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An Evaluation of Recent Evidence on Stock Market Bubbles

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I Introduction

The topic of asset price bubbles has received a large amount of professional attention. The theoretical work is exemplified by that of Obstfeld and Rogoff (1983), Tirole (1985), Diba and Grossman (1985a), and Hamilton and Whiteman (1985), while the empirical work is exemplified by that of Burmeister and Wall (1982), Flood, Garber and Scott (1984), Quah (1985), Meese (1986), West (1984, 1985a, 1985b), Diba and Grossman (1985b), Woo (1984), Scott (1985b), and Okina (1985).

The asset price bubbles we discuss are the asset market counterparts of the price-level bubbles studied by Flood and Garber (1980). The definition of a bubble depends on the model at hand, so precise definitions will have to wait until precise models have been presented. Without being very precise, though, we can say that in what follows we decompose an asset price into two components. The first is due to current and expected future *market fundamentals*, in which we list the typical set of exogenous and predetermined variables usually thought important for market price. The second is the *bubble*, which is defined to be what is left after market fundamentals have been removed from price. Bubbles may be thought of as the part of price due to self-fulfilling prophecy.

Two general types of empirical work have been interpreted as being useful in addressing the question of whether bubbles are important for asset price determination. The first follows the bubbles test of Flood and Garber (1980) and the variance bounds work of Leroy and Porter (1981) in attempting to forecast the indefinite future of market fundamentals. The second follows some of the variance bounds work of Shiller (1982) and Grossman and Shiller (1981) by examining market fundamentals only up to a fixed terminal market price. In section III of this chapter we argue that the latter method, which was not designed explicitly for bubble research, gives no information about bubbles. The results of such tests do, however, provide pertinent information regarding model specification.[1]

In the remaining sections of the chapter we discuss and extend some of the recent empirical work that is theoretically well designed to give information about asset price bubbles in aggregate stock markets. This includes some recent work by West (1984, 1985a), Diba and Grossman (1985b), and Quah (1985).

The data sets used by Quah (1985) and by Diba and Grossman (1985b) are either identical to or are subsets of the data used by West (1984, 1985a), which is the same as that used by Shiller (1981a). Further, all of these studies use an equilibrium condition to price assets that is based on the Euler equations of a risk-neutral agent. Our empirical results address the adequacy of the risk-neutral specification in empirical bubble tests, and, therefore, our results reflect on all of the studies.

After duplicating West's work, we extended it in two directions. First, because of our concern about the time series stationarity of his data, we performed his estimation using returns on stock portfolios. West used the levels of real stock prices and dividends or their first differences in his study. We found that the differences in inference between using our specification and West's were actually quite minor. This was puzzling for two reasons. West's specification requires the expected real rate of return on the stock market to be constant. Since variance bounds tests based on that specification seem to us to indicate some form of model misspecification, this representation was suspect. Also, there is a large and growing body of evidence indicating that expected rates of return on a variety of assets move through time.[2] Why, then, was West's specification indicating such a different result?

One difference between the variance bounds tests and West's Euler equation tests involves the fact that the Euler equation methods consider only temporally adjacent periods, while the variance bounds tests consider widely separated periods. If the Euler equation is incorrect, it may be that its specification error is swamped in estimation by the rational-expectations prediction error. Although the one-period specification error does not imply strong rejection of the Euler equation, it is possible that the compounded one-period specification errors that appear in variance bounds tests could lead to a rejection of the model.

In order to investigate this issue we iterated the Euler equation to equate margins across two nonadjacent periods, and we used West's data and his methods to estimate the iterated Euler equation. The iterated Euler equation was resoundingly rejected by the data, calling into question

West's interpretation of his results as indicating evidence of stock market bubbles.

An obvious potential problem with West's model was his use of a risk-neutral utility function that induces his linear estimating equations. In response to our misgivings about the assumption of risk neutrality, we estimated Euler equations for all of the utility functions in the HARA (hyperbolic absolute risk aversion) class. Our results are similar to the results we find for risk neutrality—the models seem to work marginally well only when margins for adjacent periods are explicitly equated. There is more substantial evidence against the models when the iterated Euler equations equating margins for nonadjacent periods are employed.[3]

We investigated the data in two additional ways. First, because the theory deals with after-tax returns while the data we use contain only before-tax returns, we tried to allow the estimation to tell us if differential tax treatment of dividends and capital gains might be responsible for the model's failure. The results of this part of the investigation are inconclusive. There is some evidence that agents treat dividends and capital gains differently. We also looked explicitly at return forecasting equations. The risk-neutral model implies that forecasted one-period returns should be a constant equal to the inverse of the subjective discount rate. We find that past (time-varying) dividend-asset price ratios almost surely forecast returns, which we interpret as strong evidence that the risk-neutral model is inappropriate.

Our research is reported in the following five sections. In section II we present a theoretical discussion of asset pricing in a utility-maximizing framework. In this section we are explicit about our definition of asset price bubbles. In section III we show why studies of stock-price variance bounds, which use a terminal stock market price in the way suggested in much of the variance bounds literature, give information about the adequacy of the underlying specification, but they do not give information about asset price bubbles. In section IV we discuss potential problems with interpretations of bubbles tests, and we lay out West's proposed methodology. In section V we report results concerning the usefulness of the risk-neutral utility function in developing bubbles test. We also report some additional results on nonlinear utility functions, on specifications that allow differential tax treatment of dividends and capital gains, and on the ability of past data to forecast future stock market returns. In section VI we present a summary of our views of current empirical work on bubbles in stock prices, the relation of that work to the variance bounds studies, and some suggestions about directions for future research.

II Utility Maximizing Models of Asset Prices

The purpose of this section is to set forth a simple representative agent model that is the foundation of our asset-pricing discussion. Consider a representative agent who maximizes an intertemporal utility function subject to a sequence of budget constraints. The formal problem is

$$\max_{\{c_{t+i}\}_{i=0}^{\infty}} E_t\left[\sum_{i=0}^{\infty} \rho^i U(c_{t+i})\right], \qquad 0 < \rho < 1, \tag{1}$$

subject to the sequence of budget constraints

$$c_{t+i} + p_{t+i}k_{t+i} = y + (p_{t+i} + d_{t+i})k_{t+i-1}, \qquad i = 0, 1, 2, \ldots \tag{2}$$

where c_t is consumption in period t, $U(\cdot)$ is the period utility function, ρ is the subjective discount factor, y is exogenous real endowment, k_t is the number of units of the asset purchased at time t, and the mathematical expectation operator is given by $E_t(\cdot)$.

The first order conditions for this problem can be written as

$$E_t(z_{t+i}) = \rho E_t(z_{t+i+1} + a_{t+i+1}), \qquad i = 0, 1, 2, \ldots \tag{3}$$

where $z_t \equiv U'(c_t)p_t$, the marginal utility of a unit of the asset at time t and $a_t \equiv U'(c_t)d_t$, the marginal utility of the dividend on a unit of the asset at time t.

