$E(u_{t+k,k}x_t u_{t+k-j,k}x'_{t-j}) = E\left[\left(\sum_{i=1}^{k} e_{t+i}\right)x_t\left(\sum_{h=1-j}^{k-j} e_{t+h}\right)x'_{t-j}\right] = E\left[\left(\sum_{i=1}^{k-j} e^2_{t+i}\right)x_t x'_{t-j}\right] \tag{9.24}$$

With stationary time series, the unconditional expectation of each of the $(k-j)$ terms on the right-hand side of equation (9.24) depends only upon the time interval between the variables. Hence, rather than summing e_{t+j} into the future, one can sum $x_t x'_{t-j}$ into the past:

$$E\left[\left(\sum_{i=1}^{k-j} e^2_{t+i}\right)x_t x'_{t-j}\right] = E\left[e^2_{t+1}\left(\sum_{i=0}^{k-j-1} x_{t-i}x_{t-j-i}\right)\right]. \tag{9.25}$$

Applying the same logic to all of the terms in equation (9.22) implies that

$$S_0 = E\left[e^2_{t+1}\left(\sum_{i=0}^{k-1} x_{t-i}\right)\left(\sum_{i=0}^{k-1} x_{t-i}\right)'\right] = E(v_{t+1,k}v'_{t+1,k}) \tag{9.26}$$

where

$$v_{t+1,k} \equiv e_{t+1}\left(\sum_{i=0}^{k-1} x_{t-j}\right). \tag{9.27}$$

Let $\hat{v}_{t+1,k}$ denote $v_{t+1,k}$ evaluated at the estimated residual, $\hat{e}_{t+1}$. An alternative estimator for S_0 from equation (9.26) is then

$$S^{\mathrm{b}}_T = \frac{1}{T}\sum_{t=k}^{T} \hat{v}_{t+1,k}\hat{v}'_{t+1,k}. \tag{9.28}$$

Two aspects of the estimator S_T^b are important, and both are induced by the fact that it avoids the summation of autocovariance matrices as in equation (9.23). First, the estimator is guaranteed to be positive definite. Second, if it is the summation of the autocovariance matrices that causes the poor small sample properties of the test statistics in table 9.3a, the finite sample behavior of test statistics constructed with S_T^b might be better.

The properties of this estimator are investigated in table 9.3b. The point estimates of the slope coefficients reported in the "Data" column are identical, but the standard errors are different from these in table 9.3a. In particular, the test statistic at the 24-month horizon now has an asymptotic p-value of 0.411 and the $\chi^2(5)$ statistic is only 3.37, below the median value of 4.35 in the asymptotic distribution. However, 3.37 is actually larger than the median values of the small sample distributions for the test statistics from the Null1 and Null2 simulations. Also, notice that the deterioration of the median values of the t-test statistics as the horizon increases is mitigated only slightly.

The last part of this section demonstrates how inference about the statistical significance of lagged returns as predictors of long-horizon returns can be conducted by considering the regression of one-period returns on the weighted sum of the lagged returns.[25] This specification also avoids the summation of autocovariance matrices and may therefore have better small sample properties under the null hypothesis than the estimates based on equation (9.21).

Since the compound k-period return is the sum of k one-period returns, the numerator of the regression coefficient $\beta_{k,k}$ in equation (9.21) is an estimate of $\text{cov}[\Sigma_{j=1}^{k}\ln(R_{t+j});\ \Sigma_{j=0}^{k-1}\ln(R_{t-j})]$. This covariance is the weighted sum of $(2k-1)$ autocovariances of returns separated by between one and $(2k-1)$ periods. With covariance stationary time series it follows that

$$\text{cov}\left[\sum_{j=1}^{k}\ln(R_{t+j});\ \sum_{j=0}^{k-1}\ln(R_{t-j})\right] = \text{cov}\left[\ln(R_{t+1});\ \sum_{j=1}^{2k-1}\omega_j\ln(R_{t+1-j})\right], \tag{9.29}$$

where $\omega_j = j$ for $1 \leqslant j \leqslant k$, and $\omega_j = 2k - j$ for $(k+1) \leqslant j \leqslant (2k-1)$. The sum of the covariances on the right-hand side of equation (9.29) is equal to the numerator of the slope coefficient in the following regression:

$$\ln(R_{t+t}) = \alpha_{1,2k-1} + \beta_{1,2k-1}\left[\sum_{j=1}^{2k-1}\omega_j\ln(R_{t+1-j})\right] + u_{t+1}. \tag{9.30}$$

Under the null hypothesis of constant expected returns, the error term in equation (9.30) is serially uncorrelated in contrast to $u_{t+k,k}$ in equation (9.21). The asymptotic distribution of the OLS estimator for $\kappa_{1,h} = (\alpha_{1,h}, \beta_{1,h})'$ can be derived as above. Since only the term corresponding to $j = 0$ is different from zero in equation (9.22), this specification might have better small sample properties under the null hypothesis.

The results of estimating $\beta_{1,2k-1}$ in equation (9.30) are presented in table 9.4. The small sample properties of the test statistics are better than the results of the tests in table 9.3a and are comparable to those in table 9.3b. The deterioration in the medians of the individual test statistics is again quite evident, even though the p-values for the asymptotic distribution and for the Null1 and Null2 models are quite close. There is little evidence for predictability of returns. Again, notice from the closeness of the medians for the Fads

Table 9.4 Real monthly returns, one-period regressions:

$$\ln(R_{t+1}) = \alpha_{1,2k-1} + \beta_{1,2k-1}\left(\sum_{j=1}^{2k-1} \omega_j \ln(R_{t+1-j})\right) + u_{t+1}$$

$$\omega_j = j,\ 1 \leqslant j \leqslant k;\quad \omega_j = 2k-j,\ k+1 \leqslant j \leqslant 2k-1$$

k	*Data*	*Asymp.*	*Null1*	*Fads*	*Bubble*	*Null2*	*TVRP*
Slope coefficients							
1	0.034	0.000	−0.065	−0.100	−0.340	−0.072	−0.250
	(0.052)	[0.504]	[0.491]	[0.512]	[0.436]	[0.523]	[0.552]
12	−0.201	0.000	−0.249	−0.402	−0.465	−0.287	−0.593
	(0.171)	[0.239]	[0.260]	[0.242]	[0.187]	[0.236]	[0.319]
24	−0.279	0.000	−0.294	−0.459	−0.448	−0.343	−0.789
	(0.260)	[0.283]	[0.282]	[0.311]	[0.239]	[0.251]	[0.435]
36	−0.153	0.000	−0.349	−0.510	−0.464	−0.359	−0.819
	(0.321)	[0.633]	[0.641]	[0.649]	[0.605]	[0.637]	[0.760]
48	−0.161	0.000	−0.407	−0.580	−0.489	−0.407	−0.875
	(0.336)	[0.631]	[0.648]	[0.667]	[0.618]	[0.638]	[0.753]
$\chi^2(5)$ *statistics*							
	2.85	4.35	4.57	4.79	4.58	4.58	5.74
		[0.723]	[0.731]	[0.762]	[0.732]	[0.737]	[0.851]
Coefficients of multiple correlation, R^2							
1	0.001		0.001	0.001	0.001	0.001	0.001
			[0.403]	[0.427]	[0.495]	[0.533]	[0.586]
12	0.005		0.001	0.001	0.001	0.001	0.002
			[0.183]	[0.172]	[0.269]	[0.262]	[0.369]
24	0.003		0.001	0.001	0.001	0.001	0.002
			[0.258]	[0.277]	[0.271]	[0.289]	[0.462]
36	0.001		0.001	0.001	0.001	0.001	0.001
			[0.587]	[0.596]	[0.572]	[0.590]	[0.712]
48	0.000		0.001	0.001	0.001	0.001	0.001
			[0.634]	[0.649]	[0.589]	[0.634]	[0.733]

For additional notes see table 9.3.

model and the TVRP model that these tests are likely to have very low power. We note here that the results in this section only provide evidence regarding the own predictability of returns and do not demonstrate that returns are unpredictable when additional information is used. We return to this issue in section 6.

5 Cointegration Tests

Recently, a number of authors have proposed various tests for the presence of speculative bubbles and market efficiency that are based on the idea of cointegration. In light of space constraints and related coverage elsewhere in this volume, our discussion of these techniques is brief.[26] Intuitively, two time series are defined to be cointegrated if each of the individual series is non-stationary, yet a linear combination of the two series is stationary.

