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**Perfect foresight, financial policies, and exchange-rate dynamics**

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*Abstract.* This paper formulates a model of a small open economy which has perfect foresight about the future course of financial policies and their consequences. The authorities' expenditures must satisfy a budget constraint, and the private sector discounts future tax liabilities implied by government borrowing. Further assumptions are that there is perfect capital and goods mobility; that there is full employment; and that financial policy changes are implemented with a delay. These assumptions...
guarantee that the exchange rate behaves quite differently from previous research. In particular, it is shown that there is no simple connection between the movement in the exchange rate and the state of the current account.

Prévision parfaite, politiques financières et dynamique du taux de change. Ce mémoire développe un modèle d'une petite économie ouverte où l'on aurait une vue parfaitement claire de la direction future des politiques financières et de leurs conséquences. Les dépenses gouvernementales doivent être contenues à l'intérieur d'une contrainte budgétaire et le secteur privé escompte proprement le fardeau fiscal futur impliqué par les emprunts gouvernementaux. À ceci s'ajoutent d'autres postulats: mobilité parfaite des capitaux et des biens, plein emploi, le fait aussi que les politiques financières ne peuvent s'accomplir qu'avec un certain délai. Voilà qui engendre un comportement du taux de change assez différent de ce que l'on a l'habitude d'observer dans les travaux de recherche conventionnels. En particulier, on montre qu'il n'y a pas de lien simple entre le mouvement du taux de change et l'état du compte courant.

Introduction

The volatility of exchange rates throughout the 1970s has been a major concern of policy-makers and their advisers. As a result, a substantial literature now exists on the dynamics of exchange rates and their fundamental determinants. The major conclusion of much of this research is that a wide range of economic shocks and government policies have predictable long-run effects on the exchange rate. The short-run dynamics of a flexible-exchange-rate economy are more controversial, since in that time frame expectations, wealth effects, and price rigidities may be important influences in exchange rate determination. These influences may also cause magnification or offsetting of the predicted long-run effects. In this regard some models have demonstrated that the exchange rate 'overshoots' its long-run equilibrium value in response to an increase in the money supply.

The purpose of this paper is to analyse the dynamic effects of monetary and fiscal policy in an open economy in which there is perfect foresight. This setting contrasts with most previous studies of these issues. In them the issue is one of determining the impact effects of permanent unanticipated changes
in policy variables. In such a context the assumptions of static expectations, price rigidities, and the absence of wealth effects may be justified by the relatively brief time period of the analysis and the particular structure postulated about economic agents' knowledge of policy changes.

In the present analysis the time frame is long enough that the economy achieves a steady state, and knowledge of policy changes and of the structure of the economy is taken to be complete. The assumption is that changes in policy are partially anticipated because agents become aware of an impending change, perhaps because of an announcement by government authorities, of a delayed policy implementation. In this case the assumptions of perfect foresight and price flexibility and the focus upon wealth effects seem to be appropriate. In particular, price flexibility and perfect foresight permit agents to engage in maximizing behaviour and avoid expectational errors in the process.

The model we employ is similar to Kouri's (1976) and Dornbusch's (1976b), but it differs from them in two important ways. First, in line with the discussion in Sargent (1979), we develop a more general definition of disposable income, appropriate when the authorities finance their expenditures by borrowing. Secondly, the model contains an appropriately specified government budget constraint and allows government expenditures to be exogenous.

The distinctive feature of this model is its focus on announcements of government policies which are implemented in the future. Hall (1971), Sargent and Wallace (1973), Brock (1974), and Fischer (1979) have examined this issue in closed economy models, often with surprising results. Wilson (1979), Gray and Turnovsky (1979), and Dornbusch and Fischer (1980) have examined announcement effects in open economy models. The perfect foresight assumption links today's exchange rate to policies which will be implemented in the future and forces the economy to begin to adjust to the new equilibrium immediately after the announcement of a change in policy. The dynamic path of the exchange rate during the interim between announcement and implementation is demonstrated to be of a form which may make such periods important data sources for econometricians attempting to discover the structure of the economy. The interim path is also quite different from the path following implementation. In fact, conventional models with various alternative expectational assumptions provide basically the same

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4 Obviously, the consequences of such experiments can be explored fully only within an explicitly stochastic framework. We follow the literature, however, in assuming that unforeseen changes can occur within our rational expectations model even though stochastic elements are suppressed. One consistent interpretation of these experiments is that they show the modifications in the time profiles (in a linear model) of endogenous variables when a signal provides further information to agents and changes their expectations about the future paths of exogenous variables. We have taken the extreme case of probabilities switching between zero and one in a two-outcome event, but the qualitative conclusions would be unchanged were the probabilities to change in the same direction but by a smaller amount.
story as our perfect foresight model for the period following implementation.

The paper is organized as follows. The second section presents the model of the small open economy. The next section examines the long-run impact of financial policies on the country's net foreign indebtedness and rate of depreciation of its exchange rate. While these results are standard, the results in the fourth section demonstrate how the exchange rate depends on the discounted values of any arbitrary expected future financial policies. In the process the solution extends previous analyses, in that it includes government expenditures and the wealth effects implicit in current account imbalances. The following section presents an explicit analysis of the model for delayed implementation of policy regimes for constant levels of government expenditure and constant rates of money supply growth. The time path during the period between announcement and implementation of policy is analysed in detail. The final section provides some conclusions and discusses the role of expectations in the behaviour of exchange rates in the world today.