Notice that the Euler equation generated in the example is a linear difference equation in the variable $E_t(z_{t+i})$. The equation may be interpreted as having the forcing process $E_t(a_{t+i})$ and having a root of the equation equal to ρ^{-1}. Since ρ is by assumption between zero and one, (3) is, in the conventional sense, an unstable equation. The work of Sargent and Wallace (1973) made us aware of this issue, which arises in many rational-expectations models. Sargent and Wallace proposed that researchers generally adopt a solution to models like (3) that allows a stable time path for the endogenous variable when the exogenous variables are stable. In the present model this is the solution that sets the marginal utility of current price equal to the present value of expected future dividends. We denote this solution f_t to represent the part of asset price that depends only on market fundamentals. Formally, the proposed solution to (3) is

$$f_t = \sum_{i=1}^{\infty} \rho^i E_t(a_{t+i}). \tag{4}$$

If (3) were the entire model, the solution given in (4) would be only one of an infinite number of solutions. Other solutions can be obtained by adding an arbitrary term to (4) that is the solution to the homogenous part of (3). We denote the arbitrary element at time t by b_t. Equation (3) requires that such arbitrary elements obey

$$E_t(b_{t+i}) = \rho^{-i}b_t, \qquad i = 1, 2, 3, \ldots. \tag{5}$$

In the model at hand the elements of the sequence, $b_t, b_{t+1} \ldots$, denoted $\{b_t\}$, are elements of a bubble in the market for asset k. If the innovation in the bubble at time t is denoted v_t, it follows that

$$b_T = \rho^{-(T-t)}b_t + \sum_{i=1}^{T-t} \rho^{-i}v_{t+i}. \tag{6}$$

The actual observation of z_t may therefore consist of two elements, the market fundamentals part, f_t, plus the bubble, b_t, so that

$$z_t = f_t + b_t. \tag{7}$$

A bubble in z_t produces a related bubble in market price of the asset since $z_t = p_t U'(c_t)$, and $U'(c_t)$ need not be related to the asset market bubble. in this model, the agent's maximization problem helps the researcher formulate the hypothesis that bubbles are absent from market prices. This point was stated clearly by Obstfeld and Rogoff (1983). Their argument is as follows.

The single period Euler equation given in (3) may be iterated to equate margins for any two nonadjacent periods. For instance, the margin of substitution for period t and period $t + n$ can by equated by substituting $n - 1$ future Euler equations into the current period Euler equation and appealing to the law of iterated expectations. The n-period Euler equation is

$$z_t = \rho^n E_t(z_{t+n}) + \sum_{i=1}^{n} \rho^i E_t(a_{t+i}), \tag{8}$$

and it ensures that a maximizing agent cannot increase his expected utility by rearranging his consumption between periods t and $t + n$. When n is driven to infinity in (8), the agent's optimization implies

$$z_t = \lim_{n\to\infty} \left[\rho^n E_t(z_{t+n}) + \sum_{i=1}^{n} \rho^i E_t(a_{t+i}) \right]. \tag{9}$$

The first term on the right-hand side of (9) gives the agent's current evaluation of the expected marginal utility attached to the sale of a unit of asset k indefinitely far in the future. The second term on the right-hand side of (9) is the expected utility gain attached to the strategy of holding a unit of the asset indefinitely and consuming only the stream of dividends accruing to ownership of the asset. The current utility cost of purchasing the asset is given by z_t. Therefore, an agent can be at a maximum with a buy-and-hold (forever) strategy only if the first term on the right-hand side of (9) is zero.

This example of an infinitely lived representative agent provides a special case in which bubbles are not possible in equilibrium. The agent knows that he will live forever, and he knows that everyone in the economy is identical to him. In equilibrium the asset must be priced to be held by the infinitely lived representative agent who must follow the buy-and-hold strategy. The agent can be at an equilibrium only when the marginal utility of what he gives up to buy the asset, z_t, is equal to the expected value of what he gets from holding the asset, $\sum_{i=1}^{\infty} \rho^i E_t(a_{t+i})$. Therefore, in this model, the combination of the agent's maximization and market equilibrium give the implication that the first term in (9) must be zero. This transversality condition arises as a necessary condition of the model, and one way to test this model is to test the transversality condition.

The bubble process defined by (5) and (6) is consistent with the model's Euler equation, but it is not consistent with the transversality condition. The present value of the future marginal utility of the asset price must go to zero as the discounting period goes to infinity as long as the utility value of the asset payoffs is bounded above. The present value of the expected future bubble, however, will not go to zero, since the bubble is expected to grow at the inverse of the discount factor.

Some models imply a transversality condition that is inconsistent with the presence of bubbles in asset prices. In contrast, the theoretical analysis of Tirole (1985) indicates that other models incorporating rational expectations can be perfectly consistent with asset price bubbles in some circumstances.[4] In our view, bubble tests are analogous to tests for downward sloping demand curves—not all models imply downward sloping demand curves, but some do. Many economists like to think that asset prices are determined strictly by market fundamentals, and empirical research is necessary to verify or refute this idea.

III Bubbles and Variance Bounds Tests

The purpose of this section is to show that failure of an asset-pricing model in certain variance bounds tests gives no information about bubbles. Such results are correctly interpreted as providing information about the adequacy of the underlying model. We conduct the argument using the model developed in the previous section. For this part of the argument we adopt the Euler equation, (3), and the pricing function, (7), which allows asset price bubbles. The bubble, if present, must follow the time series process described in (5). In the rest of this section, for brevity, we refer to the marginal utility of the asset price, z_t, simply as the asset price, and we refer to the marginal utility derived from the dividend paid to owners of the asset, a_t, as the dividend on the asset. This convention is not invoked in later sections.

The basic insights of the variance bounds literature are that the variance of an actual variable must be greater than or equal to the variance of its conditional expectation and that this latter variance must be greater than or equal to the variance of a forecast based on a subset of the information used by agents. To see how the existence of bubbles could lead in theory to a violation of variance bounds, consider the *ex post rational price*, which is defined as the price that would prevail if agents knew future market fundamentals with certainty and there were no bubbles. The ex post rational price is

$$z_t^* = \sum_{i=1}^{\infty} \rho^i a_{t+i}. \tag{10}$$

Notice that ex post rational price is a theoretical construct, and although it is subscripted with a t, it is neither in an agent's information set nor is it in an econometrician's information set.

The theoretical relation that is the foundation of many variance bounds tests is obtained by subtracting (7) from (10) and rearranging terms:

$$z_t^* = z_t + u_t - b_t, \tag{11}$$

where $u_t \equiv \sum_{i=1}^{\infty} \rho^i [a_{t+i} - E_t(a_{t+i})]$ is the deviation of the present value of dividends from its expected value based on time t information. By construction, u_t is uncorrelated with z_t and b_t, but z_t and b_t may be correlated with each other.

The innovation in x_t from time $t-n$ is $[x_t - E_{t-n}(x_t)]$. Then, the innovation variance and covariance operators are defined by

$$V_n(x_t) = E\{[x_t - E_{t-n}(x_t)]^2\}$$

and

$$C_n(x_t, y_t) = E\{[x_t - E_{t-n}(x_t)][y_t - E_{t-n}(y_t)]\}$$

where $E(\cdot)$ denotes the unconditional mathematical expectation. In what follows we treat n as a finite positive integer.

Applying the innovation variance operator to both sides of (11) yields

$$V_n(z_t^*) = V_n(z_t) + V_n(u_t) + V_n(b_t) - 2C_n(z_t, b_t), \tag{12}$$

which follows from the conditional orthogonality of u_t to z_t and b_t.