According to the present value model, the fundamental price must equal the expected discounted value of next period's fundamental price plus any dividend payouts. In particular, after deflating by D_t, we have

$$\frac{P_t^{\mathrm{f}}}{D_t} = E_t\left(\rho_{t+1}\left(\frac{P_{t+1}^{\mathrm{f}}}{D_{t+1}} + 1\right)\frac{D_{t+1}}{D_t}\right). \tag{9.31}$$

Assuming the transversality condition is satisfied, and using the approximation $D_{t+1}/D_t \simeq 1 + \Delta d_{t+1}$, it follows that

$$\frac{P_t^{\mathrm{f}}}{D_t} \simeq E_t\left(\sum_{j=1}^{\infty}\prod_{i=1}^{j}\rho_{t+i}(1 + \Delta d_{t+i})\right). \tag{9.32}$$

If the dividend growth rate, Δd_t, and the discount factor, ρ_t, are jointly covariance stationary, $p_t^{\mathrm{f}} \equiv \log(P_t^{\mathrm{f}})$ and d_t will be cointegrated with cointegrating vector $(1, -1)$; see Cochrane (1991a) for a formal proof. If the actual price contains a rational speculative bubble as in equation (9.13), the transversality condition underlying equation (9.32), would be violated for $P_t = P_t^{\mathrm{f}} + B_t$. In that situation, $p_t - d_t$ would be non-stationary. It is worth noting that this argument does not depend on any particular model for the equilibrium rate of return but only requires that the implied discount factor and the dividend growth rate are jointly covariance stationary.

The augumented Dickey-Fuller tests reported in table 9.5a are consistent with the hypothesis that the logarithms of real prices and real dividends are non-stationary or integrated of order one. The evidence for a unit root in d_t on the basis of the asymptotic distribution was discussed above. With p_t substituted for d_t in equation (9.4), the F-test for $\delta = 0$ and $\gamma = 0$ equals 4.34, corresponding to a p-value in the asymptotic unit root distribution of 0.795. The t-statistic for $\gamma = 0$ equals -3.01, which has an asymptotic p-value of 0.869.

Table 9.5 Real monthly stock prices and dividends

(a) Unit root tests:

$$\Delta u_t = \mu + \delta(t/T) + \rho u_{t-1} + \phi_1 \Delta u_{t-1} + \ldots + \phi_{12} \Delta u_{t-12} + e_t$$

u_t	*Data*	*Asymp.*	*Null1*	*Fads*	*Bubble*	*Null2*	*TVRP*
$p_t: t_{\rho=0}$	−3.01	−2.17	−2.19	−2.31	−1.31	−2.11	−2.33
		[0.869]	[0.862]	[0.827]	[0.960]	[0.892]	[0.795]
$p_t: \Phi_3$	4.34	2.87	2.93	3.08	3.49	2.98	4.08
		[0.795]	[0.779]	[0.755]	[0.653]	[0.765]	[0.538]
$d_t: t_{\rho=0}$	−3.46	−2.17	−2.19	−2.19	−2.19	−2.11	−2.11
		[0.953]	[0.947]	[0.947]	[0.947]	[0.958]	[0.954]
$d_t: \Phi_3$	6.19	2.87	2.90	2.90	2.90	2.95	2.99
		[0.949]	[0.933]	[0.933]	[0.933]	[0.909]	[0.923]

(b) Cointegration tests:

$$p_t = a + bd_t + u_t$$

	Data	*Asymp.*	*Null1*	*Fads*	*Bubble*	*Null2*	*TVRP*
a	3.94	–	5.26	5.25	5.21	5.26	5.28
			[1.00]	[0.991]	[0.962]	[1.00]	[0.947]
b	1.31	–	1.00	1.00	0.954	1.00	0.994
			[0.000]	[0.001]	[0.002]	[0.001]	[0.024]
$t_{\rho=0}$	−4.57	−2.21	−7.25	−3.39	−3.55	− 6.51	−4.06
$\mu = 0$		[0.999]	[0.000]	[0.954]	[0.952]	[0.054]	[0.718]
$t_{\rho=0,\, \mu\neq 0}$	−3.87	−1.56	−7.23	−3.17	−3.40	−6.42	−3.95
$a = 0,\, b = 1$		[0.999]	[0.000]	[0.848]	[0.794]	[0.029]	[0.499]

Augmented Dickey-Fuller tests for a unit root. In (a), $t_{e=0}$ refers to the t-test for $\rho = 0$, given $\delta = 0$. The joint F-test for $\rho = 0$ and $\delta = 0$ is denoted Φ_3. In (b), the coefficients a and b denote the OLS estimates from the cointegrating regression. The statistics $t_{e=0}$ give the t-tests for a unit root in the regression residuals, i.e. the null hypothesis of no cointegration. The row labelled $t_{\rho=0,\, u\neq 0}$ denotes the t-test for a unit root in $p_t - d_t$, i.e. imposing $a = 0$ and $b = 1$. The "Data" column provides the sample statistics. The "Asymp." column reports the medians and p-values in the simulated asymptotic distributions. The last five columns provide the medians from the Monte Carlo experiments under the different hypotheses with the p-values reported in square brackets.

The conformity among the p-values for the Null1 and Null2 models illustrates the robustness of the standard unit root tests to the presence of conditional heteroskedasticity as shown by Phillips (1987).

The tests for cointegration of p_t and d_t are presented in table 9.5b. Following Engle and Granger (1987), if p_t and d_t are not cointegrated, the residuals from the cointegrating regression of p_t on a constant and d_t will contain a unit root. Note that under the null hypothesis of no cointegration the regression of p_t on d_t is spurious in the sense of Granger and Newbold (1974). The asymptotic distribution of this residual- based unit root test has been formally derived by Phillips and Ouliaris (1990).[27] The 1% and 5% critical values are -3.96 and -3.37, respectively. Consistent with the predictions of the present value model and the absence of speculative bubbles, the null hypothesis of a unit root in the mean-zero residuals from the cointegrating regression is easily rejected since the test statistic is -4.57.

When we impose the cointegrating vector $(1, -1)$, the null hypothesis of a unit root in the logarithmic price/dividend ratio is also firmly rejected. Note that, when imposing the cointegrating vector, the corresponding, t-statistic should be evaluated in the standard asymptotic unit root distribution for $\rho = 0$ given $\mu = 0$. From Fuller (1976), the 1% and 5% critical values in this distribution are -3.43 and -2.86, respectively. A time series plot of the logarithmic price/dividend ratio is given in figure 9.3.

These results are counter to related findings reported in the literature, which fail to reject the null of no cointegration using annual data. The findings in

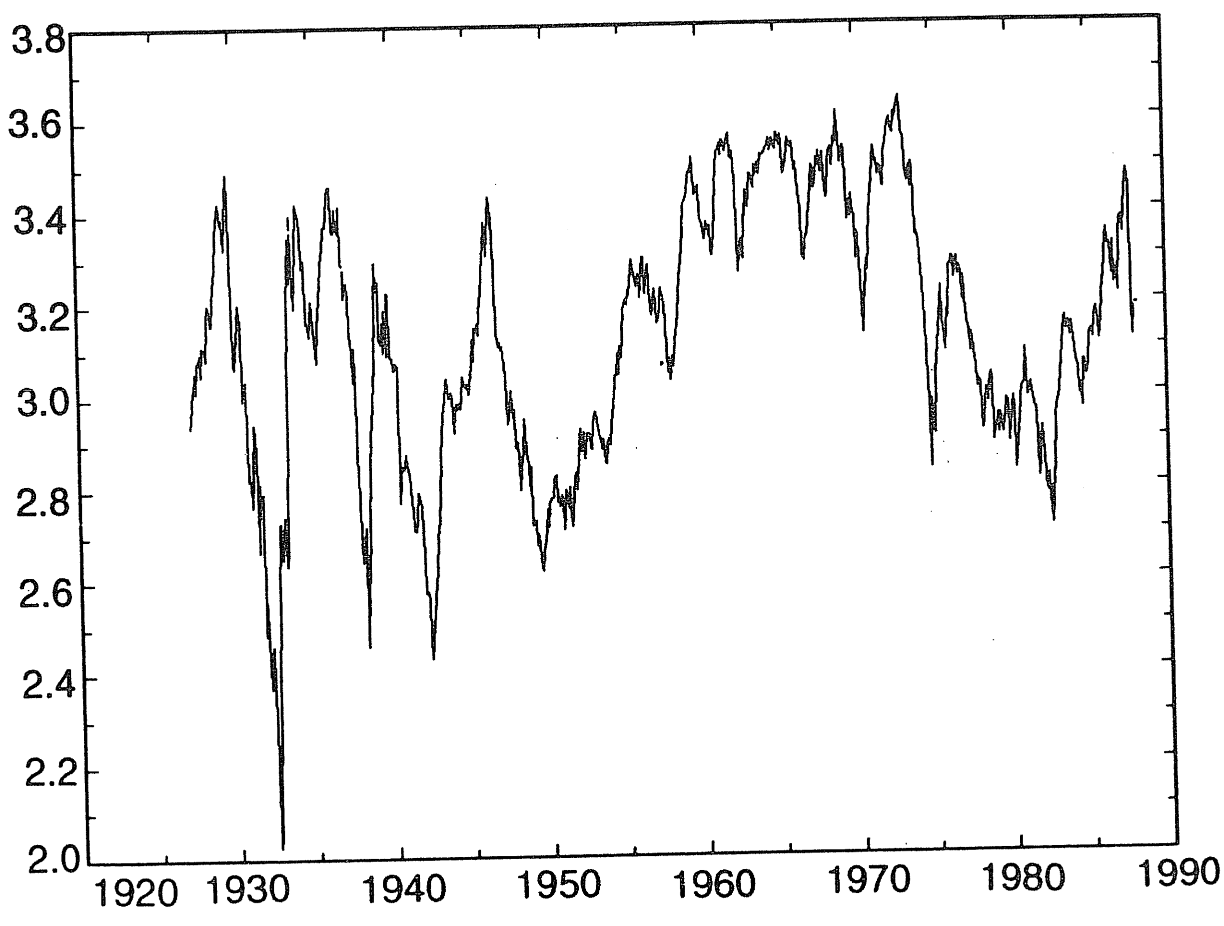

Figure 9.3 Log price/dividend ratio

table 9.5 for monthly data suggest that these results may be due to a lack of power in the tests using time-aggregated annual data, even though the annual span of the data is comparable. Note that while the p-values for the Nulll model suggest much more powerful rejections than the sample statistics, the medians and the p-values for the TVRP model are broadly consistent with the actual empirical findings. At the same time, the median values of the test statistics in the empirical distributions from the Bubble model suggest that the nominal sizes of the cointegration test are misleading for the collapsing bubble analyzed here. In fact, the medians for the Fads model, in which p_t and d_t are cointegrated, are closer to the asymptotic counterparts under the null of no cointegration than are the same statistics for the Bubble model. Additional evidence on this issue is provided in Evans (1991).

