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The Variability of Velocity in Cash-in-Advance Models

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Monetary models based on cash-in-advance constraints make strong predictions about the stochastic properties of endogenous variables such as the velocity of circulation of money, the rate of inflation, and real and nominal interest rates. We develop numerical methods to understand these predictions because the models cannot be characterized analytically. We calibrate some cash-in-advance models using driving processes estimated from U.S. time-series data to generate model predictions that are compared to sample statistics. Formulations of the models that generate variability in velocity corresponding to the U.S. data typically fail along other dimensions.

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I. Introduction

It has become quite common to introduce a demand for money in general equilibrium models through a cash-in-advance (CIA) constraint. Simple CIA models require that the money held in a given period be at least sufficient to cover perfectly anticipated expenditures. Agents facing positive nominal interest rates never hold idle cash balances in these economies, so the entire money supply turns over each period. Consequently, these models incorrectly predict that the velocity of circulation of money is always unity.

In response to this difficulty, Lucas (1984) and Svensson (1985) modify the information structure in the basic CIA setup. In Svensson's formulation of a single-agent exchange economy (which we call the "cash" model), output and the rate of growth of the nominal money stock follow a stationary Markov chain. Cash balances must be chosen before the quantity of output is known. Therefore, agents may choose to carry unspent cash across periods, and velocity can in principle vary. Lucas and Stokey (1987) further weaken the tie between money and expenditures by allowing substitution between "cash" goods and "credit" goods (ones that are not subject to the CIA constraint).[1]

These monetary models make strong predictions about the joint stochastic properties of endogenous variables such as the velocity of circulation of money, the rate of inflation, and real and nominal interest rates. Unfortunately, the precise nature of these predictions is known only for the few special cases that can be characterized analytically.[2]

This paper explores whether these models can produce realistic predictions about the stochastic properties of their endogenous variables when the exogenous driving process for consumption growth and money growth is calibrated using U.S. time-series data. To do this, we develop new numerical algorithms to solve these models without restricting the CIA constraint to be binding in all states. The stochastic process governing money growth and consumption growth is determined in two stages: we estimate bivariate vector autoregressions (VAR) using quarterly and annual data on consumption growth and money growth, and we approximate each VAR by a Markov

[1] Other recent monetary models that could be analyzed in similar investigations include those of Hartley (1988) and Marshall (1990). Marshall uses a transactions cost technology that generalizes the traditional CIA constraint so that velocity has the potential to vary.

[2] Svensson's analytical results, e.g., depend on the clearly unrealistic assumption that the state of the world is independently and identically distributed (i.i.d.). On the other hand, Giovannini (1989) and Hodrick (1989) relax the i.i.d. assumption but assume that parameters of tastes and technology are such that the CIA constraint always binds.

chain using the quadrature method of Tauchen (1987). Using each driving process, we examine the models' statistical properties in a calibration exercise similar in spirit to that of Mehra and Prescott (1985).

Motivation for our analysis comes from several sources. The most direct is Lucas and Stokey (1987, p. 492), who state that "it seems clear that to interpret empirical distributions of macroeconomic aggregates one needs an explicitly stochastic theoretical model, a model that permits the calculation of a predicted *theoretical* joint distribution of shocks and endogenously determined variables that can be compared to observed distributions." Lucas and Stokey propose a theoretical model that potentially can accomplish this objective, but they do not pursue the question of whether the model can in fact reproduce the joint distribution of shocks and endogenous variables found in the data. Another motivation for our analysis is the empirical failure of nonmonetary, representative agent, rational expectations models in Euler equation specification tests. Townsend (1987) notes that one important reason for the theoretical development of monetary models is that they supply alternative specifications that may explain the empirical failure of barter models. Cash-in-advance models of the type we consider produce one such alternative specification. These can be examined empirically using the solution methods we develop here.

To summarize the results qualitatively, we find that in the cash-only model, the predicted velocity of money is always constant. In the cash-credit model, velocity does vary because agents substitute between cash and credit goods. However, the CIA constraint for the cash good almost always binds. Moreover, the cash-credit model appears unable to generate realistic predictions about the sample moments of other key endogenous variables when parameters are set to produce reasonable variability in velocity.

It is natural to ask whether such strong negative results are robust to specification changes. To address this issue, we do an extensive sensitivity analysis, altering the information structure, the nature of the driving process, and the specification of the utility function. The sensitivity of the results to changes in information structure is of special interest because much of the difficulty in solving these models results from assuming an information structure under which velocity can in principle vary because of a slack CIA constraint. We experiment with alternative information structures by providing agents with increasingly noisy signals about the future realization of money or output. In general, such changes have negligible effects, suggesting that the more complicated information structure of these models is unlikely to be helpful empirically. The models' predictions are also highly invariant to changes in the forcing process and to a wide range of parameter variations for several utility functions. However, changing preferences to reflect "habit formation" (Ryder and Heal 1973) generates substantially different, if not more accurate, predictions.

The paper is organized as follows. Section II introduces a growth version of the cash-credit model and demonstrates that the cash model is a restricted version of it. The solution algorithm for the cash model is described in Section III, where it is shown to converge to the equilibrium, if one exists. A similar algorithm that solves the cash-credit model is discussed in Appendix B. In Section IV, we estimate VARs for consumption growth and money growth using quarterly and annual U.S. time-series data and compare each discrete approximation with the estimated process. Several unconditional moments of the models are calculated over a large range of structural parameter values and are compared to the corresponding sample statistics in Section V. Section VI presents the results of the numerical sensitivity analysis. Section VII presents conclusions.

II. The Cash-in-Advance Models

A. *The Cash-Credit Model*

In this section we set up the basic models, present the first-order conditions of the representative agent, and define a stationary equilibrium. We begin with a modified version of the Lucas and Stokey (1987) model that we call the cash-credit model. In order to compare model-generated observations to statistics estimated from actual data, we modify the model to allow for growth in the endowment process. The timing of markets and of information flows within a period is altered to conform to the Svensson (1985) model. This allows his model, which we call the cash model, to be treated as a special case of the more general cash-credit model.[3]

Consider a representative consumer exchange economy in which the aggregate nonstorable endowment at time t is y_t and the aggregate money supply in that period is X_t. The endowment can be thought of as the payoff on an asset owned by the agent, and the aggregate stock of this asset is normalized to one. Let $\omega_t \equiv X_{t+1}/X_t$ and $\gamma_t \equiv$

[3] Lucas and Stokey use a different timing convention within a period, but this difference is not substantive since the timing of events is the same as in our model. They do have a more general information structure than our base case model since they allow a noisy signal about the time $t + 1$ money supply in the time t information set rather than the perfectly revealing signal of the Svensson model. We explore such information structures in Sec. VI. Lucas (1988) derives and explores some empirical implications of Lucas and Stokey (1987).

y_t/y_{t-1}, and let $\{\gamma_t, \omega_t\}$ be a stationary ergodic Markov chain with transition probability matrix $\mathbf{\Pi}$, where the typical element Π_{ij} gives the probability of moving from state i to state j.

The agent's preferences at time t over current and future consumption of cash goods, c_{1t}, and credit goods, c_{2t}, are

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U(c_{1\tau}, c_{2\tau}). \tag{1}$$

In the base case analysis, we use the period utility function

$$U(c_{1t}, c_{2t}) = \frac{(c_{1t}^{\psi} c_{2t}^{1-\psi})^{1-\alpha} - 1}{1 - \alpha}. \tag{2}$$

Each period is divided into two subperiods. In the first subperiod, agents learn ω_t and γ_t and purchase consumption in the markets for cash and credit goods. Good 1 can be purchased only with cash, while good 2 is bought on credit. In the second subperiod, agents trade in financial assets and settle credit accounts in the securities market, as described below.

Cash and credit goods are produced according to the linear technology $c_{1t} + c_{2t} = y_t$. This technology, combined with the assumption that sellers receive payments usable in the securities market from the sale of either good, implies that the nominal price of the two goods, P_t, is the same.

The agent begins the first subperiod holding cash, M_t, and shares of stock, z_t, and chooses consumption of the cash good, c_{1t}, subject to the CIA constraint:

$$P_t c_{1t} \leq M_t. \tag{3}$$

After the goods market closes, money and stocks trade in the securities market. The nominal stock price is Q_t. The agent's sources of wealth are any unspent cash balances, receipts of nominal dividends $z_t P_t y_t$, the resale value of stockholdings $Q_t z_t$, and net lump-sum monetary transfers $(\omega_t - 1)X_t$. Hence, his budget constraint is

$$M_{t+1} + Q_t z_{t+1} = z_t P_t y_t + Q_t z_t + (\omega_t - 1)X_t + M_t - P_t(c_{1t} + c_{2t}). \tag{4}$$

The consumer, taking prices P_t and Q_t as given, chooses c_{1t}, c_{2t}, M_{t+1}, and z_{t+1} to maximize (1), subject to the flow constraints (3) and (4). In a competitive equilibrium, prices at time t adjust so that all markets clear: $c_{1t} + c_{2t} = y_t$, $z_{t+1} = 1$, and $M_{t+1} = X_{t+1}$.