While the long-run results of this paper are similar to those derived from models which postulate static or adaptive expectations, the major difference here is that the announcement of a new policy brings about an immediate depreciation of the exchange rate, no matter what the nature of the expansionary policy change or the time of its implementation. The mechanism is that expectations of a lower value for the currency in the future dictate the lower value today. The message of this analysis is clear: anticipations of future policy actions are important determinants of today's value of the exchange rate. Consequently, if policy-makers transmit constantly changing signals to agents about the future path of government policies, they can be an additional source of exchange rate volatility, even though actual policy is quite stable.

THE MODEL

Consider a small economy which takes as given the foreign currency prices of all commodities. These commodities can be aggregated into a single composite good for this analysis, and units are chosen to set the foreign currency price of this good equal to one. Under free trade the domestic currency price of the composite good is equal to the exchange rate:

\[ p = s, \]  

(1)

where \( p \) is the natural logarithm of the domestic price and \( s \) is the log of the spot exchange rate, \( S \), defined as the domestic currency price of a unit of foreign exchange. Units are chosen so that the initial value of \( s \) is zero.

5 Barro (1978b), Bilson (1978), Mussa (1978), and Flood (1979) have examined rational expectations models which demonstrate the importance of expected future monetary policies in determining the exchange rate.
All non-monetary financial assets of the small economy are assumed to be perfect substitutes for foreign assets. Thus, the country can borrow or lend at the foreign interest rate \( r^* \). Furthermore, incipient capital flows assure that the domestic nominal interest rate, \( r \), is equal to the foreign interest rate plus the expected rate of depreciation of the exchange rate, \( \dot{E} \), or

\[
r = r^* + \dot{E}.
\]

Since the expected real yield on all non-monetary assets is the same, we treat all assets as multiples of a constant-purchasing-power saving deposit.\(^7\) Such a bond has a value equal to one unit of foreign currency and rate of return \( r^* \).

The government sector of the economy provides services to households and undertakes real expenditures equal to \( G \). The government finances its expenditures with real head taxes, \( T \), and by issuing debt and money. The budget constraint of the authorities is

\[
G + r^*B_g = T + \frac{M}{S} + \dot{B}_g,
\]

where \( B_g \) is the number of outstanding government bonds and \( M \) is the stock of domestic money.

Since the residents of the country are responsible for the future taxes implied by the existence of government debt, it is assumed that government bonds are not net wealth to the public. Thus we must subtract from the public's holdings of bonds, \( B_p \), the amount of government issues in order to determine the private sector's financial wealth. Call this difference net bonds, \( B_n = B_p - B_g \), and assume that \( B_n \) is always positive. We make the conventional assumption that foreign residents do not hold the domestic money, owing to legal tender laws, which guarantee that only local money is used in domestic transactions. Private-sector, real financial wealth is therefore

\[
W + \frac{M}{S} + B_n.
\]

The real resources available to the country as a whole are the value of full employment production, \( Y_0 \), and the income from net assets, \( r^*B_n \).\(^8\) The economy accumulates net bonds if the sum of private consumption, \( C \), and government expenditures is less than the available resources:

\[
\dot{B}_n = Y_0 + r^*B_n - C - G.
\]

\(^6\) A dot above a variable denotes its first derivative with respect to time.

\(^7\) Throughout the analysis we ignore issues that arise when bonds of the domestic country are denominated in the home currency. See Boyer (1978) and Girton and Henderson (1977) for a discussion of short-term wealth effects induced by unanticipated exchange rate changes.

\(^8\) A more inclusive definition of wealth would contain the term \( Y_0/r^* \) to capture the discounted value of future income from human wealth. So doing would alter our definition of wealth by a constant.
A concept of central importance to the private sector in the division of its income between savings and consumption is disposable income. The private sector receives interest on the government debt, but it must pay taxes and maintain the real value of its money balances through savings. Disposable income is therefore

\[ Y_d = Y_0 + r^*B_p - T - (\dot{\delta})^* \frac{M}{S} - \dot{B}_k \]  

(6)

where the last term arises because new government bond issues are equivalent to increases in taxation. By substituting the government budget constraint into (5), adding expected capital losses on real balances to both sides, and taking the time derivative of the wealth definition, one sees that if consumption is equal to disposable income, the expected change in real wealth is zero.

The division of disposable income into consumption and saving is assumed to be well approximated by a Metzleric target savings function:

\[ \bar{W} = Z \left( \bar{W} - \frac{M}{S} - B_n \right) \]  

(7)

where target financial wealth, \( \bar{W} \), is taken to be proportional to disposable income, \( \bar{W} = \gamma \cdot Y_d \), where \( \gamma \cdot Z < 1 \) to yield a marginal propensity to save that is less than one. The assumption that asset demands depend upon disposable income is a conventional one, which is appropriate in either of two cases. If the government is overexpanded such that the value of its services to society is less than its costs, moving resources from the private sector to the public one causes a reduction in the value of output. By not including the value of government services in income the present approach overstates the consequences of such shifts in expenditure.\(^9\) Alternatively, holding of financial assets is motivated by the need to carry out transactions in the private sector. If the size of this sector is reduced, demand for these assets is reduced in step.

The value of an individual’s real wealth is given at a point in time, since he takes prices as given. In contrast, for the economy as a whole predetermined quantities of money and bonds are available to be held, because money is non-traded. Prices and rates of return must adjust so that these quantities are willingly held. Since there are only two assets, equilibrium in one asset market implies equilibrium in the other, owing to this wealth constraint. Asset market clearing can be described fully by equilibrium in the money market alone.

The supply of money is taken to be a function only of time, and demand for

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9 Throughout this paper parameters are defined to be positive. Modigliani (1975) discusses the importance of wealth in affecting savings, and Dornbusch (1976b) uses a specification like (7).