Even though the estimate of b from the cointegrating regression is significantly different from the implied value of unity in the TVRP model at the 0.024 level, it is interesting to note that all of the other alternatives, including the Bubble specification, result in even lower p-values for the estimated coefficient $\hat{b} = 1.31$.[28] It follows also that $\hat{a} = 3.94$ is too small compared to the results for the five simulated price processes.

To better understand the results from the simulations, suppose that real monthly dividends followed a logarithmic random walk with drift μ and normal innovations with a variance of σ^2. It follows then from equation (9.1) that if the required rate of return were constant and equal to r,

$$p_t^{\mathrm{f}} - d_t = -\ln[\exp(r - \mu - 0.5\sigma^2) - 1]. \qquad (9.33)$$

When evaluated at the sample analogues of $\mu = 0.00086$, $\sigma^2 = 0.000311$, and $r = 0.00635$, the right-hand side of equation (9.33) equals 5.23. The median values of the intercept in the cointegrating regression under each of the alternatives are all close to this value.

6 Volatility Tests

In the late 1970s researchers interested in the efficiency of asset markets shifted their focus from the predictability of returns to the volatility of prices. As always, the hypothesis of market efficiency could not be tested directly but was part of a joint hypothesis. Researchers were still required to specify a particular model of expected returns. Additionally, the predictions of price volatility from a particular model depended on the assumed time series properties of the dividend process and the information set of economic agents.

The first volatility tests were conducted by Shiller (1979; 1981a) and LeRoy and Porter (1981) using the constant expected rate of return model as the null hypothesis. The point estimates from these studies implied overwhelming rejections of market efficiency, although LeRoy and Porter's (1981) calculation of standard errors suggested that the sampling variation of the estimates implied only marginal statistical significance. Subsequent research, particularly by Flavin (1983) and Kleidon (1986a; 1986b), questioned the small sample statistical properties of these analysis. For recent surveys of this literature see West (1988), Pesaran (1991), Gilles and LeRoy (1991), and Cochrane (1991a).

In order to illustrate the issues, we focus here on a volatility test developed by Mankiw, Romer, and Shapiro (1991), which we denote the MRS test. This class of tests is designed to avoid some of the biases that plagued previous studies and to provide for more reliable statistical inference. Like many volatility tests, the MRS test recognizes that in an efficient market the price of an asset must equal the discounted conditional expected payoff from holding the asset for k periods and reselling it. That is,

$$P_t = E_t(P_t^{*k}), \tag{9.34}$$

where

$$P_t^{*k} \equiv \sum_{j=1}^{k}\left(\prod_{i=1}^{j} \rho_{t+1}\right) D_{t+j} + \left(\prod_{i=1}^{k} \rho_{t+1}\right) P_{t+k}. \tag{9.35}$$

Note that the *ex post* rational price defined in equation (9.35) is only observable at time $t + k$.[29]

Equation (9.34) implies that $P_t^{*k} - P_t$ is uncorrelated with any information available at time t. In particular, let P_t^0 denote any "naïve forecast" of the *ex post* rational price. Then, the following second-order moment condition must hold:

$$E_t[(P_t^{*k} - P_t)(P_t - P_t^0)] = 0. \tag{9.36}$$

This, in turn, implies the following condition for the corresponding uncentered second-order moments:

$$E_t[(P_t^{*k} - P_t^0)^2] = E_t[(P_t^{*k} - P_t)^2] + E_t[(P_t - P_t^0)^2]. \tag{9.37}$$

Equation (9.37) remains valid when the price constructs are deflated by any variable in the time t information set. As the results in table 9.5 indicate, such a transformation is necessary to ensure stationarity and the existence of unconditional expectations required in deriving the test statistics. In our implementation of the MRS volatility test we follow their lead and divide by P_t. Hence, from equation (9.37),

$$E_t\left[\left(\frac{P_t^{*k}}{P_t} - \frac{P_t^0}{P_t}\right)^2\right] - E_t\left[\left(\frac{P_t^{*k}}{P_t} - 1\right)^2\right] - E_t\left[\left(1 - \frac{P_t^0}{P_t}\right)^2\right] = 0. \tag{9.38}$$

Let q_{t+k} denote the corresponding sample realizations of equation (9.38):

$$q_{t+k} \equiv \left(\frac{P_t^{*k}}{P_t} - \frac{P_t^0}{P_t}\right)^2 - \left(\frac{P_t^{*k}}{P_t} - 1\right)^2 - \left(1 - \frac{P_t^0}{P_t}\right)^2. \tag{9.39}$$

The null hypothesis from equation (9.38) is then conveniently stated as $E_t(q_{t+k}) = 0$.

By the law of iterated expectations, $E(q_t) = 0$, and the sample mean of q_t, denoted $\bar{q}$, should be close to zero under the null hypothesis. The asymptotic distribution of $\bar{q}$ may be derived, as in section 4, by use of the GMM distribution theory for stationary but serially correlated and conditionally heteroskedastic processes. That is,

$$(\sqrt{T})\bar{q} \sim N(0, V_0), \tag{9.40}$$

where

$$V_0 = \sum_{j=-k+1}^{k-1} E(q_{t+k} q_{t+k-j}). \tag{9.41}$$

The sample variance may be estimated by

$$V_T = C_T(0) + \sum_{j=1}^{k-1} 2C_T(j), \tag{9.42}$$

where $C_T(j) = (1/T)\Sigma_{t=j+1}^{T}(q_{t+k}q'_{t+k-j})$. Following Mankiw, Romer, and Shapiro, the jth-order autocovariance is weighted by $(k - j)/k$ as in Newey and West (1987) in order to guarantee a positive estimate of the variance in equation (9.42). This weighting scheme is derived under the assumption that both k and T are going to infinity. Since the order of the autocorrelation is known to be k under the null hypothesis, a more appropriate truncation point with this weighting scheme might include terms of higher order than k. To circumvent this problem, the actual truncation lag could be based on the automatic lag selection procedures discussed in Andrews (1991) and Newey and West (1994).

As our measure of a naïve price prediction, we use a version of the Gordon (1962) model assuming a constant rate of return, r, and a constant dividend growth rate, μ:

$$P_t^0 = \frac{D_t}{r - \mu}. \tag{9.43}$$

The tests are reported in table 9.6a for the same five horizons as in table 3 and 4. Notice in the "Data" column that $\bar{q}$ is negative at all five horizons.[30] The asymptotic p-values are also quite small with the largest being 0.023 at the 24-month horizon. Given the strong significance for each of the individual MRS test statistics at all horizons, we did not calculate a joint test.

Several features of the MRS test are noteworthy in the simulated data. First, for both the Null1 and Null2 models, there is no bias in the test statistics in the sense that the median values of the test statistics are essentially zero. This desirable feature of the MRS volatility test arises because of its use of uncentered second moments rather than sample variances which are biased because of the necessity of estimating the sample mean. Next, notice that the p-values for the Null1, Null2, Fads, and Bubble models are all quite small. It is unlikely that any of these models could generate the sample volatility statistics. Only for the TVRP model are the p-values reasonable. Here we find values ranging from 0.142 to 0.346.