Following Svensson (1985) and Lucas and Stokey (1987), we analyze only stationary equilibria in which prices depend in a time-invariant fashion on the current state, which we assume to be the current level of the money supply, the lagged level of real endowment, and the current rates of growth of money and endowment. Furthermore, we examine only equilibrium price functions that are linear in X_t and multiplicatively separable in y_{t-1} such that $P_t = p(\gamma_t, \omega_t)X_t/y_{t-1}$ and $Q_t = q(\gamma_t, \omega_t)P_t y_{t-1}$. We assume that consumption of both goods is linear in y_{t-1}, such that $c_{1t} = c_1(\gamma_t, \omega_t)y_{t-1}$ and $c_{2t} = [\gamma_t - c_1(\gamma_t, \omega_t)]y_{t-1}$.

Define $m(\gamma_t, \omega_t) = 1/p(\gamma_t, \omega_t)$, express the constraints (3) and (4) in real terms, and let the multipliers on these constraints be μ_t and λ_t. The multipliers can be expressed as functions of the state that do not depend on the level of the money supply: $\mu_t = \mu(\gamma_t, \omega_t)y_{t-1}^{-\alpha}$ and $\lambda_t = \lambda(\gamma_t, \omega_t)y_{t-1}^{-\alpha}$. The marginal utilities of consumption of the two goods can be written as $u_1(\gamma_t, \omega_t)y_{t-1}^{-\alpha}$ and $u_2(\gamma_t, \omega_t)y_{t-1}^{-\alpha}$, where

$$u_1(\gamma_t, \omega_t) = \psi[\gamma_t - c_1(\gamma_t, \omega_t)]^{(1-\psi)(1-\alpha)} c_1(\gamma_t, \omega_t)^{\psi(1-\alpha)-1}, \tag{5a}$$

$$u_2(\gamma_t, \omega_t) = (1-\psi)[\gamma_t - c_1(\gamma_t, \omega_t)]^{(1-\psi)(1-\alpha)-1} c_1(\gamma_t, \omega_t)^{\psi(1-\alpha)}. \tag{5b}$$

With these definitions, a *stationary equilibrium* is a set of functions $\{c_1(\gamma_t, \omega_t), m(\gamma_t, \omega_t), \mu(\gamma_t, \omega_t), \lambda(\gamma_t, \omega_t), q(\gamma_t, \omega_t)\}$ such that markets clear and the following equations related to the agent's first-order conditions are satisfied:

$$c_1(\gamma_t, \omega_t) \le m(\gamma_t, \omega_t), \tag{6}$$

$$\{\mu(\gamma_t, \omega_t) \ge 0 \text{ and } \mu(\gamma_t, \omega_t)[m(\gamma_t, \omega_t) - c_1(\gamma_t, \omega_t)] = 0\}, \tag{7}$$

$$u_1(\gamma_t, \omega_t) = \lambda(\gamma_t, \omega_t) + \mu(\gamma_t, \omega_t), \tag{8}$$

$$u_2(\gamma_t, \omega_t) = \lambda(\gamma_t, \omega_t), \tag{9}$$

$$\mu(\gamma_t, \omega_t) = u_1(\gamma_t, \omega_t) - \frac{\beta E[u_1(\gamma_{t+1}, \omega_{t+1})m(\gamma_{t+1}, \omega_{t+1}) \mid \gamma_t, \omega_t]\gamma_t^{1-\alpha}}{\omega_t m(\gamma_t, \omega_t)}, \tag{10}$$

$$\lambda(\gamma_t, \omega_t)q(\gamma_t, \omega_t) = \beta E\{\lambda(\gamma_{t+1}, \omega_{t+1})[q(\gamma_{t+1}, \omega_{t+1}) + \gamma_{t+1}] \mid (\gamma_t, \omega_t)\}\gamma_t^{1-\alpha}. \tag{11}$$

The liquidity or CIA constraint is given by (6), with the conditions on the Kuhn-Tucker multiplier given in (7). Equations (8) and (9) are the first-order conditions associated with c_{1t} and c_{2t}, and equations (10) and (11) are modifications of the first-order conditions associated with M_{t+1} and z_{t+1}. An equilibrium must also satisfy the necessary conditions that expected utility and nominal wealth are finite, which is equivalent to requiring that the eigenvalues of the matrix $\mathbf{A}$, with typical element $A_{ij} = \beta\Pi_{ij}(\gamma_j)^{1-\alpha}$, lie within the unit circle.[4]

[4] It can be demonstrated that if the state space is finite, there is a unique equilibrium of this form. Giovannini and Labadie (1989) prove that the unique Markov equilibrium in the cash model has the form that we specify. Equilibrium may not be unique for a

Note that after substituting from (5), we can solve for (c_1, m, μ, λ) using only (6)–(10) and then use (11) to obtain q. In general, an analytical solution for these equations cannot be found. Hence, numerical methods must be developed to explore the predictions of the model.

B. The Cash Model

It is straightforward to demonstrate that Svensson's (1985) model is a limiting case of the model above when $\psi = 1$. Only cash goods provide utility to the agent, so $c_{2t} = 0$, equation (9) is dropped from the model, and equilibrium requires $c_{1t} = y_t$. The cash model is interesting because it provides insight into the variability of velocity that arises strictly from the precautionary demand for money because of the uncertainty about the realization of the state of the economy, without the possibility of substitution between the cash and credit goods.

C. Formulas for the Endogenous Variables

Expressions for velocity, realized real and nominal interest rates, inflation, and the growth of real balances can be calculated from the equilibrium functions above. Svensson (1985) derives such expressions for the cash model, and they are listed in table 1. It is straightforward to show that the expressions are the same for the cash-credit model.

Although the only assets outstanding in the economy are money and the endowment stock, any other state-contingent claim can be priced similarly using the market-clearing condition that it be in zero net supply. When calculating returns for other assets such as risk-free nominal bonds, we assume that they are traded in the securities market after the goods market closes. Hence, the payoff received at time t from a one-period bond purchased at time $t - 1$ is not available for consumption purchases until time $t + 1$.

III. A Solution Algorithm

Both models are solved by a similar method, but the algorithm is most straightforward for the cash model. Since the algorithm does

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countably infinite state space. A stationary equilibrium will not exist for ω sufficiently small or β too large. When ω is very low in many states, money has a high real rate of return. If the return is high enough, the agent tries to postpone consumption perpetually, and markets cannot clear.

TABLE 1

EXPRESSIONS FOR ENDOGENOUS VARIABLES

Consumption velocity	$v(\gamma_i, \omega_i) = \dfrac{\gamma_i}{m(\gamma_i, \omega_i)}$
Inflation rate (plus one)	$\pi(\gamma_i, \omega_i \mid \gamma_j, \omega_j) = \dfrac{m(\gamma_i, \omega_i)\omega_i}{m(\gamma_j, \omega_j)\gamma_i}$
Nominal interest rate	$i(\gamma_i, \omega_i) = \dfrac{E\{\mu(\gamma_j, \omega_j)[1/\pi(\gamma_i, \omega_i \mid \gamma_j, \omega_j)] \mid (\gamma_i, \omega_i)\}}{E\{\lambda(\gamma_j, \omega_j)[1/\pi(\gamma_i, \omega_i \mid \gamma_j, \omega_j)] \mid (\gamma_i, \omega_i)\}}$
Realized real interest rate	$r(\gamma_i, \omega_i \mid \gamma_j, \omega_j) = \dfrac{1 + i(\gamma_i, \omega_i)}{\pi(\gamma_i, \omega_i \mid \gamma_j, \omega_j)} - 1$
Growth rate of real balances	$m_g(\gamma_i, \omega_i \mid \gamma_j, \omega_j) = \dfrac{m(\gamma_j, \omega_j)\gamma_i}{m(\gamma_i, \omega_i)} - 1$

NOTE.—The current state of the Markov chain is (γ_i, ω_i), and $(\gamma_i, \omega_i \mid \gamma_j, \omega_j)$ denotes a transition from the current state to the state (γ_j, ω_j) in the next period.

not rely on a contraction principle, it has the advantage that the speed of convergence does not depend explicitly on the discount factor, and it accommodates discount factors greater than one.[5]

Let γ_i and ω_i indicate the values of γ and ω in state i, $i = 1, \ldots, n$. With the equilibrium of the form described above, the system (6), (7), (8), and (10) reduces to two equations in two unknown functions, $\mu(\gamma_i, \omega_i)$ and $m(\gamma_i, \omega_i)$:

$$\gamma_i \leq m(\gamma_i, \omega_i), \quad \{\mu(\gamma_i, \omega_i) \geq 0, \mu(\gamma_i, \omega_i)[\gamma_i - m(\gamma_i, \omega_i)] = 0\} \; \forall i, \tag{12}$$

$$\mu(\gamma_i, \omega_i) = \gamma_i^{-\alpha} - \frac{\beta E[\gamma_j^{-\alpha} m(\gamma_j, \omega_j) \mid \gamma_i, \omega_i]\gamma_i^{1-\alpha}}{\omega_i m(\gamma_i, \omega_i)} \quad \forall i. \tag{13}$$

The expectation in (13) is conditioned on state (γ_i, ω_i). The algorithm takes the following steps in which $m_h(\gamma_i, \omega_i)$ and $\mu_h(\gamma_i, \omega_i)$, for $h = 0$, 1, indicate values of the functions at different steps.