10 For further discussion of the effects on resource allocation of shifts in expenditure between the private and public sectors see Floyd (1979).
real balances is assumed to be a standard multiplicative function of the opportunity cost of holding money and a scale variable, assumed to be real bond holdings.\textsuperscript{11}

\[ m - s = -\eta (r^* + (\delta)^e) + b_n. \]  

(8)

Given the expected rate of change of the exchange rate, it is asset market equilibrium which determines the level of the exchange rate in accordance with the asset market approach.

Taking the time derivative of (4), using the target wealth definition, and substituting (6) and (3) into (7) gives the time derivative for net bond holdings:

\[ \dot{B}_n = Z \cdot \left[ \gamma Y_0 + \gamma r^* B_n - \gamma G + \gamma (\mu - \delta)^e \frac{M}{S} - \frac{M}{S} - B_n \right] + (\delta - \mu) \frac{M}{S}, \]  

(9)

where \( \mu = \bar{M}/M \). and it is assumed that \( \gamma r^* < 1 \).

By log-linearizing this equation we obtain:

\[ \dot{b}_n = \gamma_0 - \gamma_1 \cdot g - \gamma_2 \cdot \mu - \beta_1 (m - s) - \beta_2 \cdot b_n, \]  

(10)

where \( g, m, \) and \( b_n \) are the natural logarithms of government expenditures, nominal money balances, and net bond holdings respectively, with units chosen so that \( g \) and \( m \) equal zero at the initial equilibrium. Denoting the initial values of \( B_n \) and of \( b_n \) by \( B_{n_0} \) and \( b_{n_0} \), we can write the \( \gamma \)'s and \( \beta \)'s as the following expressions of the underlying parameters:

\[ \delta_0 = B_{n_0} (1 - b_{n_0}) \]

\[ \delta_1 = B_{n_0} \]

\[ \gamma_0 = (Z/\delta_1) \left[ \gamma Y_0 - (1 - \gamma r^*) \delta_0 - (1 + \gamma) + (\gamma - 1/Z) r^* \right] \]

\[ \gamma_1 = Z \gamma/\delta_1 \]

\[ \gamma_2 = (1 - Z \gamma)/\delta_1 \]

\[ \beta_1 = (Z + (1 - Z \gamma)/\eta)/\delta_1 \]

\[ \beta_2 = Z (1 - \gamma r^*) - (1 - Z \gamma)/(\delta, \eta). \]

The sign of \( \beta_2 \) is ambiguous: none the less, all comparative static multipliers have determinate signs. It is to these results that we turn first.

\textsuperscript{11} This formulation is a standard one; see, for example, Girton and Henderson (1977). Such a specification emphasizes the dependence of money demand upon income and thereby on wealth. Barro (1978a) has postulated that expenditure is a more appropriate argument in money demand. With such a specification an increase in government expenditures could lead to an initial appreciation of the exchange rate. See in 15.

\textsuperscript{12} The log-linearization of such expressions is used quite often in the literature. See, for example, Johnson (1972) and subsequent work in the monetary approach.
THE LONG-RUN EQUILIBRIUM

The long-run consequences of alternative financial and government expenditure policies for this small economy are straightforward. In such a time period all real variables, such as real balances and net bond holdings, are constant. In contrast, all nominal variables, such as the exchange rate, are changing at the rate dictated by the rate of monetary growth.

Solving (9) and (10) under these conditions yields

$$\begin{align*}
\frac{m - s}{\bar{\mu}, \bar{g}} &= \frac{\gamma_0 - \gamma_1 \bar{g} - \beta_2 \eta r^* - Z \eta (1 - \gamma r^*) \bar{\mu}}{\beta_1 + \beta_2}, \\
\frac{\bar{b}^n}{\bar{\mu}, \bar{g}} &= \frac{\gamma_0 - \gamma_1 \bar{g} + \beta_1 \eta r^* + (Z \eta / \delta_1) \bar{\mu}}{\beta_1 + \beta_2},
\end{align*}$$

(11)

where $\bar{\mu}$ and $\bar{g}$ are the long-run constant government policies. This result shows that an increase in government expenditures depresses both real money balances and net bond holdings, since the expenditures reduce disposable income. In contrast, an increase in the rate of growth of the money supply primarily alters the composition of real wealth. Thus, real balances fall, owing to the increased opportunity cost, but net bonds rise.

Figure 1 presents these results in a diagram introduced by Kouri (1976). The values of the logarithms of real balances and real bond holdings consistent with asset market equilibrium, for an expected rate of exchange rate depreciation equal to a particular rate of monetary expansion, $\bar{\mu}$, are shown as the locus of points $LL$. The locus of points for which accumulation of net bonds is zero for particular levels of $\bar{\mu}$ and government expenditure is $BB$. For convenience $\beta_2$ has been assumed positive in drawing this diagram. The slopes of $LL$ and $BB$ are

$$\frac{d(m - s)}{db_n} \bigg|_{LL} = 1 \quad \text{and} \quad \frac{d(m - s)}{db_n} \bigg|_{BB} = \frac{-\beta_2}{\beta_3}.$$  

Above and to the right of $BB$ real net bonds are being decumulated ($b_n < 0$); below and to its left they are being accumulated ($b > 0$). Since the only way that the economy can alter $b_n$ is through the foreign sector (in the absence of economic growth), these areas can be identified with the state of the current account. Specifically, above $BB$ there is a deficit in the current account; below there is surplus; and on the locus, the current account is balanced. The long-run equilibrium is at $E_0$ for the initial government policies. Given these policies, that is the only point for which real balances and net bond holdings are consistent with a balanced current account and with equilibrium in the asset markets, given correct expectations.