The orthogonality condition in equation (9.36) also forms the basis for Scott's (1985) regression test of the present value model. If equation (9.34) is true, a regression of $(P_t^{*k} - P_t)$ on anything in the time t information set should have insignificant coefficients. Note, again, that deflation of the price series by some trending variable W_t in the time t information set does not formally change this null hypothesis but is desirable to ensure stationarity. In particular, consider the regression specified without a constant term as

$$\frac{P_t^{*k} - P_t}{W_t} = \beta\left(\frac{P_t - P_t^0}{W_t}\right) + e_t. \tag{9.44}$$

The OLS estimate for β is

$$\hat{\beta} = \frac{\frac{1}{T}\sum_{t=1}^{T}[(P_t^{*k} - P_t)/W_t][(P_t - P_t^0)/W_t]}{\frac{1}{T}\sum_{t=1}^{T}[(P_t - P_t^0)/W_t]^2}. \tag{9.45}$$

For $W_t = P_t$ the numerator of $\hat{\beta}$ equals $(1/2)\bar{q}$. Mankiw, Romer, and Shapiro (1991) note that their volatility test has an advantage and a disadvantage relative to the regression test. The disadvantage is that regression coefficients

Table 9.6 Volatility tests

(a) Second-moment volatility tests:

$$q_{t+k} \equiv \left(\frac{P_t^{*k}}{P_t} - \frac{P_t^0}{P_t}\right)^2 - \left(\frac{P_t^{*k}}{P_t} - 1\right)^2 - \left(1 - \frac{P_t^0}{P_t}\right)^2$$

k	*Data*	*Asymp.*	*Null1*	*Fads*	*Bubble*	*Null2*	*TVRP*
1	−0.007	0.000	−0.001	−0.002	−0.002	−0.001	−0.005
	(0.002)	[0.001]	[0.000]	[0.000]	[0.041]	[0.008]	[0.346]
12	−0.032	0.000	−0.002	−0.003	−0.006	−0.001	−0.009
	(0.003)	[0.000]	[0.000]	[0.000]	[0.173]	[0.013]	[0.142]
24	−0.059	0.000	−0.001	−0.007	−0.010	−0.001	−0.034
	(0.026)	[0.023]	[0.000]	[0.000]	[0.009]	[0.000]	[0.331]
36	−0.082	0.000	−0.001	−0.010	−0.014	−0.001	−0.041
	(0.031)	[0.008]	[0.000]	[0.000]	[0.012]	[0.000]	[0.307]
48	−0.112	0.000	−0.001	−0.012	−0.018	−0.001	−0.046
	(0.041)	[0.006]	[0.000]	[0.000]	[0.021]	[0.003]	[0.266]

(b) Diagnostics on *ex post* rational prices and dividend yields

Horizon	*Data*	*Null1*	*Fads*	*Bubble*	*Null2*	*TVRP*
Second moment of ex post *rational price relative to naïve price*						
1	0.142	0.005	0.016	0.030	0.004	0.047
	(0.023)	[0.000]	[0.000]	[0.093]	[0.008]	[0.180]
12	0.117	0.004	0.013	0.024	0.003	0.041
	(0.020)	[0.000]	[0.000]	[0.017]	[0.003]	[0.187]
24	0.155	0.023	0.036	0.049	0.022	0.056
	(0.028)	[0.000]	[0.000]	[0.114]	[0.015]	[0.140]
36	0.144	0.033	0.042	0.055	0.028	0.061
	(0.028)	[0.000]	[0.000]	[0.118]	[0.017]	[0.153]
48	0.141	0.040	0.048	0.061	0.033	0.063
	(0.029)	[0.000]	[0.000]	[0.115]	[0.030]	[0.155]
Second moment of ex post *rational price relative to unity*						
1	0.003	0.001	0.001	0.001	0.001	0.002
	(0.001)	[0.000]	[0.000]	[0.045]	[0.027]	[0.274]
12	0.003	0.001	0.001	0.001	0.001	0.001
	(0.001)	[0.000]	[0.000]	[0.197]	[0.021]	[0.166]
24	0.069	0.021	0.027	0.027	0.018	0.042
	(0.016)	[0.000]	[0.000]	[0.083]	[0.023]	[0.295]
36	0.080	0.029	0.036	0.038	0.024	0.052
	(0.018)	[0.000]	[0.000]	[0.085]	[0.035]	[0.316]
48	0.107	0.035	0.044	0.047	0.030	0.061
	(0.028)	[0.000]	[0.000]	[0.074]	[0.033]	[0.269]
Second moment of naïve price relative to unity						
	0.146	0.005	0.016	0.031	0.004	0.049
	(0.023)	[0.000]	[0.000]	[0.092]	[0.008]	[0.184]

Mankiw, Romer, and Shapiro (1991) volatility tests. The *ex post* rational price is denoted P_t^{*k}, and the naïve price is P_t^0. The results reported in (a) test the mean of q_{t+k} equal to zero. The three second-moment components of q_{t+k} are reported in (b). The columns report the medians for the sample statistics with the p-values in square brackets.

and the coefficient of multiple correlation, R^2, have natural interpretations in terms of the predictability of returns. The advantage is that regression tests may be more systematically biased.

Equation (9.38) implies two volatility inequalities which serve as diagnostics for the models:

$$E\left[\left(\frac{P_t^{*k}}{P_t} - \frac{P_t^0}{P_t}\right)^2\right] \geqslant E\left[\left(\frac{P_t^{*k}}{P_t} - 1\right)^2\right], \tag{9.46}$$

$$E\left[\left(\frac{P_t^{*k}}{P_t} - \frac{P_t^0}{P_t}\right)^2\right] \geqslant E\left[\left(1 - \frac{P_t^0}{P_t}\right)^2\right]. \tag{9.47}$$

Table 9.6b reports the sample analogue for the left-hand side of equations (9.46) and (9.47) as *ex post* rational price relative to naïve price. The realization of the right-hand side of equation (9.46) is denoted *ex post* rational price relative to unity, while the sample realization of the right-hand side of equation (9.47) is referred to as naïve price relative to unity. Interestingly, it is the latter quantity that is the primary source of evidence for excess volatility in the data. From inspection of the p-values for the various models, it follows that only the TVRP model is consistent with the data at a 10% significance level.

The right-hand side of equation (9.47) is a measure of the variability in the dividend/price ratio. The naive Gordon (1962) model predicts that the dividend/price ratio should be a constant. There are three sources of movement in the TVRP model that improve on this counterfactual prediction. First, information other than the current level of dividends is useful for predicting future dividends, which is true for all the models. Second, the time variation in the conditional variance of dividends matters to investors who are forecasting the levels of dividends, which is true for Null2 as well. The third feature is that the required rate of return is not a constant.[31]

As noted above, there is also a direct relation between the volatility of prices and the predictability of returns. To further understand this relation consider the following argument. Assuming a constant discount factor ρ, and $W_t = P_t$, it follows from equation (9.35) that for $k = 2$,

$$\frac{P_t^{*2}}{P_t} - 1 = \rho\,\frac{D_{t+1}}{P_t} - 1 + \rho^2\left(\frac{D_{t+2} + P_{t+2}}{P_t}\right). \tag{9.48}$$

Add and subtract $\rho(P_{t+1}/P_t)$ to the right-hand side of equation (9.48) and multiply the second term on the right-hand side by (P_{t+1}/P_{t+1}). The result is

$$\begin{aligned}\frac{P_t^{*2}}{P_t} - 1 &= \rho\left(\frac{D_{t+1} + P_{t+1}}{P_t}\right) - 1 + \rho^2\left(\frac{P_{t+1}}{P_t}\right)\left(\frac{D_{t+2} + P_{t+2}}{P_{t+1}}\right) - \rho\,\frac{P_{t+1}}{P_t}. \\ &= (\rho R_{t+1} - 1) + \rho\left(\frac{P_{t+1}}{P_t}\right)(\rho R_{t+2} - 1).\end{aligned} \tag{9.49}$$

Clearly, if the null hypothesis is true and expected returns have a constant mean equal to ρ^{-1}, $(P_t^{*2}/P_t - 1)$ is not predictable.

Let $x_t \equiv (1 - P_t^0/P_t)$, and let $u_{t+1} \equiv (\rho R_{t+1} - 1)$. Substituting these definitions into equation (9.49) and the results into equation (9.36) with its variables deflated by P_t yields

$$E_t[(u_{t+1} + \rho(P_{t+1}/P_t)u_{t+2})x_t] = 0. \tag{9.50}$$

It is apparent that both the MRS volatility test and Scott's regression test examine the null hypothesis that a variable in the time t information set cannot predict returns at various horizons.

As in section 4, stationarity of the variables in equation (9.50) allows us to reorganize the equation and express its unconditional expectation as

$$E[u_{t+1}(x_t + \rho(P_t/P_{t-1})x_{t-1})] = 0. \tag{9.51}$$

This formulation of the volatility test makes it transparent that it is predictability of one-period returns that is being tested, and from the definition of x_t, any predictability of returns is due to a filtered measure of dividend yields. The sample counterparts to the unconditional expectation of equation (9.50) and equation (9.51) only differ by the first and last observations. Of course, estimates of the variance of the sample mean might be very different in small samples. The variance for the estimator based on equation (9.50) may be calculated as in equation (9.42). However, the variance for the estimator based on equation (9.51) avoids the overlapping data problem and the summation of the autocovariances and may be calculated analogously to equation (9.28). The small sample properties of the resulting test statistics will therefore differ. We conjecture that reorganizing the test statistics as in equation (9.51) to avoid the overlapping data problem will result in superior small sample behavior.

7 Conclusion

The analysis in this chapter provides a partial survey of the econometric methods employed in testing for own temporal dependencies in the distribution of asset returns. It also addresses the relationship of these dependencies to the concept of market efficiency. We intentionally worked only with price, dividend, and return data, and avoided the use of other macroeconomic variables that might help explain the evolution of returns over time. We focused the discussion on tests that have been primarily designed for broadly defined asset categories. A large literature in finance analyzes cross-sectional differences in returns. Fama and French (1992) and Ferson (1995) provide recent contributions to this literature. Our somewhat narrow focus was motivated in part by space considerations and by the relevance of expected return variability for issues in macroeconomics.