Suppose that somehow a researcher could develop very good measurements of the variance of the ex post rational price, z_t^*, and of the variance of market price, z_t. Suppose further that it was found that ex post rational price had a smaller variance than market price. Since the variance of both u_t and b_t must be nonnegative, such a finding could only be rationalized, within the framework of the model, by a positive conditional covariance between the bubble and z_t. Therefore, as long as the model is correct, and as long as the variance of ex post rational price and the variance of market price are measured appropriately, a finding of $V_n(z_t) > V_n(z_t^*)$ can be interpreted as evidence of bubbles.

The difference between the theoretical exercise described above and its practical implementation arises in the construction of an observable counterpart to z_t^*. Because it is impossible to measure ex post rational price since it depends on the infinite future, researchers typically measure a related variable that we call $\hat{z}_t$. Since actual price and dividend data are available for a sample of observations on $t = 0, 1, \ldots T$, researchers use

$$\hat{z}_t \equiv \sum_{i=1}^{T-t} \rho^i a_{t+i} + \rho^{T-t} z_T, \qquad t = 0, 1, \ldots, T-1, \tag{13}$$

in place of z_t^*. Notice from (10) and (13) that

$$\hat{z}_t = z_t^* - \rho^{T-t} z_T^* + \rho^{T-t} z_T, \tag{14}$$

which implies from (11) that

$$\hat{z}_t = z_t^* + \rho^{T-t}(b_T - u_T). \tag{15}$$

Since u_T is the innovation in the present value of dividends between time T and the infinite future, it is uncorrelated with all elements of the time T information set, which includes the time t information set. Since b_T depends on the evolution of the stochastic bubble between t and T from (6), it is not orthogonal to time t information.

Notice what happens when (15) is solved for z_t^*, and the result is substituted into (11). After slight rearrangement, one obtains

$$\hat{z}_t = z_t + w_t \tag{16}$$

where

$$w_t = (u_t - \rho^{T-t}u_T) + (\rho^{T-t}b_T - b_t). \tag{17}$$

Equation (16) is the empirical counterpart of (11) and forms the basis of the usual variance bounds tests. The only important difference between our version of (16) and that of previous researchers is that we have allowed explicitly for rational stochastic bubbles in our derivation.

Application of the innovation variance operator to (16) gives

$$V_n(\hat{z}_t) = V_n(z_t) + V_n(w_t) + 2C_n(z_t, w_t). \tag{18}$$

The important point concerning (18) is that the innovation covariance between z_t and w_t is zero. To understand why, consider the nature of the composite disturbance W_t. First, as noted above, both u_t and u_T are uncorrelated with z_t since z_t is in the time t information set, which is a subset of the T information set. Second, and most important, the combined term $\rho^{T-t}b_T - b_t$ is uncorrelated with the time t information set, even though each term separately is not orthogonal to time t information. This follows from (6) because $\rho^{(T-t)}b_T - b_t = \sum_{i=1}^{T-t} \rho^{-i}v_{t+i}$, which is orthogonal to all time t information including z_t. Hence, $C_n(z_t, w_t) = 0$.

Therefore, (18) takes the form

$$V_n(\hat{z}_t) = V_n(z_t) + V_n(w_t), \tag{19}$$

from which it follows that

$$V_n(\hat{z}_t) \geq V_n(z_t), \tag{20}$$

by the nonnegativity of $V_n(w_t)$. Recall that (20) is derived in the presence of rational stochastic bubbles.

In a study of actual data, an assumption must be made about the form of the marginal utility of consumption foregone when purchasing an asset and the marginal utility realized from consuming the the dividends and capital value of the asset. A popular assumption in some applied work is that the marginal utility of consumption is a positive constant whose value is immaterial to agents' decisions. A finding, in applied work, that an asset-pricing model violates inequality (20) is evidence of model misspecification. Many mistakes can arise in the choice of utility function, the choice of observation period, the treatment of taxes, or some other misspecification, but the violation of (20) cannot be due to rational asset price bubbles since (20) was derived in a model that allowed bubbles.

Research that does not use the terminal price as above in variance bounds tests of stock price volatility, such as Leroy and Porter (1981), could, in principle, find variance bounds violations attributable to rational stock price bubbles. Of course, these models could also violate variance bounds if misspecified in any of the ways mentioned above.

IV Testing for Bubbles

In the previous section we demonstrated that some volatility tests, which were not originally proposed as bubble tests, are not well-designed tests of bubbles. In this section we discuss some tests that were conceived explicitly to test for bubbles. We also provide a warning about the interpretation of such tests.

A Warning about Bubble Tests

In virtually all modern economic models, expectations of agents about the future play an important role in decision making. Empirical implementation of these models is complicated by the fact that expectations are not observable directly. The investigator must model agents' expectations in terms of observable variables; he substitutes his model of expectations for the unobservable true expectations. Once the final model of actual data is estimated, with the restrictions from expectations imposed, inference can be carried out conditionally on having modeled expectations correctly. If the model of expectations is flawed, incorrect inference can result. This problem is particularly serious in bubble tests, but it is not just in these tests that the problem arises.

The typical rational-expectations econometric methodology involves using the assumption of rational expectations and an assumed time series model for the exogenous driving processes. These assumptions allow the researcher to use historical data to substitute for the unobserved expectations variables. Suppose that the assumed time series model is incorrect and that historical time series data on market fundamentals are a poor reflection of agents' beliefs about the future evolution of data. For example, if in order to finance expansion a profitable firm has been paying no dividends and retaining all profits throughout its finite history, the firm's nonexistent dividend history gives no information about the dividends that the firm is capable of paying in the future. Consequently, the dividend history provides no information about the value of a share in that firm to an investor.

If the market knows that the firm will not be paying dividends for some time, market equilibrium requires that the expected real value of the firm rise at a rate equal to the expected real rate of interest appropriate for the riskiness of that firm. This circumstance creates a debilitating problem for a researcher interested in testing for bubbles. If the investigator assumes that it is appropriate to infer the market fundamentals price from historical dividends, he would infer that the fundamental value of the firm is zero. He would also ascribe all movements in the firm's value to a bubble, since bubbles, in the type of model presented above, are characterized by arbitrary price movements whose expected rate of change is equal to the real rate of interest.

This is an obvious, simple example of a problem in testing for bubbles that may assume a much more complex form. Stated more generally, the issue is that it seems very difficult to disentangle bubbles from the possibility that agents may be anticipating, with some finite probability, some eventual change in the underlying economic environment. Flood and Garber (1980) discussed this problem in their original bubble tests, and Hamilton and Whiteman (1985) have recently also addressed the problem. In later work, Flood and Garber (1983) referred to agents' beliefs in possible future alterations of the economic environment as *process switching*. We adopt that terminology here.

Since dividend policy is arbitrary in simple models of the firm, the problem of process switching seems particularly devastating here. By working with over one hundred years of data from the Standard and Poor's data set, Shiller and West tried to circumvent the problem in two ways. First, they used a data set with a long intertemporal dimension. Second, the data set is for a large aggregate of firms rather than for an individual firm. Intuitively, both features of the data seem useful in avoiding the process-switching pitfall in interpreting the data, but at a formal level neither seems to help very much. Having a long intertemporal dimension does not guarantee that the sample includes either a large sample of process switches or that the stochastic process governing such switches is modeled appropriately. Further, if dividend policy for one firm is arbitrary, then dividend policy for a large aggregate of firms will generally also be arbitrary. Hence, aggregation of dividends does not provide much formal help in avoiding problems of interpretation induced by process switching.