This diagram can be used to demonstrate the long-run results of the model.
An increase in \( g \) shifts \( BB \) to the left (to \( B'B' \)), since the initial equilibrium values now correspond to a current account deficit. Thus, long-run equilibrium requires a lower level of wealth to balance the current account, and at the new equilibrium, \( E_1 \), the reduction in wealth is consistent with portfolio balance. Both real balances and net bond holdings are lowered by this policy.

In contrast, an increase in the rate of growth of the money supply causes both the \( LL \) and the \( BB \) loci to shift. The rightward shift in \( LL \) (to \( L' L' \)) is caused by a reduction, at any level of net bond holdings, in the desired level of real balances, owing to the higher inflation rate. This policy change, with real government spending held constant, causes head taxes to be reduced by the amount of the revenue from inflation so that the government’s budget stance is unaffected. Thus, disposable income is independent of the rate of growth of the money supply (since we are ignoring the welfare loss due to smaller real balance holdings). This conclusion assumes that desired real wealth is unchanged with higher inflation, so that reduced demand for money must be matched exactly by an increased demand for bonds. As a result, the higher rate of monetary expansion causes the \( BB \) locus to shift to the left (to \( B'' B'' \)) by a smaller amount than \( LL \)'s movement, thereby establishing a new equilibrium at \( E_2 \), so that \( b_n \) is higher than at the old steady state and \( m - s \) is lower.

This conclusion can be contrasted with Kouri's (1976, 299) where an increase in the rate of monetary growth has an ambiguous effect on long-run real bond holdings. This conclusion is dependent upon his assumption that
real taxes are constant so that an increase in $\tilde{\mu}$ requires real government expenditures to rise to keep the budget balanced. Had we made the assumption that $\tilde{g}$ was a residual variable, the identical result would have arisen from our model. Such a policy would then cause $BB$ to shift by an amount which can be smaller or larger than $LL$'s, so that a new equilibrium such as $E_3$ is established. The change in $b_n$ between $E_0$ and $E_3$ is of ambiguous sign.

This discussion has been concerned with an economy in the steady state, where the questions of interest concern wealth and portfolio composition. The next section of the paper investigates short-run dynamic problems.

**The Short-Run Equilibrium**

In this section we examine the instantaneous equilibrium solution of the model, under the hypothesis that agents have perfect foresight of the future path of the economy. This hypothesis in a model without uncertainty is the natural counterpart to the rational expectations assumption in models which identify sources of uncertainty explicitly. For the purposes of this section, $T_0$ is merely a time at which the initial stock of net bond holdings is known.

The instantaneous equilibrium of the economy under the perfect foresight assumption can be found by substituting the actual rate of change of the exchange rate for the expected rate of change in (10). Thus, equations (9) and (10) become a system of linear, constant coefficient, differential equations with time-dependent forcing functions. Whereas the diagram of the previous section requires constant long-run values of $\mu(\tau)$ and $g(\tau)$, the solution of the system for general time profiles of monetary and fiscal policies is: \[13\]

\[\begin{align*}
\sigma(t) &= C_2 q_2 e^{\lambda_2(t-T_0)} + \frac{q_1}{q_1 - q_2} q_2 \int_{t}^{\infty} [r^* + \lambda_1 m(\tau) + q_2 (\gamma_0 - \gamma_1 g(\tau) - \gamma_2 \mu(\tau))] e^{-\lambda_1(t-\tau)} d\tau \\
&\quad + \frac{q_2}{q_1 - q_2} \int_{-\infty}^{t} [r^* + \lambda_2 m(\tau) + q_1 (\gamma_0 - \gamma_1 g(\tau) - \gamma_2 \mu(\tau))] e^{-\lambda_2(t-\tau)} d\tau \\
b_n(t) &= C_2 e^{\lambda_2(t-T_0)} + \frac{1}{q_1 - q_2} \int_{t}^{\infty} [r^* + \lambda_1 m(\tau) + q_2 (\gamma_0 - \gamma_1 g(\tau) - \gamma_2 \mu(\tau))] e^{-\lambda_2(t-\tau)} d\tau \\
&\quad - \gamma_2 \mu(\tau)) e^{-\lambda_1(t-\tau)} d\tau + \frac{1}{q_1 - q_2} \int_{-\infty}^{t} [r^* + \lambda_2 m(\tau) + q_1 (\gamma_0 - \gamma_1 g(\tau) - \gamma_2 \mu(\tau))] e^{-\lambda_2(t-\tau)} d\tau,
\end{align*}\]

See Brauer and Nohel (1973) for a discussion of the solution technique. Sargent (1979) provides a detailed discussion of the choice of limits of integration and of the determination of the arbitrary constants of the homogeneous solution.
where
\[
\lambda_1 = \frac{1}{2} \left( -\beta_2 + \frac{1}{\eta} + \left( \left( \beta_2 - \frac{1}{\eta} \right)^2 + \frac{4}{\eta} (\beta_1 + \beta_2) \right)^{1/2} \right) > 0,
\]
\[
\lambda_2 = \frac{1}{2} \left( -\beta_2 + \frac{1}{\eta} - \left( \left( \beta_2 - \frac{1}{\eta} \right)^2 + \frac{4}{\eta} (\beta_1 + \beta_2) \right)^{1/2} \right) < 0,
\]
\[
q_1 = \frac{\lambda_1 + \beta_2}{\beta_1} > 0, \quad \text{and} \quad q_2 = \frac{\lambda_2 + \beta_2}{\beta_1} < 0.
\]

These inequalities hold no matter what the sign of \( \beta_2 \).