Much of the literature on testing for market efficiency has proceeded under the convenient assumption that rational asset pricing requires a constant rate of return. While simple short-horizon serial correlation tests often cannot reject this hypothesis, the idea that equilibrium required rates of returns may vary through time has been in the literature since the early discussions of LeRoy (1973) and Merton (1973) and the general equilibrium models of Lucas (1978) and Breeden (1979). The latter models are often referred to as consumption-based capital asset pricing models.

Although the time-varying risk premium model postulated in our simulations was not rigorously derived from a rational expectations model, its performance in the simulations was broadly consistent with the empirical findings pertaining to US stock returns. In contrast, the alternative models assuming a constant discount factor or modifications to incorporate fads or bubbles in asset prices were grossly inconsistent with some aspects of the data. These results illustrate the importance of explicitly recognizing the presence of a time-varying risk premium in tests for market efficiency.

The particular functional form for our time-varying risk premium model related the expected return on the market to the change in the conditional variance of future dividends. This formulation was motivated by the analysis in Abel (1988) and Hodrick (1989). Such an approach is radically different from much of the empirical literature that has been devoted to testing restrictions implied by the consumption-based capital asset pricing model. In spite of its theoretical appeal, the consumption-based CAPM does not perform well empirically, as exemplified by the many tests following the approach of Hansen and Singleton (1982; 1983). The basic problem stems from the fact that consumption is much too smooth compared to the variability of most financial asset returns. This is formally documented in Hansen and Jagannathan (1991) who derive bounds on intertemporal marginal rates of substitution using asset market data. In a related context, Pesaran and Potter (1991) argue that predictability of negative excess returns is inconsistent with most equilibrium consumption-based asset pricing models. While the consumption-based asset pricing models generally fail specification tests, the more recent developments in Cochrane (1991b) and Braun (1991) suggest that alternative dynamic asset pricing models based on the investment decisions of firms may provide a better explanation for the observed time-varying risk premiums.

The ready availability of data on asset returns and the declining cost of computing have generated a large literature documenting the stylized facts of financial markets. The current challenge facing financial economists is to develop models that are consistent with the observed variability in the distributions of returns. Variability of expected returns appears to be a necessary aspect of rational explanations of these phenomena. We also conjecture that understanding the link between conditional volatility of market fundamentals and variation in required expected returns that arises from risk aversion of economic agents will be critical to success in this endeavor.

Appendix

Forecast expressions for $E_t(d_{t+j})$ and $\mathrm{var}_t(d_{t+j})$ from the estimated AR(12)-ARCH(12) model for the dividend growth rate in equation (9.5) are most easily evaluated by expressing the model in first-order companion form in the logarithmic levels. That is,

$$\begin{bmatrix} d_t \\ d_{t-1} \\ \vdots \\ d_{t-12} \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} \phi_1 + 1 & \phi_2 - \phi_1 & \dots & \phi_{12} - \phi_{11} & -\phi_{12} \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & & & & \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix} \begin{bmatrix} d_{t-1} \\ d_{t-2} \\ \vdots \\ d_{t-13} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

or more compactly,

$$\mathbf{d}_t = \boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{d}_{t-1} + \boldsymbol{\epsilon}_t, \tag{9.52}$$

where μ denotes the constant, and ϕ_i the ith autoregressive parameter for Δd_t in equation (9.5). By repeated substitution in equation (9.52),

$$\mathbf{d}_{t+j} = \sum_{i=0}^{j-1} \boldsymbol{\Phi}^i \boldsymbol{\mu} + \sum_{i=0}^{j-1} \boldsymbol{\Phi}^i \boldsymbol{\epsilon}_{t+j-i} + \boldsymbol{\Phi}^j \mathbf{d}_t.$$

Define $\mathbf{e}_i$ to be a 13×1 basis vector of zeros except for unity in the ith element. Also, define $\psi_i \equiv \mathbf{e}_1' \boldsymbol{\Phi}^i \mathbf{e}_1$. Then,

$$d_{t+j} = \mu \sum_{i=0}^{j-1} \mathbf{e}_1' \boldsymbol{\Phi}^i \mathbf{e}_1 + \sum_{i=0}^{j-1} \mathbf{e}_1' \boldsymbol{\Phi}^i \mathbf{e}_1 \varepsilon_{t+j-i} + \mathbf{e}_1' \boldsymbol{\Phi}^j \sum_{i=1}^{13} \mathbf{e}_i d_{t+1-i} \tag{9.53}$$

$$= \mu \sum_{i=0}^{j-1} \psi_i + \sum_{i=0}^{j-1} \psi_i \varepsilon_{t+j-i} + \mathbf{e}_1' \boldsymbol{\Phi}^j \sum_{i=1}^{13} \mathbf{e}_i d_{t+1-i}.$$

It follows now directly from equation (9.53) that

$$E_t(d_{t+j}) = \mu \sum_{i=0}^{j-1} \psi_i + \mathbf{e}_1' \boldsymbol{\Phi}^j \sum_{i=1}^{13} \mathbf{e}_i d_{t+1-i}, \tag{9.54}$$

and

$$\operatorname{var}_t(d_{t+j}) = \sum_{i=0}^{j-1} \psi_i^2 E_t(\varepsilon_{t+j-i}^2). \tag{9.55}$$

Evaluation of the expression for the conditional variance of d_{t+j} requires forecasts from the ARCH(12) conditional variance process for ε_j. Following Baillie and Bollerslev (1992), the ARCH(12) model is conveniently expressed in first-order companion form as

$$\begin{bmatrix} \varepsilon_t^2 \\ \varepsilon_{t-1}^2 \\ \vdots \\ \varepsilon_{t-11}^2 \end{bmatrix} = \begin{bmatrix} \omega \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} \alpha_1 & \alpha_2 & \dots & \alpha_{12} \\ 0 & 0 & \dots & 0 \\ \vdots & & & \\ 0 & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{t-1}^2 \\ \varepsilon_{t-2}^2 \\ \vdots \\ \varepsilon_{t-12}^2 \end{bmatrix} + \begin{bmatrix} \nu_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

or compactly as

$$\boldsymbol{\epsilon}_t^2 = \boldsymbol{\omega} + \boldsymbol{\Gamma}\boldsymbol{\epsilon}_{t-1}^2 + \boldsymbol{\nu}_t, \tag{9.56}$$

where $\nu_t \equiv \varepsilon_t^2 - \sigma_t^2$. Note that $E_{t-1}(\nu_t) = 0$, and ν_t is readily interpreted as the innovation to the conditional variance for ε_t. Analogous to the expression for d_{t+j} in equation (9.53), it follows by repeated substitution in equation (9.56) and post-multiplication with the 12×1 basis vector e_1 that

$$\varepsilon^2_{t+j} = \omega \sum_{i=0}^{j-1} e_1'\Gamma^i e_1 + \sum_{i=0}^{j-1} e_1'\Gamma^i e_1 \boldsymbol{\nu}_{t+j-i} + e_1'\Gamma^j \sum_{i=1}^{12} e_i \varepsilon^2_{t+1-i}\,. \tag{9.57}$$

By the law of iterated expectations $E_t(\boldsymbol{\nu}_{t+j}) = 0$ for all $j > 0$, and

$$E_t(\varepsilon^2_{t+j}) = \omega \sum_{i=0}^{j-1} \mathbf{e}_1'\Gamma^i \mathbf{e}_1 + \mathbf{e}_1'\Gamma^j \sum_{i=1}^{12} \mathbf{e}_i \varepsilon^2_{t+1-i}\,. \tag{9.58}$$

Combining equations (9.55) and (9.58) we get

$$\mathrm{var}_t(d_{t+j}) = \sum_{i=0}^{j-1} \psi_i^2 \left[\omega \sum_{h=0}^{j-i-1} (\mathbf{e}_1'\Gamma^h \mathbf{e}_1) + \sum_{h=1}^{12} (\mathbf{e}_1'\Gamma^{j-i}\mathbf{e}_h)\varepsilon^2_{t+1-h} \right]. \tag{9.59}$$

This completes the derivation of the forecast formula used in the evaluation of equation (9.9) for $j \leqslant J$.

Notes

We thank Geert Bekaert, John Cochrane, Martin D. Evans, Stephen F. LeRoy, Robert J. Shiller, Kenneth D. West, and seminar participants at the Australasian Meetings of the Econometric Society, Monash University, the NBER Asset Pricing Group, the University of Kentucky, and the University of North Carolina for helpful comments.

1 The estimates in Fama and French (1988a) for monthly US stock returns imply that for three to five year returns up to 40% of the variability is predictable. The evidence in Kim, Nelson, and Startz (1991) suggests that much of this predictability may be driven by data around the time of the Great Depression.

2 We do not discuss these techniques here because of related coverage in this volume.

3 The simulation approach employed in this chapter was inspired by the earlier work of Mattey and Meese (1986) which could be usefully read in conjunction with the present study.