For these reasons, we interpret tests of the no bubbles hypothesis as actually being tests of the hypothesis of no bubbles *and* no process switching. Of course, conditional on no process switching, the tests may be interpreted as tests of the no bubbles hypothesis.

Tests under the Alternative Hypothesis of Bubbles

Early tests for bubbles were conducted on data from European hyperinflations following World War I. Flood and Garber (1980), Burmeister and

Wall (1984), and Flood, Garber, and Scott (1984) estimate an equation of money market equilibrium while simultaneously estimating a money-supply forcing process.

There is a close relation between these early price-level models, which allow bubbles, and the asset-pricing models discussed above. In the early models the log of the price level played the role currently being played by the marginal utility value of the asset, the log of the money supply played the role currently taken by the utility value of dividend payments, and a transformation of the semielasticity of money demand with respect to expected inflation played the role currently taken by the constant discount rate, ρ.

There are some important differences among the early studies in empirical implementation of bubble tests. Flood and Garber (1980) did a time series estimation of a nonstochastic bubble; Burmeister and Wall (1984) did a time series estimation of a specific stochastic bubble while relaxing some strong identifying restrictions Flood and Garber made about the nature of the forcing process; and Flood, Garber, and Scott (1984) combined time series and cross section data to test for a nonstochastic bubble simultaneously inhabiting a number of post–World War I hyperinflations.

There is also an important similarity in these studies. In each case the researchers desired to test the hypothesis that bubbles are absent from the data while estimating under the alternative hypothesis that bubbles are present. The Flood and Garber and the Burmeister and Wall studies both attempt time series asymptotic tests of the null hypothesis that bubbles are absent from the data. They desired to test the statistical significance of the parameters associated with the bubble against the null hypothesis that these parameters are zero. The difficulty with such tests is that the statistics used to test for bubbles must be derived under the alternative hypothesis that allows for bubbles. It is well known that the asymptotic distribution of test statistics in situations such as the presence of bubbles (exploding regressors) is difficult to derive and that standard tests are almost certainly not applicable.[5]

Flood, Garber, and Scott (1984) try to avoid the time series problem by estimating with panel data. The conceptual experiment yielding the asymptotic distributions involves letting the size of the cross section in the panel become very large, and this would produce well-behaved asymptotic parameter distributions in large samples if the cross-sectional errors satisfy the appropriate orthogonality conditions. The problem in applying this methodology is that the number of simultaneous hyperinflations was not actually very large. The size of the cross section in Flood, Garber, and Scott was only three.

West's Bubble Tests

Prompted by some ideas presented in Blanchard and Watson (1982), West (1984, 1985a, 1985b) developed bubble tests that circumvent the problems associated with obtaining limiting distributions described above. West's insight was to conduct all estimation under the null hypothesis of no bubbles. Under the null, standard asymptotic distribution theory applies for all parameter estimates, and tests of the no bubbles hypothesis may be conducted in large samples using these distributions. The nonstationarity of bubbles affects West's tests only because asymptotic distributions of the parameter estimates are not well behaved under the alternative hypothesis. Consequently, the power of his tests is unknown. This problem, though, appears in all econometric work that allows for a variety of unspecified alternative hypotheses and is not specific to West's tests.

West's first application of his bubble tests was to annual aggregate stock prices, and he interpreted his results as providing overwhelming evidence of the presence of economically important stochastic bubbles in the stocks comprising Shiller's (1981) modified Dow-Jones data and the Standard and Poor's index.

Since a large portion of our empirical work involves extensions and modifications of West's work, we now present a stylized version of his methods. Also, since our research as well as West's involves data from the stock market, we discuss the issues in the context of the example examined above. The goal of West's research is to test the hypothesis that every element in the series $\{b_t\}$ is zero, where the series $\{b_t\}$ contains the bubble elements from a specific model of an asset price series.

The first step in West's methodology is to estimate and test the specification given in (3), the Euler equation for adjacent periods. West's methods require the investigator to specify the agent's utility function, and in most of his work he assumed a risk-neutral representative agent. With risk neutrality an agent's marginal utility of consumption is constant across time and is known to all agents. Hence, the marginal utility terms divide out of each side of (3) to yield

$$p_t = \rho E_t(p_{t+1} + d_{t+1}) \tag{3a}$$

where p_{t+i} is the real price of the asset at time $t + i$ and d_{t+i} is the real dividend paid by the asset at time $t + i$ to purchasers of the asset at $t + i - 1$. The model provides no guidance to the researcher in determining the appropriate deflator to convert nominal asset prices and nominal dividends into real terms. West followed Shiller (1981) and deflated nominal stock prices and nominal dividends by a producer price index.[6]

West examines four aspects of (3a) to determine its consistency with the data. The first involves a specification test of the overidentifying restrictions. West estimated (3a) using Hansen's (1982) generalized method of moments (GMM), which is an instrumental variable technique that delivers overidentifying restrictions when the number of instrumental variables exceeds the number of parameters to be estimated. The specification test of the overidentifying restrictions involves examination of a chi-square statistic. The second specification test involves examining serial correlation of the residuals using the procedures described in Pagan and Hall (1983). The third test checks the stability of estimated coefficients by testing for midsample shifts in the coefficients. The fourth way the specification was examined involved checking the quality and reasonableness of the estimated parameters. Are the standard errors relatively small and do the point estimates correspond to reasonable economic values? Do the estimates change with changes in the instruments?

Step two of the methodology involves estimating a prediction equation for real dividends as a function of past dividends and possibly a linear trend. One of the nice aspects of West's work is that he is able to test for bubbles without taking a stand on the econometric exogeneity of any variables. He is able to carry out the tests as long as he has correctly identified the order of the lagged dividends required to forecast future dividends with a white noise error. Real dividends may depend on many contemporaneous and lagged variables not explicitly included in the forecasting equation. The methodology simply requires that the dividend forecasting equation be taken to be the projection of current dividends onto lagged dividends, which are assumed to be contained in the information set used by agents in making their predictions of future dividends. Other variables that might have entered a more primitive dividend equation have implicitly been solved out in the projection process.

The dividend forecasting equation is also subjected to a battery of tests. These include testing for midsample coefficient shifts, testing for first-order serial correlation following the Pagan and Hall procedures, and calculating the Box-Pierce Q statistic testing simultaneously for first- and higher-order serial correlation. If process switching is important, it could be manifest in the stability of the coefficients of the forecasting equation.

The third step in the methodology involves modeling the asset price in two ways. The two should be equivalent if there are no asset price bubbles. The first asset price model involves parameters estimated in the first two steps. From the work of Hansen and Sargent (1980, 1982), a closed-form expression for the market fundamentals portion of asset price is available once the econometrician takes a stand on the information set conditioning the expectation operator in (3a), the parameters entering the forecasting equation for future dividends, and the discount parameter in the agent's utility function. In West's method these parameters and their distributions are obtained in the first two steps. The second asset price equation involves estimating an unconstrained regression of asset price on the information used to form the dividend forecasts. As long as there are no bubbles, the parameters constructed from (3a) and the dividend forecasting equation ought not to be significantly different from the parameters estimated in the unconstrained regression. If a bubble is present in asset price, however, and as long as the bubble has a nonzero mean or is correlated with past dividends, the parameters calculated in the unconstrained regression will not be unbiased estimates of the parameters constructed from (3a). A Hausman (1978) test is appropriate to test the significance of the measured differences between the two asset price models.