The initial condition on net bond holdings at \( T_0 \) is used to determine \( C_2 \), the arbitrary coefficient of the homogeneous part of the solution associated with the negative root of the system. The arbitrary coefficient modifying the part of the homogeneous solution associated with the positive root has been set to zero in order that the exchange rate moves towards an equilibrium value when financial policies are constant through time. The limits of integration have been determined in a similar manner, so that the integrals have convergent values for financial policy regimes that are process consistent.\(^{14}\)

An alternative expression for the exchange rate is:
\[
s(t) = q_2 b_n(t) + \int_t^\infty \left[ r^* + \lambda_1 m(\tau) + q_2 (\gamma_0 - \gamma_1 g(\tau) - \gamma_2 \mu(\tau)) \right] \times e^{-\lambda_1 (\tau - t)} d\tau. \tag{13}
\]

This expression is a convenient one for analyzing the impact effect of unanticipated alterations in policy. At \( T_0 \), \( b_n \) is given and \( q_2 \) is merely a constant. Therefore the entire effect of a change in \( m \) or \( g \) is captured by its influence upon the value of the single, forward-looking integral. The normalized eigen-vector \( q_2 \) is negative, so that the coefficients of both \( m(\tau) \) and \( g(\tau) \) in the integrand are positive, implying that announcements at time \( T_0 \) of increases in the money supply, its rate of growth, or government spending, to be instituted either currently or in the future, cause the exchange rate at \( T_0 \) to depreciate in an instantaneous jump.\(^{15}\) The extent of the depreciation depends upon the nature and the time profiles of the policies pursued and upon the length of time between the announcement and its implementation. The next section emphasizes the latter point for specific time profiles for \( g \) and \( \mu \).

\(^{14}\) This expression is used by Flood and Garber (1980) to characterize monetary regimes for which this integral converges.

\(^{15}\) This model does not include a mechanism for an initial appreciation in response to an increase in government expenditures, because the authorities cannot influence the values of the arguments in the money demand function. One way of restoring this mechanism is to suppose that money demand depends upon expenditure (including that of the authority).
DYNAMICS FOLLOWING ANNOUNCEMENT OF FUTURE POLICY

The last section determined the perfect foresight path of this economy for general time profiles of the authorities' policies. The discussion in this section makes the assumption that government expenditures and the rate of growth of the money supply are constants which change only once, at time $T_1$. For this time profile it is possible to employ the conclusions of the third section above (since the long-run equilibria are well-defined) and to derive explicit expressions for the movement over time of the exchange rate and net bond holdings, using the solution methods of the last section. The entire time paths are investigated, and their characteristics are shown using a simple diagrammatic analysis.

Announcements of future implementation

Assume that the authorities have been pursuing a policy of constant expenditures, $\bar{g}_1$, and constant money growth, $\bar{\mu}_1$, over the indefinite past. Agents in the economy expect these policies to continue through the future, so that the economy is in long-run equilibrium characterized by equation (11).

Consider the consequences of the following change in policy. At time $T_0$, agents become aware, perhaps through an announcement by the authorities, that the old policy will continue until $T_1$; starting then, higher rates of expenditure and money growth, $g_2$ and $\mu_2$ respectively, will be instituted and pursued indefinitely into the future. Although the new policy was unanticipated before $T_0$, thereafter the public believes with certainty that starting at $T_1$, the new policy will be followed indefinitely into the future.$^{16}$

By straightforward application of the solution techniques from the previous section the solutions for $s(t)$ and $b_n(t)$ for $T_0 \leq t \leq T_1$ are the following:

$$s(t) = m(t) - \left( \frac{m - s}{\bar{\mu}_1 + \bar{g}_1} \right) + I(T_1)e^{-\lambda_1(T_1 - T_0)}$$

$$\times \left\{ \frac{q_1e^{\lambda_1(t - T_0)} - q_2e^{\lambda_2(t - T_0)}}{q_1 - q_2} \right\}, \quad (14)$$

and

$$b_n(t) = \left( \frac{b_n}{\bar{\mu}_1 + \bar{g}_1} \right) + I(T_1)e^{-\lambda_1(T_1 - T_0)}\left\{ \frac{e^{\lambda_1(t - T_0)} - e^{\lambda_2(t - T_0)}}{q_1 - q_2} \right\}, \quad (15)$$

16 The analysis of the economy's response to announced changes in government policies presented here is correct if it is understood that these changes were unanticipated before $T_0$. Thereafter, the economy follows the new dynamic path forecast by the perfect foresight model if there are no further unanticipated shocks. Gray and Turnovsky (1979) and Wilson (1979) analyse the Dornbusch (1976a) model in a similar type of thought experiment. See fn. 4 for a discussion of these experiments.
where \( I(T_1) = (1/\lambda_1)[\bar{m}_2 - \bar{m}_1](1 - q_2\gamma_2 - q_2\gamma_1(\bar{g}_2 - \bar{g}_1)) \), which is the value of the forward-looking integral evaluated from \( T_1 \) to infinity for the differential of monetary and fiscal policies from their previous values. The solutions after the implementation of the policies, that is, for \( T_1 \leq t \), are the following:

\[
s(t) = m(t) - \left( \frac{m - s}{\bar{m}_2, \bar{g}_2} \right) - \left[ (m(T_1) - s(T_1)) - \left( \frac{m - s}{\bar{m}_2, \bar{g}_2} \right) \right] e^{\lambda_2(t - T_1)},
\]

(16)

and

\[
b_n(t) = \left( \frac{\bar{b}_n}{\bar{m}_2, \bar{g}_2} \right) + \left[ b_n(T_1) - \bar{b}_n \left( \frac{\bar{m}_2, \bar{g}_2}{} \right) \right] e^{\lambda_2(t - T_1)}.
\]

(17)

The purpose of the discussion in the rest of this section is to point out some of the salient characteristics of these solutions. Of particular importance is an outlining of how the initial jump in the exchange rate and its subsequent movement depend upon the length of time between \( T_0 \) and \( T_1 \). Also important is a determination of whether overshooting of the exchange rate can occur.