Step 1.—Set $m_0(\gamma_i, \omega_i) = \gamma_i$ for all i. (This is equivalent to assuming that the CIA constraint binds in all states.)

Step 2.—Use (13) to solve for $\mu_0(\gamma_i, \omega_i)$ for each state i. If $\mu_0(\gamma_i, \omega_i) \geq 0$ for all i, this is an equilibrium. If not, go to step 3.

Step 3.—If $\mu_0(\gamma_i, \omega_i) < 0$ for all i, stop since no equilibrium exists. (See lemma 2 in App. A.) Otherwise, for any state (γ_i, ω_i) in which $\mu_0(\gamma_i, \omega_i) \geq 0$, set $m_1(\gamma_i, \omega_i) = m_0(\gamma_i, \omega_i)$. If $\mu_0(\gamma_i, \omega_i) < 0$, set $m_1(\gamma_i, \omega_i)$ in the denominator of the right-hand side of (13) so that $\mu(\gamma_i, \omega_i) = 0$ when (13) is solved using the previous values of $m_0(\gamma_i, \omega_i)$ in the numerator of the right-hand side.

[5] Kocherlakota (1988) shows that in a growth model, a discount factor greater than one cannot be dismissed a priori.

Step 4.—Use (13) to solve for each $\mu_1(\gamma_i, \omega_i)$ using $m_1(\gamma_i, \omega_i)$ on the right-hand side. If $\mu_1(\gamma_i, \omega_i) \geq 0$ for all i, this is an equilibrium. If not, set $\mu_0(\gamma_i, \omega_i) = \mu_1(\gamma_i, \omega_i)$ and $m_0(\gamma_i, \omega_i) = m_1(\gamma_i, \omega_i)$ for all i and repeat step 3.

THEOREM 1. The algorithm converges to a stationary equilibrium if one exists.

The proof of theorem 1 is in Appendix A.

A similar algorithm solves the cash-credit model (see App. B).[6] The programs also check that the necessary conditions of finite wealth and utility are satisfied.

IV. Data and Estimation of the Vector Autoregressions

This section describes the calibration of the two Markov processes obtained from VARs, estimated with quarterly (1959:I–1987:IV) and annual (1950–86) data, using the procedures developed by Tauchen (1987). The sources of the data are described in Appendix C. Real per capita consumption is the sum of consumption in 1982 dollars of nondurables and services divided by total population. The corresponding price level is the sum of the current dollar series divided by consumption measured in 1982 dollars. The per capita money stock is M2 divided by total population.

We choose M2 as the monetary aggregate for the following reason. A first-order Markov process in the growth rates of money and endowments implies a stationary velocity in the models. Since M1 velocity appears to be nonstationary over the sample period, the models would be rejected immediately.[7] The velocity and rate of growth of M2, on the other hand, appear to be stationary. Thus, in this sense, M2 is the more appropriate monetary aggregate for calibrating these simple CIA models. The sensitivity analyses (reported below) also suggest robustness of the results to potential gross misspecifications in the money growth process.

One problem with using M2 as the monetary aggregate is that its average velocity is in general smaller than that predicted by the models. One way to adjust for this is to assume that real balances exceed the amount spent on the cash good by a constant fraction (see Eckstein and Leiderman 1988), since part of M2 can be considered a form of saving. Since the coefficient of variation of velocity (the

[6] The computer programs that solve the models are written in Gauss and are available from the authors.

[7] Marshall (1990) notes that M1 growth may be nonstationary. He therefore works with a monetary transactions cost technology that incorporates technological change.

ratio of the standard deviation to the mean) is not affected by scaling factors, while the standard deviation would be, we focus on the coefficient of variation of velocity as a measure of the volatility of velocity.

In the theory above, the exogenous process is assumed to be first-order Markov, but our method can accommodate a higher-order process. Hence, in each case, the appropriate order of the VAR was assessed by examination of the Schwarz (1978) criterion and by likelihood ratio tests. The estimation of the first-order VARs with diagnostic test statistics is reported in table 2. A first-order VAR is adequate in both cases. The Schwarz criterion always suggests the lower-dimensional model, and the marginal levels of significance of the test statistics indicate that the restricted models are generally not rejected at standard levels of significance.

Tauchen (1987) describes a quadrature procedure that constructs approximating Markov chains for VARs. This procedure chooses grid points and transition probabilities so as to match the conditional moments of the estimated (Gaussian) VAR. Application of his method to the quarterly data VAR using 16 states provides a good approximation. We check this by estimating the VAR using data generated from the Markov chain. The Markov counterpart to the VAR is also reported in table 2. The two VARs correspond very closely.

Applying Tauchen's method to the annual VAR also requires a 16-state Markov chain, which produces a close fit between the actual VAR in panel B of table 2 and the VAR estimated using data generated from the Markov chain reported there.

The next section compares simulation results to sample statistics calculated from quarterly and annual data on velocity, real and nominal interest rates, inflation, and real balances. Velocity is calculated as the ratio of nominal consumption to nominal money balances. Ex post real interest rates are calculated by subtracting one from one plus the nominal interest rate divided by one plus the inflation rate, which is the ratio of the price level at time $t + 1$ to the price level at time t. Real balances are the nominal money supply divided by the price level.

V. Predictions of the Models

Using the algorithms of Section III and the Markov chains of Section IV allows us to investigate the predictions of the models for the joint distribution of the endogenous and exogenous variables by simulation. The main question we address is whether the models can generate statistics consistent with sample moments computed from U.S. time-series data.

Since we have weak a priori beliefs about the preference parame-

TABLE 2

ESTIMATED VARs AND THEIR MARKOV COUNTERPARTS

Dependent Variable	Coefficients on				Covariance Matrix		Test Statistics			Likelihood Ratio Test†	
	Constant	ω_{t-1}	γ_{t-1}	R^2	σ	$\rho_{\omega\gamma}$	SC(1)*	SC(2)	SC(3)	1 vs. 2	2 vs. 3
					A. Quarterly Data						
ω_t	.501	.623	−.166	.375	.00636	.009	−20.401	−20.038	−19.740	1.576	9.757
	(.134)	(.074)	(.118)							[.813]	[.048]
γ_t	.655	.109	.238	.079	.00489						
	(.103)	(.057)	(.091)								
					Estimated Markov Counterpart						
ω_t	.508	.613	−.113		.00628	.009					
γ_t	.655	.109	.237		.00489						
					B. Annual Data						
ω_t	.742	.685	−.400	.489	.02243	−.443	−16.439	−16.095	−15.840	1.495	4.063
	(.350)	(.116)	(.321)							[.827]	[.398]
γ_t	.680	.091	.239	.084	.01072						
	(.163)	(.054)	(.150)								
					Estimated Markov Counterpart						
ω_t	.756	.635	−.362		.02220	−.462					
γ_t	.680	.091	.239		.01072						

* SC(j) is the value of the Schwarz (1978) criterion for lag length j. The statistic is calculated as eq. (16.6.7) of Judge et al. (1985, p. 687). The appropriate lag length is the minimum of SC(j).
† The likelihood ratio tests lag length j vs. length $j + 1$. The marginal level of significance of this test is in brackets The statistics incorporate the degrees of freedom correction recommended by Sims (1980).

ters, and to allow the models the greatest chance of success, we calculate several first and second unconditional moments for some variables of interest over a large parameter range, using the base case utility specification given in equation (2). For the annual simulations the parameter ranges are $\beta \in \{.9, .92, \ldots, 1\}$, $\alpha \in \{0, .5, \ldots, 9.5\}$, and $\psi \in \{.2, .4, \ldots, 1\}$.

The unconditional moments we examine include the coefficient of variation of velocity and the correlations of velocity with money growth, output growth, and the nominal interest rate. We also examine the means and standard deviations of real and nominal interest rates, inflation, and real balance growth; we calculate the correlations of inflation with money growth, consumption growth, and the nominal interest rate. In all, we consider 15 statistics. Tables 3 and 4 summarize the results of this analysis. For each individual statistic, the tables show the maximum and minimum attainable predictions over the entire range of parameter values. The parameters generating the maximum and minimum predictions are in parentheses next to the estimated values. For comparison, the last two columns list the corresponding sample statistics from the annual data with an asymptotic standard error.[8]

Table 3 illustrates the overall poor performance of the cash model. The sample value falls outside the range attainable from the cash model for 12 out of 15 statistics. When allowance is made for asymptotic standard errors around the point estimates of the sample statistics, the model's predictions are still not within two sample standard errors for three of the sample values. Most important, the model predicts virtually no variation in velocity.