4 Although we illustrate the tests for market efficiency using stock market data, the same ideas apply equally well to tests of other present value relations such as the rational expectations theory of the term structure of interest rates; see Campbell and Shiller (1987), for example. We also note that dividends are not the only way of distributing value to shareholders, as documented by Bagwell and Shoven (1989). Dividends are also a decision variable of management which may choose to smooth them or to have a liquidating dividend. Marsh and Merton (1986) note that such behaviour can cause serious problems with econometric analysis of present value relations.

5 Even though the annualized dividend series exhibit seasonality, it is worth noting that the degree of own temporal dependence is substantially reduced when com-

pared to the real monthly growth rate in the raw dividend series. For instance, the Ljung and Box (1978) portmanteau test statistic, defined formally below, for up to 12th-order serial correlation in Δd_t equals 342.7 compared to 4042.7 for $\Delta\ln(ND_t)$.

6 The autocorrelations for the squared residuals at lags 1, 3, 6, and 12 equal 0.066, 0.245, 0.120, and 0.164 respectively, and all but lag 1 exceed the 5% critical value of $1.96/\sqrt{T} = 0.073$ under the null of $\hat{\varepsilon}_t^2$ iid through time.

7 The maximized value of the conditional normal log-likelihood function for the model in equation (9.5) equals -4472.4 compared to -4644.8 for the homoskedastic normal AR(12) model restricting the four ARCH coefficients to be zero. The resulting likelihood ratio test statistic for no ARCH equals 344.8, which is highly significant.

8 While a closed-form expression for the infinite discounted sum in equation (9.1) may be derived using the methods of Hansen and Sargent (1980; 1981) in the case of a constant discount rate (see e.g. West, 1987b), the presence of time-varying discount rates coupled with time-varying conditional variances renders a closed-form solution infeasible.

9 Let $\zeta(L) = \phi(L)^{-1}$ denote the lag polynomial in the infinite moving average representation for the AR(12) model with parameters ζ_i, where $\phi(L)\Delta d_t = \mu + \varepsilon_t$. It follows that

$$\mathrm{var}_t(d_{t+j+1}) = \mathrm{var}_t(d_{t+j}) + \sum_{h=0}^{j}\sum_{i=0}^{h}\zeta_h\zeta_{h-i}E_t(\varepsilon_{t+j+1-h}^2).$$

Thus, in the limit,

$$\xi = \sigma^2\sum_{h=0}^{\infty}\sum_{i=0}^{h}\zeta_h\zeta_{h-i},$$

where σ^2 equals the unconditional variance of ε_t. Also, $\eta = \mu\phi(1)^{-1}$.

10 From equations (9.11) and (9.12) it follows that $\mathrm{var}[\Delta\ln(P_t)] = \mathrm{var}[\Delta\ln(P_t^f)] + \mathrm{var}[\Delta\ln(F_t)] = \mathrm{var}[\Delta\ln(P_t^f)] + 2\sigma_u^2(1+\phi)^{-1}$. Setting $\phi = 0.98$ and requiring that $2\sigma_u^2(1+\phi)^{-1}$ equals 25% of $\mathrm{var}[\Delta\ln(P_t)]$ implies $\sigma_u^2 = 0.330\mathrm{var}[\Delta\ln(P_t^f)]$.

11 Our stochastic bubbles do not collapse to zero because, as Diba and Grossman (1988) note, the theoretical impossibility of a rational negative bubble rules out a zero-mean innovation in a bubble starting at zero.

12 We also imposed two other restrictions on the bubble process. We required that the bubble continue with probability one if its current value is less than the reversion value under a collapse, and we set the maximum probability of collapse at 0.99. Flood and Hodrick (1986; 1990) provide a discussion of tests for bubbles.

13 Let $\hat{\mu}_i$ denote the ith centered sample moment. Then $b_3 \equiv \hat{\mu}_3/(\hat{\mu}_2)^{3/2}$ and $b_4 \equiv \hat{\mu}_4/(\hat{\mu}_2)^2$.

14 Under the null of iid the standardized test statistics, $[\sqrt{(T/6)}]b_3$ and $[\sqrt{(T/24)}](b_4 - 3)$, should both be the realization of a standard normal distribution; see e.g. Jarque and Bera (1980).

15 Let ρ_i denote the ith sample autocorrelation. The Ljung and Box (1978) test for up to Nth-order serial dependence is then given by $Q_N = (T+2)T[\rho_1^2/(T-1) + \rho_2^2/(T-2) + \ldots + \rho_N^2/(T-N)]$, where T denotes the sample size. Under the null hypothesis of iid observations, Q_N has an asymptotic chi-squared distribution with N degrees of freedom.

16 The variance ratio statistic in equation (9.18) is consistently estimated by $\hat{v}(k) = \{1 + 2[(k-1)\rho_1 + (k-2)\rho_2 + \ldots + \rho_{k-1}]\}/k$. Under the null hypothesis of iid observations, with $k = 12$ and $T = 720$, the standard error for the variance ratio statistic in the asymptotic normal distribution with mean one equals 0.140. A heteroskedasticity consistent standard error may be calculated from White's (1980) covariance matrix estimator for the sample autocorrelations; see Lo and MacKinlay (1988) for further details.

17 The martingale model is sometimes incorrectly referred to as the random walk model. Whereas the random walk model assumes iid innovations, a martingale difference sequence only stipulates that the innovations be serially uncorrelated, or white noise.

18 Clearly, increasing the importance of the fad from our *a priori* specification of 25% would help explain the variability of returns, but it may cause problems for the model along other dimensions. Similarly, by increasing the innovation variance of the bubble, it would be possible to match the sample variance of returns.

19 Let μ_t and σ_t^2 denote the conditional mean and variance functions, with gradients $\nabla\mu_t$ and $\nabla\sigma_t^2$, respectively. The asymptotic covariance matrix for the quasi-maximum-likelihood estimator, $\hat{\Xi}_T$, is then consistently estimated by $A_T B_T^{-1} A_T$, where

$$A_T = \sum_{t=1}^{T} \nabla\mu_t(\nabla\mu_t)'\sigma_t^{-2} + 0.5\nabla\sigma_t^2(\nabla\sigma_t^2)'\sigma_t^{-4},$$

$$B_T = \sum_{t=1}^{T} \nabla l_t(\Xi)\nabla l_t(\Xi)',$$

$$\nabla l_t(\Xi) = \nabla\mu_t\sigma_t^{-2}\varepsilon_t + 0.5\nabla\sigma_t^2\sigma_t^{-4}(\varepsilon_t^2 - \sigma_t^2),$$

all evaluated at $\hat{\Xi}_T$.

20 A possible explanation for this phenomenon and the estimate of $\alpha + \beta$ close to one is provided by the continuous time approximation arguments given in Nelson (1992) and Nelson and Foster (1994). The apparent strong persistence in the conditional variance could also be a result of stochastic regime changes as in the analysis of Cai (1994).

21 In the EGARCH(1,1) model the conditional variance is given by

$$\ln(\sigma_t^2) = \omega + \theta z_{t-1} + \gamma(|z_{t-1}| - E(|z_{t-1}|)) + \beta\ln(\sigma_{t-1}^2),$$

where $z_t \equiv \varepsilon_t\sigma_t^{-1}$ denotes the standardized innovations. Alternative asymmetric conditional variance formulations include the model in Glosten, Jagannathan, and Runkle (1993) and the specifications in Engle and Ng (1993). Recent evidence in Gallant, Rossi, and Tauchen (1992) and Andersen (1992) explores structural links between conditional volatility and volume. Space considerations prevent us from pursuing these specifications here.

22 If the one-period returns are serially uncorrelated, it is possible to solve explicitly for the parameters in the corresponding moving average representation for $u_{t+k,k}$ as a function of the overlap, as in Baillie and Bollerslev (1990) and Richardson and Smith (1991).

23 Richardson and Stock (1989) develop an alternative asymptotic distribution theory based on a functional central limit theorem. They argue that if T is the sample size

and k is the forecast horizon, a better asymptotic approximation to the finite sample distribution is obtained by letting (k/T) go to a non-zero constant rather than to zero as in the conventional asymptotic distribution theory. Inference under this alternative distribution theory provides little support for the hypothesis that lagged long-horizon returns predict future long-horizon returns. Nelson and Kim (1993) and Hodrick (1992) use Monte Carlo simulations and Goetzmann and Jorion (1992) use bootstrap techniques to investigate the regressions of dividend yields as predictors of long-horizon returns. These authors also find that the small sample properties of estimators are not well approximated by conventional asymptotic distribution theory.

24 Lars Hansen suggested this estimator, which is a heteroskedastic counterpart to the covariance matrix in Richardson and Smith (1991). This section draws heavily from Hodrick (1992).

25 Jegadeesh (1990) uses this logic and the fads alternative hypothesis to derive the test with the largest asymptotic slope for investigating long-horizon predictability of returns on the basis of lagged returns. He argues that using the one-period return as the dependent variable and the sum of k lagged returns as the regressor is a superior way to conduct inference. The choice of k depends on the share of the variance of returns attributed to the transitory components in prices.