These topics can be approached most conveniently using the diagrammatic tools of the third section above.

**A diagrammatic analysis**

Figure 1 shows how the long-run steady-state values of real balances and net bond holdings can be viewed as corresponding to the intersection of \( LL \) and \( BB \). Thus, in figure 2 the original policy of \( \bar{m}_1 \) and \( \bar{g}_1 \) establishes an initial steady state at \( E_0 \). The new steady-state equilibrium is at a point such as \( E_1 \) for pure fiscal policy, \( E_2 \) for pure monetary policy, or at some intermediate point for a combination of policies. (Note that for convenience only it is assumed that all new long-run equilibria lie along the line \( LL' \) which is to be derived below.) This assumption establishes the nature of the long-run equilibria. The problem now is to establish the time path connecting them.

It is well known that monetary models tend to have saddle-point equilibria.\(^\text{17}\) The present model is no exception, as evidenced by the fact that the characteristic roots are of opposite sign. Through any equilibrium run two loci, called steady-state paths, which have the property that if the economy is ever on such a locus, it stays on the locus indefinitely (as long as the economy is not forced off the locus by an unanticipated change in policy). The stable branch of the steady-state path defines the only locus on which the economy moves ever closer to the long-run equilibrium. On all other loci, including the unstable branch, it moves indefinitely far away.

Since the present model is linear, the steady-state paths are straight lines.

\(^\text{17}\) See Brock (1975) or Sargent and Wallace (1973) for discussions of perfect foresight monetary growth models.
whose slopes are equal to minus the values of the normalized characteristic vectors, because these vectors have the property that along them the derivative of the displacement of the endogenous variables of the economy from their long-run equilibria is proportional to the displacement itself. This property guarantees that once the economy is on a characteristic vector, it will travel linearly along the vector, unless a new long-run equilibrium is established by an unanticipated change in policy. Clearly, the economy's path in transition to a long-run equilibrium can never cross a characteristic vector unless the long-run equilibrium is altered.

In figure 2, the characteristic vectors associated with the unstable and stable roots of the system for the initial equilibrium are drawn as $UU$ and $VV$, respectively. The slopes of these vectors satisfy

$$\frac{d(m-s)}{db_n} \bigg/ \frac{d(m-s)}{db_n}$$

$$= -q_1 < \frac{-\beta_2}{\beta_1} = \frac{d(m-s)}{db_n} \bigg/ \frac{d(m-s)}{db_n}$$

$$UU$$

and

$$\frac{d(m-s)}{db_n} \bigg/ \frac{d(m-s)}{db_n}$$

$$= -q_2 < 1 = \frac{d(m-s)}{db_n} \bigg/ \frac{d(m-s)}{db_n}$$

$$VV$$

The line $V'V'$ drawn parallel to $VV$ is the stable characteristic vector for the
new steady-state equilibria, $E_1$ or $E_2$. Four dynamical arrows corresponding to equilibrium $E_0$ are included in figure 2 to indicate the path of the economy in the transition to a new steady state when the path is not on a characteristic vector. Which of these arrows describes the movement of the economy depends on the type of policy change considered, and the path of the economy will always be consistent with the solutions in (12) and, for the present case, in equations (14)–(17).

Consider the initial equilibrium of the economy after the announcement at time $T_0$. Since money is a non-traded asset, net bond holdings cannot change in the initial jump to the new equilibrium path. Rather, the exchange rate changes instantaneously to establish the appropriate level of real balances. From (14) the impact effect on the exchange rate is

$$s(T_0) = m(T_0) - \left( \frac{m - s}{b_1, T_1} \right) + I(T_1)e^{-\lambda_1(T_1 - T_0)}. \quad (18)$$

This indicates that announcement of expansionary monetary or fiscal policies cause an instantaneous depreciation of the exchange rate. Real balances decline to a point like $F_0$ in figure 2 even though the money supply, its rate of growth, and government spending are unchanged at $T_0$. The depreciation is larger the closer the date of implementation is to the time of announcement and the larger the announced increases in government policies are.

After the initial jump in the exchange rate it cannot be anticipated to move discontinuously since this would imply an infinitely large instantaneous expected return on holding money. The attempt by agents to move out of real balances depreciates the exchange rate, thus eliminating the possibility of anticipated jumps. This behaviour implies that the exchange rate moves continuously for all $t > T_0$.\(^\text{18}\) After the initial jump the economy follows the dynamical arrows (diagrammatic representation of (9) and (10)) in figure 2, moving first to the $V'V'$ locus, then moving along $V'V'$ to the new equilibrium.\(^\text{18}\)

The time path between $T_0$ and $T_1$

The linearity of the model and the integration operator allow an interesting interpretation of the solutions (14) and (15) for the movement of the economy during the period between announcement and implementation of new policies. As (14) and (15) demonstrate, the divergences of real balances and net bond holdings from their former equilibrium levels depend on the term $I(T_1)e^{-\lambda_1(T_1 - T_0)}$, which measures the changes in monetary and fiscal policies discounted to $T_0$ with discount rate $\lambda_1$, and on the exponential terms in brackets. The exponential functions and the term that precedes them arise from the solution to (9) and (10) where the endogenous variables are considered to be real balances and real bond holdings as deviations from their initial equilibria and the forcing functions are zero since policies are constant.