Table 4 reports similar results for the cash-credit model. The model fails to reproduce 10 of the 15 point estimates of the statistics and cannot reproduce two of the sample statistics after allowance is made for standard errors. In contrast to the cash model, inclusion of a credit good does generate substantial variation in velocity for some parameter values.

The implicit test of the models in tables 3 and 4 is extremely weak because it asks only whether the models can replicate moments taken one at a time. Ideally, all moments of the model would be close to *all* the sample moments for a *fixed* set of preference parameters. Thus although the cash-credit model generates plausible values of the coef-

[8] The asymptotic standard errors of the unconditional sample moments are calculated following the suggestions in Hansen and Jagannathan (this issue). Each statistic is a nonlinear function, $f(y)$, of an arbitrarily serially correlated time series, y. If V is the variance of y, we calculate the standard error of $f(y)$ as $[\nabla f(y)' V \nabla f(y)]^{0.5}$, where $\nabla f(y)$ is the gradient of $f(y)$. The variance V is calculated using the method of Newey and West (1987).

TABLE 3

CASH MODEL SIMULATION RESULTS VS. SAMPLE VALUES (Annual Data, 1950–87)

$\beta = \{.9, .92, \ldots, .98, 1.0\}$; $\alpha = \{0, .5, \ldots, 9.5\}$

	Min	Min (α, β)	Max	Max (α, β)	Sample Value	Standard Error
cv[v]	.0000	(.95, 1.0)	.0009	(1.0, 1.0)	.0456	.0097**
corr[v, γ]	−.1585	(1.5, 1.0)	.0000	(9.5, 1.0)	−.5000	.1447**
corr[v, ω]	.0000	(9.5, 1.0)	.0711	(0, .98)	−.0668	.2263*
corr[v, i]	.0000	(2.5, 1.0)	.1555	(0, .98)	.5348	.2245*
$E[\pi]$	.0389	All	.0389	All	.0434	.0079*
$\sigma[\pi]$	.0297	All	.0297	All	.0283	.0061*
$E[i]$	.0594	(0, .98)	.3901	(9.5, .90)	.0587	.0094*
$\sigma[i]$	.0182	(0, .98)	.0537	(9.5, .90)	.0323	.0076
$E[r]$	.0201	(1.0, 1.0)	.3377	(9.5, .90)	.0148	.0053*
$\sigma[r]$	.0116	(4.0, 1.0)	.0218	(9.5, .90)	.0200	.0046
$E[m_g]$	.0203	All	.0203	All	.0164	.0063*
$\sigma[m_g]$	.0450	(9.5, 1.0)	.0474	(1.0, 1.0)	.0336	.0061*
corr[π, ω]	.9227	(1.0, 1.0)	.9254	(9.5, 1.0)	.3421	.1191**
corr[π, i]	.9165	(0, .98)	.9274	(5.0, .98)	.7689	.0805*
corr[π, r]	−.8812	(0, .90)	.4445	(9.5, .98)	−.1808	.1904

NOTE.—π is inflation; i is the nominal interest rate; r is the real interest rate; ω is the money growth rate; γ is the consumption growth rate; m_g is the real balances growth rate. Correlation of any variable and money growth is contemporaneous (e.g., corr[v, ω] = corr[v_t, ω_{t-1}]). No equilibrium exists for $\beta = 1$ and $\alpha \leq .5$

* The sample value falls outside the possible range predicted by the model.

** The sample value falls outside the possible range by more than two standard errors.

TABLE 4

CASH-CREDIT MODEL SIMULATION RESULTS VS. SAMPLE VALUES (Annual Data, 1950–87)

$\beta = \{.9, .92, \ldots, .98, 1.0\}$; $\alpha = \{0, .5, \ldots, 9.5\}$; $\psi = \{.2, .4, .6, .8\}$

	Min	Min(α, β, ψ)	Max	Max (α, β, ψ)	Sample Value	Standard Error
cv[v]	.0056	(0, .96, .8)	.0510	(9.5, .9, .2)	.0456	.0097
corr[v, γ]	−.3319	(0, .98, .8)	.5157	(9.0, .92, .4)	−.5000	.1447*
corr[v, ω]	.6190	(0, .98, .8)	.7520	(7.5, .92, .8)	−.0668	.2263**
corr[v, i]	.7342	(9.5, .96, .4)	.9879	(0, .9, .6)	.5348	.2245*
$E[\pi]$	.0389	(1.0, .9, .8)	.0389	(9.5, .9, .2)	.0434	.0079*
$\sigma[\pi]$	.0290	(1.0, .9, .8)	.0529	(9.5, .9, .2)	.0283	.0061*
$E[i]$	.0594	(0, .98, .6)	.3902	(9.5, .9, .8)	.0587	.0094*
$\sigma[i]$	.0181	(0, .98, .6)	.0638	(9.5, .9, .8)	.0323	.0076
$E[r]$	.0201	(1.0, 1.0, .8)	.3390	(9.5, .9, .2)	.0148	.0053*
$\sigma[r]$	.0138	(3.0, 1.0, .8)	.0675	(9.5, .9, .2)	.0200	.0046
$E[m_g]$	.0203	(2.0, 1.0, .8)	.0213	(9.5, .9, .2)	.0157	.0064*
$\sigma[m_g]$	.0648	(0, .96, .8)	.3232	(9.5, .9, .2)	.0334	.0060**
corr[π, ω]	.4665	(9.5, .9, .2)	.8746	(2.0, 1.0, .8)	.3421	.1191*
corr[π, i]	.4518	(9.5, .9, .2)	.8867	(3.5, 1.0, .8)	.7689	.0805
corr[π, r]	−.8393	(0, .9, .8)	.3004	(9.5, 1.0, .8)	−.1808	.1904

NOTE.—See also table 3. The algorithm failed to converge for 13 out of the 400 possible parameter specifications.

ficient of variation of velocity, it is not a successful model. In particular, matching the variation in velocity in this parameter range requires β close to one, ψ close to zero, and α very large. For these preference parameters, the expected real interest rate is approximately 20 percent per year. When the expected real interest rate is more realistic (smaller than 3 percent per year), the highest coefficient of variation of velocity that the model generates is approximately .03 (the sample average is .0456). Thus it appears impossible to reconcile the low value of average interest rates with the volatility of velocity in this framework.

Tables 5 and 6 present the same information for the quarterly implementation.[9] Although a planning period of one quarter in a CIA model might seem more plausible than a planning period of a year, both the cash and cash-credit models perform more poorly when calibrated with quarterly data.

A. *Analysis*

The evidence to this point strongly suggests an inability of the cash model to produce variability in velocity when driven by a realistic forcing process. Here we informally examine a condition that must be satisfied in the cash model if there is to be any variability in velocity, and we show why in fact it is unlikely to hold.

Notice from (12) and (13) that if the CIA constraint is always binding,

$$\mu_t = \gamma_t^{-\alpha}\left[1 - \beta E_t\left(\frac{\gamma_{t+1}^{1-\alpha}}{\omega_t}\right)\right] > 0. \tag{14}$$

In early CIA models in which cash needs are fully anticipated, the expression in brackets in equation (14) equals the nominal interest rate divided by one plus the nominal interest rate. Hence, for these models, positive nominal interest rates imply that the CIA constraint always binds. Although the expression for nominal interest rates with the alternative timing of events in this paper is different, equation (14) can still be used to assess whether the CIA constraint always binds.[10]

In terms of equation (14), the CIA constraint will always bind if

[9] Hodrick, Kocherlakota, and Lucas (1989) provide results for $\beta > 1$, for both annual and quarterly data. The models perform no better in this region.

[10] We thank Andrew Atkeson for the insight that changing the timing within a period does not change the states of the world in which the CIA constraint binds since eq. (14) depends only on the preferences of agents and the time-series properties of the exogenous forcing processes. Checking that eq. (14) is always positive is equivalent to following the first two steps of our algorithm.

TABLE 5

CASH MODEL SIMULATION RESULTS VS. SAMPLE VALUES (Quarterly Data, 1959:II–1988:I)

$\beta = \{.975, .98, \ldots, .995, 1.000\}$; $\alpha = \{0, .5, \ldots, 1, \ldots, 9.5\}$

	Min	Min(α, β)	Max	Max(α, β)	Sample Value	Standard Error
cv[v]	.0000	All	.0000	All	.0398	.0061**
corr[v, γ]	.0000	All	.0000	All	−.3420	.1112**
corr[v, ω]	.0000	All	.0000	All	−.1634	.1243*
corr[v, i]	.0000	All	.0000	All	.6208	.1526*
$E[\pi]$	.0141	All	.0141	All	.0122	.0014*
$\sigma[\pi]$	.0085	All	.0085	All	.0074	.0012*
$E[i]$	.0195	(1.0, 1.0)	.0933	(9.5, .975)	.0151	.0014**
$\sigma[i]$	.0041	(0, .990)	.0122	(9.5, .975)	.0069	.0013
$E[r]$	.0054	(1.0, 1.0)	.0781	(9.5, .975)	.0030	.0011**
$\sigma[r]$	.0051	(3.0, 1.0)	.0075	(9.5, .975)	.0062	.0010
$E[m_g]$	.0054	All	.0054	All	.0052	.0016*
$\sigma[m_g]$	.0205	All	.0205	All	.0098	.0014**
corr[π, ω]	.8042	All	.8042	All	.1844	.1012**
corr[π, i]	.7811	(9.5, .995)	.8167	(.5, .995)	.6192	.0942*
corr[π, r]	−.9113	(0, .985)	.4570	(9.5, .990)	−.5047	.1392

NOTE.—See also table 3. Equilibrium does not exist for (α, β) = (0, .995), (0, 1), and (.5, 1).