26 See Campbell and Shiller (1987; 1988a; 1988b), Cochrane (1991a; 1992), Craine (1991), Evans (1991), and Froot and Obstfeld (1991) for more complete discussions and applications.

27 The likelihood ratio test in Johansen (1988; 1991) provides an alternative non-residual-based testing procedure for cointegration.

28 As noted in note 4, if the importance of dividends as a means of distributing cash to shareholders has declined systematically, one would expect that the slope coefficient in the cointegrating regression would exceed one. Asymptotically valid standard errors for the estimated cointegrating vector that would allow a standard t-test of the hypothesis $b = 1$ could be calculated as in Stock and Watson (1993) by including leads and lags of Δd_t on the right-hand side of the cointegrating regression of p_t on d_t.

29 This contrasts with Shiller's (1981a) definition of *ex post* rational price in which $k = \infty$ in equation (9.34). Shiller develops a measurable counterpart by substitution of $k_t = T - t$ for k in equation (9.35), where T is the end of the sample observed by the econometrician. Flood and Hodrick (1990) refer to equation (9.34) as an interated Euler equation.

30 We use the sample mean return for r and the sample mean of the dividend growth rate for μ, rather than preselected values, in the construction of the naïve prices.

31 Several authors including Campbell (1991), Cochrane (1992), and LeRoy and Parke (1992) have reformulated volatility tests to examine the variance of the price/dividend ratio.

References

Abel, A.B. (1988), "Stock Prices under Time-Varying Dividend Risk: An Exact Solution in an Infinite Horizon General Equilibrium Model," *Journal of Monetary Economics*, 22, 375–93.

Andersen, T.G. (1992), "Return Volatility and Trading Volume in Financial Markets: An Information Flow Interpretation of Stochastic Volatility," manuscript, Department of Finance, J. L. Kellogg Graduate School of Management, Northwestern University.

Andrews, D.W.K. (1991), "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation," *Econometrica*, 59, 817–58.

Bagwell, L.S. and J.B. Shoven (1989), "Cash Distributions to Shareholders," *Journal of Economic Perspectives*, 3, 129–40.

Baillie, R.T. (1989), "Econometric Tests of Rationality and Market Efficiency," *Econometric Reviews*, 8, 151–86.

Baillie, R.T. and T. Bollerslev (1990), "A Multivariate Generalized ARCH Approach to Modeling Risk Premia in Forward Foreign Exchange," *Journal of International Money and Finance*, 9, 309–24.

Baillie, R.T. and T. Bollerslev (1992), "Prediction in Dynamic Models with Time Dependent Conditional Variances," *Journal of Econometrics*, 52, 91–113.

Bekaert, G. (1992), "Empirical Analyses of Foreign Exchange Markets: General Equilibrium Perspectives," PhD dissertation, Department of Economics, Northwestern University.

Bekaert, G. and R.J. Hodrick (1992), "Characterizing Predictable Components in Equity and Foreign Exchange Rates of Return," *Journal of Finance*, 47, 467–509.

Black, F. (1976), "Studies of Stock Market Volatility Changes," *Proceedings of the American Statistical Association, Business and Economic Statistics Section*, 177–81.

Blanchard, O.J. and M.W. Watson (1982), "Bubbles, Rational Expectations and Financial Crises," in P. Wachtel (ed.), *Crises in the Economic and Financial Structure: Bubbles, Bursts and Shocks*, Lexington, Mass.

Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics*, 31, 307–27.

Bollerslev, T. (1987), "A Conditionally Heteroskedastic Time Series Model for Speculation Prices and Rates of Return," *Review of Economics and Statistics*, 69, 542–7.

Bollerslev, T., R.Y. Chou and K.F. Kroner (1992), "ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence," *Journal of Econometrics*, 52, 5–59.

Bollerslev, T. and R.F. Engle (1993), "Common Persistence in Conditional Variances," *Econometrica*, 61, 167–86.

Bollerslev, T. and J.M. Wooldridge (1992), "Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time Varying Covariances," *Econometric Reviews*, 11, 143–72.

Braun, P. (1991), "Asset Pricing and Capital Investment: Theory and Evidence," manuscript, Department of Finance, J. L. Kellogg Graduate School of Management, Northwestern University.

Breeden, D.T. (1979), "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," *Journal of Financial Economics*, 7, 265–96.

Cai, J. (1994), "A Markov Model of Switching-Regime ARCH," *Journal of Business and Economic Statistics*, 12, 309–16.

Campbell, J.Y. (1991), "A Variance Decomposition of Stock Returns," *Economic Journal*, 101, 152–79.

Campbell, J.Y. and R.J. Shiller (1987), "Cointegration and Tests of Present Value Models," *Journal of Political Economy*, 95, 1062–88.

Campbell, J.Y. and R.J. Shiller (1988a), "The Dividend–Price Ratio and Expectations of Future Dividends and Discount Factors," *Review of Financial Studies*, 1, 195–228.

Campbell, J.Y. and R.J. Shiller (1988b), "Stock Prices, Earnings, and Expected Dividends," *Journal of Finance*, 43, 661–76.

Christie, A.A. (1982), "The Stochastic Behavior of Common Stock Variances: Value, Leverage and Interest Rate Effects," *Journal of Financial Economics*, 10, 407–32.

Cochrane, J.H. (1991a), "Volatility Tests and Efficient Markets: A Review Essay," *Journal of Monetary Economics*, 27, 463–85.

Cochrane, J.H. (1991b), "Production-Based Asset Pricing and the Link between Stock Returns and Economic Fluctuations," *Journal of Finance*, 46, 207–34.

Cochrane, J.H. (1992), "Explaining the Variance of Price Dividend Ratios," *Review of Financial Studies*, 5, 243–80.

Craine, R. (1991), "Rational Bubbles: A Test," manuscript, Department of Economics, University of California, Berkeley.

Cumby, R.E. and J. Huizinga (1992), "Testing the Autocorrelation Structure of Disturbances in Ordinary Least Squares and Instrumental Variables Regressions," *Econometrica*, 60, 185–95.

Diba, B.T. and H.I. Grossman (1988), "The Theory of Rational Bubbles in Stock Prices," *Economic Journal* 98, 746–54.

Dickey, D.A. and W.A. Fuller (1981), "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," *Econometrica*, 49, 1057–72.

Diebold, F.X. (1986), "Testing for Serial Correlation in the Presence of ARCH," *Proceedings of the American Statistical Association, Business and Economic Statistics Section*, 323–8.

Domowitz, I. and H. White (1982), "Misspecified Models with Dependent Observations," *Journal of Econometrics*, 20, 35–58.

Engle, R.F. (1982), "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation," *Econometrica*, 50, 987–1008.

Engle, R.F. and G. Gonzalez-Rivera (1991), "Semiparametric ARCH Models," *Journal of Business and Economic Statistics*, 9, 345–59.

Engle, R.F. and C.W.J. Granger (1987), "Cointegration and Error Correction: Representation, Estimation and Testing," *Econometrica*, 55, 251–76.

Engle, R.F. and V.K. Ng (1993), "Measuring and Testing the Impact of News on Volatility," *Journal of Finance*, 48, 1749–78.

Evans, G.W. (1991), "Pitfalls in Testing for Explosive Bubbles in Asset Prices," *American Economic Review*, 81, 922–30.

Fama, E.F. (1965), "The Behavior of Stock Market Prices," *Journal of Business*, 38, 34–105.

Fama, E.F. (1970), "Efficient Capital Markets: A Review of Theory and Empirical Work," *Journal of Finance*, 25, 383–417.

Fama, E.F. (1991), "Efficient Capital Markets: II," *Journal of Finance*, 46, 1575–617.

Fama, E.F. and K.R. French (1988a), "Permanent and Temporary Components of Stock Prices," *Journal of Political Economy*, 96, 246–73.

Fama, E.F. and K.R. French (1988b), "Dividend Yields and Expected Stock Returns," *Journal of Financial Economics*, 22, 3–25.

Fama, E.F. and K.R. French (1992), "The Cross-Section of Expected Stock Returns," *Journal of Finance*, 47, 427–65.

Ferson, W.E. (1995), "Theory and Empirical Testing of Asset Pricing Models," in R.A. Jarrow, W.T. Ziemba and V. Maksimovic (eds), *The Handbook of Finance*, North-Holland, Amsterdam, forthcoming.

Flavin, M. (1983), "Excess Volatility in Financial Markets: A Reassessment of the Empirical Evidence," *Journal of Political Economy*, 91, 929–56.

Flood, R.P. and R.J. Hodrick (1986), "Asset Price Volatility, Bubbles, and Process Switching," *Journal of Finance*, 41, 831–42.

Flood, R.P. and R.J. Hodrick (1990), "On Testing for Speculative Indeterminacies in Models with Rational Expectations," *Journal of Economic Perspective*, 4, 85–101.

Froot, K.A. and M. Obstfeld (1991), "Intrinsic Bubbles: The Case of Stock Prices," *American Economic Review*, 81, 1189–214.

Fuller, W.A. (1976), *Introduction to Statistical Time Series*, New York: John Wiley and Sons.