\(^{18}\) In light of the continuity of $b_*$ and $m$ in the present context, $s$ is continuous as well from (8).
at the old equilibrium level during the interim. The solution to this homogeneous differential equation system requires two conditions, either initial or terminal, to determine the arbitrary constants. The first condition is the initial condition that the instantaneous deviation of real bond holdings from their former equilibrium value is zero. The other condition, rephrasing equation (13) at time \( T_1 \), is that
\[
(m(T_1) - s(T_1)) - \left( \frac{m - s}{\mu_1, g_1} \right) = -q_2 \left( b_n(T_1) - b_n \left( \frac{\mu_2, g_2}{\mu_1, g_1} \right) \right) - I(T_1), \tag{19}
\]
which is an alternative way of stating that the economy must be on the \( V'V' \) line at \( T_1 \) in figure 2 so as to be in position to converge to the new long-run equilibrium along the new steady-state path.

As (19) indicates, any combination of policies that has a discounted value of deviations from the previous equilibrium path equal to \( I(T_1) \) will produce the same dynamic path during the interim period. This is true regardless of whether the new policies are to be constant in the future. For our experiments we assume that constant values for the rate of growth of the money supply and government expenditure will be implemented and that the combination has value equal to \( I(T_1) \), which places the final equilibrium on \( V'V' \) in figure 2.

Since an increase in government spending with an overextended government sector, an increase in the rate of monetary growth, or inflationary finance all cause real balances to decline in the long run, the transition path between \( T_0 \) and \( T_1 \) is associated with a depreciation of the exchange rate in excess of the rate of growth of real balances and a current account surplus which allows accumulation of real bond holdings. Differentiating (14) gives the rate of growth of the exchange rate between \( T_0 \) and \( T_1 \):
\[
\dot{s}(t) = \mu_1 + I(T_1)e^{-k_1(t-T_0)} \left\{ \frac{q_1 \lambda_1 e^{-k_1(t-T_0)} - q_2 \lambda_2 e^{-k_2(t-T_0)}}{q_1 - q_2} \right\}, \tag{20}
\]
and differentiating (15) gives the current account surplus:
\[
b_n(t) = I(T_1)e^{-k_1(t-T_0)} \left\{ \frac{\lambda_1 e^{-k_1(t-T_0)} - \lambda_2 e^{-k_2(t-T_0)}}{q_1 - q_2} \right\}. \tag{21}
\]
After the announcement of increases in the government policies, the rate of change of the exchange rate is unambiguously greater than the rate of monetary growth and the current account is in surplus. Evaluating (20) and (21) as \( t \) increases from \( T_0 \) to \( T_1 \) indicates that the rate of depreciation of the exchange rate increases over time and that the current account surplus (CAS) is always positive and increasing. These results are presented in figures 3 and 4.

Clearly, the length of time \( T_1 - T_0 \) influences the transition path. If this
difference is zero, announcement and enactment of policies occur simultaneously, and the economy jumps immediately to $F_1$ in figure 2. The longer the time between announcement and enactment, the smaller is the initial vertical jump away from $E_0$, and at $T_1$ the closer is the economy to $F_2$, the intersection of the unstable vector, $UU$, and the new stable path, $V'V''$.

If there is a delay between announcement and implementation of new policies, the economy has time to prepare for the eventual effects of the new policy. That is, the reduction in real balances is less than with an immediate implementation. None the less, this reduction puts real wealth below its target level, which is unchanged during this period, which causes agents to save positive amounts, thereby creating a surplus in the current account. With monetary policy this mechanism continues after the implementation, so that the surplus persists. In contrast, the implementation of fiscal policy alters the
target level of wealth, so that the current account turns to deficit after the policy is in place.

The economy after policy enactment

After implementation of the new policies at $T_1$, the economy moves monotonically over time towards its new steady state along the $V''V'$ line in figure 2. From the above discussion and consideration of (16) and (17) we can determine the time path of the exchange rate and the current account surplus after $T_1$. For an increase in the rate of monetary growth the long-run equilibrium is at $E_2$ in figure 2. Real balances and net bond holdings at time $T_1$ are below their long run equilibria, indicating that the rate of change of the exchange rate is less than the new rate of monetary growth $\bar{\mu}_2$. Over time the rate of depreciation increases to $\bar{\mu}_4$. Similarly, net bond holdings at $T_1$ are smaller than their eventual steady-state value, indicating that real bond holdings must increase over time. The economy runs a current account surplus, which declines over time as the new wealth position is approached. Figure 3 presents the path of the economy in this case.

An increase in government spending produces a long-run equilibrium at $E_4$ in figure 2. At $T_1$ the economy has a higher level of real balances and bond holdings than in the new steady state. Consequently, it deaccumulates both assets over time. The increase in government spending increases aggregate demand and the current account moves into deficit as consumers dissave. The rate of inflation is greater than the unchanged rate of monetary growth, $\bar{\mu}_1$, and it declines over time back to $\bar{\mu}_4$ as the economy approaches the new equilibrium. Figure 4 presents the paths of the rate of depreciation of the exchange rate and the current account surplus for the pure fiscal policy.

Kouri (1976) noted the strong prediction of his perfect foresight model that along the steady-state path real balances and real bond holdings always move in the same direction. This fact is true in our model only after the implementation of policy. Consequently, observing a current account surplus (deficit) and a depreciating (appreciating) exchange rate could not be taken as prima facie evidence against the perfect foresight model. Indeed, both variables are endogenous, as this paper amply demonstrates.