The steps in West's methodology contain an important sequential aspect. Only if the first two steps deliver correct equations does the third step test for bubbles. Formally, the bubbles test is conditional on having correct specifications for the Euler equation and the dividend forecasting equation. If either the Euler equation or the dividend forecasting equation is incorrect, there is no reason to expect an asset-pricing function constructed from incorrect elements to be close to the unconstrained pricing function.

This methodology is applied by West (1984, 1985a) to a stock market model of a long data series of aggregated stock prices and dividends. His finding is that there is strong evidence of bubbles in aggregate stock prices. These findings intrigued us for several reasons. First, if the findings held up under additional scrutiny, they would be strong evidence of either expected process switching or of asset price bubbles, and neither possibility is particularly attractive. Second, we suspected that this linear Euler equation featuring a constant rate of return is not appropriate. Although West works with a long time series of annual data, which are considerable different from the quarterly or monthly post–World War II data in Hansen and Singleton (1982, 1983), the strength of the evidence against the constant real rate of return model in postwar data seems overwhelming. Third, we suspected that his data do not satisfy the assumption of time series stationarity necessary to conduct inference in the manner he proposed.

In the next section we use data provided to us by West to demonstrate that his interpretation of his results is almost surely incorrect.[7] We show that the data indicate it is very likely that his basic model is misspecified. His test for no bubbles is actually a test of a joint hypothesis that includes correct model specification and absence of bubbles. Since it is likely that the model is misspecified, failure of a test of this joint hypothesis does not give much evidence that bubbles are present. Of course, failure of the test is not inconsistent with bubbles; it simply does not give much information about bubbles.

V New Empirical Analyses

The data we use consist of annual real stock price indices and associated real dividend payments for two time series. The first set of series is for the Standard and Poor's data for the years 1871–1980, and the second is for a modified Dow-Jones index for the years 1928–1978. Nominal magnitudes are deflated by the Bureau of Labor Statistics wholesale price index. The stock price data are the daily averages for each January, and the dividends are those that accrue during a year.[8]

We first replicated the results in West's table IA. Since we were concerned that first differencing the levels of the data would not be sufficient to provide a stationary time series process, we estimated the Euler equation in return form using a set of instruments that ought to be stationary in a growing real economy. The first equation estimated was

$$1 = \rho E_t(R_{t+1}) \tag{21}$$

where $R_{t+1} \equiv (p_{t+1} + d_{t+1})/p_t$, the return at time $t + 1$.

We also employed a GMM estimation using a constant and three lags of the dividend-price ratio, d_t/p_t, as instruments. The results are reported in table 5.1. The usefulness of the instrument set, as measured by its ability to predict the returns, is discussed later in this section. Equations 1 and 5 in table 5.1 report the results of estimating the Euler equation of the risk-neutral utility function. Our results are very similar to those of West even though our instruments are different and we estimated the Euler equation in return form while he estimated either in levels or in first differences.

The discount rate, ρ, is very precisely and very plausibly estimated. The estimated value using the Standard and Poor's data (specification 5) with lagged dividend-price ratios as instruments is 0.9155 with a standard error of 0.0138. The estimate using the modified Dow-Jones data (specification 1) is 0.9171 with a standard error of 0.0268. As West mentions, the discount rate estimates are quite close to the inverse of the average return on the stock market over the estimation period. That the discount rate is precisely and plausibly estimated, however, is only part of the story. The chi-square statistic that tests the overidentifying restrictions indicates

Table 5.1
GMM estimation of Euler equation

$1 = E_t\rho\{[U'(c_{t+1})/U'(c_t)][(p_{t+1} + d_{t+1})/p_t]\}$

Instruments: $(1, d_t/p_t, d_{t-1}/p_{t-1}, d_{t-2}/p_{t-2})$
Data set (equations 1–4): modified Dow-Jones (1931–1978)

1. Utility function $U(c_t) = c_t$ (risk neutral)
$\hat{\rho} = 0.9171$; S.E. $= 0.0268$; M.L.S. $= 0.000$; $\chi^2(3) = 6.6461$; M.L.S. $= 0.084$

2. Utility function $U(c_t) = ln(c_t)$ (log utility)
$\hat{\rho} = 0.9446$; S.E. $= 0.0278$; M.L.S. $= 0.000$; $\chi^2(3) = 8.8779$; M.L.S. $= 0.031$

3. Utility function $U(c_t) = [1/(1-\alpha)]c_t^{1-\alpha}$ (CRRA)
$\hat{\rho} = 0.8622$; S.E. $= 0.0470$; M.L.S. $= 0.000$; $\hat{\alpha} = -1.8663$; S.E. $= 2.0173$; M.L.S. $= 0.355$; $\chi^2(2) = 6.7852$; M.L.S. $= 0.034$

4. Utility function $U(c_t) = 1 - (1-\alpha)\exp(-\alpha c_t)$ (CARA)
$\hat{\rho} = 0.8639$; S.E. $= 0.0423$; M.L.S. $= 0.000$; $\hat{\alpha} = -0.5064$; S.E. $= 0.4791$; M.L.S. $= 0.291$; $\chi^2(2) = 6.4260$; M.L.S. $= 0.040$

Data set (equation 5): Standard and Poor's (1874–1980)

5. Utility function $U(c_t) = c_t$ (risk neutral)
$\hat{\rho} = 0.9155$; S.E. $= 0.0138$; M.L.S. $= 0.000$; $\chi^2(3) = 8.8499$; M.L.S. $= 0.034$

Note: Standard errors are denoted S.E. and marginal levels of significance are denoted M.L.S. Standard errors are calculated under the null hypothesis with allowance for conditional heteroscedasticity as in Hansen and Singleton (1982).

mixed evidence concerning the model. The test statistic is $\chi^2(3) = 8.8499$ with an associated marginal level of significance of 0.034 for the Standard and Poor's data and $\chi^2(3) = 6.6461$ with an associated marginal level of significance of 0.084 for the modified Dow-Jones data.

These results are not very different from those reported by West in his table IA, when he estimated his model in levels. He found that the model performed poorly in levels for the Standard and Poor's data, and he attributed this to possible nonstationarity in prices and dividends. Consequently, he reestimated the model with some of the equations in first differenced form and other equations remaining in levels. The chi-square statistics in this instance are much more favorable to the model. We simply do not follow the logic of West's procedure. Prices and dividends were differenced to allow for possible nonstationarity in levels due to linear growth. The Euler equation, however, is estimated in level form. If prices and dividends are indeed nonstationary, the Euler equation ought also to be estimated in a form that takes satisfactory account of this nonstationarity. This is a problem that has been confronted in the literature previously, for example, Hansen and Singleton (1982, 1983), and we have adopted the typical solution—estimation of the Euler equation in return form.