TABLE 6

CASH-CREDIT MODEL SIMULATION RESULTS VS. SAMPLE VALUES (Quarterly Data, 1959:II–1988:I)

$\beta = \{.9, .92, \ldots, .98, 1.0\}$; $\alpha = \{0, .5, \ldots, 9.5\}$; $\psi = \{.2, .4, .6, .8\}$

	Min	Min(α, β, ψ)	Max	Max(α, β, ψ)	Sample Value	Standard Error
cv[v]	.0014	(0, .99, .8)	.0166	(9.5, .975, .2)	.0398	.0061**
corr[v, γ]	−.0692	(0, .975, .6)	.6639	(9.5, .975, .6)	−.3420	.1112**
corr[v, ω]	.5667	(9.5, .975, .6)	.6189	(2.5, .975, .2)	−.1634	.1243**
corr[v, i]	.8894	(9.5, .975, .6)	.9997	(0, .975, .6)	.6208	.1526*
$E[\pi]$	.0141	All	.0141	All	.0122	.0014*
$\sigma[\pi]$	.0076	(8.5, .985, .8)	.0147	(9.5, .975, .2)	.0074	.0012*
$E[i]$	.0195	(1.0, 1.0, .8)	.0933	(9.5, .975, .6)	.0151	.0014**
$\sigma[i]$	.0041	(0, .99, .6)	.0132	(9.5, .975, .6)	.0069	.0013
$E[r]$	.0054	(1.0, 1.0, .8)	.0783	(9.5, .975, .2)	.0030	.0011**
$\sigma[r]$	.0050	(3.0, 1.0, .8)	.0192	(9.5, .975, .2)	.0062	.0010
$E[m_g]$	.0054	(6.0, 1.0, .8)	.0055	(9.5, .975, .2)	.0052	.0016*
$\sigma[m_g]$	.0224	(2.5, 1.0, .8)	.1099	(9.5, .975, .2)	.0098	.0014**
corr[π, ω]	.1791	(9.5, .975, .2)	.7784	(5.5, 1.0, .8)	.1844	.1012
corr[π, i]	.0913	(9.5, .975, .2)	.7808	(2.0, 1.0, .8)	.6192	.0942
corr[π, r]	−.8930	(0, .975, .8)	.0695	(9.5, 1.0, .8)	−.5047	.1392

NOTE —See also table 3 Equilibrium does not exist for (α, β) = (0, 995), (0, 1), and (5, 1) money growth and consumption growth are high and not too variable (assuming $\alpha > 1$). From the VAR analysis of the previous section, we know that the conditional expectation of consumption growth is not constant. But how much must it vary in order that the CIA constraint can be slack in some states? Suppose that we consider γ_t to be unconditionally lognormally distributed and unforecastable. Then given estimates of the mean and variance of $\ln(\gamma)$, the expression in brackets in equation (14) can be evaluated at various preference parameters for the entire time series of money growth rates. For example, with quarterly data and $\alpha = 2$, the estimates are $E[\ln(\gamma)] = .0049$, $\sigma^2[\ln(\gamma)] = (.0051)^2$, and $E(\gamma^{1-\alpha}) = .9951$. With $\beta = .99$, the maximum value of $\beta E(\gamma^{1-\alpha})/\omega_t$ for all realizations of money growth is .9867. Hence, unless the conditional expected value of $\gamma^{1-\alpha}$ increases by at least .0133, the CIA constraint will always bind.

To assess roughly how likely such a change is, we examine the unconditional variance of $\gamma^{1-\alpha}$ since the unconditional variance of a time series provides an upper bound on the variance of the conditional mean. For the parameters given above, the unconditional variance is $(.0152)^2$, which makes an increase of .0133 in the conditional mean somewhat unlikely. Hence, consumption growth would have to be quite predictable, which it is not, in order for there to be any possibility for a nonbinding CIA constraint in the cash model.

For the cash-credit model, the numerical analysis reveals that adding a choice between cash and credit goods is sufficient to generate variation in velocity but that the CIA constraint for the cash good almost always binds. The condition similar to equation (14) for the cash-credit model under which the CIA constraint always binds is

$$\mu_t = u_{1t}\left\{1 - \beta E_t\left[\frac{g(\gamma_{t+1}, \omega_{t+1})\gamma_{t+1}^{1-\alpha}}{g(\gamma_t, \omega_t)\omega_t}\right]\right\} > 0, \tag{15}$$

where $g(\gamma_t, \omega_t) = [c_1(\gamma_t, \omega_t)/\gamma_t]^{\psi(1-\alpha)}\{1 - [c_1(\gamma_t, \omega_t)/\gamma_t]\}^{(1-\psi)(1-\alpha)}$. Since equation (15) depends on the endogenous consumption choices of the agent, whether this condition holds cannot be examined analytically. Only by solving numerically for the equilibrium functions as above can we assess the importance of this source of variability.

VI. Sensitivity Analysis

The question remains whether the models fail to explain the data because of our particular assumptions or whether the failure reflects more fundamental problems with the models. In this section we examine the robustness of the models' predictions to changes in the information structure, the driving process, and the utility specification.

A. *Varying the Agent's Information*

An important contribution of Lucas and Stokey (1987) was to allow for state-contingent uncertainty about the future fundamentals of the economy. This feature creates the possibility of a slack CIA constraint, which is one mechanism that could generate variation in velocity in the cash-credit model. It is useful to see whether the velocity estimates are sensitive to the assumed information structure for two reasons: it suggests whether misspecification of the information set can explain the base case results, and if the results are not sensitive to information, the computational complexity in future empirical work could be reduced by assuming that next period's cash needs are fully anticipated.

To examine whether the assumed precision of the agent's information about next period's money growth or output growth can have a significant effect on the predictions of the model, we expand the state space to include a noisy signal about next period's money supply or output. This requires a slight modification of the timing assumptions and a reestimation of the VAR for the driving process.

In the models above, agents receive information about the time t output and the time $t + 1$ money stock at the beginning of period t, and the actual transfer of money occurs in the securities market at time t. To consider the effects of a signal about the monetary transfer, we assume that it occurs after the close of the time t securities market, since otherwise the agent would learn from his own transfer. This timing is similar to that in Lucas and Stokey (1987).

First, assume that agents in the asset market at time t receive a signal about the growth of the money supply between the current period and next period equal to the true money growth rate plus i.i.d. noise: $S_t \equiv \omega_t + \theta_t$, with $\theta_t \sim N(0, \sigma_\eta^2)$ and uncorrelated with the time t information set. Time t information includes S_t, ω_{t-1}, and γ_t (recall that $\omega_{t-1} = X_t/X_{t-1}$ and $\gamma_t = y_t/y_{t-1}$), and the evolution of the stationary component of the state can be estimated with the following VAR:

$$\begin{aligned} S_{t+1} &= a_0 + a_1 S_t + a_2\omega_{t-1} + a_3\gamma_t + \epsilon_{St}, \\ \omega_t &= b_0 + b_1 S_t + b_2\omega_{t-1} + b_3\gamma_t + \epsilon_{\omega t}, \\ \gamma_{t+1} &= c_0 + c_1 S_t + c_2\omega_{t-1} + c_3\gamma_t + \epsilon_{\gamma t}. \end{aligned} \tag{16}$$

We want to estimate this VAR and discretize the distribution as before, varying σ_θ to reflect different degrees of uncertainty about next period's money supply. It is straightforward to calculate the resulting coefficients and covariance matrix of the VAR as a function of σ_θ and the matrix of independent variables (see App. D).