Financial policies and overshooting of the exchange rate

There has been particular interest in the possibility of overshooting of the exchange rate. In the context of the original investigation (Dornbusch, 1976a and its extensions Wilson, 1979, Gray and Turnovsky, 1979, and Dornbusch and Fischer 1980), a single step increase in the quantity of money was examined, which led to a very simple definition of the phenomenon. Overshooting occurred when the exchange rate increased by more in the short run than it did in the long run.

In the present framework where there is no well-defined long-run exchange rate, overshooting needs to be defined more generally. 'Overshooting' is said
to occur in the present model if the exchange rate moves beyond the path to which it asymptotes. If instead the exchange rate does not move so far, it is said to 'undershoot.'

This question can be rephrased in terms of real balances. In the steady state real balances are constant. In the movement to the steady state the change in real balances determines whether over- or undershooting occurs. The reason is that if after $T_1$ real balances rise, the exchange rate must have moved beyond the steady-state path in order to depress the value of real balances further in the short run than the long run. In contrast, if after $T_1$ real balances must fall, undershooting occurs because the rate of change of the exchange rate must be faster than the rate of monetary growth in order to reduce real balances.

With this simple rule it is possible to show in figure 2 what policies cause the exchange rate to overshoot. Any policy whose long-run equilibrium lies to the left of $F_1$ on $V^TV$ has undershooting of the exchange rate. Any policy whose long-run equilibrium lies to the right of $F_2$ has overshooting of the exchange rate. Policies with long-run equilibria between these two points cannot be classified unambiguously. Nevertheless, it is clear that the longer the delay of the implementation, the closer the economy at time $T_1$ to point $F_2$. This result shows that an increased rate of monetary growth necessarily causes overshooting, since no matter how long a delay there is after $T_1$, individuals wish to substitute further away from money towards net bonds. In contrast, a pure fiscal policy causes undershooting, since after $T_1$ individuals dissave, reducing both real balances and net bond holdings. In an intermediate case, for example, where increased expenditures are financed by more rapid money growth, it is ambiguous whether undershooting will occur.

A final experiment of interest is an increase in the money supply. If such a transfer is carried out when $T_1 = T_0$, the exchange rate rises equiproportionately, thus restoring full equilibrium. (This finding shows that the impulse which causes undershooting in models with sluggish price adjustment does not do so in the present equilibrium model.) An increase which is delayed causes undershooting, since net bonds are accumulated in the interim, and they must be dissaved afterwards to restore the old equilibrium in real magnitudes.

**Conclusions**

This paper has discussed the influence of financial policies on the small open economy in a full employment model with flexible prices, perfect foresight, and an appropriate definition of disposable income. Throughout the analysis we ignored discussions of the optimal level of the provision of government services, assuming either that agents place no value on the services or that

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19 Flood (1979) uses this definition.
private sector resource allocation provides a motivation for holding financial assets. The experiments are therefore consistent with a model in which the government sector is overextended. The analysis examined the general perfect foresight solution, demonstrating the influence of anticipated future monetary and fiscal policy on the exchange rate and the balance of payments.

The definition of disposable income is central to the conclusion that tax- or bond-financed government expenditures have equivalent effects. In both cases increases in expenditure cause the domestic currency to depreciate and the desired stock of net bond holdings to fall. Specifically, the exchange rate undershoots in the sense that its initial change is in the same direction but smaller than its final movement. This conclusion tends to hold also for money financed expenditures. The more that transfer payments are the source of monetary growth, the more likely will the exchange rate overshoot.

The effects of the assumption of perfect foresight were highlighted by consideration of partially anticipated policy changes. The two portions of the economy's path in this case are quite distinct. Perfect foresight has its most important influences during the period between which agents become aware of a change in policy and its implementation. With such expectations this delay provides time for the economy to prepare for the new policy but does not necessarily cause the adjustment process to be more gradual. An interesting conclusion in this respect is that the process of adjustment during this period is of a very specific form, regardless of the nature of the policy enacted. Any policy change produces a transition path which is a scalar multiple of the path for any other policies. In all cases considered in this paper, since we analysed only expansionary financial policies, the current account moves into surplus during the interim. This fact is true whether the final equilibrium has a higher or lower value for net bond holdings. As a result, a delay in implementation can actually increase the adjustment required subsequent to the enactment.

The major point of the model is that anticipations of future government financial policies are important determinants of today's exchange rate. The finding that the movement in the economy prior to an anticipated change in policy is not well-described by the earlier literature on the monetary approach is an important one. After the implementation of new constant policies, it is the process of asset accumulation which drives the economy during the adjustment to the steady state. This mechanism, familiar from the earlier work, can be understood in the present context with minimal modification. The message of the paper is therefore that understanding the actual movement in exchange rates requires knowledge of anticipated future government policies and how these anticipations affect the economy. Forecasts of future policy will be a major influence upon exchange rate volatility if government policy makers do not provide clear signals to agents about their future choices for policy.
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Rational expectations and the optimal foreign exchange regimes

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Abstract. This paper evaluates the optimality of fixed and flexible rates under rational expectations and wage and prices stickiness. The optimality of fixed or flexible rates depends crucially on how efficient and accurate contemporaneous information on exchange rate or money supply can be dissipated to firms and households. If current money supply or exchange rate is perfectly observable, the two exchange rate regimes are identical in spite of wage and prices stickiness.

Anticipations rationnelles et régimes de taux de change optimaux. Ce mémoire étudie l’optimalité de régimes de taux de change fixes ou fluctuants quand les anticipations sont rationnelles et qu’il y a rigidité du niveau de salaire et des prix. L’optimalité de