We see no reason to difference our instruments or to difference the returns on the stock market. Even in an exponentially growing economy, stock market returns and dividend price ratios are stationary. Consequently, our interpretation of the data indicates that the risk-neutral specification does not work at all well for the Standard and Poor's data and works only marginally better for the modified Dow-Jones data. On the basis of these results and the tests in West's paper, there are grounds for proceeding cautiously with bubble tests based on the linear Euler equation.

Nonlinear Euler Equations

A number of recent studies have estimated nonlinear Euler equations, and a natural question is how well do some popular nonlinear period utility functions explain the current data? In table 5.1 we report our results for three nonlinear period utility functions: $U(c_t) = ln(c_t)$ (logarithmic utility), $U(c_t) = (1 - \alpha)^{-1} c_t^{(1-\alpha)}$ (constant relative risk aversion), and $U(c_t) = 1 - (1/\alpha)\exp(-\alpha c_t)$ (constant absolute risk aversion). Since we want to compare the performance of these utility functions against the performance of the linear alternative, while giving the linear alternative the benefit of the doubt, we conduct the comparison using the Modified Dow-Jones data in which the risk-neutral model performed best.

The results of this investigation are presented in table 5.1 specifications 2–4. The data set is the Modified Dow-Jones data 1931–1978 along with real per capita consumption figures for the United States.[9]

Three points about the results are noteworthy. First, the discount rate is estimated approximately as precisely and reasonably in all three specifications of nonlinear utility functions as in the case of the linear utility function. All of the estimates of the discount rate are within two standard errors of the estimate for the constant relative risk aversion utility function. Second, the tests of the overidentifying restrictions for the nonlinear utility functions are all above the chi-square statistic for the linear utility function. In fact, for the nonlinear utility functions, the Euler equation model would be rejected at standard confidence levels. Third, for the nonlinear utility functions of the constant relative risk aversion and constant absolute risk aversion types, the free parameter in the utility function is very imprecisely estimated.

Iterated Euler Equation Estimation

These results seem to us to point in the direction of the linear utility function as providing the most nearly adequate description of the data in this class of utility functions. Of course, the utility function could be complicated in a wide variety of ways, but an investigation of such complications is beyond the scope defined for this study.

While the results thus far, on the Dow-Jones data set, point in the direction of not rejecting the linear utility function at traditional levels of significance, there remains one problem: Even if the linear Euler equation is fairly close to the true Euler equation, is it close enough to the true Euler equation to use in bubble tests? The potential problem arises because bubble tests do not simply use the Euler equation once; they use the Euler equation iterated an indefinite number of times. Suppose, for example, that using the linear utility function in place of the true utility function induces a small specification error into the Euler equation that is difficult to detect. Bubble tests require iteration of the Euler equation over and over with future Euler equations projected onto the current information set. It might be that this minor specification error, when summed over indefinitely many periods, becomes a quite formidable mistake. Certainly, we have no formal proof of such a proposition in mind, for it may also be true that the summation of the specification errors causes cancellation such that the sum over lots of specification errors is less formidable than any single error.[10]

One way to proceed empirically to investigate the importance of this issue is to iterate the Euler equation a second period as in the derivation of (8). The iterated Euler equation was subjected to the same type of testing procedure used for the noniterated equation. Since the modified Dow-Jones data set previously was the most favorable environment for the risk-neutral utility function, we started our investigation using the Dow-Jones data. Table 5.2 reports the results. We estimated the Euler equation for the four period utility functions used above. In all cases the $\chi^2(3)$ statistic rose as compared with the noniterated equation, and in all cases the chi-square statistic indicates dramatic rejection of the equation. Most interesting is the large increase in the chi-square statistic for the risk-neutral utility function. Recall that previously, with these data, the noniterated risk-neutral utility function appeared to provide the best explanation of the functions we investigated. Now, with one iteration of the Euler equation, the chi-square statistic with three degrees of freedom jumps dramatically from 6.6461 to 35.5453, indicating almost sure rejection of the risk-neutral model in these data.

Different Discount Rates for Dividends and Capital Gains

One possibly important objection to the way we have used the data is that we, like most other investigators, have used pre-tax returns to estimate

Table 5.2
GMM estimation of once iterated Euler equation

$1 = E_t\rho\{\rho[U'(c_{t+2})/U'(c_t)][(p_{t+2} + d_{t+2})/p_t] + [U'(c_{t+1})/U'(c_t)](d_{t+1}/p_t)\}$

Instruments: $(1, d_t/p_t, d_{t-1}/p_{t-1}, d_{t-2}/p_{t-2})$
Data set (equations 1–4): modified Dow-Jones (1931–1978)

1. Utility function $U(c_t) = c_t$ (risk neutral)
$\hat{\rho} = 0.8429$; S.E. = 0.0102; M.L.S. = 0.000; $\chi^2(3) = 35.5453$; M.L.S. = 0.000

2. Utility function $U(c_t) = ln(c_t)$ (log utility)
$\hat{\rho} = 0.9460$; S.E. = 0.0235; M.L.S. = 0.000; $\chi^2(3) = 13.7362$; M.L.S. = 0.003

3. Utility function $U(c_t) = [1/(1-\alpha)]c_t^{1-\alpha}$ (CRRA)
$\hat{\rho} = 0.8743$; S.E. = 0.0811; M.L.S. = 0.000; $\hat{\alpha} = -0.8023$; S.E. = 3.4361; M.L.S. = 0.815;
$\chi^2(2) = 8.2965$; M.L.S. = 0.016

4. Utility function $U(c_t) = 1 - (1-\alpha)\exp(-\alpha c_t)$ (CARA)
$\hat{\rho} = 0.8691$; S.E. = 0.0716; M.L.S. = 0.000; $\hat{\alpha} = -0.0195$; S.E. = 0.7640; M.L.S. = 0.980;
$\chi^2(2) = 9.3902$; M.L.S. = 0.009

Data set (equation 5): Standard and Poor's (1874–1980)

5. Utility function $U(c_t) = c_t$ (risk neutral)
$\hat{\rho} = 0.9361$; S.E. = 0.0115; M.L.S. = 0.000; $\chi^2(3) = 10.787$; M.L.S. = 0.013

Note: See table 5.1.

behavior which depends on after-tax returns. If dividends and capital gains were taxed at equal constant uniform rates, the estimated discount rates could simply be interpreted as after-tax discount rates, equal to the primitive discount rate times one minus the tax rate. There are three problems though. First, tax rates are not constant; second, dividends are not subject to a flat tax rate; and third, dividends and capital gains are not taxed in the same way.

We do not treat the first two problems. We tried, however, to make a crude correction for the unequal taxation of dividends and capital gains. Our idea was simply to split the return into its capital gain component and its dividend yield component and to estimate separate discount rates for the two elements of the return. We estimated only the Euler equation for the risk-neutral utility function, and we estimated only in the Standard and Poor's data set. Table 5.3 gives the results for both the noniterated and the iterated versions of the Euler equation. The discount rates are now not very precisely estimated for discounting the dividend yield, but they continue to be quite precisely estimated for the capital gain component of the return. The hypothesis that the two discount rates are equal is not strongly supported for either estimation. In fact, the point estimate of the discount rate attached to the dividend yield is negative.