Gallant, A.R., P.E. Rossi and G. Tauchen (1992), "Stock Prices and Volume," *Review of Financial Studies*, 5, 199–242.

Gallant, A.R. and G. Tauchen (1989), "Seminonparametric Estimation of Conditionally Constrained Heterogeneous Processes: Asset Pricing Applications," *Econometrica*, 57, 1091–120.

Goetzmann, W.N. and P. Jorion (1992), "Testing the Predictive Power of Dividend Yields," manuscript, Graduate School of Business, Columbia University.

Gilles, C. and S.F. LeRoy (1991), "Econometric Aspects of the Variance-Bounds Tests: A Survey," *Review of Financial Studies*, 4, 753–91.

Glosten, L.R., R. Jagannathan and D. Runkle (1993), "On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks," *Journal of Finance*, 48, 1779–1801.

Gordon, M.J. (1962), *The Investment, Financing and Valuation of the Corporation*, Irwin, Homewood, IL.

Granger, C.W.J. and P. Newbold (1974), "Spurious Regressions in Econometrics," *Journal of Econometrics*, 2, 111–20.

Hansen, L.P. (1982), "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica*, 50, 1029–54.

Hansen, L.P. and R.J. Hodrick (1980), "Forward Rates as Optimal Predictors of Future Spot Rates: An Econometric Analysis," *Journal of Political Economy*, 88, 829–53.

Hansen, L.P. and R. Jagannathan (1991), "Implications of Security Market Date for Models of Dynamic Economies," *Journal of Political Economy*, 99, 225–62.

Hansen, L.P. and T.J. Sargent (1980), "Formulating and Estimating Dynamic Linear Rational Expectations Models," *Journal of Economic Dynamics and Control*, 2, 7–46.

Hansen, L.P. and T.J. Sargent (1981), "Linear Rational Expectations Models for Dynamically Interrelated Variables," chapter 8 in R.E. Lucas and T.J. Sargent (eds), *Rational Expectations and Econometric Practice*, University of Minnesota Press, Minneapolis.

Hansen, L.P. and K.J. Singleton (1982), "Generalized Instrumental Variables Estimation in Nonlinear Rational Expectations Models," *Econometrica*, 50, 1269–86.

Hansen, L.P. and K.J. Singleton (1983), "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns," *Journal of Political Economy*, 91, 249–65.

Hodrick, R.J. (1989), "Risk, Uncertainty and Exchange Rates," *Journal of Monetary Economics*, 23, 433–59.

Hodrick, R.J. (1992), "Dividend Yields and Expected Stock Returns: Alternative Procedures for Inference and Measurement," *Review of Financial Studies*, 5, 357–86.

Jarque, C.M. and A.K. Bera (1980), "Efficient Tests for Normality, Homoskedasticity and Serial Independence of Regression Residuals," *Economics Letters*, 6, 255–9.

Jegadeesh, N. (1990), "Evidence of Predictable Behavior of Security Returns," *Journal of Finance*, 45, 881–98.

Johansen, S. (1988), "Statistical Analysis of Cointegration Vectors," *Journal of Economic Dynamics and Control*, 12, 231–54.

Johansen, S. (1991), "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models," *Econometrica*, 59, 1551–80.

Kim, M.J., C.R. Nelson and R. Startz (1991), "Mean Reversion in Stock Prices? A Reappraisal of the Empirical Evidence," *Review of Economic Studies*, 58, 515–28.

Kleidon, A.W. (1986a), "Variance Bounds Tests and Stock Price Valuation Models," *Journal of Political Economy*, 94, 953–1001.

Kleidon, A.W. (1986b), "Bias in Small Sample Tests of Stock Price Rationality," *Journal of Business*, 59, no. 2, part 1, 237–62.

LeRoy, S.F. (1973), "Risk Aversion and the Martingale Property of Stock Prices," *International Economic Review*, 14, 436–46.

LeRoy, S.F. and W.R. Parke (1992), "Stock Price Volatility: Tests Based on the Geometric Random Walk," *American Economic Review*, 82, 981–92.

LeRoy, S.F. and R.D. Porter (1981), "The Present-Value Relation: Tests Based on Implied Variance Bounds," *Econometrica*, 49, 555–74.

Ljung, G.M. and G.E.P. Box (1978), "On a Measure of Lack of Fit in Time Series Models," *Biometrika*, 65, 297–303.

Lo, A.W. and A.C. MacKinlay (1988), "Stock Market Prices Do Not Follow Random Walks: Evidence from a Simple Specification Test," *Review of Financial Studies*, 1, 41–66.

Lucas, R.E. (1978), "Asset Prices in an Exchange Economy," *Econometrica*, 46, 1429–45.

Mandelbrot, B. (1963), "The Variation of Certain Speculative Prices," *Journal of Business*, 36, 394–419.

Mankiw, N.G., D. Romer and M.D. Shapiro (1991), "Stock Market Forecastability and Volatility: A Statistical Appraisal," *Review of Economic Studies*, 58, 455–77.

Marsh, T.A. and R.C. Merton (1986), "Dividend Variability and Variance Bounds Tests for the Rationality of Stock Market Prices," *American Economic Review*, 76, 483–503.

Mattey, J. and R. Meese (1986), "Empirical Assessment of Present Value Relations," *Econometric Reviews*, 5, 171–233.

Merton, R.C. (1973), "An Intertemporal Capital Asset Pricing Model," *Econometrica*, 41, 867–87.

Nelson, C. and M.J. Kim (1993), "Predictable Stock Returns: The Role of Small Sample Bias," *Journal of Finance*, 48, 641–61.

Nelson, D.B. (1991), "Conditional Heteroskedasticity in Asset Returns: A New Approach," *Econometrica*, 59, 347–70.

Nelson, D.B. (1992), "Filtering and Forecasting with Misspecified ARCH Models I: Getting the Right Variance with the Wrong Model," *Journal of Econometrics*, 52, 61–90.

Nelson, D.B. and D.P. Foster (1994), "Asymptotic Filtering Theory for Univariate ARCH Models," *Econometrica*, 62, 1–41.

Newey, W.K. and K.D. West (1987), "A Simple Positive Semi-Definite Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55, 703–8.

Newey, W.K. and K.D. West (1994), "Automatic Lag Selection in Covariance Matrix Estimation," *Review of Economic Studies*, 61, 631–53.

Pesaran, M.H. (1991), "On the Volatility and Efficiency of Stock Prices," *Cuadernos Economicos de ICE*, 49.

Pesaran, M.H. and S.M. Potter (1991), "Equilibrium Asset Pricing Models and Predictability of Excess Returns: Theory and Evidence," manuscript, Department of Applied Economics, University of Cambridge.

Phillips, P.C.B. (1987), "Time Series Regression with Unit Roots," *Econometrica*, 55, 277–302.

Phillips, P.C.B. and S. Ouliaris (1990), "Asymptotic Properties of Residual Based Tests for Cointegration," *Econometrica*, 58, 165–93.

Poterba, J. and L.H. Summers (1988), "Mean Reversion in Stock Prices: Evidence and Implications," *Journal of Financial Economics*, 22, 27–59.

Richardson, M. (1993), "Temporary Components of Stock Prices: A Skeptic's View," *Journal of Business and Economic Statistics*, 11, 199–207.

Richardson, M. and T. Smith (1991), "Test of Financial Models in the Presence of Overlapping Observations," *Review of Financial Studies*, 4, 227–54.

Richardson, M. and J.H. Stock (1989), "Drawing Inferences from Statistics Based on Multi-Year Asset Returns," *Journal of Financial Economics*, 25, 323–48.

Scott, L.O. (1985), "The Present Value Model of Stock Prices: Regression Tests and Monte Carlo Results," *Review of Economics and Statistics*, 67, 599–605.

Shiller, R.J. (1979), "The Volatility of Long-Term Interest Rates and Expectations Models of the Term Structure," *Journal of Political Economy*, 87, 1190–1219.

Shiller, R.J. (1981a), "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?," *American Economic Review*, 71, 421–36.

Shiller, R.J. (1981b), "The Use of Volatility Measures in Assessing Market Efficiency," *Journal of Finance*, 36, 291–304.

Shiller, R.J. (1984), "Stock Prices and Social Dynamics," *Brookings Papers on Economic Activity*, 2, 457–510.

Stock, J.H. and M. Watson (1993), "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems," *Econometrica*, 61, 783–820.

Summers, L.H. (1986), "Does the Stock Market Rationally Reflect Fundamental Values?," *Journal of Finance*, 41, 591–601.

Weiss, A.A. (1986), "Asymptotic Theory for ARCH Models: Estimation and Testing," *Econometric Theory*, 2, 107–31.

West, K.D. (1987a), "A Note on the Power of Least Squares Tests for a Unit Root," *Economics Letters*, 24, 249–52.

West, K.D. (1987b), "A Specification Test for Speculative Bubbles," *Quarterly Journal of Economics*, 102, 553–80.

West, K.D. (1988), "Bubbles, Fads and Stock Price Volatility Tests: A Partial Evaluation," *Journal of Finance*, 43, 639–56.

White, H. (1980), "A Heteroskedasticity Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity," *Econometrica*, 48, 817–38.