To test the sensitivity of the model predictions, we set σ_θ equal to

TABLE 7

RESULTS WITH NOISY SIGNALS IN THE CASH-CREDIT MODEL

	Signal about Money Growth			Signal about Output Growth		
	$.01\sigma_\omega$	$.5\sigma_\omega$	$2\sigma_\omega$	$.01\sigma_\gamma$	$.5\sigma_\gamma$	$2\sigma_\gamma$
($\alpha = 1, \psi = .1$):						
cv[v]	.014	.016	.016	.015	.016	.016
$E[\pi]$	.042	.048	.045	.058	.049	.053
$E[r]$	.030	.029	.030	.028	.029	.030
corr[v, i]	.965	.979	.979	.890	.974	.978
($\alpha = 1, \psi = .9$):						
cv[v]	.002	.002	.002	.002	.002	.002
$E[\pi]$	.042	.048	.044	.058	.049	.053
$E[r]$	.029	.029	.030	.028	.029	.029
corr[v, i]	.965	.979	.979	.890	.974	.978
($\alpha = 4, \psi = .1$):						
cv[v]	.017	.019	.019	.029	.020	.019
$E[\pi]$	.042	.048	.045	.059	.049	.053
$E[r]$	.087	.088	.092	.083	.088	.089
corr[v, i]	.822	.876	.877	.710	.853	.871
($\alpha = 4, \psi = .9$):						
cv[v]	.002	.002	.002	.004	.002	.002
$E[\pi]$	.042	.048	.044	.058	.049	.053
$E[r]$	.087	.088	.092	.082	.088	.088
corr[v, i]	.821	.876	.876	.710	.852	.871

NOTE.—Annual data, 1958–87; $\beta = .99$.

0.01, 0.5, and 2 times the standard deviation of the innovation in money growth from the VAR (σ_ω). The equilibrium conditions are unchanged except that ω_t in equation (13) must be inside the expectation operator. The results in table 7 indicate that the coefficient of variation of velocity, the correlation between velocity and the nominal interest rate, and the unconditional expectations of inflation and the real interest rate are relatively insensitive to the agent's information about future money growth.

Similarly, we allow agents at time t to observe a signal $S_t = \gamma_{t+1} + \theta_t$ of next period's output growth, where θ_t is distributed as above. In this case, σ_θ is set to 0.01, 0.5, and 2 times the unconditional standard deviation of output growth. Again, the precision of the signal has little effect on the predictions of the model, as reported in table 7. We conclude that the specification of information has a minimal effect on the predictions of the cash-credit model.

B. CES and Habit Formation Preferences

It is possible that the cash-credit model generates a larger range of predictions for an alternative utility specification. The functional form in equation (2) is restrictive because it implies a unitary intra-

TABLE 8

CASH-CREDIT MODEL SIMULATION RESULTS VS. SAMPLE VALUES (Annual Data, 1950–87; CES Preferences)

$\beta = \{.92, .94, \ldots, .98, 1.0\}$; $\alpha = \{0, .5, \ldots, 9.5\}$; $\eta = \{.5, .75, .8, .9, .95, .97\}$

	Min	Min(α, β, η)	Max	Max(α, β, η)	Sample Value	Standard Error
cv[v]	.0234	(0, .98, .5)	.1572	(9.5, .92, .8)	.0456	.0097
corr[v, γ]	−.4235	(0, .98, .8)	.3955	(9.0, .98, .5)	−.5000	.1447*
corr[v, ω]	.5777	(0, .92, .97)	.7481	(7.0, .92, .5)	−.0668	.2263**
corr[v, i]	.7715	(9.0, .98, .5)	.9960	(0, .92, .97)	.5348	.2245*
$E[\pi]$	.0390	(0, .98, .5)	.0461	(9.5, .92, .8)	.0434	.0079
$\sigma[\pi]$	.0325	(0, .98, .5)	.1273	(9.5, .92, .8)	.0283	.0061*
$E[i]$	.0579	(1.0, 1.0, .97)	.3599	(9.5, .92, .75)	.0587	.0094
$\sigma[i]$	.0013	(0, .94, .97)	.0547	(8.5, .94, .5)	.0323	.0076
$E[r]$	.0202	(0, .98, .5)	.3193	(9.5, .92, .8)	.0148	.0053*
$\sigma[r]$	.0265	(0, .98, .5)	.1665	(9.5, .92, .8)	.0200	.0046*
$E[m_g]$	.0206	(0, .98, .5)	.0285	(9.5, .92, .8)	.0157	.0064*
$\sigma[m_g]$	.1759	(0, .98, .5)	1.0380	(9.5, .92, .8)	.0334	.0060**
corr[π, ω]	−.1591	(9.5, .98, .97)	.5627	(0, .98, .5)	.3421	.1191
corr[π, i]	−.1153	(9.5, .98, .97)	.5705	(0, .98, .5)	.7689	.0805**
corr[π, r]	−.9968	(0, .92, .97)	−.7336	(7.0, 1.0, .5)	−.1808	.1904**

NOTE.—See also table 3. No equilibrium exists for $\beta = 1$ and $\alpha \leq .5$ The algorithm failed to converge for 61 out of 600 possible parameter specifications

temporal elasticity of substitution between the cash and credit goods for all ψ. If agents regard cash and credit goods as closer substitutes than is implied by this utility function, velocity may vary more as they substitute more freely between the two goods. To assess this conjecture, we consider a constant elasticity of substitution (CES) alternative,

$$u(c_{1t}, c_{2t}) = \frac{(c_{1t}^{\eta} + c_{2t}^{\eta})^{(1-\alpha)/\eta} - 1}{1 - \alpha},$$

in which the intratemporal elasticity of substitution is $1/(1 - \eta)$. Note that this utility specification does not nest the Cobb-Douglas base case we consider earlier. We examine the model's predictions for $\eta \in \{.5, .75, .8, .9, .95, .97\}$ in table 8. A comparison with table 4 indicates no great improvement in the predictions of the model overall. Indeed, the model's predictions for the correlations of inflation with real and nominal interest rates are worse than before. As expected, though, the potential for velocity to vary is increased.

If utility this period depends on the increase in consumption over that in the previous period, preferences in the cash good model can be written as

$$u(c_t) = \frac{\{c_t - b[\min(\gamma)]c_{t-1}\}^{1-\alpha} - 1}{1 - \alpha}, \tag{17}$$

TABLE 9

Cash Good Model Simulation Results vs. Sample Values (Annual Data, 1950–87; Habit Formation Preferences)

$\beta = \{.92, .94, \ldots, .98, 1.0\}$; $\alpha = \{0, .5, \ldots, 9.5\}$; $b = \{0, .2, .4, .6, .8\}$

	Min	Min(α, β, b)	Max	Max(α, β, b)	Sample Value	Standard Error
cv[v]	.0000	(9.5, 1.0, .8)	.3001	(3.5, .98, .8)	.0456	.0097
corr[v, γ]	−.8422	(3.0, 1.0, .8)	.0000	(9.5, 1.0, .8)	−.5000	.1447
corr[v, ω]	.0000	(9.5, 1.0, .8)	.2480	(1.0, 1.0, .6)	−.0668	.2263*
corr[v, i]	.0000	(4.5, 1.0, .2)	.8342	(3.0, 1.0, .8)	.5348	.2245
$E[\pi]$	.0389	(6.5, .96, .4)	.4304	(3.5, .98, .8)	.0434	.0079
$\sigma[\pi]$	.0297	(6.5, .96, .4)	3.9709	(3.5, .98, .8)	.0283	.0061*
$E[i]$	.0592	(1.0, 1.0, .8)	.3599	(9.5, .92, 0)	.0587	.0094*
$\sigma[i]$	.0182	(0, .98, .6)	.0954	(9.5, .92, .6)	.0323	.0076
$E[r]$	.0201	(1.0, 1.0, 0)	.4773	(3.5, .98, .8)	.0148	.0053*
$\sigma[r]$	.0116	(4.0, 1.0, 0)	3.6304	(9.5, 1.0, .8)	.0200	.0046
$E[m_g]$	.0204	(9.5, 1.0, .8)	.3767	(3.5, .98, .8)	.0157	.0064*
$\sigma[m_g]$	.0450	(9.5, 1.0, .8)	19.000	(3.5, .98, .8)	.0334	.0060*
corr[π, ω]	−.3386	(2.5, 1.0, .8)	.9254	(9.5, 1.0, .8)	.3421	.1191
corr[π, i]	−.4345	(2.5, 1.0, .8)	.9274	(5.0, .98, 0)	.7689	.0805
corr[π, r]	−.9667	(1.0, 1.0, .8)	.5510	(9.5, .94, .4)	−.1808	.1904

Note.—See also table 3. No equilibrium exists for $\beta = 1$ and $\alpha \leq .5$

where b can be varied between zero and one to reflect the degree of habit formation.[11] In equation (17) we scale b by the minimum growth rate of consumption to ensure that preferences are well defined. The effect of these preferences on the range of predicted values is dramatic. Table 9 reports ranges for α and β as in table 3, and $b \in \{0, .2, .4, .6, .8\}$ such that the marginal utility of consumption is positive. The model can generate predictions that lie within one standard error of all 15 statistics; nine of the sample statistics lie within the range of possible moments generated by the model. Intuitively, these preferences induce extremely high risk aversion as b approaches one even when α is small because the point of infinite marginal utility in the current-period utility function is based on last period's consumption. Such preferences generate a large precautionary demand for money in some states, which reduces the velocity of money and increases its volatility. This interpretation is consistent with the fact that with the standard preferences of equation (2), the cash good model can generate large fluctuations in velocity and inflation (e.g., for values of $\alpha >$ 400 and $\beta > 1.15$).