Table 5.3
GMM estimation of unequal discount rates Euler equation

Noniterated Euler equation
$1 = E_t\{[\rho_1(d_{t+1})/p_t) + \rho_2(p_{t+1}/p_t)][U'(c_{t+1})/U'(c_t)]\}$

Instruments: $(1, d_t/p_t, d_{t-1}/p_{t-1}, d_{t-2}/p_{t-2})$
Data set: Standard and Poor's (1974–1980)

1. Utility function $U(c_t) = c_t$ (risk neutral)
$\rho_1 = -1.9597$; S.E. = 1.4844; M.L.S. = 0.187
$\rho_2 = 1.0565$; S.E. = 0.0745; M.L.S. = 0.000
$\chi^2(2) = 3.4813$; M.L.S. = 0.175

Hypothesis test: H_0: $\rho_1 = \rho_2$ vs. H_1: $\rho_1 \neq \rho_2$
Wald statistic* = 3.7512; M.L.S. = 0.053

Once iterated Euler equation risk neutrality
$1 = E_t\{\rho_1(d_{t+1})/p_t) + \rho_1\rho_2(d_{t+2}/p_t) + \rho_2^2(p_{t+2}/p_t)\}$

(106 observations)
$\rho_1 = -2.4349$; S.E. = 1.6234; M.L.S. = 0.134
$\rho_2 = 1.1047$; S.E. = 0.0846; M.L.S. = 0.000
$\chi^2(2) = 2.2313$; M.L.S. = 0.328

Hypothesis test: H_0: $\rho_1 = \rho_2$ vs. H_1: $\rho_1 \neq \rho_2$
Wald statistic* = 4.3002; M.L.S. = 0.038

*Wald statistic = $(\hat{\rho}_1 - \hat{\rho}_2)^2/[V(\hat{\rho}_1) + V(\hat{\rho}_2) - 2C(\hat{\rho}_1, \hat{\rho}_2)] = \chi^2(1)$

Note: See table 5.1.

Return Forecasting Equations

Underlying all of our empirical work is the first stage forecasting equation for returns. If the risk-neutral utility function describes the data, then expected returns should be a constant equal to the inverse of the discount factor. Our estimation procedure requires that past information is useful in forecasting returns. No element of that past information set, other than a constant, should be helpful in predicting returns if the risk-neutral model is correct. In table 5.4 we present estimates of some linear regressions of stock market returns on some predetermined variables and constants. The GMM estimates we reported above implicitly used forecasting equations based on lagged dividend-price ratios, and here we present both those forecasting equations and some forecasting equations based on lagged dividend-price ratios and on lagged returns. These regressions are reported for both the Standard and Poor's and modified Dow-Jones data sets.

The interesting statistic obtained in all of these regressions is the $\chi^2(3)$ statistic that tests the hypothesis that the estimated coefficients on all of the

Table 5.4
Estimation of return forecasting equations

Data set: Standard and Poor's (1874–1980)

Equation 1. $R_t = a_0 + a_1 d_{t-1}/p_{t-1} + a_2 d_{t-2}/p_{t-2} + a_3 d_{t-3}/p_{t-3} + e_{1t}$

Coefficient	Estimate	S.E.	z	M.L.S.
a_0	0.9428	0.0606	15.5654	0.0000
a_1	−0.2746	1.3626	−0.2015	0.8403
a_2	3.2704	1.6222	2.0160	0.0438
a_3	−0.2943	1.5804	−0.1862	0.8523

H_0: $a_1 = a_2 = a_3 = 0$; $\chi^2(3) = 9.418$; M.L.S. = 0.024; $R^2 = 0.032$; D.W. = 1.953

Equation 2. $R_t = b_0 + b_1 R_{t-1} + b_2 d_{t-1}/p_{t-1} + b_3 d_{t-2}/p_{t-2} + e_{2t}$

Coefficient	Estimate	S.E.	z	M.L.S.
b_0	0.9030	0.1936	4.6633	0.0000
b_1	0.0292	0.1538	0.1903	0.8491
b_2	0.0725	1.9222	0.0377	0.9699
b_3	2.7903	1.7239	1.6187	0.1055

H_0: $b_1 = b_2 = b_3 = 0$; $\chi^2(3) = 9.263$; M.L.S. = 0.026; $R^2 = 0.032$; D.W. = 1.987

Data set: modified Dow-Jones (1931–1978)

Equation 3. $R_t = c_0 + c_1 d_{t-1}/p_{t-1} + c_2 d_{t-2}/p_{t-2} + c_3 d_{t-3}/p_{t-3} + e_{3t}$

Coefficient	Estimate	S.E.	z	M.L.S.
c_0	0.8171	0.1133	7.214	0.0000
c_1	0.7896	1.9022	0.4151	0.6800
c_2	5.0456	2.0796	2.4260	0.0094
c_3	−0.6237	2.0888	−0.2986	0.7667

H_0: $c_1 = c_2 = c_3 = 0$; $\chi^2(3) = 11.917$; M.L.S. = 0.008; $R^2 = 0.076$; D.W. = 2.153

Equation 4. $R_t = f_0 + f_1 R_{t-1} + f_2 d_{t-1}/p_{t-1} + f_3 d_{t-2}/p_{t-2} + e_{4t}$

Coefficient	Estimate	S.E.	z	M.L.S.
f_0	1.1142	0.2908	3.8316	0.0004
f_1	−0.2636	.2222	−1.1861	0.2419
f_2	−2.3591	2.9354	−0.8036	0.4259
f_3	7.2999	2.3440	3.1142	0.0032

H_0: $f_1 = f_2 = f_3 = 0$; $\chi^2(3) = 17.578$; M.L.S. = 0.005; $R^2 = 0.1032$; D.W. = 1.942

Note: See table 5.1. All standard errors are estimated using the Hansen-White correction for conditional heteroscedasticity. The z statistic, the ratio of an estimated coefficient to its standard error, is distributed as a standard normal in large samples. The R^2 is adjusted for degrees of freedom.

Table 5.5
Estimation of compound return forecasting equations

Data set: Standard and Poor's (1874–1980)

Equation 1. $R_{t+1,2} = a_0 + a_1 d_{t-1}/p_{t-1} + a_2 d_{t-2}/p_{t-2} + a_3 d_{t-3}/p_{t-3} + e_{t+1,2}$

Coefficient	Estimate	S.E.	z	M.L.S.
a_0	0.7716	0.1116	6.9144	0.0000
a_1	3.2430	2.2468	1.4430	0.1489
a_2	−0.5367	2.1492	−0.2497	0.8028
a_3	5.0880	1.9029	2.6739	0.0075

H_0: $a_1 = a_2 = a_3 = 0$; $\chi^2(3) = 13.462$; M.L.S. = 0.004; $R^2 = 0.101$;

Equation 2. $R_{t+2,3} = a_0 + a_1 d_{t-1}/p_{t-1} + a_2 d_{t-2}/p_{t-2} + a_3 d_{t-3}/p_{t-3} + e_{t+2,3}$

Coefficient	Estimate	S.E.	z	M.L.S.
a_0	0.6295	0.1513	4.1587	0.0000
a_1	1.3261	2.5997	0.5101	0.6100
a_2	6.6862	1.7869	3.7417	0.0002
a_3	4.3317	2.8192	1.5365	0.1244

H_0: $a_1 = a_2 = a_3 = 0$; $\chi^2(3) = 24.568$; M.L.S. = 0.000; $R^2 = 0.186$;

Equation 3. $R_{t+3,4} = a_0 + a_1 d_{t-1}/p_{t-1} + a_2 d_{t-2}/p_{t-2} + a_3 d_{t-3}/p_{t-3} + e_{t+3,4}$

Coefficient	Estimate	S.E.	z	M.L.S.
a_0	0.4433	0.2436	1.8198	0.0718
a_1	7.5549	3.2868	2.2985	0.0236
a_2	6.9082	0.8026	8.6072	0.0000
a_3	3.5789	3.4461	1.0385	0.3015

H_0: $a_1 = a_2 = a_3 = 0$; $\chi^2(3) =$ * M.L.S. = *; $R^2 = 0.232$;

Note: A * indicates that the matrix was not positive definite.

time-varying regressors are zero. These chi-square statistics have small marginal levels of significance ranging from the largest of 0.032 for Standard and Poor's data with a lagged return included to 0.0005 for the Dow-Jones data with a lagged return included. In our view these simple linear regressions give overwhelming evidence that the risk-neutral model does not adequately describe the data.