Again, though, the model is not successful when we require that it simultaneously fit the data on several dimensions. Parameters that

[11] These preferences were first suggested by Ryder and Heal (1973) and are used by Constantinides (1990) in an attempt to resolve the equity premium puzzle.

generate realistic variability of velocity also generate unrealistically high variability of inflation and real interest rates. Any parameter specification that generates a coefficient of variation of velocity larger than .04 (the sample estimate is .0456) also produces a standard deviation of inflation larger than .05 and a standard deviation of real interest rates larger than .06. The sample estimates of the latter quantities are .03 (standard deviation .008) and .02 (standard deviation .005).

C. *Sensitivity to the Forcing Process and Sample Period*

We test the robustness of the models' predictions using the quarterly data by (*a*) varying the parameters of the forcing process and (*b*) truncating the data at 1979:II. Neither variation substantially changes the ranges reported in tables 3–6. In particular, the basic inability of the cash model to generate the observed variability of velocity remains.

Recall that we estimate the following VAR to calibrate the forcing process in the model:

$$\gamma_t = A_{10} + A_{11}\gamma_{t-1} + A_{12}\omega_{t-1} + \epsilon_{\gamma t},$$

$$\omega_t = A_{20} + A_{21}\gamma_{t-1} + A_{22}\omega_{t-1} + \epsilon_{\omega t}.$$

In order to consider the robustness of the results for the quarterly data to potential misspecification of this driving process, we look at 18 variations of the VAR. In each of these experiments, we alter one parameter of the estimated VAR and adjust the constants so as to leave the unconditional means of γ and ω unchanged. The first six experiments involve manipulating the variance/covariance matrix of the forecast errors. We doubled and tripled the variances of the forecast errors, set the covariance of the forecast errors equal to zero, and changed the sign (but not the absolute value) of the covariance of the forecast errors. The other experiments increase the regression coefficients by one and two standard errors, and decrease them by one standard error.

These mean-preserving changes have little effect on the expected values of inflation, real balance growth, and interest rates. Although there are some changes in the second moments of the variables, the effects are small in terms of changing the ranges in tables 5 and 6. Qualitatively, increasing the predictability of consumption growth, the variability of consumption growth, or the variability of monetary growth in the cash-credit model causes the CIA constraint to bind less frequently and increases the coefficient of variation of velocity. In the cash model, tripling the variance of the forecast error of monetary growth increases the coefficient of variation of velocity from zero to

.0012; for all the other experiments the CIA constraint is always binding.

In a separate set of experiments, we also look at the effects of changing the intercepts of the VARs. It is important to note that while the second moments are not greatly affected, the predictions of the model for the expectations of inflation and real balance growth are quite sensitive to these parameters.

A strong assumption underlying our results is that quarterly consumption and money growths follow a stationary process from 1959 to 1987. Since many researchers argue that there was a fundamental change in the monetary regime in October 1979, we reconstructed tables 3–6 using data from the period 1959:I–1979:II. We find that the model performs *worse* over this period in the sense that for all four tables, a larger number of sample estimates fall outside the ranges consistent with the model.

VII. Conclusions

Lucas (1984) and Svensson (1985) show that adding uncertainty about future cash needs can in principle allow velocity to vary in CIA models. Whether this change in the information structure has practical implications for the models' predictions is an empirical question. We investigate this issue by calibrating the model and using driving processes estimated from U.S. time-series data to generate sample statistics. A striking and robust result is that the model predicts essentially constant velocity.

Why a precautionary demand for cash balances fails to generate variation in velocity in the calibrated model can be understood by considering the choice between holding an additional unit of cash and investing in an interest-bearing bond. In this model, the benefit of the former is that money provides liquidity services in the next period, while the bond cannot be converted into consumption until two periods hence. Velocity varies when agents hold more cash than necessary for current expenditures in some states. However, if nominal interest rates are sufficiently high and if the variation in the marginal utility of consumption across future states is sufficiently small, agents economize on cash balances and hold just enough money to cover purchases in all future states.

One way to interpret the failure of the model in this context is to conclude that there is too little variability in the aggregate consumption data to explain velocity in a representative consumer, CIA model, unless risk aversion is extremely high. This is similar to the reason for the equity premium puzzle offered by Mehra and Prescott (1985).

Despite the empirical failure of the cash model in our investigation, its basic insights may still prove useful. If, for instance, agents face individual as well as aggregate uncertainty about future consumption because of incomplete markets, the model may underestimate the variability of velocity, even if a CIA model describes each individual's decision correctly. Therefore, it is possible that a similar model that seriously treated the aggregation problem could produce reasonable velocity predictions.

Including credit goods, as in Lucas and Stokey (1987), generates variability in velocity even without a complicated information structure. Still, the cash-credit model is unable to generate realistic predictions about the joint distribution of the first and second moments of other variables. Thus it appears that the cash-credit model, at least in its simple form, cannot provide a satisfactory characterization of the data. We emphasize, though, that we have not exhaustively tested CIA models as a class nor have we compared their performance to any of the popular alternatives, which might perform just as poorly when embedded in a similarly stylized model.[12]

As mentioned earlier, much of the computational complexity in implementing these models comes from assuming an information structure that accommodates a slack CIA constraint. Our experiments with changing the information structure strongly suggest that, in practice, both the cash and cash-credit models will predict a binding CIA constraint. This finding strengthens the results of previous authors who have assumed a binding CIA constraint[13] and justifies making this simplifying assumption in future empirical applications of CIA constraints.

The calibration procedure of this paper is fairly easy to implement, and it quickly reveals the properties of these models under a variety of scenarios. It provides an inexpensive way to assess the probable value of a theoretical model in explaining data and to understand the basic properties of the model. For instance, plots of the sample statistics generated by the cash-credit model reveal that although the model is highly nonlinear, the response of the sample statistics varies fairly linearly in the model parameters. Our procedure thus serves as a useful complement to more formal statistical tests such as the generalized method of moments developed by Hansen (1982).[14]

[12] One such test is performed by Eckstein and Leiderman (1988), who find that money in the utility function outperforms the cash-credit model when Israeli data are used.

[13] Backus, Gregory, and Zin (1989) and Labadie (1989) investigate the empirical predictions of the cash model for asset prices, assuming that the CIA constraint always binds.

[14] Additional simulation results and graphical analyses of the models are available in Hodrick et al. (1989). Finn, Hoffman, and Schlagenhauf (1988) test the equity pricing implications of the Svensson model using generalized method of moments. They find that these restrictions of the model are not rejected by monthly data.

Appendix A

Proof of Theorem 1

LEMMA 1. Let $\{m_k(\gamma_i, \omega_i)\ \forall\ i\}$, $k = 0, 1, 2, \ldots$, be a sequence generated by the procedure described above. The sequence is nondecreasing.

Proof. On any iteration k, for states (γ_i, ω_i) in which $\mu_{k-1}(\gamma_i, \omega_i) \geq 0$, $m_k(\gamma_i, \omega_i) = m_{k-1}(\gamma_i, \omega_i)$ in step 3. For states (γ_j, ω_j) in which $\mu_{k-1}(\gamma_j, \omega_j) < 0$, then $m_k(\gamma_j, \omega_j)$ must be increased in the denominator of the right-hand side of equation (13) until $\mu_k(\gamma_j, \omega_j) = 0$. Q.E.D.

LEMMA 2. In the sequence of $\{\mu_k(\gamma_i, \omega_i)\}$, $k = 0, 1, 2, \ldots$, generated by the algorithm, if it ever occurs that $\mu_k(\gamma_i, \omega_i) < 0$ for all i, the multipliers will always be negative, and the algorithm does not converge.

Proof. Say that $\mu_k(\gamma_i, \omega_i) < 0$ for all i. Then by lemma 1, $m_{k+1}(\gamma_i, \omega_i) > m_k(\gamma_i, \omega_i)$ for all i. Therefore, on the next iteration, the numerator of the expectation in equation (13) increases. Using $m_{k+1}(\gamma_i, \omega_i)$ in the denominator of the expectation and solving for $\mu_{k+1}(\gamma_i, \omega_i)$ in equation (13) imply that $\mu_{k+1}(\gamma_i, \omega_i)$ is again less than zero since μ was equal to zero when the numerator of the expectation was smaller and the denominator was the same. Thus $m_{k+2}(\gamma_i, \omega_i) > m_{k+1}(\gamma_i, \omega_i)$, and the stopping condition is never satisfied. Q.E.D.