Since the iterated Euler equation specification gave the strongest evidence against the null hypothesis of constant expected real returns, we investigated whether the same instruments used in the specification tests in table 5.4 were useful in predicting the compound return across several periods into the future. Table 5.5 reports regressions of the compound return $R_{t+j,j+1}$, for $j = 1, 2, 3$, on a constant and the lagged dividend price ratios. The notation for the compound returns indicates that they are the product of the $j + 1$ one-period returns from time t to time $t + j$. Notice that the value of the chi-square statistic with three degrees of freedom testing the hypothesis of a constant expected two-period compound return is 13.462, which is larger than its analogue in table 5.4, equation (1). Similarly, the chi-square statistic testing the same hypothesis for the compound three-period return has a value of 24.568, which is even larger.

Unfortunately, the algorithm for computing the optimal weighting matrix needed in the calculation of the estimated GMM covariance matrix of the parameters does not constrain the estimated variance-covariance matrix of the estimated coefficients to be positive definite; and in computing the four-period compound return, the matrix was not positive definite. Since the effective degrees of freedom in 107 observations with an overlap of four is quite small, we did not choose to use one of the proposed procedures that does impose a positive definite construction. Since all of the estimation relies on asymptotic distribution theory, the results may be sensitive to sample size.

Summary and Conclusions

Some researchers have concluded that aggregate stock prices in the United States are too volatile to be explained rationally by movements in market fundamentals. Some have also concluded that stock prices may contain rational bubbles. In section III we show that failure of certain variance bounds tests conveys no information about rational bubbles. An incorrectly specified model, however, will generally fail a typical variance bounds test. In section V we examine the specification of the model usually used in variance bounds tests and in bubble tests. We find that the model used in the previous studies is inadequate to explain the data. As noted in section IV, the formal tests that have been carried out on these data are actually tests of the joint hypothesis of (i) the adequacy of the model, (ii) no process switching, and (iii) no bubbles. The joint hypothesis is rejected very strongly, and conditional on having the correct model and no process switching, the rejection has been taken to be evidence of bubbles. Since we find the model to be inadequate, we conclude that the bubble tests do not give much information about bubbles—since the model is inadequate, the null hypothesis should be rejected even if bubbles are not present.

Testing for bubbles requires an unrejected asset-pricing model that explains expected rates of return. Our results, as well as other empirical analyses such as Hansen and Singleton (1983) for example, present what we think is a convincing case that conditional expected returns on stock prices fluctuate through time. The profession is now attempting to reconcile such empirical results with theory and is searching in a number of different directions for the right model. Eichenbaum and Hansen (1985) and Dunn and Singleton (1985) try to save the representative agent Euler equation by adding the service flow from durable goods to the utility function. Garber and King (1984) argue that preference shocks may be necessary before we will be able to have an unrejected model. Grossman, Melino, and Shiller (1985) incorporate taxes and, along with Christiano (1984), explore the estimation of continuous-time models with discrete-time data. Others, such as Mehra and Prescott (1985), argue that the representative agent paradigm must be abandoned in favor of models with differential information sets across agents in order to explain the expected return premium that equity commands over bills.

To this list of research areas and problems we must add the standard caveat that the data may not be generated by ergodic processes that render invalid standard asymptotic inference. In such an environment, learning, possibly about government policies, may be an important contributing factor to time variation in expected returns. Whatever the eventual resolution of the problem, it is worth remembering that tests for bubbles are joint tests of no bubbles and no process switching and that bubble tests require an unrejected asset-pricing model.

Data Appendix

1. Stock market data were provided to us by Kenneth West who obtained the data from Robert Shiller. Two data series were used:

(a) The Standard and Poor's data for 1871–1981 with p_t defined to be the January price divided by the wholesale price index for January. Dividends paid during the year are assumed to accrue to the January holder of the stock. The sum of dividends paid during the year is deflated by the average of that year's wholesale price index and was available from 1871 to 1980.

(b) The (Shiller) Modified Dow-Jones index 1928–1979 with prices and dividends constructed and dated as in (a) above.

Both of these data sets are discussed in more detail in Shiller (1981a).

In our tables we report results for returns labeled Standard and Poor's 1874–1980 and Modified Dow-Jones 1931–1978. The year of a return is denoted by the dividend used in its construction. Estimation begins three years after the beginning of the data sets since we used three lags of the dividend price ratio as instruments.

2. The nonlinear utility functions all required a real per capita consumption measure. We used U.S. real per capita consumption of nondurables and

services. Aggregate consumption of nondurables and services were obtained from the *Economic Report of the President 1984* and were put into per capita terms by dividing by U.S. population taken from the same source. These data were then put into real terms by dividing by the wholesale price index (1967 = 100), which was taken from various issues of the *Handbook of Cyclical Indicators.*

Notes

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1. Mankiw, Romer and Shapiro (1985) mention this point in their derivation of an unbiased volatility tests. Some of section II incorporates material from Flood and Hodrick (1986), which discusses the issue in depth.

2. Huizinga and Mishkin (1984) is just one example that investigates movement in expected returns on a variety of risky assets over various time periods.

3. Our results match well with those of other researchers such as Hansen and Singleton (1982, 1983), Eichenbaum and Hansen (1985), and Scott (1985a) who report difficulty in finding an adequate representative-agent utility function to use in asset pricing.

4. Tirole (1985) explores the existence of speculative bubbles in an overlapping generations economy, which is an alternative dynamic model to the representative agent paradigm discussed in this chapter.

5. Domowitz and Muus (1985) have some new results concerning asymptotic distribution theory for exploding regressors, which may prove useful in future work on this subject.

6. Shiller (1984) finds similar results when deflating by the consumption deflator for services and nondurables.

7. West provided us with the data that he had obtained from Shiller. The data were partially constructed by Shiller, and they are described in Shiller (1981a).

8. The data are described in more detail in the Data Appendix. Estimation was done with a GMM program supplied by Kenneth Singleton. The standard errors of the statistics are calculated as in Hansen and Singleton (1982, pp. 1276–1277), and they allow for conditional heteroscedasticity.

9. The consumption data were obtained from the *Economic Report of the President, 1984* and are described in the Data Appendix.

10. Without specifying the true utility function, we could make no formal progress on this issue, and if we knew the true utility function, we would have used it in the first place.

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