Proof of theorem 1 (by contradiction).—Let $\{m^*(\gamma_i, \omega_i), \mu^*(\gamma_i, \omega_i), i = 1, \ldots, n\}$ be the equilibrium defined by equations (12) and (13). By lemma 1, $m_k(\gamma_i, \omega_i)$ is a nondecreasing sequence, so that if the algorithm fails to converge, the sequence must jump over $m^*(\gamma_i, \omega_i)$. For all (γ_i, ω_i) such that $\mu_k(\gamma_i, \omega_i) \geq 0$, $m_{k+1}(\gamma_i, \omega_i) = m_k(\gamma_i, \omega_i)$, so the equilibrium is not jumped in these states. Assume that $m_{k+1}(\gamma_h, \omega_h) > m^*(\gamma_h, \omega_h)$ for a subset of states $\{(\gamma_h, \omega_h)\}$ but that $m_k(\gamma_i, \omega_i) \leq m^*(\gamma_i, \omega_i)$ for all i. Then $\mu_k(\gamma_h, \omega_h) < 0$, and $m_{k+1}(\gamma_h, \omega_h) > m^*(\gamma_h, \omega_h) > m_k(\gamma_h, \omega_h)$. Solving for $\mu(\gamma_h, \omega_h)$ in equation (13) with $m^*(\gamma_h, \omega_h)$ in place of $m_{k+1}(\gamma_h, \omega_h)$ but all else the same implies $\mu(\gamma_h, \omega_h) < 0$, since $m_{k+1}(\gamma_h, \omega_h)$ is determined to set $\mu(\gamma_h, \omega_h) = 0$. Consider the effect of increasing $m_k(\gamma_i, \omega_i)$ to $m^*(\gamma_i, \omega_i)$ for all i in the numerator of the expectation in equation (13), and solve again for $\mu(\gamma_h, \omega_h)$ with $m^*(\gamma_h, \omega_h)$ in the denominator. Algebra establishes that the resulting $\mu(\gamma_h, \omega_h) < 0$, which contradicts that $m^*(\gamma_h, \omega_h)$ is an equilibrium.

Appendix B

Solution Algorithm for the Cash-Credit Model

First, the system (5)–(10) reduces to the following three-equation system:

$$\begin{gathered} c_1(\gamma_i, \omega_i) \leq m(\gamma_i, \omega_i), \\ \{\mu(\gamma_i, \omega_i) \geq 0 \text{ and } \mu(\gamma_i, \omega_i)[c_1(\gamma_i, \omega_i) - m(\gamma_i, \omega_i)] = 0\}, \end{gathered} \tag{B1}$$

$$u_1(\gamma_i, \omega_i) = u_2(\gamma_i, \omega_i) + \mu(\gamma_i, \omega_i), \tag{B2}$$

$$\mu(\gamma_i, \omega_i) = u_1(\gamma_i, \omega_i) - \frac{\beta E_t[u_1(\gamma_j, \omega_j)m(\gamma_j, \omega_j) \mid \gamma_i, \omega_i]\gamma_i^{1-\alpha}}{\omega_i m(\gamma_i, \omega_i)}. \tag{B3}$$

The algorithm solves for the functions $c_1(\gamma_i, \omega_i)$, $m(\gamma_i, \omega_i)$, and $\mu(\gamma_i, \omega_i)$. The notation c_{10} or m_0 refers to the value of c_1 or m on an initial iteration, and c_{11} or m_1 refers to c_1 or m on the subsequent iteration.

Step 1.—Set $m_0(\gamma_i, \omega_i) = c_{10}(\gamma_i, \omega_i) = \psi\gamma_i$ for all i. This implies $\mu(\gamma_i, \omega_i) = 0$.

Step 2.—Use (B3) to solve for $\mu_0(\gamma_i, \omega_i)$. If $\mu_0(\gamma_i, \omega_i) = 0$ for all (γ_i, ω_i), this is an equilibrium. If not, go to step 3.

Step 3.—If $\mu_0(\gamma_i, \omega_i) < 0$ in all states, stop since the algorithm will never converge. For any state (γ_h, ω_h) in which $\mu_0(\gamma_h, \omega_h) \leq 0$, use (B2) to define $c_{11}(\omega_h, \gamma_h)$ such that $\mu(\gamma_h, \omega_h) = 0$, and solve for $m_1(\gamma_h, \omega_h)$ in (B3) with $\mu(\gamma_h, \omega_h) = 0$. If $c_{11}(\gamma_h, \omega_h)$ is larger than $m_1(\gamma_h, \omega_h)$, reduce $c_{11}(\omega_h, \gamma_h)$ to $m_1(\gamma_h, \omega_h)$. On all future iterations, this procedure will determine c_1 and m in this state. For any state (γ_i, ω_i) in which $\mu_0(\gamma_i, \omega_i) > 0$, substitute for μ in (B3) using (B2), and solve (B3) for $m_1(\gamma_i, \omega_i) = c_{11}(\gamma_i, \omega_i)$, using the vectors $m_0(\gamma_i, \omega_i)$ and $c_{10}(\gamma_i, \omega_i)$ from the previous iteration in the expectation on the right-hand side of (B3).

Step 4.—Use (B3) to solve for $\mu_1(\gamma_i, \omega_i)$ using $m_1(\gamma_i, \omega_i)$ and $c_{11}(\gamma_i, \omega_i)$ on the right-hand side. If $\mu_1(\gamma_i, \omega_i) \geq 0$ in all states and $m_1(\gamma_i, \omega_i) = m_0(\gamma_i, \omega_i)$ and $c_{11}(\gamma_i, \omega_i) = c_{10}(\gamma_i, \omega_i)$, this is an equilibrium. If not, set $c_{10}(\gamma_i, \omega_i) = c_{11}(\gamma_i, \omega_i)$ and $m_0(\gamma_i, \omega_i) = m_1(\gamma_i, \omega_i)$ and repeat step 3.

Appendix C

Data Sources

Quarterly data come from Citibase, and their acronyms are listed below. National Income and Product Accounts is denoted NIPA.

Money stock (FM2): Average of three months
Population (POP): Average of three months
Consumption of nondurables in 1982 (current) dollars (GCN82 [GCN]): NIPA
Consumption of services in 1982 (current) dollars (GCS82 [GCS]): NIPA
Nominal interest rates (FGYM3): 3-month Treasury bill yield

Annual data come from the 1987 *Economic Report of the President* unless otherwise indicated.

Consumption of nondurables and services in 1982 (current) dollars: table B-2 (B-1); 1987 observations from the 1988 report
Population: table B-31; 1986–87 observations from the 1988 report
Money stock: M2; 1948–83 from Balke and Gordon (1986); 1984–87 from 1988 report
Nominal interest rate: Commercial paper rate for 4–6-month maturity; 1950–87 from 1988 report, table B-71.

Appendix D

Derivation of Modified VARs for Information Experiments

Here we demonstrate how the parameters of the VAR can be estimated with information on the true process and knowledge of the variance of the noise process as in equations (16). Suppose that there exist $T + 2$ observations of output growth and money growth. Define $\mathbf{X}$ to be a $T \times 4$ matrix with the following four columns: column 1, vector of ones; column 2, observations of

ω beginning with the second period; column 3, observations of ω beginning with the first period; column 4, observations of γ beginning with the second period. Let $\mathbf{Y}$ be a vector containing T observations of ω beginning with the third period. Let $\boldsymbol{\Sigma}$ be a 4×4 matrix with a single nonzero element, $\Sigma_{22} = \sigma_\eta^2$. Then a consistent estimator of the vector $\mathbf{c}$ in equations (16) is $\mathbf{c} = (\mathbf{X}'\mathbf{X} + T\boldsymbol{\Sigma})^{-1}\mathbf{X}'\mathbf{Y}$ (see Chow 1983, pp. 105–6). We can similarly estimate the vectors $\mathbf{a}$ and $\mathbf{b}$.

By decomposing S_t in equations (16), we can write the error terms as $\{a_1\theta_t + \epsilon_{St}, b_1\theta_t + \epsilon_{\omega t}, c_1\theta_t + \epsilon_{\gamma t}\}$, where $\epsilon_{\omega t}$ and $\epsilon_{\gamma t}$ are the innovations from the full-information VAR and $S_t = \omega_t + \theta_t$. Since θ_t is uncorrelated with the time t information set, the covariance matrix is

$$\begin{bmatrix} (a_1^2\sigma_\eta^2 + \sigma_S^2) & (a_1b_1\sigma_\eta^2 + \sigma_{S\omega}) & (a_1c_1\sigma_\eta^2 + \sigma_{S\gamma}) \\ (a_1b_1\sigma_\eta^2 + \sigma_{S\omega}) & (b_1^2\sigma_\eta^2 + \sigma_\omega^2) & (b_1c_1\sigma_\eta^2 + \sigma_{\omega\gamma}) \\ (a_1c_1\sigma_\eta^2 + \sigma_{S\gamma}) & (b_1c_1\sigma_\eta^2 + \sigma_{\omega\gamma}) & (c_1^2\sigma_\eta^2 + \sigma_\gamma^2) \end{bmatrix},$$

where $\sigma_S^2 = \sigma_\omega^2 + \sigma_\eta^2$, $\sigma_{S\omega} = \sigma_\omega^2$, and $\sigma_{S\gamma} = \sigma_{\omega\gamma}$.

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